On the spectrum of CFTs with weakly broken higher-spin symmetry

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Based mainly on SG, Kirilin arXiv:1601.01310; SG, Guru-Charan, Kirilin, Prakash, Skvortsov arXiv:1610.08472; SG, Kirilin, Skvortsov arXiv: 1701.06997

Also work on closely related topics with L. Fei, I. Klebanov, G. Tarnopolsky

Unitarity bounds

- A classic result in CFT_d are the unitarity bounds. For a spin-*s* operator $J_{\mu_1\mu_2\cdots\mu_s}$ (in the symm. traceless of *SO(d)*):

$$\Delta_s \geq d - 2 + s\,, \qquad s \geq 1$$

- For a scalar operator $\mathcal{O}$:

$$\Delta_0 \geq \frac{d}{2} - 1$$

- Operators that saturate these inequalities belong to *short representations* of the conformal algebra.
- E.g., for a scalar operator, the shortening condition is just the free wave equation $\partial^2 \mathcal{O} = 0$, so $\mathcal{O}$ is a free conformal scalar field ($\Delta_0 = d/2 - 1$)

Conserved currents

- For a spin-*s* operator, saturation of the inequality bound implies that $J_{\mu_1\mu_2\cdots\mu_s}$ is a *conserved current*

$$\partial^\mu J_{\mu\mu_2\cdots\mu_s} = 0$$

- The cases *s=1* and *s=2* are familiar in any CFT. The case of exactly conserved currents of *s>2* is realized in free field theories. E.g., a free scalar field theory has the conserved currents of the form

$$J_{\mu_1\cdots\mu_s} = \sum_{k=0}^{s} c_{sk}\partial_{\{\mu_1}\cdots\partial_{\mu_k}\phi\partial_{\mu_{k+1}}\cdots\partial_{\mu_s\}}\phi$$

The HS currents in free scalar theory

- It is convenient to introduce a null "polarization vector" z^μ and construct the index-free object

$$\hat{J}_s(x,z) = J_{\mu_1\mu_2\cdots\mu_s} z^{\mu_1} z^{\mu_2} \ldots z^{\mu_s}$$

- Writing (define $\hat{\partial} \equiv z^\mu \partial_\mu$):

$$\hat{J}_s(x,z) = \sum_{k=0}^{s} c_{sk} \hat{\partial}^k \phi \hat{\partial}^{s-k} \phi = f_s(\hat{\partial}_1, \hat{\partial}_2)\phi(x_1)\phi(x_2)|_{x_{1,2}\to x}$$

- Imposing conservation and using the free scalar equation of motion, the function $f_s(u,v)$ can be fixed explicitly in terms of Gegenbauer polynomials

$$\hat{J}_s(x,z) = \left(\hat{\partial}_1 + \hat{\partial}_2\right)^s C_s^{\frac{d-3}{2}} \left(\frac{\hat{\partial}_1 - \hat{\partial}_2}{\hat{\partial}_1 + \hat{\partial}_2}\right) \phi(x_1)\phi(x_2)|_{x_{1,2}\to x}$$

Exact HS symmetry

- Exactly conserved currents J_s -> symmetries generated by conserved charges Q_s. E.g. $[Q_s, \phi] \sim \partial^{s-1}\phi$

$$\text{HS algebra:} \qquad [Q_{s_1}, Q_{s_2}] = \sum Q_s$$

- Infinite dimensional. A charge of *s>2* requires whole tower.

$$\text{E.g.:} \quad [Q_4, Q_4] \sim Q_2 + Q_4 + Q_6$$

- Exact HS symmetry is very constraining. Assuming a CFT with a *s=4* exactly conserved current, one may show
  - Full tower of conserved HS charges is present in the theory
  - Correlation functions are fixed to be those of a free field theory

Maldacena, Zhiboedov '11

Free O(N) model

- Let us consider *N* free massless scalars $\partial^2\phi^i = 0, \qquad i = 1, \dots, N$
- The conserved currents

$$\hat{J}^{ij} = \sum_{k=0}^{s} c_{sk}\hat{\partial}^{s-k}\phi^i\hat{\partial}^k\phi^j$$

decompose in irreps of the global O(N) symmetry

$$J_s^{ij} = J_s^{(ij)} + J_s^{[ij]} + J_s$$

- Let us focus on the *O(N) singlet sector*, which is relevant in the AdS/CFT applications of this model. Then, the singlet conserved currents J_s of all even spins and the scalar operator $J_0 = \phi^i\phi^i$ are the only "single-trace" in the singlet sector of the free O(N) model

AdS Higher-spin/vector model duality

- The singlet sector of the free O(N) scalar CFT is conjectured to be dual to the Vasiliev HS gravity theory in AdS_{d+1} [Klebanov, Polyakov ’02]

- In a series of works (’90-’92), Vasiliev discovered a fully non-linear, consistent theory of interacting massless *higher-spin gauge fields*. Non-zero cosmological constant is essential: vacuum is AdS or dS space. No smooth flat-space limit
- Originally developed in 4-dimensional space-time, but generalizations to arbitrary dimensions were later constructed [Vasiliev ‘03]

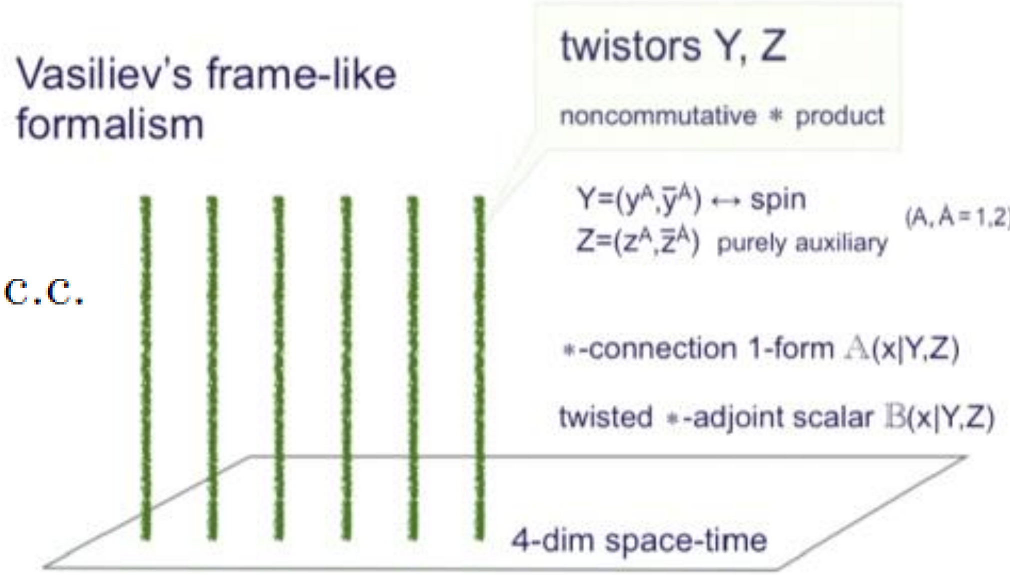
Vasiliev theory in 4d

- It involves a remarkable construction using ideas from twistor theory and non-commutative field theory

Vasiliev's equations:

$$dA + A * A = e^{i\theta_0} B * \kappa \, dz^\alpha dz_\alpha + \text{c.c.}$$

$$dB + A * B - B * \pi(A) = 0$$



- The equations of motion have a vacuum solution corresponding to anti-de Sitter (or de Sitter) space-time
- Spectrum of the theory can be found by solving linearized perturbations around vacuum

Spectrum of Vasiliev theory

- One finds

$$\text{Spectrum}: \qquad s = 1, 2, 3, \ldots, \infty \qquad \text{gauge fields}$$
$$s = 0, \quad m^2 = -2/\ell^2_{AdS} \quad \text{scalar}$$

- Crucial feature: consistency with non-linear gauge symmetries requires that the spectrum contains an *infinite tower* of higher spin fields
- Spectrum is just like a single Regge trajectory. Much simpler than full-fledged string theory
- A truncation to a minimal higher-spin theory with even spins s=0,2,4,6,…∞ is possible
- A massless spin 2 field is always part of the spectrum: the graviton!

Vasiliev Higher-Spin Gravity

- So, Vasiliev theory may be viewed as a peculiar *theory of gravity* which generalizes Einstein's theory by including an infinite set of massless fields of all spins
- Classical equations of motion are complicated (infinitely many auxiliary fields), but fully known. Interactions in principle can be read-off order by order

$$d\mathcal{A} + \mathcal{A} * \mathcal{A} = e^{i\theta_0} B * \kappa \, dz^{\alpha} dz_{\alpha} + e^{-i\theta_0} B * \bar{\kappa} \, d\bar{z}^{\dot{\alpha}} d\bar{z}_{\dot{\alpha}}$$

$$dB + \mathcal{A} * B - B * \pi(\mathcal{A}) = 0$$

- Structure of the 4d equations is almost completely fixed by the higher-spin symmetry, up to the free parameter θ_0
  - $\theta_0 = 0$: "Type A" Theory
  - $\theta_0 = \pi/2$: "Type B" Theory
  - General $0 < \theta_0 < \pi/2$: Parity Breaking family of theories

AdS Higher-spin/vector model duality

- In AdS/CFT dictionary, conserved currents are dual to gauge fields (bulk gauge symm. $\leftrightarrow$ boundary global symm.)

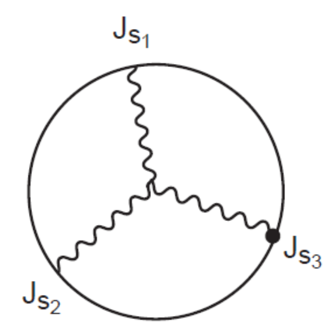
Single trace operators in CFT	$\Leftrightarrow$	Single particle states in AdS
$J_s, \qquad \partial \cdot J_s = 0$	$\Leftrightarrow$	Massless HS gauge field
J_0	$\Leftrightarrow$	Scalar field with $m^2 = \Delta_0(\Delta_0 - d)$

- In our case Δ_0=1 and we get for the bulk scalar m^2 = -2 (in units of AdS radius). This is precisely the value predicted by the Vasiliev equations
- The single-trace operator spectrum of the free vector model is precisely in one-to-one correspondence with the spectrum of (type A) Vasiliev theory
- Multi-trace operators correspond to multi-particle bulk states

$$\text{E.g.: } (\phi^i\phi^i)(\phi^j\partial\cdots\partial\phi^j) \quad \Leftrightarrow \quad \text{Two-particle state in AdS}$$

Higher-spin AdS/CFT

- So we come at the natural conclusion that Vasiliev higher-spin gravity in AdS should be exactly equivalent to a *free* vector model CFT in one less dimension
- Comparing large N scaling of CFT correlation functions to bulk Witten diagrams implies that the bulk Newton's constant is $G_N \sim 1/N$ in these HS/vector model dualities
- Note that both sides of the duality are in principle accessible perturbatively in 1/N: a "weak/weak" AdS/CFT duality
- Some tests:
  - 3-point functions [SG, Yin '09-'11]
  - Symmetry arguments [Maldacena, Zhiboedov '11]
  - Matching one-loop partition functions [SG, Klebanov '13; SG, Klebanov, Safdi '14; SG, Klebanov, Tseytlin '14; Gunaydin, Skvortsov, Tran '16; SG, Klebanov, Tan '16…]



Weakly broken HS symmetry in CFT

- Consider an interacting CFT which admits a small parameter *g* that controls the breaking of the higher-spin currents

$$\partial \cdot J_s = g K_{s-1}$$

- The parameter *g* may be a power of 1/N in the large N expansion, a power of ε in a Wilson-Fisher type fixed points, a marginal coupling constant, etc.
- When *g* is small, the weakly broken symmetry can still be used to constrain correlation functions (*Maldacena, Zhiboedov '12*: Three-point functions in large *N* CS theories with scalars/fermions)

Anomalous dimensions of broken currents

- When the currents are non-conserved, they acquire an anomalous dimension

$$\Delta_s = d - 2 + s + \gamma_s$$

- Using conformal symmetry and the non-conservation equation, the leading order anomalous dimensions can be obtained by a *tree-level* calculation: $\gamma_s \propto g^2 \langle K_{s-1} K_{s-1} \rangle_{g=0}$
- Recall that conformal invariance implies for a spin-*s* primary

$$\langle \hat{J}_s(x_1, z_1) \hat{J}_{s'}(x_2, z_2) \rangle = \delta_{ss'} C_s(g) \frac{(I_{\mu\nu} z_1^\mu z_2^\nu)^s}{(x_{12}^2)^{\Delta_s}}$$

$$I_{\mu\nu} = \eta_{\mu\nu} - 2\frac{x_{12}^\mu x_{12}^\nu}{x_{12}^2}$$

Anomalous dimensions of broken currents

- Directly differentiating the 2-point conformal structure and using $\partial \cdot J_s = g K_{s-1}$, one finds the relation

$$g^2 \hat{x}^2 \frac{\langle \hat{K}_{s-1}(x_1) \hat{K}_{s-1}(x_2) \rangle}{\langle \hat{J}_s(x_1) \hat{J}_s(x_2) \rangle} = -\gamma_s(g^2) s(s+d/2-2) \big[(s+d/2-1)(s+d-3) + \gamma_s(g^2)(s^2+sd/2-2s+d/2-1) \big]$$

- This allows to gain one order in perturbation theory. To leading order, we just need tree-level two-point functions

$$\gamma_s \sim g^2 \frac{\langle K_{s-1} K_{s-1} \rangle_{g=0}}{\langle J_s J_s \rangle_{g=0}}$$

[*Anselmi '98; Henn, Jarczak, Sokatchev '05*]

Some general comments

$$\partial \cdot J_s = g K_{s-1}$$

- The operator K_{s-1} is a conformal primary in the representation $(\Delta = d - 1 + s, s - 1)$ in the *g=0* theory
- The non-conservation equation is the statement that, when the coupling is switched on, the short representation $(\Delta = d - 2 + s, s)$ *combines* with the $(\Delta = d - 1 + s, s - 1)$ to form a *long multiplet* (the non-conserved current)

$$(\Delta = d - 2 + s + \gamma_s, s) \simeq (\Delta = d - 2 + s, s) \oplus (\Delta = d - 1 + s, s - 1)$$

``Multiplet recombination"

Much recent work on related ideas: [*Rychkov, Tan '15; Basu, Krishnan '15; Gosh et al '15; Bertolini et al '16; …*]

Bulk interpretation

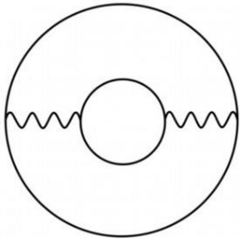
- In the AdS bulk, this corresponds to a HS version of the Higgs mechanism: the massless gauge field combines with a spin-(*s-1)* massive field to form a massive spin-*s* field.
- In a large *N* theory, we can distinguish two cases, depending on whether K_{s-1} is a *single-trace* operator or *multi-trace*
- If K_{s-1} is single-trace:
  - It is dual to a single-particle state in the bulk
  - Higgsing occurs *classically*
  - Anomalous dimensions are non-zero at planar level
  - Typical example: adjoint theories (e.g. Yang-Mills)
  - Requires understanding coupling of Vasiliev's HS fields to (infinite towers?) of matter fields

Quantum breaking

- If K_{s-1} is multi-trace, e.g. $K_{s-1} \sim \sum JJ$
  - Higgs is a multi-particle state
    [*Girardello, Porrati, Zaffaroni '02*]
  - Anomalous dimensions arise at non-planar level, $\gamma_s \sim O(1/N)$
  - Masses are generated via bulk loop effects

$$(\Delta_s + s - 2)(\Delta_s + 2 - d - s) = m_s^2 \ell_{AdS}^2$$

$$m_s^2 \ell_{AdS}^2 \approx (2s + d - 4)\gamma_s \sim O(1/N)$$



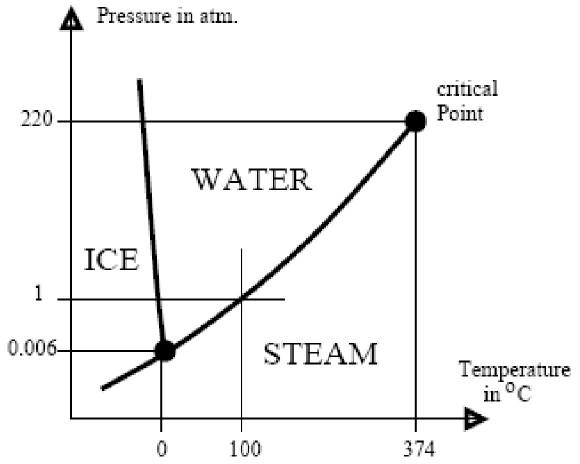
$G_N \sim 1/N$

  - Do not need additional fields on top of Vasiliev's spectrum
- Typical examples are Large *N* vector models:
  - Critical O(N) model
  - Gross-Neveu model
  - Conformal QED with N_f flavors, CP^N model
  - Large N Chern-Simons vector models

The interacting O(N) vector model

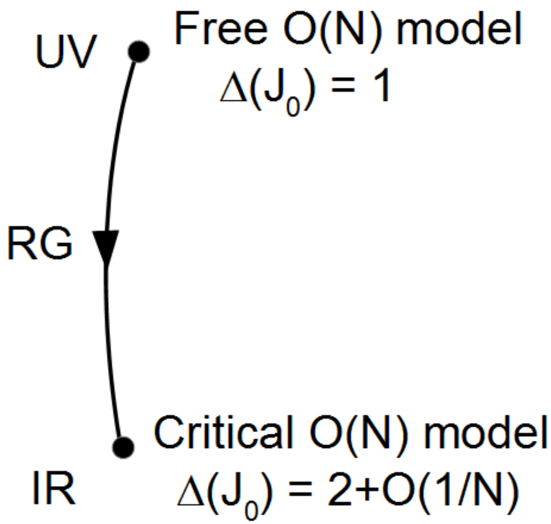
$$S = \int d^3x \left[\frac{1}{2}\left(\partial_\mu \phi^i\right)^2 + \frac{\lambda}{4}(\phi^i\phi^i)^2\right]$$

- At low energies, the field theory flows to an interacting CFT (provided mass term suitably tuned). This is the Wilson-Fisher IR fixed point
- N=1 corresponds to the 3d Ising CFT
- Strongly coupled in d=3. May be studied by large N expansion, or Wilson-Fisher epsilon-expansion



The interacting O(N) vector model

- At large N, the quartic interaction can be viewed as a "double-trace" deformation of the free theory
- The scalar bilinear $\phi^i\phi^i$ has $\Delta=2+O(1/N)$ in the IR CFT
- Relevant entry of the AdS/CFT dictionary [Klebanov, Witten] implies that the AdS dual of the interacting O(N) model is the *same* Vasiliev theory dual to free vector model, with an alternate choice of boundary condition for the bulk scalar [Klebanov, Polyakov]



$\Leftrightarrow$ *Type A HS gravity* $\Delta=1$ b.c.

$\Leftrightarrow$ *Type A HS gravity* $\Delta=2$ b.c.

Weakly-broken higher-spin symmetry

- At large N, using the classical equations of motion, one finds that the HS breaking at the IR fixed point has the structure

$$\partial \cdot J_s \sim \frac{1}{\sqrt{N}} \sum_{s'<s} \partial^n J_{s'} \partial^m J_0$$

- As anticipated earlier, the operator on the right-hand side of non-conservation equation is "double-trace" (it is dual to a two-particle state in the bulk). Then $\gamma_s \sim O(1/N)$
- Can show that for spin 2 the divergence is exactly zero: there is no double-trace operator with the required quantum numbers. Stress-tensor exactly conserved

The HS anomalous dimensions

- The explicit anomalous dimensions can be obtained as explained above from a classical calculation

$$\gamma_s = 2\gamma_\phi \frac{(s-1)(d+s-2) - \frac{\Gamma(d+1)\Gamma(s+1)}{2(d-1)\Gamma(d+s-3)}}{(d/2+s-2)(d/2+s-1)}$$

where $\gamma_\phi \sim 1/N$ is the anomalous dimension of ϕ (this can also be similarly fixed purely by eq. of motion and conformal symmetry [Skvortsov '15; SG, Kirilin '16])

- This formula agrees with (and provides independent test of) the result obtained by Lang and Ruhl ('93) using 4-point functions and OPE. Recently, same result was reproduced by direct loop diagram calculations [*Hikida-Wada '16*]

Anomalous dimensions

- In d=3, result takes the simple-looking form

$$\gamma_s = 2\gamma_\phi \frac{2(s-2)}{2s-1} = \frac{16(s-2)}{3\pi^2(2s-1)} \frac{1}{N}$$

corresponding to AdS_4 masses $m_s^2 \ell_{\rm AdS}^2 \approx \frac{16(s-2)}{3\pi^2 N}$

- The large spin behavior is, in general d:

$$\gamma_s = 2\gamma_\phi - 2\gamma_\phi \frac{\Gamma(d+1)}{2(d-1)} \frac{1}{s^{d-2}} - 2\gamma_\phi \frac{d(d-2)}{4} \frac{1}{s^2} + \dots$$

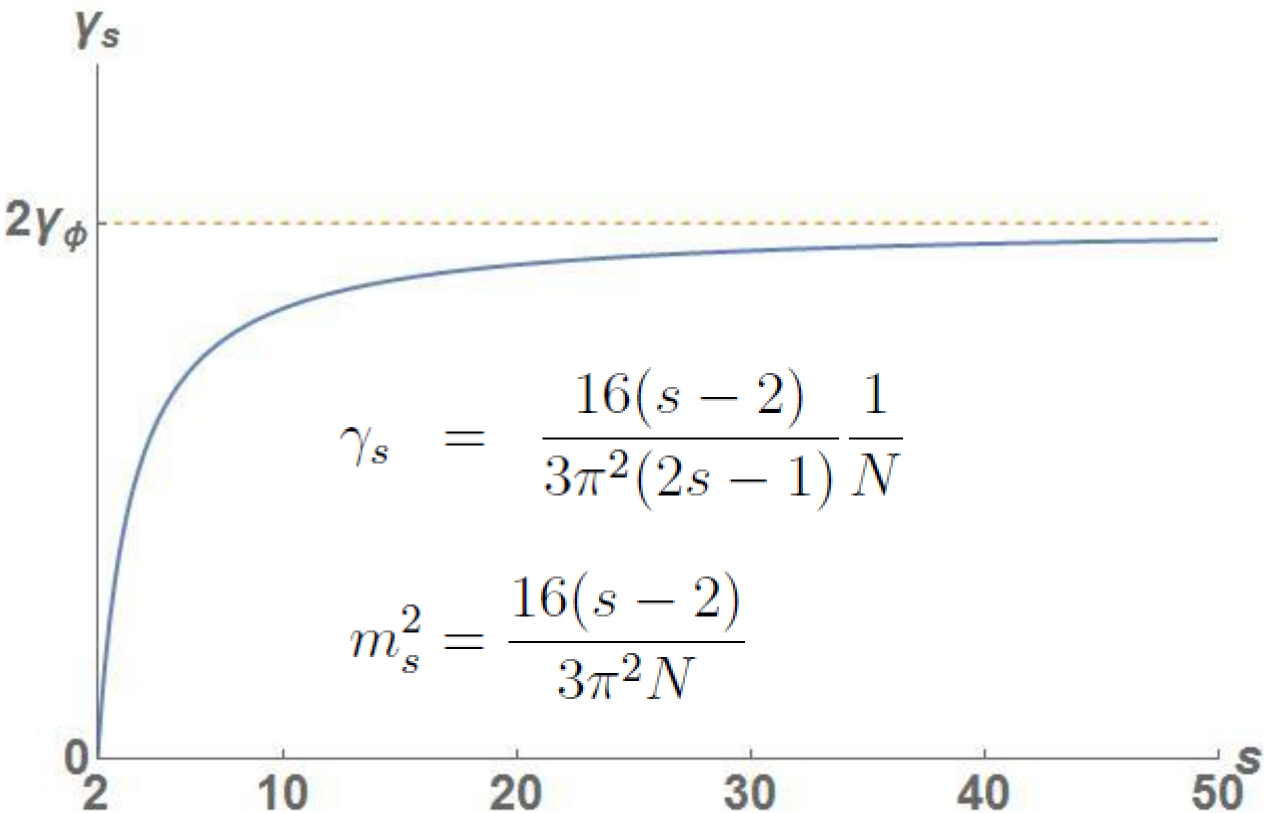
In agreement with general CFT results

Callan, Gross '73. Nachtmann '73 $\Delta_s \quad \to \quad s + 2\Delta_\phi$

Komargodski, Zhiboedov '12

Fitzpatrick et al '12

d=3 Higher-Spin anomalous dimensions



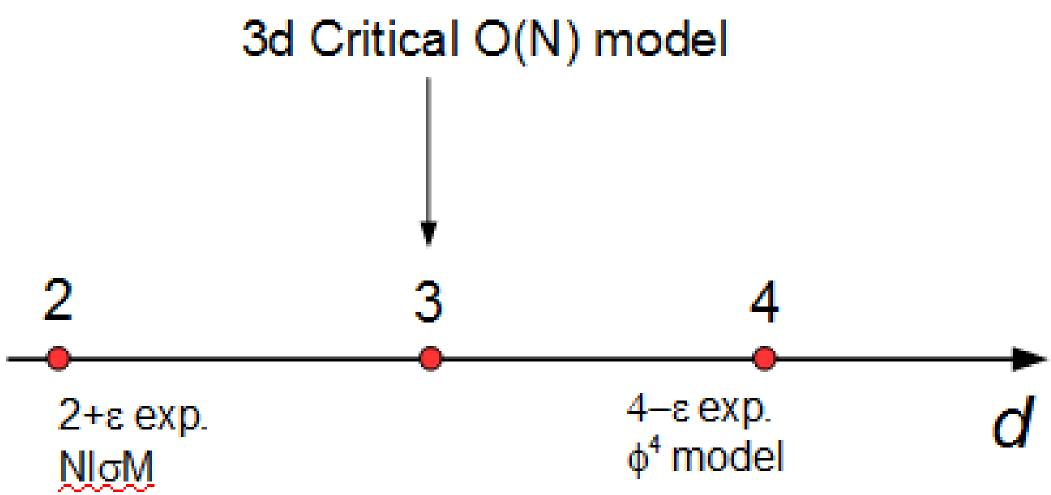
$$\gamma_s = \frac{16(s-2)}{3\pi^2(2s-1)}\frac{1}{N}$$

$$m_s^2 = \frac{16(s-2)}{3\pi^2 N}$$

$$\gamma_\phi = \frac{4}{3\pi^2}\frac{1}{N}$$

Higher-spin symmetry at finite N?

- HS symmetry is weakly broken at large *N*. What happens at finite *N*?
- For finite N, we can use the *epsilon-expansion*: formally study the theory as a function of continuous dimension *d*



- Can compute scaling dimensions perturbatively in ε near *d*=4 and *d*=2, and use combined data to obtain estimates at *d*=3 ("two-sided" *Pade* approximants)

HS scaling dimensions in d=3

- Combining d=4-ε and d=2+ε, as well as some input from large *N* expansion, we obtain the following estimates [SG, Kirilin '16]

N	3	4	5	6	10	20
$\gamma_{s=4}$ (Padé$_{[3,2]}$)	0.0261	0.0257	0.0208	0.0195	0.0158	0.0082
$\gamma_{s=6}$ (Padé$_{[3,2]}$)	0.0318	0.0310	0.0258	0.0240	0.0191	0.0100
$\gamma_{s=8}$ (Padé$_{[3,2]}$)	0.0342	0.0332	0.0278	0.0259	0.0206	0.0110
$\gamma_{s=10}$ (Padé$_{[3,2]}$)	0.0353	0.0343	0.0289	0.0269	0.0214	0.0115

- *N=1* and *N=2* have to be treated separately: NLσM cannot be used in this case. Ordinary Pade's give

$$\gamma^{N=1}_{s=4} = 0.0240\,, \quad \gamma^{N=1}_{s=6} = 0.0300 \quad \gamma^{N=2}_{s=4} = 0.0252\,, \quad \gamma^{N=2}_{s=6} = 0.0315$$

$$\gamma^{N=1}_{s=8} = 0.0324\,, \quad \gamma^{N=1}_{s=10} = 0.0336 \quad \gamma^{N=2}_{s=8} = 0.0340\,, \quad \gamma^{N=2}_{s=10} = 0.0353$$

Approximate HS symmetry?

- In all cases, anomalous dimensions appear to be very small. Approximate HS symmetry at finite *N* ?
- Other quantities apparently very close to free field value (even though the CFT is not weakly coupled).

E.g. for *N=1* (3d Ising)

$$\Delta_\phi \approx 0.518$$

$$c_T^{\text{3d Ising}} / c_T^{\text{3d free scalar}} \approx 0.9466$$

[*El-Showk et al '13*]

$$F_{3d\ \text{Ising}} / F_{3d\ \text{free sc}} \approx 0.976$$

[*SG, Klebanov '14*]

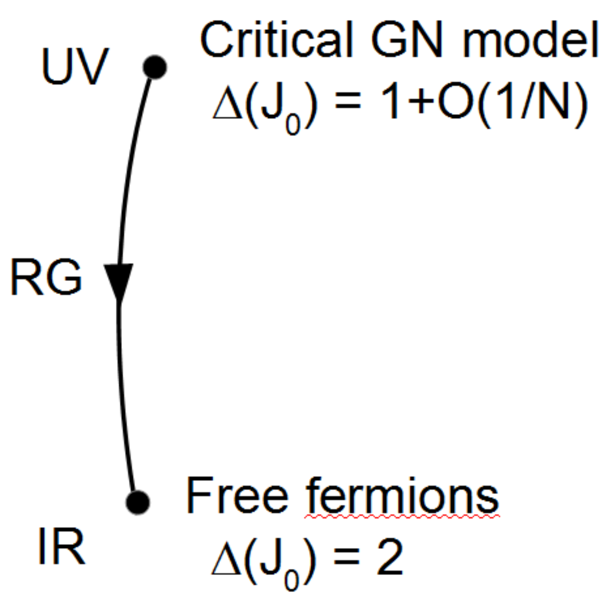
[*Fei, SG, Klebanov,Tarnopolsky '15*]

- Are all these facts related to the approximate HS symmetry? Can we use it to constrain further CFT data?

The critical Gross-Neveu model

$$S = \int d^d x \left(\bar{\psi} \partial\!\!\!/ \psi + \frac{1}{2} g (\bar{\psi}\psi)^2 \right)$$

- In 2<d<4, it defines an interacting CFT. 3d theory accessible by large N expansion, or ε-expansion near *d*=2 and *d*=4
- It also has a higher-spin gravity dual, following very similar arguments to scalar model [Sezgin, Sundell; Leigh, Petkou]



⇔ *Type B HS gravity* $\Delta=1$ b.c.

HS symm. weakly broken $\gamma_s \sim O(1/N)$

⇔ *Type B HS gravity* $\Delta=2$ b.c.

Chern-Simons vector models

- Interesting generalization: couple the scalar or fermion vector models to U(N) (or O(N)) Chern-Simons gauge theory

$$S = \frac{ik}{4\pi} S_{CS} + \int d^3x \bar{\psi} \not{D} \psi \qquad S_{CS} = \int d^3x \epsilon^{\mu\nu\rho} \mathrm{Tr}(A_\mu \partial_\nu A_\rho - \frac{2i}{3} A_\mu A_\nu A_\rho)$$

$$S = \frac{ik}{4\pi} S_{CS} + \int d^3x \left(D_\mu \bar{\phi} D^\mu \phi + \frac{\lambda_6}{N^2} (\bar{\phi}\phi)^3 \right)$$

- Natural in AdS/CFT context: gives explicit way to implement restriction to U(N) (or O(N)) singlet sector
- Chern-Simons level *k* is quantized, does not run. We still get CFTs even in the presence of the gauge interactions

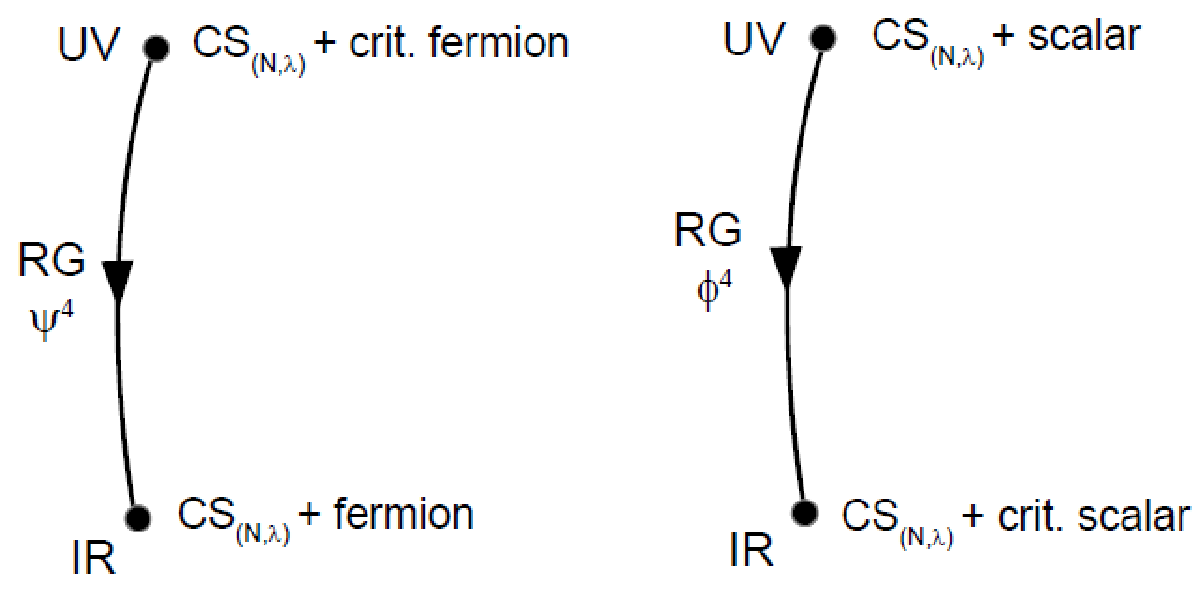
[*SG et al ‘11; Aharony et al ’*11]

Chern-Simons vector models

- We will mostly work in the large N ‘t Hooft limit

$$N, k \to \infty\,, \qquad \lambda \equiv \frac{N}{k} \quad \text{fixed}$$

- We can add quartic fermion or scalar self-interactions. This yields generalizations of the Wilson-Fisher and Gross-Neveu critical theories



Chern-Simons vector models

- The spectrum of "single-trace" operators is as simple as in the ungauged versions (CS term does not add any new local operators)

$$\mathrm{CS-fermion}: \quad \tilde{j}_0 = \bar{\psi}\psi \qquad j_s \sim \bar{\psi}\gamma D^{s-1}\psi$$

$$\mathrm{CS-scalar}: \quad j_0 = \bar{\phi}\phi \qquad j_s \sim \bar{\phi}D^s\phi$$

- In the critical models, the spectrum is the same, with the dimension of the scalar operators changed as usual due to the "double-trace" flow

Weakly broken HS symmetry

- A remarkable fact is that at large N, and *for any finite 't Hooft coupling*, the higher-spin operators are still approximately conserved

$$\partial \cdot j_s \sim \sum_{s_1,s_2} \frac{1}{N} f^{(3)}_{s,s_1,s_2}(\lambda)\partial^n j_{s_1}\partial^m j_{s_2} + \sum_{s_1,s_2,s_3} \frac{1}{N^2} f^{(4)}_{s,s_1,s_2,s_3}(\lambda)\partial^n j_{s_1}\partial^m j_{s_2}\partial^p j_{s_3}$$

- So they acquire anomalous dimensions starting at 1/N order, for any λ (unlike familiar example of YM theory in d=4)

$$\Delta_s = s + 1 + \frac{\gamma_s^{(1)}(\lambda)}{N} + \ldots$$

- *Conjecture*: the AdS duals must be the parity breaking Vasiliev higher-spin theories with: [*SG et al '11; Aharony et al '11*]

$$\theta_0 \quad \Leftrightarrow \quad \lambda$$

Weakly broken HS symmetry

- One can also show that the spin-zero bilinears have scaling dimensions $\Delta=2+\gamma_0(\lambda)/N+\ldots$ in the CS+fermion and CS+critical scalar, and $\Delta=1+\gamma_0(\lambda)/N+\ldots$ in the CS+scalar and CS+critical fermion
- The weakly broken HS symmetry can be used to fix all planar 3-point functions [Maldacena, Zhiboedov; Aharony, Gur-Ari, Yacoby]

$$\langle j^{\rm f}_{s_1} j^{\rm f}_{s_2} j^{\rm f}_{s_3}\rangle = \tilde{N}\left[\frac{1}{1+\tilde{\lambda}^2}\langle j_{s_1} j_{s_2} j_{s_3}\rangle_{\rm fer} + \frac{\tilde{\lambda}^2}{1+\tilde{\lambda}^2}\langle j_{s_1} j_{s_2} j_{s_3}\rangle_{\rm sc} + \frac{\tilde{\lambda}}{1+\tilde{\lambda}^2}\langle j_{s_1} j_{s_2} j_{s_3}\rangle_{\rm odd}\right]$$

$$\langle j^{\rm b}_{s_1} j^{\rm b}_{s_2} j^{\rm b}_{s_3}\rangle = \tilde{N}\left[\frac{1}{1+\tilde{\lambda}^2}\langle j_{s_1} j_{s_2} j_{s_3}\rangle_{\rm sc} + \frac{\tilde{\lambda}^2}{1+\tilde{\lambda}^2}\langle j_{s_1} j_{s_2} j_{s_3}\rangle_{\rm fer} + \frac{\tilde{\lambda}}{1+\tilde{\lambda}^2}\langle j_{s_1} j_{s_2} j_{s_3}\rangle_{\rm odd}\right]$$

With: $\tilde{N} = 2N\dfrac{\sin(\pi\lambda)}{\pi\lambda}\,, \qquad \tilde{\lambda} = \tan(\dfrac{\pi\lambda}{2})$

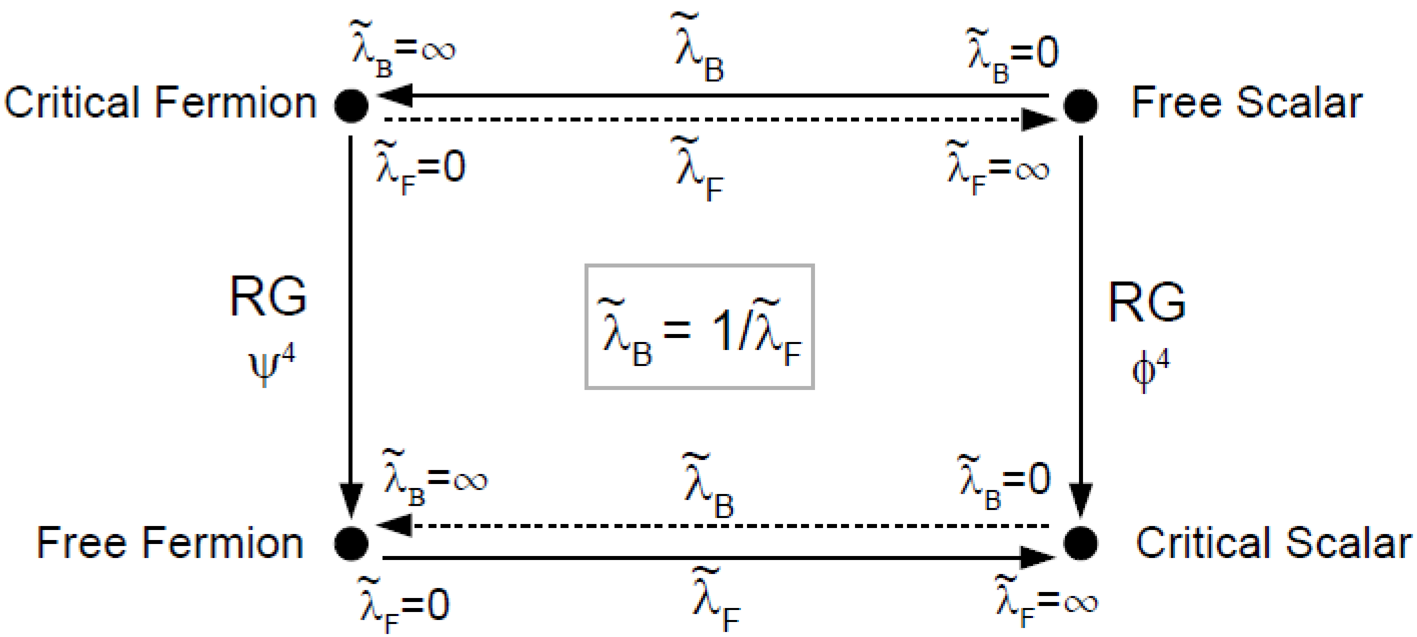
Bose/fermi duality

- Note that as one increases the parameter $\tilde{\lambda}$, fermion structures are turned into (critical) scalar structures and vice-versa! Expression mapped to each other by $\tilde{N}_b = \tilde{N}_f,\, \tilde{\lambda}_b^2 = 1/\tilde{\lambda}_f^2$, or:

$$\lambda_b = \lambda_f - \text{sign}(\lambda_f)\,, \qquad \frac{N_b}{|\lambda_b|} = \frac{N_f}{|\lambda_f|}$$

- Same structure of the 3-point functions is obtained from the bulk Vasiliev theory [SG, Yin '11] . Duality manifest in the bulk: fermionic and bosonic theory have the same bulk dual up to a relabeling of the θ_0 parameter (θ_0 -> $\pi/2-\theta_0$)

"3d Bosonization duality"



$$\tilde{N} = 2N\frac{\sin(\pi\lambda)}{\pi\lambda}\,, \qquad \tilde{\lambda} = \tan(\frac{\pi\lambda}{2})$$

3d Bosonization duality

- Passed various tests at leading order at large N:
  - 2- and 3- point functions [Aharony, Gur-Ari, Yacoby]
  - Thermal free energy on R^2 [Minwalla et al; Aharony, SG, Gur-Ari, Maldacena, Yacoby]
  - Relation to SUSY dualities [Gur-Ari, Yacoby]

- Written in terms of rank and level of the CS theory, this duality amounts to a generalization of level-rank duality of the CS theory

$$U(N)_{\kappa-1/2}\ \ \mathrm{CS-Fermion} \quad \Leftrightarrow \quad U(|\kappa|)_{-N}\ \ \mathrm{Critical\ CS-Scalar}$$

3d Bosonization duality

$$U(N)_{\kappa-1/2} \ \ \mathrm{CS-Fermion} \quad \Leftrightarrow \quad U(|\kappa|)_{-N} \ \ \mathrm{Critical\ CS-Scalar}$$

- It is plausible that this duality holds for finite *N, k* [Aharony et al]
- Abelian versions of these dualities were recently investigated [*Seiberg et al; Karch, Tong; Murugan, Nastase*], and shown to be part of a "web" of dualities in 3d CFTs (including "particle/vortex" duality)
- An abelian example

$$U(1)_{-1/2} \ \ \mathrm{Fermion} \quad \Leftrightarrow \quad O(2) \ \mathrm{Wilson\text{-}Fisher\ Scalar}$$

Anomalous dimensions of the HS currents

- The structure of the HS non-conservation equation can be used, by same methods as before, to extract the explicit expressions for the anomalous dimensions of the weakly broken HS operators [SG, Guru-Charan, Kirilin, Prakash, Skvortsov '16]
- In both fermionic and scalar model, we find the result

$$\gamma_s = \frac{1}{\tilde{N}}\left(a_s \frac{\tilde{\lambda}^2}{1+\tilde{\lambda}^2} + b_s \frac{\tilde{\lambda}^2}{(1+\tilde{\lambda}^2)^2}\right) + O(\frac{1}{N^2})$$

$$= \frac{\pi\lambda}{2N\sin(\pi\lambda)}\left(a_s \cos^2(\frac{\pi\lambda}{2}) + \frac{b_s}{4}\sin^2(\pi\lambda)\right)$$

$$a_s = \begin{cases} \frac{16}{3\pi^2}\frac{s-2}{2s-1}\,, & \text{for even } s\,, \\ \frac{32}{3\pi^2}\frac{s^2-1}{4s^2-1}\,, & \text{for odd } s\,, \end{cases}$$

$$b_s = \begin{cases} \frac{2}{3\pi^2}\left(3g(s) + \frac{-38s^4+24s^3+34s^2-24s-32}{4s^4-5s^2+1}\right)\,, & \text{for even } s \\ \frac{2}{3\pi^2}\left(3g(s) + \frac{20-38s^2}{4s^2-1}\right)\,, & \text{for odd } s \end{cases}$$

$$g(s) = \sum_{n=1}^{s}\frac{1}{n-1/2}$$

Anomalous dimensions of the HS currents

- The expression vanishes for s=1 and s=2 as expected
- It is precisely consistent with the bose/fermi duality under $\tilde{\lambda} \leftrightarrow 1/\tilde{\lambda}$
- Note at large spin, the anomalous dimensions now *grow logarithmically* with spin, as expected in a gauge theory

$$\gamma_s \simeq \frac{1}{\tilde{N}} \frac{\tilde{\lambda}^2}{(1+\tilde{\lambda}^2)^2} \frac{2}{\pi^2} \log s = \frac{\lambda \sin(\pi\lambda)}{4\pi N} \log s$$

- This logarithmic dependence disappears in the strong coupling limit, where the scaling dimensions turn into those of the critical O(k-N) (or GN) model, consistently with the 3d bosonization duality

$$\gamma_s \overset{\lambda\to 1}{\simeq} \frac{1}{2(k-N)} a_s$$

Conclusion

- The Vasiliev theory in AdS is conjectured to be exactly dual to simple vector model CFTs. Perhaps the *simplest* version of AdS/CFT duality
- Higher-spin theories provide exact gravitational duals not only to free theories, but also to interacting CFTs such as the Wilson-Fisher O(N) model, the Gross-Neveu model, and generalizations involving Chern-Simons gauge fields
- These models have weakly broken higher-spin symmetry, and the leading anomalous dimensions can be efficiently fixed by purely classical calculations
- It would be interesting to further study the constraints imposed by the weakly broken HS symmetry (scaling dimensions of composite operators, 4 point functions,…)

Conclusion

- New non-perturbative dualities, such as the "*3d bosonization*", were motivated and suggested by the weakly broken HS symmetry at large N and the holographic duality to higher-spin gravity
- Extrapolation of ε-expansion results to d=3 suggest that the anomalous dimensions of the HS operators in the 3d Ising and related CFTs are rather small even for finite N, indicating a still present weakly broken HS symmetry
- Higher-spin theories in AdS might be relevant more broadly for applications of AdS/CFT to condensed matter

THANK YOU!

d=4-ε

$$S=\int d^dx\Big(\frac{1}{2}\partial_\mu\phi^i\partial^\mu\phi^i+\frac{\lambda}{4}(\phi^i\phi^i)^2\Big)$$

$$\beta(\lambda)=-\epsilon\lambda+\frac{(N+8)\lambda^2}{8\pi^2} \qquad \lambda_*=\tfrac{8\pi^2}{N+8}\epsilon+O(\epsilon^2)$$

- We can use classical equation of motion to deduce the form of non-conservation $\partial\cdot J_s=\lambda_* K_{s-1}$, and we get the anomalous dimensions

$$\gamma_s=\frac{\epsilon^2(N+2)}{2(N+8)^2}\Big(1-\frac{6}{s(s+1)}\Big)$$

In agreement with Wilson, Kogut '74

d=2+ε

$$S = \int d^d x \Big(\frac{1}{2} \partial_\mu \phi^i \partial^\mu \phi^i + \sigma(\phi^i \phi^i - \frac{1}{g^2}) \Big)$$

$$\beta = \frac{\epsilon}{2} g - (N-2) \frac{g^3}{4\pi} \qquad\qquad g_*^2 = \frac{2\pi\epsilon}{N-2}$$

- From the HS non-conservation $\partial \cdot J_s = g_*^2 K_{s-1}$ we get

$$\gamma_s = \frac{\epsilon^2}{N-2} \left(\frac{1}{s} - \frac{1}{2} + \sum_{k=1}^{s-2} \frac{1}{k} \right)$$

- Note: large spin behavior is logarithmic in this case

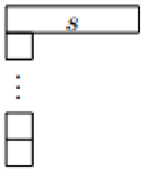
$$\gamma_s \overset{s \gg 1}{\simeq} \frac{\epsilon^2}{N-2} \log(s)$$

Fermionic CFT

- Consider the free fermionic CFT of N_f Dirac fermions

$$S = \int d^3x \bar{\psi}_i \gamma^\mu \partial_\mu \psi^i \,, \qquad i = 1, \ldots, N_f$$

- The single-trace, $U(N_f)$ invariant, operators consist of a $\Delta=2$ scalar operator $\tilde{j}_0 = \bar{\psi}\psi$ and a tower of exactly conserved higher-spin currents $j_s \sim \bar{\psi}\gamma\partial^{s-1}\psi$ of all integer spins
- The $U(N_f)$ singlet sector of this free CFT is conjecturally dual to the so-called Vasiliev ``type B" theory in AdS_4, which has single-particle spectrum in one-to-one corresp. with CFT
- This duality can actually be defined in any *d*, but the spectrum then also includes towers of hook-type mixed symmetry representations



Critical Gross-Neveu model

- An interacting fermionic CFT can be defined starting from the GN model

$$S = \int d^d x \left(\bar{\psi} \partial\!\!\!/ \psi + \frac{1}{2} g (\bar{\psi}\psi)^2 \right)$$

- It has perturbative UV fixed points in d=2+ε ($N = N_f$ tr1 below)

$$\beta = \epsilon g - (N-2)\frac{g^2}{2\pi} + \ldots \qquad\qquad g_* = \frac{2\pi}{N-2}\epsilon$$

- Scaling dimensions may be computed in the epsilon-expansion, e.g.

$$\Delta_\psi = \frac{1}{2} + \frac{1}{2}\epsilon + \frac{N-1}{4(N-2)^2}\epsilon^2 - \frac{(N-1)(N-6)}{8(N-2)^3}\epsilon^3 + \mathcal{O}(\epsilon^4)$$
$$\Delta_\sigma = 1 - \frac{1}{N-2}\epsilon - \frac{N-1}{2(N-2)^2}\epsilon^2 + \frac{N(N-1)}{4(N-2)^3}\epsilon^3 + \mathcal{O}(\epsilon^4)\,, \qquad \sigma \sim \bar{\psi}\psi$$

GN model at Large N

- The UV fixed point can be also accessed by large N techniques, for arbitrary *d*, similar to the scalar model

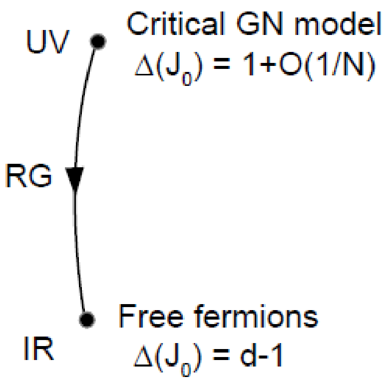
$$S = \int d^d x \left(\bar{\psi} \partial\!\!\!/ \psi + \frac{\sigma}{\sqrt{N}} (\bar{\psi}\psi) - \frac{1}{2}\sigma^2 \right)$$

- In the UV limit in 2<d<4, we can drop quadratic term and develop 1/N expansion with the $\sigma\bar{\psi}\psi$ interaction, and the induced 2-point function of σ, which is found to be

$$\langle\sigma\sigma\rangle = \frac{C_{\sigma\sigma}}{x_{12}^2} \qquad C_{\sigma\sigma} = -\frac{2(d-2)\Gamma(d-1)\sin\left(\frac{\pi d}{2}\right)}{\pi\Gamma\left(\frac{d}{2}\right)^2}$$

GN model at Large N

- At the UV fixed point, σ (which plays the role of $\bar{\psi}\psi$), becomes a scalar primary of scaling dimension 1+O(1/N)



- The interacting UV fixed point is conjectured to be dual to the Vasiliev type B theory with alternate boundary condition on the bulk (pseudo)scalar (*Sezgin-Sundell, Leigh-Petkou*)
- The classical equation of motion $\partial\!\!\!/ \psi = -\frac{1}{\sqrt{N}}\psi\sigma$ can be efficiently used to extract the scaling dimension of ψ and of the higher-spin operators

Large N scaling dimensions

- The scaling dimension of the fundamental fermion is found to be

$$\gamma_\psi = \frac{C_{\sigma\sigma}}{2d} = -\frac{(d-2)\Gamma(d-1)\sin\left(\frac{\pi d}{2}\right)}{d\pi\Gamma\left(\frac{d}{2}\right)^2}$$

- After working out the non-conservation equation $\partial \cdot \hat{J}_s = \frac{1}{\sqrt{N}}\hat{K}_{s-1}$ for arbitrary spin, and computing the 2-point function of the descendant operators, we find the result

$$\gamma_s = 2\gamma_\psi \frac{(s-1)(d+s-2) - \frac{\Gamma(d+1)\Gamma(s+1)}{2(d-1)\Gamma(d+s-3)}}{(d/2+s-2)(d/2+s-1)}$$

- Agrees with an old result of *Muta-Popovic '77*. Remarkably, it is equal to the critical scalar result up to the overall factor of γ_ψ. A priori not obvious from the calculation, should be better understood.

4–ε: the Gross-Neveu-Yukawa model

- The UV fixed point of the GN model has an alternative description (a ``UV completion") as the IR fixed point of the GNY model in d=4-ε (*Zinn Justin '91; Hasenfratz et al '91*)

$$\mathcal{L}_{\rm GNY} = \frac{1}{2}(\partial_\mu \sigma)^2 + \bar\psi_j \not{\partial} \psi^j + g_1 \sigma \bar\psi_j \psi^j + \frac{1}{24} g_2 \sigma^4$$

- There are IR stable fixed points for any N

$$\beta_{g_1^2} = -\epsilon g_1^2 + \frac{N+6}{16\pi^2} g_1^4$$
$$\beta_{g_2} = -\epsilon g_2 + \frac{1}{8\pi^2}\left(\frac{3}{2}g_2^2 + N g_2 g_1^2 - 6N g_1^4\right)$$

$$(g_1^*)^2 = \frac{16\pi^2 \epsilon}{N+6}$$
$$g_2^* = 16\pi^2 R\epsilon,$$

$$R = \frac{24N}{(N+6)[(N-6)+\sqrt{N^2+132N+36}]}$$

- The IR CFT is equivalent to the UV fixed point of GN model (scaling dimensions of operators can be explicitly matched).

GNY model

- The classical equations of motion

$$\partial\!\!\!/ \psi^i = -g_1 \sigma \psi^i$$

$$\partial^2 \sigma = g_1 \bar{\psi}^i \psi^i + \frac{g_2}{6} \sigma^3$$

can be used to fix the anomalous dimensions of the near-free fields

$$\gamma_\psi = \frac{1}{32\pi^2} g_1^2 \,, \qquad \gamma_\sigma = \frac{N}{32\pi^2} g_1^2$$

$$\Delta_\psi = \frac{d-1}{2} + \gamma_\psi = \frac{3}{2} - \frac{N+5}{2(N+6)} \epsilon$$

$$\Delta_\sigma = \frac{d-2}{2} + \gamma_\sigma = 1 - \frac{3}{N+6} \epsilon \,.$$

- They precisely agree with large N results in GN model, expanded near d=4 (cheked up to order $1/N^3$)

HS dimensions from GNY model

- Novel feature: in the free theory in the UV, there are *two* towers of conserved HS operators, $J_s^\psi \sim \bar{\psi}\gamma\partial^{s-1}\psi$ and $J_s^\sigma \sim \sigma\partial^s\sigma$ (as σ is a nearly free scalar in GNY description)
- They mix under RG flow. In our approach, the mixing can be described by computing the classical non-conservation equations $\partial \cdot J_s^\psi = K_{s-1}^\psi$ $\partial \cdot J_s^\sigma = K_{s-1}^\sigma$ and evaluating 2-point functions of the descendants
- Mixing matrix is found to be

$$\frac{g_1^2}{16\pi^2}\begin{bmatrix} \frac{(s-1)(s+2)}{s(s+1)} & \frac{-2\sqrt{N}}{\sqrt{s(s+1)}} \\ \frac{-2\sqrt{N}}{\sqrt{s(s+1)}} & N \end{bmatrix}$$

Eigenvalues:

$$\gamma_s = \frac{g_1^2}{16\pi^2}\frac{-2+(N+1)s(1+s)\pm\sqrt{4+s(1+s)(-4+20N+(N-1)^2s+(N-1)^2s^2)})}{2s(1+s))}$$

HS dimensions from GNY model

- One eigenvalue corresponds to the tower of the nearly conserved HS operators that include the stress tensor

$$\gamma_s = \frac{\epsilon}{N}\frac{(s-2)(s+3)}{s(s+1)} + O(\frac{\epsilon}{N^2})$$

Precisely matches the large N Gross-Neveu result

- The other eigenvalue

$$\Delta_2 = d - 2 + s + \gamma_2 = 2 + s - 2\frac{\epsilon}{N}\frac{3s^2+3s-2}{s(1+s)} + O(\frac{\epsilon}{N^2})$$

should match the dimension of the ``double-trace" operator $\sigma\partial^s\sigma \sim \bar{\psi}\psi\partial^s\bar{\psi}\psi$ in the GN model description

- Combining 2+ε and 4-ε expansions, one should be able to obtain estimates for the d=3 scaling dimensions in the critical fermionic CFT_3 at finite N