The Causal Structure of Superfluid Dark Matter

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Based on arXiv.2105.02241, with

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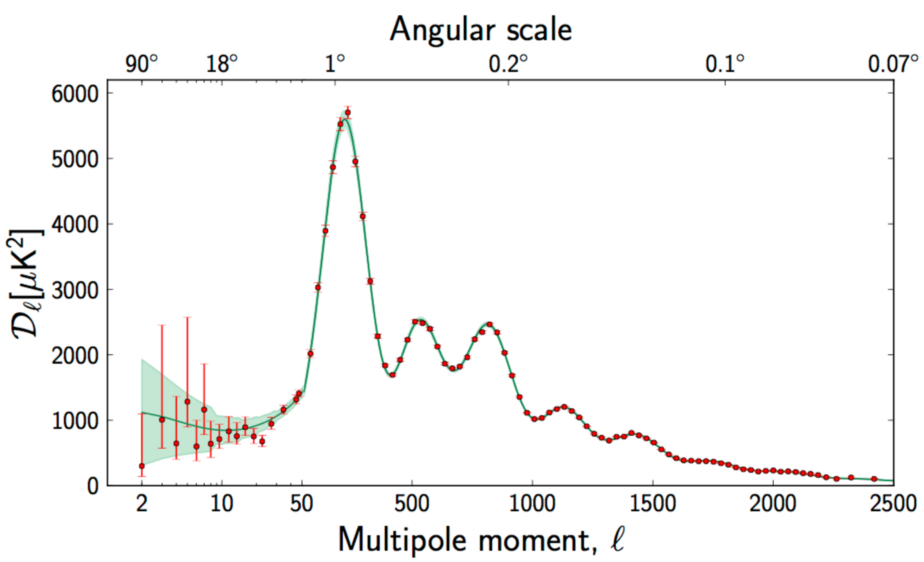
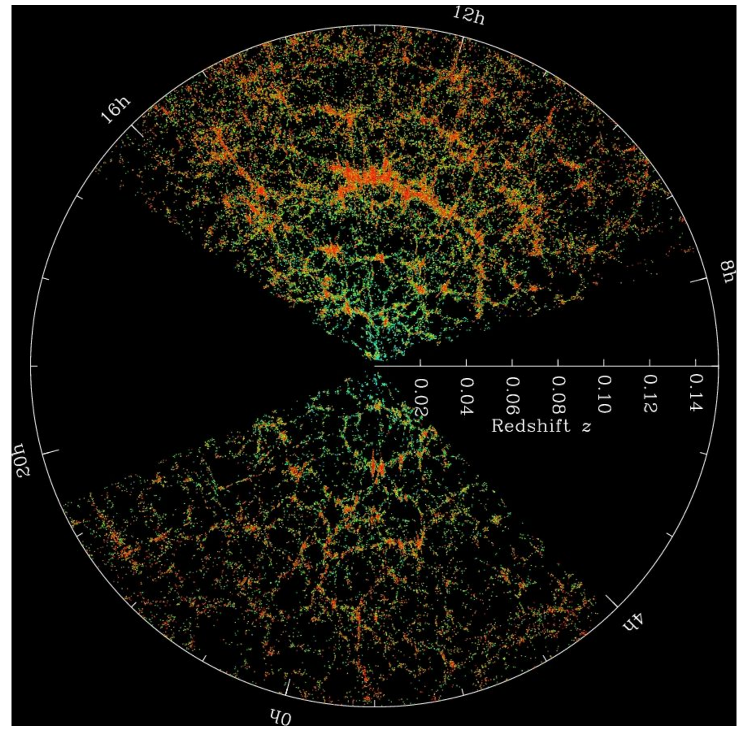
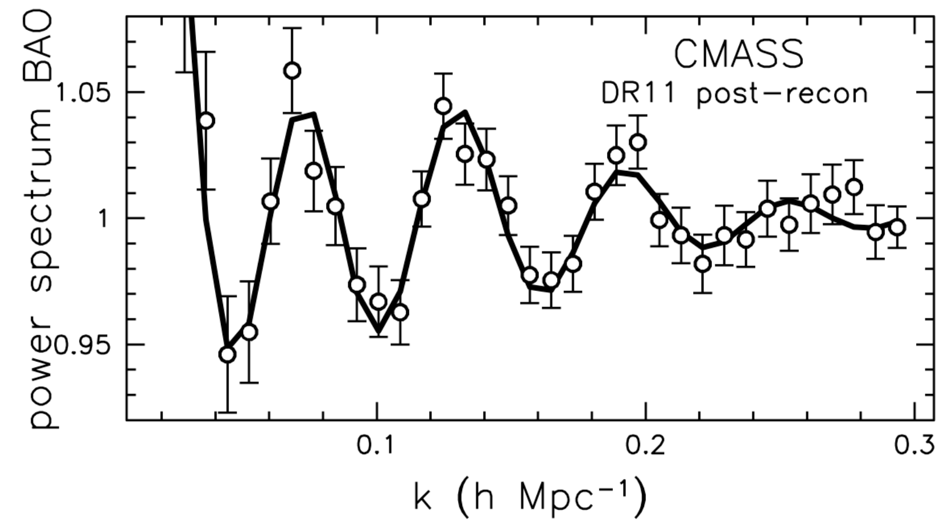
Tremendous Success of CDM on Large Scales



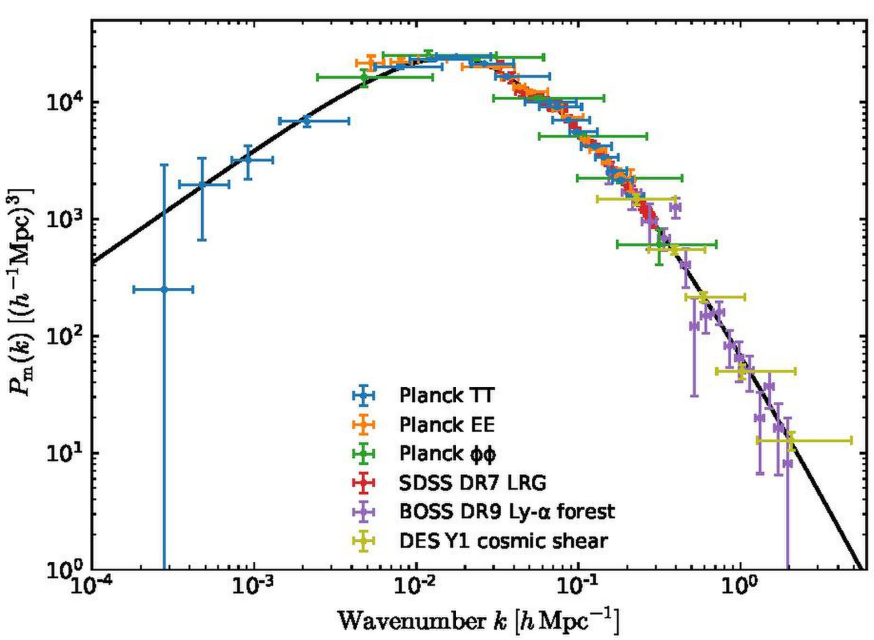
CMB (Planck)



Large Scale Structure (SDSS)

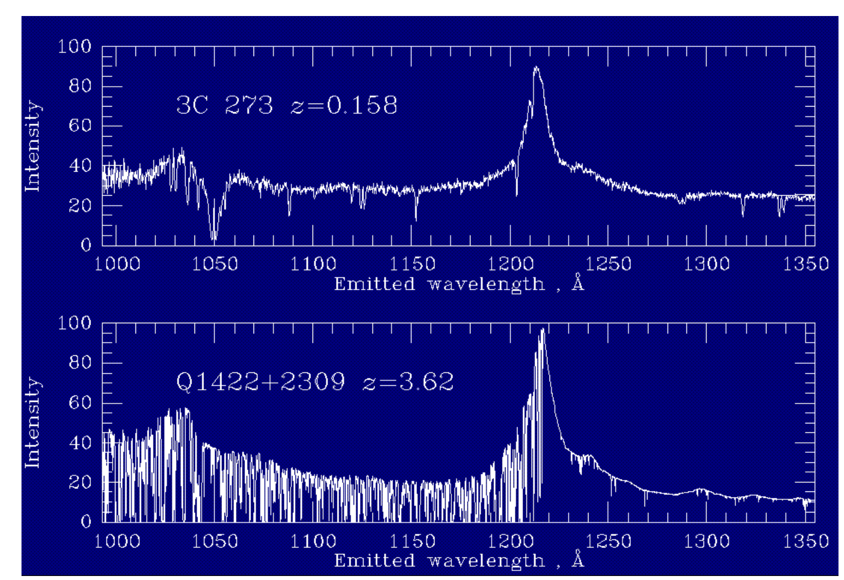


BAO (BOSS)



Concordance (Planck)

Galaxy Clustering (Hubble)

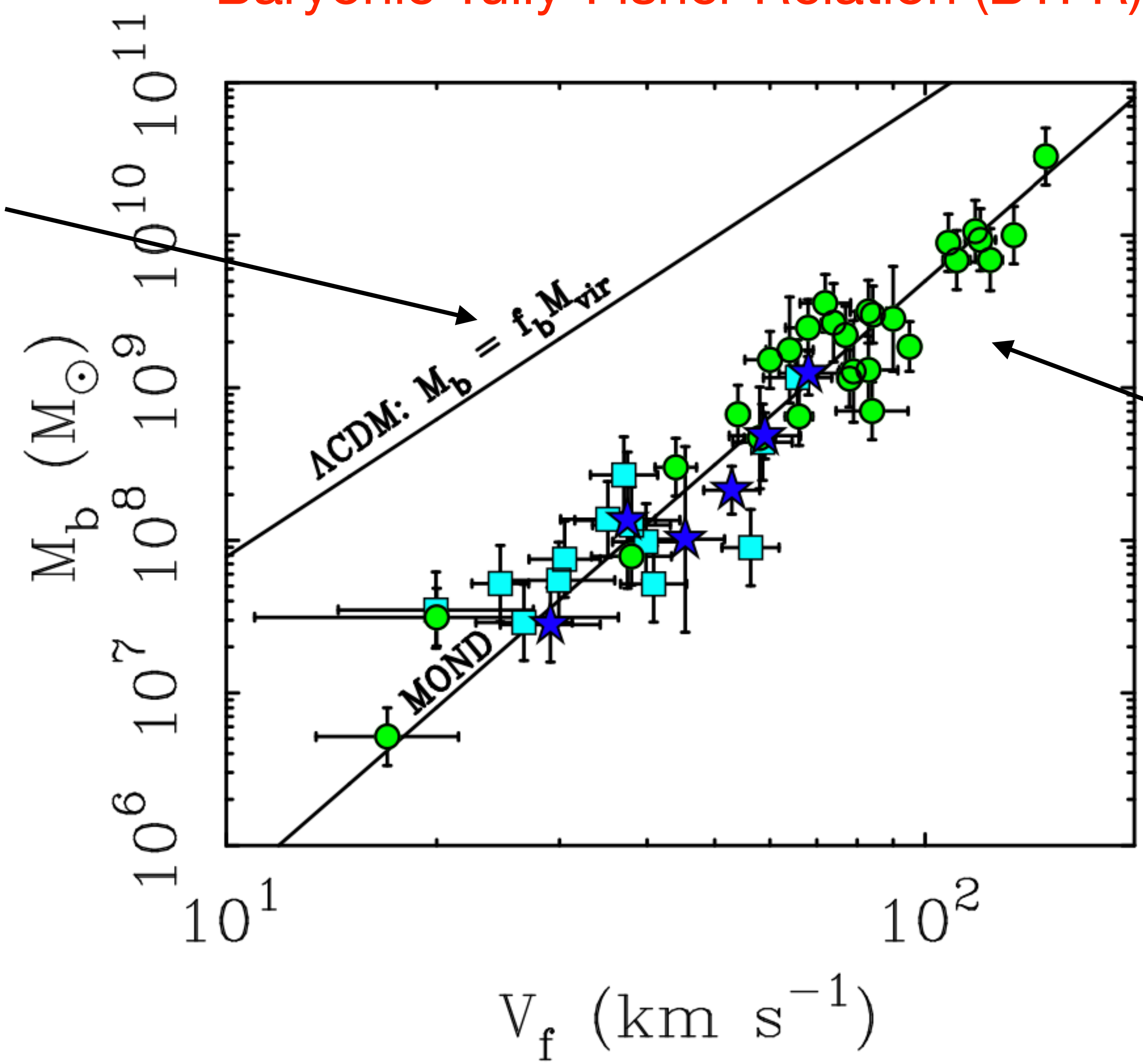


Lyman Alpha Forest (Keck)

Possible Difficulties with CDM on Galactic Scales?

Baryonic Tully-Fisher Relation (BTFR)

$M_b \propto v_f^3$?



$M_b \propto v_f^4$

(from McGaugh 2011)

Modify Gravity on Galactic Scales (MOND)?

$$a \propto \frac{M_{enc}}{R^2}$$

If instead: $$\frac{v^2}{R} = a \propto \sqrt{\frac{M_{enc}}{R^2}} \implies \boxed{M_b \propto v_f^4}$$

(Milgrom)

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(Milgrom)

Implementing this is very difficult:

The unique, causal, Lorentz invariant, theory of massless spin 2 particles, at large distances, is general relativity

(Feynman, Weinberg, Deser,…)

So one needs to add new degrees of freedom, namely scalars, to mediate a new long range (peculiar) interaction

Simplistic Attempt

$$X \equiv \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi$$

$$S = -\int d^4x \sqrt{-g} \left[F(X, \varphi) - \tilde{\beta} \varphi T_{\mathrm{B}} + \frac{\mathcal{R}}{16\pi G} + \mathcal{L}_{SM} \right]$$

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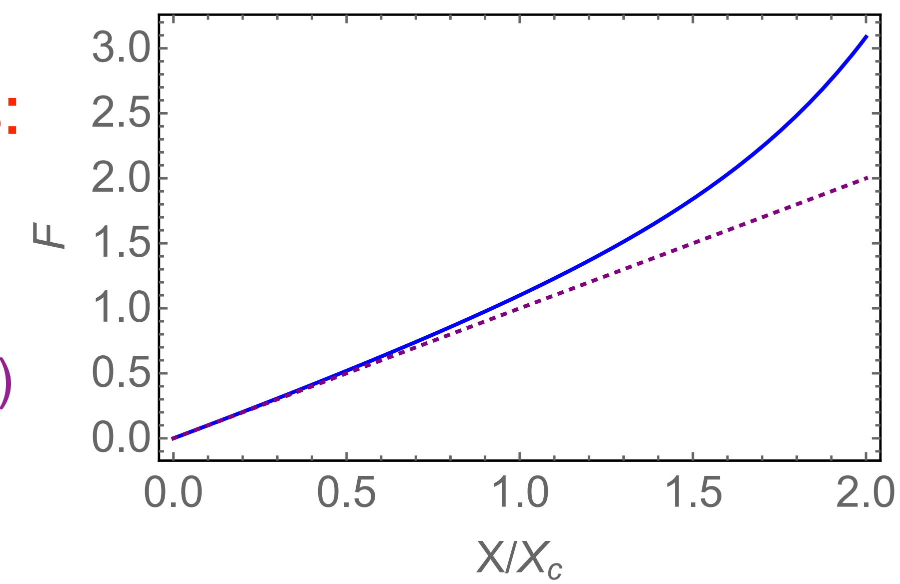
Introduce function *F* with 2 different asymptotic regimes:



Low densities/large scales $F = X$ (canonical)

(3/2 scaling)

High densities/galactic scales $\boxed{F = \tilde{\alpha}\, X \sqrt{|X|}}$ Mediates a MOND-like force

$$-\frac{3\tilde{\alpha}}{2^{3/2}} \nabla \cdot (\nabla\varphi |\nabla\varphi|) = \tilde{\beta}\, T_B \qquad \mathbf{a} \propto -\mathrm{sign}(\tilde{\alpha}) \sqrt{\frac{M_{enc}}{R^2}}\, \hat{r}$$

Simplistic Attempt

Two Problems

$$F = \tilde{\alpha}\, X \sqrt{|X|}$$

Theoretical: High energy perturbations on top of the MONDian solution are superluminal (related details later)

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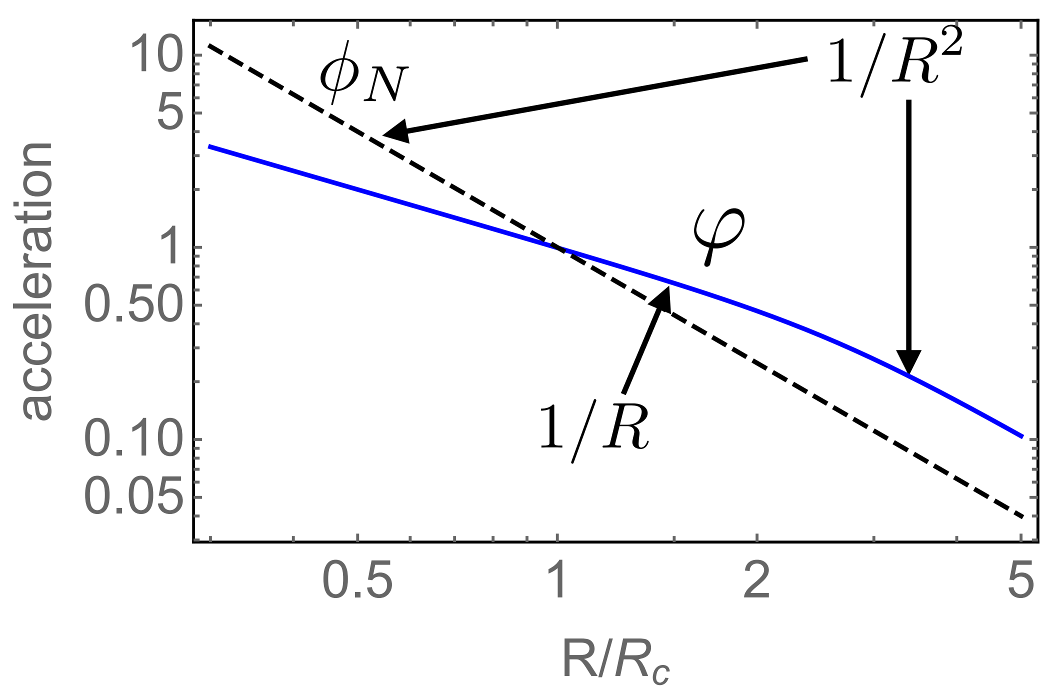
$$F = X$$

Phenomenological: Although the scalar becomes canonical at large scales, it introduces another $1/r^2$ force. So it is difficult to consistently obtain the desired galactic and large scale behaviors



Sophisticated Attempt - SuperFluid Dark Matter (SFDM)

Clever idea: Use Spontaneous Symmetry Breaking

Complex Scalar Dark Matter

$U(1)$ Symmetry

Φ

$$X = g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi^*$$

Berezhiani, Khoury 2015

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$U(1)$ Symmetry

Example:

$$F = \frac{1}{2}\left(X + m^2|\Phi|^2\right) + \frac{\Lambda^4}{6(\Lambda_c^2 + |\Phi|^2)^6}\left(X + m^2|\Phi|^2\right)^3$$

Reproduces CDM on large scales

Allows for phase transition to superfluid at galactic densities

$$\Phi = \rho\, e^{i(\theta + mt)}$$

Goldstone θ can act as long-ranged force mediator

Berezhiani, Khoury 2015

Sophisticated Attempt - SuperFluid Dark Matter (SFDM)

$$\Phi = \rho\, e^{i(\theta + mt)}$$

Slowly varying phase θ and modulus ρ around superfluid condensate

$$X + m^2|\Phi|^2 = (\nabla\rho)^2 - 2\,m\,\rho^2\,Y \qquad \text{with} \qquad Y \equiv \dot{\theta} - m\,\phi_N - \frac{(\nabla\theta)^2}{2m}$$

At tree-level, can integrate out heavy modulus (Higgs mode)

$$\rho^2 = \Lambda\sqrt{2m|Y|}$$

(3/2 scaling)

Find low energy effective action for Goldstone is

$$F_{\text{eff}} = -\frac{2\Lambda(2m)^{3/2}}{3} Y\sqrt{|Y|}$$

By coupling to baryons, can mediate MOND-like force — reproduce BTFR, and CDM on large scales

Berezhiani, Khoury 2015

Analysis of High Energy Perturbations ε_j

Decompose into components $\Phi = (\phi_1 + i\,\phi_2)/\sqrt{2}$

Expand around superfluid $\phi_j = \phi_j^b + \varepsilon_j \qquad (j = 1, 2) \qquad \left(F' \equiv \frac{\partial F}{\partial X}\right)$

Linear equation of motion for high energy perturbations

$$\sum_{j=1}^{2} \left[F' \eta^{\mu\nu} \delta^{ij} + F'' \partial^\mu \phi_i^b \partial^\nu \phi_j^b\right] \partial_\mu \partial_\nu \varepsilon_j = 0$$

Diagonalize to obtain Higgs normal mode perturbations and associated effective metric

$$\psi = \partial^\mu \phi_1^b \partial_\mu \varepsilon_1 + \partial^\mu \phi_2^b \partial_\mu \varepsilon_2$$

$$G_\phi^{\mu\nu} \partial_\mu \partial_\nu \psi = 0 \qquad \boxed{G_\phi^{\mu\nu} = F' g^{\mu\nu} + F''(\partial^\mu \phi_1^b \partial^\nu \phi_1^b + \partial^\mu \phi_2^b \partial^\nu \phi_2^b)}$$

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Causal Propagation?

Obtain eigenvalues of effective metric $$G_\phi^{\mu\nu} = F' g^{\mu\nu} + F''(\partial^\mu \phi_1^b \partial^\nu \phi_1^b + \partial^\mu \phi_2^b \partial^\nu \phi_2^b)$$

Conditions for hyperbolicity

$$(A) \quad A \equiv F' > 0$$

$$(B) \quad B \equiv F' + 2XF'' > 0$$

(Aharanov, Komar, Susskind; Wald; Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi; Bruneton;...)

Condition for subluminality

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Evaluate in SFDM model $A > 0$ $\quad B = \dfrac{4\, m^3\, \Lambda^4\, Y}{\rho^8}$ $\quad C = \dfrac{2\, m\, \Lambda^4\, Y}{\rho^{10}}$

MOND regime $\quad Y \approx -\dfrac{(\nabla\theta)^2}{2m} \implies$ $B < 0$ (ghost) and $C < 0$

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Causal Propagation? - General Analysis

Obtain eigenvalues of effective metric $$G_\phi^{\mu\nu} = F' g^{\mu\nu} + F''(\partial^\mu \phi_1^b \partial^\nu \phi_1^b + \partial^\mu \phi_2^b \partial^\nu \phi_2^b)$$

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Most general form $$F = (X + m^2|\Phi|^2) \sum_{n=0} g_n \frac{\Lambda^{2n} \left(X + m^2|\Phi|^2\right)^n}{(\Lambda_c^2 + |\Phi|^2)^{3n}}$$

We proved that when g_n allow MOND regime $(\tilde{\alpha} > 0)$ $\Longrightarrow$ $B < 0$ (ghost) and $C < 0$

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Conclusions

Superfluid Dark Matter is a novel way (only known?) to obtain success of CDM on large scales and success of MOND on galactic scales

We studied a general class of models, and proved that high energy perturbations always violate hyperbolicity — ghost like behavior — in MOND regime

Intermediate regions can exhibit forms of superluminality.
(There are problems in related models too.)

Open question: is there ANY other model free of these theoretical problems?
Alternatively, can one rigorously show that CDM reproduces BTFR, etc?

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