A Reconstruction Conjecture: Deciphering the Structure of the Dark Sector from the Matter Power Spectrum

Fei Huang

ITP CAS, and UC Irvine

arXiv: 2001.02193, 2101.10337

in collaboration with

Keith Dienes, Jeff Kost, Kevin Manogue, Shufang Su, Brooks Thomas

05/25/2021

PHENO 2021

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See the next talk by Kevin Manogue

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Basic idea…

early-universe dynamics

dark-matter phase-space distribution $f(p)$

linear matter power spectrum $P(k)$

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early-universe dynamics

dark-matter phase-space distribution $f(p)$

linear matter power spectrum $P(k)$

?

reconstructed $f(p)$

- **What can we learn from the matter power spectrum P(k)?**
- **To what extent is an inversion possible?**

Phase-Space Distribution

For any particle species in the universe, its properties can be described through its phase space distribution $f(p,t)$

$$f(\vec{x},\vec{p},t) \approx f(p,t)$$

homogeneity and isotropy

$$n(t) \equiv g\int \frac{d^3p}{(2\pi)^3} f(p,t)$$

$$\rho(t) \equiv g\int \frac{d^3p}{(2\pi)^3} E f(p,t)$$

$$P(t) \equiv g\int \frac{d^3p}{(2\pi)^3} \frac{p^2}{3E} f(p,t)$$

$$w(t) \equiv \frac{P(t)}{\rho(t)}$$

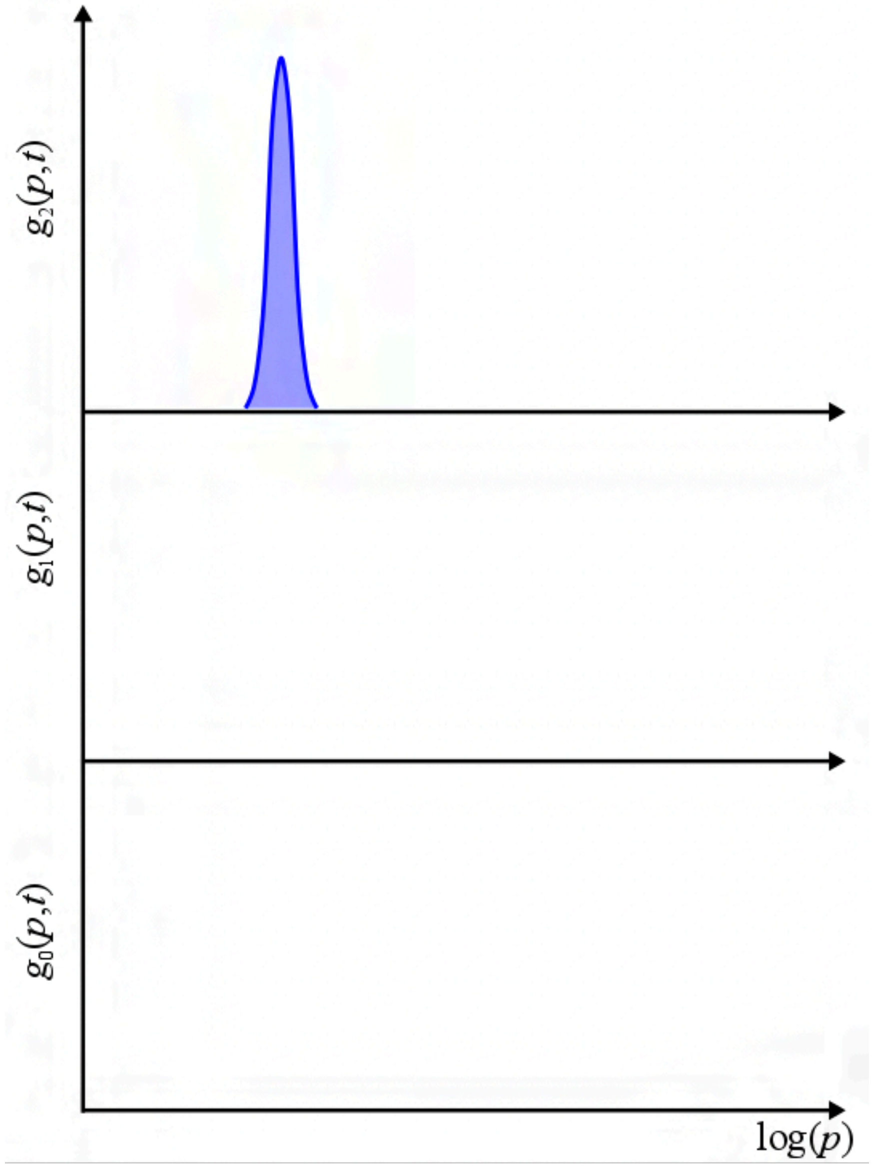
Often, $f(p,t)$ is assumed to be thermal. However, this need not be the case. In fact, $f(p,t)$ could take any reasonable functional form.

Picturing the evolution

It turns out it is very convenient to use the rescaled distribution w.r.t. $\log p$

$$N(t) \sim n(t)a^3 \sim \int d\log p \, \boxed{(ap)^3 f(p,t)}$$

$$\boxed{g(p,t) \equiv a^3(t) p^3 f(p,t)}$$



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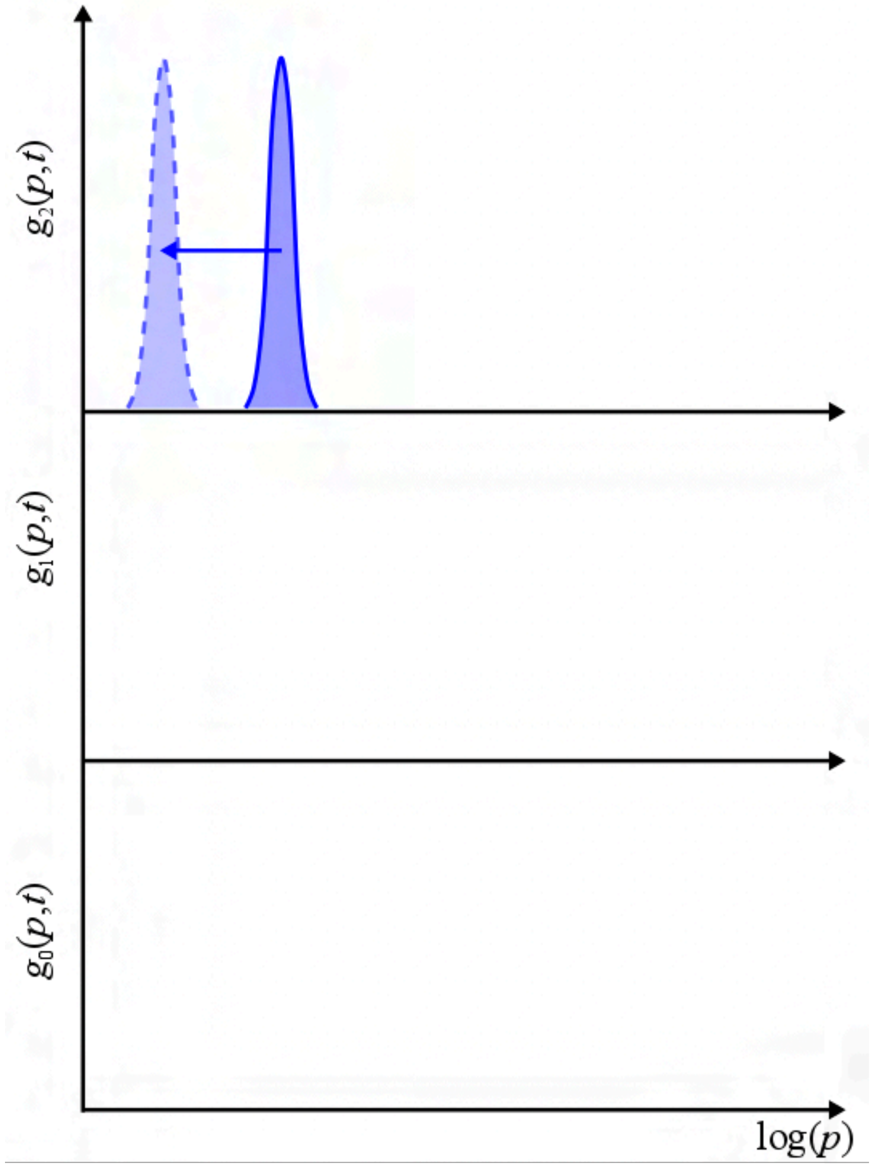

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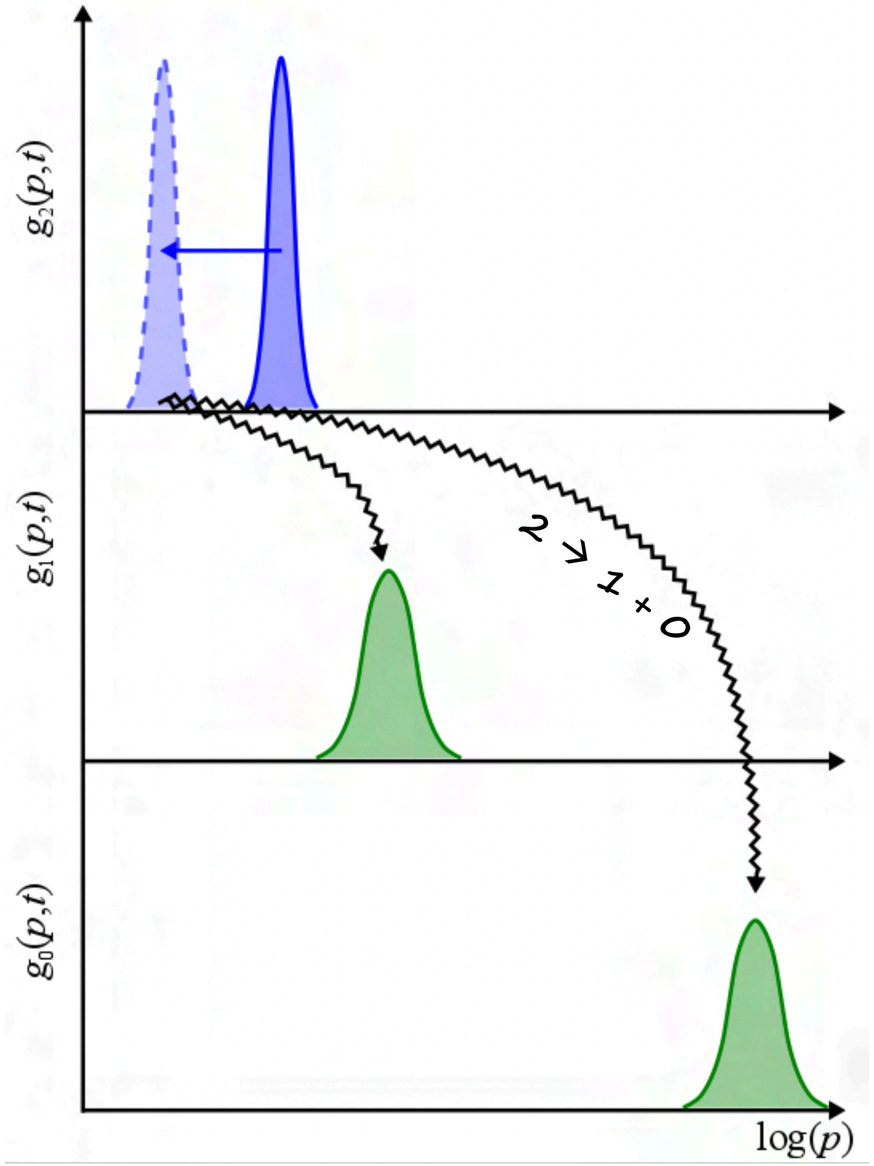

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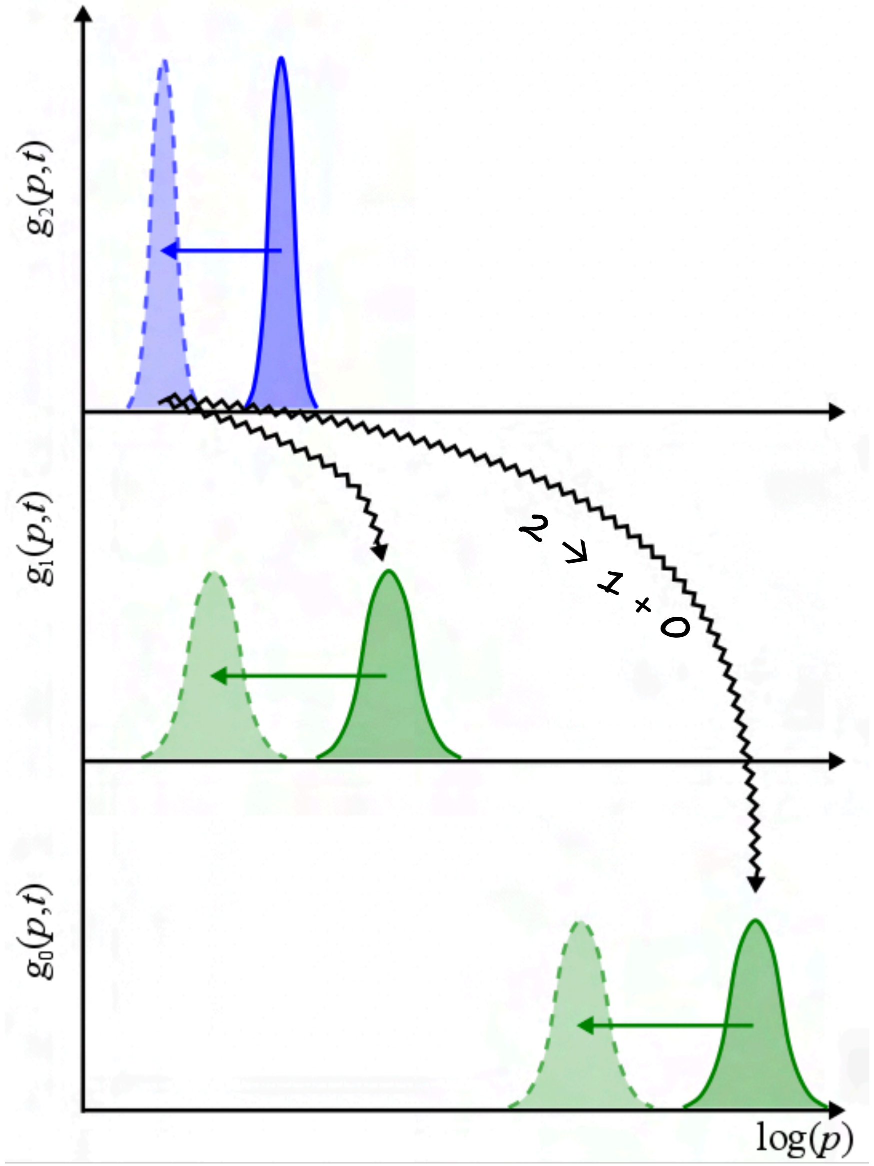

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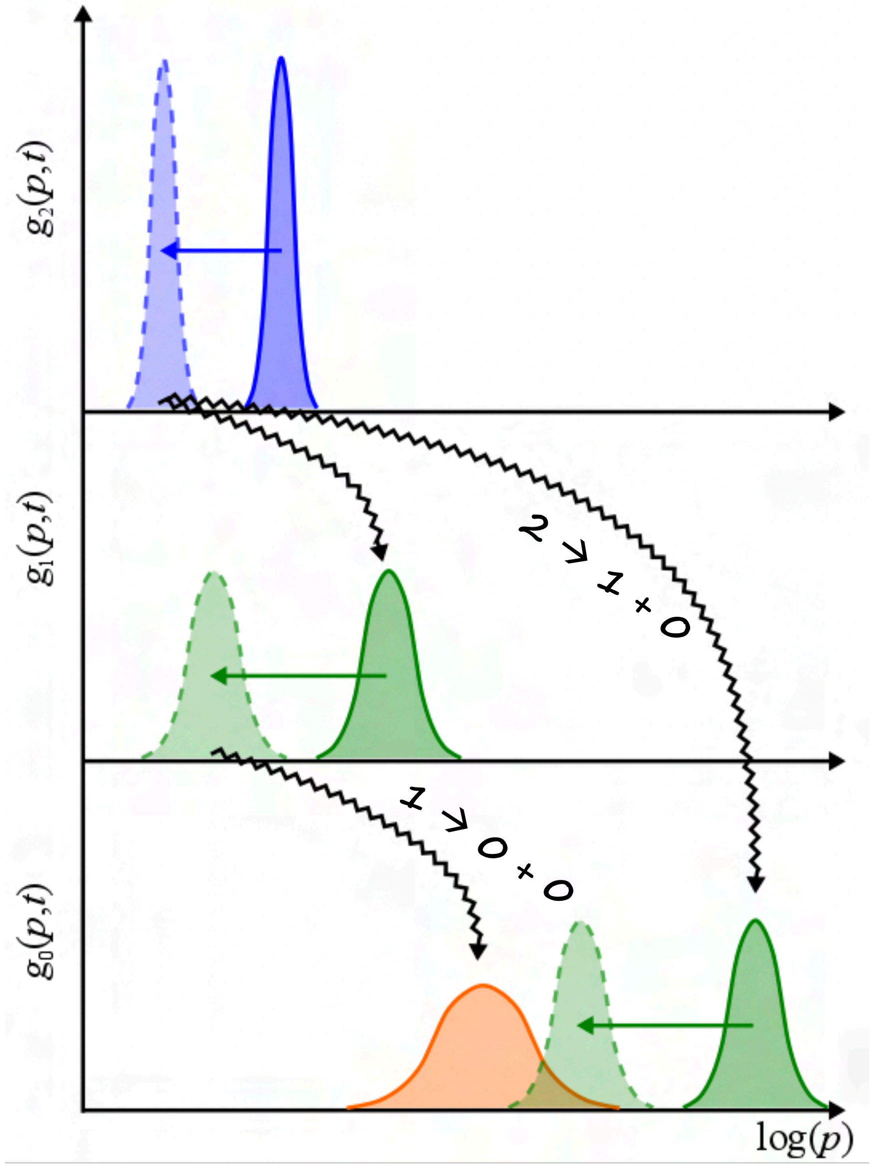

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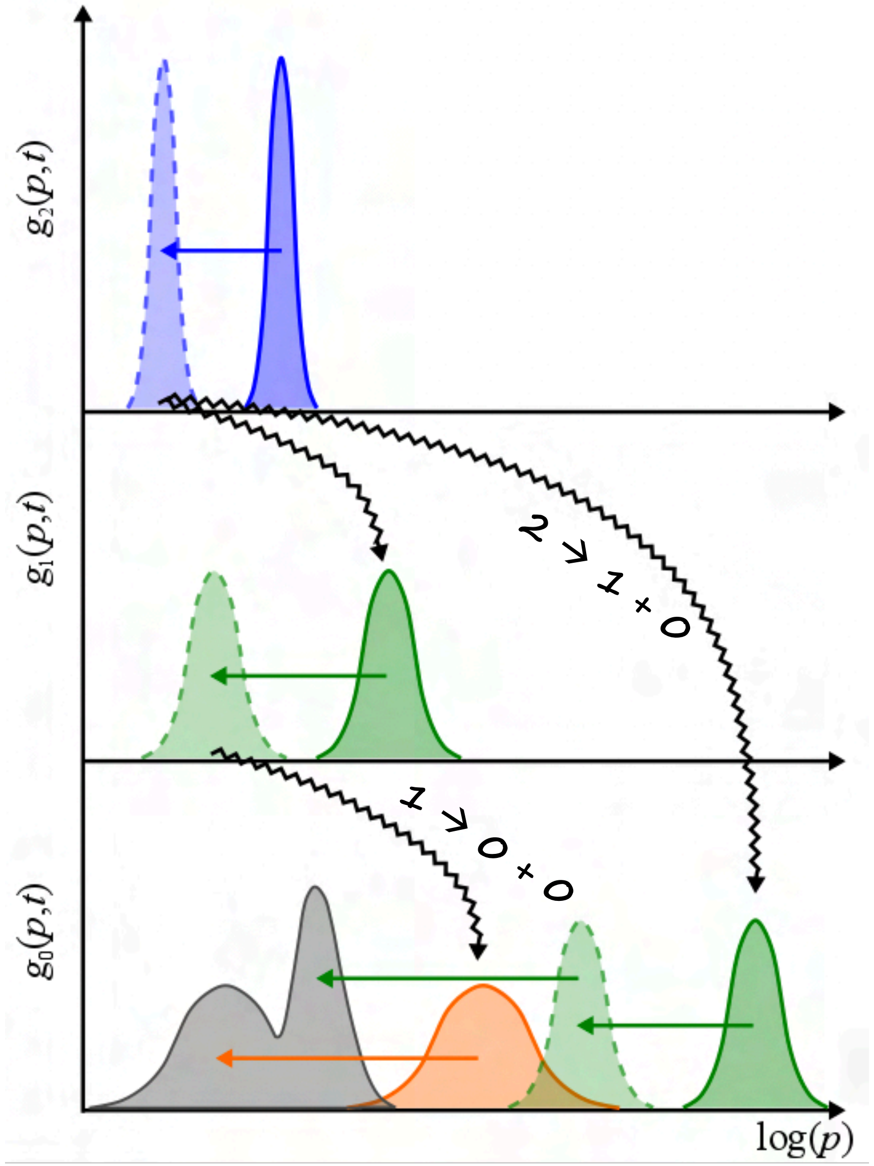

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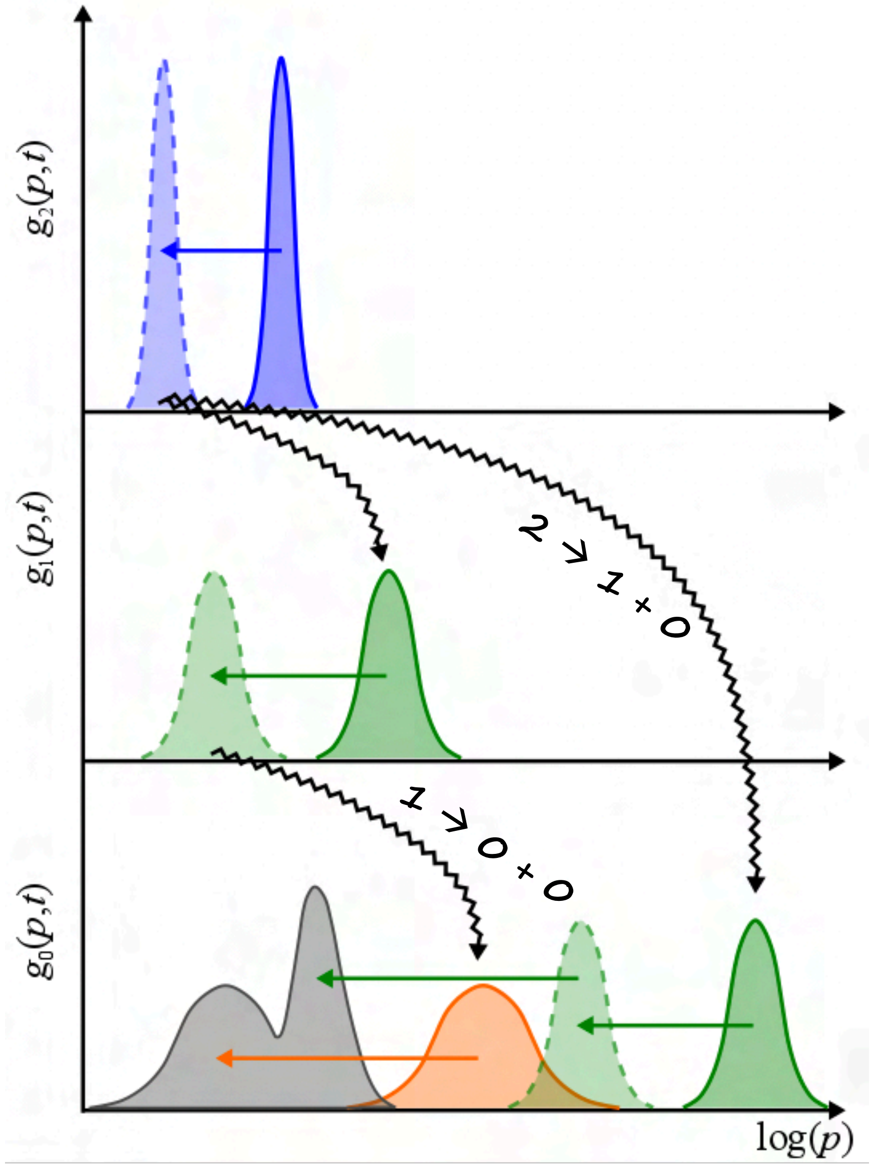
If deposits occur at different times during the cosmological history, a non-trivial, multi-modal distribution can result at present time!



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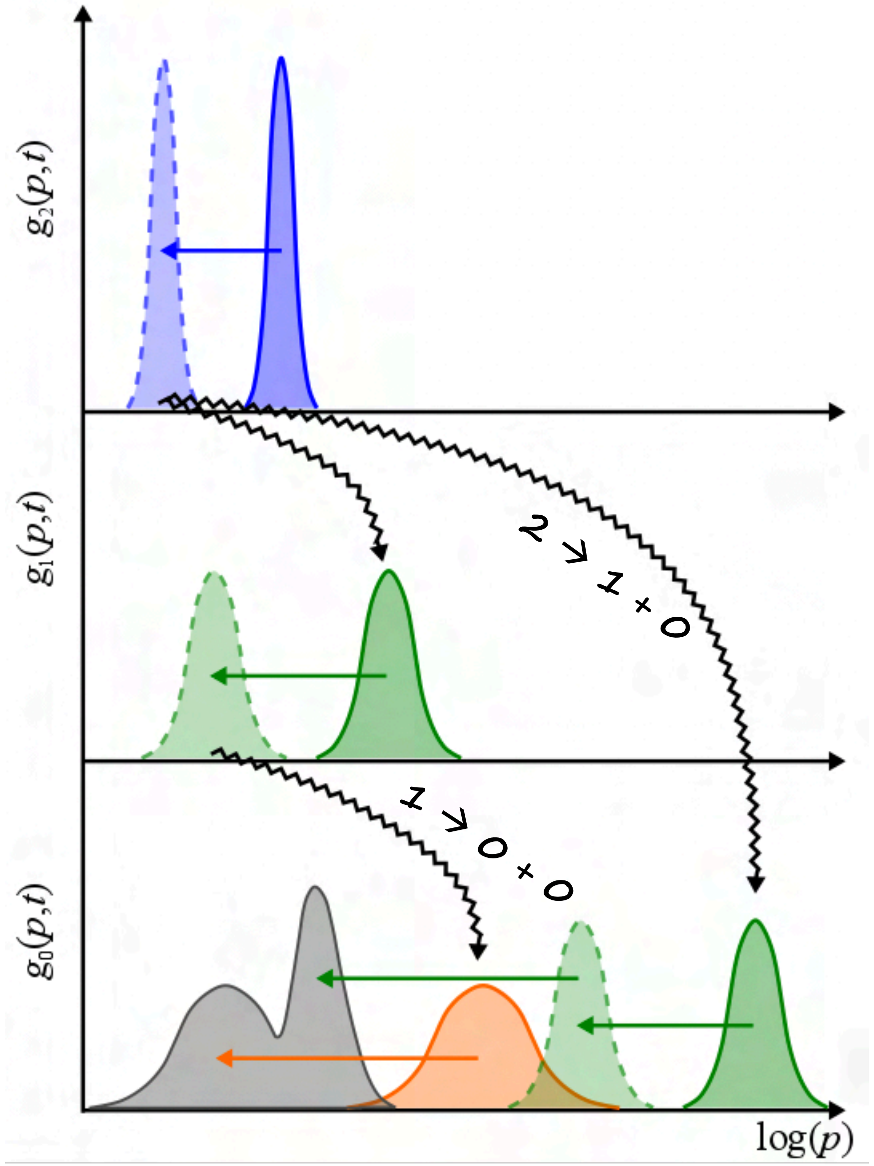
A non-trivial DM phase-space distribution at late times can represent the imprint of complex dynamics at earlier points in the cosmological history.

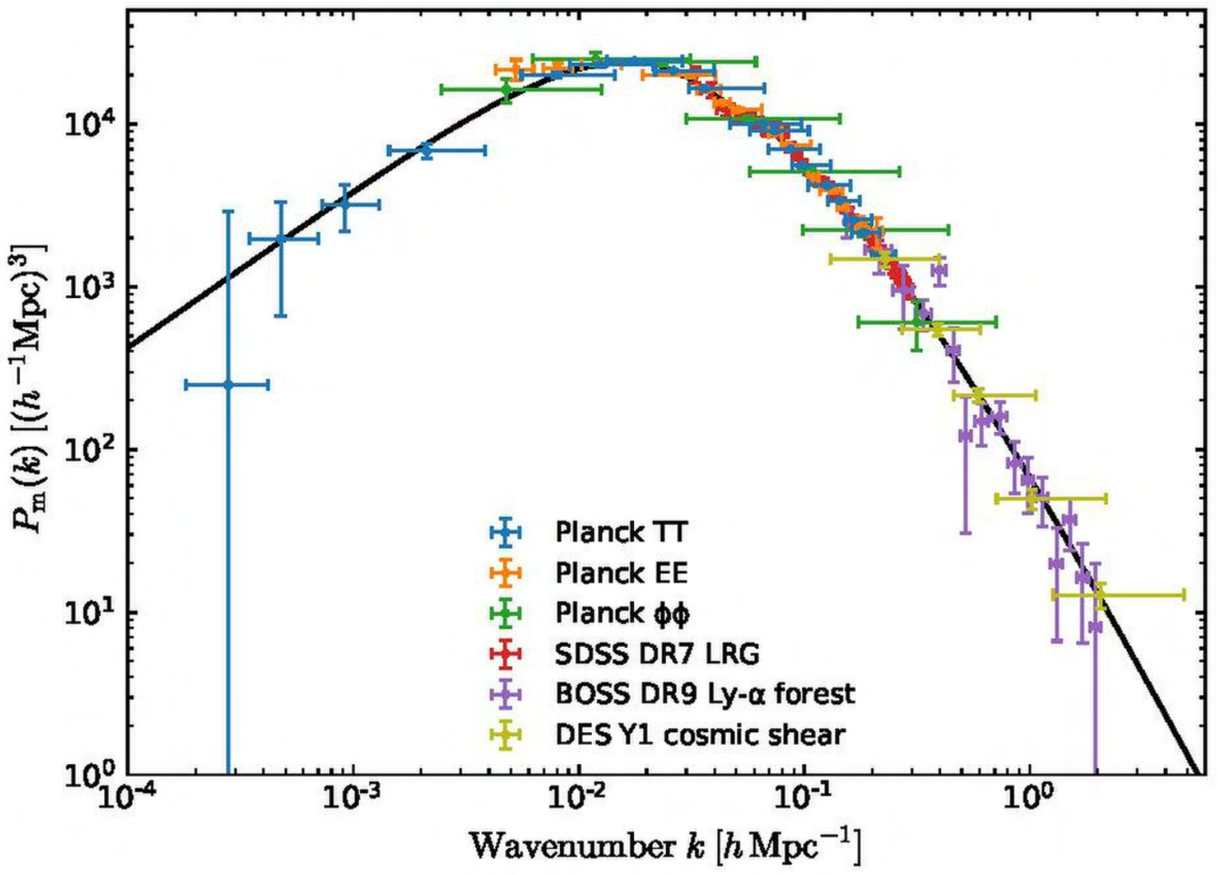


Why is the DM phase-space distribution important?

It turns out that the ***formation of structure (clusters, galaxies, etc.) is sensitive to the velocity of DM!***

Structure formation is ***suppressed*** if DM has non-negligible velocity and therefore deviates from what is expected for CDM!

In fact, the cosmic structure carries an ***imprint*** of the DM velocity distribution.



e.g., in the linear regime, can be reflected in the shape of the matter power spectrum $P(k)$.

Since structure formation depends on gravity only...

- **Studying the relation between the DM phase-space distribution and the large-scale structure enables us to learn about DM from its gravitational interaction only.**
- **This provides a way to learn about the dark sector even if the dark sector does not interact with the SM at all, except through gravity!**

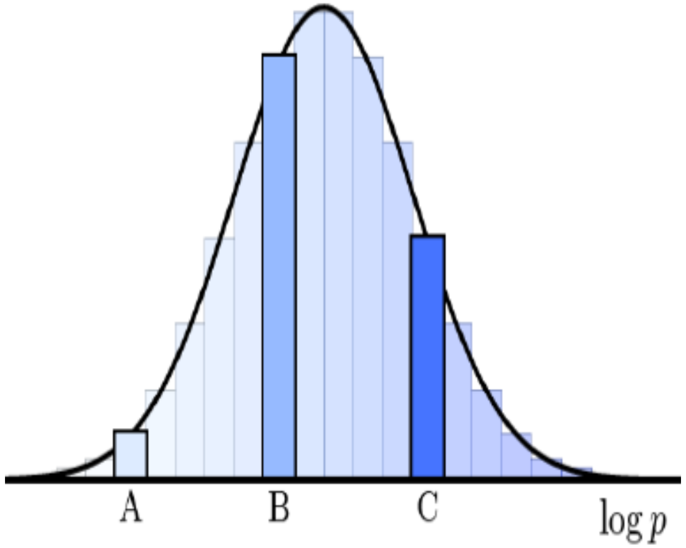
To study the impact of non-negligible velocities on *P(k)*, a standard approach is to define a single "free-streaming horizon" as a benchmark scale below which structure is suppressed

$$k_{\mathrm{FSH}} \equiv \left[\int_{t_{\mathrm{prod}}}^{t_{\mathrm{now}}} dt\, \frac{\langle v(t)\rangle}{a(t)}\right]^{-1}$$

Relies on ***averaging***, not suitable for non-trivial or multi-modal distributions – average velocity might ***NOT*** be able to capture all the features in the distribution.

In some cases, the distribution might not even contain any DM particle with velocity $\langle v\rangle$!

Our approach



We begin by considering ***momentum slices*** through our dark-matter packet, relating each slice of momentum *p* to a corresponding value $k_{\rm hor}(p)$.

$$k_{\rm hor}(p) \equiv \xi \left[\int_{t_{\rm prod}}^{t_{\rm now}} dt \, \frac{v(t)}{a(t)}\right]^{-1}$$

Normally, $k_{\rm hor}$ would be interpreted as defining the ***minimum*** value of k which can be affected by dark matter in that momentum slice.

However, we shall instead take the defining relation for $k_{\rm hor}(p)$ as defining a ***mapping*** between the p-variable of $g(p)$ and the k-variable of $P(k)$.

$$p \to k$$

Our approach

In other words, we ***identify*** $k_{\mathrm{hor}}(p)$ with k and thereby consider $g(p)$ as having a corresponding profile in k-space:

$$g(p) \to \tilde{g}(k)$$

It then follows

$$N(t) \sim \int d \log p \;\; g(p) = \int d \log k \;\; \tilde{g}(k)$$

Thus $\tilde{g}(k)$ describes a **dark-matter distribution in k-space**!

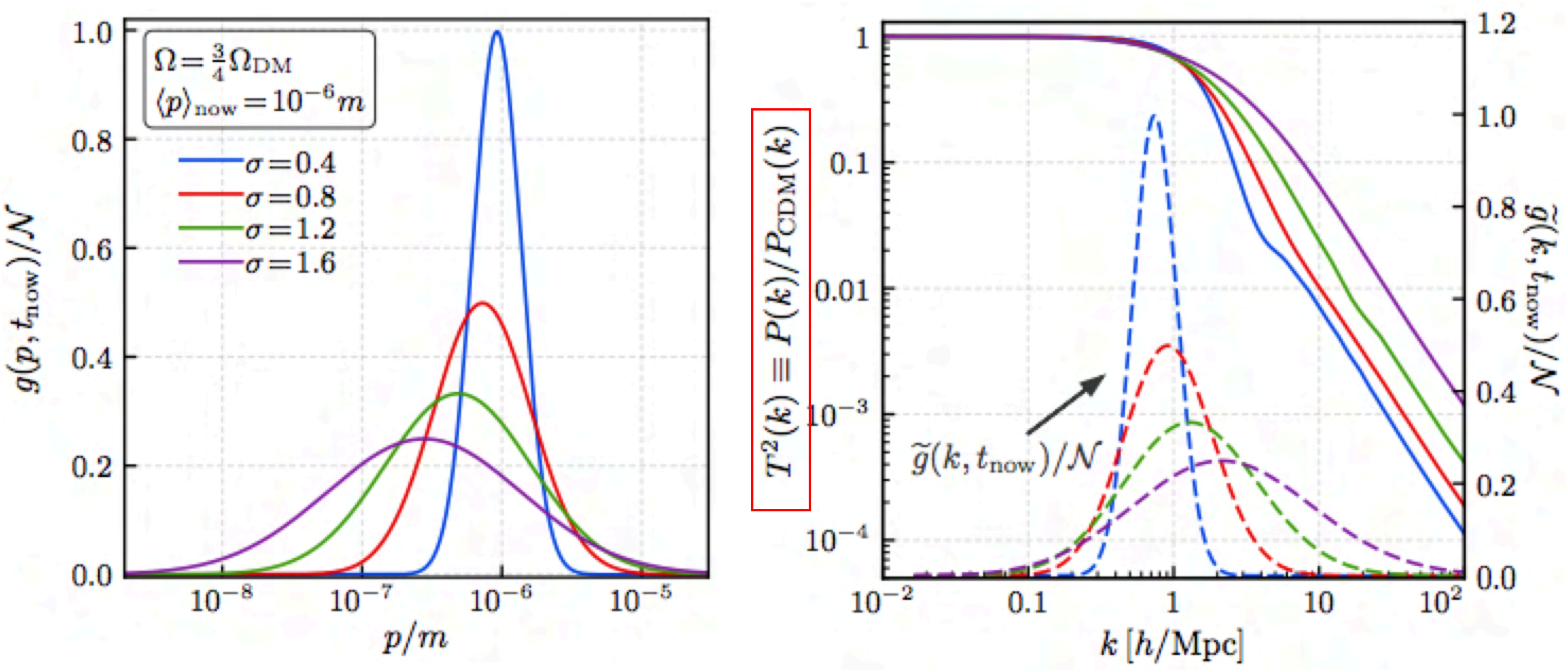
Moreover, because this $\tilde{g}(k)$ lives in the same space as $P(k)$, these two functions can even be plotted together along the same axis!

Now it makes sense to ask:
Can we discover/conjecture any relation between *these* two functions?

Examining the relations

Vary width with average/area fixed

(a complementary CDM component is added to get the total DM abundance)



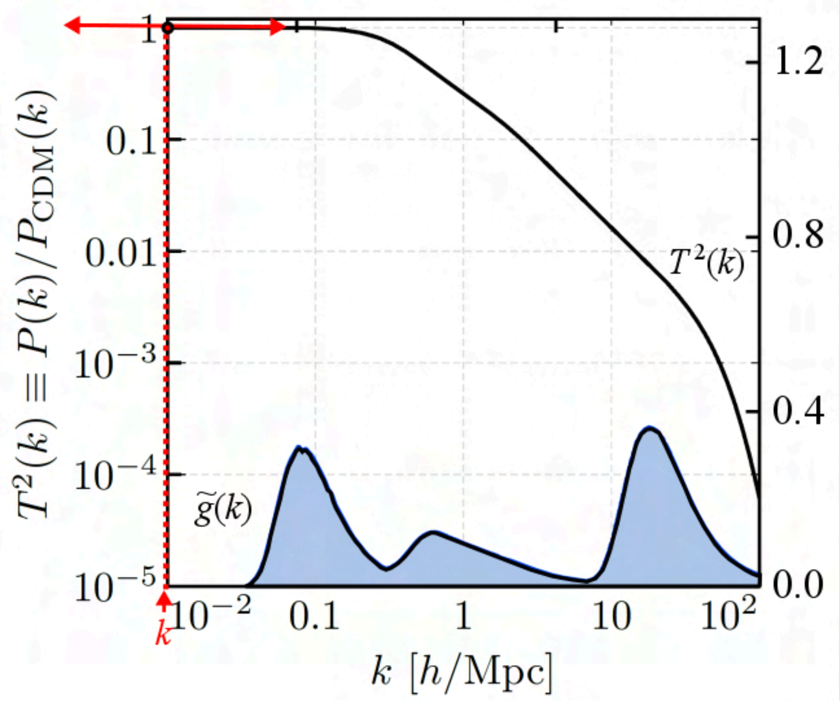
- The amount of suppression differs, but the ***slope*** at large k is essentially *unaffected* by widths! This suggests the ***accumulated abundance is correlated with the slope***, *NOT* with the net suppression.

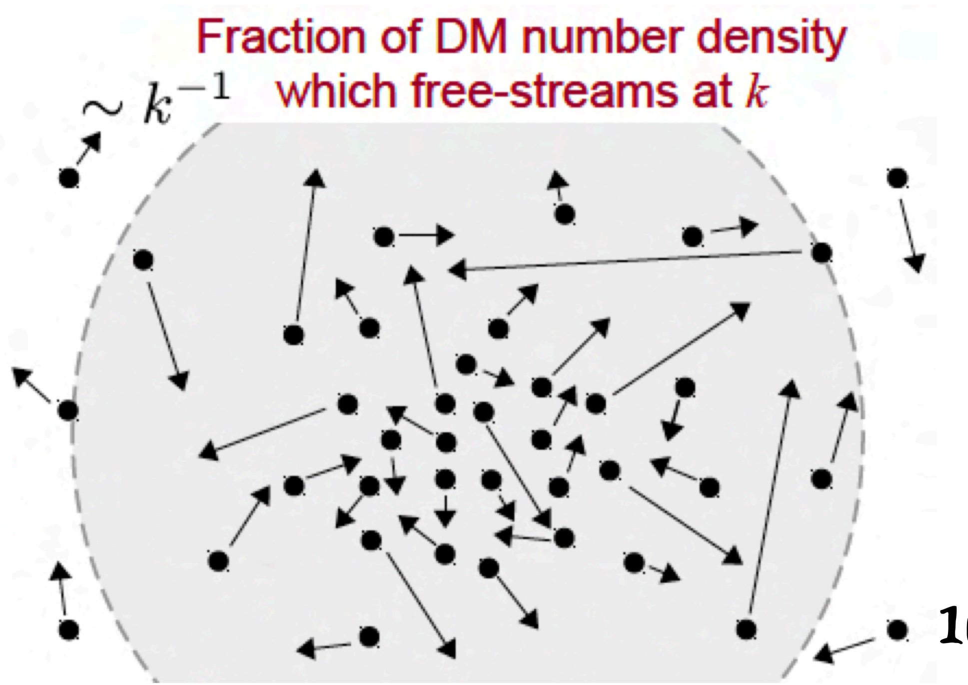
Relating $\tilde{g}(k)$ and $T^2(k)$

We found the ***slope*** of the transfer function at a particular scale k is related to the amount of DM particles that is able to freestream a distance larger than $\sim 1/k$,
i.e., the fraction of DM particles that is effectively "hot" relative to the scale k

We define the ***hot-fraction function***,

$$F(k) \equiv \frac{\int_{-\infty}^{\log k} \tilde{g}(k')\, d\log k'}{\int_{-\infty}^{\infty} \tilde{g}(k')\, d\log k'}$$





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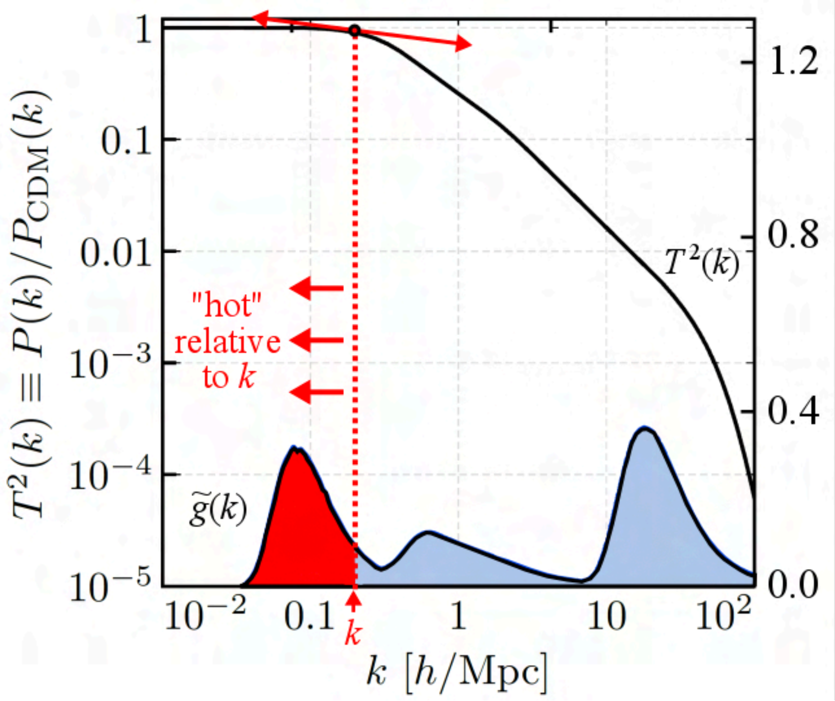

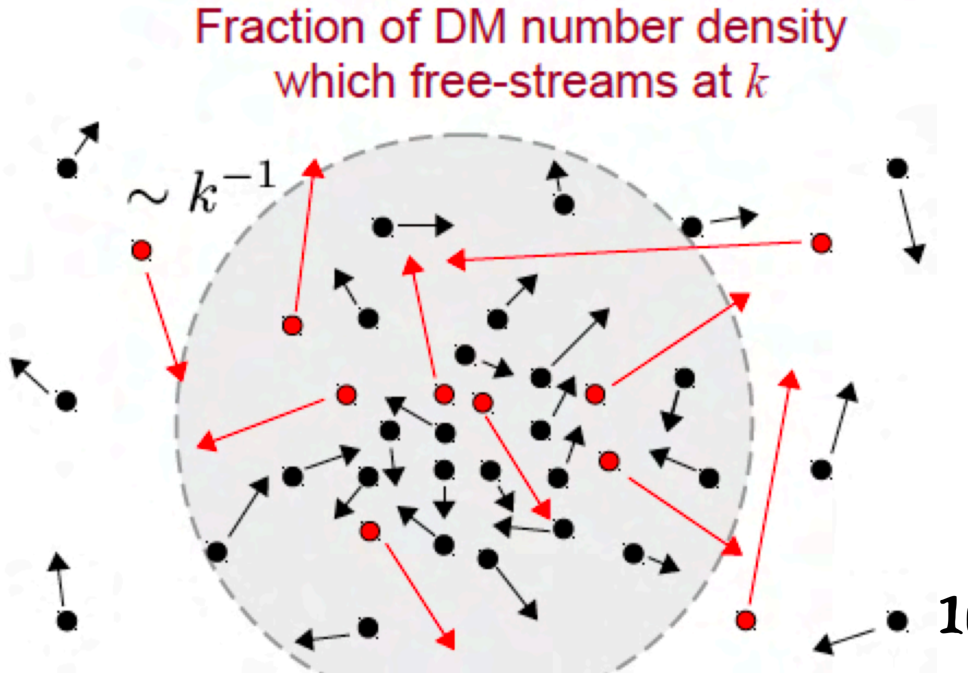

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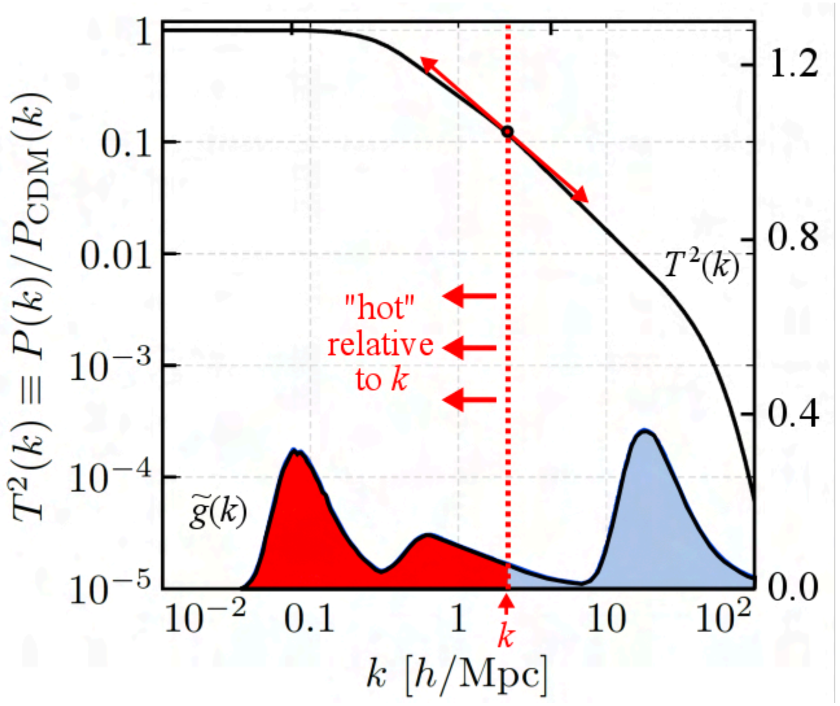

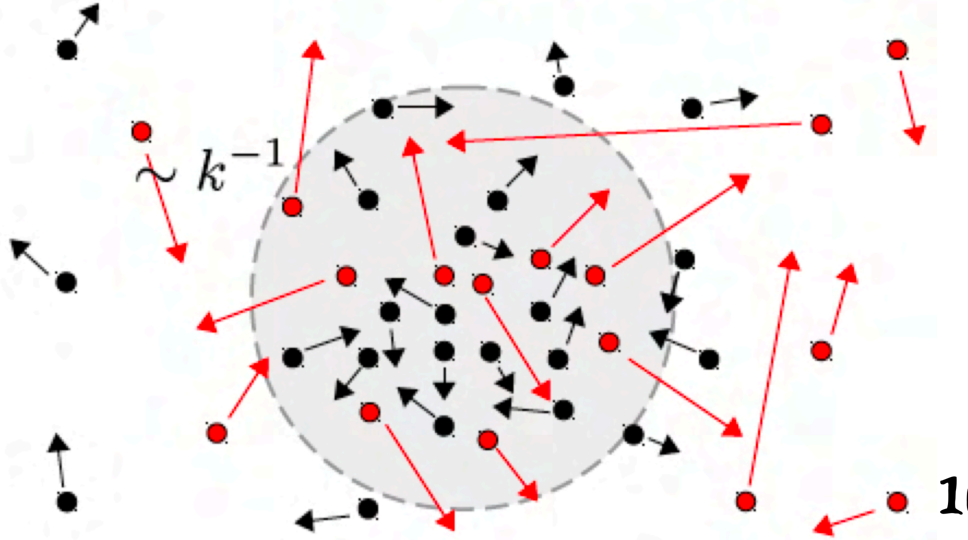

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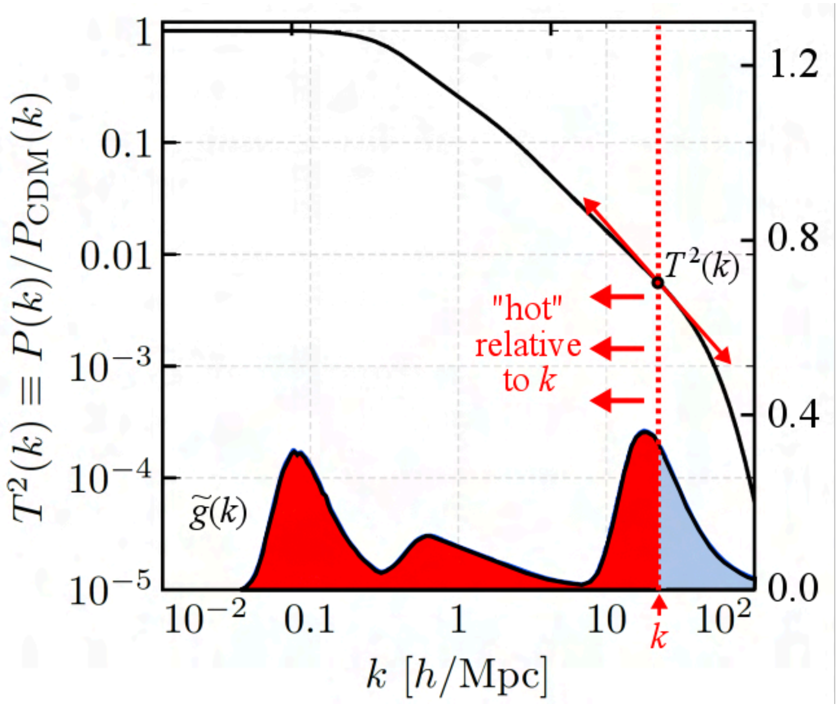
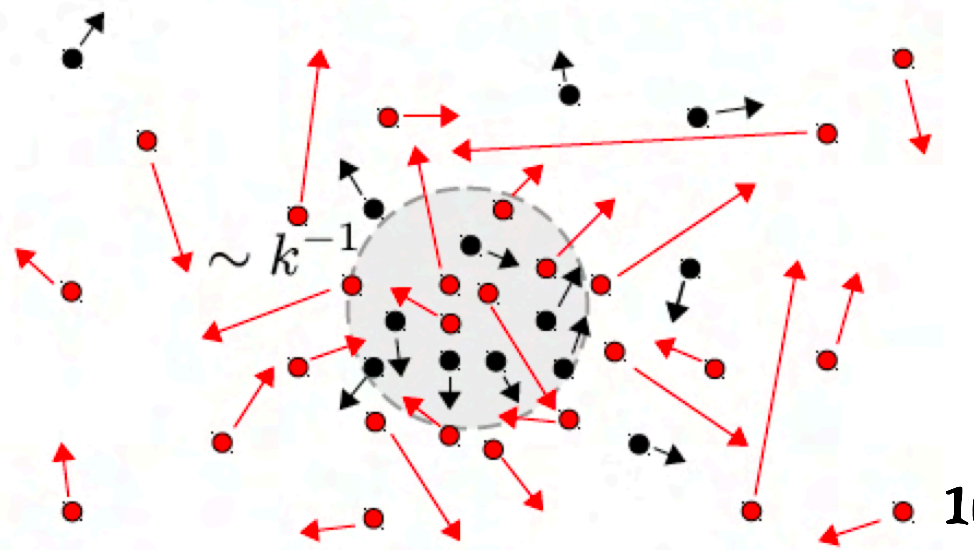


Fraction of DM number density which free-streams at k



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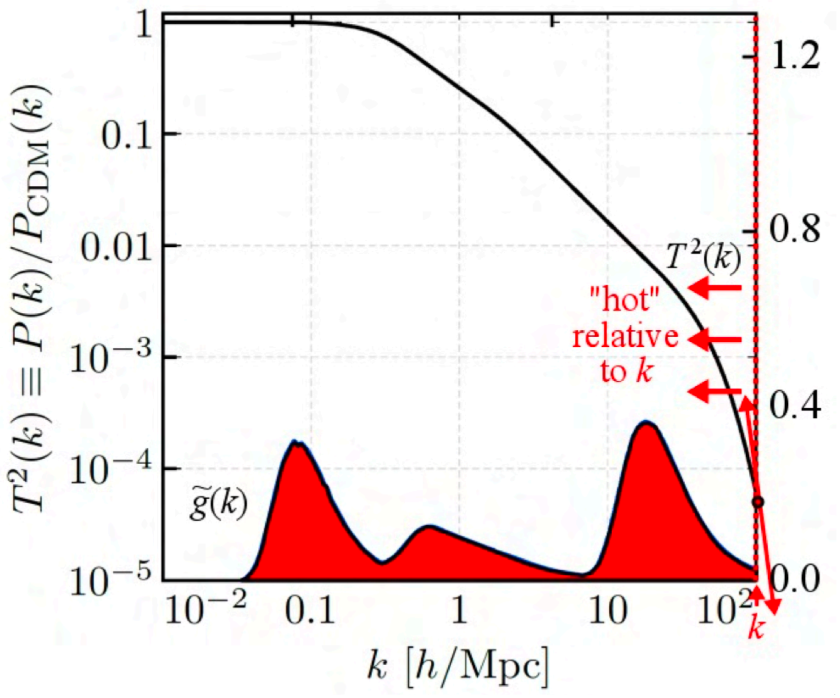

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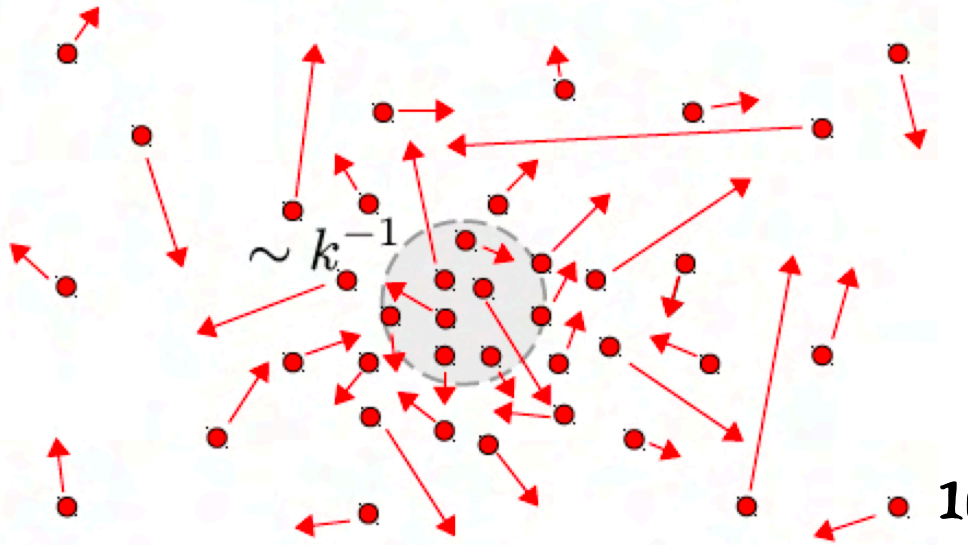

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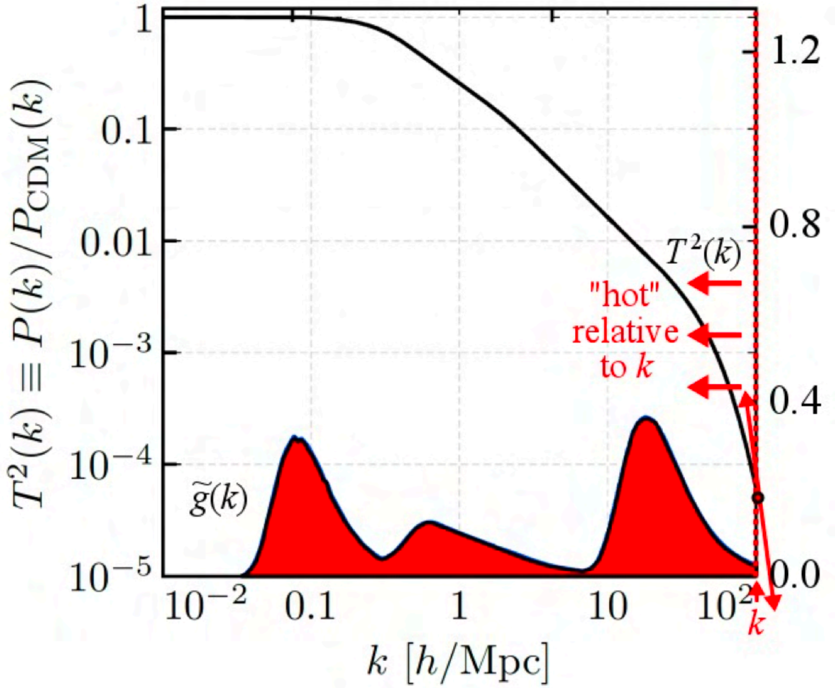

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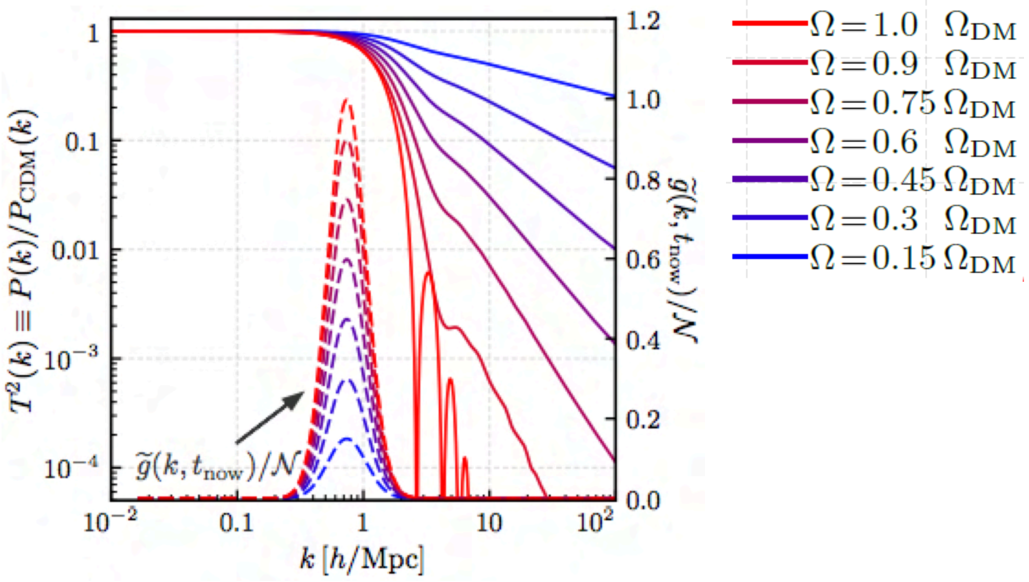

$$\left| \frac{d\log T^2}{d\log k} \right| \approx [F(k)]^2 + \frac{3}{2} F(k)$$

relation holds to very high precision!

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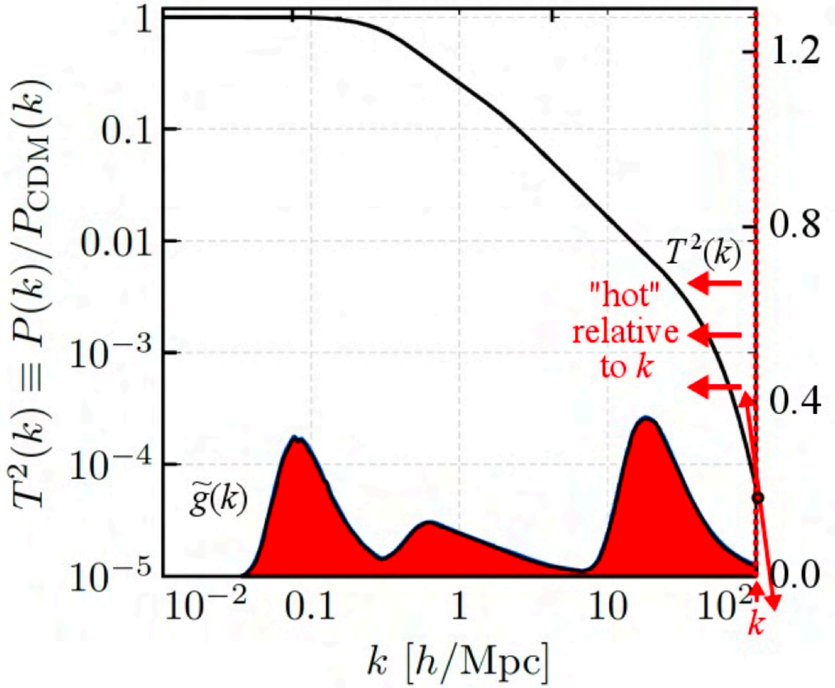

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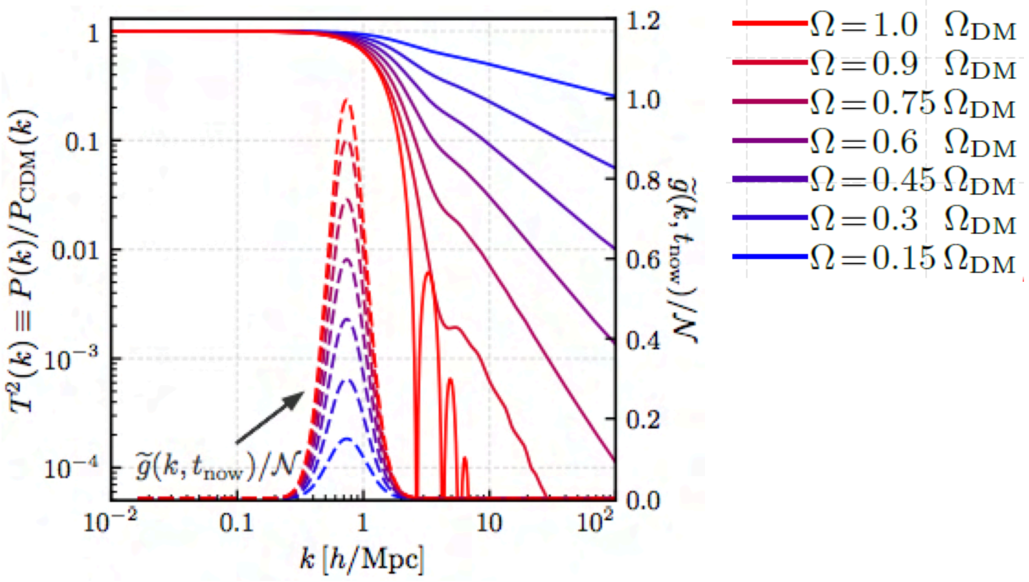

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$$\frac{\tilde{g}(k)}{\mathcal{N}} \approx \frac{1}{2}\left(\frac{9}{16} + \left|\frac{d\log T^2}{d\log k}\right|\right)^{-1/2} \left|\frac{d^2\log T^2}{(d\log k)^2}\right|$$

This allows us to "resurrect" $\tilde{g}(k)$ directly from the transfer function $T^2(k)$!

An Illustrative Model

Dark ensemble consists of *N+1* real scalars ϕ_j with $j = 0,1,\ldots N$, and a mass spectrum:

$$m_j = m_0 + j^\delta \Delta m$$

Lagrangian:

$$\mathcal{L} = \sum_{\ell=0}^{N} \left(\frac{1}{2}\partial_\mu \phi_\ell \partial^\mu \phi_\ell - \frac{1}{2} m_\ell^2 \phi_\ell^2 - \sum_{i=0}^{\ell}\sum_{j=0}^{i} c_{\ell i j}\, \phi_\ell \phi_i \phi_j \right) + \cdots$$

The trilinear coupling:

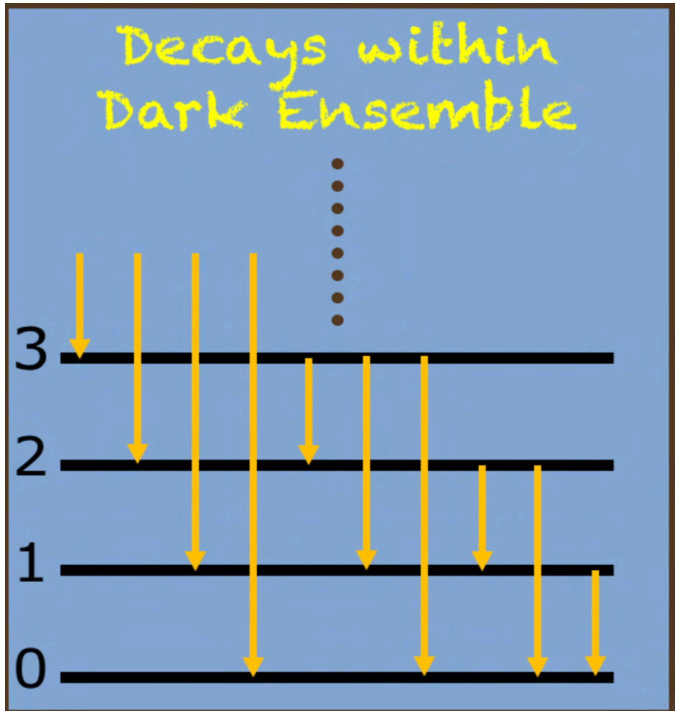
$$c_{\ell i j} = \mu R_{\ell i j} \left(\frac{m_\ell - m_i - m_j}{\Delta m}\right)^{r} \left(1 + \frac{|m_i - m_j|}{\Delta m}\right)^{-s} \Theta(m_\ell - m_i - m_j)$$

difference between parent and products

difference between products

Positive r → Decays with more kinetic energy
Negative r → Decays more marginal (less phase space)

Positive s → Decay products tend to have similar masses
Negative s → Decay products tend to have different masses

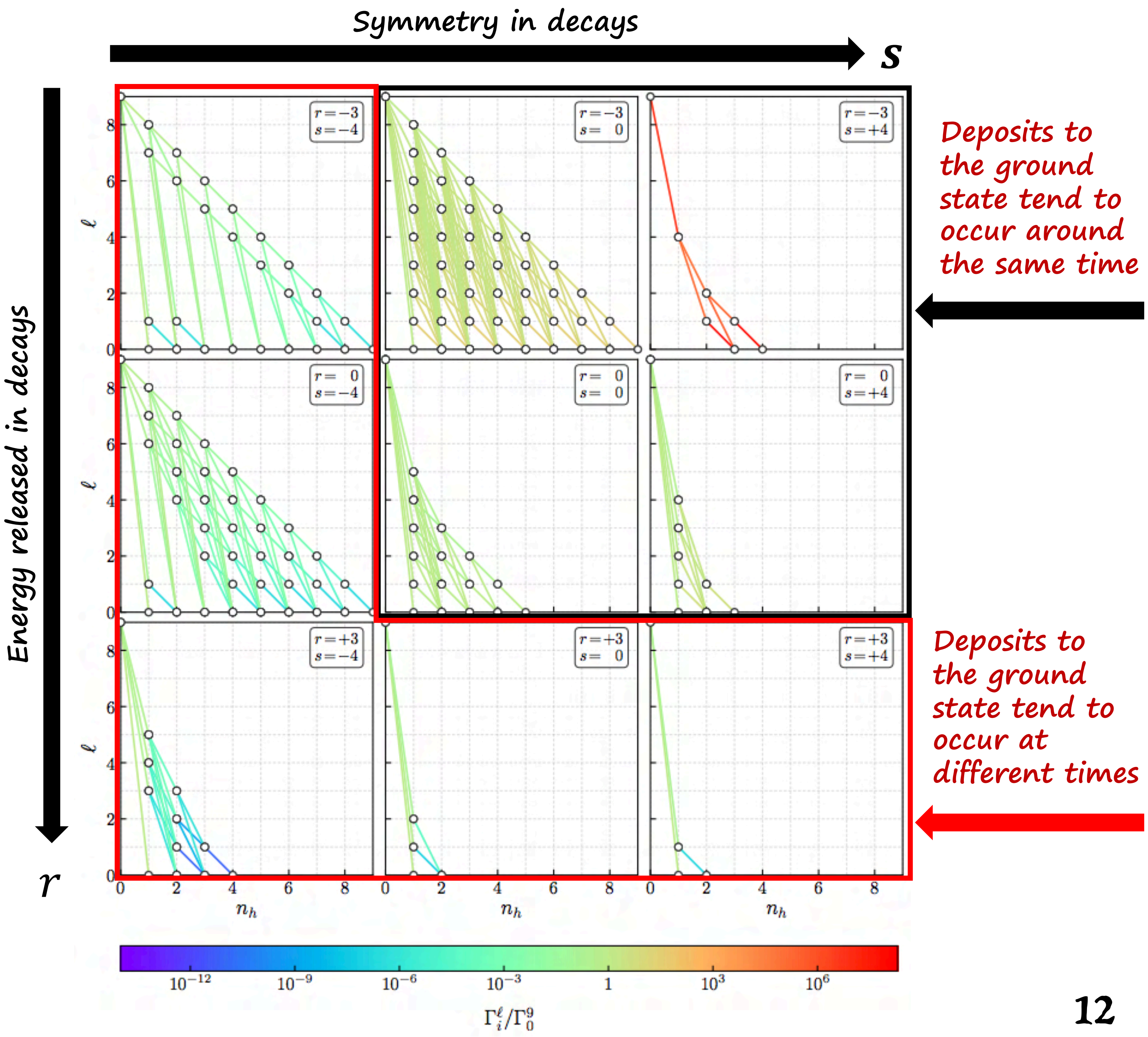


In our analysis we considered ***N=9*** ***(10 states)***

Illustrative Model: Decay chains

The figure shows how decays proceed step by step from a heavy state to the ground state. Only major decay chains are shown.

Many different patterns of decay chains could emerge!



Final Phase-Space Distribution

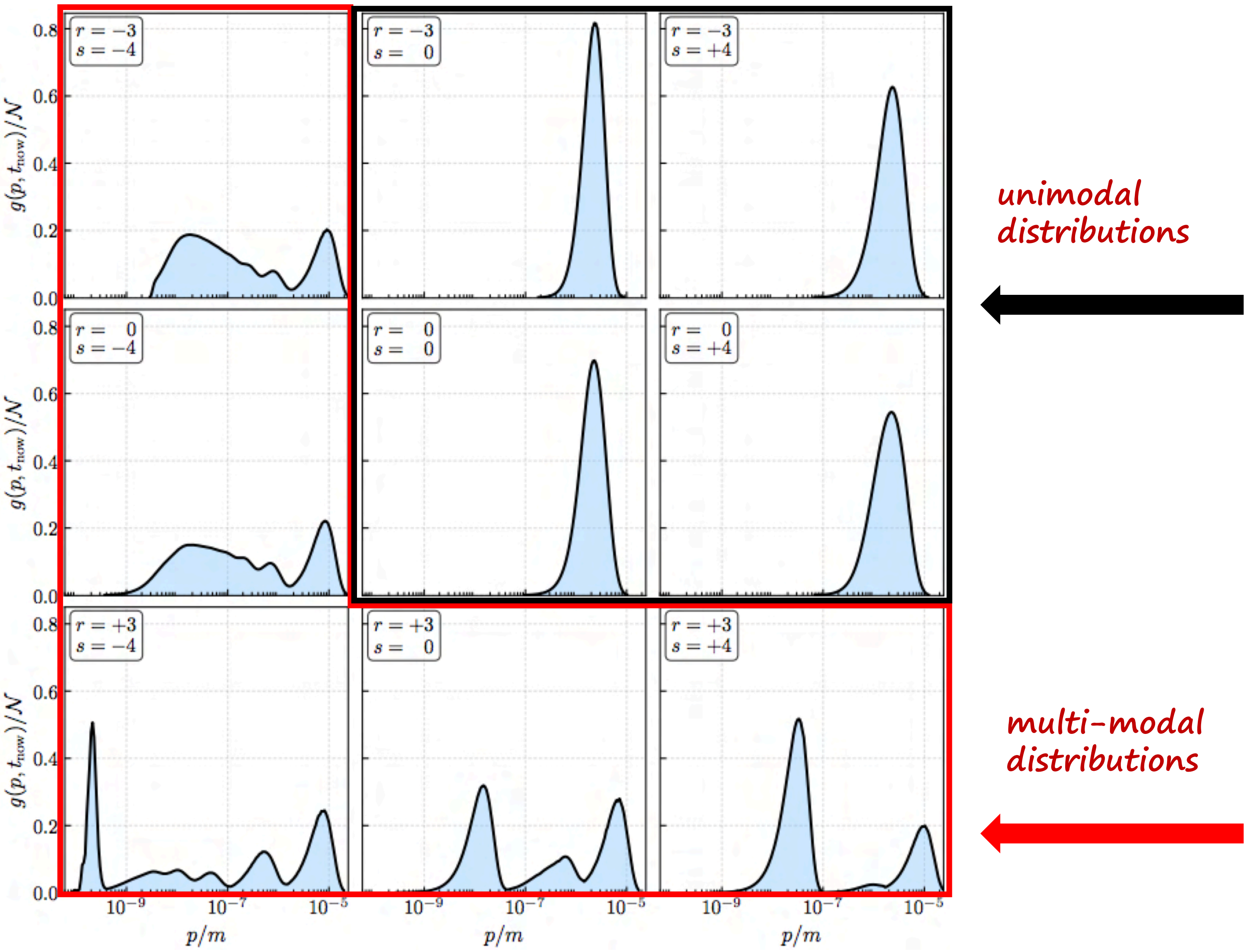
Numerically solve the Boltzmann equation assuming only the heaviest state is populated initially.

A rich variety of distributions emerges!

As expected!

- Cases in which decay chains land on the ground state at ***similar timescales*** tend to produce ***unimodal distributions***

- ***Multi-modal*** distributions could result if ***timescales*** of different decay chains ***differ significantly***



Reconstruction Test

To what extent can we "***resurrect***" the DM phase-space distribution from the transfer function?

Recall our conjecture…

$$\frac{\widetilde{g}(k)}{\mathcal{N}} \approx \frac{1}{2}\left(\frac{9}{16}+\left|\frac{d\log T^2}{d\log k}\right|\right)^{-1/2}\left|\frac{d^2\log T^2}{(d\log k)^2}\right|$$

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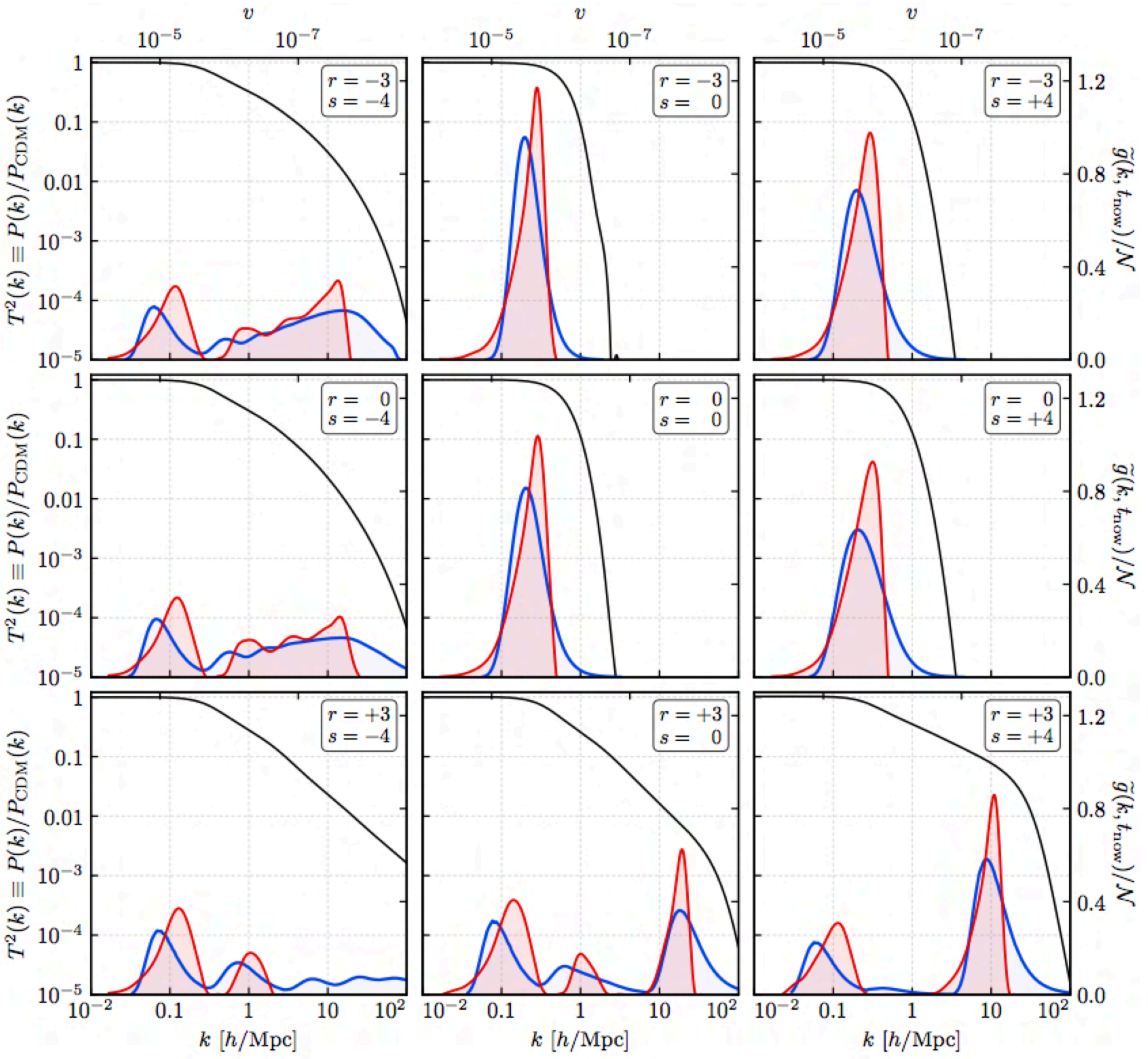
Blue: original DM distribution in k-space
Red: reconstruction directly from $T^2(k)$

Our reconstruction is surprisingly accurate for a **variety** of possible DM distributions.
Able to resurrect the **salient features** of the original distribution!



Conclusions

- Early-universe processes could leave ***identifiable patterns*** in the phase-space distribution $g(p)$ of dark matter which are then ***imprinted*** on the cosmic structure.
- The DM phase-space distribution $g(p)$ is ***correlated*** with the matter power spectrum $P(k)$ through the ***hot-fraction function*** $F(k)$.
- We proposed a ***reconstruction conjecture*** which enables us to reproduce the DM phase-space distribution. The reconstruction conjecture is simple and allows us to ***resurrect the salient features*** of the phase-space distribution directly from $P(k)$.
- The reconstruction conjecture is local, i.e., partial reconstruction could be obtained from incomplete information.
- Since structure formation relies on gravity only, such approach allows us to learn about dark-sector dynamics even ***if the dark sector has only gravitational couplings to the SM***.
- The dark sectors of string theory generically include unstable Kaluza-Klein towers, thus could potentially lead to multi-modal distributions and non-trivial $P(k)$. This provides motivation to measure/bound $P(k)$ with increased precision.