

Glueball dark matter in $SU(N)$ lattice gauge theory

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In Collaboration with

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Based on

N. Yamanaka et al., Phys. Lett. B **813**, 136056 (2021)

N. Yamanaka et al., Phys. Rev. D **102**, 054507 (2020)

N. Yamanaka et al., PoS LATTICE2019 (2019) 013

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Motivation

Dark matter is representing a significant fraction of the energy content of the Universe

But, we do not know what it is...

Many candidate theories are under discussion

(WIMPs, axions, blackholes, entropic gravity ...)

Let us consider the **SU(N) Yang-Mills theory** : $\mathcal{L}_{\text{YM}} = -\frac{1}{4}F_a^{\mu\nu}F_{\mu\nu,a}$

Generation of mass scale, logarithmic dependence, no important fine-tuning

⇒ Theory with very high **naturalness**

Lightest particles are **glueballs** ! ⇒ SU(N) glueballs are candidate of DM

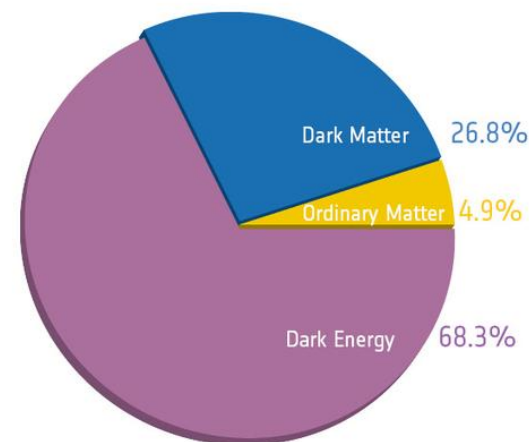
Important feature of DM : **Self-interaction**

(= DM-DM scattering)

DM self-interaction is constrained by observations (collisions of galaxies, structure formation)

⇒ We need to evaluate

the **interglueball force**!



Bullet cluster : collision of galaxies

Object of study

Glueballs of SU(N) Yang-Mills theory are good candidates of dark matter

In this work, we study the **interglueball scattering** on **lattice** which is the only way to quantify nonperturbative physics of nonabelian gauge theory.

The Yang-Mills theory depends only on the scale parameter Λ (given N_c): can we determine Λ from observation?

Object:

In this work, we study the interglueball scattering of SU(2) Yang-Mills theory on lattice, and set constraint on its scale parameter Λ .

We consider the **SU(2) pure Yang-Mills** theory

- Standard SU(2) plaquette action :

Lattice spacings : $\beta = 2.1, 2.2, 2.3, 2.4, 2.5$

Volume : $10^3 \times 12 \sim 16^3 \times 24$

Confs. generated with pseudo-heat-bath method (1 M confs.)

- Use SX-ACE (@RCNP, Osaka U.), vector machine

- Improvement of glueball operator : APE smearing

We use all space-time translational and cubic rotational symmetries to effectively increase the statistics

(like the all-mode average for meson and baryon observables)

Reduction of the statistical error w/ cluster decomposition principle

Scale determination

We do not know the scale of the YM theory, so we leave it as a free parameter Λ
Nevertheless, all quantities calculated on lattice depend on Λ
 \Rightarrow **We express all quantities in unit of Λ** (and finally constrain Λ from other data).

Relation between Λ and string tension:

$$\begin{aligned}\frac{\Lambda_{\overline{MS}}}{\sqrt{\sigma}} &= 0.503(2)(40) + \frac{0.33(3)(3)}{N^2} \\ &= 0.586(41) \quad (\text{for SU}(2))\end{aligned}$$

Fitted from the analysis
of the running coupling

C. Allton et al., JHEP 0807 (2008) 021
M. Teper, Acta Phys. Polon. B 40 (2009) 3249

String tension in SU(2) YM :

β	$a\sqrt{\sigma}$
2.1	0.608(16)
2.2	0.467(10)
2.3	0.3687(22)
2.4	0.2660(21)
2.5	0.1881(28)

M. Teper, Phys. Lett. B 397 (1997) 223; hep-th/9812187

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String tension in SU(2) YM :

β	$a\sqrt{\sigma}$	a (in unit of Λ^{-1})
2.1	0.608(16)	0.356(27)
2.2	0.467(10)	0.273(20)
2.3	0.3687(22)	0.216(15)
2.4	0.2660(21)	0.156(11)
2.5	0.1881(28)	0.110(8)

\Rightarrow Lattice spacing is now expressed in unit of Λ

Glueball operator and operator improvement

0^{++} glueball operator:

$$\Phi = \sum_{\text{cube}} \left\{ \text{Diagram} - \langle \text{Diagram} \rangle \right\}$$

Glueball has vacuum expectation value
 → Subtract
 Sum over cubic rotational invariance

APE smearing :

$U^{(n+1)}$ so as to maximize $\text{Re Tr} [U^{(n+1)} V^{(n)\dagger}$

where $V^{(n)} = \alpha x \uparrow + \text{Diagram}$

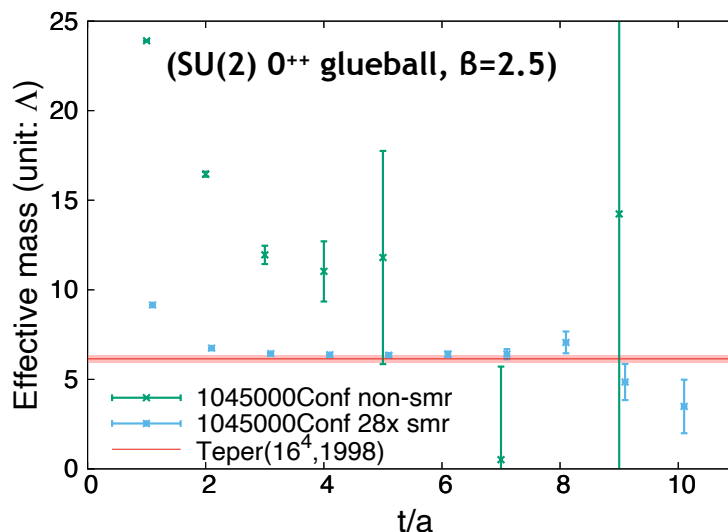
⇒ Gaussian spread: $2\sqrt{\frac{n}{4+\alpha}}$
 (in lattice unit)

Ape Collaboration, PLB 192 (1987) 163
 N. Ishii et al., PRD 66, 094506 (2002)

Optimal parameters
 for SU(2), $\beta=2.5$:

$$n = 28$$

$$\alpha = 2.0$$

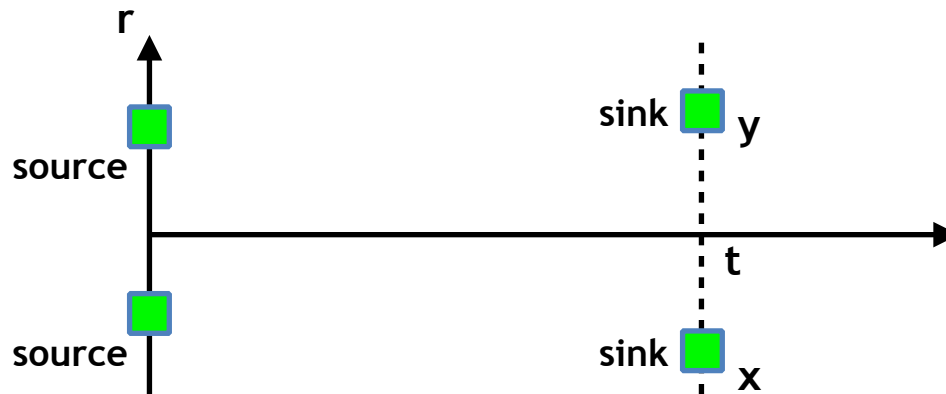


Nambu-Bethe-Salpeter amplitude

The information of the scattering is included in the following n-point correlator (Nambu-Bethe-Salpeter amplitude):

$$C_{\phi\phi}(t, \mathbf{x} - \mathbf{y}) \equiv \frac{1}{V} \sum_{\mathbf{r}} \langle 0 | T[\phi(\mathbf{x} + \mathbf{r}, t)\phi(\mathbf{y} + \mathbf{r}, t) \cdot \mathcal{J}(0)] | 0 \rangle$$

$\mathcal{J}(0)$: source op.



- 2-gluon state **mixes with all other multi-gluon states**:
⇒ The source may be chosen as 1-body, 2-body, etc, on convenience.
We choose **1-body source**, signal is noisier with higher-body source.
- The NBS amplitude **obeys the Schroedinger equation** below inelastic threshold

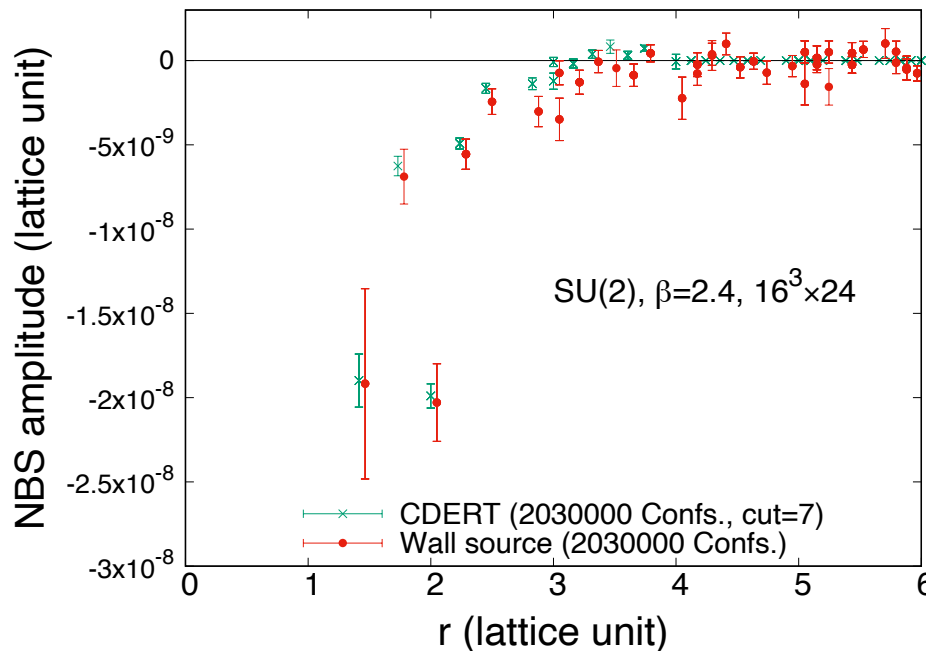
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Result of NBS amplitude calculation:



● 2-gluon

⇒ The source

We choose

● The NBS amplitude
threshold

states:

convenience.

2-body source.

or inelastic


Extract the **interglueball potential** from the NBS amplitude by inversely solving Schroedinger equation

$$\left[\frac{1}{4m_\phi} \frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} + \frac{1}{m_\phi} \nabla^2 + \frac{(\mathbf{r} \times \nabla)^2}{2m_\phi r^2} \right] R(t, \mathbf{r}) = \int d^3 \mathbf{r}' U(\mathbf{r}, \mathbf{r}') R(t, \mathbf{r}')$$

$$R(t, \mathbf{r}) \equiv \frac{C_{\phi\phi}(t, \mathbf{r})}{e^{-2m_\phi t}}$$

N. Ishii et al., PLB 712 (2012) 437.

- Crucial advantage : **do not need ground state saturation**

 Almost mandatory to use time-dependent HAL method for the glueball analysis, since the glueball correlator becomes **very noisy before ground state saturation**

- Inelastic threshold for glueball = $3m_\phi$: high enough to use low t


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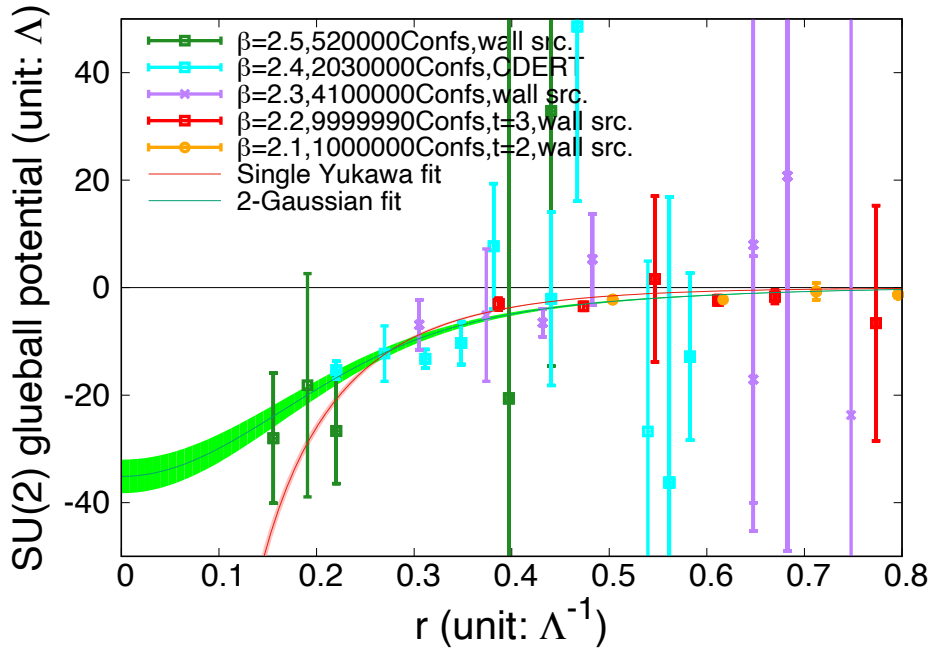
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 Almost mandatory to use time-dependent HAL method for the glueball analysis, since the glueball correlator becomes **very noisy before ground state saturation**

- Inelastic threshold for glueball = $3m_\phi$: high enough to use low t
- **Subtract centrifugal force** for removing higher angular momenta

Result



We test two fitting forms:

● Yukawa fit:

$$V_Y(r) = V_1 \frac{e^{-m_\phi r}}{4\pi r}$$

$$V_1 = -231 \pm 8 \quad \chi^2 \text{ d.o.f.} = 1.3$$

● 2-Gaussian fit:

$$V(r) = V_1 e^{-\frac{(m_\phi r)^2}{8}} + V_2 e^{-\frac{(m_\phi r)^2}{2}}$$

$$V_1 = (-8.5 \pm 0.5)\Lambda$$

$$V_2 = (-26.6 \pm 2.6)\Lambda \quad \chi^2 \text{ d.o.f.} = 0.9$$

DM cross section is derived from phase shift calculated with the potentials

$$\rightarrow \sigma_{\text{tot}} = \frac{4\pi}{k^2} \sin^2[\delta(k \rightarrow 0)]$$

Yukawa: $\sigma_{\text{tot}} = (2.5 - 4.7)\Lambda^{-2} \text{ (stat.)}$

2-Gaussian: $\sigma_{\text{tot}} = (14 - 51)\Lambda^{-2} \text{ (stat.)}$

$\rightarrow \sigma_{\text{tot}} = (2 - 51) \Lambda^{-2} \text{ (stat. and sys.)}$

(sys. due to fitting forms)

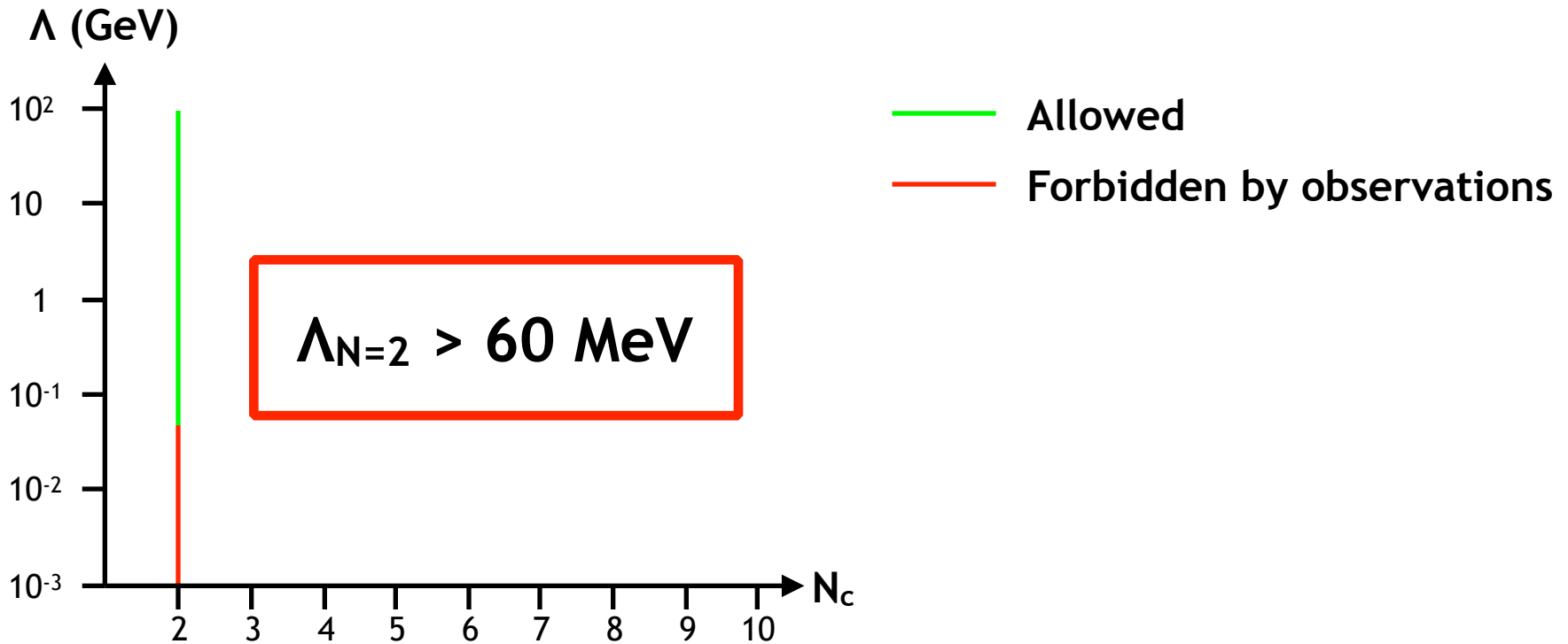
Constraint on $SU(N)$ YM scale parameter from DM X section

Observational constraints:

$$\frac{\sigma_{\text{tot}}}{m_{\phi}} < 1.0 \text{ cm}^2/\text{g}$$

Robust constraint from galactic cluster shape, collisions (upper limit)

A. H. Peter et al., MNRAS 430, 81 (2013), 430, 105 (2013); S. W. Randall et al., APJ 679, 1173 (2008).



N_c vs. scale parameter (Λ) diagram

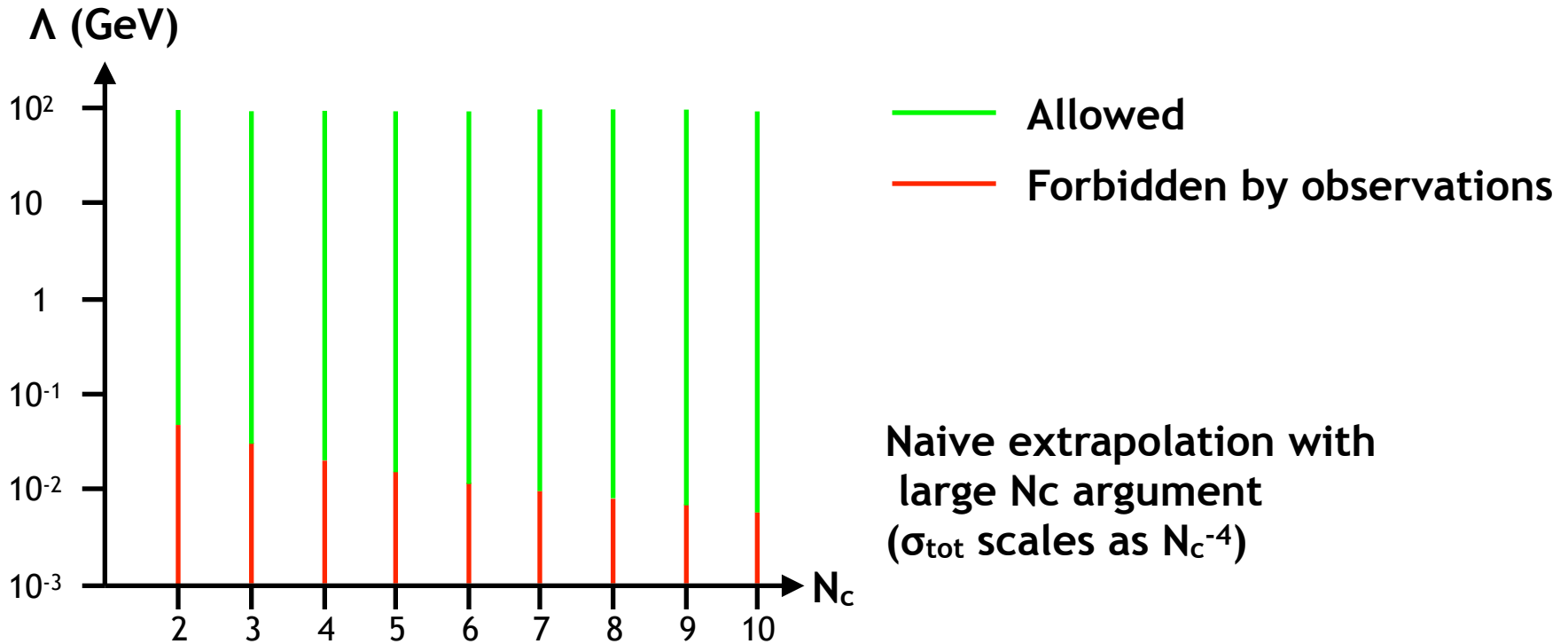
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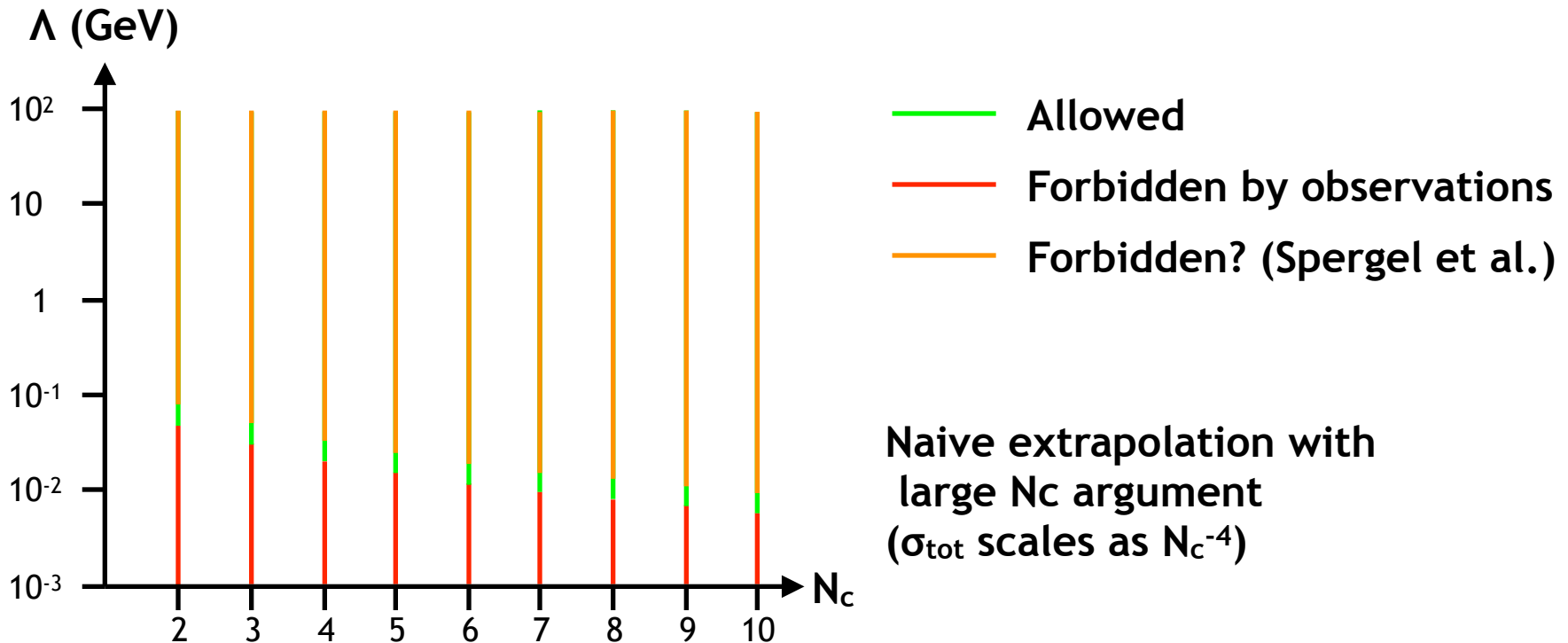
Observational constraints: $0.45 \text{ cm}^2/\text{g} < \frac{\sigma_{\text{tot}}}{m_\phi} < 1.0 \text{ cm}^2/\text{g}$

Robust constraint from galactic cluster shape, collisions (upper limit)

A. H. Peter et al., MNRAS 430, 81 (2013), 430, 105 (2013); S. W. Randall et al., APJ 679, 1173 (2008).

Constraint from Spergel et al. (lower limit), under discussion?

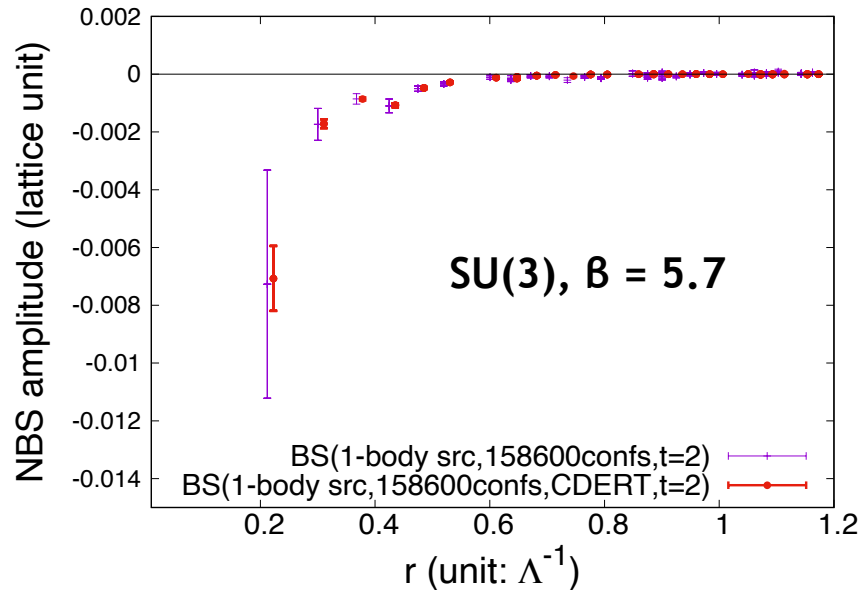
D. N. Spergel et al., PRL 84, 3760 (2000).



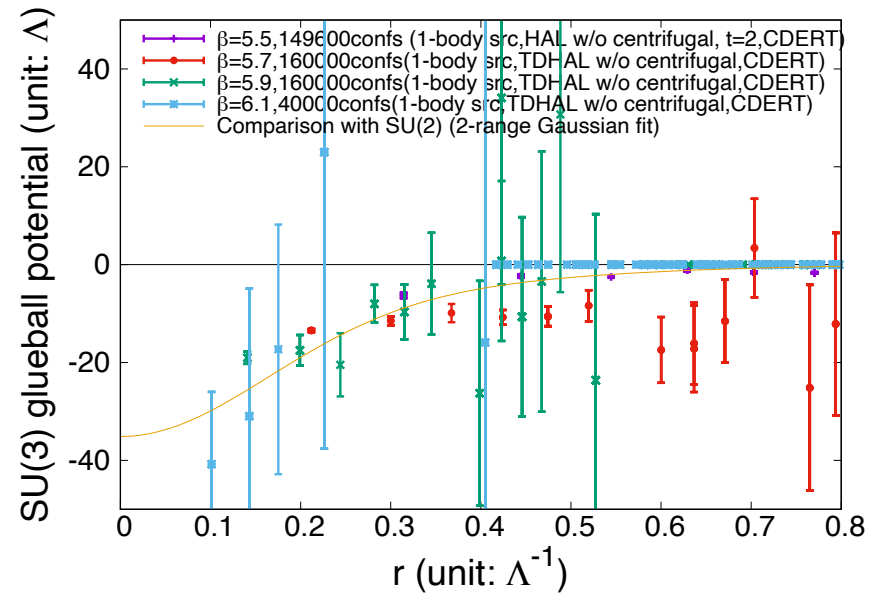
N_c vs. scale parameter (Λ) diagram

SU(3) Yang-Mills result (preliminary)

NBS amplitude:



Potential:



- The CDERT is very efficient in reducing statistical error
- The removal of centrifugal force makes the potential attractive, like SU(2)
- The value of the SU(3) interglueball potential is close to the SU(2) one

Summary

- Glueballs of the SU(N) Yang-Mills theory are good candidates of dark matter, study of **self-interaction** is important.
- We calculated the interglueball potential in the SU(2) Yang-Mills theory on lattice using the **Time-dependent HALQCD method**.
- We **removed the centrifugal force** : the interglueball potential is attractive.
- We calculated the scattering phase shift and derived the interglueball cross section, and we could constrain Λ of SU(2) YMT for the 1st time from observational data : **$\Lambda > 60$ MeV**.
- Preliminary result of SU(3) YMT looks consistent with SU(2).

Homeworks:

- Calculations for $N_c > 2$ on-going : extrapolate to large N_c .
- Lattice artifacts to be discussed.

