B-modes from Post-inflationary Gravitational Waves Sourced by Axionic Instabilities at Cosmic Reionization

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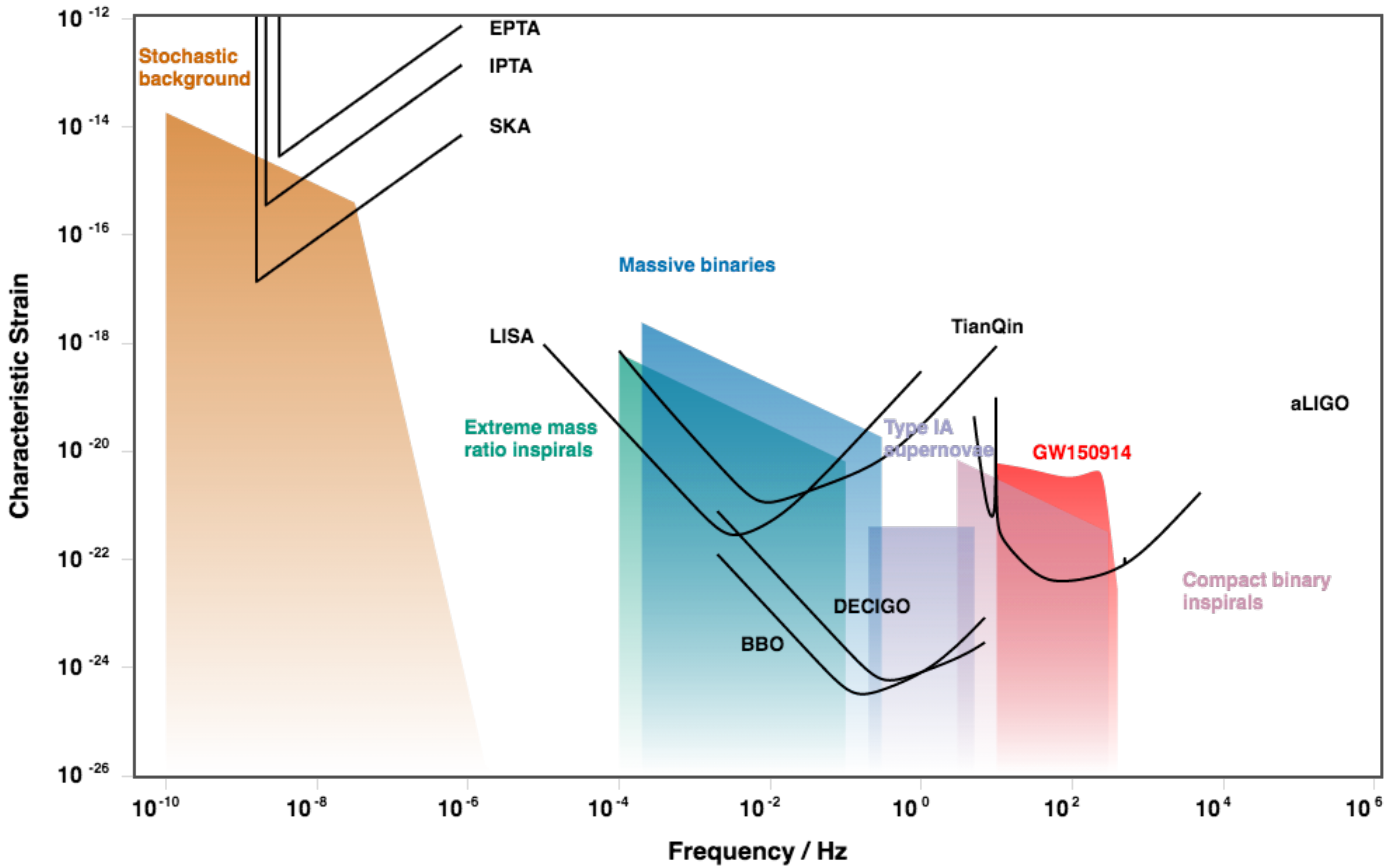
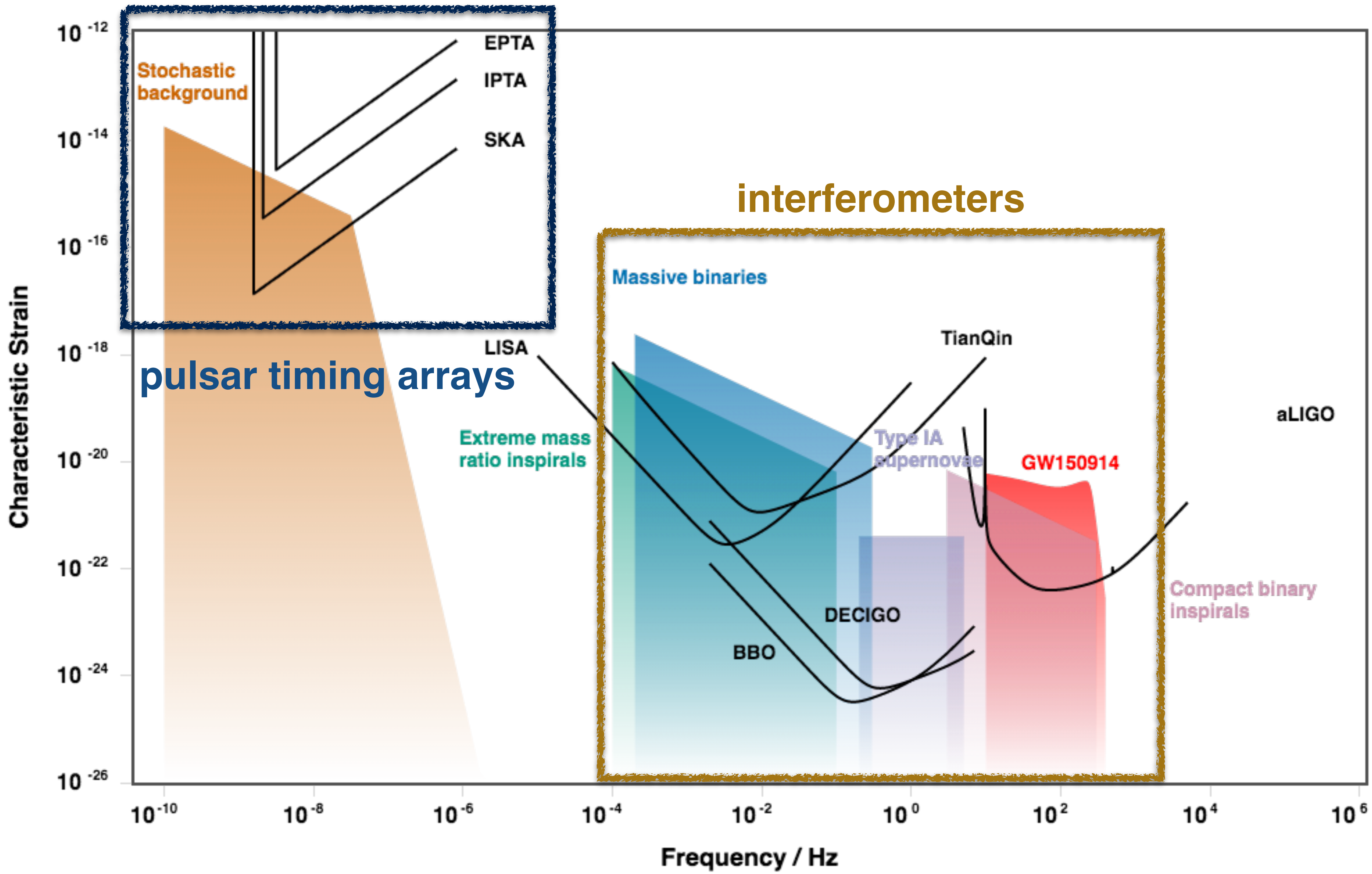
based on 2104.08284, w/ Michael Geller and Yuhsin Tsai

ALP meets GW



[from gwplotter.com]

ALP meets GW



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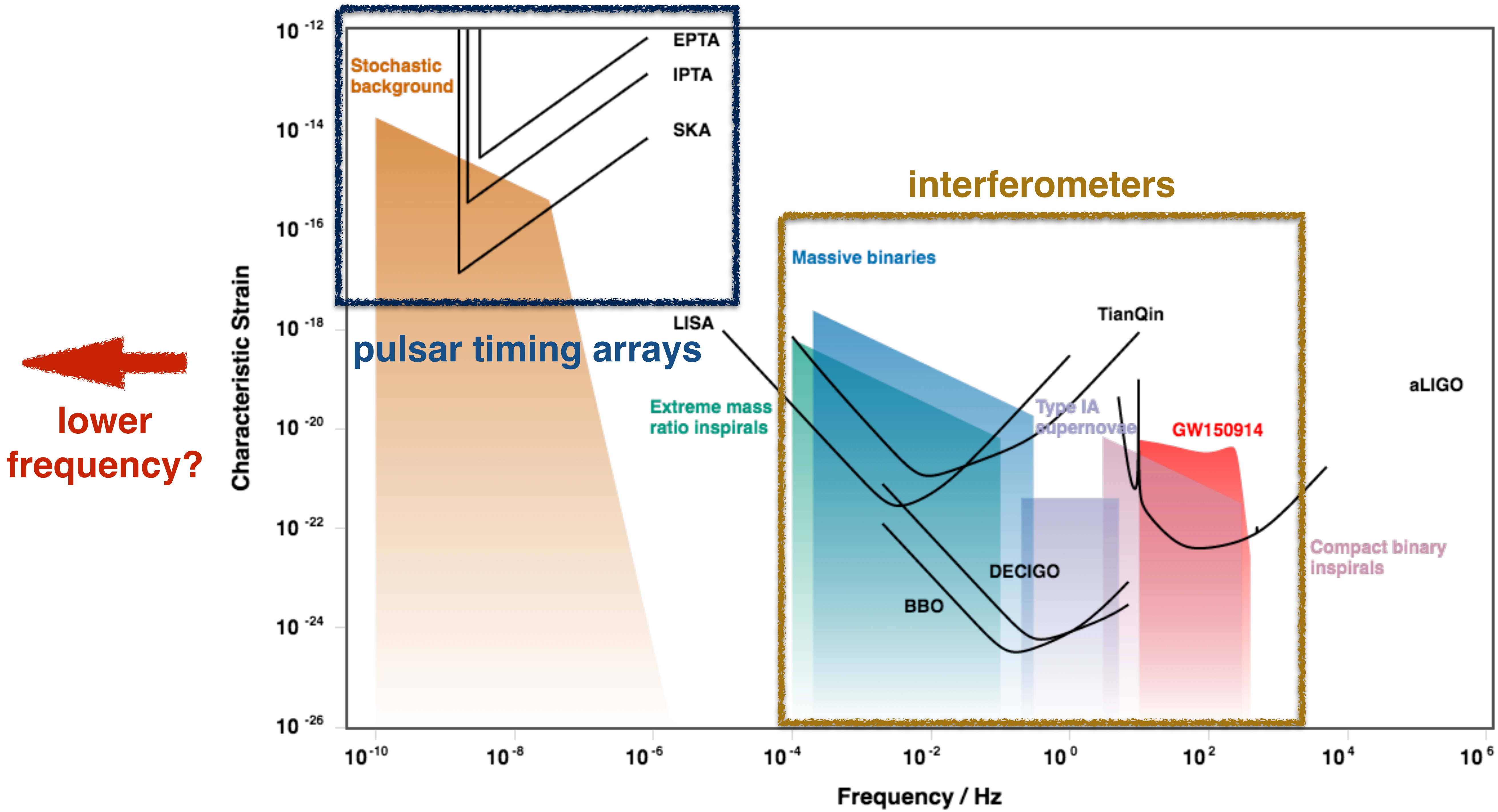
ALP meets GW

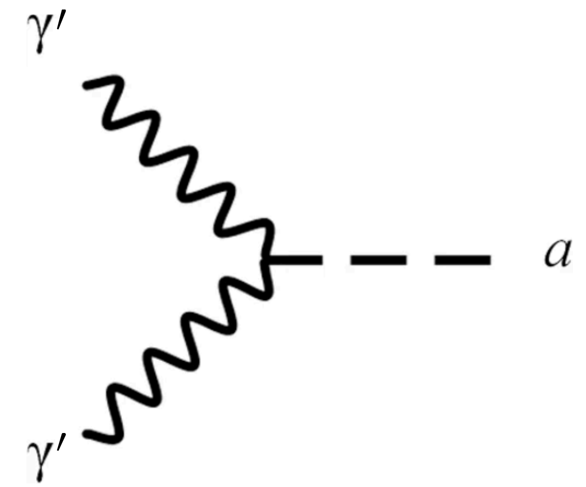


[from gwplotter.com]

ALP meets GW

- **Gravitational waves (GWs) with (way) lower frequency are relevant for CMB**
  - ➡ wavelength comparable to Hubble patch size

- **Axion-like particles (ALPs) can be the sources of the GW**
  - ➡ coupled to dark photons
  - ➡ tachyonic instability leads to exponential enhancement of the gauge modes



Tachyonic Instability

- **Consider a Lagrangian of ALPs and dark photons**

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\alpha}{4f}\phi F_{\mu\nu}\tilde{F}^{\mu\nu},$$

$$V(\phi) = \frac{1}{2}m^2\phi^2$$

- **The EOMs then write**

Negative for $0 < k < \alpha|\phi'|/f$

$$v''_\pm(k,\tau) + \omega^2_\pm(k,\tau)v_\pm(k,\tau) = 0,\ \omega^2_\pm(k,\tau) = k^2 \mp k\alpha\phi'/f$$

$$\phi'' + 2aH\phi' + a^2\frac{\partial V}{\partial\phi} = \frac{\alpha}{f}a^2\mathbf{E}\cdot\mathbf{B}$$

a mass term will easily spoil everything

Grow exponentially

EM back reaction

[for spacially inhomogeneous ALP see Schwaller et. al. 2012.11584]

Tachyonic Instability

- Consider a Lagrangian of ALPs and dark photons

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\alpha}{4f}\phi F_{\mu\nu}\tilde{F}^{\mu\nu},$$

$$V(\phi) = \frac{1}{2}m^2\phi^2$$

- The dark sector doesn't couple to or mix with the SM sector

- Also, we assume the dark photons are not produced from inflation but just live in the Bunch-Davis vacuum (i.e. plane wave)

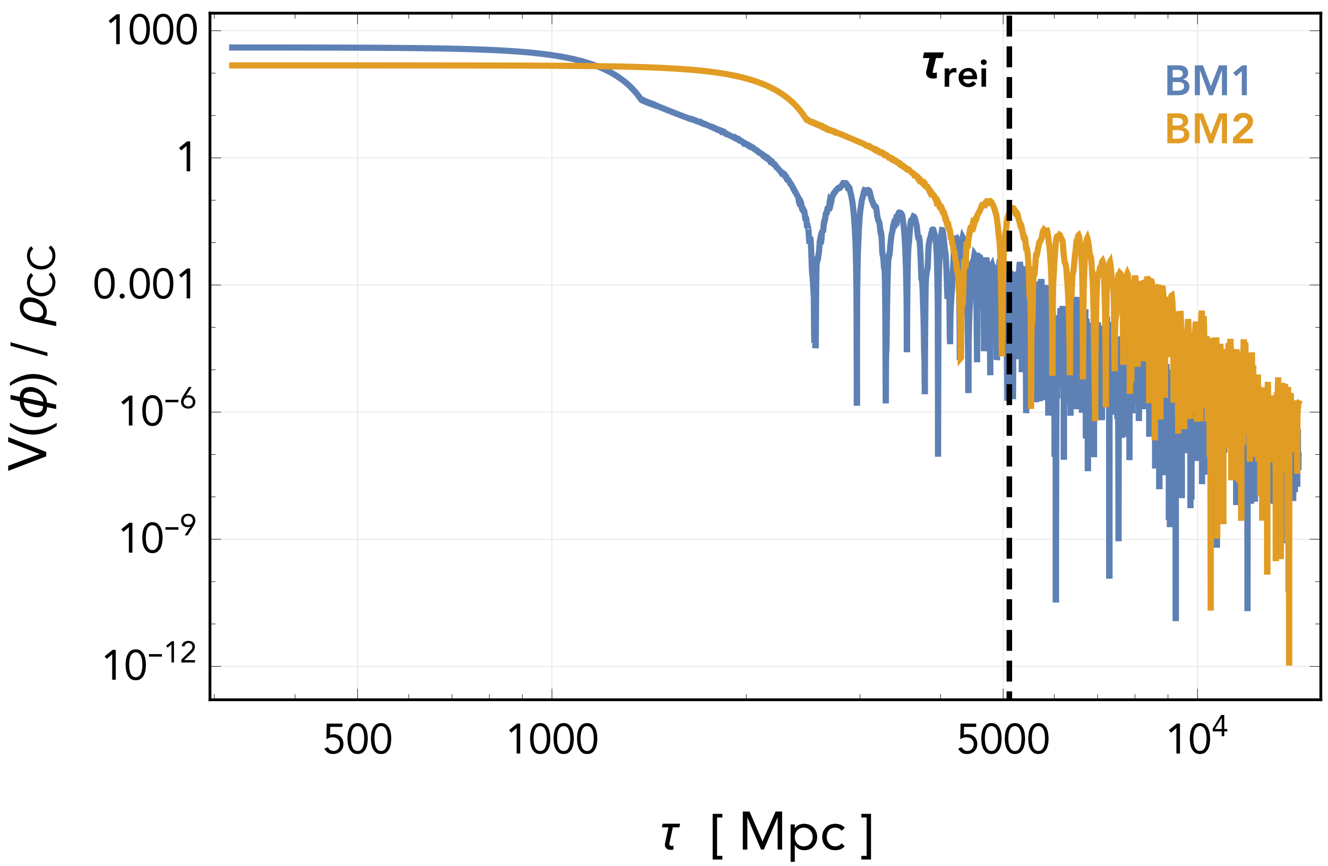
Tachyonic Instability

- **Setup**
  - 100 discretized gauge modes, logarithmically spaced in k
  - the k-range determined by trial-and-error
  - initial condition: plane wave for gauge modes and $\phi_i = f$
  - parametrize $m = \Lambda^2 / f$

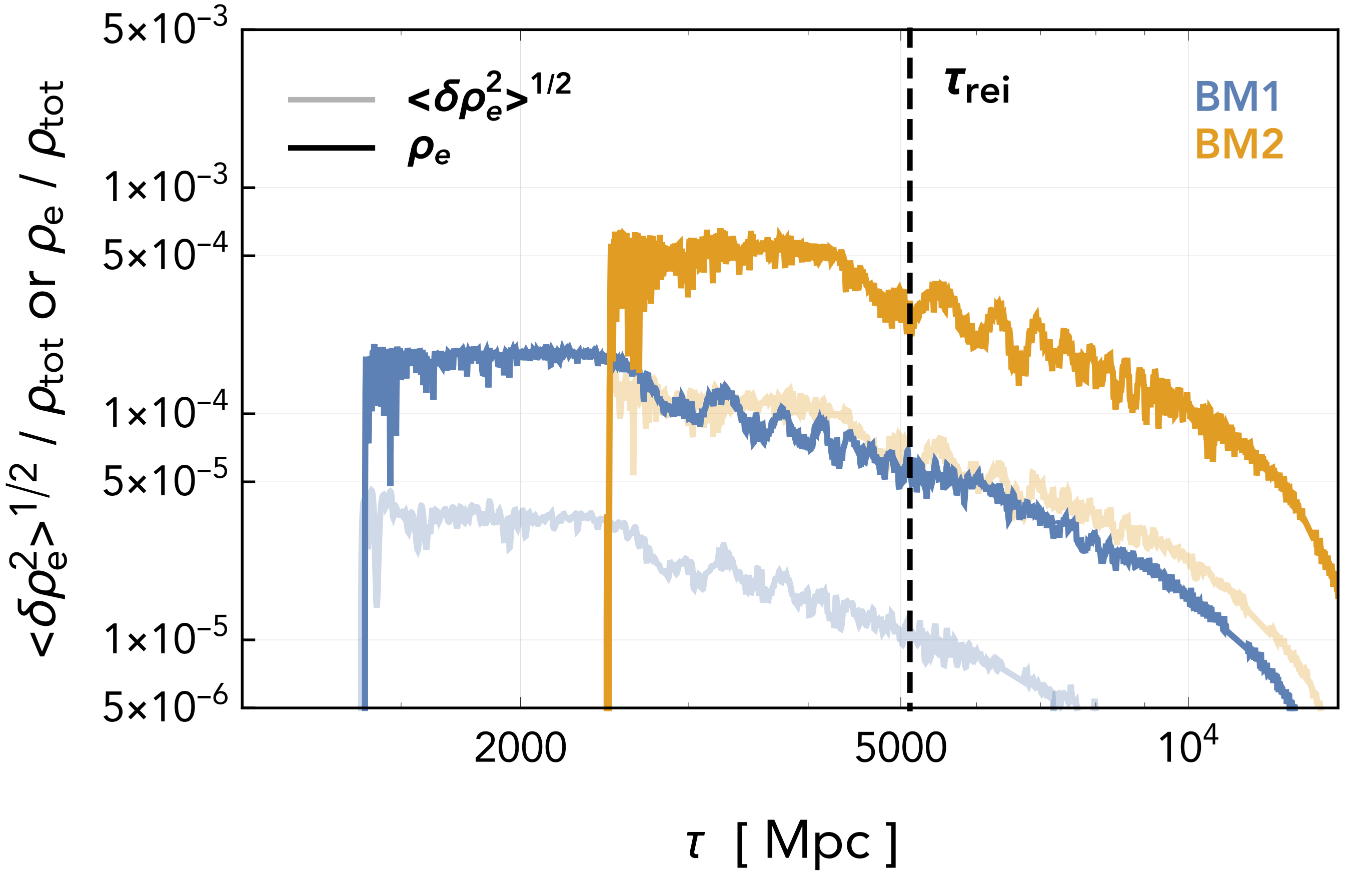
knob of the signal strength

	m (eV)	$k_{\rm max}$ (Mpc^{-1})	$\Lambda_{\rm bound}$	α
BM1	4×10^{-30}	0.94	15 meV	400
BM2	8.8×10^{-31}	0.78	9 meV	400

Tachyonic Instability



Axion rolling is irregular due to the back reaction term



Not much of the total energy budget but have large fluctuations

The CMB anisotropy spectrum

- **The dark sector also influences the CMB anisotropy spectrum (TT), through both the tensor and scalar perturbation**

$$\Theta = \frac{\delta T}{T}, \quad a_{lm} = \int d\mathbf{n} Y^*_{lm}(\mathbf{n})\Theta(\mathbf{n}), \quad C_l^{TT} = \frac{1}{2l+1}\sum_m |a_{lm}|^2$$

- **Scalar: mainly integrated Sachs-Wolfe (ISW)**

$$C_l^{TT} = 16\pi \int d\tau' d\tau'' \int \mathcal{D}k\mathcal{D}k' \, j_l[k'(\tau_0 - \tau')] j_l[k''(\tau_0 - \tau'')] \underline{\langle \Phi'(\tau', k')\Phi'(\tau'', k'')\rangle}$$

new contribution to the scalar spectrum

[Gorbunov & Rubakov, *Introduction to the theory of the early universe: Cosmological perturbations and inflationary theory*]

The CMB anisotropy spectrum

- **The scalar potential is obtained by solving the linear Einstein equations**

$$\delta'_m + \theta_m = 3\Phi' \,,$$

$$\theta'_m + \frac{a'}{a}\theta_m = -\Phi \,,$$

$$k^2\Phi + 3\frac{a'}{a}\Phi' + 3\left(\frac{a'}{a}\right)^2 \Phi = -4\pi G_N \, a^2(\delta\rho_e + \delta\rho_m)$$

- **Solve the above equations with Green's Function method**
  - ➡ the dark sector energy density perturbation as the source
  - ➡ boundary condition $G_\Phi(\tau,\tau) = a(\tau)/(3a'(\tau))$

The CMB anisotropy spectrum

- **Tensor perturbations**

$$C_l^{TT} = \frac{9\pi}{2}\frac{(l+2)!}{(l-2)!}\int \mathcal{D}k\mathcal{D}k' \cdot \langle \left\{ \int_{\tau_r}^{\tau_0} d\tau\, h'_{ij}(k,\tau)\frac{j_l[(\tau_0-\tau)k]}{(\tau_0-\tau)^2 k^2} \right\}^2 \rangle$$

- **Again, solve the tensor spectrum above w/ GF**

the stress tensor

$$\bar{h}''_{ij} + \left(k^2 - \frac{a''}{a}\right)\bar{h}_{ij} = \frac{2}{M_{Pl}^2} a\,\underline{\Pi_{ij}(\mathbf{k},\tau)}\,,$$

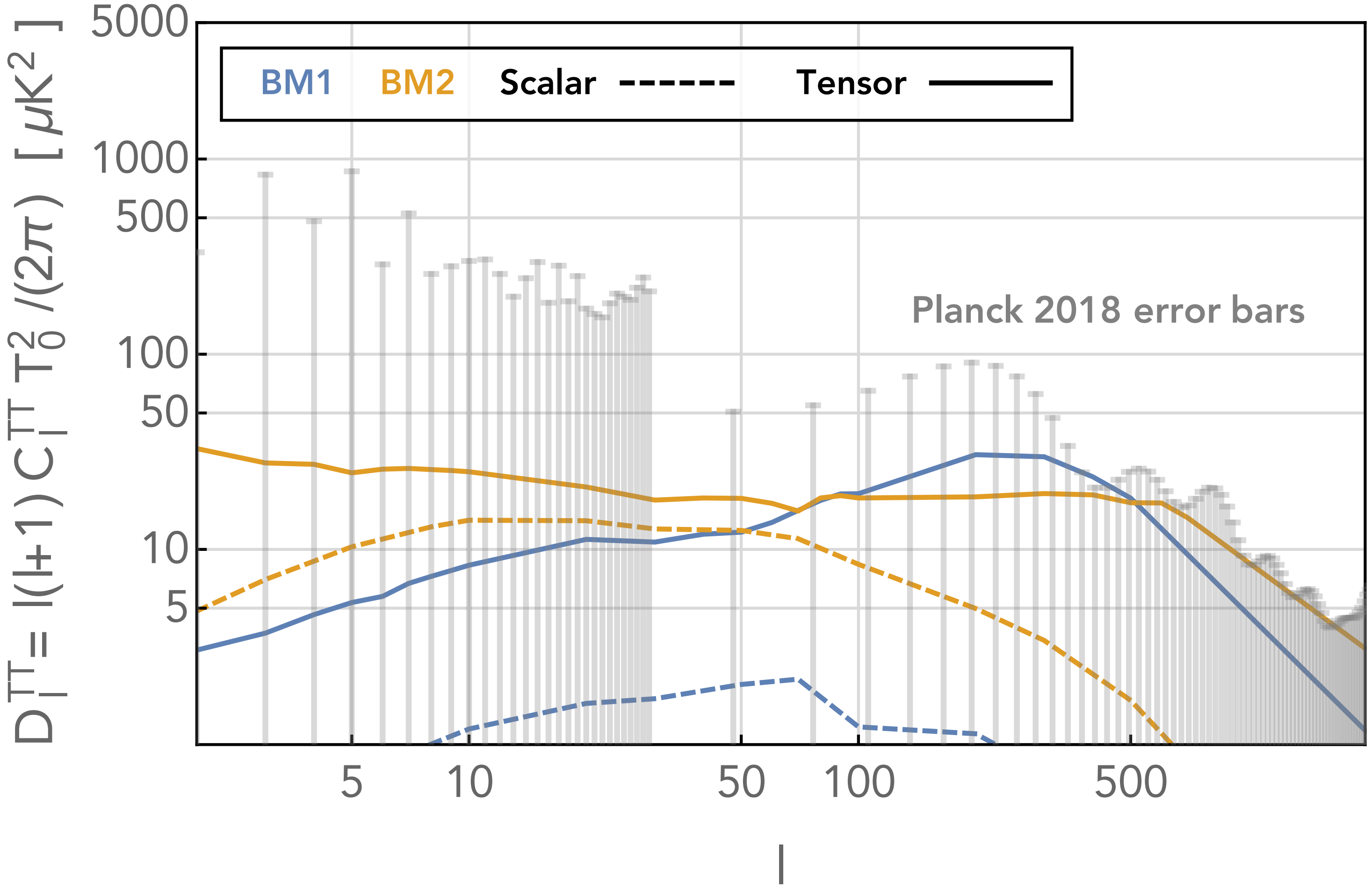
$$\Pi_{ij}(\mathbf{k},\tau) = \int \frac{d^3q}{(2\pi)^3}\Theta_{ij}^{\lambda_1\lambda_2}(\mathbf{q},\mathbf{k})\,\mathcal{S}_{\lambda_1\lambda_2}(\mathbf{q},\mathbf{k},\tau)$$

gauge modes

$$\mathcal{S}_{\lambda_1\lambda_2}(\mathbf{q},\mathbf{k},\tau) \equiv -\frac{1}{a^2}\left[\underbrace{\lambda_1\lambda_2\,|\mathbf{q}||\mathbf{k}-\mathbf{q}|\,v_{\lambda_1}(\mathbf{q})v_{\lambda_2}(\mathbf{k}-\mathbf{q})}_{\text{magnetic}} + \underbrace{v'_{\lambda_1}(\mathbf{q})v'_{\lambda_2}(\mathbf{k}-\mathbf{q})}_{\text{electric}}\right]$$

[for detailed derivation see e.g. Schwaller et. al. 1811.01950]

The CMB anisotropy spectrum



➡ Compared with the error bar of the binned Planck 2018 result

CMB B-modes

- **Of course also B-mode signals, as we have tensor perturbations**

$$\mathcal{P}_{ab} = -\{\nabla_a \nabla_b\}\mathcal{P}_E - \{\epsilon_a^c \nabla_b \nabla_c\}\mathcal{P}_B$$

$$\mathcal{P}_E(\mathbf{n}) = \sqrt{2}\sum_{lm}\sqrt{\frac{(l-2)!}{(l+2)!}}a_{lm}^E Y_{lm}(\mathbf{n}), \quad \mathcal{P}_B(\mathbf{n}) = \sqrt{2}\sum_{lm}\sqrt{\frac{(l-2)!}{(l+2)!}}a_{lm}^B Y_{lm}(\mathbf{n})$$

$$C_l^{XY} = \frac{1}{2l+1}\sum_m \langle a_{lm}^X a_{lm}^{Y*}\rangle$$

- **We focus on the contributions from around reionization (z~8)**

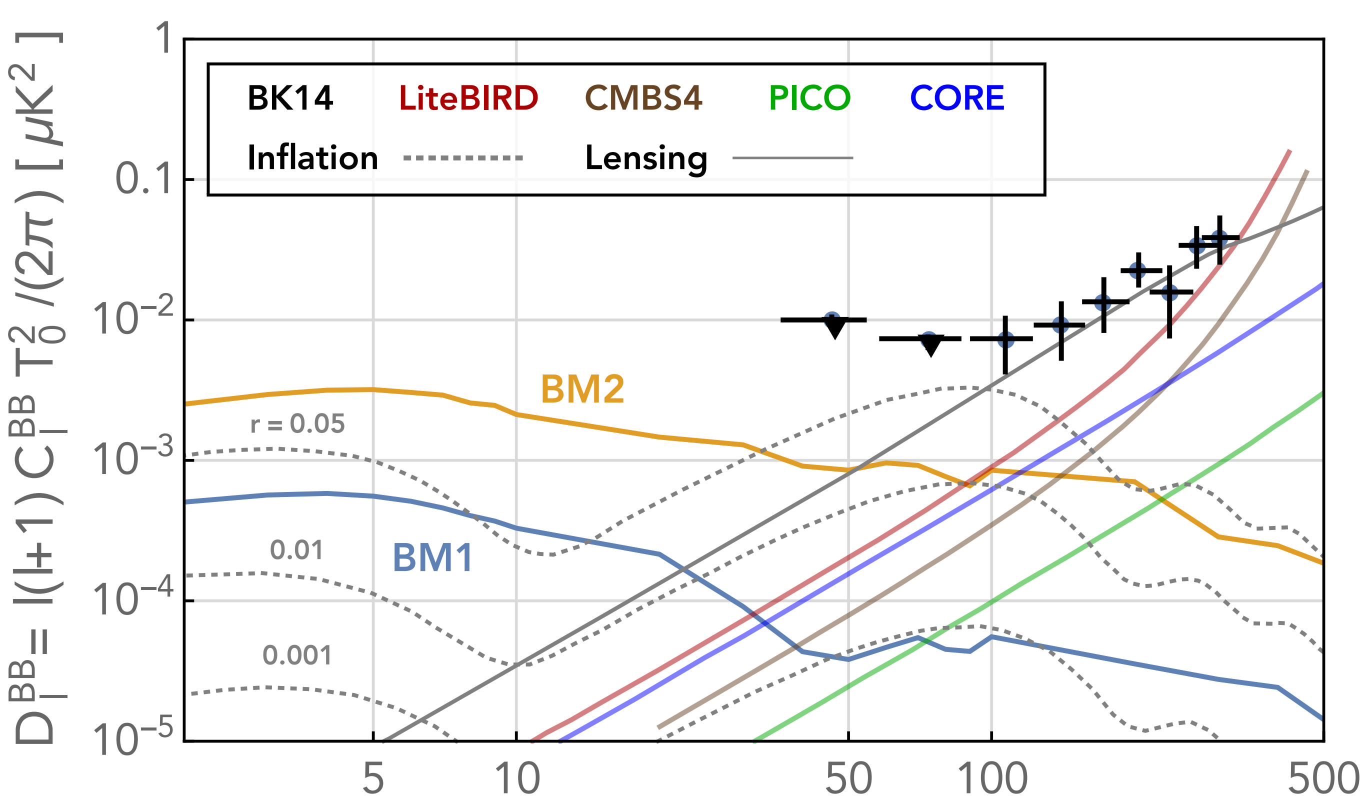
[for contribution around recombination, see Weiner et. al. 2008.01732]

CMB B-modes

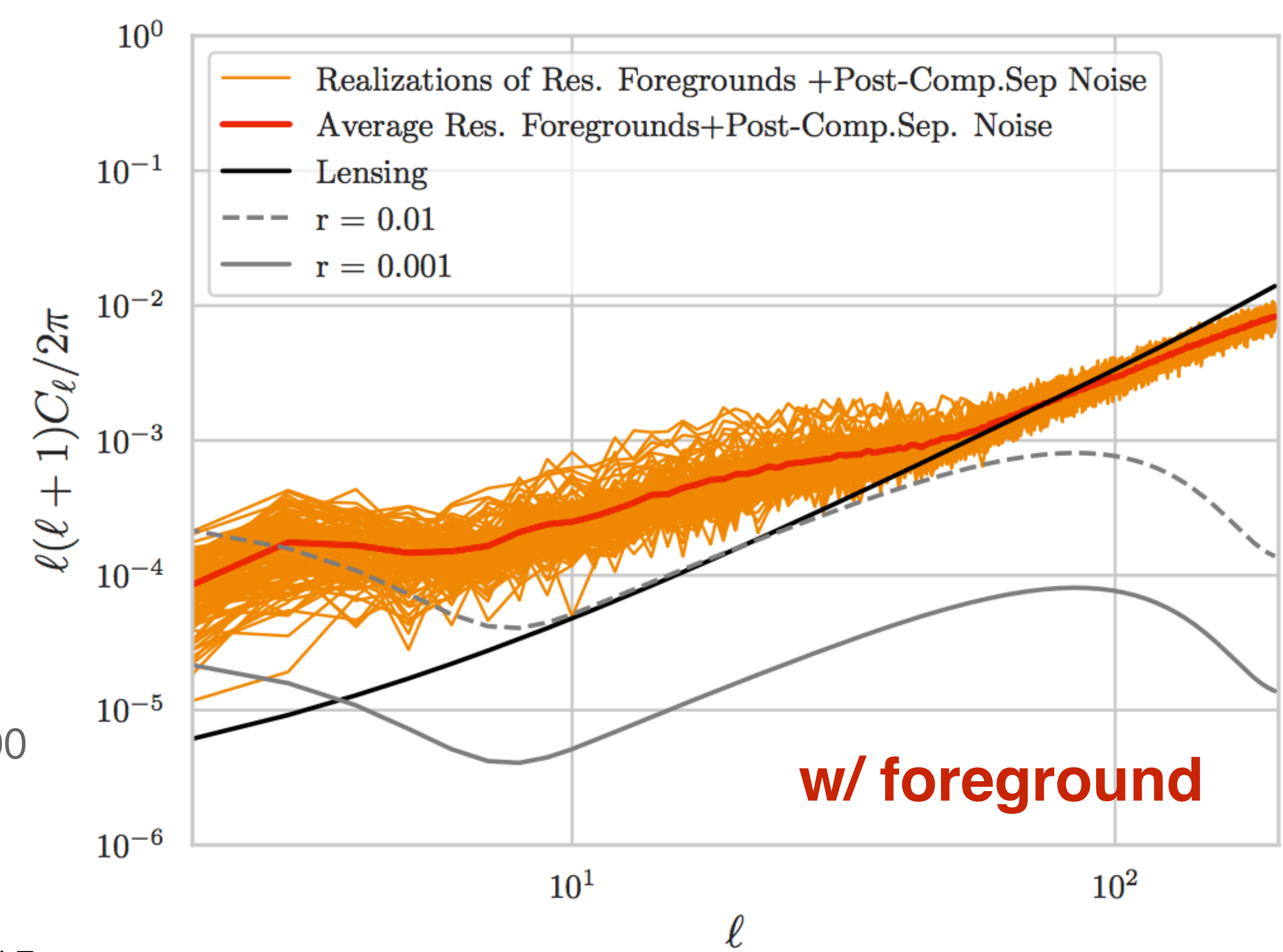
- **The B-mode can be calculated as**

$$C_l^{BB} = 36\pi\, \mathcal{T}_{\rm rei}^2 \int \mathcal{D}k\mathcal{D}k'\, \mathcal{J}_{l,B}^2(k) \cdot \langle \left\{ \int_{\tau_{\rm rec}}^{\tau_{\rm rei}} d\tau\, h'_{ij}(k,\tau) \frac{j_2[(\tau_{\rm rei}-\tau)\,k]}{(\tau_{\rm rei}-\tau)^2\,k^2} \right\}^2 \rangle$$

$$\mathcal{J}_{B,l}(k) = \frac{l+2}{2l+1} j_{l-1}(\kappa) - \frac{l-1}{2l+1} j_{l+1}(\kappa),\ \kappa = (\tau_0 - \tau_{\rm rei})k$$



compare with detector noise



[Campeti et. al. 2007.04241]

Conclusion

- **We have studied the CMB power spectra generated by ALPs via a tachyonic instability at around the cosmic reionization epoch**

- **The signal can leave a visible imprint on B-mode to the next generation experiments, and meanwhile stays compatible with the current CMB anisotropy measurements**

- **Roads ahead:**
  - non-Gaussianity?
  - more to exploit from chiral GW?

Back up

Quantizing the gauge modes

- **The dark photon field in Coulomb gauge**

$$X_i = \int \mathcal{D}k \left(\epsilon_{+i}(\mathbf{k}) v_+(\tau, k) \hat{\mathbf{a}}_+(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{x}} + h.c. \right) ,$$
$$X_0 = 0 ,$$

$$\mathbf{k} \cdot \epsilon_\pm = 0, \ \mathbf{k} \times \epsilon_\pm = \mp i k \epsilon_\pm, \ \epsilon_\pm \cdot \epsilon_\pm = 0, \ \epsilon_\pm \cdot \epsilon_\mp = 1$$

- **Using these we will have**

$$\langle \mathbf{E} \cdot \mathbf{B} \rangle = -\sum_{\lambda=\pm} \lambda \int \frac{k^2 dk}{2\pi^2 a^4} \, \mathrm{Re} \left[v_\lambda^*(k,\tau) v_\lambda'(k,\tau) \right]$$

- **Energy density fluctuations of the dark sector**

$$\delta\rho_e \approx \frac{1}{2} \delta \left[(\partial_0 X_i)^2 \right] + \frac{1}{4} \delta \left[X^{ij} X_{ij} \right] , \ \delta[\mathcal{O}] = \mathcal{O} - \langle \mathcal{O} \rangle$$

Amplitude of the spectra

- **Spectrum amplitudes are sensitive to the values of the parameters**
  - Parametrizing $m = \Lambda^2/f$
  - The dark photon energy density

$$\rho_X = \frac{1}{2a^4} \int \mathcal{D}k \left(|v'(k)|^2 + k^2 |v(k)|^2 - k \right)$$

  - As tachyonic production quickly transfer the axion energy into dark photon, we expect $|v| \propto \Lambda^2$
  - The power counting of the mode functions in these spectra suggests that when **fixing m**, the spectrum amplitudes scale as Λ^8

The E-modes?

- **There are also the E-modes, from both tensor and scalar. Similar to the calculations before**

  - tensor

$$\mathcal{J}_{B,l}(k) \to \mathcal{J}_{E,l}(k) = \frac{(l+2)(l+1)}{(2l+1)(2l-1)} j_{l-2}(\kappa) - \frac{6(l+2)(l-1)}{(2l+3)(2l-1)} j_l(\kappa) + \frac{l(l-1)}{(2l+3)(2l+1)} j_{l+2}(\kappa)$$

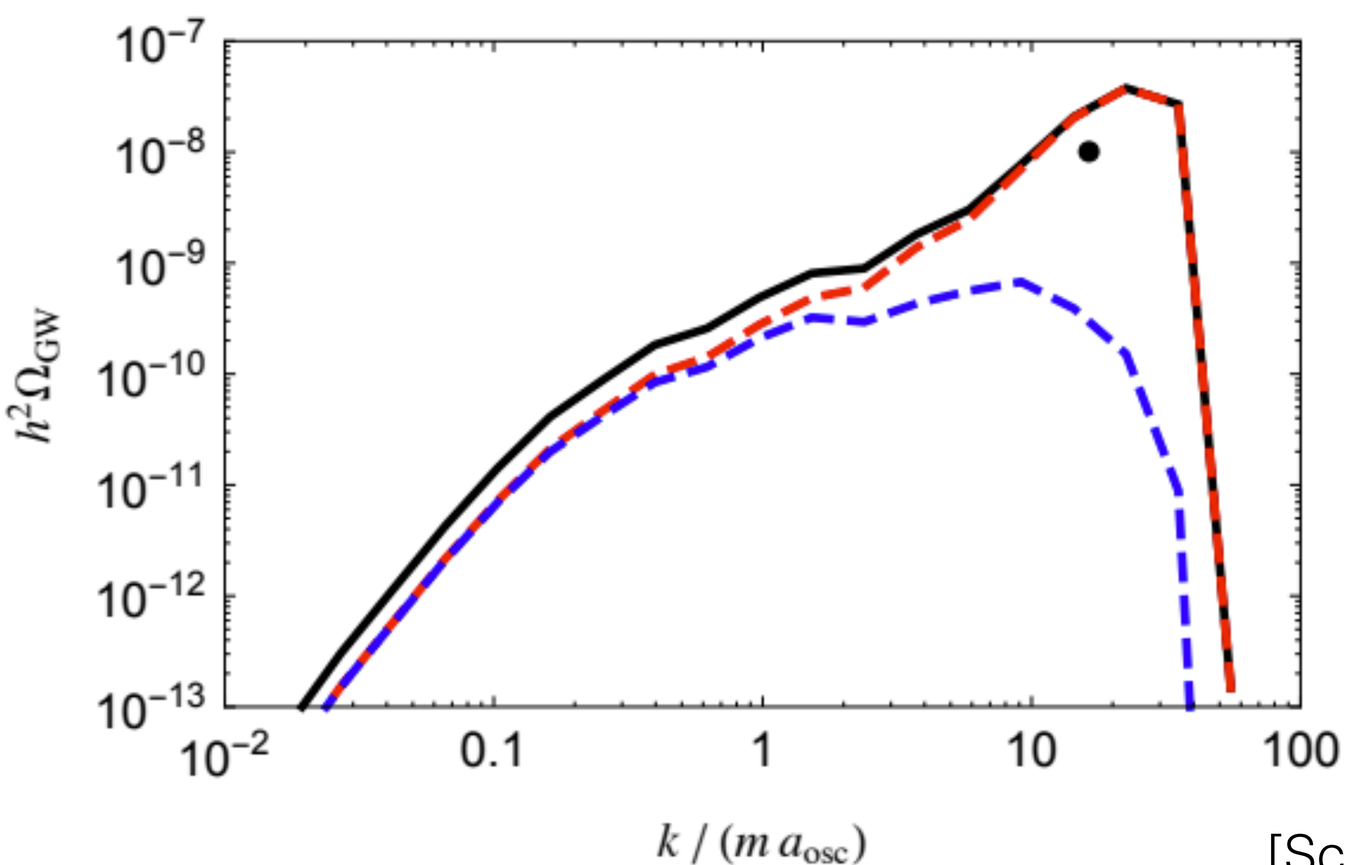
  - scalar

$$C_l^{EE} = \frac{9\pi}{2}\mathcal{T}_{\rm rei}^2 \frac{(l+2)!}{(l-2)!} \int \mathcal{D}k\mathcal{D}k' \langle \Phi(\tau_{\rm rei})\Phi(\tau_{\rm rei})\rangle \cdot j_2^2(k\tau_{\rm rei}) \cdot \frac{j_l^2[(\tau_0-\tau_{\rm rei})k]}{k^4(\tau_0-\tau_{\rm rei})^4}$$

- **Not as constrained as the TT spectrum**

Chiral GW

- E- and B-modes are at odd under parity so in the standard calculation we usually don’t expect <EB>
- A non-zero <EB> may indicate direct CPV for photons, but not necessary. Chiral GW can be another source



[Schwaller et. al. 1811.01950]