The Weak Eightfold Way: $SU(3)_W$ unification of the electroweak interactions

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Once upon a time there was a star $\sin^2\theta_W$. Everybody was parading the star under the banner gauge unification and quark-lepton unification, leaving behind a big desert. All led to proton decay.

The proton hasn't decayed and slowly the star was forgotten. Recently it as resurfaced with a new banner: No GUT, no quark-lepton unification!. I am a TeV-star!

. $\sin^2\theta_W$ without GUT

- $\sin^2\theta_W = g'^2/(g'^2+g^2)$ with $g' \in U(1)_Y$ and $g \in SU(2)$.
- Prediction for $\sin^2\theta_W \Rightarrow g' = ??g$. Depends only on gauge unification of $SU(2)$ and $U(1)_Y$.
- The form of unification depends on an independent prediction for $\sin^2\theta_W$. Could this prediction come from TeV-scale physics?
- Unexpected twist:Dirac Quantization Condition on an electroweak monopole $\Rightarrow \sin^2\theta_W = 1/4$ at the mass scale of the monopole (2-3 TeV).
- Renormalization of $\sin^2\theta_W = 1/4$ down to M_Z gives $\sin^2\theta_W(M_Z) \approx 0.231 - 0.232$.
- Electroweak monopole?

Electroweak monopole and $\sin^2\theta_W$

- For EW $SU(2)$, the existence of an electroweak monopole depends on the existence of a real Higgs triplet (topological argument).
- Which model contains a real Higgs triplet and for what reasons?
- A model of non-sterile, electroweak-scale right-handed neutrinos: ν_Rs are members of right-handed mirror fermion doublets $l_R^M = \begin{pmatrix} \nu_R \\ e_R^M \end{pmatrix}$. P. Q. Hung, "A Model of electroweak-scale right-handed neutrino mass," Phys. Lett. B **649**, 275-279 (2007).
- 3 steps: 1) ν_R Majorana mass from coupling to a complex Higgs triplet $\tilde{\chi}$: $g_{M_R} v_M$ ($\langle\tilde{\chi}\rangle = v_M$); 2) Z boson width: $M_R > M_Z/2$; 3) Value of v_M badly violates custodial symmetry which guarantees $M_W = M_Z \cos\theta_W$ at tree level!
- Cure: Introduce, in addition, a real Higgs triplet $\xi \Rightarrow$ Custodial symmetry is restored!

Electroweak monopole and $\sin^2\theta_W$

- Details in PQH, 'Topologically stable, finite-energy electroweak-scale monopoles," Nucl. Phys. B **962**, 115278 (2021).
- Magnetic charge given by Topological Quantization Condition: $g_M = (1/g)n$. Magnetic field at large distances: $B_i = \frac{g_M}{r^2}\hat{r}_i = \frac{\sin\theta_W}{er^2}\hat{r}_i$
- Dirac Quantization Condition for an electron going around the monopole: $eg_M = m/2$.
- Consistency requires ($m = 1$): $\sin^2\theta_W = 1/4$ J. Ellis, P. Q. Hung and N. Mavromatos, "An Electroweak Monopole, Dirac Quantization and the Weak Mixing Angle," [arXiv:2008.00464 [hep-ph]]. (August 2, 2020).
- A related work within the framework of GUT: G. Lazarides and Q. Shafi, "Electroweak monopoles and magnetic dumbbells in grand unified theories," Phys. Rev. D **103**, 095021 (2021) [arXiv:2102.07124 [hep-ph]]. v1 (Feb 14, 2021) quoted our papers and v2 made them disappear!

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- $\sin^2\theta_W = 1/4$ implies that $g'^2 = g^2/3 \Rightarrow$ Unification of $SU(2)$ and $U(1)_Y$ at some scale $M_U \sim O(TeV)$.
- Simplest possibility: $SU(3)_W \to SU(2) \times U(1)_Y$.
- Comparing $D_\mu = \partial_\mu + \imath g_U(\frac{\lambda^a}{2})A^a_\mu$ of $SU(3)_W$ with $D_\mu = \partial_\mu + \imath g(\frac{\tau^i}{2})W^i_\mu + \imath g'(\frac{Y}{2})B_\mu$ of $SU(2) \times U(1)_Y$, and identifying A^8_μ with B_μ, one obtains $g_U(\frac{\lambda_8}{2}) = g'(\frac{Y}{2})$. Explicitly $\pm\frac{Y}{2} = diag(\pm\frac{1}{2}, \pm\frac{1}{2}, \mp 1)$.
- With $\lambda_8/2 = diag(1,1,-2)/2\sqrt{3} \Rightarrow g = -\sqrt{3}g' = g_U \Rightarrow \sin^2\theta_W = 1/4$.
- Requirements: 1) Fermion representations should contain all SM degrees of freedom; 2) $SU(3)_W$ is anomaly-free.

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P. Q. Hung, "The Weak Eightfold Way: $SU(3)_W$ unification of the electroweak interactions," [arXiv:2101.09607 [hep-ph]].

- Gauge bosons: $(\frac{\lambda^a}{2})A^a_\mu = \begin{pmatrix} \frac{W_{3\mu}}{2} + \frac{B_\mu}{2\sqrt{3}}, & W^+_\mu, & \frac{X^+_\mu}{\sqrt{2}} \\ W^-_\mu, & -\frac{W_{3\mu}}{2} + \frac{B_\mu}{2\sqrt{3}}, & \frac{X^0_\mu}{\sqrt{2}} \\ \frac{X^-_\mu}{\sqrt{2}}, & \frac{\bar{X}^0_\mu}{\sqrt{2}}, & -\frac{B_\mu}{\sqrt{3}} \end{pmatrix}$.
- Representations (m, n) with dimension $(m+1)(n+1)(m+n+2)/2$ are classified under $T_{3W} = -T_W, .., T_W)$ and $Y/2$ with $T_W = (p+q)/2$, $\frac{Y_q}{2} = \frac{p}{2} - \frac{q}{2} + \frac{1}{3}(n-m)$ for quarks, and $\frac{Y_l}{2} = \frac{3p}{2} - \frac{3q}{2} + (n-m)$ for leptons, with $0 \le p \le m$; $0 \le q \le n$.
- All SM fermions are written as left-handed fields (just as in GUT).
- In analogy with the old flavor $SU(3)$: $T^\pm_W = \frac{\lambda_1 \pm \imath\lambda_2}{2}$; $U^\pm_W = \frac{\lambda_6 \pm \imath\lambda_7}{2}$; $V^\pm_W = \frac{\lambda_4 \pm \imath\lambda_5}{2}$; $W^\pm_\mu = \frac{W^1_\mu \mp \imath W^2_\mu}{\sqrt{2}}$; $X^\pm_\mu = \frac{A^4_\mu \mp \imath A^5_\mu}{\sqrt{2}}$; ${X^0}_\mu = \frac{A^6_\mu \mp \imath A^7_\mu}{\sqrt{2}}$. Couplings: $\frac{1}{\sqrt{2}}(T^\pm_W W^\pm_\mu + V^\pm_W X^\pm_\mu + U^\pm_W {X^0}_\mu)$

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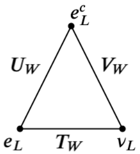
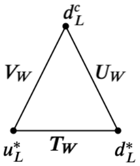
lepton: $\bar{\mathbf{3}}^l_L$=(0,1). $p = q = 0$ and $n = 1$, $m = 0$.



down quark: $\bar{\mathbf{3}}^d_L$=(0,1). $1/3$ ($p = q = 0$) and $-1/6$ ($p = 0$, $q = 1$).



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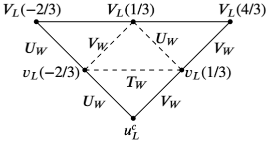
up quark: $\mathbf{6}_L$=(2,0). $m = 2$ ($p = 0$) and $n = 0$ ($q = 0$).



- Anomaly coefficients: $Tr[\{T^R_a, T^R_b\}T^R_c] = d_{abc}A(R)$
- $A(\bar{\mathbf{3}}^l_L) = A(\bar{\mathbf{3}}^d_L) = -1$ and $A(\mathbf{6}_L) = 7$. With color factors: $A_{tot} = -1 - 3 + 21 = 17$. NOT ANOMALY-FREE!
- Simplest option: representations with the *same dimensionality* but with *opposite anomaly coefficients* ⇒ Right-handed multiplets ⇒ Mirror fermions of the EW-ν_R model!

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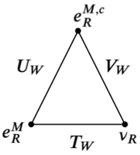

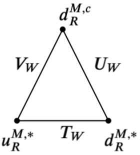

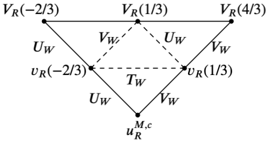

Conclusion

- Unification of $SU(2)$ and $U(1)_Y$ leads to the predictions: 1) $\sin^2\theta_W = 1/4$; 2) Existence of mirror fermions and, hence non-sterile, electroweak-scale right-handed neutrinos; 3) Vector-like quarks with unconventional electric charges $\mathbf{V} = (V_{L,R}(-2/3), V_{L,R}(1/3), V_{L,R}(4/3))$ and $\mathbf{v}= (\upsilon_{L,R}(-2/3), \upsilon_{L,R}(1/3))$.
- A rich phenomenology involving vector-like quarks, X-gauge bosons and BSM scalars (not shown here).