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High-quality axions in solutions to the μ problem

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Based on work with Steve Martin, `arXiv:hep-ph/2106.*****`

Supersymmetric extensions of DFSZ-type axion

Consider MSSM content + two gauge-singlet chiral superfields X and Y with

$$W_{\rm I} \supset \frac{\lambda_\mu}{M_P} X Y H_u H_d + \frac{\lambda}{6 M_P} X^3 Y.$$

The PQ charges of $(X, Y) = (-1, 3)$ such that the PQ charge of $H_u H_d$ is -2.

The λ term along with the soft terms stabilize the potential, and the total scalar potential has a local minimum for

$$\langle X \rangle \sim \langle Y \rangle \sim \sqrt{m_{\rm soft} M_P} \equiv M_{\rm int}.$$

where $m_{\rm soft} \sim \rm TeV$ scale, and $M_{\rm int}$ could therefore be in the range

$$10^9 \,{\rm GeV} \lesssim M_{\rm int} \lesssim 10^{12} \,{\rm GeV}.$$

Thus giving rise to an invisible QCD axion of the DFSZ type, and the low-energy theory contains

$$\mu = \frac{\lambda_\mu}{M_P} \langle XY \rangle \sim m_{\rm soft},$$

solving the μ problem and the strong CP problem! (Kim-Nilles mechanism)

Base models[†]

Three other possibilities.

Base model	Superpotential terms	PQ charges of (X, Y)
B_{I}	$XYH_uH_d + X^3Y$	$(-1, 3)$
B_{II}	$X^2H_uH_d + X^3Y$	$(1, -3)$
B_{III}	$Y^2H_uH_d + X^3Y$	$(-1/3, 1)$
B_{IV}	$X^2H_uH_d + X^2Y^2$	$(1, -1)$

And, the PQ charges of the MSSM superfields in all four base models (and extensions)[‡]

	H_u	H_d	q	ℓ	$\overline{u}$	$\overline{d}$	$\overline{e}$
PQ	$-2c_\beta^2$	$-2s_\beta^2$	Q_q	Q_ℓ	$2c_\beta^2 - Q_q$	$2s_\beta^2 - Q_q$	$2s_\beta^2 - Q_\ell$

(Can extend with the seesaw mechanism.)

$g_{A\gamma}$ suppressed in all four base models!

[†] B_{I} proposed in H. Murayama, H. Suzuki, T. Yanagida Phys. Lett. B **291**, 418-425 (1992); B_{II} in K. Choi, E. J. Chun, J. E. Kim hep-ph/9608222; B_{III}, B_{IV} in S. P. Martin hep-ph/0005116;

[‡] After imposing the orthogonality condition on Slide 20, in terms of two free parameters Q_q, Q_ℓ, and the ratio of the Higgs VEVs $\tan\beta = s_\beta/c_\beta$.

$$U(1)_{\mathrm{PQ}} \xrightarrow{\text{PQ breaking}} Z_{N_{\mathrm{DW}}} \text{ discrete symmetry}$$

N_{DW}: Domain Wall (DW) number that corresponds to the number of discrete set of inequivalent degenerate minima of the axion potential.

(Calculated in terms of $[SU(3)_c]^2 - U(1)_{\mathrm{PQ}}$ anomaly N.)

Problem: Formation of topological defects such as stable DWs, due to the different possible phases of the axion, which dominate the universe[†]

Solutions:

- If PQ breaking happens before inflation
- $N_{\mathrm{DW}} = 1$ (our focus)

$N_{\mathrm{DW}} \neq 1$ in all four base models.

[†] See e.g. P. Sikivie Phys. Rev. Lett. **48**, 1156-1159 (1982)

Base model extensions

Consistent with gauge coupling unification, we consider the following extensions:

- $5 + \overline{5}$ at TeV or $M_{\rm int}$
- $10 + \overline{10}$ at TeV
- $10 + \overline{10}$ at $M_{\rm int}$
- $(5 + \overline{5})$ or $(10 + \overline{10})$ at TeV, $(5 + \overline{5})$ or $(10 + \overline{10})$ at $M_{\rm int}$

(last two items:) $\Big\}\ N_{\rm DW} = 1$ possible

Here,

$$\overline{5} = \underbrace{(\overline{3}, 1, 1/3)}_{\overline{D}} + \underbrace{(1, 2, -1/2)}_{L}$$

$$10 = \underbrace{(3, 2, 1/6)}_{Q} + \underbrace{(\overline{3}, 1, -2/3)}_{\overline{U}} + \underbrace{(1, 1, 1)}_{\overline{E}}$$

We allow for different components of the $5 + \overline{5}$ and/or $10 + \overline{10}$ to have different mass source terms.

> Also, extensions with $N_{\rm DW} = 1$ give rise to enhanced low-energy axion couplings!

Vectorlike superpotential mass terms

Assuming the same mechanism that gives a μ term also gives masses to vectorlike pairs of chiral superfields $\Phi + \overline{\Phi}$.

TeV scale masses:

$$W_{\text{mass}} = \begin{cases} \frac{\lambda_\Phi}{M_P} XY\Phi\overline{\Phi}, \\ \frac{\lambda_\Phi}{2M_P} X^2\Phi\overline{\Phi}, \\ \frac{\lambda_\Phi}{2M_P} Y^2\Phi\overline{\Phi}, \end{cases}$$

Intermediate scale masses:

$$W_{\text{mass}} = \begin{cases} \lambda_\Phi X\Phi\overline{\Phi}, \\ \lambda_\Phi Y\Phi\overline{\Phi}. \end{cases}$$

Mass terms fix the PQ charge of the terms $\Phi\overline{\Phi}$ which in turn fix the low-energy axion couplings, independent of the Yukawa terms.

Interlude: The axion quality problem

Higher dimensional operators that explicitly violate global $U(1)_{\mathrm{PQ}}$ are expected from quantum gravity. Such operators can displace the QCD θ parameter away from 0 at the minimum of the scalar potential, reintroducing the strong CP problem.

In our case, consider

$$W = \frac{\kappa}{M_P^{p-3}} X^j Y^{p-j}$$

that contributes to the axion potential (with soft terms), giving rise to:

$$|\theta_{\mathrm{eff}}| \quad = \quad \frac{\delta f_A^{p+2}}{\Lambda_{\mathrm{QCD}}^4 M_P^{p-2}},$$

with a dimensionless quantity δ, and f_A identified with M_{int}.

With $\Lambda_{\mathrm{QCD}} \sim 0.2$ GeV, $\delta \sim 1$, and $|\theta_{\mathrm{eff}}| \lesssim 10^{-10}$, we need $f_A \lesssim (4 \times 10^9, 10^{12})$ GeV if superpotential terms with $p = (8, 12)$ are allowed.

Therefore, need to forbid $X^j Y^{p-j}$ with $p < 7$.†

†odd p can be bumped up to next integer by imposing an additional Z_2 symmetry under which X and Y are odd.

Non-R and R discrete Z_n symmetries and anomaly constraints

	gauginos	W	chiral superfield Φ	fermion in Φ
Z_n^R charge (mod n)	r	$2r$	z_Φ	$z_\Phi - r$

For non-R symmetry $r = 0$, and for R-symmetry $0 < r < n/2$.

In both cases, $z_\Phi = 0, 1, \ldots, n-1$.

With a normalization where $Z_n^R \times G_{\mathrm{SM}} \times G_{\mathrm{SM}}$ anomalies are integers, we impose the following anomaly free conditions:†

$$A_2 = A_3 = \rho_{\mathrm{GS}} \pmod{n},$$

for the weaker condition, with the additional stronger condition

$$A_1 = 5A_3 = 5\rho_{\mathrm{GS}} \pmod{n},$$

which does not require the Green-Schwarz (GS) mechanism if $\rho_{\mathrm{GS}} = 0$.

† Treating Z_n^R as a subgroup of a spontaneously broken anomaly-free continuous $U(1)$ symmetry, see e.g. L. E. Ibanez arXiv:hep-ph/9210211

Examples with non-R Z_n symmetries: Base models

Stronger constraints with $\rho_{\rm GS} \neq 0$: (Here, $m = 0, 1, 2$)

Model	Z_n	X	H_u	p	$\rho_{\rm GS}$
$\mathrm{B_{III}}$	36	1	$8 + 12m$	12	18
$\mathrm{B_{IV}}$	36	3	$4 + 12m$	8	18

Lots of other cases that satisy the weaker constraint with $\rho_{\rm GS} \neq 0$.

Examples with Z_n^R symmetries: Base models

Stronger constraints with $\rho_{\rm GS} = 0$: Some examples,

Model	Z_n^R	r	X	H_u	p
$\rm B_{III}$	54	3	5	$1 + 18m$	10
$\rm B_{IV}$	12	1	8	$1 + 4m$	7

Stronger constraints with $\rho_{\rm GS} \neq 0$: As a special case, we found a Z_{24}^R symmetry with $SU(5)$ invariance†

Model	Z_n^R	r	X	H_u	p	$\rho_{\rm GS}$
$\rm B_{II}$	24	1	11	1	10	18
$\rm B_{III}$	24	1	5	1	10	18

We, however, do not impose $SU(5)$ invariance.‡

† Proposed and studied for the MSSM in H. M. Lee et al. 1102.3595, and was found in K. J. Bae, H. Baer, V. Barger, D. Sengupta 1902.10748 and H. Baer, V. Barger, D. Sengupta 1810.03713 to extend to base models $\rm B_{II}$ and $\rm B_{III}$ with suppression $p = 10$, and to base models $\rm B_{I}$ and $\rm B_{IV}$ only with suppression $p = 7$.

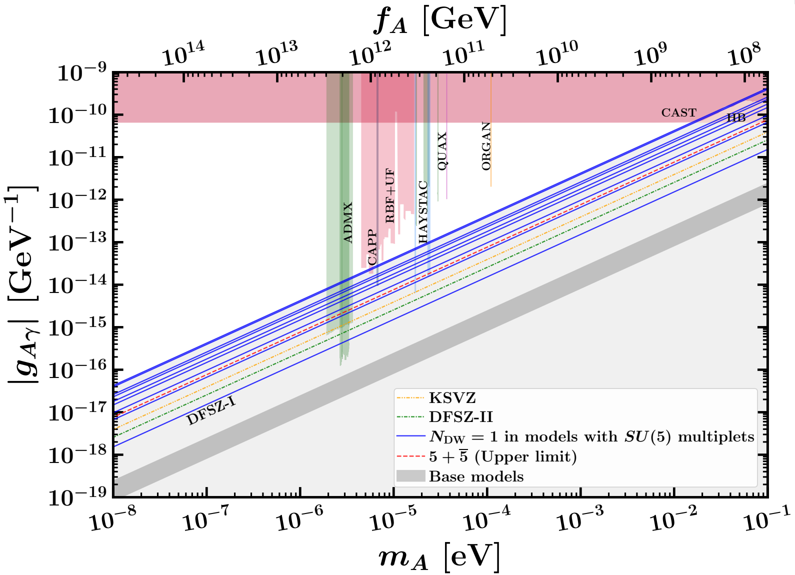
‡ We note that all of the symmetries we find can be made consistent with a partial unification with a Pati-Salam $SU(4)_{\rm PS} \times SU(2)_L \times U(1)_R$.

Stronger constraints: Examples,

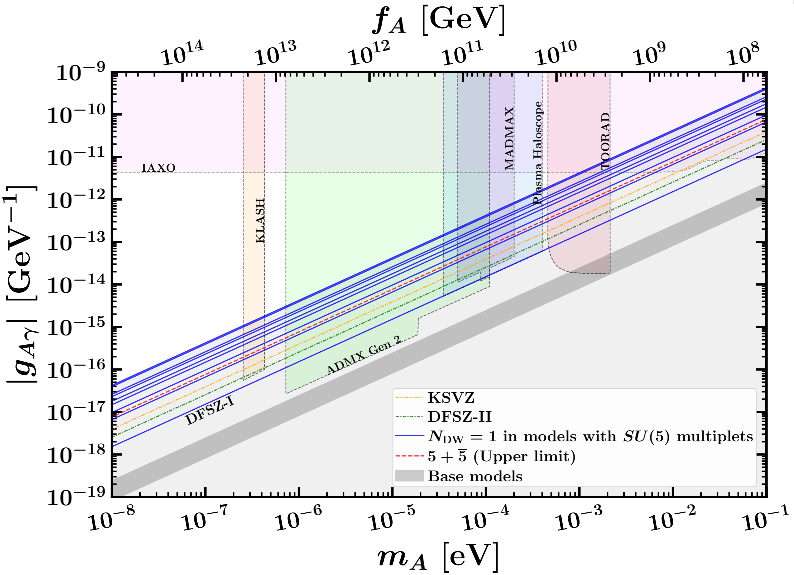
Base	Extension	Z_n^R	r	X	H_u	p	ρ_{GS}
$\mathrm{B_I}$	$XYD\overline{D} + X^2L\overline{L}$	34	1	31	15	12	16
$\mathrm{B_{II}}$	$Y^2D\overline{D} + Y^2L\overline{L}$	108	6	11	$22 + 36m$	20	0
$\mathrm{B_{III}}$	$X^2Q\overline{Q} + X^2U\overline{U} + Y^2E\overline{E}$	42	0	1	$8 + 14m$	14	18
$\mathrm{B_{IV}}$	$XD\overline{D} + YL\overline{L}$	20	0	1	8	12	5

Can find an anomaly-free ordinary or R Z_n symmetry for each model with/without the GS mechanism, thus giving rise to a quality axion!

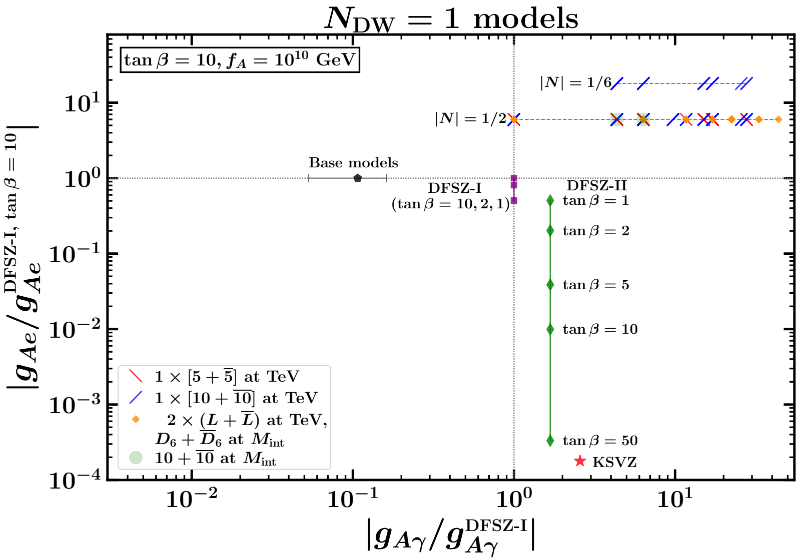
Axion-photon coupling (limits)

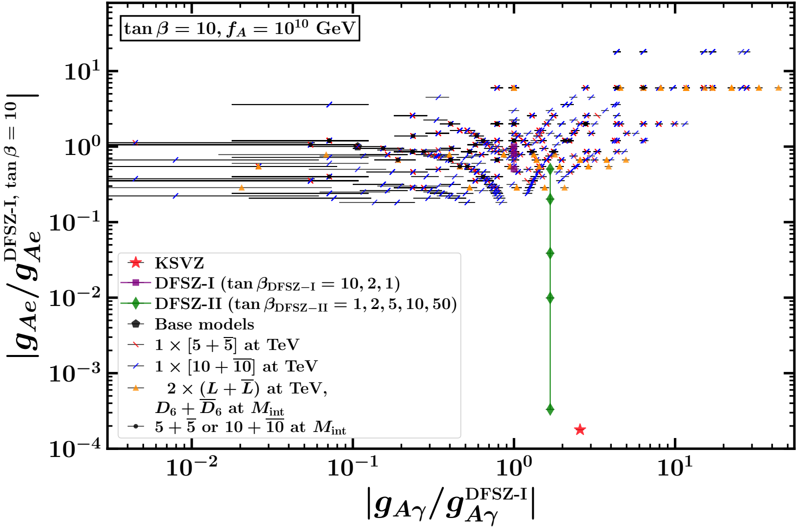


Axion-photon coupling (projections)



$\left|g_{Ae}/g_{Ae}^{\text{DFSZ-I},\tan\beta=10}\right|$ vs. $\left|g_{A\gamma}/g_{A\gamma}^{\text{DFSZ-I}}\right|$





$\tan\beta = 10, f_A = 10^{10}$ GeV
$|g_{Ae}/g_{Ae}^{\text{DFSZ-I}, \tan\beta=10}|$
$|g_{A\gamma}/g_{A\gamma}^{\text{DFSZ-I}}|$
KSVZ
DFSZ-I ($\tan\beta_{\text{DFSZ-I}} = 10, 2, 1$)
DFSZ-II ($\tan\beta_{\text{DFSZ-II}} = 1, 2, 5, 10, 50$)
Base models
$1 \times [5 + \overline{5}]$ at TeV
$1 \times [10 + \overline{10}]$ at TeV
$2 \times (L + \overline{L})$ at TeV, $D_6 + \overline{D}_6$ at M_{int}
$5 + \overline{5}$ or $10 + \overline{10}$ at M_{int}

Conclusion

While SUSY itself addresses the EW hierarchy problem, we considered extensions that

- have high-quality QCD axions within the reach of future axion searches
- simultaneously solve the μ problem
- evade cosmological domain wall problem
- can provide neutrino masses
- maintain gauge coupling unification
- have no dangerous cosmological relics

BACKUP SLIDES

Lightning review: The strong CP problem

Non-trivial QCD vacuum structure requires the term:

$$\mathcal{L}_{\rm QCD} \supset \theta \frac{g_s^2}{32\pi^2} G^{a\mu\nu} \tilde{G}^a_{\mu\nu},$$

where the QCD vacuum angle θ is expected to be $\mathcal{O}(1)$.

> "Everything not forbidden is compulsory."

However, experimentally:

$$|\theta| \lesssim 10^{-10}.$$

Why so small? – strong CP problem

Peccei-Quinn (PQ) solution: promote θ to a dynamical field

Lightning review: Peccei-Quinn (PQ) solution

Consider a global $U(1)_{\mathrm{PQ}}$ axial symmetry:

$$\partial_\mu j^\mu_{\mathrm{PQ}} = \underbrace{\frac{g_s^2 N}{16\pi^2} G^{a\mu\nu}\tilde{G}^a_{\mu\nu}}_{\text{QCD anomaly}} + \underbrace{\frac{e^2 E}{16\pi^2} F^{\mu\nu}\tilde{F}_{\mu\nu}}_{\text{EM anomaly}},$$

with left-handed fermions with PQ charge Q_f, $SU(3)_c$ index $T(R_f)$, and EM charge q_f contributing to:

$$\begin{aligned} N &= \mathrm{Tr}\left[Q_f T(R_f)\right], \\ E &= \mathrm{Tr}\left[Q_f q_f^2\right]. \end{aligned}$$

$U(1)_{\mathrm{PQ}}$ can be spontaneously broken by scalars with PQ charge Q_s

$$\varphi_s \supset \frac{v_s}{\sqrt{2}} e^{i a_s / v_s}.$$

With $V^2 = \sum_s Q_s^2 v_s^2$, the axion field is given by:

$$A = \frac{1}{V}\sum_s Q_s v_s a_s.$$

Ensuring the axion is massless at tree-level by imposing:

$$\sum_s Y_s Q_s v_s^2 = 0,$$

where Y_s: weak hypercharge of φ_s.

QCD vaccum term now becomes:

$$\mathcal{L}_{\rm QCD} \supset \left(\theta + \frac{A}{f_A}\right) \frac{g_s^2}{32\pi^2} G^{a\mu\nu} \tilde{G}^a_{\mu\nu},$$

with the axion decay constant

$$f_A \equiv \frac{V}{2N}.$$

Under $U(1)_{\rm PQ}$ transformations:

$$A \to A + ({\rm constant})\ f_A,$$

Thus solving the strong CP problem.

Lightning review: Low-energy axion couplings†

$$\mathcal{L}^A_{\rm int} \supset \frac{1}{4} g_{A\gamma} A F^{\mu\nu} \tilde{F}_{\mu\nu} - \sum_{f=e,n,p} i g_{Af} A \overline{\Psi}_f \gamma_5 \Psi_f$$

where,

$$g_{A\gamma} = \frac{\alpha_e}{2\pi f_A}\left(c_\gamma - 1.92(4)\right),$$

$$g_{Ae} = \frac{m_e}{f_A}\left[c_e + \frac{3\alpha_e^2}{4\pi^2}\left(c_\gamma \log\frac{f_A}{m_e} - 1.92(4)\log\frac{\rm GeV}{m_e}\right)\right],$$

$$\text{with} \qquad c_\gamma = \frac{E}{N}, \quad c_e = \frac{Q_e + Q_{\overline{e}}}{2N}.$$

Benchmarks:‡

$$\begin{aligned} \text{KSVZ} &: \quad c_\gamma = 0, \quad c_e = 0, \\ \text{DFSZ-I} &: \quad c_\gamma = 8/3, \quad c_e = s_\beta^2/3, \\ \text{DFSZ-II} &: \quad c_\gamma = 2/3, \quad c_e = -c_\beta^2/3, \end{aligned}$$

where $\tan\beta = s_\beta/c_\beta$ is the ratio of Higgs VEVs in the DFSZ models.

† Axion can accidentally decouple from photons if $E/N \approx 1.92$.

‡ J. E. Kim Phys. Rev. Lett. **43**, 103 (1979); M. A. Shifman, A. I. Vainshtein, V. I. Zakharov Nucl. Phys. B **166**, 493-506 (1980); M. Dine, W. Fischler, M. Srednicki Phys. Lett. B **104**, 199-202 (1981); A. R. Zhitnitsky Sov. J. Nucl. Phys. **31**, 260 (1980)

Quixotic extension

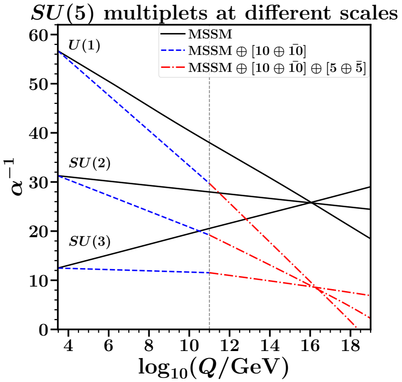
Consistent with gauge coupling unification, we can also consider

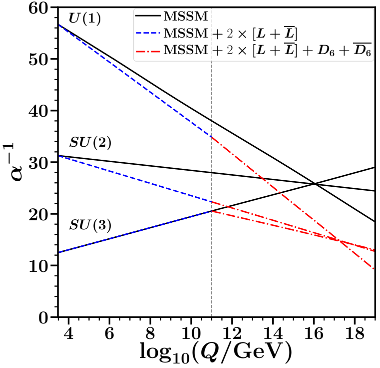
D_6LL models: $2 \times (L + \overline{L})$ at TeV, $D_6 + \overline{D}_6$ at $M_{\rm int} \sim 10^{11}$ GeV ($N_{\rm DW} = 1$ possible)

where

$$D_6 + \overline{D}_6 \quad = \quad (6, 1, 1/3) + (\overline{6}, 1, -1/3) \quad \text{is an exotic quix pair}$$

Gauge coupling unification





$N_{\rm DW} = 1$ in base model extensions

$$N_{\rm DW} \equiv \text{minimum integer} \left(2N \sum_s \frac{n_s Q_s v_s^2}{V^2} \right),$$

where $n_s \in \mathbb{Z}$.[†] Using the above formula:

$$N_{\rm DW} = \begin{cases} \text{minimum integer } |2Nn_x| \text{ in } \mathrm{B_I}, \mathrm{B_{II}}, \mathrm{B_{IV}}, \text{ and extensions,} \\ \text{minimum integer } |6Nn_x| \text{ in } \mathrm{B_{III}} \text{ and extensions.} \end{cases}$$

Clearly, $N_{\rm DW} \neq 1$ in all four base models.

In the base model extensions,

$$\text{For } N_{\rm DW} = 1: \quad N = \begin{cases} \pm\frac{1}{2} \text{ in model extensions of } \mathrm{B_I}, \mathrm{B_{II}}, \text{ and } \mathrm{B_{IV}}, \\ \pm\frac{1}{6} \text{ in model extensions of } \mathrm{B_{III}}. \end{cases}$$

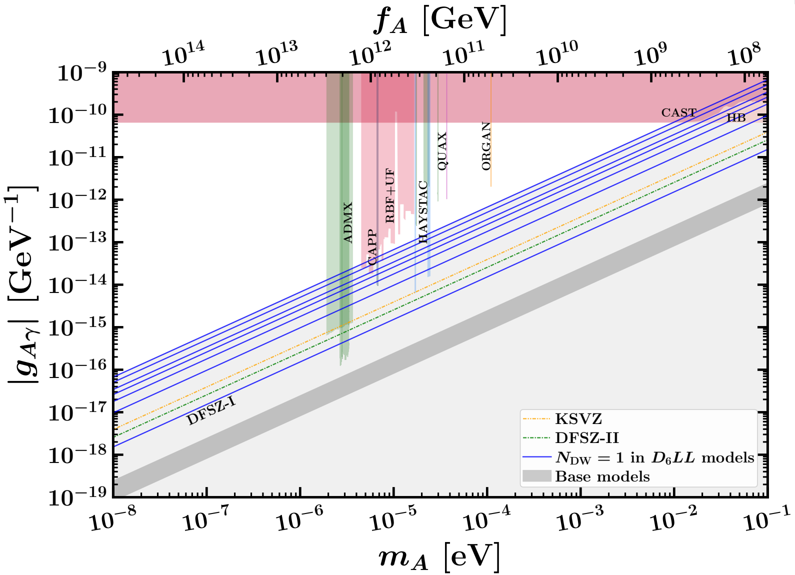
[†] See A. Ernst, A. Ringwald, C. Tamarit 1801.04906

Weaker constraint with $\rho_{\mathrm{GS}} \neq 0$: Lots of cases, e.g., a Z_{22} symmetry[†]

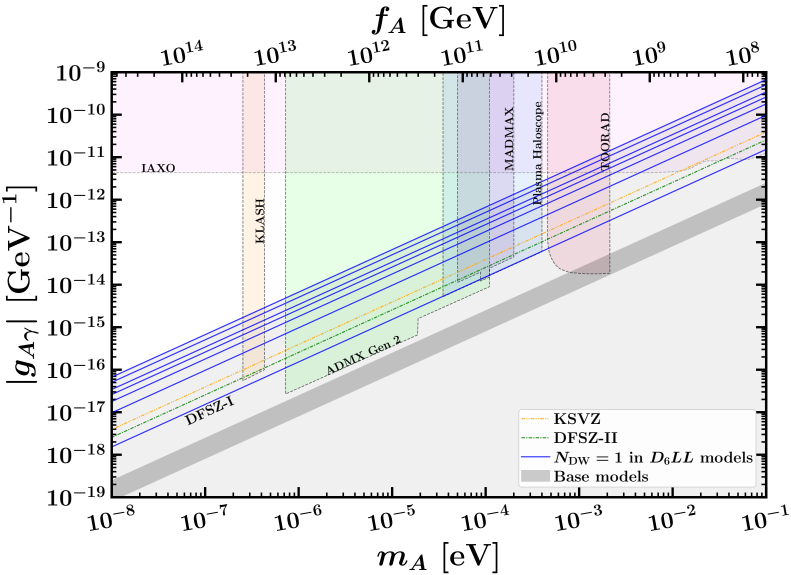
Model	Z_n	X	H_u	p	ρ_{GS}
B_{IV}	22	2	2	11	12

[†] proposed and studied in K. S. Babu, I. Gogoladze, K. Wang hep-ph/0212245.

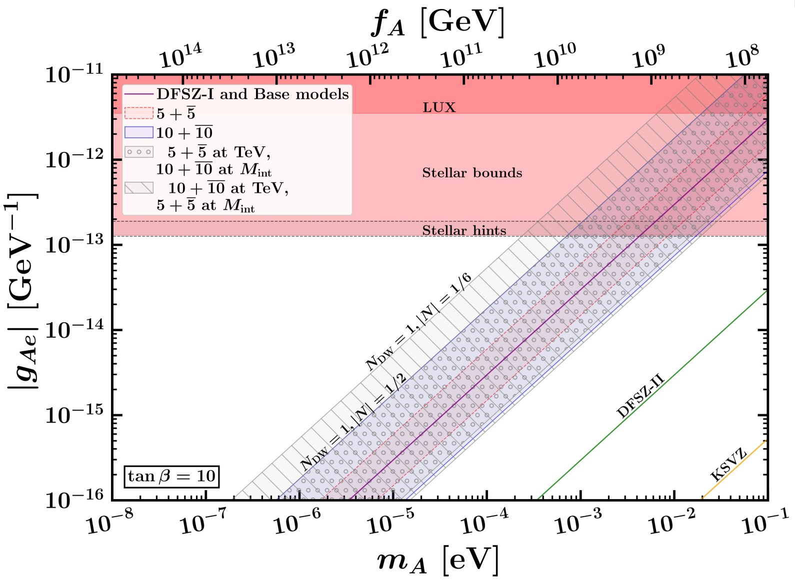
Axion-photon coupling (limits) with an exotic quix pair

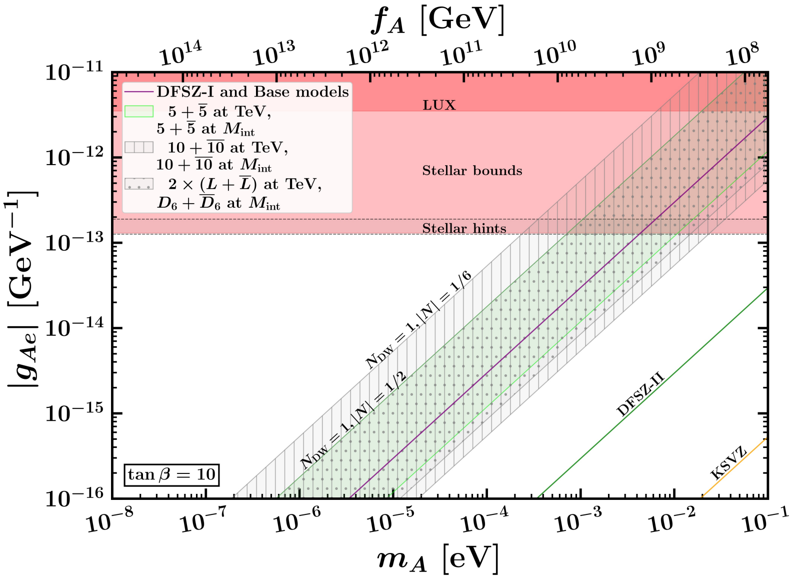


Axion-photon coupling (projections) with an exotic quix pair



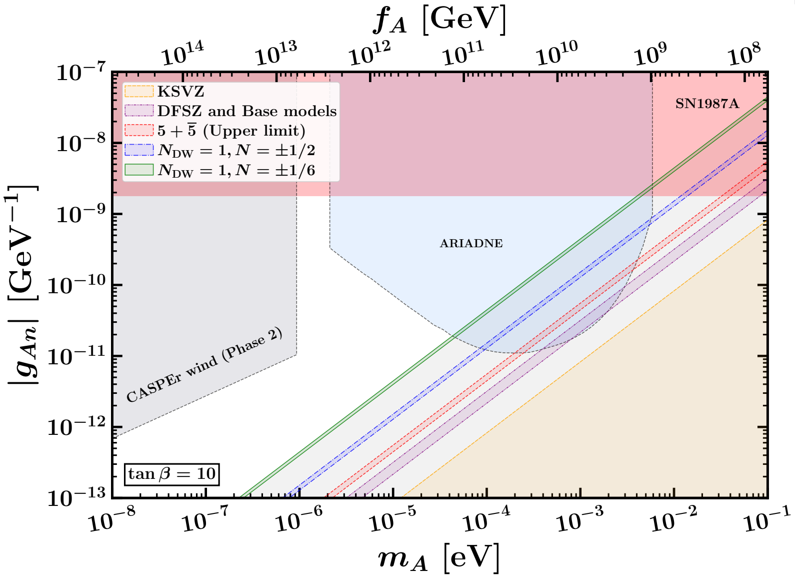
Axion-electron coupling





f_A [GeV]
10^{14} 10^{13} 10^{12} 10^{11} 10^{10} 10^{9} 10^{8}
$|g_{Ae}|$ [GeV^{-1}]
10^{-11} 10^{-12} 10^{-13} 10^{-14} 10^{-15} 10^{-16}
m_A [eV]
10^{-8} 10^{-7} 10^{-6} 10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1}
DFSZ-I and Base models
$5+\overline{5}$ at TeV,
$5+\overline{5}$ at $M_{\rm int}$
$10+\overline{10}$ at TeV,
$10+\overline{10}$ at $M_{\rm int}$
$2\times(L+\overline{L})$ at TeV,
$D_6+\overline{D}_6$ at $M_{\rm int}$
LUX
Stellar bounds
Stellar hints
$N_{\rm DW}=1, |N|=1/6$
$N_{\rm DW}=1, |N|=1/2$
DFSZ-II
KSVZ
$\tan\beta = 10$

Axion-neutron coupling



Axion-proton coupling