Strings, Fields and Geometry

THE STRING THEORY UNIVERSE

Strings and Things

- Fundamental strings: D=10 Superstrings, D=26 Bosonic Strings, D=6 Little Strings

- T-duality, double field theory formulation

- 1-Branes. BPS: D-strings. Magnetic strings in D=5 SYM, Self-dual strings in D=2 (2,0) theory

- Effective string field theory description

- BPS 1-branes: stringy fields

- Double Field Theory for Little Strings

(2,0) SCFT in 6-D

- Abelian case: 2-form B, H=*H, 5 scalars
- ADE gauge group: no local field theory
- On S^1, gives 5D SYM $g_{YM}^2 = R$
- Coulomb branch: BPS strings
- 6D strings → 5D W-bosons and monopole-strings
- On T^2, 4D SYM, S-duality from T^2 diffeos.

Non-abelian (2,0) tensor multiplets

6-D (2,0) theory with fields taking values in 3-algebra
Has vector field taking values in 3-algebra

Lambert and Papageorgakis

Variant with fields taking values in Lie algebra
Has vector field C^m

CMH and Lambert

Field equations imply C is constant, and
all fields annihilated by $C^m \nabla_m$

Choose vev for C and dimensionally reduce in C direction: get 5-D SYM

5D SYM dressed up in 6D

Strings on a Torus

- States: momentum p, winding w
- String: Infinite set of fields $\psi(p, w)$
- Fourier transform to doubled space: $\psi(x, \tilde{x})$
- "Double Field Theory" from closed string field theory. Some non-locality in doubled space
- Subsector? e.g. $g_{ij}(x, \tilde{x}),\ b_{ij}(x, \tilde{x}),\ \phi(x, \tilde{x})$
- T-duality is a manifest symmetry

Double Field Theory

CMH & Zwiebach

- Double field theory on doubled torus
- General solution of string theory: involves doubled fields $\psi(x, \tilde{x})$
- *Real* dependence on *full* doubled geometry, dual dimensions not auxiliary or gauge artifact. Double geom. *physical* and *dynamical*
- *Strong constraint* restricts to subsector in which extra coordinates auxiliary: get conventional field theory locally. Recover Siegel's duality covariant formulation of (super)gravity

Constraints

$$L_0 - \bar{L}_0 = 0 \qquad\qquad p_i w^i = N - \bar{N}$$

For fields with

$$\mu \equiv N - \bar{N} = 0$$

$$\Delta\psi = 0 \qquad\qquad \Delta \equiv \frac{\partial^2}{\partial x^i \partial \tilde{x}_i}$$

Weak constraint. Each field is sum of terms each depending on half the coordinates.
Intrinsically non-local, as p,w mutually non-local

Strong constraint truncates to sector where all fields depend on same half of the coordinates.
Truncates to local theory in doubled space.

DFT gives O(D,D) covariant formulation

O(D,D) Covariant Notation

$$X^M \equiv \begin{pmatrix} \tilde{x}_i \\ x^i \end{pmatrix} \qquad \eta_{MN} = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

$$\Delta \equiv \frac{\partial^2}{\partial x^i \partial \tilde{x}_i} = \frac{1}{2}\partial^M \partial_M$$

Generalised Metric Formulation

Hohm, H &Z

$$\mathcal{H}_{MN} \;=\; \begin{pmatrix} g^{ij} & -g^{ik}b_{kj} \\ b_{ik}g^{kj} & g_{ij} - b_{ik}g^{kl}b_{lj} \end{pmatrix} .$$

2 Metrics on double space $\mathcal{H}_{MN},\ \eta_{MN}$

Eff. String Field Theory

Field in loop space $\Phi[X(\sigma)]$

Functional of loop $X^m(\sigma)$

Action is functional integral $\int DX^m(\sigma) L$

e.g.

$$L = \int d\sigma \left(\frac{1}{e(\sigma)} \frac{\delta \Phi}{\delta X^m(\sigma)} \frac{\delta \Phi}{\delta X^m(\sigma)} + \frac{1}{\alpha'} e(\sigma) \Phi^2 + \dots \right)$$

$$e = \sqrt{X'^2} \qquad X'^m = \frac{dX^m}{d\sigma}$$

Abelian Symmetry

$$\Phi \to e^{i\alpha}\Phi \qquad \alpha[X(\sigma)]$$

$$\alpha = \int d\sigma \Lambda_m(X(\sigma)) X'^m$$ Reparameterisation invariant.

1-form on space-time $\Lambda_m(X)$

Covariant derivative

$$\frac{\mathcal{D}\Phi}{\mathcal{D}X^m(\sigma)} \to e^{i\alpha} \frac{\mathcal{D}\Phi}{\mathcal{D}X^m(\sigma)}$$

uses 2-form gauge field $B_{mn}(X)$

$$\delta B_{mn} = \partial_m \Lambda_n - \partial_n \Lambda_m$$

$$\frac{\mathcal{D}\Phi}{\mathcal{D}X^m(\sigma)} = \frac{\delta\Phi}{\delta X^m(\sigma)} - iB_{mn}X'^m\Phi$$

$$X^m(\sigma) = x^m + w^m\sigma + \sum_n a_n^m e^{in\sigma}$$

Closed Strings $\quad \sigma \sim \sigma + 2\pi$

If X^m periodic $\; X^m \sim X^m + 2\pi R_m$

then w^m is winding with winding number n_m

$$w^m = n_m R_m$$

String field becomes function

$$\Phi[X(\sigma)] \to \phi(x^m, w^m, a_n^m)$$

Can also formally consider infinite strings in w^m direction

$$\sigma \in \mathbb{R}$$

Reparameterisation invariant if

$$X'^m(\sigma)\frac{\delta\Phi[X(\sigma)]}{\delta X^m(\sigma)} = 0$$

Straight Strings

$$X^m(\sigma) = x^m + w^m \sigma$$

No oscillations. Typically “ground state”

$$\Phi[X(\sigma)] \to \phi(x^m, w^m)$$

Reparameterisation invariance condition becomes

$$w^m \frac{\partial \phi}{\partial x^m} = 0 \qquad \text{or} \qquad w^m p_m \phi = 0$$

Field in D-1 dimensional transverse space

$$x^m = (x^\mu, y), w^m = (0, w) \qquad \text{gives} \qquad \phi(x^\mu)$$

Supersymmetric Strings

BPS 1-branes in supersymmetric theories have Green-Schwarz effective action Hughes,Liu, Polchinski

Strings in superspace give effective string fields

$$\Phi[X(\sigma),\Theta(\sigma)]$$

BPS configurations

$$X^m(\sigma) = x^m + w^m\sigma \qquad \Theta(\sigma) = \theta$$

BPS Fields $\phi(x^m, w^m, \theta)$

Expanding in fermion zero modes gives supermultiplet of straight strings

Multiplet of D-1 dimensional fields

Stringy Fields

Multiplet of fields $\phi(x^m, w^m)$

For spacetime $\mathbb{R}^{n,1} \times T^d$ get finite energy only if w^m is torus vector, giving winding mode

For fields with same w^m, dimensional reduction in w^m direction gives field theory in D-1 dimensional transverse space.

Gives interpretation of Lambert-Papageorgakis theory with vector C^m as stringy fields with $w^m=C^m$

U(N) Little String Theory

- N NS5-branes in limit $g_s \to 0, \quad M_s$ fixed
- IIA: N M5-branes on S^1; $R \to 0,\ M_s^2 = M_p^3 R$ fixed
- IIB: N D5-branes in limit $g_s \to \infty, \quad M_s$ fixed
- Low energy effective theory $E << M_S$
- IIB: (1,1) SUSY D=6 U(N) SYM $g_{YM}^2 = M_s^2$
- IIA: (2,0) U(N) SCFT

Little T-duality

- IIA LST on S^1 radius R = IIB LST on S^1 radius 1/R
- Momentum modes of IIB LST= winding modes of IIA LST. Massive YM multiplets, KK modes of D=6 SYM.
- Momentum modes of IIA LST= winding modes of IIB LST. Massive tensor multiplets, KK modes of D=6 (2,0) theory.
- On T^d, T-duality $O(d, d; \mathbb{Z})$

U(1) IIA LST

D=6 (2,0) multiplet $E << M_S$ $B_{mn}, \phi^i, \lambda^a$ $H = *H$

On S[1], radius R, m'th KK mode has momentum $p = \frac{m}{R}$

$x^m = (x^\mu, y)$ $B_{mn} = (B_{\mu\nu}, B_{\mu y} = C_\mu)$

5-D Massive multiplet, mass p $B^{(p)}_{\mu\nu}, C^{(p)}_\mu, \phi^{(p)}, \lambda^{(p)}$

$$*dB^{(p)} = pB^{(p)} + dC^{(p)}$$

$p = 0:$ $*dB^{(0)} = dC^{(0)}$ Dual potentials

$p \neq 0 : C^{(p)}$ Stucekelberg field

U(1) IIB LST

D=6 (1,1) multiplet $E << M_S$

On S[1], radius $\tilde{R}$, $\tilde{m}$'th KK mode has momentum $\tilde{p} = \frac{\tilde{m}}{\tilde{R}}$

$x^m = (x^\mu, \tilde{y})$ $\tilde{A}_m = (\tilde{A}_\mu, \tilde{\chi} = \tilde{A}_{\tilde{y}})$

5-D Massive multiplet, mass $\tilde{p}$ $\tilde{A}_\mu^{(\tilde{p})}, \tilde{\chi}^{(\tilde{p})}, \tilde{\varphi}^{(\tilde{p})}, \tilde{\psi}^{(\tilde{p})}$

$$S \sim \int (\tilde{F}^{(\tilde{p})})^2 + (\tilde{p}\tilde{A}^{(\tilde{p})} + d\tilde{\chi}^{(\tilde{p})})^2$$

$\tilde{p} = 0 :$ Massless vector + scalar

$\tilde{p} \neq 0 : \chi^{(\tilde{p})}$ Stucekelberg field

IIA LST

Momentum modes: tensor mults

$$B^{(p)} \qquad p = \frac{m}{R}$$

Winding modes: vector mults

$$A^{(w)} \qquad w = nR \qquad w \neq 0$$

IIB LST

Momentum modes: vector mults

$$\tilde{A}^{(\tilde{p})} \qquad \tilde{p} = \frac{\tilde{m}}{\tilde{R}}$$

Winding modes: tensor mults

$$\tilde{B}^{(\tilde{w})} \qquad \tilde{w} = \tilde{n}\tilde{R} \qquad \tilde{w} \neq 0$$

IIA LST

Momentum modes: tensor mults

$$B^{(p)} \qquad p = \frac{m}{R}$$

Winding modes: vector mults

$$A^{(w)} \qquad w = nR \qquad w \neq 0$$

IIB LST

Momentum modes: vector mults

$$\tilde{A}^{(\tilde{p})} \qquad \tilde{p} = \frac{\tilde{m}}{\tilde{R}}$$

Winding modes: tensor mults

$$\tilde{B}^{(\tilde{w})} \qquad \tilde{w} = \tilde{n}\tilde{R} \quad \tilde{w} \neq 0$$

Include $\tilde{B}^{(0)}, A^0 \qquad d\tilde{B}^{(0)} = *d\tilde{A}^{(0)}, dA^{(0)} = *dB^{(0)}$

IIA LST

Momentum modes: tensor mults

$$B^{(p)} \qquad p = \frac{m}{R}$$

Winding modes: vector mults

$$A^{(w)} \qquad w = nR$$

IIB LST

Momentum modes: vector mults

$$\tilde{A}^{(\tilde{p})} \qquad \tilde{p} = \frac{\tilde{m}}{\tilde{R}}$$

Winding modes: tensor mults

$$\tilde{B}^{(\tilde{w})} \qquad \tilde{w} = \tilde{n}\tilde{R}$$

$$d\tilde{B}^{(0)} = *d\tilde{A}^{(0)}, dA^{(0)} = *dB^{(0)}$$

IIA LST

Momentum modes: tensor mults

$$B^{(p)} \qquad p = \frac{m}{R}$$

Winding modes: vector mults

$$A^{(w)} \qquad w = nR$$

IIB LST

Momentum modes: vector mults

$$\tilde{A}^{(\tilde{p})} \qquad \tilde{p} = \frac{\tilde{m}}{\tilde{R}}$$

Winding modes: tensor mults

$$\tilde{B}^{(\tilde{w})} \qquad \tilde{w} = \tilde{n}\tilde{R}$$

Dual formulations of same system

$$\tilde{R} = \frac{1}{R} \qquad \tilde{w} = p,\ \tilde{p} = w \qquad \tilde{B}^{(\tilde{w})} = B^{(p)},\ \tilde{A}^{(\tilde{p})} = A^{(w)}$$

Momentum modes can be combined to 6-D fields

$$B(x^\mu, y) = \sum_p B^{(p)}(x^\mu) e^{ipy}$$

Winding modes $A^{(w)}(x^\mu)$

On T^d, winding modes specified by lattice vector w^m

Winding field $A(x^m, w^m)$ $w^m \frac{\partial}{\partial x^m} A = 0$

$$A = A(x^m + \sigma w^m)$$

6-D fields + winding fields

Dimensionally reduce in w^m direction: 5-D field on transverse space. Get 5-D supermultiplet.

Winding fields combine to form 6-D fields in dual space

$$A(x^\mu, \tilde{y}) = \sum_w A^{(\tilde{p})}(x^\mu) e^{iw\tilde{y}}$$

Weakly constrained DFT $\quad B(x^\mu, y), \; A(x^\mu, \tilde{y})$

IIB LST: $\quad \tilde{A}(x^\mu, y), \; \tilde{B}(x^\mu, \tilde{y})$

Combine into field on doubled space $\quad X^M = (x^\mu, y, \tilde{y})$

$$B_{MN}(X^P) \qquad \partial_y \partial_{\tilde{y}} B = 0$$

$$B = B^0(x^\mu) + B^+(x^\mu, y) + B^-(x^\mu, \tilde{y})$$

$$B^+(x^\mu, y) \sim B(x^\mu, y), \tilde{A}(x^\mu, y)$$

$$B^-(x^\mu, \tilde{y}) \sim A(x^\mu, \tilde{y}), \tilde{B}(x^\mu, \tilde{y})$$

$$B^+_{\mu\nu} = B_{\mu\nu}, B^+_{\mu y} = B_{\mu y} = C_\mu, B^+_{\mu\tilde{y}} = \tilde{A}_\mu, B^+_{y\tilde{y}} = \tilde{\chi}$$

$$B^-_{\mu\nu} = \tilde{B}_{\mu\nu}, B^-_{\mu\tilde{y}} = \tilde{C}_\mu, B^-_{\mu y} = A_\mu, B^-_{\tilde{y}y} = A_{\tilde{y}} = \chi$$

$$H_{MNP} = \partial_{[M} B_{NP]}$$

Metric $\mathcal{H}_{MN} dX^M dX^N = \eta_{\mu\nu} dx^\mu dx^\nu + R^2 dy^2 + R^{-2} d\tilde{y}^2$

Action $S \sim \int d^7X \; H_{MNP} H^{MNP}$

Constraints $*H^+ = d\tilde{y} \wedge H^+ \quad *H^- = dy \wedge H^-$

$$*H^0 = d\tilde{y} \wedge H^0 = dy \wedge H^0$$

Fully Doubled Formulation

Theory on T^6 gives doubled theory on T^{12} $\quad B_{PQ}(X^M)$

Metric of signature (10,2) $\quad \mathcal{H}_{MN} \sim diag(g, g^{-1})$

$$S \sim \int d^{12}X \, H_{MNP} H^{MNP}$$

Metric, signature (6,6) $\quad \eta_{MN}$

Constraint $\quad \eta^{MN} \partial_M \partial_N B_{PQ} = 0$

B is sum of terms, each depending on half the coordinates

For each term, there is a split $T^{12} \to T^6 + \tilde{T}^6$

$$X^M \to \begin{pmatrix} x^m \\ \tilde{x}_m \end{pmatrix}$$

x^m coordinates on T^6 that is null wrt η_{MN}
and signature (5,1) wrt $\mathcal{H}_{MN}$

Volume form $\Omega = \frac{1}{6!}\epsilon_{m_1 \ldots m_6} dx^{m_1} \wedge \ldots dx^{m_6}$

Fields depend on x $B^{\Omega}(x^m)$

$$*H^{\Omega} = (*\Omega) \wedge H$$

Sum of such terms for different choices of Ω

Non-abelian interactions?

IIB LST: multiplets $\tilde{A}(x^m)$ with 6D SYM interactions

IIA LST: multiplets $A(x^m, w^m)$ with 5D SYM interactions

Tensor multiplets $B(x^m)$, $\tilde{B}(x^m, w^m)$ still problematic

Interaction between A and B multiplets involves interaction between momentum and winding, so expected to be non-local. Doubled picture might help?

(2,0) Theory?

Subsector of IIA LST, no T-duality

Consider Coulomb branch, with gauge group broken to $U(1)^r$

Compactified on S^1, get 5D SYM with $U(1)^r$ massless subsector. W-boson and monopole string for each root.

Lifts to 6D: r massless abelian tensor multiplets
Multiplet of stringy fields for each root.

Conclusions

- *Stringy fields* give description of winding states, interpretation of Lambert-Papageorgakis multiplet
- Combine with fields to give local fields in doubled space
- Theory of particles and strings
- Generalisation to brane fields
- Weakly constrained DFT non-local, as p,w mutually non-local. In LST etc, these give electric and magnetic charges. Can DFT non-locality help with understanding (2,0) SCFT and LST non-locality?