F-theory, M5-branes and N=4 SYM with Varying Coupling

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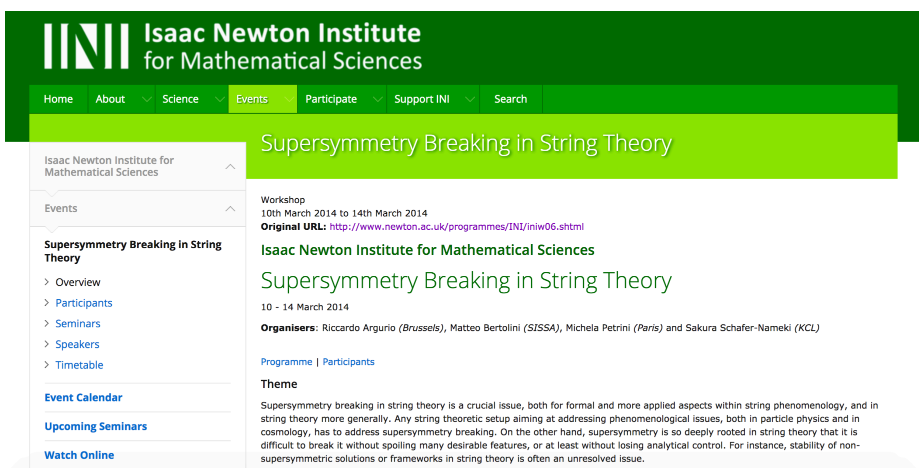
COST Conference Milano, February 23, 2017

`1610.03663` with Benjamin Assel
`1612.05640` with Craig Lawrie, Timo Weigand



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10 - 14 March 2014
Organisers: Riccardo Argurio (Brussels), Matteo Bertolini (SISSA), Michela Petrini (Paris) and Sakura Schafer-Nameki (KCL)
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Theme
Supersymmetry breaking in string theory is a crucial issue, both for formal and more applied aspects within string phenomenology, and in string theory more generally. Any string theoretic setup aiming at addressing phenomenological issues, both in particle physics and in cosmology, has to address supersymmetry breaking. On the other hand, supersymmetry is so deeply rooted in string theory that it is difficult to break it without spoiling many desirable features, or at least without losing analytical control. For instance, stability of non-supersymmetric solutions or frameworks in string theory is often an unresolved issue.

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Cosmology:
Daniel Baumann (DAMTP), Enrico Pajer (Utrecht)
Phenomenology:
David Shih (Rutgers), Florian Staub (CERN)
Geometry:
David Morrison (UCSB), Andreas Braun (Oxford)
Effective Actions:
Mariana Grana (Saclay), Hagen Triendl (CERN)
Speakers at the Workshops include:
Steve Abel (Durham)
Paolo Creminelli (ICTP)
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Plan

Goal: Understanding the D3-brane sector in F-theory.

I. F-theory in a Flash

  # Geometric progress

  # 8d, 6d, 4d, 2d

II. D3-brane Sector:

  # D3s in F/M5s in M

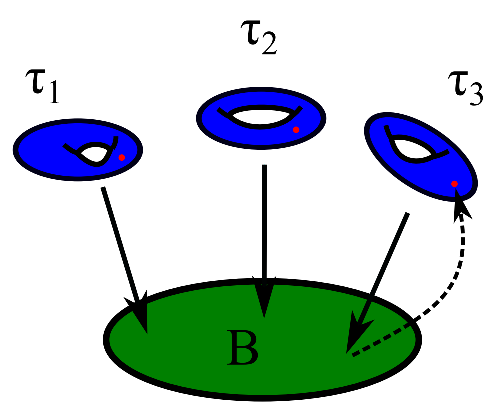
  # Defect theories

  # New chiral 2d $(0,2)$ Theories

I. F-theory in a Flash

F-theory and Elliptic Fibrations

F-theory on an elliptically fibered Calabi-Yau Y^{τ}_{6-d} results in $N=1$ vacua in $\mathbb{R}^{1,2d-1}$, with $\tau = C_0 + ie^{-\phi}$ axio-dilaton and B the spacetime of Type IIB string theory:



$$\begin{array}{ccc} \mathbb{E}_\tau & \rightarrow & Y^{\tau}_{6-d} \\ & & \downarrow \\ & & B_{5-d} \end{array}$$

⇒ $\mathbb{E}_\tau$ fibers = Tori $\mathbb{C}/\mathbb{Z} \oplus \tau\mathbb{Z}$ with marked point O. There exists a "zero section" σ_0: $B \rightarrow \mathbb{E}_\tau : b \mapsto O$

⇒ For such fibrations there is a Weierstrass form with $O = [0, 1, 1]$

$$y^2 = x^3 + fxw^4 + gw^6 \qquad [w, x, y] \in \mathbb{P}(1, 2, 3)$$

Effective Theory from M/F duality

$$\begin{tikzcd}
\text{M-theory on } Y_{6-d} \arrow[r, "\mathrm{Vol}(\mathbb{E}_\tau)=R_M R_A \to 0"] \arrow[d, "R_M"'] & \text{F-theory on } Y_{6-d} \arrow[d] \\
\mathbb{R}^{1,d-2} \arrow[r, "R_A \sim \frac{1}{R_B} \to 0"] & \begin{array}{c} N=1 \text{ Gauge Theory} \\ \text{with Gravity on } \mathbb{R}^{1,d-1} \end{array}
\end{tikzcd}$$

Gauge bosons and Singular Fibers

Reduce M-theory 3-form along $(1,1)$ forms $\omega^{(1,1)}$ in fiber:

$$C_3 = \omega^{(1,1)} \wedge A$$

Additional (1,1) forms from singularities:

- Elliptic curve is $y^2 = x^3 + fxw^4 + gw^6$ singular if

$$\Delta = 4f^3 + 27g^2 = 0$$

  Here Δ depends on base:

$$\Delta(z) = O(z^n) \quad \Leftrightarrow \quad z = 0 \text{ is surface } S \subset B$$

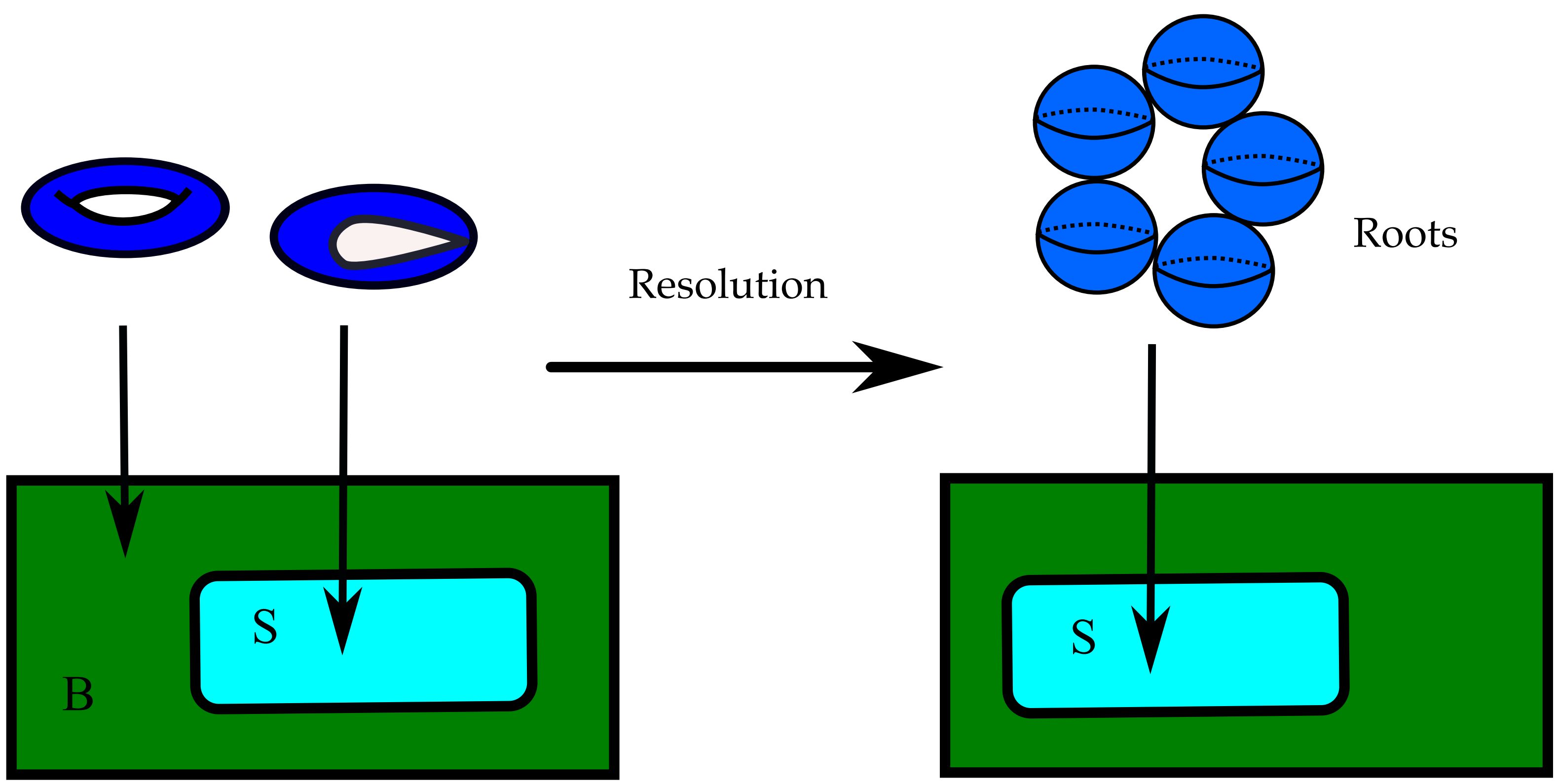
- Kodaira fibers from resolutions of singular fibrations
- Physics:
  Syncs with 7-branes intuition in IIB, which sources F_9 and $\tau \sim \log(x - x_0)$ undergoes monodromy $SL_2\mathbb{Z}$

F-theory and Singular Fibers: Codim 1

Kodaira classification of singular fibers: Resolution of singularities results in collection of rational curves $\mathbb{P}^1$ which intersect in ADE-type Dynkin diagrams:

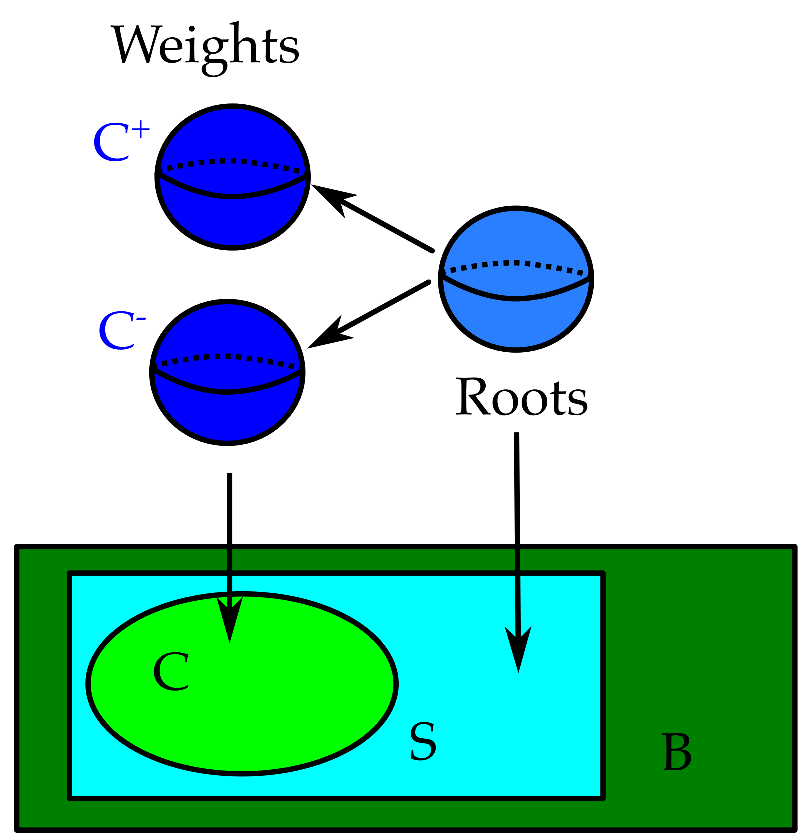
Affine roots of $\mathfrak{g} \leftrightarrow$ Fibral rational curves

Gauge bosons from $C_3 = A_i \wedge \omega_i^{(1,1)}$ and wrapped M2



F-theory and Singular Fibers: Codim 2

Codim 2: Roots can split into weights of matter representation R of $\mathfrak{g}$:



E.g. $SU(n)$, fundamental weights L_i: $\alpha_i = L_i + (-L_{i+1})$.

How exactly this happens: as systematically understood as Kodaira in codim 1 [Hayashi, Lawrie, Dave Morrison, SSN]

F-theory on elliptic CY

F-theory on an elliptically fibered Calabi-Yau

$$\mathbb{E}_\tau \to Y^\tau_{6-d} \to B_{5-d}$$

results in $N = 1$ vacua in $\mathbb{R}^{1,2d-1}$ with the geometric singularities above codim i in the base have the following physical interpretation:

Codim$_\mathbb{C}$	$\mathbb{P}^1$s in Fiber	7-brane Gauge Theory
1	Simple roots	Gauge algebra $\mathfrak{g}$
2	Weights for Reps **R**	Matter in **R**
3	Splitting gauge invariantly	Cubic interactions
4	Further gauge invariant splitting	Quartic interactions

2n-dimensional F-theory Vacua

Fiber geometry relatively universal. Base B

- 8d: K3, Geometry highly constrained $B_1 = \mathbb{P}^1$ etc.
- 6d: Classification of $N = (1, 0)$ SCFTs, Geometry of B_2 constrained (Grassi, Gross)+ Anomalies
- 4d: $N = 1$, MSSM/GUTs, Geometry of B_3 largely unconstrained and freedom of Fluxes
- 2d: $N = (0, 2)$ gauge theories + gravity, Geometry of B_4 largely unconstrained, freedom of Fluxes and necessity of D3-branes (tadpole).

II. D3-brane Sector in F-theory

4d $N = 4$ SYM with varying τ

F-theory is IIB with varying τ, where there is also a self-duality group $SL_2\mathbb{Z}$, which descends upon D3-branes to the Montonen-Olive duality group of $N = 4$ SYM.

4d $N = 4$ SYM has an $SL_2\mathbb{Z}$ duality group acting on the complexified coupling

$$\tau = \frac{\theta}{2\pi} + i\frac{4\pi}{g^2}\,, \qquad \tau \to \frac{a\tau + b}{c\tau + d}\,,$$

$ad - bc = 1$ and integral. Incidentally: the gauge group G maps to the Langlands dual group $G^\vee$.

Usually, we consider τ constant in the 4d spacetime.

Coming from F-theory, it's very natural to ask whether we can define a version of $N = 4$ SYM with varying τ, compatible with the $SL_2\mathbb{Z}$ action.

$\Rightarrow$ Network of 3d walls, 2d and 0d duality defects in $N = 4$.

Duality Defects

Variation of τ without singular loci are trivial. So the interesting physics will happen along the 4d space-time where τ is singular.

$\Rightarrow$ around such singular loci, τ will undergo an $SL_2\mathbb{Z}$ monodromy.

Usual lore: τ as the complex structure of an elliptic curve $\mathbb{E}_\tau$
$\Rightarrow$ Lift to M5-branes

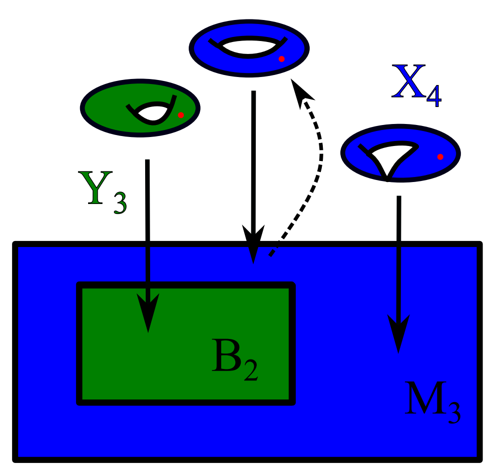
$\Rightarrow$ Setup: elliptic fibration over the 4d spacetime with $N = 4$ SYM in the bulk and duality defects (2d), which can intersect in 0d.

Key: M5-brane point of view

$\{$6d $(2,0)$ theory on $\mathbb{E}_\tau \times \mathbb{R}^4\}$ = $\{N = 4$ SYM on $\mathbb{R}^4$ with coupling τ $\}$

So the setup that we will study is:

$\{$6d $(2,0)$ theory on a singular elliptic fibration$\}$
= $\{$ 4d $N = 4$ SYM with varying τ and duality defects$\}$



Setups:

\# Setup 1:
τ varies over 4d space (with Benjamin Assel)
$\Rightarrow Y_3$ elliptic three-fold $\subset$ elliptic CY4

\# Setup 2:
τ varies onver a 2d space: 2d $(0, p)$ scfts (with C. Lawrie, T. Weigand)
$\Rightarrow$ D3s on curves in the base of CYn.

In both setups: M5-brane point of view will be instrumental.

The 6d $(2,0)$ Theory

Lorentz and R-symmetry:

$$SO(1,5)_L \times Sp(4)_R \subset OSp(6|4)$$

Tensor multiplet:

$$\begin{aligned} \mathcal{B}_{MN}: &\qquad (\mathbf{15},\mathbf{1}) \qquad \text{with selfduality } \mathcal{H} = d\mathcal{B} = *_6\mathcal{H} \\ \Phi^{\widehat{m}\widehat{n}}: &\qquad (\mathbf{1},\mathbf{5}) \\ \rho^{\widehat{m}}: &\qquad (\bar{\mathbf{4}},\mathbf{4}) \end{aligned}$$

Abelian EOMs:

$$\mathcal{H}^- = d\mathcal{H} = 0\,, \qquad \partial^2\Phi^{\widehat{m}\widehat{n}} = 0\,, \qquad \not{\partial}\rho^{\widehat{m}} = 0\,.$$

Setup 1: M5-branes on Elliptic 3-folds

[Assel, SSN]

An elliptic fibration $\mathbb{E}_\tau \to Y_3 \to B$ (Y not CY) has metric

$$ds^2_Y = \frac{1}{\tau_2}\left((dx+\tau_1 dy)^2 + \tau_2^2 dy^2\right) + g^B_{\mu\nu} db^\mu db^\nu \,.$$

Pick a frame e^a for the base B and

$$e^4 = \frac{1}{\sqrt{\tau_2}}(dx+\tau_1 dy)\,, \qquad e^5 = \sqrt{\tau_2} dy\,.$$

Let Y_3 be a Kähler three-fold, so the holonomy is reduced to $U(3)_L$:

$$SO(6)_L \to U(3)_L$$
$$\mathbf{4} \to \mathbf{3}_1 \oplus \mathbf{1}_{-3}\,.$$

On a curve space: Killing spinor equation with ∇_M connection

$$(\nabla_M - A^R_M)\eta = 0$$

R-symmetry background $\Rightarrow$ constant spinor wrt twisted connection.

M5-branes on Elliptic 3-folds: Twist

\# Standard geometric twist: $U(1)_L$ with $U(1)_R$

$$
\begin{gathered}
Sp(4)_R \rightarrow SU(2)_R \times U(1)_R \\
\mathbf{4} \rightarrow \mathbf{2}_1 \oplus \mathbf{2}_{-1} \,.
\end{gathered}
$$

\# Topological Twist

$$
T_{U(1)_{\text{twist}}} = (T_{U(1)_L} - 3T_{U(1)_R})
$$

implies that the supercharge decomposes as

$$
\begin{aligned}
SO(6)_L \times Sp(4)_R \quad &\rightarrow \quad SU(3)_L \times SU(2)_R \times U(1)_{\text{twist}} \times U(1)_R \\
(\mathbf{4},\mathbf{4}) \quad &\rightarrow \quad (\mathbf{3},\mathbf{2})_{-2,1} \oplus (\mathbf{3},\mathbf{2})_{4,-1} \oplus (\mathbf{1},\mathbf{2})_{-6,1} \oplus (\mathbf{1},\mathbf{2})_{0,-1}
\end{aligned}
$$

$\Rightarrow (\mathbf{1},\mathbf{2})_{0,-1}$ give two scalar supercharges

Now consider 6d spacetime as $\mathbb{E}_\tau \to Y_3 \to B_2$ with coordinates $x^0, \cdots, x^5$, and (x^4, x^5) the directions of the elliptic fiber.

The spin connection along $U(1)_L$ is

$$\Omega^{U(1)_L} = -\frac{1}{6}(\Omega^{01} + \Omega^{23} + \Omega^{45}),$$

and the twist corresponds to turning on the background gauge field

$$A^{U(1)_R} = -3\Omega^{U(1)_L}.$$

The base B_2 is Kähler as well, so the holonomy lies in $U(1)_\ell \times SU(2)_\ell \subset U(3)_L$ with the $U(1)$ generators given by

$$T_L = T_\ell + 2T_{45}$$

Key: $SO(2)_{45}$ rotation is along the fiber, and the non-trivial fibration is characterized through a connection in this $SO(2)_{45}$ direction and the spin connection is

$$\mathcal{A}_D = \omega^D = -\frac{\partial_a \tau_1}{4\tau_2} e^a$$

Duality Twist

This means: from the 4d point of view the topological twisting requires

$$\mathcal{A}_D = \omega^D = -\frac{\partial_a \tau_1}{4\tau_2} e^a$$

The associated $U(1)$ is in fact what is known as the "bonus symmetry" of abelian $N = 4$ SYM [Intrilligator] and we recovered the duality twist of N=4 SYM [Martucci] from the M5-brane theory.

The bonus symmetry exists for the abelian $N = 4$ SYM and acts as follows on the supercharges for $ab - cd = 1$

$$\begin{aligned} Q^{\dot m} &\to e^{-\frac{i}{2}\alpha(\gamma)} Q^{\dot m} \\ \tilde{Q}^{m} &\to e^{\frac{i}{2}\alpha(\gamma)} \tilde{Q}^{m} \end{aligned} \qquad \text{where} \qquad e^{i\alpha(\gamma)} = \frac{c\tau + d}{|c\tau + d|}$$

$$\phi^{\widehat{i}} \to \phi^{\widehat{i}}, \qquad \lambda_+^{\dot m} \to e^{-\frac{i}{2}\alpha(\gamma)} \lambda_+^{\dot m}, \qquad \lambda_-^{m} \to e^{\frac{i}{2}\alpha(\gamma)} \lambda_-^{m}$$

$$F_{\mu\nu}^{(\pm)} \to e^{\mp i\alpha(\gamma)} F_{\mu\nu}^{(\pm)} \qquad F^{(\pm)} \equiv \sqrt{\tau_2}\left(\frac{F \pm \star F}{2}\right)$$

Duality Twisted $N = 4$ SYM from 6d

6d topological twist + dim reduction to B gives an $N = 4$ SYM with varying τ over a Kähler base B

$$
\begin{aligned}
S^{U(1)}_{\text{total}} &= \frac{1}{4\pi} \int_B \tau_2 F_2 \wedge \star F_2 - i\tau_1 F_2 \wedge F_2 \\
&\quad + \frac{8}{\pi} \int_B \bar{\partial} \star \psi^{\alpha}_{(1,0)}\, \chi_{(0,0)\alpha} - \partial \psi^{\alpha}_{(1,0)} \wedge \rho_{(0,2)\alpha} - \partial_{\mathcal{A}} \star \tilde{\psi}^{\dot{\alpha}}_{(0,1)}\, \tilde{\chi}_{(0,0)\dot{\alpha}} + \bar{\partial}_{\mathcal{A}} \tilde{\psi}^{\dot{\alpha}}_{(0,1)} \wedge \tilde{\rho}_{(2,0)\dot{\alpha}} \\
&\quad - \frac{1}{4\pi} \int_B \bar{\partial} \varphi^{\alpha\dot{\alpha}} \wedge \star \partial \varphi_{\alpha\dot{\alpha}} + 2\bar{\partial}_{\mathcal{A}} \sigma_{(2,0)} \wedge \star \partial_{\mathcal{A}} \tilde{\sigma}_{(0,2)}
\end{aligned}
$$

and non-abelian extension (see paper with Ben Assel).

The twisted fields are form fields and sections of the $\mathcal{A}_D$ bundle specified by the charges:

	$F_2^{(\pm)}$	$\varphi^{\alpha\dot{\alpha}}$	$\sigma_{(2,0)}$	$\tilde{\sigma}_{(0,2)}$	$\chi^{\alpha}_{(0,0)}$	$\tilde{\chi}^{\dot{\alpha}}_{(0,0)}$	$\psi^{\alpha}_{(1,0)}$	$\tilde{\psi}^{\dot{\alpha}}_{(0,1)}$	$\rho^{\alpha}_{(0,2)}$	$\tilde{\rho}^{\dot{\alpha}}_{(2,0)}$
$L_D^{q/2}$	∓ 2	0	-2	2	0	-2	0	2	0	-2

Singular Elliptic Curves and Defects

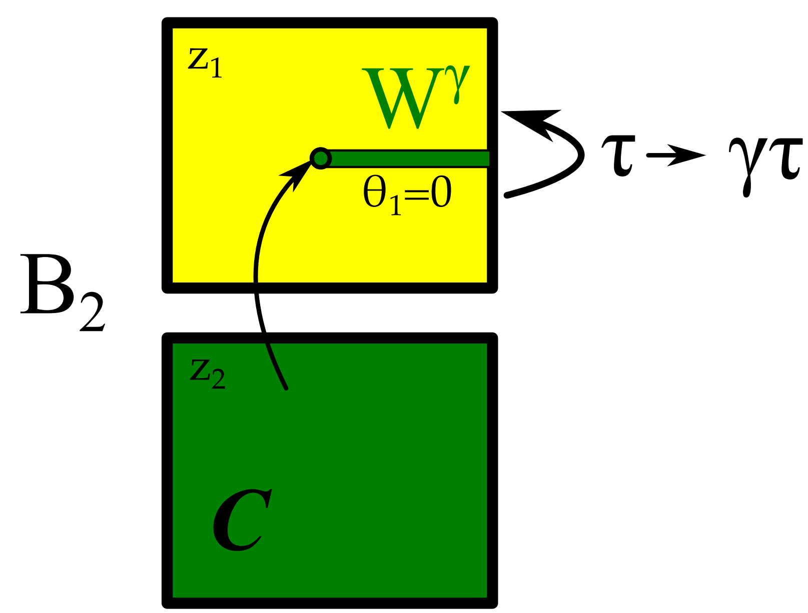
We can describe the elliptic fibration by $\mathbb{E}_\tau$ in terms of a Weierstrass model

$$y^2 = x^3 + fx + g$$

f and g sections $K_B^{-2/-3}$ and the singular loci are

$$\Delta = 4f^3 + 27g^2 = 0\,.$$

Close to a singular locus $z_2 = 0$, $\tau \sim i \log z_2 + \cdots$ with a branch-cut in the complex plane z_2. For the M5 this is relevant along $\Delta \cap B$:



Gauge theoretic description of walls and defects

Locally we can cut up $B = \cup B_i$ and W_{ij} 3d walls between these regions, where τ has a branch-cut.

Define

$$F_D = \tau_1 F + i\tau_2 \star F$$

then the action of $\gamma \in SL_2\mathbb{Z}$ monodromy on the gauge field is

$$(F_D^{(j)}, F^{(j)})\Big|_{W_{ij}} = \gamma(F_D^{(i)}, F^{(i)})\Big|_{W_{ij}}$$

This maps the gauge part $S_F = -\frac{i}{4\pi}\int_B F \wedge F_D$ to itself, except for an offset on the 3d wall (see also [Ganor])

$$S^\gamma_{W_{ij}} = -\frac{i}{4\pi}\int_{W_{ij}} \left(A^{(i)} \wedge F_D^{(i)} - A^{(j)} \wedge F_D^{(j)}\right)$$

E.g. $\gamma = T^k$ this is a level k CS term.

Chiral Duality Defects

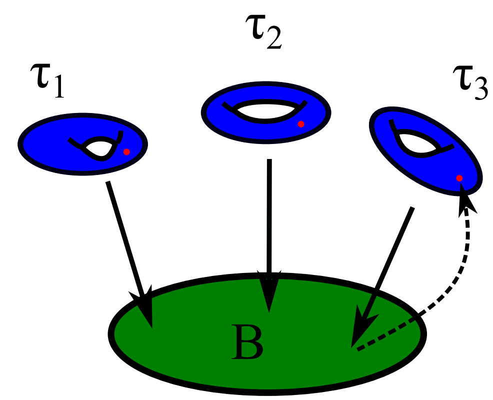
The wall action S^γ is neither supersymmetric nor gauge invariant. At the boundary of the wall $\partial W = \mathcal{C}$ this induces chiral dofs: e.g. for the T^k wall this is simply a chiral WZW model with β_i, $i = 1, \cdots, k$, with $\star_2 d\beta_i = i d\beta_i$ [Witten]

$$S_{\mathcal{C}} = \sum_{i=1}^{k} -\frac{1}{8\pi}\int_{\mathcal{C}} \star_2 (d\beta_i - A)\wedge(d\beta_i - A) - \frac{i}{4\pi}\int_{\mathcal{C}} \beta_i\, F$$

Under gauge transformations $A \to A + d\Lambda$, $\beta_i \to \beta_i + \Lambda$ this generates $\int F\Lambda$ which cancels the anomaly from the 3d wall.

Duality Defects from M5-branes

From the elliptic fibration and M5-brane we can apply this to any γ:



Singular fibers resolve into collections of $S^2 = \mathbb{P}^1$s, intersecting in affine ADE Dynkin diagrams.
Each resolution spheres gives rise to an $\omega^{(1,1)}$ form, along which we can expand $\mathcal{B}$

$$d\mathcal{B} = \sum_{i=1}^{k-1} \left(\partial_z b_i dz \wedge \omega^i_{(1,1)} + \partial_{\bar{z}} b_i d\bar{z} \wedge \omega^i_{(1,1)} \right)$$

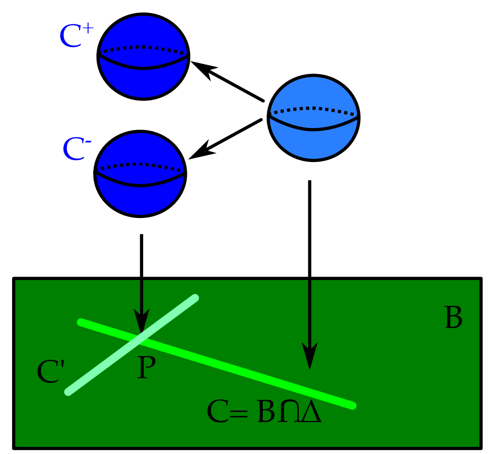
Imposing self-duality, and redefining the basis of chiral modes b_i with the "section" of the elliptic fibration, identifies these modes with β_i.

Point-defects

These chiral $(0,2)$ supersymmetric defects can intersect at points

$$P_{\alpha\beta} = \{z_\alpha = z_\beta = z = 0\} = \mathcal{C}_\alpha \cap \mathcal{C}_\beta = B \cap \Delta_\alpha \cap \Delta_\beta$$

Geometrically: Kodaira fiber $\mathbb{P}^1$s become further reducible $\mathbb{P}^1_i \to C_+ + C_-$



Duality defects form network and at intersections:

$$\left(\int_{C^+} + \int_{C^-}\right)\mathcal{B} = \int_{\mathbb{P}^1_i} \mathcal{B} \qquad \to \qquad \beta_+ + \beta_- = \beta_i$$

Such point-intersections are generic e.g. in CY4.

Example:

D3-branes wrapping B_2 intersecting discriminant loci in

$$\Delta_1 \cap B = \mathcal{C} \leftrightarrow SU(n) \qquad \Delta_2 \cap B = \tilde{\mathcal{C}} \leftrightarrow SU(m)$$

E.g. fibers are given in terms of simple roots F_i, $i = 0, 1, \cdots, n-1$ and $\tilde{F}_j$, $j = 0, 1, \cdots, m-1$ and there are chiral modes localized on each curve

$$\mathcal{C}: \qquad \beta_i\,, \qquad i = 0,1,2,3,4\,, \qquad \tilde{\mathcal{C}}: \qquad \tilde{\beta}_i\,, \qquad i = 0,1,2$$

The fibers in codim 2 split as, e.g. for $SU(5)$ and $SU(3)$: $C^{\pm}_{ij} \equiv \pm(L_i + \tilde{L}_j)$.

$$\mathcal{C}\ :\quad \begin{cases} F_0 & \to F_0' + C^-_{\tilde{3}1} \\ F_1 & \to F_1 \\ F_2 & \to C^+_{\tilde{2}2} + C^-_{\tilde{2}3} \\ F_3 & \to F_3 \\ F_4 & \to C^-_{\tilde{1}5} + C^+_{\tilde{1}4} \end{cases} \qquad \tilde{\mathcal{C}}\ :\quad \begin{cases} \tilde{F}_0 & \to \tilde{F}_0' + C^-_{\tilde{1}5} \\ \tilde{F}_1 & \to C^+_{\tilde{1}4} + F_3 + C^-_{\tilde{2}3} \\ \tilde{F}_2 & \to C^+_{\tilde{2}2} + F_1 + C^-_{\tilde{3}1} \end{cases}$$

In codim 3 the $SU(5)$ and $SU(3)$ singularities collide at points $P = \mathcal{C} \cap \tilde{\mathcal{C}}$ in B:

$$\left(F_{\mathcal{C}} \sum_{i=0}^{4} \beta_i \right) \Bigg|_P = F_{\mathcal{C}} \left(\beta^+_{\tilde{3}5} + \beta^-_{\tilde{3}1} + \beta_1 + \beta^+_{\tilde{2}2} + \beta^-_{\tilde{2}3} + \beta_3 + \beta^-_{\tilde{1}5} + \beta^+_{\tilde{1}4} \right) \Bigg|_P$$

and likewise

$$\left(F_{\tilde{\mathcal{C}}} \sum_{i=0}^{2} \tilde{\beta}_i \right) \Bigg|_P = F_{\tilde{\mathcal{C}}} \left(\beta^+_{\tilde{3}5} + \beta^-_{\tilde{1}5} + \beta^+_{\tilde{1}4} + \beta_3 + \beta^-_{\tilde{2}3} + \beta^+_{\tilde{2}2} + \beta_1 + \beta^-_{\tilde{3}1} \right) \Bigg|_P$$

Locally, this enhances the flavor symmetry of the 2d chiral models to $SU(n+m)$.

More generally: What happens for $\gamma \in SL_2\mathbb{Z}$? Defect theory will be chiral cft, with flavor symmetry dictated by the singular fiber geometry.

Setup 2: New 2d $(0,2)$ Theories

Consider now $N=4$ SYM on $\mathbb{R}^{1,1}\times C$, with τ-varying only over the curve C:

$$SO(1,4)_L \to SO(1,1)_L \times U(1)_L$$

and to preserve supersymmetry, consider $U(1)_R \subset SU(4)_R$:

$$SU(4)_R \quad \to \quad \begin{array}{rl} SO(4)_T \times \underline{U(1)_R} & \qquad \text{CY}_3 \text{ Duality-Twist: } (0,4) \\ SU(2)_R \times \underline{U(1)_R} \times SO(2)_T & \qquad \text{CY}_4 \text{ Duality-Twist: } (0,2) \\ SU(3)_R \times \underline{U(1)_R} & \qquad \text{CY}_5 \text{ Duality-Twist: } (0,2) \end{array}$$

Geometric embedding corresponds to D3-branes on $C\times\mathbb{R}^{1,1}$ with

$$C \subset B_{n-1} = \text{Base of the elliptic } CY_n$$

CY_n Duality-Twist:

$$T_C^{\text{twist}} = \frac{1}{2}(T_C + T_R) \qquad T_D^{\text{twist}} = \frac{1}{2}(T_D + T_R)\,.$$

Example: CY_4-Duality Twist of $N=4$ SYM

$$
\begin{aligned}
& SU(2)_R \times SO(1,1)_L \times U(1)_C^{\text{twist}} \times U(1)_D^{\text{twist}} \times SO(2)_T \times U(1)_R \\
A: \quad & \mathbf{1}_{2,0,*,0,0} \oplus \mathbf{1}_{-2,0,*,0,0} \oplus \mathbf{1}_{0,1,*,0,0} \oplus \mathbf{1}_{0,-1,*,0,0} \\
& = v_+ \oplus v_- \oplus \bar{a} \oplus a \\
\phi: \quad & \mathbf{1}_{0,0,0,2,0} \oplus \mathbf{1}_{0,0,0,-2,0} \oplus \mathbf{2}_{0,\frac{1}{2},\frac{1}{2},0,1} \oplus \mathbf{2}_{0,-\frac{1}{2},-\frac{1}{2},0,-1} \\
& = \bar{g} \oplus g \oplus \varphi \oplus \bar{\varphi} \\
\Psi: \quad & \mathbf{2}_{1,\frac{1}{2},\frac{1}{2},1,0} \oplus \mathbf{1}_{1,1,1,-1,1} \oplus \mathbf{1}_{1,0,0,-1,-1} \oplus \mathbf{2}_{-1,-\frac{1}{2},\frac{1}{2},1,0} \oplus \mathbf{1}_{-1,0,1,-1,1} \oplus \mathbf{1}_{-1,-1,0,-1,-1} \\
& = \mu_+ \oplus \psi_+ \oplus \gamma_+ \oplus \rho_- \oplus \lambda_- \oplus \beta_- \\
\widetilde{\Psi}: \quad & \mathbf{2}_{1,-\frac{1}{2},-\frac{1}{2},-1,0} \oplus \mathbf{1}_{1,-1,-1,1,-1} \oplus \mathbf{1}_{1,0,0,1,1} \oplus \mathbf{2}_{-1,\frac{1}{2},-\frac{1}{2},-1,0} \oplus \mathbf{1}_{-1,0,-1,1,-1} \oplus \mathbf{1}_{-1,1,0,1,1} \\
& = \tilde{\mu}_+ \oplus \tilde{\psi}_+ \oplus \tilde{\gamma}_+ \oplus \tilde{\rho}_- \oplus \tilde{\lambda}_- \oplus \tilde{\beta}_- \,.
\end{aligned}
$$

Geometric identification:

$$N_{C/B_3} : (q_C^{\text{twist}}, q_D^{\text{twist}}) = \left(-\frac{1}{2}, -\frac{1}{2}\right)$$

and q_D charge give power of $\mathcal{L}_D = K_B^{-1}|_C$

Fermions	Bosons	(0,2) Multiplet	Zero-mode Cohomology
μ_+ $\tilde{\mu}_+$	φ $\bar{\varphi}$	Chiral Conjugate Chiral	$h^0(C, N_{C/B_3})$
$\tilde{\psi}_+$ $\tilde{\psi}_+$	a $\bar{a}$	Chiral Conjugate Chiral	$h^0(C, K_C \otimes \mathcal{L}_D) = g - 1 + c_1(B_3) \cdot C$
γ_+ $\tilde{\gamma}_+$	g $\bar{g}$	Chiral Conjugate Chiral	$h^0(C) = 1$
ρ_- $\tilde{\rho}_-$	— —	Fermi Conjugate Fermi	$h^1(C, N_{C/B_3}) = h^0(C, N_{C/B_3}) - c_1(B_3) \cdot C$
β_- $\tilde{\beta}_-$	— —	Fermi Conjguate Fermi	$h^1(C) = g$
λ_- $\tilde{\lambda}_-$	v_+ v_-	Vector	0

Central Charges

From the $N = 4$ with duality twist, the zero modes contribute

$$\begin{aligned} c_R =& 3(g + c_1(B_3) \cdot C + h^0(C, N_{C/B_3})) \\ c_L =& 3(g + h^0(C, N_{C/B_3})) + c_1(B_3) \cdot C \end{aligned}$$

However, this neglets the contributions from the singularities (unless B_3 is Calabi-Yau itself and the fibration is trivial).

Quick F-theory excursion: singular loci of τ correspond to 7-branes, and additional defect modes are $3 - 7$ strings.

Direct computation from 6d $(2, 0)$ or anomalies, on the elliptic surface $\mathbb{E}_\tau \to C$ times $\mathbb{R}^{1,1}$ (much like in the earlier discussion) yields

$$\delta c_L^{\text{defects}} = 8 c_1(B) \cdot C.$$

Discussion of other cases:

\# CY_3 Duality twist $N = (0,4)$:

$c_R = 3C \cdot C N_c^2 + 3c_1(B) \cdot C N_c + 6\,, \qquad c_L = 3C \cdot C N_c^2 + 6c_1(B) \cdot C N_c + 6$

This is dual to M5-branes on elliptic surfaces in CY three-folds, i.e. MSW-string. Computation of elliptic genera see e.g. [Haghighat, Murthy, Vandoren, Vafa].

\# CY_5 Duality twist:

$$
\begin{aligned}
c_L &= 3(g + h^0(C, N_{C/B_4}) - 1) + 9c_1(B_4) \cdot C \\
c_R &= 3(g + c_1(B_4) \cdot C + h^0(C, N_{C/B_4}) - 1)
\end{aligned}
$$

Application to 2d $(0,2)$ vacua from CY_5 compactifications of F-theory [SSN, Weigand], [Apruzzi, Hassler, Heckman, Melnikov]. Tadpole cancellation requires D3-branes wrapped on curves in the class

$$C = \frac{1}{24} c_4(Y_5)|_{B_4}$$

BPS-equations and Hitchin moduli space

For τ constant, $N = 4$ SYM on $C \times \mathbb{R}^{1,1}$ with Vafa-Witten twist, gives rise to a sigma-model into the Hitchin moduli space, which for the abelian case is just flat connections [Bershadsky, Johansen, Sadov, Vafa].

In all duality-twisted theories the BPS equations imply

$$\mathcal{F}_{\mathcal{A}} = \frac{1}{2}\Big(\bar{\partial}_{\mathcal{A}}(\sqrt{\tau_2}a) - \partial_{\mathcal{A}}(\sqrt{\tau_2}\bar{a})\Big) = 0$$

where the internal components of the gauge field $a, \bar{a}$ are

$$\begin{aligned} \sqrt{\tau_2}\bar{a} &\in \Gamma(\Omega^{0,1}(C, \mathcal{L}_D^{-1})) \\ \sqrt{\tau_2}a &\in \Gamma(\Omega^{0,0}(C, K_C \otimes \mathcal{L}_D)) \end{aligned}$$

In particular, for this abelian setup, the theory is a sigma-model into $U(1)_D$-twisted flat connections. $\rightarrow$ duality twisted Hitchin moduli space

Non-abelian Generalizations

We have seen: M5-brane point of view is useful in systematically assessing the contributions from singularities of τ, i.e. 2d defects.

So far this was $N = 4$ with $U(1)$ gauge group. To non-abelianize, there is a conceptual problem: bonus $U(1)_D$ is only known to exist in abelian $N = 4$, so the direct dimensional reduction from 4d is not really possible.

Start instead in 6d and reduce on an elliptic fibration: as shown in [Assel, SSN] this theory can be non-abelianized. E.g. $C \subset K3^\tau$ get non-abelian version of the heterotic string.

M2-branes on $C \times \mathbb{R}$ give rise to Super-QM: i.e. twisted version of the Bagger-Lambert-Gustavsson theory on C.

Summary

\# M5 on an elliptic three-fold give rise to N=4 SYM with varying τ, and a network of intersecting duality defects.

\# If τ varies only over a curve, we get 2d $(0,p)$ scfts from this, which correspond to N=4 on a curve with varying τ. These correspond to M5-branes on elliptic surfaces. Example: MSW-string in CY3.
These 2d $(0,p)$ SCFTs have interesting "F-theory" AdS-duals, i.e. varying-τ IIB solutions [Haghigat, Murthy, Vafa, Vandoren] [Couzens, Martelli, SSN, Wong]

$$AdS_3 \times S^3 \times CY_3^{\tau}$$

Similarly: AdS_3 solutions for 2d $(0,2)$ theories from CY4 in F-theory.