Deformations, Moduli Stabilisation and Loop-Corrected Gauge Couplings for Particle Physics on D-Branes

Gabriele Honecker

Cluster of Excellence PRISMA & Institut für Physik, JG|U Mainz

based on `arXiv:1702.WXYZ[hep-th]`
with **Isabel Koltermann & Wieland Staessens**

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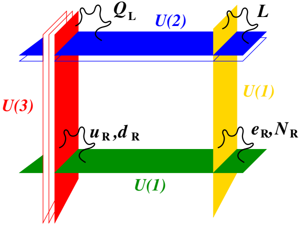
Motivation: Moduli Dependences of String Vacua



- plethora of **D6-brane** models with **MSSM** or **GUT** spectrum
- Kähler moduli v_i: $G_N^{(4)}(v_i, g_{\text{string}}, M_{\text{string}})$
- complex structure moduli ζ_j: $\frac{1}{g_a^2}(\zeta_j, g_{\text{string}})$
- @ 1-loop: $\frac{1}{g_a^2}(v_i, \zeta_j, g_{\text{string}})$
- ...

$$\int_{\mathbb{R}^{1,9}} e^{-2\Phi} R_{(10)} \to \boxed{\frac{\text{Vol}_6}{g_{\text{string}}^2}} \int_{\mathbb{R}^{1,3}} R_{(4)} \qquad \int_{\mathbb{R}^{1,3}\times\Pi_3^a} e^{-\Phi}\, \text{tr} F_{\mu\nu} F^{\mu\nu}_{(7)} \to \boxed{\frac{\text{Vol}_3}{4\, g_{\text{string}}}} \int_{\mathbb{R}^{1,3}} \text{tr} F_{\mu\nu} F^{\mu\nu}_{(4)}$$

Standard *a priori* assumption:

- moduli not stabilised - unless fluxes
  - no known particle physics vacua beyond twisted torus

Argue here:

- presence of D-branes stabilises (part of the c.s.) moduli

Motivation cont'd: Deformations & 1-Loop Effects

here: IIA string theory with D6-branes; IIB via mirror Kähler $\leftrightarrow$ complex str.

Field theoretic expectation:

- SUSY D6-branes wrap *special Lagrangian (sLag)* 3-cycles
- deformations either:
  - ~~SUSY~~ via $\langle \zeta_j \rangle \propto$ FI parameter $\leftarrow$ modulus stabilised
  - constitute flat directions of $\frac{1}{g_a^2}$ $\leftarrow$ stronger/weaker possible

1-loop gauge thresholds:

- can also take *negative* values
- compete with deformations

…so far only explicitly known for simple backgrounds

Questions:

- how many stabilised moduli?
- can $M_{\rm string} \ll M_{\rm GUT}$ appear?

Outline

- Motivation
- D6-brane set-up & moduli
- Description of deformations
- 1-loop results
- Conclusions & Outlook

D6-Brane Set-Up: Type IIA Orientifolds

***special Lagrangians*:**

- needed for SUSY D6-branes
- symplectic geometry & *sLag*s - very limited knowledge

see e.g. Joyce '01-03; Morrison, Plesser '15

- need also O6-planes for model building
  - IIA requires anti-holomorphic involution on CY_3

partial results for hypersurfaces in weighted projective spaces by Palti '09

- **here:** $\boxed{T^6/\Gamma}$ as background (with $\mathbb{Z}_2 \times \mathbb{Z}_2 \subset \Gamma$)
  - **advantage:**
    - geometry (e.g. metric) well-known
    - CFT tools for vector-like spectrum & effective field theory
  - **disadvantage:**
    - not all *singularities* resolvable/deformable$\leftarrow$duality arguments$\lightning$?
    - tiny class of backgrounds with set of CY_3

D6-Brane Set-Up: $T^6/(\Gamma \times \Omega\mathcal{R})$ with $\mathbb{Z}_2 \times \mathbb{Z}_2 \subset \Gamma$

notation: $T^6 = \otimes_{i=1}^3 T^2_{(i)}$

$T^4_{(i)} = T^2_{(j)} \times T^2_{(k)}$

Hodge numbers:

- singularities at $\mathbb{Z}_2 \times \mathbb{Z}_2$ fixed loci: $e^{(i)}$ at each $\mathbb{C}^2_{(i)}/\mathbb{Z}_2^{(i)}$
- $\mathbb{Z}_2^{(j,k)}$ acts with phase $\eta = \pm 1$ on $\mathbb{Z}_2^{(i)}$-twisted sector:

  without/with discrete torsion

  - $\eta = +1$: 2-cycle per singularity
  - $\boxed{\eta = -1}$: 3-cycle instead $e^{(i)} \otimes$ [1-cycle on $T^2_{(i)}$]
    - suitable for D6-brane model building
    - $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ has $(h^{11}, h^{21}) = (3_{\text{bulk}}, 3_{\text{bulk}} + 3 \times 16_{\mathbb{Z}_2})$
    - can add $\mathbb{Z}_3$ symmetry for model building:

      $\boxed{T^6/(\mathbb{Z}_2 \times \mathbb{Z}_6)} \simeq (T^2_{(1)} \times T^4_{(1)}/\mathbb{Z}_6)/\mathbb{Z}_2^{(3)}$ has

      $(h^{11}, h^{21}) = (3_{\text{bulk}} + 8_{\mathbb{Z}_3} + 2 \times 4_{\mathbb{Z}_6'}, 1_{\text{bulk}} + [6 + 2 \times 4]_{\mathbb{Z}_2} + [2+2]_{\mathbb{Z}_3 + \mathbb{Z}_6})$

Fractional 3-cycles:

- unimodular lattice contains $\boxed{\Pi^{\text{frac}} = \frac{\Pi^{\text{bulk}} + \sum_i \Pi^{\mathbb{Z}_2^{(i)}}}{4}}$
- $\Pi^{\mathbb{Z}_2^{(i)}}$ couples to $\leq h^{21}_{\mathbb{Z}_2^{(i)}}$ deformation moduli

Deformations & Orientifold Symmetry

General considerations:

- $\boxed{\Pi^{\text{frac}} = \frac{\Pi^{\text{bulk}} + \sum_i \Pi^{\mathbb{Z}_2^{(i)}}}{4}}$ couples to (some) twisted moduli
- N D6-branes $\rightsquigarrow U(N)$
  - $U(1)$ D-term vanishes $\Leftrightarrow \Pi^{\text{frac}}$ is *sLag*
  - related twisted moduli stabilised $\Leftrightarrow$ FI term for $\langle \zeta_j \rangle \neq 0$
- not all couplings generate D-term: e.g. $USp(2N)$ or $SO(2N)$
  - no D-term $\Leftrightarrow \Pi^{\text{frac}}$ is *sLag* for any deformation

Orientifold symmetry:

- IIA requires anti-holomorphic involution $\mathcal{R}$
- $(h^{11}_+, h^{11}_-) = (4_{\mathbb{Z}_6'}, 3_{\text{bulk}} + 8_{\mathbb{Z}_3} + 4_{\mathbb{Z}_6'})$ - one $\mathbb{Z}_6'$ sector does not have any scalars that could resolve the singularities
- $(h^{21}+1)$ $\Omega\mathcal{R}$-even + $(h^{21}+1)$ $\Omega\mathcal{R}$-odd 3-cycles
  - deformation SUSY $\Leftrightarrow$ only $\Omega\mathcal{R}$-even contribution to $\Pi^{\mathbb{Z}_2^{(i)}}$

Deformations con't: Hypersurface Technique

Blaszczyk, **G.H.**, Koltermann '14-15 & **G.H.**, Koltermann, Staessens '17

- use $\mathbb{P}^2_{112}$ with coord. (x_i, v_i, y_i) to describe $T^2_{(i)}$

$$-y_i^2 + F_i(x_i, v_i) \stackrel{!}{=} 0 \qquad F_i(x_i, v_i) = 4 v_i x_i^3 - g_2^{(i)} v_i^3 x_i - g_3^{(i)} v_i^4$$

  - $g_3^{(i)} = 0$: square torus
  - $g_2^{(i)} = 0$: hexagonal lattice

- take product and impose $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry: $y_i \stackrel{\mathbb{Z}_2}{\to} -y_i$

$$(T^2)^3/(\mathbb{Z}_2 \times \mathbb{Z}_2) \simeq \{-y^2 + F_1 F_2 F_3 = 0\} \qquad \text{with } y \equiv y_1 y_2 y_3$$

- deform fixed points $\alpha\beta \in T^4_{(i)} \equiv T^2_{(j)} \times T^2_{(k)}$:

$$-y^2 + F_1 F_2 F_3 + \sum_{i \neq j \neq k \neq i} \sum_{\alpha\beta} \varepsilon^{(i)}_{\alpha\beta} F_i \, \delta F_j^{(\alpha)} \, \delta F_k^{(\beta)} \stackrel{!}{=} 0$$

with $\delta F_j^{(\alpha)}(x_j, v_j)$ also polynomials of degree 4

deformation method based on Vafa, Witten '95

Deformations con't: *sLags*

***sLags*:** probe $\boxed{\mathrm{Re}\int_{\Pi_a}\Omega_3 > 0 \quad \text{and} \quad \mathrm{Im}\int_{\Pi_a}\Omega_3 = 0}$

- specify calibration via anti-holomorphic involution $\sigma_{\mathcal{R}}$
- in general too complicated $\rightsquigarrow$ concentrate on anti-linear maps

$$\begin{pmatrix} x_i \\ v_i \end{pmatrix} \to A \begin{pmatrix} \overline{x}_i \\ \overline{v}_i \end{pmatrix}, \quad y_i \to e^{i\beta}\overline{v}_i \quad \text{s.t.} \quad \overline{A}A = \mathbb{1}, \quad \sigma_{\mathcal{R}}(F_i(x_i, v_i)) = e^{-2i\beta}F(\overline{x}_i, \overline{v}_i)$$

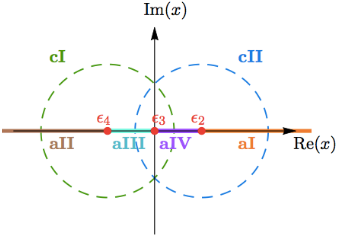
- two different calibrations β and $\beta + \pi$ per given A
  $\rightsquigarrow$ integration path Π_a specified by (A_a, β_a)
- impose SUSY products of one-cycles @ orbifold point

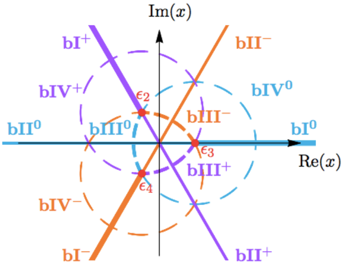
$\mathbb{Z}_3 \times \Omega\mathcal{R}$ **symmetry:**

- $\mathbb{Z}_3$: need for hexagonal tori
  & identification of $\mathbb{Z}_2 \times \mathbb{Z}_2$ deformations $\varepsilon^{(i)}_{\alpha\beta}$ under $\mathbb{Z}_3$
- $\Omega\mathcal{R}$-invariance: only **real** deformation parameters

Lagrangian lines on square untilted torus T^2				
(n^i, m^i)	displacement	condition in x	label	picture
$\pm(1,0)$	0	$\epsilon_2 \leq x$	aI	
	1	$\epsilon_4 \leq x \leq \epsilon_3$	aIII	
	continuous	$\lvert x - \epsilon_4 \rvert^2 = 2\epsilon_4^2 + \epsilon_2\epsilon_3$	cI	
$\pm(0,1)$	0	$x \leq \epsilon_4$	aII	
	1	$\epsilon_3 \leq x \leq \epsilon_2$	aIV	
	continuous	$\lvert x - \epsilon_2 \rvert^2 = 2\epsilon_2^2 + \epsilon_4\epsilon_3$	cII	

Lagrangian lines on hexagonal torus T^2				
(n^i, m^i)	displacement	condition in x	label	picture
$\pm(1,0)$	0	$1 \leq x$	bI^0	
	1	$\lvert x-1\rvert^2 = 3, \mathrm{Re}(x) \leq -1/2$	$bIII^0$	
$\pm(-1,2)$	0	$x \leq 1$	bII^0	
	1	$\lvert x-1\rvert^2 = 3, -1/2 \leq \mathrm{Re}(x)$	bIV^0	
$\pm(0,1)$	0	$1 \leq \xi^2 x$	bI^-	
	1	$\lvert \xi^2 x-1\rvert^2 = 3, \mathrm{Re}(\xi^2 x) \leq -1/2$	$bIII^-$	
$\pm(2,-1)$	0	$\xi^2 x \leq 1$	bII^-	
	1	$\lvert \xi^2 x-1\rvert^2 = 3, -1/2 \leq \mathrm{Re}(\xi^2 x)$	bIV^-	
$\pm(1,-1)$	0	$1 \leq \xi x$	bI^+	
	1	$\lvert \xi x-1\rvert^2 = 3, \mathrm{Re}(\xi x) \leq -1/2$	$bIII^+$	
$\pm(1,1)$	0	$\xi x \leq 1$	bII^+	
	1	$\lvert \xi x-1\rvert^2 = 3, -1/2 \leq \mathrm{Re}(\xi x)$	bIV^+	





G.H., Koltermann, Staessens '17

Deformations cont'd: Examples on $\mathbb{Z}_2 \times \mathbb{Z}_6$ Backgrd.

Ecker, **G.H.**, Staessens '14-15

- extensive computer scans boil down to prototypes

$$SU(3)_a \times USp(2)_b \times U(1)_Y \times SU(4)_h \times \mathbb{Z}_3 \subset U(3)_a \times USp(2)_b \times U(1)_c \times U(1)_d \times U(4)_h$$

$$SU(3)_a \times USp(2)_b \times USp(2)_c \times \begin{cases} \mathbb{Z}_3 & \textbf{I} \\ \widetilde{U(1)}_{B-L} & \textbf{II} \end{cases} \subset U(3)_a \times USp(2)_b \times USp(2)_c \times U(h)^2$$

$$SU(4)_a \times USp(2)_b \times USp(2)_c \times \begin{cases} SU(6) & \textbf{I} \\ SU(2) & \textbf{II} \end{cases} \subset U(4)_a \times USp(2)_b \times USp(2)_c \times U(h)$$

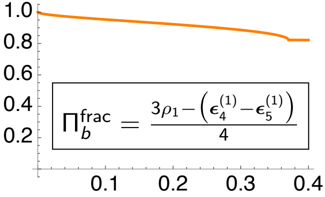
- equivalent @ orbifold point:
  - closed & open string spectrum
  - gauge couplings (tree + 1-loop)

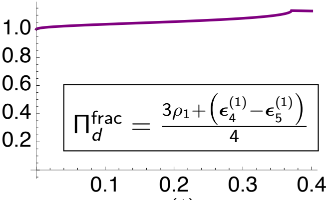
Counting of stabilised complex structure moduli & flat directions with $1/g^2_{\mathrm{D6}_{x\in\{a,b,c,d,h_{1,2}\}}}$ dependence

$\mathbb{Z}_2 \times \mathbb{Z}_6$	ϱ	$\varepsilon^{(1)}_{0,1,2}$	$\varepsilon^{(1)}_{3}$	$\varepsilon^{(1)}_{4+5}$	$\varepsilon^{(1)}_{4-5}$	$\varepsilon^{(2)}_{1,2}$	$\varepsilon^{(2)}_{3,4}$	$\varepsilon^{(3)}_{1,2}$	$\varepsilon^{(3)}_{3,4}$	$\#_{\text{stab}}$
MSSM	none		a, c, h		$[b, d]_{\text{flat}}$	c, d	none	a, d, h	none	6
L-R I	none	$h_{1,2}$	$a, d, h_{1,2}$	a, d	$[b, c]_{\text{flat}}$	$h_{1,2}$	none	a, d	none	9
L-R II	none	$h_{1,2}$	$a, d, h_{1,2}$	a, d	$[b, c]_{\text{flat}}$	$[a, b, c, d, h_{1,2}]_{\text{flat}}$	none	$a, d, h_{1,2}$	none	7
L-R IIb	none	$h_{1,2}$	$a, d, h_{1,2}$	a, d	$[b, c]_{\text{flat}}$	$[a, b, c, d]_{\text{flat}}$	$[h_{1,2}]_{\text{flat}}$	a, d	$h_{1,2}$	9
L-R IIc	none		a, d	a, d	$[b, c, h_{1,2}]_{\text{flat}}$	h_1	h_2	a, d, h_1	h_2	10
PS I	none		a, h		$[b, c]_{\text{flat}}$	$[a, b, c, h]_{\text{flat}}$	none	a, h	none	4
PS II	none	h	a, h	a	$[b, c]_{\text{flat}}$	$[a, b, c, h]_{\text{flat}}$	none	a, h	none	7

Deformations: Flat Direction of MSSM Example

- $USp(2)_b$ and $U(1)_d \subset U(1)_Y$ feel flat direction $\varepsilon^{(1)}_{4-5} < 0$





  - weak interaction on b slightly stronger for $\varepsilon^{(1)}_{a-5} \neq 0$
  - hypercharge slightly weaker

Caveat on 'stabilised' moduli:

- compensation by charged matter *vev* in D-terms
  - only bifundamentals under $U(N) \times U(N)$ $(cd^{(\prime)})_{\rm MSSM}$
  - only if string derived field theory couplings exist

1-Loop Corrections to Gauge Couplings @ Orbifold Point

Why?

- deformations by **twisted** moduli change couplings @ tree-level
- **untwisted** moduli enter 1-loop corrections
  - only known at orbifold point
  - independent of *absolute* choice of $\mathbb{Z}_2 \times \mathbb{Z}_2$ eigenvalues
  - can - *in principle* - contribute to hierarchy $M_{\text{string}} \ll M_{\text{GUT}}$

G.H., Ripka, Staessens '12

Unusual feature here:

- net O6-plane charge along **one** direction on $T^2_{(1)}$
- Einstein-Hilbert $\rightsquigarrow \frac{M^2_{\text{Planck}}}{M^2_{\text{string}}} = \frac{4\pi}{g^2_{\text{string}}} v_1 v_2 v_3$ with $v_1 = R_1^{(1)} R_2^{(1)}$
- DBI action $\rightsquigarrow \frac{8\pi^2}{g^2_{SU(3)_a/SU(4)_h,\text{tree}}} = \frac{\pi}{2\sqrt{6}} \sqrt{\frac{R_1^{(1)}}{R_2^{(1)}}} \frac{M_{\text{Planck}}}{M_{\text{string}}}$

$\rightsquigarrow M_{\text{string}} \ll M_{\text{Planck}}$ can be compensated by $R_2^{(1)} \gg R_1^{(1)}$

. . . spoilt by 1-loop effects??

- **Kähler moduli** v_i enter via parallel D6-branes

$$\Lambda_{0,0}(v) \equiv -\frac{1}{4\pi} \ln(\eta(iv)) \stackrel{v\to\infty}{\longrightarrow} \frac{v}{48}$$

$$\Lambda_{\tau,\sigma}(v) \equiv -\frac{1}{4\pi} \ln\left(e^{-\frac{\pi\sigma^2 v}{4}} \frac{|\vartheta_1(\frac{\tau - i\sigma v}{2}, iv)|}{\eta(iv)} \right) \stackrel{v\to\infty}{\longrightarrow} \frac{\left[3(1-\sigma)^2 - 1\right] v}{48} - \delta_{\sigma,0} \frac{\ln[2\sin(\frac{\pi\tau}{2})]}{4\pi}$$

  - **sign** depends on discrete brane data (τ, σ)

- **sum** over sector-by-sector ($\mathcal{A}$nnulus + $\mathcal{M}$öbius strip) e.g.

$$\frac{\Delta^{\mathcal{A}+\mathcal{M},\mathbf{MSSM}}_{SU(3)_a}}{2} \stackrel{v\gg 1}{\longrightarrow} 2\pi\, v_1 + \frac{\pi}{3}\left(v_2 + c^{\frac{1}{2}}_{1,1}(v_2 - v_3) \right) - \ln\left(\left(\frac{R^{(1)}_1}{R^{(1)}_2} v_1 \right)^6 v_2 \right) - 12$$

$$\frac{\Delta^{\mathcal{A}+\mathcal{M},\mathbf{MSSM}}_{SU(4)_h}}{2} \stackrel{v\gg 1}{\longrightarrow} 2\pi\, v_1 + \frac{\pi}{3}\left(-v_2 + c^{\frac{1}{2}}_{1,1}(v_2 - v_3) \right) - \ln\left(\left(\frac{R_1}{R_2} v_1 \right)^6 v_2^{-1} \right) - 7$$

  - $v_1 \gg 1$ disfavoured by ${}^1/_{g_a^2} \sim \mathcal{O}(1) \rightsquigarrow R^{(1)}_1 \ll R^{(1)}_2$ ↯
  - for $v_2 \gg 1$, $SU(4)_h$ more strongly coupled than $SU(3)_a$
  - for $v_1 \approx 1$ and $v_2 = v_3 \lesssim 6$: $M_{\text{string}} \sim M_{\text{GUT}}$, $\Delta^{\mathcal{A}+\mathcal{M},\mathbf{MSSM}}_{SU(3)_a} < 0$

G.H., Koltermann, Staessens to appear

Conclusions

- **deformations** of T^6/Γ with $\mathbb{Z}_2 \times \mathbb{Z}_2 \subset \Gamma$ can be described by **hypersurface formalism**
- **identifications & reality** conditions of deformation parameters under $\mathbb{Z}_3 \times \Omega\mathcal{R}$
- **D6-brane** couples $\Omega\mathcal{R}$-**odd** $\rightsquigarrow$ **stabilisation** @ orbifold point
- *all* D6-branes couple (at most) $\Omega\mathcal{R}$-**even** $\rightsquigarrow$ **flat** direction
  - mild change in gauge couplings for small deformation
  - lifts degeneracies of tree-level gauge couplings
  - depends on *absolute* $\mathbb{Z}_2 \times \mathbb{Z}_2$ eigenvalues
- **competing** effect: **1-loop** corrections
  - only computable @ orbifold point
  - depend on *relative* $\mathbb{Z}_2 \times \mathbb{Z}_2$ eigenvalues
  - concrete models disfavour very low M_{string}

Outlook

Open questions:

- low M_{string} disfavoured:
  generic feature or artefact of concrete model?
- any (fine-tuned) model **matching all pheno** couplings?
- needed: **exact** results for low-energy **effective field theory**
  - @ orbifold point:
    not even Möbius strip contributions for $\delta \frac{1}{g_a^2}$ fully understood;
    Yukawas ...
  - generic CY_3 ???

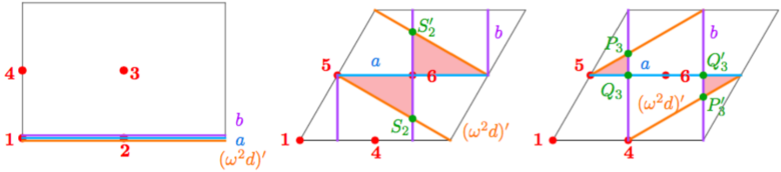
Appendix

Ex: MSSM on rigid D6-branes on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_6 \times \Omega\mathcal{R})$

Ecker, **G.H.**, Staessens '15

D6-brane configuration of a global 5-stack MSSM model on the aAA lattice

	wrapping #	$\frac{\text{Angle}}{\pi}$	$\mathbb{Z}_2^{(i)}$ eigenv.	$(\vec{\tau})$	$(\vec{\sigma})$	gauge gr.
a	$(1,0;1,0;1,0)$	$(0,0,0)$	$(+++)$	$(0,1,1)$	$(0,1,1)$	$U(3)$
b	$(1,0;-1,2;1,-2)$	$(0,\frac{1}{2},-\frac{1}{2})$	$(--+)$	$(0,1,0)$	$(0,1,0)$	$USp(2)$
c	$(1,0;-1,2;1,-2)$	$(0,\frac{1}{2},-\frac{1}{2})$	$(-+-)$	$(0,1,1)$	$(0,1,1)$	$U(1)$
d	$(1,0;-1,2;1,-2)$	$(0,\frac{1}{2},-\frac{1}{2})$	$(+--)$	$(0,0,1)$	$(0,0,1)$	$U(1)$
h	$(1,0;1,0;1,0)$	$(0,0,0)$	$(--+)$	$(0,1,1)$	$(0,1,1)$	$U(4)$



- **Green-Schwarz** mech.: $\prod_{x\in\{a,c,d,h\}} U(1)_x \to$ $\boxed{U(1)_Y \times \mathbb{Z}_3}$
  - **perturbatively** $U(1)^3_{\text{massive}} \supset U(1)_{PQ} = U(1)_c - U(1)_d$
  - **non-pert.** only: $\boxed{SU(3) \times SU(2) \times U(1)_Y \times \mathbb{Z}_3 \times SU(4)_{\text{hidden}}}$

Ex: MSSM spectrum on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_6 \times \Omega\mathcal{R})$

Ecker, **G.H.**, Staessens '15

- **MSSM matter** of $\boxed{(SU(3)_a \times USp(2)_b \times SU(4)_h)^{(U(1)_{PQ}),\mathbb{Z}_3}_{U(1)_Y}}$:

$$3 \times \left[(\mathbf{3},\mathbf{2},\mathbf{1})^{(0),0}_{1/6} + \underbrace{2 \times (\overline{\mathbf{3}},\mathbf{1},\mathbf{1})^{(1),1}_{1/3} + (\mathbf{3},\mathbf{1},\mathbf{1})^{(1),1}_{-1/3}} + (\overline{\mathbf{3}},\mathbf{1},\mathbf{1})^{(1),1}_{-2/3}\right]$$

$$+ 3 \times \left[\underbrace{(\mathbf{1},\mathbf{2},\mathbf{1})^{(1),1}_{1/2} + 2 \times (\mathbf{1},\mathbf{2},\mathbf{1})^{(1),1}_{-1/2}} + (\mathbf{1},\mathbf{1},\mathbf{1})^{(-2),1}_{0} + (\mathbf{1},\mathbf{1},\mathbf{1})^{(0),0}_{1}\right]$$

$$= 3 \times \left[Q_L + \overbrace{2 \times d_R + \overline{d_R}} + u_R\right] + 3 \times \left[\overbrace{H_u/\overline{L} + 2 \times L} + \nu_R + e_R\right]$$

- **Higgses**: $3 \times \left[(\mathbf{1},\mathbf{2},\mathbf{1})^{(1),1}_{1/2} + h.c.\right] + \tilde{2} \times \left[(\mathbf{1},\mathbf{2},\mathbf{1})^{(-1),2}_{1/2} + h.c.\right]$
- **axions:** $3 \times [\Sigma^{cd} + \tilde{\Sigma}^{cd}] = \mathbf{3} \times \left[(\mathbf{1},\mathbf{1},\mathbf{1})^{(-2),1}_{\mathbf{0}} + \mathbf{h.c.}\right]$

- **SM vector-like states:** $(5_{\mathbf{Anti}_b} + 4_{\mathbf{Adj}_c} + 5_{\mathbf{Adj}_d}) \times (\mathbf{1},\mathbf{1},\mathbf{1})^{(0),0}_{0} +$

$$+ \left[2 \times (\mathbf{3},\mathbf{1},\overline{\mathbf{4}})^{(0),1}_{1/6} + (\mathbf{3},\mathbf{1},\mathbf{4})^{(0),2}_{1/6} + 2 \times (\mathbf{3_A},\mathbf{1},\mathbf{1})^{(0),0}_{1/3} + h.c.\right]$$

$$+ \left[3 \times (\mathbf{1},\mathbf{1},\mathbf{1})^{(0),0}_{1} + (\mathbf{1},\mathbf{1},\mathbf{1})^{(-2),1}_{1} + 2 \times (\mathbf{1},\mathbf{1},\mathbf{6_A})^{(0),1}_{0} + h.c.\right]$$

$$+ 3 \times (\mathbf{1},\mathbf{2},\mathbf{4})^{(0),2}_{0} + 6 \times (\mathbf{1},\mathbf{1},\overline{\mathbf{4}})^{(-1),0}_{-1/2} + 3 \times (\mathbf{1},\mathbf{1},\overline{\mathbf{4}})^{(-1),0}_{1/2} + 3 \times (\mathbf{1},\mathbf{1},\mathbf{4})^{(-1),1}_{1/2}$$