# T-branes, monopole operators and S-duality

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ICTP

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Based on: A. Collinucci, S.G., R. Savelli and R. Valandro arXiv:1603.00062[hep-th]
A. Collinucci, S.G. and R. Valandro, to appear.

# Branes and F/M-theory geometry



A stack of $N$ $D_p$ branes supports a $U(N)$ gauge theory and the vev of the scalars $\Phi_i$ in the vectormultiplet parametrizes the position of the branes.
In M/F-theory these data (eigenvalues of $\Phi_i$) are encoded in the geometric properties of the background.

In the case of D7 branes we have the BPS equation $[\Phi, \Phi^{\dagger}] \sim F_A$ and if we turn on the gauge flux we can consider a non diagonalizable Higgs field! S. Cecotti, C. Cordova, J. Heckman, C. Vafa '10.

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# T-branes in M-theory and probes



One way to characterize compactifications of F-theory is in terms of a dual description in M-theory:

$$\text{M-theory on X} \sim \text{F-theory on } S^1 \times X.$$

On a stack of D6 branes there are three scalars $\Phi_i$. A T-brane is defined by $[\langle \Phi_i \rangle, \langle \Phi_j \rangle] \neq 0$. We consider the case of nilpotent vev for $\Phi_{D6} = \Phi_1 + i\Phi_2$.

Since we don't have a definition of T-brane in M-theory, we consider the 3d theory on a 2-brane probing a T-brane background. For simplicity we will restrict to ADE singularities.

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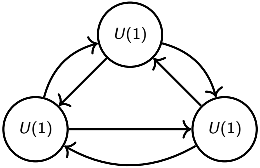
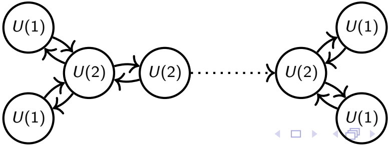
# D6 branes and ADE singularities

**Theory A:** (D2 on top of N D6 branes) SQED with N flavors.
**Theory B:** (D2 at a $A_{N-1}$ singularity) circular quiver with N abelian gauge groups




**Theory A:** (D2 on top of N D6 and O6 plane) $SU(2)$ SQCD with N flavors.
**Theory B:** (D2 brane probing a singularity of type $D_N$) unitary quiver with affine $D_N$ shape.



# $\mathcal{N} = 4$ multiplets and mirror symmetry



- Vectormultiplet: $(A_\mu, \sigma, \Phi)$.
- Hypermultiplet: $(\Phi_1, \Phi_2)$.
- Monopole operators: $d\gamma = *dA$, $W_\pm = e^{\sigma \pm i\gamma}$

**Mirror Symmetry (K. Intriligator, N. Seiberg '96)**

*Duality between $\mathcal{N} = 4$ theories exchanging Coulomb and Higgs branches.*

In the $D2$ theory, $\langle \Phi_{D6} \rangle$ is interpreted as the mass $m_i^j Q_j \tilde{Q}^i$. Under mirror symetry a T-brane is mapped in theory B to

$$\delta\mathcal{W} = mW_{i,+}.$$

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# Abelian theories and mirror symmetry



Consider SQED with 2 flavors and monopole superpotential

$$\mathcal{W} = -\phi\,\mathrm{Tr}(q\tilde{q}) + W_{+}$$

Integrating out the massive flavor in the mirror

$$\mathcal{W} = -\phi(S_1 + S_2) - S_1 S_2 Q\tilde{Q}$$

and mirroring again we get a deformed XYZ model

$$\mathcal{W} = -\phi\,\mathrm{Tr}\,M - X\det M;\;\; M = \begin{pmatrix} S_1 & Y \\ Z & S_2 \end{pmatrix}$$

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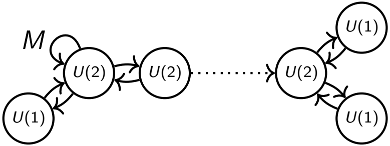
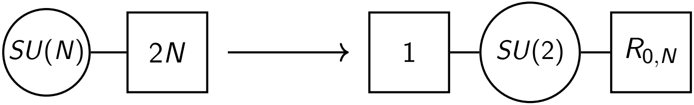


# Nonabelian SQCD and S-duality

**S-duality** for $\mathcal{N} = 2$ $SU(N)$ SQCD with 2N flavors in 4d:




$R_{0,N}$ is a SCFT with $SU(2) \times SU(2N)$ global symmetry.

- $R_{0,2}$ consists of three $SU(2)$ doublets;
- $R_{0,N\geq 3}$ is described by N M5 branes wrapping a three-punctured sphere.

**Higgs branch operators of** $R_{0,N}$**:** $\mu_{SU(2)}$, $\mu_{SU(2N)}$

**Chiral ring relation for** $R_{0,N}$**:** $\mu^2_{SU(2N)} = \mathrm{Tr}(\mu^2_{SU(2)}) I_{2N\times 2N}$

**By dimensional reduction we get a duality in 3d!**

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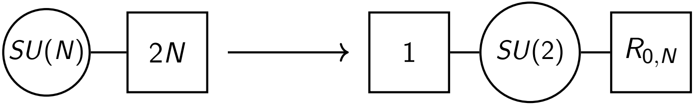


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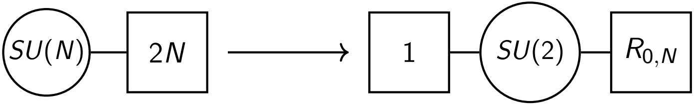


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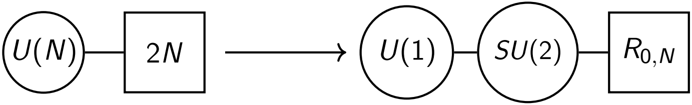
$U(1)_B$ of SQCD is mapped to $U(1)$ acting on $SU(2)$ doublet

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$$\mathcal{W} = -\phi \,\mathrm{Tr}\, M - X \det M + \mathrm{Tr}[\Phi_{SU(2)}(M - \mu_{SU(2)})]$$

From F-terms we get: $M = \mu_{SU(2)}$, $M^2 = 0 \longrightarrow \mu^2_{SU(2N)} = 0$.

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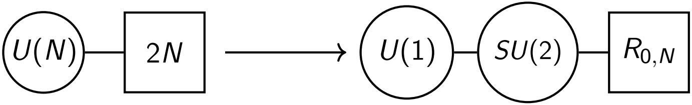


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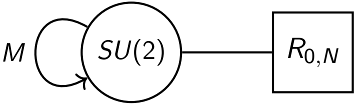


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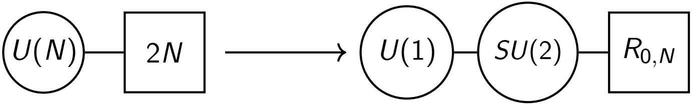


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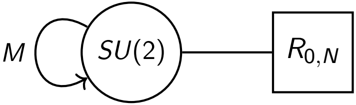


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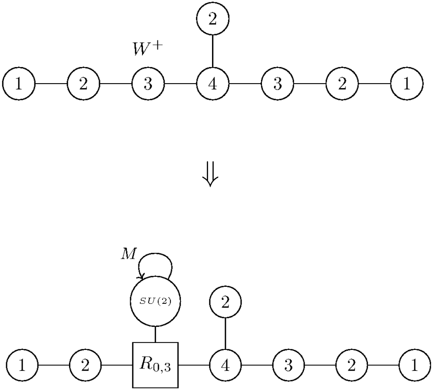
# Higgs branch and resolutions






$\mu^2_{SU(6)} = 0 \rightarrow$ Higgs Branch ($E_7$ singularity) is not deformed.

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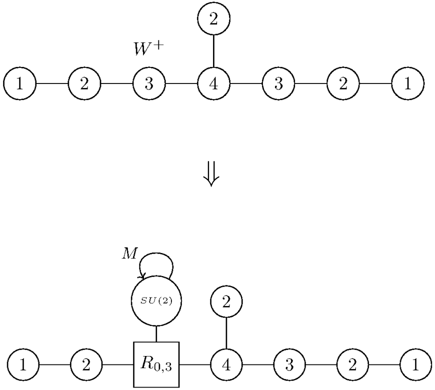


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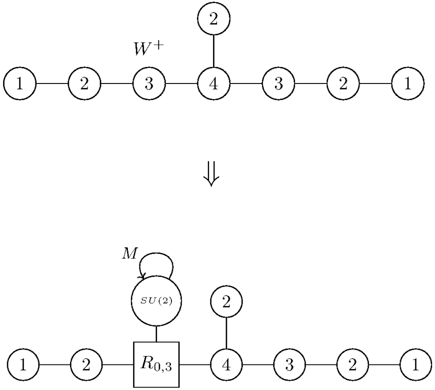


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# Concluding remarks



We proposed a method to understand the properties of T-branes through the wordvolume theory of a brane probing the geometry.

The problem reduces to understanding quivers with monopole superpotential terms. This can be approached combining mirror symmetry and 4d dualities.

T-branes do not deform the geometry but obstruct resolutions!

It would be interesting to apply this method to more complicated backgrounds/brane systems.

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