

Challenges for an axion explanation of the muon $g - 2$ measurement

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Let us sketch the type of BSM that could explain $(g-2)_\mu$:

- Absence of $(g-2)_e$, $a_e^{\text{EXP}} - a_e^{\text{SM}} = (4.8 \pm 3.0) \times 10^{-13}$

$$\Delta a_e \sim \left(\frac{m_e}{m_\mu}\right)^2 \Delta a_\mu \approx 5.9 \times 10^{-14}$$

- Constraints from EDM, $|d_e| < 1.1 \times 10^{-29}$ e cm and $|d_\mu| < 1.9 \times 10^{-19}$ e cm

$$|d_e| \sim \frac{e}{2m_\mu} \Delta a_\mu \approx 2.3 \times 10^{-22} \text{ e cm},$$

$$|d_\mu| \sim \frac{m_e}{m_\mu} d_\mu \approx 1.1 \times 10^{-24} \text{ e cm}.$$

- Constraints from flavor violating processes $\text{Br}(\mu \rightarrow e\gamma) < 4.2 \times 10^{-13}$

$$\text{Br}(\mu \rightarrow e\gamma) \sim \frac{6\pi^2 e^2}{G_F^2 m_\mu^4} (\Delta a_\mu)^2 \approx 2.0 \times 10^{-3}.$$

New Physics solutions — a crude classification

In terms of the new ingredient:

- New scalars
- New vectors
- New fermions
- New pNGB

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makes the naturalness problem worse, unless embedded in a bigger theory that addresses it

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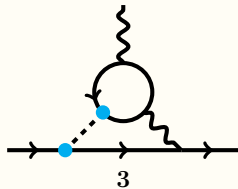
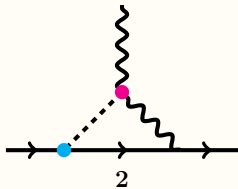
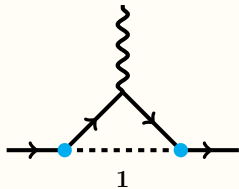


benign to the naturalness problem by themselves

Axion EFT

Axion EFT – Cont'd

$$\mathcal{L}_{\text{eff}} \supset \frac{c_{ii}}{2} \frac{\partial_\mu a}{f_a} (\bar{l}_i \gamma^\mu \gamma^5 l_i) - V(a) + c_{\gamma\gamma} \frac{\alpha}{4\pi} \frac{a}{f_a} F_{\mu\nu} \tilde{F}^{\mu\nu} + c_{\gamma\gamma;2} \frac{\alpha}{4\pi} \frac{\partial^2 a}{f_a^3} F_{\mu\nu} \tilde{F}^{\mu\nu}$$



$$\Delta a_\mu^{(1)} \propto -\frac{c_{\mu\mu}^2}{16\pi^2},$$

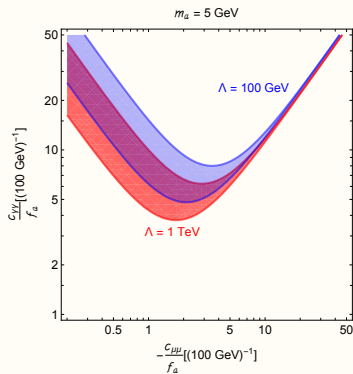
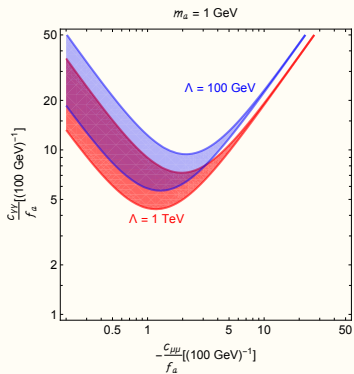
$$\Delta a_\mu^{(2)} \propto -\frac{c_{\mu\mu} c_{\gamma\gamma} \alpha}{16\pi^3},$$

$$\Delta a_\mu^{(3)} \propto -\frac{c_{\mu\mu} c_{ii} \alpha}{16\pi^3},$$

We consider two scenarios

- $c_{\mu\mu} \neq 0, \quad c_{\gamma\gamma} \neq 0$
- $c_{\mu\mu} \neq 0, \quad c_{ee} \neq 0$

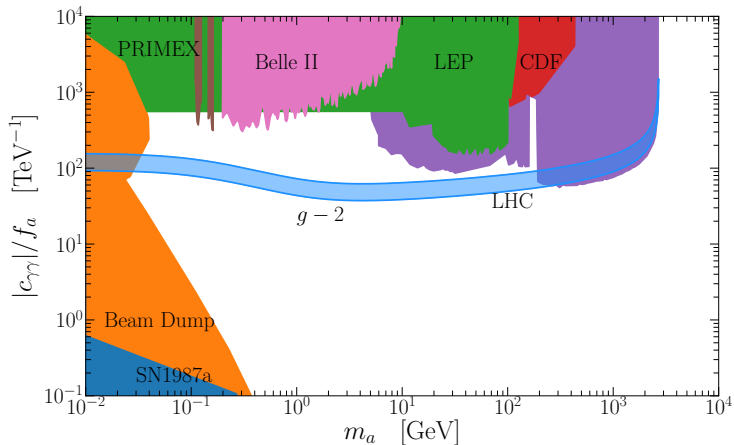
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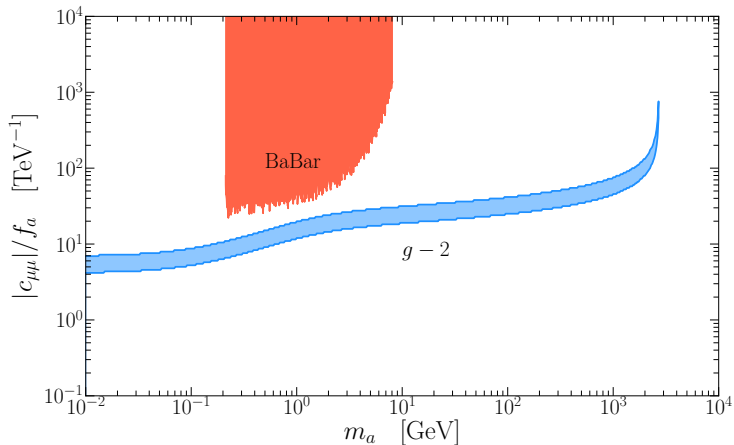
$$c_{\gamma\gamma}/c_{\mu\mu} < 0, \quad m_a \in (40 \text{ MeV} - 200 \text{ GeV})$$

$$\left| \frac{f_a}{c_{\gamma\gamma}} \right| \lesssim (10 - 25) \text{ GeV}, \quad \left| \frac{f_a}{c_{\mu\mu}} \right| \lesssim 100 \text{ GeV} .$$

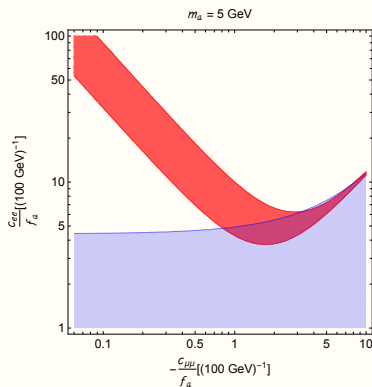
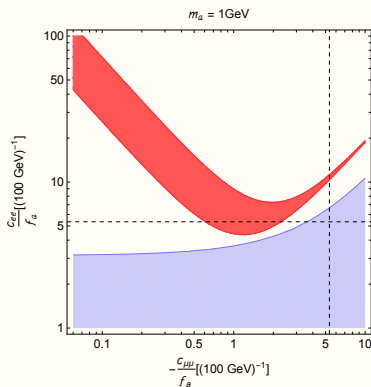
Collider Bounds of Axion EFT



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- $c_{\mu\mu} \neq 0, \quad c_{ee} \neq 0$



$$m_a \gtrsim 2 \text{ GeV}, \quad c_{ee}/c_{\mu\mu} < 0,$$

$$\left| \frac{f_a}{c_{\mu\mu}} \right| \lesssim 100 \text{ GeV}, \quad \left| \frac{f_a}{c_{ee}} \right| \lesssim 25 \text{ GeV} \quad \text{for } m_a = 5 \text{ GeV}.$$

Model completion and challenges

- μ is directly charged under PQ
- μ is neutral under PQ , but coupled to other fields charged under $U(1)_{PQ}$.

l is PQ charged – sketch

Outline of the ingredients:

- Scalars content: $H_1, H_2, \dots; \Phi$
- All break PQ
- $H_1, H_2 \dots$ break EW
- Φ is SM singlet

\Rightarrow For $v_1 \sim v_2 \sim \dots \sim v_\Phi$, $v_{EW}^2 = v_1^2 + v_2^2 + \dots$, $f_a \sim v_{EW}^2 + v_\Phi^2$. In case of invisible axion, one only needs to make $v_\Phi \gg v_{EW}$.

\Rightarrow For our purpose, $f_a \sim |c_{\gamma\gamma,ee}| 25 \text{ GeV}$, we need $f_a \ll v_{EW}$!

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\Rightarrow One solution is to have $v_1 \gg v_2, v_\Phi$, so the component eaten by Z is (mostly) H_1 , and the axion is (mostly) a linear combination of H_2 and Φ . This gives $v_{EW} \sim v_1$, $f_a \sim v_2, v_\Phi$.

ℓ is PQ charged – details

Field	$SU(2)_L$	$U(1)_Y$	$U(1)_{PQ}$
H_l	2	$-\frac{1}{2}$	+1
H_q	2	$+\frac{1}{2}$	+1
Φ	1	0	+1
U^c	1	$-\frac{2}{3}$	-1
D^c	1	$+\frac{1}{3}$	+1
E^c	1	+1	-1

$$V_{scalar} = V_0(|H_l|, |H_q|, |\Phi|, |H_l H_q|) + \lambda_{q\ell\Phi} H_l H_q \Phi^{\dagger 2}$$

$$V_{int} = y_u H_q Q U^c + y_d H_q^\dagger Q D^c + y_e H_l L E^c + \text{h.c.}$$

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H_q couples to quarks

H_l couples to leptons

Identify the pNGB's

- $(v_q^2\theta_q - v_l^2\theta_l)$ is eaten by Z
- $(\theta_l + \theta_q - 2\theta_\Phi)$ gets massive due to $\lambda_{ql\Phi} H_l H_q \Phi^{\dagger 2}$
- light axion mode a

$$a = \frac{1}{f_a} \left(v_\Phi^2 \theta_\Phi + 2 \frac{v_q^2 v_l^2}{v_{EW}^2} (\theta_l + \theta_q) \right)$$
$$\approx \frac{1}{\sqrt{v_\Phi^2 + 4v_l^2}} (v_\Phi^2 \theta_\Phi + 2v_l^2 \theta_l) + \mathcal{O} \left(\frac{v_{l,\Phi}}{v_{EW}} \right)$$

ℓ is PQ charged – axion couplings

$$\begin{aligned}\mathcal{L}_{af} &\supset (\partial_\mu \theta_l) E^{c\dagger} \bar{\sigma}^\mu E^c + (\partial_\mu \theta_q) U^{c\dagger} \bar{\sigma}^\mu U^c - (\partial_\mu \theta_q) D^{c\dagger} \bar{\sigma}^\mu D^c \\ &\mapsto \underbrace{\frac{2}{f_a} \frac{v_q^2}{v_{EW}^2}}_{-c_{\mu\mu}} (\partial_\mu a) E^{c\dagger} \bar{\sigma}^\mu E^c + \frac{2}{f_a} \frac{v_l^2}{v_{EW}^2} (\partial_\mu a) (U^{c\dagger} \bar{\sigma}^\mu U^c - D^{c\dagger} \bar{\sigma}^\mu D^c),\end{aligned}$$

$$\begin{aligned}\mathcal{L}_{a\gamma} &\supset 3(\theta_l + \theta_q) \frac{\alpha}{4\pi} F_{\mu\nu} \tilde{F}^{\mu\nu} \\ &\mapsto \underbrace{\frac{6}{f_a} \frac{v_q^2 + v_l^2}{v_{EW}^2}}_{c_{\gamma\gamma}} \frac{\alpha}{4\pi} a F_{\mu\nu} \tilde{F}^{\mu\nu} = \frac{6}{f_a} \frac{\alpha}{4\pi} a F_{\mu\nu} \tilde{F}^{\mu\nu}.\end{aligned}$$

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Observations:

- $v_q \sim v_{EW} > v_l \sim v_\Phi$, axions dominantly couples to ℓ and γ
- no $a - g$ coupling b/c U^c and D^c have opposite PQ
- $c_{\mu\mu}$ has the opposite sign as $c_{\gamma\gamma}$

l is PQ charged – challenges

2HDM + radial mode of Φ :

- $v_l, v_\Phi \ll v_{EW}$
- Close to alignment limit, s.t. couplings are SM-like
- Radial mode of Φ could mix with SM higgs, $h \rightarrow aa$
- Extra ingredients

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Lepton-specific 2HDM: 1106.0034, PRD41 3421, 1207.4835, 1305.2424,
Global EW fit on 2HDM: 1803.01853

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Recipe:

- ℓ doesn't couple to axion directly
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- mass-mixing $\ll \chi$ mass
- ℓ inherits "fractional" PQ charge

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- Axions are benign candidates for light modes
- From EFT point of view, axion could address $(g - 2)_\mu$
- Yet, completing the model leads to serious challenges:
 - it requires large axion coupling, $f/c_{ae}, f/c_{a\mu}, f/c_{a\gamma} \gtrsim (100 \text{ GeV})^{-1}$
 - light charged modes $\sim \mathcal{O}(10) - \mathcal{O}(100) \text{ GeV}$
 - light neutral modes that mixes with Higgs
 - direct contribute to $(g - 2)_\mu$ from extra ingredient