

Model-independent considerations of dark sectors

With R. Contino, K. Max, and ongoing work with R. Contino, M. Costa, S. Verma, M. Reece and C. Cesarotti Based on 2012.08537 + 21xx(s)





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- Several BSM scenarios naturally give hidden sectors at low energies.
- Dark matter may be part of a sector with its own particles and

dynamics.

The range of possibilities is vast!





Portals to the Hidden sector

For heavy mediators, we can write the portal as a contact interaction.

Natural to consider gauge invariant operators

Operator
 $H^{\dagger}H$
$B_{\mu u}$
ℓH
$J^{SM}_{\mu} = ar{\psi} \gamma^{\mu} \psi, H^{\dagger} i \overleftrightarrow{D}_{\mu} H$
$O^{ m SM}_{\mu u}=F^i_{\mulpha}F^{lphai}_ u,D_\mu H^\dagger D_ u H,ar\psi\gamma_\mu D_ u\psi$
$O_{SM} = \bar{\psi} i \not\!\!\!D \psi, D_{\mu} H^{\dagger} D^{\mu} H, F_{\mu\nu} F^{\mu\nu}, F_{\mu\nu} \tilde{F}^{\mu\nu}, \bar{\psi}_L H \psi_R, (H^{\dagger} E^{\mu\nu}) = 0$
$\beta m + \gamma + \gamma - \mu - \gamma - \mu - \gamma + \mu - \mu - \gamma + \mu - \mu$



Dimension



Can we use general principles of Quantum Field Theories to write down a minimal set of operators (and hence) portals) that *must* exist?

Assuming the hidden sector is a local and natural QFT, we can make progress.



Portals to the Hidden sector

Consider a dark sector characterized by two scales (with a possible hierarchy between them).

The sector can be weakly or strongly coupled.

The dynamics between the scales is approximately scale invariant.



What are the lowest dimension operators?

- Operators for conserved currents only. J_{μ} , $T_{\mu\nu}$
- A natural separation of scales in the hidden

sector: no highly relevant scalar operators. \mathcal{O}_{Δ} , $\Delta \leq 4$







Portals to the Hidden sector

These are the portals expected on general principles of gauge invariance, Lorentz invariance and locality.

<i>D</i> < 6	$\frac{\kappa_O}{\Lambda_{\rm UV}^{\Delta-2}} (H^{\dagger}H) \mathcal{O}_{\Delta}$	Natura
<i>D</i> = 6	$\frac{\kappa_J}{\Lambda_{\rm UV}^2} J_{\mu}^{\rm SM} J_{\rm DS}^{\mu}$	Reaso
<i>D</i> = 8	$\frac{\kappa_T}{\Lambda_{\rm UV}^4}T_{\rm DS}^{\mu\nu}O_{\mu\nu}^{\rm SM}$	Minima

e.g.

- Confining SU(N) gauge sectors with heavy matter content.
- Heavy scalar and a light fermion.
- Extended RS models with more than 2 branes.





ral	Irrelevant Portals
onable	Less elusive new sectors with marginal or relevant portals have been thoroughly studied in the literature
nal	e.g. Twin Higgs/Neutral-Naturalness, Unparticles, Higgs portal, dark photon

Easy to construct examples: see 2012.08537 for details (also backup slides)

An example

Consider a DS with a dim-8 portal interaction with the SM

$$\frac{\kappa_T}{\Lambda_{\rm UV}^4} \left(F^{\mu}_{\alpha} F^{\alpha\nu} + \bar{e}\gamma^{\mu} D^{\nu} e \right) T^{\rm DS}_{\mu\nu}$$

The probe of such a portal is the process:

$$e^+e^-
ightarrow \gamma + \mathrm{DS}$$



The importance of threshold contribution depends on dimensionality of portal and energy range probed (c.f with relevant portals)



• Largest contribution from events with highest momentum in the DS, away from the threshold.

Necessitates assessing the EFT validity carefully.



Standard Model



Experimental Probes

(without specifying explicit field content)



Continuum Regime: being inclusive on the final state allows using optical theorem. Signal is Missing Energy.

$$\sum_{n} \int d\Phi_{D} |\langle \Omega | \mathcal{O}_{\rm HS} | n \rangle |^{2} = 2 \operatorname{Im} \langle \Omega | \mathcal{O}_{\rm HS} \mathcal{O}_{\rm HS} | \Omega \rangle$$
$$\langle \mathcal{O}_{\rm HS}(p) \mathcal{O}_{\rm HS}(-p) \rangle \sim \frac{c}{16\pi^{2}} \left(p^{2\Delta - 4} + p^{2\Delta - 6} \Lambda_{\rm IR}^{2} + \dots + \Lambda_{\rm IR}^{2\Delta - 4} \right)$$

With some additional assumptions, this can be used for probing LLP signals as well. Need: $au_{
m LLP}$, $\langle n_{
m LLP}
angle$, $\langle \gamma_{
m LLP}
angle$.



If the UV contribution is dominant, this does not probe the DS directly. (e.g. no condition on Λ_{IR} , rather only on Λ_{UV}).







Relevant experimental probes

DS direct production

- Z and Higgs decays
- Non-resonant production at LEP and LHC (mono-X searches)
- High Intensity experiments (Fixed-target, beam dump, FC Meson decays)
- Supernova and Stellar evolution
- Positronium lifetime



DS virtual exchange

- Fifth Force experiments
- EWPT







 $m Log_{10}(\Lambda_{IR}/\Lambda_{UV})$



Future experimental proposals cover a significant part of the parameter space





NA64

$$\sigma(eN \to eN + DS) = 0.8 \times 10^{-41} \text{cm}^2 \ \left(c_J \kappa_J^2\right) \left(\frac{500 \text{ GeV}}{\Lambda_{\text{UV}}}\right)^4$$

$$E137$$

$$\sigma(eN \to eN + DS) = 0.4 \times 10^{-43} \text{cm}^2 \left(c_J \kappa_J^2\right) \left(\frac{500 \text{ GeV}}{\Lambda_{\text{UV}}}\right)^4$$

- Weaker constraints than colliders.
- **DV** nature of signal allows a higher **UV** reach but in limited IR range.
- High POT compensates for low incident E.

 10^{2}

AUV (GeV) 10^{1}

 10^{0}

Other constraints:

Meson decays

	Annihilation decays	$\Lambda_{ m UV} < 2.3~{ m GeV}(c_J\kappa_J^2)^{1/4}$
	Radiative decays	$\Lambda_{\mathrm{UV}} < 83 \ \mathrm{GeV} (c_J \kappa_J^2)^{1/4}$
		$\Lambda_{\rm UV} < 1.3 \ { m GeV} (c_{\mathcal{O}} \kappa_{\mathcal{O}})^{1/2}$
Ener	gy Loss	
	SN	$\Lambda_{\rm UV} < 400 { m GeV}(c_J(\kappa_J^{(nn)}))$
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Posit	ronium lifetime	
	to DS	$\Lambda_{\mathrm{UV}} < 346~\mathrm{MeV}(c_J\kappa_J^2)^{1/2}$
	to DS + photon	$\Lambda_{\mathrm{UV}} < 3.6 \ \mathrm{MeV} (c_T \kappa_T^2)^{1/2}$

Fifth force

 $\Lambda_{
m UV} < 0.2~{
m MeV}~(c_J\kappa_J^2)^{1/2}$

$$\begin{aligned} & \underbrace{\mathsf{Standard Model}}_{\mathsf{H}dden \,\mathsf{Sector}} \\ & \underbrace{\mathsf{/}^4, \ \Lambda_{\mathrm{IR}} \lesssim 0.6 \ \mathrm{MeV}(c_J \kappa_J^2)^{-0.18}}_{4, \ \Lambda_{\mathrm{IR}} \ll 90 \ \mathrm{MeV}(c_J \kappa_J^2)^{-0.17}} \\ & \underbrace{\mathsf{/}^4, \ \Lambda_{\mathrm{IR}} \ll 90 \ \mathrm{MeV}(c_C \kappa_{\mathcal{O}}^2)^{-0.17}}_{1/4, \ \Lambda_{\mathrm{IR}} \ll 800 \ \mathrm{MeV}(c_C \kappa_{\mathcal{O}}^2)^{-0.1}} \\ & \underbrace{\mathsf{/}^{-1/4}, \ \Lambda_{\mathrm{IR}} \ll 800 \ \mathrm{MeV}(c_J \kappa_{\mathcal{O}}^2)^{-0.19}}_{1/4, \ \Lambda_{\mathrm{IR}} \ll \min(T_{\mathrm{HB}}, 10 \ \mathrm{MeV}(c_J (\kappa_J^{(nn)})^2)^{-0.23}} \\ & \underbrace{\mathsf{N}^{-1/4}, \ \Lambda_{\mathrm{IR}} \ll 3 \ \mathrm{MeV}(c_J \kappa_J^2)^{-0.19}}_{1/4, \ \Lambda_{\mathrm{IR}} \ll 3 \ \mathrm{MeV}(c_J \kappa_J^2)^{-0.19}} \end{aligned}$$

$$^{/8}, \ \Lambda_{\rm IR} \ll 0.4 \ {
m MeV}(c_T \kappa_T^2)^{-0.1}$$

$$^{/4}, \ \Lambda_{\mathrm{IR}} \ll 1 \ \mathrm{keV}$$

Take UV physics to be heavier than ~ TeV. What reach do we have on the IR scale?

Lessons so far:

Take UV physics to be heavier than ~ TeV. What reach do we have on the IR scale?

Probably an altogether different kind of probe is needed to reach lower IR values.

Summary

- Big chunks of parameter space unconstrained at the moment.
- Future experimental proposals are crucial to probe them.
 - DV searches are the most constraining when applicable.
 - Future Higgs/Z factories will constrain the mixing portals strongly.
- Several usual probes are UV sensitive and do not constrain DS directly.
- Cosmological considerations will provide a complimentary probe, but usually come with more assumptions.

Thank you!

Extra slides

Example: strong coupling Pure (confining) YM

A confining $SO(N_{DC})$ gauge group with a singlet (Majorana) $N(1_0)$ and a doublet (Dirac) $L(2_{-1/2})$.

$$egin{aligned} \Delta \mathcal{L} &= - \, rac{1}{4g_{DC}^2} \mathcal{G}_{\mu
u} \mathcal{G}^{\mu
u} + ar{L}(i D\!\!\!\!/ - m_L) L + rac{1}{2} ar{N}(i D\!\!\!\!/ - m_N) \ &- \left(y_L \, ar{N} P_L L H + y_R \, ar{N} P_R L H + h.c.
ight), \end{aligned}$$

For $m_L, m_N \gg \Lambda_{DC}$, the low energy is a pure YM dark sector, with portals

$$\sim \frac{\alpha_{DC}}{4\pi} (|y_L|^2 + |y_L|^2)$$

 $N(V_{V})$

 $y_R|^2) \mathcal{G}_{\mu\nu} \mathcal{G}^{\mu\nu} H^{\dagger} H$

$$\frac{\kappa_O}{\Lambda_{\rm UV}^{\Delta-2}} (H^{\dagger}H) \mathcal{O}_{\Delta}$$

 $\Lambda_{\rm IR} \sim \Lambda_{\rm DC}$

 $\Lambda_{\rm UV} \sim m_L, \, m_N$

 ${\cal O}\sim {\cal G}_{\mu
u}\,{\cal G}^{\mu
u}$

 $\kappa \sim \alpha y^2$

Example: weak coupling Free Fermion (FF)

A SM-neutral Majorana fermion χ and a scalar ϕ with hyper charge -1, both odd under a dark parity.

$$\Delta \mathcal{L} = (D_{\mu}\phi)^{\dagger}(D^{\mu}\phi) + \frac{1}{2}\bar{\chi}(i\partial \!\!\!/ - m_{\chi})\chi + (y\,\bar{e}_R\phi\chi + h.c.) - m_{\phi}^2\phi^{\dagger}\phi - \lambda_{\phi}(\phi^{\dagger}\phi)^2 \,.$$

For $m_{\phi} \gg m_{\chi}$, we can integrate out the scalar.

 $\sim \frac{y^2}{m_{\phi}^2} \bar{e}_R \gamma_{\mu} e_R \, \bar{\chi} \gamma^{\mu} \gamma^5 \chi$

$$\begin{split} &\frac{\kappa_J}{\Lambda_{\rm UV}^2} J_{\mu}^{\rm SM} J_{\rm DS}^{\mu} \\ &\Lambda_{\rm UV} \sim m_{\phi} \\ &\Lambda_{\rm IR} \sim m_{\chi} \\ &J_{\mu} \sim \bar{\chi} \gamma^{\mu} \gamma^5 \chi \\ &\kappa \sim y^2 \end{split}$$

Example: 5D RS dark sector

5D RS scenario with SM on UV brane, and brane localized interactions

$$\int d^4x \sqrt{-g} \left(M_0^2 R + \frac{1}{\Lambda_{\rm UV}^2} R_{\mu 5\nu 5} T_{SM}^{\mu\nu} \right) \,. \label{eq:gamma_star}$$

can be thought as the low energy limit of a multi brane RS theory (Agashe et al, 1608.00526)

After a KK decomposition, the interaction between the SM and the DS states (= KK gravitons) in the 4D effective action is captured by the operator

$$\thicksim \frac{(k^3/M_5^3)}{M_5^4} T^{DS}_{\mu\nu} T^{\mu\nu}_{SM}$$

where $T^{DS}_{\mu\nu}$ excites KK gravitons.

 $\Lambda_{
m UUV}$

Refer to 2012.08537 for a comprehensive list of the models, with other details.

 $\frac{\kappa_T}{\Lambda^4} T^{\rm SM}_{\mu\nu} T^{\mu\nu}_{\rm DS}$ $\Lambda_{\rm UV} \sim {\rm UV} {\rm \ brane}$ $\Lambda_{\rm IR} \sim {\rm IR} {\rm \ brane}$ $T_{\mu\nu} \sim G_{MN}$ $\kappa \sim k^3/M_5^3$

EFT validity:

Resonant vs Non-Resonant portals

To be in the continuum limit and to be in the validity of portal interactions to arise from integration out heavy physics:

$$\Lambda_{\mathrm{IR}}^2 \ll p_{\mathrm{DS}}^2 \ll \Lambda_{\mathrm{UV}}^2$$

Depending on the process, constraints from data (from a given range in pDS) should only constrain the UV and IR scales that satisfy the criteria.

In practice, this depends on the nature of the portal:

 Λ_{IR}

High-intensity experiments

Standard lore: such experiments are a good probe of dark sectors. Small coupling is compensated by high intensity.

e.g. Fixed target experiment (NA64), Beam dump experiment (E137), Meson decays (BESIII, BABAR)

Cross-section for DS production scales as: $\sigma \sim$

A naive estimate for the number of events gives:

$$\frac{N_{collider}}{N_{target}} = \frac{\sigma_{collider} \mathcal{L}_{collider}}{\sigma_{target} \mathcal{L}_{target}} \sim 10^{-3} \left(\frac{E_{collider}}{E_{target}}\right)^{2D-10} \left(\frac{\mathcal{L}_{collider}}{100 \, \text{fb}^{-1}}\right) \left(\frac{10^{20}}{N_{\text{POT}}}\right)$$

D > 5 : colliders will give more stringent bounds

$$\sim (c\kappa^2/E^2)(E/\Lambda_{\rm UV})^{2D-8}$$

Using $\mathcal{L}_{\mathrm{target}} = N_{\mathrm{POT}} \, \ell
ho, \, \ell = 10 \ \mathrm{cm}, \,
ho = 10^{23}$

NA64 (at CERN)

- A high energy electron beam (E0 = 100 GeV) hits a lead target (ECAL).
- DS excitation decay outside the detector (the HCAL) if sufficiently long lived.
- The HCAL itself is used to veto any hadronic activity (nucleus breaking, deep inelastic scattering).
- POT ~ 10^11

E137 (at SLAC)

- A 20 GeV electron beam hits aluminum plates.
- The particles produced by the collision must traverse a hill of 179 m in thickness before reaching a 204 m-long open region followed by a detector.
- POT ~ 10^20

ad target (ECAL). f sufficiently

Process: eN -> eN + ME

The constraints on dark photon model can be used to constrain the portal:

$$\begin{aligned} \sigma(eN \to eN + DS) &= \frac{\kappa_J^2}{\Lambda_{\rm UV}^4} \frac{1}{4E_0 m_N} \frac{1}{2\pi} \int dp_{DS}^2 \int d\Phi_3 \,\mathcal{M}_{\mu} \mathcal{M}_{\nu}^* \,G(t) \\ &\times 2 \,{\rm Im} \left[i \langle 0 | T(J_{DS}^{\mu}(p_{DS}) J_{DS}^{\nu}(-p_{DS})) | 0 \rangle \right] \,, \end{aligned}$$

For a dark photon model: $\pi \delta(p_{
m DS}^2 - m_{\gamma'}^2) \sum_{
m pol} \epsilon_\mu \epsilon_
u^*$

$$\frac{d\sigma}{dp_{DS}^2}(eN \to eN + DS) = \frac{\kappa_J^2 c_J}{\Lambda_{\rm UV}^4} \frac{p_{DS}^2}{96\pi^2} \frac{\sigma(eN \to eI}{(\varepsilon e)}$$

 $\frac{N+A_D}{)^2}$

DS with a JJ portal is equivalent to DP with a mass between 0.1-10 GeV.

(max momentum transfer is ~ 1 GeV², so deep inelastic scattering is not relevant.)

NA64

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Stability of hierarchy

$\mathcal{O}H^{\dagger}H$

Higgs legs can be closed into a loop, giving a contribution that is a marginal deformation of the CFT

$$\kappa_O \lesssim 16\pi^2 \left(\frac{\Lambda_{\rm IR}}{\Lambda_{\rm UV}}\right)^{4-\Delta_{\mathcal{O}}}$$
 (UV contribution)

$$\kappa_O \lesssim \frac{\Lambda_{\rm IR}^2}{v_{\rm EW}^2} \left(\frac{\Lambda_{\rm IR}}{\Lambda_{\rm UV}}\right)^{2-\Delta_{\mathcal{O}}}$$

(EW contribution)

Assuming mediators heavier than EW scale,

$\Lambda_{\rm UV} > 4 \,\pi \, v_{\rm EW}$

The UV contribution is bigger, unless some UV mechanism that suppresses it.

Importance of UV effects 1-loop DS tree-level DS exchange exchange

 $E < \Lambda IR$, the contributions from both DS and UV states are local and parametrized by dim-6 operators. As such, they are qualitatively indistinguishable at low energy. Furthermore, for $D \ge 5$ the UV threshold correction is always larger than the RG running, which in turn dominates (for D even) over the IR thresholds. For 4 < D < 5, instead, the DS exchange gives only an IR threshold contribution, which can (depending on the UV dynamics) be larger than the one generated by heavy mediators at AUV.

 $E > \Lambda IR$, then for $D \ge 5$ the UV threshold corrections are larger than the long-distance effects, which in turn are larger than the RG running. In principle, one could distinguish experimentally the long-distance from local effects, since the former induce a nonanalytic dependence of the cross section on the energy. For 4 < D < 5, the DS exchange generates only a long-distance contribution, which can win over the UV effect induced by heavy mediators.

To summarize, UV thresholds are expected to give the most important virtual effects for $D \ge 5$; portals with 4 < D < 5, instead, generate only long-distance (for $E > \Lambda IR$) or IR threshold (for $E < \Lambda IR$) corrections, and can give the largest indirect contribution.

For virtual effects parametrized by local operators, both DS and UV (mediator) states contribute. It can be a tree level or a loop effect.

but also

contact term to regulate the divergences in the DS EFT, from mediators. A threshold effect.

 $\sqrt{10}V$

Consider *n* SM external legs, at *l* loops

$$\begin{split} \Delta c_6(\Lambda_{\rm UV}) &\sim g_{SM}^{n-4} \frac{\kappa^2}{\Lambda_{\rm UV}^2} \frac{c}{16\pi^2} \left(\frac{g_{SM}^2}{16\pi^2}\right)^{\ell} \\ \Delta c_6(\mu) &\sim g_{SM}^{n-4} \frac{\kappa^2}{\Lambda_{\rm UV}^2} \frac{c}{16\pi^2} \left(\frac{g_{SM}^2}{16\pi^2}\right)^{\ell} \left(\frac{\bar{\Lambda}^2}{\Lambda_{\rm UV}^2}\right)^{D-5} \log \frac{\mu}{\Lambda_{\rm UV}} \\ \Delta c_6(\Lambda_{\rm IR}) &\sim g_{SM}^{n-4} \frac{\kappa^2}{\Lambda_{\rm UV}^2} \frac{c}{16\pi^2} \left(\frac{g_{SM}^2}{16\pi^2}\right)^{\ell} \left(\frac{\bar{\Lambda}^2}{\Lambda_{\rm UV}^2}\right)^{D-5} \end{split}$$

$$\frac{\Delta R}{R} \sim \frac{\kappa^2 c}{16\pi^2 g_{SM}^2} \left(\frac{g_{SM}^2}{16\pi^2}\right)^\ell \left(\frac{s}{\Lambda_{\rm UV}^2}\right)^{D-4} \quad \text{(long range)}$$

