Linear quivers and non-Abelian T-duals

Yolanda Lozano (U. Oviedo)

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1. Introduction & motivation: NATD in AdS/CFT

Non-Abelian T-duality (NATD) has proved to be very useful as a solution generating technique in AdS/CFT

Its realization in the CFT remains however quite unknown

Interestingly, some examples suggest that, contrary to its Abelian counterpart, NATD may change the CFT:

NATD of $AdS_5 \times S^5$: Gaiotto & Maldacena geometry (dual to N=2 SCFTs (Gaiotto theories))

NATD of $AdS_5 \times T^{1,1}$: Bah, Beem, Bobev, Wecht geometry (dual to N=1 SCFTs (Sicilian quivers))

Indeed, contrary to its Abelian counterpart, NATD has not been proven to be a symmetry of string theory

Applying NATD to an AdS/CFT pair, a new AdS background is generated which may have associated a different CFT dual, which, moreover, may only exist in the strong coupling regime

This will be the focus of this talk

Based on:
- Y.L., Carlos Núñez, 1603.04440
- Y.L., Niall Macpherson, Jesús Montero, Carlos Núñez, 1609.09061
- Y.L., Carlos Núñez, Salomón Zacarías, in progress

Outline:

1. Introduction and motivation: NATD in AdS/CFT

2. Basics of NATD: i) NATD vs Abelian T-duality
   ii) NATD as a solution generating technique

3. The Sfetsos-Thompson $AdS_5 \times S^2$ background

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   3.2. ST as a GM geometry

4. The $AdS_4 \times S^2 \times S^2$ example

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2. Basics of NATD: i) NATD vs Abelian T-duality

Using the string sigma-model Rocek and Verlinde proved that Abelian T-duality is a symmetry to all orders in g_s and α'

(Buscher'88; Rocek, Verlinde'92)

The extension to arbitrary wordsheets determines the global properties of the dual variable:

$$\theta \in [0, 2\pi] \quad \xrightarrow{\text{T}} \quad \tilde{\theta} \in [0, 2\pi]$$

In the non-Abelian case neither proof works

Variables living in a group manifold are substituted by variables living in its Lie algebra

$$g \in SU(2) \quad \xrightarrow{\text{NAT}} \quad \chi \in \mathbb{R}^3$$

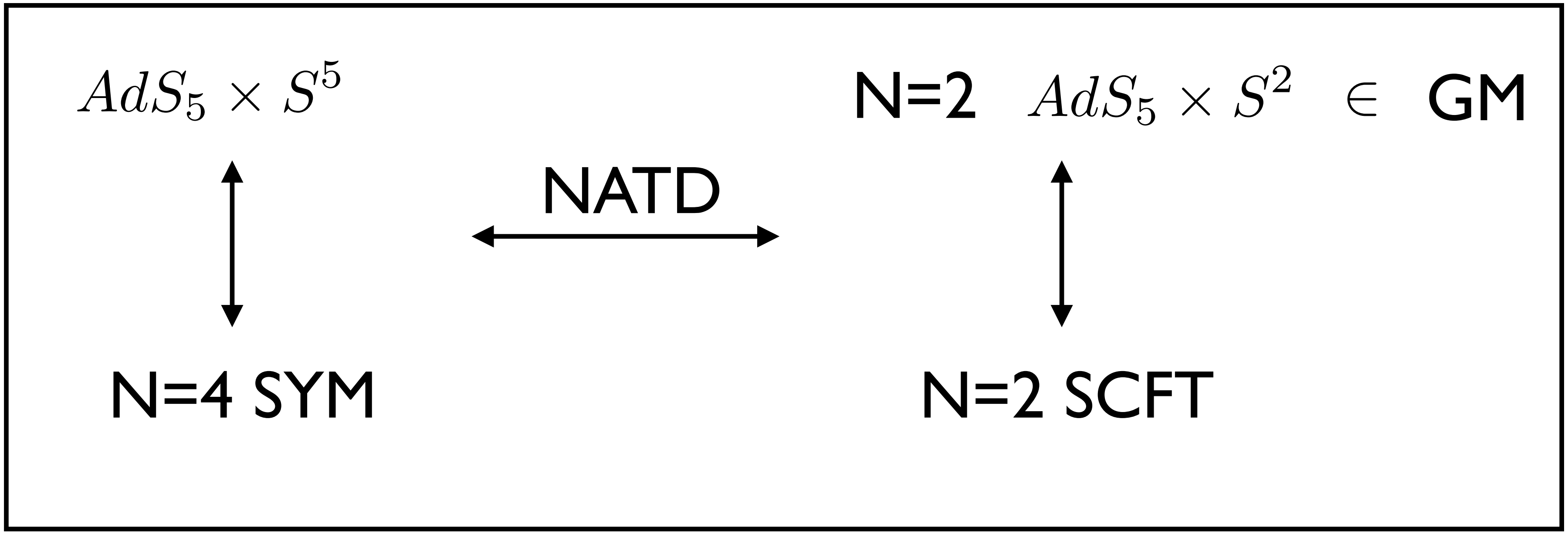
In the absence of global information the new variables remain non-compact

ii) NATD as a solution generating technique

Need to know how the RR fields transform

Sfetsos and Thompson (2010) extended Hassan's derivation in the Abelian case, implementing the relative twist between left and right movers in the bispinor formed by the RR fields, to the non-Abelian case

3. The ST $AdS_5 \times S^2$ background



(Sfetsos, Thompson'10)

- Gaiotto-Maldacena geometries encode the information about the dual CFT
- Useful example to study the CFT realization of NATD

- Take the $AdS_5 \times S^5$ background

$$ds^2 = ds^2_{AdS_5} + L^2\Big(d\alpha^2 + \sin^2\alpha d\beta^2 + \cos^2\alpha ds^2(S^3)\Big)$$

$$F_5 = 8L^4 \sin\alpha \cos^3\alpha d\alpha \wedge d\beta \wedge \mathrm{Vol}(S^3)$$ + Hodge dual

- Dualize it w.r.t. one of the $SU(2)$ symmetries

In spherical coordinates adapted to the remaining $SU(2)$:

$$ds^2 = ds^2_{AdS_5} + L^2\Big(d\alpha^2 + \sin^2\alpha d\beta^2\Big) + \frac{dr^2}{L^2\cos^2\alpha} + \frac{L^2\cos^2\alpha r^2}{r^2 + L^4\cos^4\alpha} ds^2(S^2)$$

$$B_2 = \frac{r^3}{r^2 + L^4\cos^4\alpha}\mathrm{Vol}(S^2), \qquad e^{-2\phi} = L^2\cos^2\alpha(L^4\cos^4\alpha + r^2)$$

$$F_2 = L^4 \sin\alpha \cos^3\alpha d\alpha \wedge d\beta, \qquad F_4 = B_2 \wedge F_2$$

- New Gaiotto-Maldacena geometry
- What about r ?
  - Background perfectly smooth for all $r \in \mathbb{R}^+$
  - No global properties inferred from the NATD
  - How do we interpret the running of r to infinity in the CFT?
- Singular at $\alpha = \pi/2$ where the original S^3 shrinks (due to the presence of NS5-branes)

This is the tip of a cone with S^2 boundary $\rightarrow$

Large gauge transformations $B_2 \rightarrow B_2 - n\pi \mathrm{Vol}(S^2)$
for $r \in [(n-1)\pi, n\pi]$

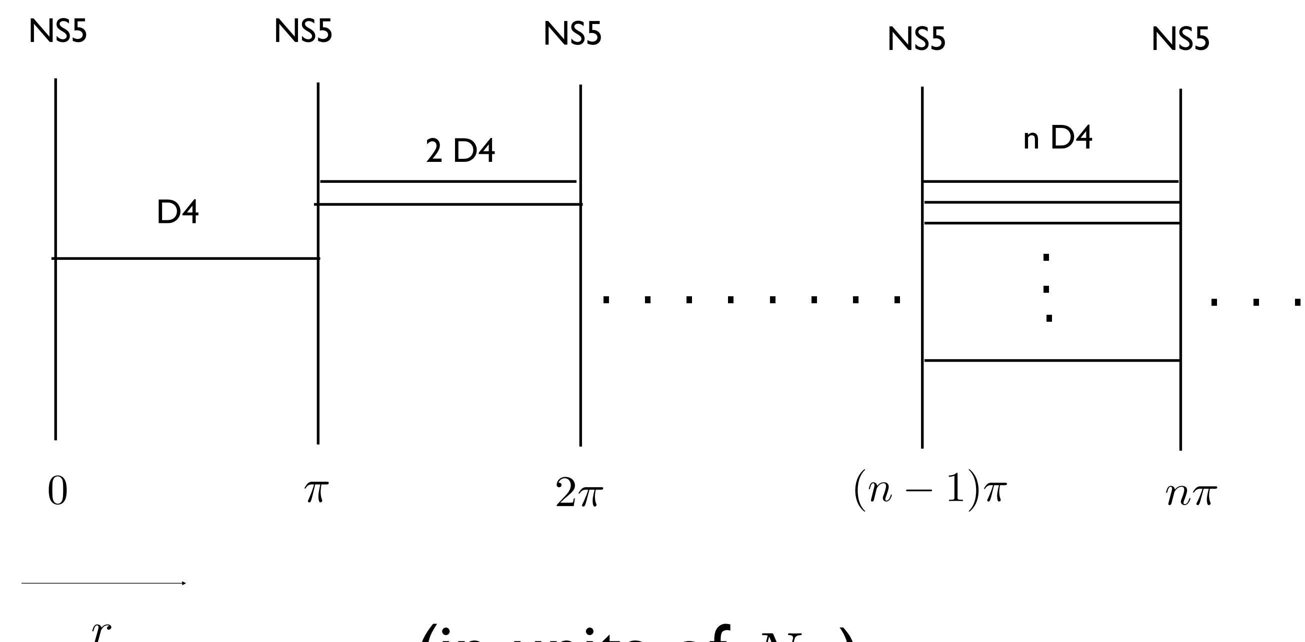
This modifies the Page charges such that $N_4 = nN_6$ in each $[(n-1)\pi, n\pi]$ interval

We have also N_5 charge, such that every time we cross a π interval one unit of NS5 charge is created

This is compatible with a D4/NS5 brane set-up:

D4: $\mathbb{R}^{1,3}, r$

NS5: $\mathbb{R}^{1,3}, \alpha, \beta$



(in units of N_6)

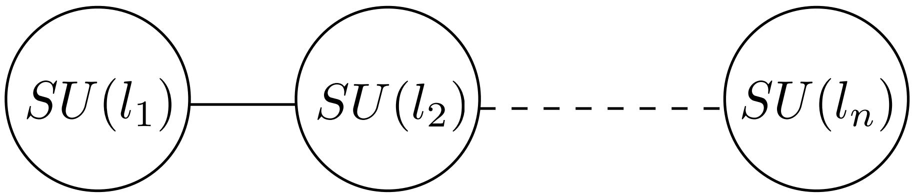
These D4/NS5 brane set-ups realize 4d $\mathcal{N}=2$ field theories with gauge groups connected by bifundamentals (Witten'97)

Having the D4 finite extension in the r direction, the field theory living in them is 4d at low energies, with effective gauge coupling:

$$\frac{1}{g_4^2} \sim r_{n+1} - r_n$$

For l_n D4-branes in $[r_n, r_{n+1}]$ the gauge group is $SU(l_n)$ and there are (l_{n-1}, l_n) and (l_n, l_{n+1}) hypermultiplets.

The field theory is then described by a quiver



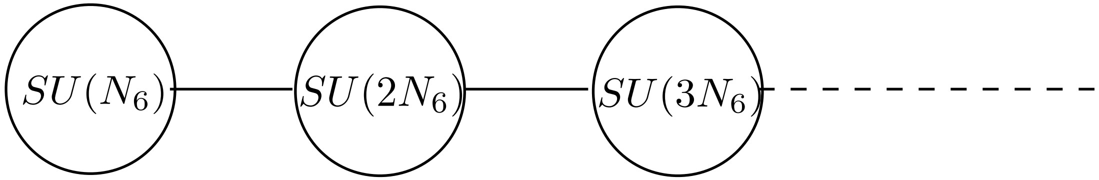
The bifundamentals contribute to the $SU(l_n)$ beta function as $l_{n-1} + l_{n+1}$ flavors.

The beta function thus vanishes at each interval if

$$2l_n = l_{n+1} + l_{n-1}$$

This condition is satisfied by our brane configuration, which has $l_n = nN_6$

It corresponds to an infinite linear quiver:



This is in agreement with Gaiotto-Maldacena

3.1 Short review of GM geometries

Generic backgrounds dual to 4d N=2 SCFTs.

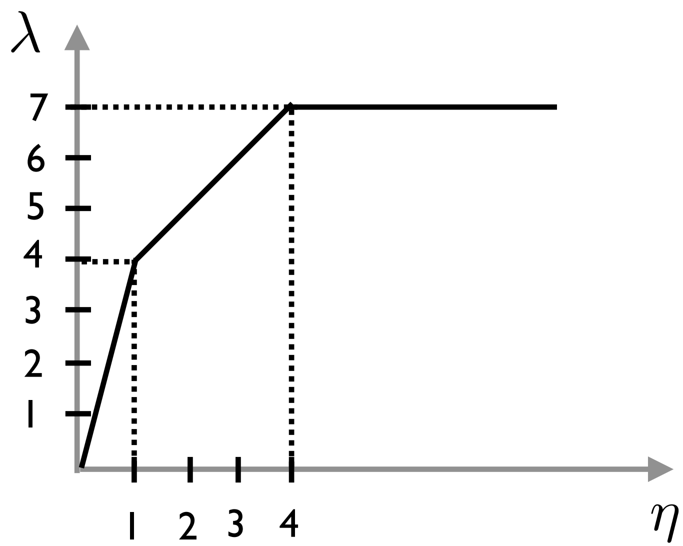
Described in terms of a function $V(\sigma,\eta)$ solving a Laplace eq. with a given charge density $\lambda(\eta)$ at $\sigma=0$

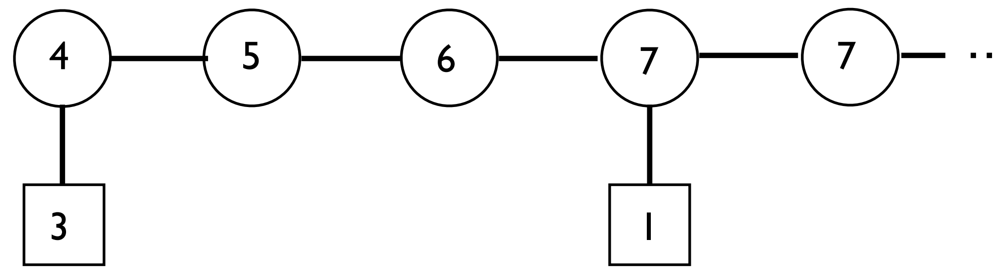
$$\partial_\sigma[\sigma\partial_\sigma V]+\sigma\partial_\eta^2 V=0, \qquad \lambda(\eta)=\sigma\partial_\sigma V(\sigma,\eta)|_{\sigma=0}$$

Regularity and quantization of charges impose strong constraints on the allowed form of $\lambda(\eta)$, which encodes the information of the dual CFT:

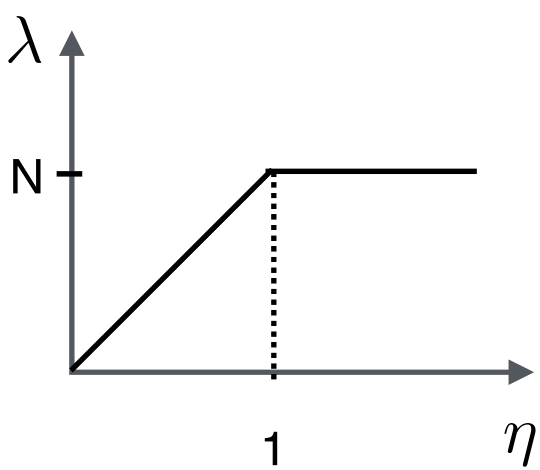
- A $SU(n_i)$ gauge group is associated to each integer value of $\eta=\eta_i$, with n_i given by $\lambda(\eta_i)=n_i$
- A kink in the line profile corresponds to extra k_i fundamentals attached to the gauge group at the node n_i

For example:

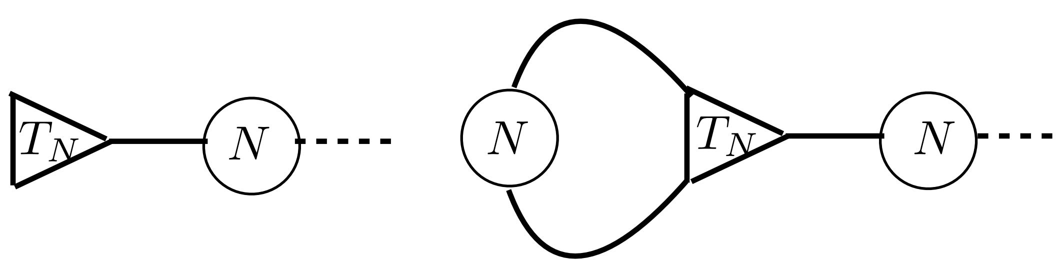




The Maldacena-Nunez solution:



$$\lambda_{MN}(\eta) = \frac{N}{2}(|\eta+1| - |\eta-1|)$$



Interesting for our work:

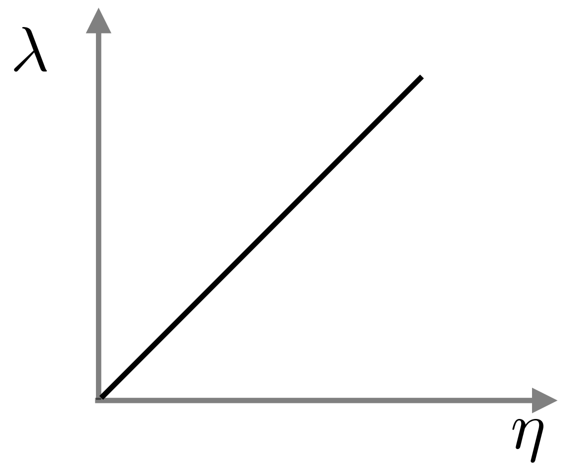
Following Reid-Edwards and Stefanski'10 (see also Aharony, Berdichevsky, Berkooz'12) , the MN solution can be taken as a building block for N=2 IIA solutions: Any allowed profile of the line charge density can be viewed as a sum of suitably re-scaled and shifted λ_{MN} profiles

We can use this to *complete* the NATD solution

3.2. The NATD as a GM geometry

GM geometry with $\lambda(\eta) = \eta \; , \; \eta \sim r \; , \; \sigma = \sin\alpha$

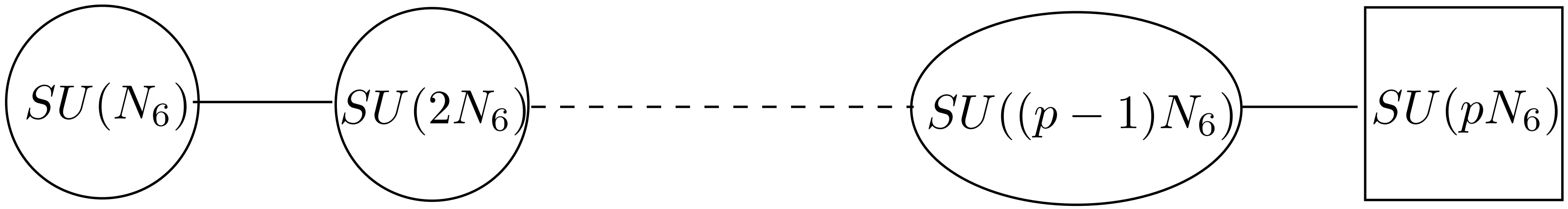
$\lambda(\eta) = \eta \Rightarrow$ Infinite linear quiver, consistent with the brane set-up:



Next, we will complete the quiver and, using holography, *complete* the geometry (both for large r and at the singularity)

Example in which the field theory *informs* the geometry

A natural way to complete the quiver is by adding fundamentals:



This completion reproduces correctly the value of the holographic central charge:

From the geometry:

$$c_{NATD} \sim V_{int} \sim \int_0^{\eta_*} f(\eta) d\eta = \frac{N_6^2 N_5^3}{12}$$

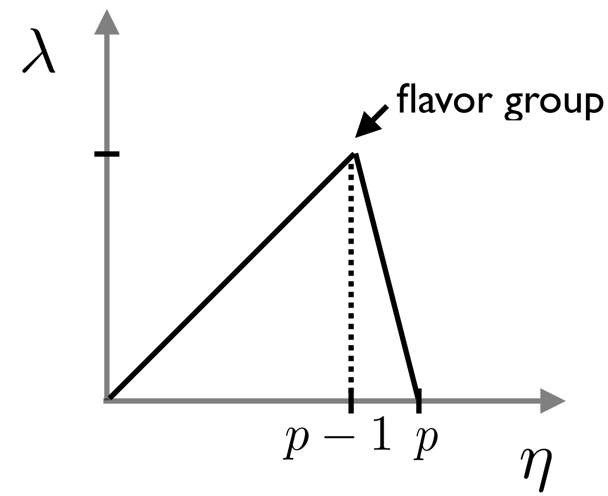
(Klebanov, Kutasov, Murugan'08)

In the field theory we can use: $c = \frac{1}{12}(2n_v + n_h)$ (Shapere, Tachikawa'08)

This gives

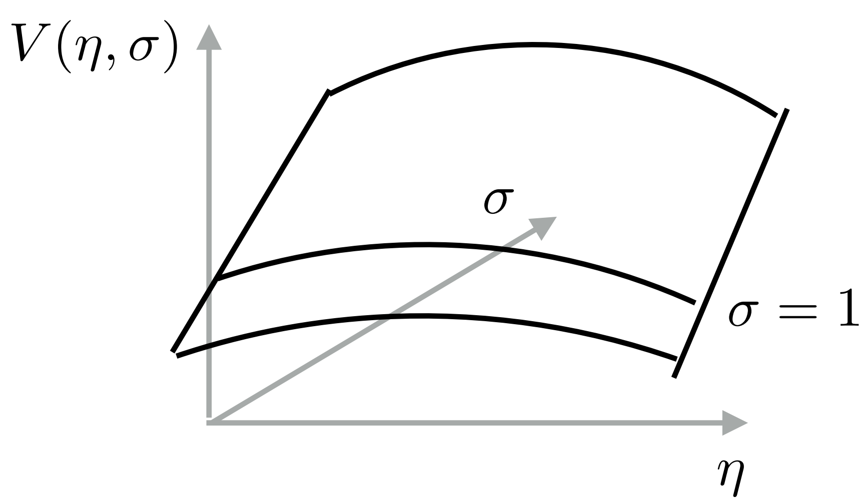
$$c = \frac{N_6^2 p^3}{12}\left[1 - \frac{1}{p} - \frac{2}{p^2 N_6^2} + \frac{2}{N_6^2 p^3}\right] \approx \frac{N_6^2 p^3}{12}$$

In the geometry, the completed quiver corresponds to



$$\frac{\lambda(\eta)}{N_6} = \begin{cases} \eta & 0 \leq \eta \leq p-1 \\ (1-p)\eta + (p^2 - p) & (p-1) \leq \eta \leq p \end{cases}$$

This charge density can be obtained as a superposition of MN solutions:



This superposition completes the NATD solution, and removes the singularity

The singularity can be interpreted as a result of cutting the space at $\sigma = 1$

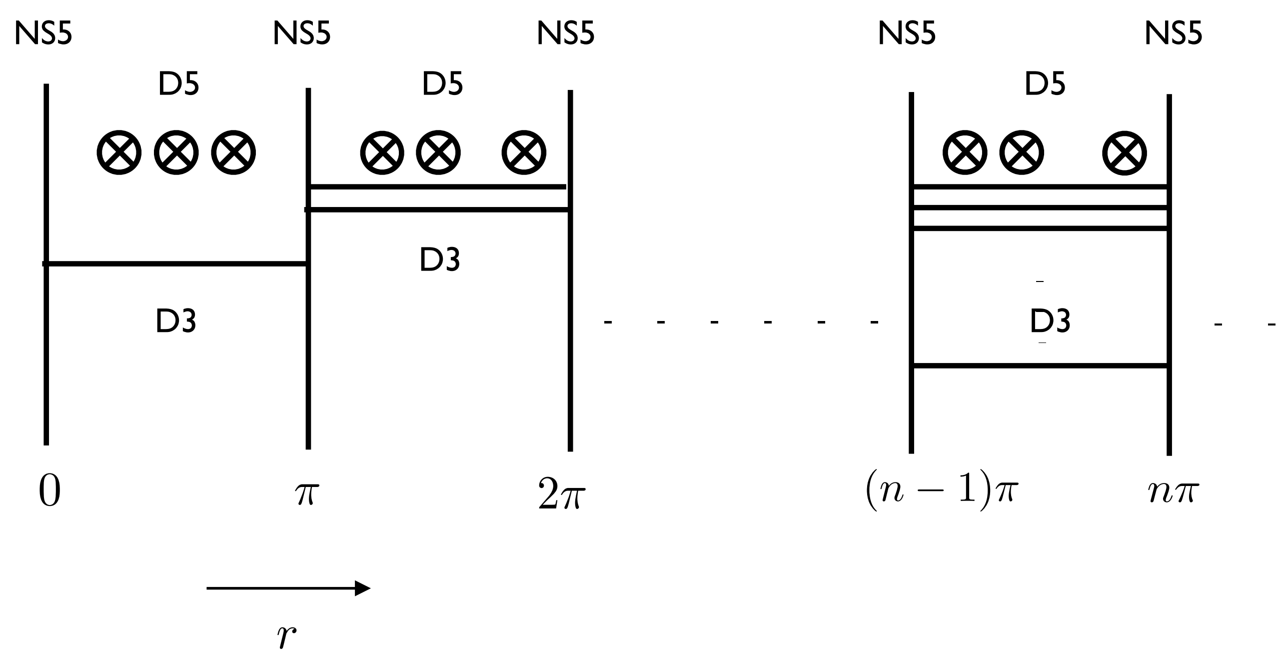
Can we find other examples where the NATD solution belongs to a classification with known field theory dual to check these ideas?

4. The $AdS_4 \times S^2 \times S^2$ example

Non-Abelian T-duality on a reduction to IIA of $AdS_4 \times S^7/\mathbb{Z}_k$

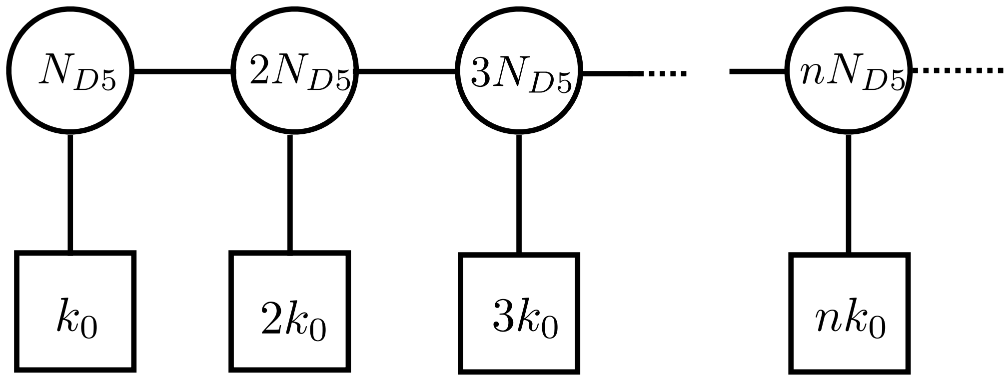
$\rightarrow$ IIB $AdS_4 \times S^2 \times S^2$ background, N=4 SUSY, in the classification of D'Hoker, Estes and Gutperle'07

Analysis of charges: (D3,NS5,D5) brane set-up:



Gaiotto and Witten'08: 3d N=4 $T_\rho^{\hat{\rho}}(N)$ theories

$T^{\hat{\rho}}_{\rho}(N)$ field theories flow to CFTs in the infrared if the partitions satisfy certain conditions, that are satisfied by our brane set-up



The holographic duals of these CFTs are known (Assel, Bachas, Estes and Gomis'11)

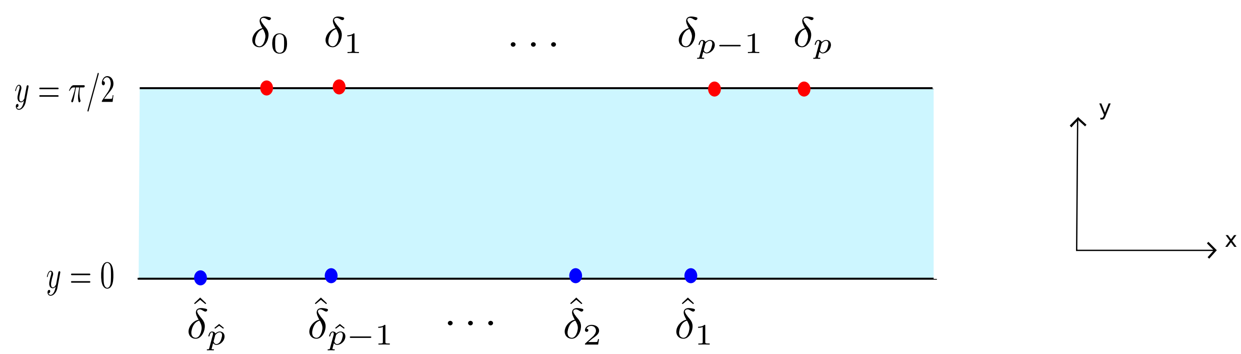
They belong to the general class of $AdS_4 \times S^2 \times S^2$ geometries in D'Hoker, Estes and Gutperle'07

These are fibrations of $AdS_4 \times S^2 \times S^2$ over a Riemann surface that can be completely determined from two harmonic functions $h_1(z,\bar{z}), h_2(z,\bar{z})$

Assel, Bachas, Estes and Gomis'11 showed how to determine these functions from the (D3, NS5, D5) brane set-ups associated to $T_\rho^{\hat\rho}(N)$ theories:

$$h_1 = -\frac{1}{4}\sum_{a=1}^{p} N_5^a \log\tanh\left(\frac{i\frac{\pi}{2}+\delta_a - z}{2}\right) + cc$$

$$h_2 = -\frac{1}{4}\sum_{b=1}^{\hat p} \hat N_5^b \log\tanh\left(\frac{z-\hat\delta_b}{2}\right) + cc$$



The positions of the D5 and NS5 branes are determined, in turn, from the linking numbers of the configuration:

$$\hat\delta_b - \delta_a = \log\tan\left(\frac{\pi}{2}\frac{l_a \hat l_b}{N}\right)$$

The h_1, h_2 functions computed from our *completed* brane set-up agree with those associated to the non-Abelian T-dual geometry in the region $x, y \sim 0$, far from the location of the branes

The non-Abelian T-dual arises as a result of zooming-in in a particular region of the *completed* solution

This completion smoothes out the singularities and defines the geometry globally

The free energy of the completed solution satisfies the bound $N^2 \log N$ found by Assel, Estes, Yamazaki'12

5. Conclusions

- NATD geometries dual to infinite linear quivers

  $\rightarrow$ Different CFTs after NATD

  D3 $\rightarrow$ (D4,NS5)   (D2,D6) $\rightarrow$ (D3,NS5,D5)

- Quivers completed, and thereof the geometries, to define the CFTs

- NATD as a zooming-in in a patch of the completed geometry

  Penrose limit of superstar solution (in progress)

- General pattern?

  $AdS_6 \times S^4$: (D4,D8) system $\rightarrow$ (D5,NS5,D7)

  $AdS_6 \times S^2$ IIB solutions recently classified by D'Hoker, Gutperle, Karch and Uhlemann'16

THANKS!