
# Aspects of black holes and holography in gauged supergravity

Alessandra Gnecchi

24/02/2017
@TheStringTheoryUniverseCOST

*JHEP 1609 (2016) 090 + wip*

*with U.Gursoy, O.Papadoulaki, C. Toldo*

KU LEUVEN

# Outline

- Motivations/Intro
- $AdS_4$ black holes from gauged Supergravity
- Holographic renormalization and black hole thermodynamics setup
- Phase space and the QCP
    - T=0
    - T>0

# *Motivations*

- String/M-theory origin
  - ➡ Black hole micro-states structure
- Applications of AdS/CFT
  - ➡ Phase transitions in dual theories

Black holes in Supergravity in flat space

1. Supersymmetric properties related to electric-magnetic duality structure of the solutions
2. Rich BPS spectrum (wall crossing, multicenter states..)
3. Origin from D/M-branes constituents in 10/11D under control

Holography for BPS states in AdS ?

# *BPS black holes*

Static background, abelian and scalar fields

$$S = \int d^4x \left( -\frac{R}{2} + g_{i\bar{\jmath}} \partial_\mu z^i \partial^\mu \bar{z}^{\bar{\jmath}} + \mathrm{Im}\mathcal{N}_{\Lambda\Sigma} F^{\Lambda}_{\mu\nu} F^{\Lambda\,\mu\nu} + \right.$$
$$\left. + \frac{1}{2\sqrt{-g}} \mathrm{Re}\mathcal{N}_{\Lambda\Sigma} \epsilon^{\mu\nu\rho\sigma} F^{\Lambda}_{\mu\nu} F^{\Sigma}_{\rho\sigma} - V_g \right)$$

Interpolating geometry + scalar fields

$$ds^2 = -e^{2U(r)} dt^2 + e^{-2U(r)} (dr^2 + e^{2\psi(r)} d\Omega^2)$$

BPS black holes as attractor points

[Ferrara, Gibbons, Kallosh, Strominger, '95,'96]

$$AdS_2 \times S^2 \longleftrightarrow \mathrm{R}^{1,3}$$

$$\partial_{\mathrm{i}} |\mathcal{Z}(\mathrm{p}, \mathrm{q}, \mathrm{z}^{\mathrm{j}}, \bar{\mathrm{z}}^{\mathrm{j}})| = 0$$

$AdS_4$ BPS black holes as holographic RG flows

$$AdS_2 \times S^2 \longleftrightarrow AdS_4$$

[Cacciatori, Klemm '09]

*Translates radial equations in algebraic relations that capture the physics of the (dual) theory*

# *SUSY structure at the horizon*

| | | | |
|---|---|---|---|
| $AdS_2 \times S^2$ | $g = 0$ | $\partial_i \lvert\mathcal{Z}\rvert = 0$ | $\mathcal{N} = 2$ |
| | $g \neq 0$ | $\partial_i \frac{\lvert\mathcal{Z}\rvert}{\lvert\mathcal{W}\rvert} = 0$ | $\mathcal{N} = 1$ |

$$\mathcal{W} = g\mathcal{P}^x_M \mathcal{V}^M \qquad \mathcal{Z} = q_\Lambda L^\Lambda - p^M M_\Lambda$$

- Killing spinor equations at $AdS_2$ x $S^2$ show a topological twist at the horizon for the $S^2$ factor
- The Killing spinor does not depend on the $S^2$ coordinates

$$\hat{\nabla}_{\underline{a}}\epsilon^i_\pm \mp \frac{1}{2\ell_{AdS}}\epsilon^{ij}\gamma_{\underline{a}}\epsilon_{j\pm} = 0 \qquad AdS_2$$

$$\hat{\nabla}_{\hat{a}}\epsilon^i_\pm + \frac{1}{2}\mathcal{V}_{\hat{a}}{}^i{}_j\epsilon^j_\pm = 0 \qquad S^2$$

[de Wit, Van Zalk, ’11]

⟶ *No more Bertotti-Robinson geometry, only N=1 SUSY*

# *BPS AdS₄ black holes*

[Cacciatori, Klemm '09]

Finite horizon SUSY Solution requires:

- magnetic charge
- or magnetic gauging

$$g_\Lambda p^\Lambda - \tilde{g}^\Lambda q_\Lambda = \kappa$$

$$AdS_2 \times \Sigma_\kappa$$

$AdS_2 x S_2$ → $mAdS_4$

*1/4 BPS* states

[Dall'Agata, AG, Hristov, Vandoren]

[Klemm, Vaughan] [Halmagyi, Erbin, Vanel]

[Klemm, Marrani, Petri, Rabbiosi, Santoli..]

[Chow, Compère]

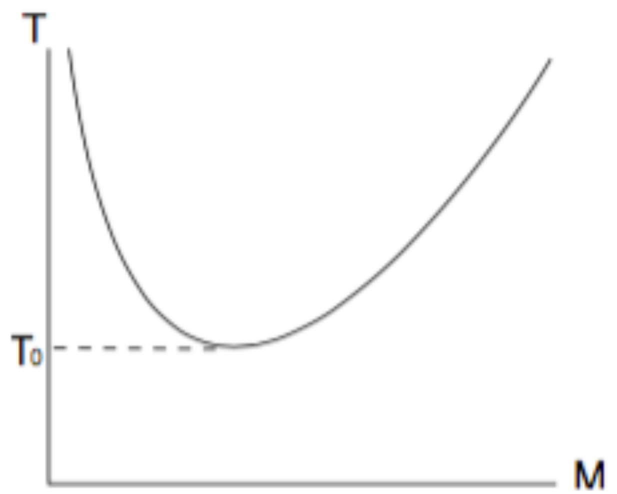
**Extensions/applications**

- Microstate counting
- Holography [Benini, Hristov, Zaffaroni]
- Hypermultiplets [Meessen, Ortin] [Halmagyi, Petrini, Zaffaroni] [Chimento, Faedo, Klemm, Nozawa, Toldo, Monten]
- 10-11D Uplift [Tomasiello, Katmadas]

# Black hole physics can teach about strongly coupled field theories

Investigate quantum critical phases of strongly coupled solid state systems

Hawking-Page transition for a black hole in anti de Sitter spacetime ['83]




Holographic interpretation as confinement/de-confinement phase transition [Witten '98]

## Goal: construct analytic examples

Branches of small/large black holes [Hristov, Vandoren, Toldo, '13]

Black branes phase transition in temperature [Caldarelli, Christodoulu, Papadimitriou, Skenderis, '16]

## *Non-extremal black holes/branes*

[Duff, Liu, Klemm, Vaughan, Toldo, Vandoren]

R-symmetry gauged Supergravity solutions

$$ds^2 = -e^{K} f(r) dt^2 + e^{-K} \frac{dr^2}{f(r)} + e^{-K} \frac{r^2}{\ell_{AdS}^2} (dx^2 + dy^2) \ ,$$

Metric functions, gauge fields and scalar fields

$$e^{-K} = \sqrt{H_0 H_1^3} \qquad f(r) = \left( \frac{c_1}{r} + \frac{c_2}{r^2} + \frac{r^2}{\ell_{AdS}^2} e^{-2K} \right)$$

$$A^{\Lambda} = \frac{1}{2} p^{\Lambda} (x dy - y dx) \ , \qquad \Lambda = 0, 1$$

$$H_{\Lambda} = 1 + \frac{Q_{\Lambda}}{r} \ , \qquad e^{\varphi} = \frac{H_1}{H_0}$$

*Non BPS, non extremal solutions do not have restrictions on the electric or magnetic charges*

# *Relations with String/M-theory*

M-theory on $AdS_4 \times S_7$ ⟶ dual to ABJM

effective theory:

N=8 4dim Supergravity with SO(8) gauging

*truncation to $U(1)^4 \in SO(8)$*

N=2 R-symmetry gauged supergravity with prepotential

$$F(X^\Lambda) = 2i\sqrt{X^0 X^1 X^2 X^3}$$

*even simpler setup, one scalar field, two vectors*

general T=0 solutions, might not be BPS

*known embedding in N=8 gives BF instabilities*

[Donos, Gauntlett, Pantelidou, '11-'12]

*any N=2 dual construction of the same gravity model?*

# *Holographic renormalization*

Counterterm action $$I_{ct,can} = \int_{\partial\mathcal{M}_0} d^3x\sqrt{h}\ (W(\varphi) + W_0\mathcal{R})$$

Field expansion at the boundary

$$\varphi \sim e^{-\Delta_- r/\ell}(\varphi_-(x) + ...) + e^{-\Delta_+ r/\ell}(\varphi_+(x) + ...)$$

Scalar mass $m_\varphi^2 = -\frac{2}{\ell_{AdS}^2}$

Both modes are renormalizable in the window $$-9/4 \leq m_\varphi^2 \ell_{AdS}^2 \leq -9/4 + 1$$

Dual operator conformal dimensions $$\Delta_- = 1\ , \qquad \Delta_+ = 2$$

$$\varphi_+ = \lambda\varphi_-^2$$

***The solutions of electric and magnetic black holes are dual to marginal multitrace deformations***

[Papadimitriou, 2007]

SUSY compatibility?
N=8 SUGRA boundary term required
[Freedman et al. ‘16]

$$\delta S_{SUSY} \sim \lambda \int \mathcal{O}^3$$

# *Thermodynamic ensemble*

Euclidean path integral formulation of gravity at the semiclassical level:

[Gibbons, Hawking ’76 , York, ’86]

$$Z = \int d[g_{\mu\nu}] d[\phi] \exp\{i I_e[g_{\mu\nu}, \phi]\}$$

The partition function defines a free energy, which, within a saddle point approximation, corresponds to the Euclidean on-shell action

$$-\beta F = \ln Z = i I_e[g^*, \phi^*]$$

What are the thermodynamic variables?

# *Thermodynamic ensemble*

Electric configuration: the bare on-shell action corresponds to the free energy for the grand canonical ensemble F(T, χ)

$$F(T, \chi) = M - TS - q_\Lambda \chi^\Lambda$$

Magnetic configuration: the bare on-shell action corresponds to the free energy for the grand canonical ensemble F(T, p)

$$F(T, p^\Lambda) = M - TS$$

Adding boundary terms on the action changes the boundary conditions. In Supergravity, that corresponds to an *electric-magnetic duality rotation*

# *Good singularity*

Planar black holes compete with a thermal gas solution

Black branes in the limit where the black hole coincide with the singularity [Gubser,2000]

$$g_{tt}(r_h) = 0 \qquad r_h \to r_s$$

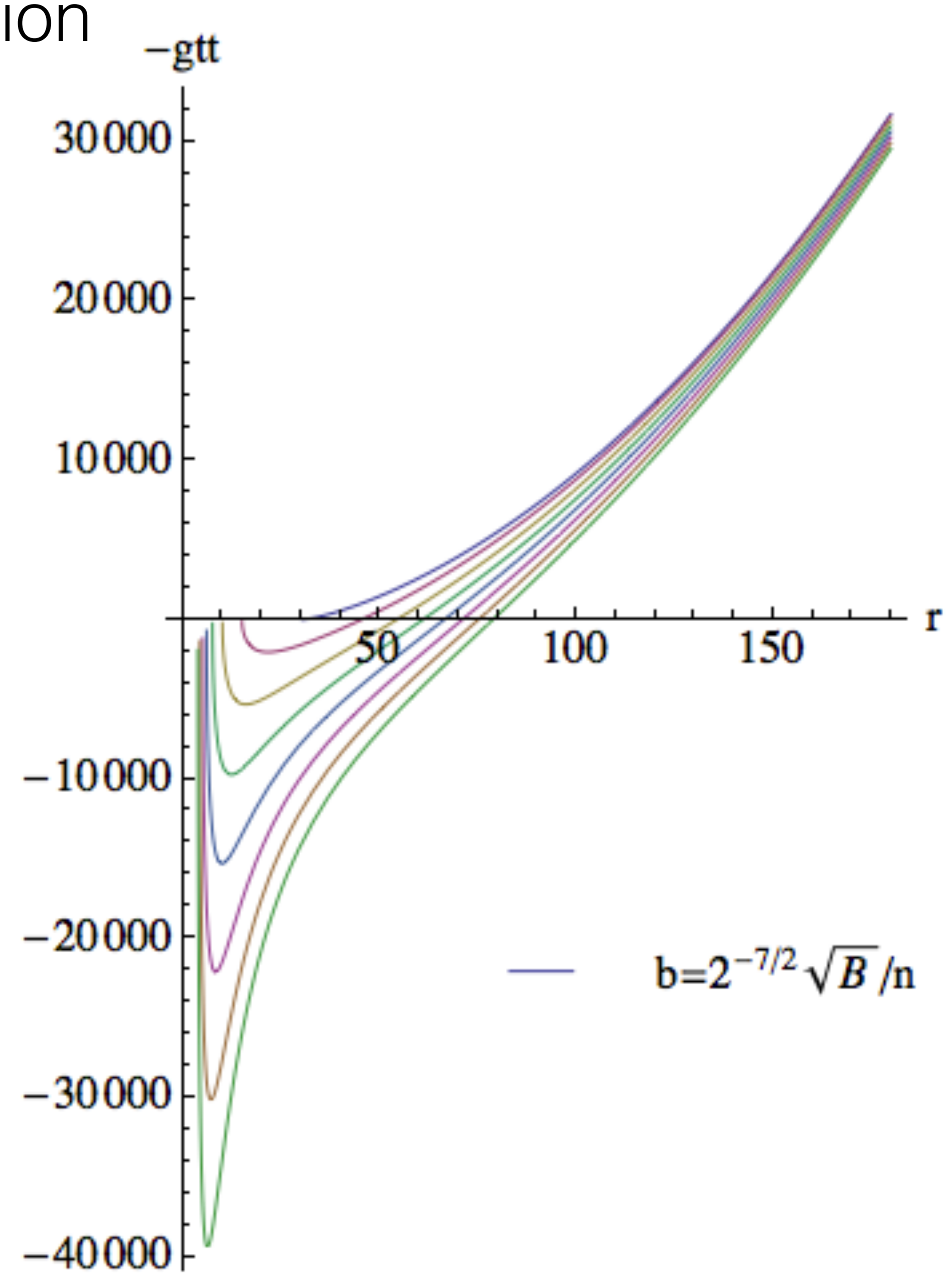
$$g_{xx} = g_{yy} = \sqrt{(r-3b)(r+b)^3}$$

Family of black branes with horizon $r_h = 3b + \epsilon$

Horizon condition $g_{tt}(3b+\epsilon) = 0$

$$|B| = 8\sqrt{2}b^2 + \frac{(6b^2+\chi^2)\epsilon}{\sqrt{2}b} + \mathcal{O}(\epsilon^2)$$

*The electric charge of the solution is* ***q = 0***




## *Good singularity*

Define a thermal gas as the subset in parameter space where

$$|B| = 8\sqrt{2}b^2 \ , \qquad \chi \quad \text{finite}$$

Because of this relation <u>the dependence on χ drops from the metric:</u>

$$\begin{aligned} ds^2_{TG} &= e^{-\sqrt{6}\varphi}(r^2+6br+21b^2)dt^2 - e^{\sqrt{6}\varphi}dr^2(r^2+6br+21b^2)^{-1} + \\ &\quad -e^{\sqrt{6}\varphi}(r-3b)^2(dx^2+dy^2) \ , \\ e^{\sqrt{6}\phi} &= \left(\frac{r+b}{r-3b}\right)^{3/2} \ . \end{aligned}$$

Thermodynamic variables (B, χ, T)

In particular, temperature T and electric potential χ are moduli of the thermal gas solution

Possible resolution of the singularity from string theory

## Black brane

$$F_{bb} = M_{bb} - TS_{bb} + q_{bb}\chi_{bb}$$

Mass $$M = \frac{B^2 - q^2}{4b}$$

Thermodynamic potential

$$dF_{bb} = -S_{bb}dT + q_{bb}d\chi_{bb} + m_{bb}dB$$

## Thermal gas

$$b_{TG} = +2^{-\frac{7}{4}}\sqrt{|B|}$$

*Free energy depends only on the magnetic charge B*

$$F_{TG} = M_{TG} = \frac{B^2}{4b_{TG}} = 2^{-\frac{1}{4}}|B|^{\frac{3}{2}}$$

$$dF_{TG} = m_{TG}dB$$

Qualitative different magnetization

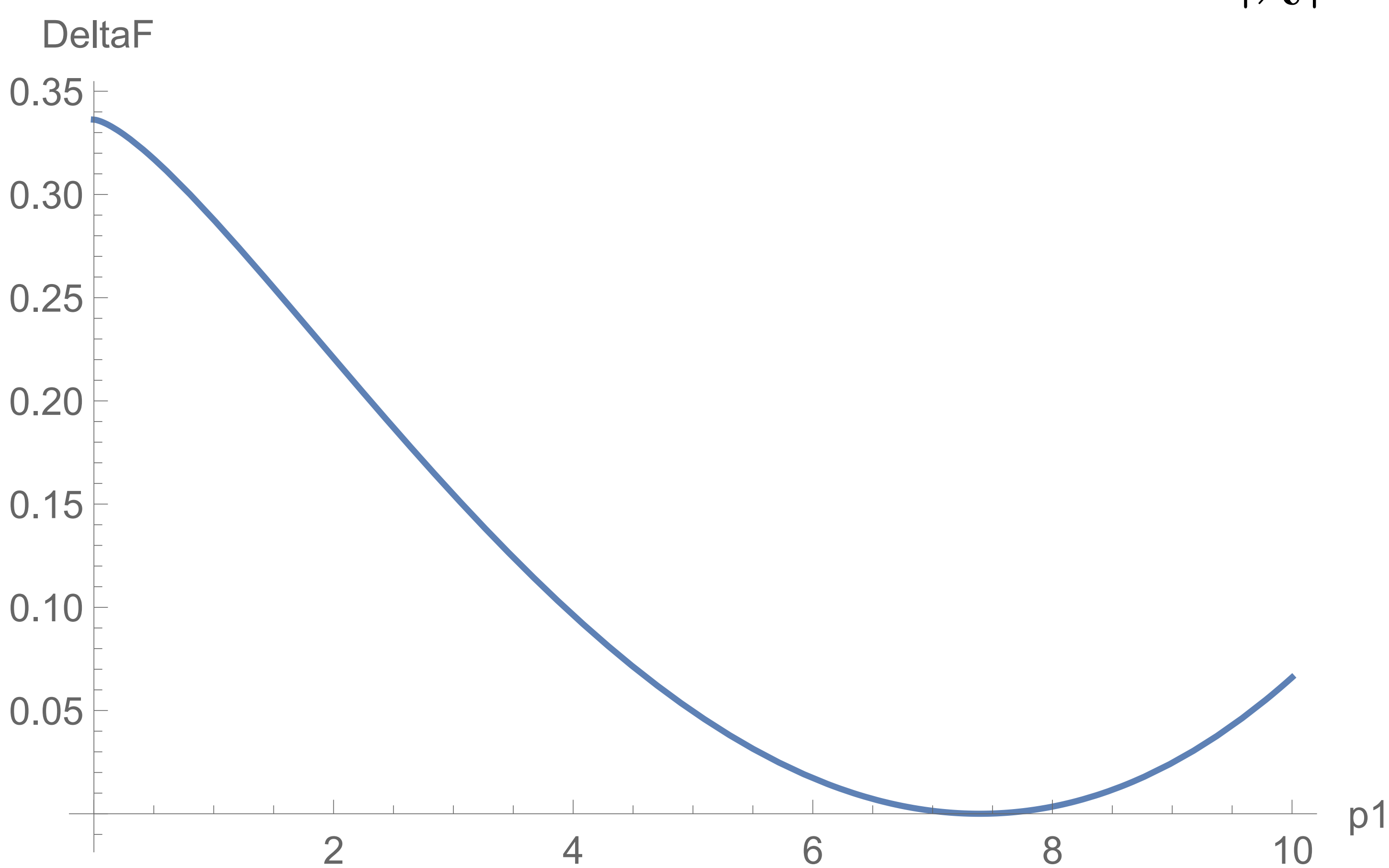
$$m_{bb} = \frac{\partial F_{bb}}{\partial B}\Big|_{\chi,T} = 3\sqrt{\frac{3}{2}}\,\frac{B}{|\chi|}$$

$$m_{TG} = 3\,2^{-\frac{5}{4}}\sqrt{|B|}\,\mathrm{sgn}(B)$$

# *Phase space, T=0*

At T=0 it is possible to solve analytically for the free energy in terms of the thermodynamic variables

$$\Delta F = F_{bb} - F_{TG} = \frac{27B^2 + 32\chi^4}{24\sqrt{6}|\chi|} - 2^{-\frac{1}{4}}|B|^{\frac{3}{2}}$$




We find a critical point for every value of χ

$$|B_*| = \frac{4\sqrt{2}}{3}\chi^2$$

The QCP is second order

$$\Delta F = 3\sqrt{\frac{3}{2}}\frac{(B-B_*)^2}{|\chi|} + \mathcal{O}\left((B-B_*)^3\right)$$

# *Comments on the BPS point*

BPS solution obtained by setting

$$g_{tt} = e^K (r^2 - r_h^2)^2 \ , \qquad g_\Lambda p^\Lambda - g^\Lambda q_\Lambda = 0$$

BPS black holes in ungauged supergravity are stable. They minimize the mass in the parameter space.

$$M \geq |Z|$$

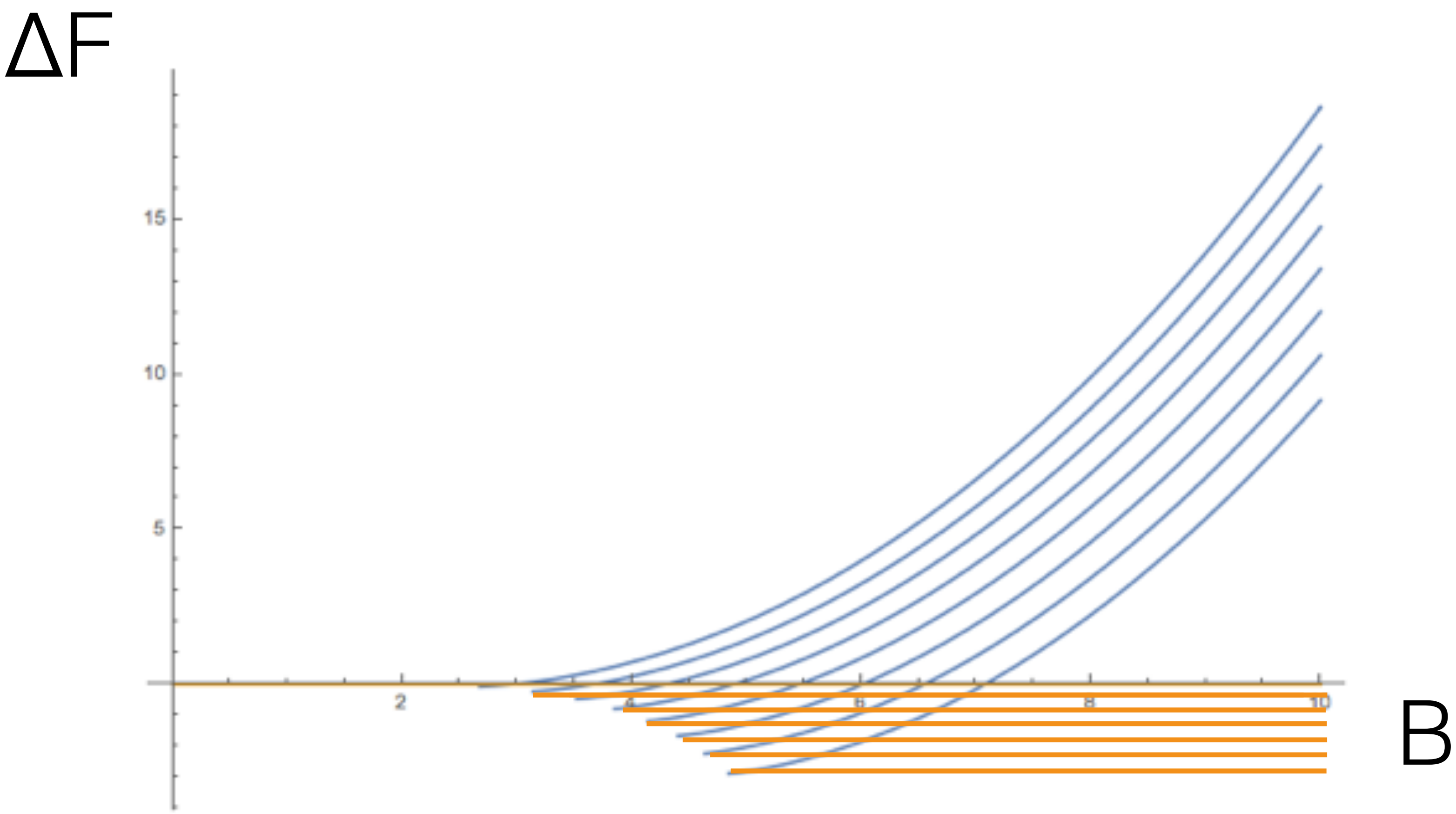
In the canonical ensemble the on-shell action at zero temperature is the mass, so it is consistent

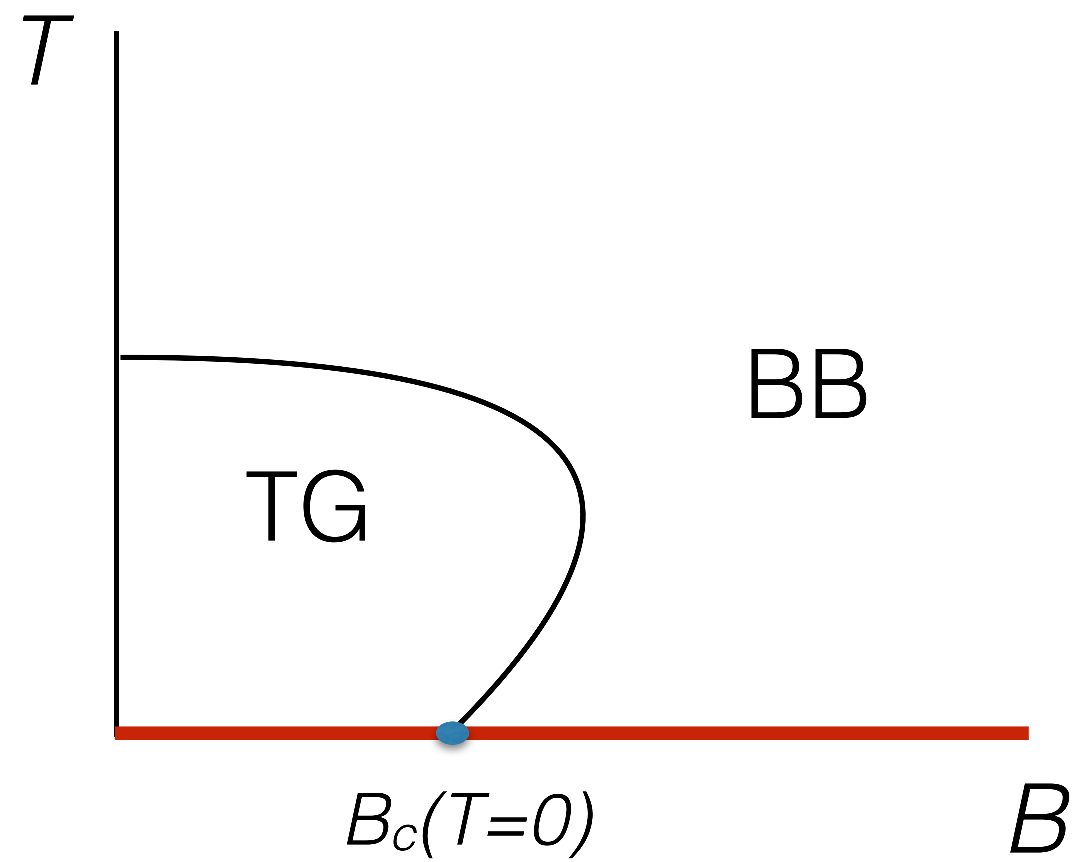
$$F(T=0) = M \qquad F_{extremized} \Leftrightarrow M_{extremized}$$

We work instead in a mixed ensemble, and we find a phase transition which thus seems to be related to the choice of different thermodynamical ensemble. No contradiction, in principle.

*T > 0 phase space*

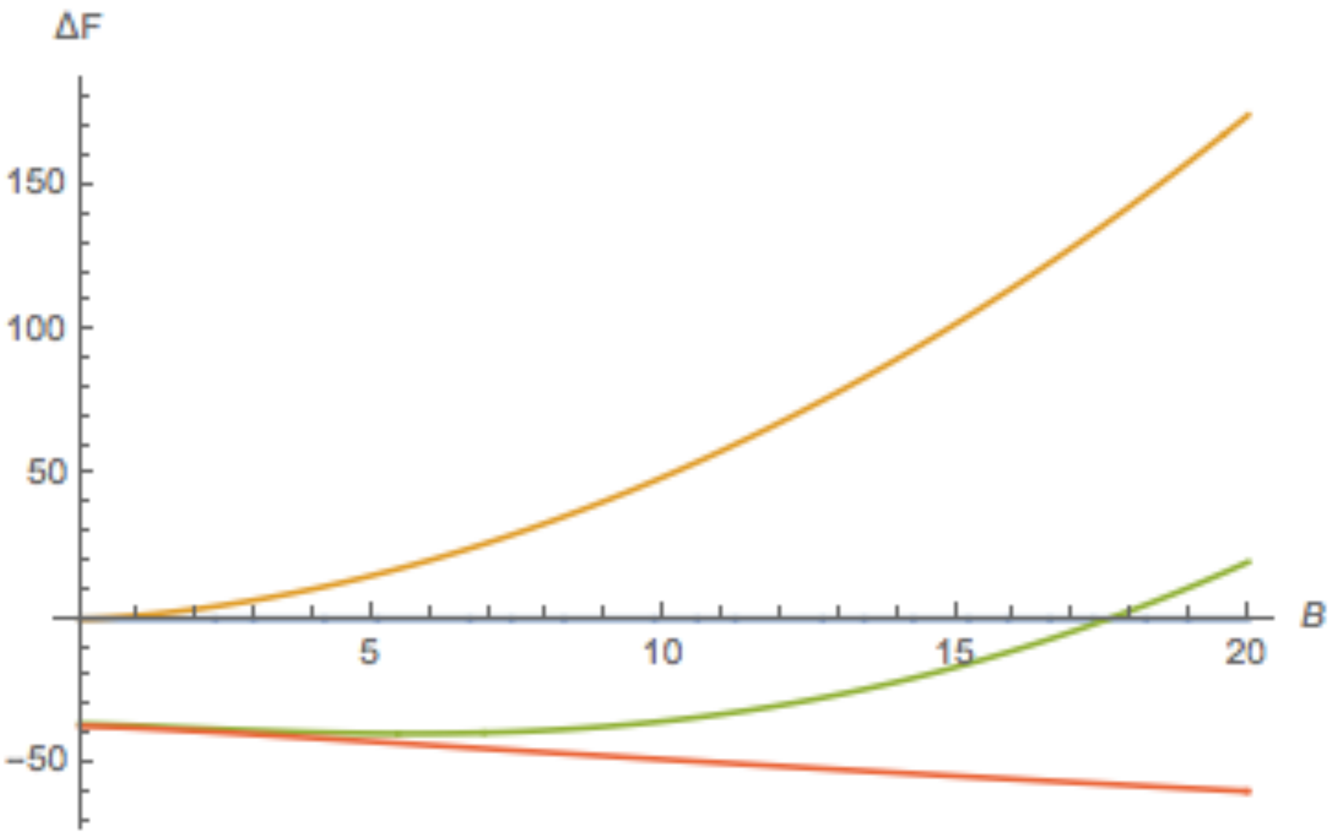
Low temperature

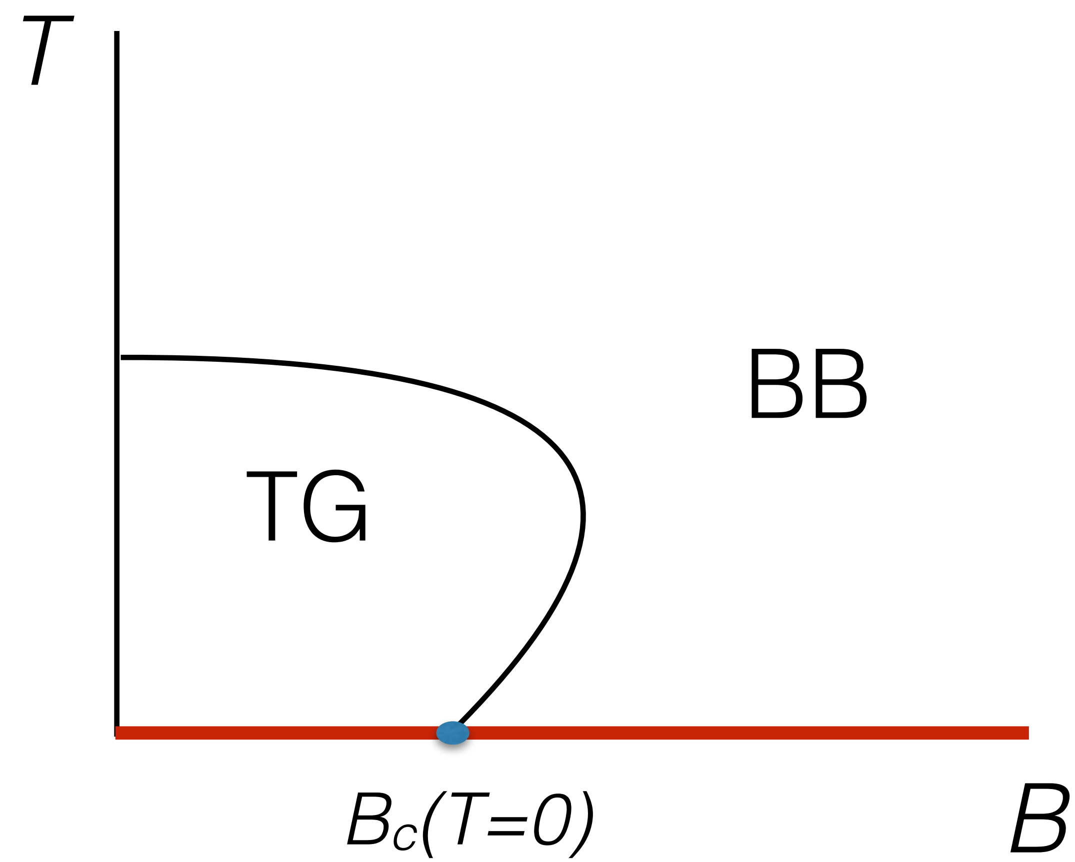


# *T > 0 phase space*

## High temperature



# *Study of QCP*

Quantum criticality emerges at the locus B= Bc(χ)

Information on the nature of the critical point from spectrum of the dual theory
[Gursoy, Kiritsis, Nitti '07]

Scalar fluctuations of the background: acting with a bosonic operator on vacuum, $\mathcal{O}_\Delta$

Holographically induce fluctuations of the corresponding bosonic bulk field on the background

Effective action for a scalar perturbation $m^2 = 0\ ,\ \phi(r,x) = \xi(r) e^{-i\omega t + \vec{k}\cdot\vec{x}}$

$$S_{fluc} = \int d^4x \sqrt{-g} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi^* =$$

$$= \int dr d^3x \sqrt{-g} \left\{ g^{rr} |\partial_r \xi_\omega(r)|^2 + \omega^2 g^{00} |\xi_\omega(r)|^2 \right\}$$

Eigenvalue problem for normalizable modes both in the UV and IR (singularity)

## *Study of QCP*

- The thermal gas has no normalizable modes for arbitrary small ω: gapped system

$$\xi_0 \sim \epsilon^{-1} \qquad \epsilon = r - r_s$$

*Crucial constraint on the TG: releasing the relation* $|B| = 8\sqrt{2}b^2$ *introduces normalizable modes!!*

- Black branes have QNM spectrum discrete frequencies, the lowest **$|\omega|\sim T$**

- Lowering the temperature one can reach arbitrary small energies **$|\omega|\sim\epsilon$**, with separations also **$|\Delta\omega|\sim\epsilon$**.

- The T=0 case can be studied separately: $(x = r - r_h)$

$$\xi_\omega(x) \sim D e^{i\frac{\omega}{f_0 x}} \ , \qquad f(r) = f_0(B,\chi)(r-r_h)^2 + \mathcal{O}(r-r_h)^3$$

*Checked also that there is no confinement/deconfinement phase transition*

# *conclusions and outlook*

- Analytic study of black holes in holographic systems
- Existence of a good singularity solution whose origin might me clarified by a string theory uplift
- Presence of a QCP between a gapped and a gapless phase

- Extend the study to quantum critical region
- An N=2 supersymmetric field theory dual?

— Thank you!