The String Field Theory vertex, gluing and wrapping

Romuald A. Janik

Jagiellonian University
Kraków

work with Z. Bajnok

Outline

Motivation

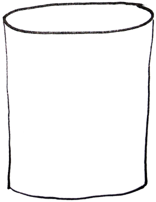
- We have a very good understanding of **the spectrum** of a string on $AdS_5 \times S^5$
- This is due to **the integrability** of the worldsheet theory

Key question:

- How to describe string **interactions** for a generic integrable worldsheet theory
- Previously we knew how to proceed only for a **free** worldsheet theory
  - massless free bosons and fermions in the case of flat spacetime
  - massive free bosons and fermions in the case of pp-wave background geometry
- These methods do not generalize to the interacting QFT case

Motivation

- We have a very good understanding of **the spectrum** of a string on $AdS_5 \times S^5$
- This is due to **the integrability** of the worldsheet theory

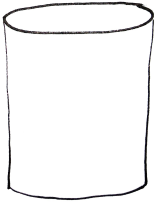


Key question:

- How to describe string **interactions** for a generic integrable worldsheet theory
- Previously we knew how to proceed only for a **free** worldsheet theory
  - massless free bosons and fermions in the case of flat spacetime
  - massive free bosons and fermions in the case of pp-wave background geometry
- These methods do not generalize to the interacting QFT case

Motivation

- We have a very good understanding of **the spectrum** of a string on $AdS_5 \times S^5$
- This is due to **the integrability** of the worldsheet theory

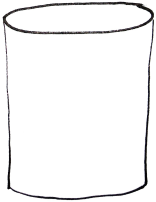


Key question:

- How to describe string **interactions** for a generic integrable worldsheet theory
- Previously we knew how to proceed only for a **free** worldsheet theory
  - massless free bosons and fermions in the case of flat spacetime
  - massive free bosons and fermions in the case of pp-wave background geometry
- These methods do not generalize to the interacting QFT case

Motivation

- We have a very good understanding of **the spectrum** of a string on $AdS_5 \times S^5$
- This is due to **the integrability** of the worldsheet theory

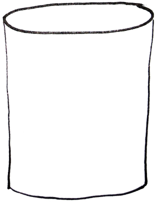


Key question:

- How to describe string **interactions** for a generic integrable worldsheet theory
- Previously we knew how to proceed only for a **free** worldsheet theory
  - massless free bosons and fermions in the case of flat spacetime
  - massive free bosons and fermions in the case of pp-wave background geometry
- These methods do not generalize to the interacting QFT case

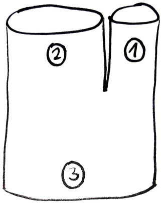
Motivation

- We have a very good understanding of **the spectrum** of a string on $AdS_5 \times S^5$
- This is due to **the integrability** of the worldsheet theory



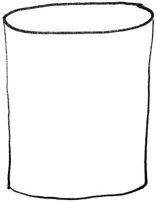
Key question:

- How to describe string **interactions** for a generic integrable worldsheet theory
- Previously we knew how to proceed only for a **free** worldsheet theory
  - massless free bosons and fermions in the case of flat spacetime
  - massive free bosons and fermions in the case of pp-wave background geometry
- These methods do not generalize to the interacting QFT case



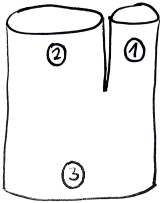
Motivation

- We have a very good understanding of **the spectrum** of a string on $AdS_5 \times S^5$
- This is due to **the integrability** of the worldsheet theory



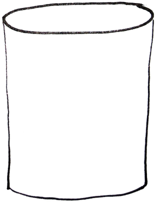
Key question:

- How to describe string **interactions** for a generic integrable worldsheet theory
- Previously we knew how to proceed only for a **free** worldsheet theory
  - massless free bosons and fermions in the case of flat spacetime
  - massive free bosons and fermions in the case of pp-wave background geometry
- These methods do not generalize to the interacting QFT case



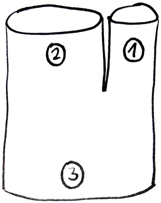
Motivation

- We have a very good understanding of **the spectrum** of a string on $AdS_5 \times S^5$
- This is due to **the integrability** of the worldsheet theory



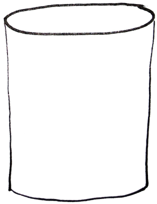
Key question:

- How to describe string **interactions** for a generic integrable worldsheet theory
- Previously we knew how to proceed only for a **free** worldsheet theory
  - massless free bosons and fermions in the case of flat spacetime
  - massive free bosons and fermions in the case of pp-wave background geometry
- These methods do not generalize to the interacting QFT case



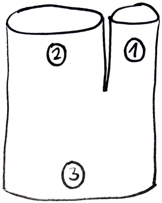
Motivation

- We have a very good understanding of **the spectrum** of a string on $AdS_5 \times S^5$
- This is due to **the integrability** of the worldsheet theory



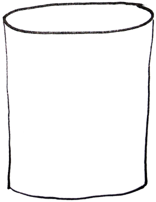
Key question:

- How to describe string **interactions** for a generic integrable worldsheet theory
- Previously we knew how to proceed only for a **free** worldsheet theory
  - massless free bosons and fermions in the case of flat spacetime
  - massive free bosons and fermions in the case of pp-wave background geometry
- These methods do not generalize to the interacting QFT case



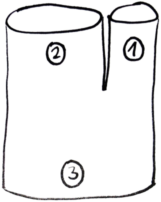
Motivation

- We have a very good understanding of **the spectrum** of a string on $AdS_5 \times S^5$
- This is due to **the integrability** of the worldsheet theory



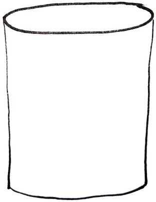
Key question:

- How to describe string **interactions** for a generic integrable worldsheet theory
- Previously we knew how to proceed only for a **free** worldsheet theory
  - massless free bosons and fermions in the case of flat spacetime
  - massive free bosons and fermions in the case of pp-wave background geometry
- These methods do not generalize to the interacting QFT case



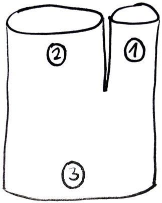
Motivation

- We have a very good understanding of **the spectrum** of a string on $AdS_5 \times S^5$
- This is due to **the integrability** of the worldsheet theory



Key question:

- How to describe string **interactions** for a generic integrable worldsheet theory
- Previously we knew how to proceed only for a **free** worldsheet theory
  - massless free bosons and fermions in the case of flat spacetime
  - massive free bosons and fermions in the case of pp-wave background geometry
- These methods do not generalize to the interacting QFT case



size of the cylinder $L \equiv J$ charge under a $U(1)_R$ subgroup

- Handling QFT on a finite size cylinder is very difficult
- **First decompactify**
  1. Now one can formulate crossing equations
  2. solve YBE, unitarity and crossing to get the S-matrix
- **Handle large cylinders** ($\equiv$ power law corrections in L)
  – Bethe ansatz equations
- **Handle exponential corrections** $\sim e^{-mL}$
  1. single wrapping: Lüscher corrections
  2. multiple wrapping: Thermodynamic Bethe Ansatz (TBA)

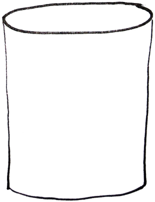
size of the cylinder $L \equiv J$ charge under a $U(1)_R$ subgroup

- Handling QFT on a finite size cylinder is very difficult
- **First decompactify**
  1. Now one can formulate crossing equations
  2. solve YBE, unitarity and crossing to get the S-matrix
- **Handle large cylinders** ($\equiv$ power law corrections in L)
  – Bethe ansatz equations
- **Handle exponential corrections** $\sim e^{-mL}$
  1. single wrapping: Lüscher corrections
  2. multiple wrapping: Thermodynamic Bethe Ansatz (TBA)

size of the cylinder $L \equiv J$ charge under a $U(1)_R$ subgroup

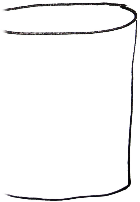
- Handling QFT on a finite size cylinder is very difficult
- **First decompactify**
  1. Now one can formulate crossing equations
  2. solve YBE, unitarity and crossing to get the S-matrix
- **Handle large cylinders** ($\equiv$ power law corrections in L)
  – Bethe ansatz equations
- **Handle exponential corrections** $\sim e^{-mL}$
  1. single wrapping: Lüscher corrections
  2. multiple wrapping: Thermodynamic Bethe Ansatz (TBA)

size of the cylinder $L \equiv J$ charge under a $U(1)_R$ subgroup



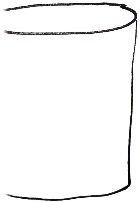
- Handling QFT on a finite size cylinder is very difficult
- **First decompactify**
  1. Now one can formulate crossing equations
  2. solve YBE, unitarity and crossing to get the S-matrix
- **Handle large cylinders** ($\equiv$ power law corrections in L)
  – Bethe ansatz equations
- **Handle exponential corrections** $\sim e^{-mL}$
  1. single wrapping: Lüscher corrections
  2. multiple wrapping: Thermodynamic Bethe Ansatz (TBA)

size of the cylinder $L \equiv J$ charge under a $U(1)_R$ subgroup



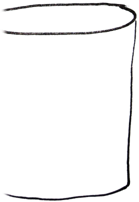
- Handling QFT on a finite size cylinder is very difficult
- **First decompactify**
  1. Now one can formulate crossing equations
  2. solve YBE, unitarity and crossing to get the S-matrix
- **Handle large cylinders** ($\equiv$ power law corrections in L)
  – Bethe ansatz equations
- **Handle exponential corrections** $\sim e^{-mL}$
  1. single wrapping: Lüscher corrections
  2. multiple wrapping: Thermodynamic Bethe Ansatz (TBA)

size of the cylinder $L \equiv J$ charge under a $U(1)_R$ subgroup



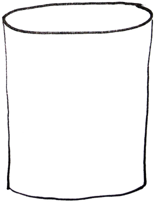
- Handling QFT on a finite size cylinder is very difficult
- **First decompactify**
  1. Now one can formulate crossing equations
  2. solve YBE, unitarity and crossing to get the S-matrix
- **Handle large cylinders** ($\equiv$ power law corrections in L)
  – Bethe ansatz equations
- **Handle exponential corrections** $\sim e^{-mL}$
  1. single wrapping: Lüscher corrections
  2. multiple wrapping: Thermodynamic Bethe Ansatz (TBA)

size of the cylinder $L \equiv J$ charge under a $U(1)_R$ subgroup



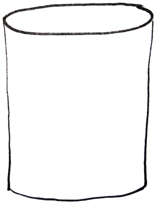
- Handling QFT on a finite size cylinder is very difficult
- **First decompactify**
  1. Now one can formulate crossing equations
  2. solve YBE, unitarity and crossing to get the S-matrix
- **Handle large cylinders** ($\equiv$ power law corrections in L)
  – Bethe ansatz equations
- **Handle exponential corrections** $\sim e^{-mL}$
  1. single wrapping: Lüscher corrections
  2. multiple wrapping: Thermodynamic Bethe Ansatz (TBA)

size of the cylinder $L \equiv J$ charge under a $U(1)_R$ subgroup



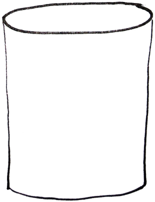
- Handling QFT on a finite size cylinder is very difficult
- **First decompactify**
  1. Now one can formulate crossing equations
  2. solve YBE, unitarity and crossing to get the S-matrix
- **Handle large cylinders** ($\equiv$ power law corrections in L)
  – Bethe ansatz equations
- **Handle exponential corrections** $\sim e^{-mL}$
  1. single wrapping: Lüscher corrections
  2. multiple wrapping: Thermodynamic Bethe Ansatz (TBA)

size of the cylinder $L \equiv J$ charge under a $U(1)_R$ subgroup



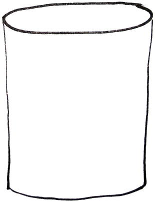
- Handling QFT on a finite size cylinder is very difficult
- **First decompactify**
  1. Now one can formulate crossing equations
  2. solve YBE, unitarity and crossing to get the S-matrix
- **Handle large cylinders** ($\equiv$ power law corrections in L)
  – Bethe ansatz equations
- **Handle exponential corrections** $\sim e^{-mL}$
  1. single wrapping: Lüscher corrections
  2. multiple wrapping: Thermodynamic Bethe Ansatz (TBA)

size of the cylinder $L \equiv J$ charge under a $U(1)_R$ subgroup



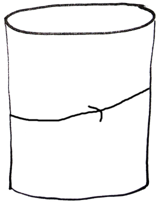
- Handling QFT on a finite size cylinder is very difficult
- **First decompactify**
  1. Now one can formulate crossing equations
  2. solve YBE, unitarity and crossing to get the S-matrix
- **Handle large cylinders** ($\equiv$ power law corrections in L)
  – Bethe ansatz equations
- **Handle exponential corrections** $\sim e^{-mL}$
  1. single wrapping: Lüscher corrections
  2. multiple wrapping: Thermodynamic Bethe Ansatz (TBA)

size of the cylinder $L \equiv J$ charge under a $U(1)_R$ subgroup



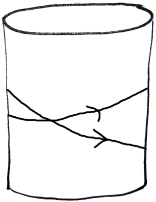
- Handling QFT on a finite size cylinder is very difficult
- **First decompactify**
  1. Now one can formulate crossing equations
  2. solve YBE, unitarity and crossing to get the S-matrix
- **Handle large cylinders** ($\equiv$ power law corrections in L)
  – Bethe ansatz equations
- **Handle exponential corrections** $\sim e^{-mL}$
  1. single wrapping: Lüscher corrections
  2. multiple wrapping: Thermodynamic Bethe Ansatz (TBA)

size of the cylinder $L \equiv J$ charge under a $U(1)_R$ subgroup



- Handling QFT on a finite size cylinder is very difficult
- **First decompactify**
  1. Now one can formulate crossing equations
  2. solve YBE, unitarity and crossing to get the S-matrix
- **Handle large cylinders** ($\equiv$ power law corrections in L)
  – Bethe ansatz equations
- **Handle exponential corrections** $\sim e^{-mL}$
  1. single wrapping: Lüscher corrections
  2. multiple wrapping: Thermodynamic Bethe Ansatz (TBA)

size of the cylinder $L \equiv J$ charge under a $U(1)_R$ subgroup



- Handling QFT on a finite size cylinder is very difficult
- **First decompactify**
  1. Now one can formulate crossing equations
  2. solve YBE, unitarity and crossing to get the S-matrix
- **Handle large cylinders** ($\equiv$ power law corrections in L)
  – Bethe ansatz equations
- **Handle exponential corrections** $\sim e^{-mL}$
  1. single wrapping: Lüscher corrections
  2. multiple wrapping: Thermodynamic Bethe Ansatz (TBA)

How to perform these steps for other observables (and for the string vertex)?

Decompactifying the string vertex

Two approaches:

1. **Decompactified string vertex** — Bajnok, RJ
2. **The hexagon approach** — Basso, Komatsu, Vieira

Decompactifying the string vertex

Two approaches:

1. **Decompactified string vertex** Bajnok, RJ
2. **The hexagon approach** Basso, Komatsu, Vieira

Decompactifying the string vertex

Two approaches:

1. **Decompactified string vertex** Bajnok, RJ
2. **The hexagon approach** Basso, Komatsu, Vieira

Decompactifying the string vertex

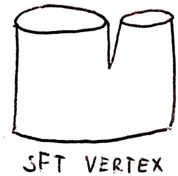
Two approaches:

1. **Decompactified string vertex** — Bajnok, RJ
2. **The hexagon approach** — Basso, Komatsu, Vieira

Decompactifying the string vertex

Two approaches:

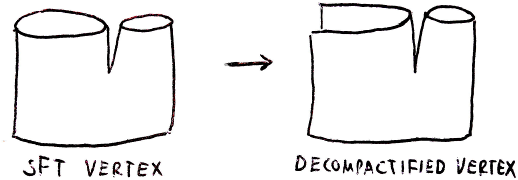
1. **Decompactified string vertex** — Bajnok, RJ
2. **The hexagon approach** — Basso, Komatsu, Vieira



Decompactifying the string vertex

Two approaches:

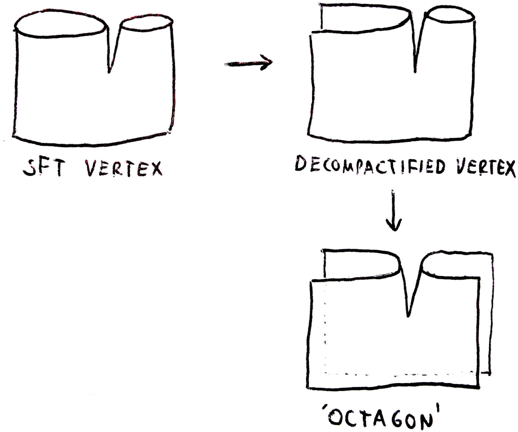
1. **Decompactified string vertex** — Bajnok, RJ
2. **The hexagon approach** — Basso, Komatsu, Vieira



Decompactifying the string vertex

Two approaches:

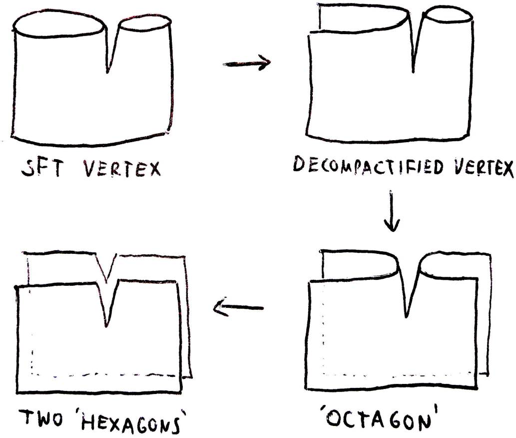
1. **Decompactified string vertex** — Bajnok, RJ
2. **The hexagon approach** — Basso, Komatsu, Vieira



Decompactifying the string vertex

Two approaches:

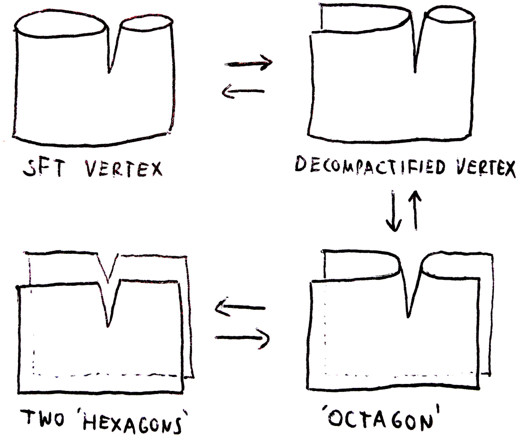
1. **Decompactified string vertex** — Bajnok, RJ
2. **The hexagon approach** — Basso, Komatsu, Vieira

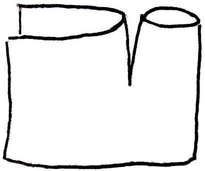


Decompactifying the string vertex

Two approaches:

1. **Decompactified string vertex** — Bajnok, RJ
2. **The hexagon approach** — Basso, Komatsu, Vieira



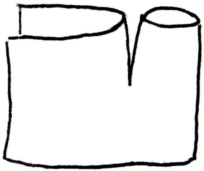


Decompactified string vertex

- Axioms solved for pp-wave
  $\longrightarrow$ reproduce the exact result
  (includes all wrappings w.r.t. size of the remaining closed string)
- Solutions exist for some relativistic integrable QFT's (like sinh-Gordon)
- No full solution yet for $AdS_5 \times S^5$
- We are still lacking a solution of ordinary form factor axioms for $AdS_5 \times S^5$

Hexagon

- Somewhat miraculously exact solution exists for $AdS_5 \times S^5$
- Efficient for calculations in the perturbative regime!
  (wrapping appear at high loop orders)
- 'half wrappings'

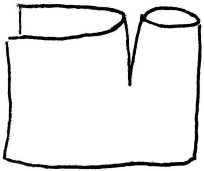


Decompactified string vertex

- Axioms solved for pp-wave
  $\longrightarrow$ reproduce the exact result
  (includes all wrappings w.r.t. size of the remaining closed string)
- Solutions exist for some relativistic integrable QFT's (like sinh-Gordon)
- No full solution yet for $AdS_5 \times S^5$
- We are still lacking a solution of ordinary form factor axioms for $AdS_5 \times S^5$

Hexagon

- Somewhat miraculously exact solution exists for $AdS_5 \times S^5$
- Efficient for calculations in the perturbative regime!
  (wrapping appear at high loop orders)
- 'half wrappings'

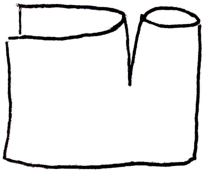


Decompactified string vertex

- Axioms solved for pp-wave
  $\longrightarrow$ reproduce the exact result
  (includes all wrappings w.r.t. size of the remaining closed string)
- Solutions exist for some relativistic integrable QFT's (like sinh-Gordon)
- No full solution yet for $AdS_5 \times S^5$
- We are still lacking a solution of ordinary form factor axioms for $AdS_5 \times S^5$

Hexagon

- Somewhat miraculously exact solution exists for $AdS_5 \times S^5$
- Efficient for calculations in the perturbative regime!
  (wrapping appear at high loop orders)
- 'half wrappings'

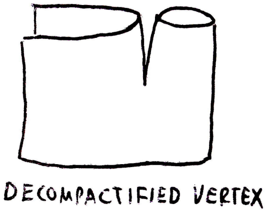


Decompactified string vertex

- Axioms solved for pp-wave
  $\longrightarrow$ reproduce the exact result
  (includes all wrappings w.r.t. size of the remaining closed string)
- Solutions exist for some relativistic integrable QFT's (like sinh-Gordon)
- No full solution yet for $AdS_5 \times S^5$
- We are still lacking a solution of ordinary form factor axioms for $AdS_5 \times S^5$

Hexagon

- Somewhat miraculously exact solution exists for $AdS_5 \times S^5$
- Efficient for calculations in the perturbative regime!
  (wrapping appear at high loop orders)
- 'half wrappings'

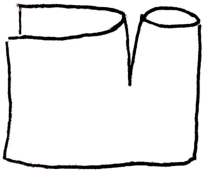


Decompactified string vertex

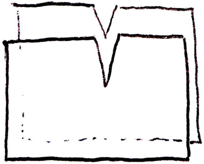
- Axioms solved for pp-wave
  $\longrightarrow$ reproduce the exact result
  (includes all wrappings w.r.t. size of the remaining closed string)
- Solutions exist for some relativistic integrable QFT's (like sinh-Gordon)
- No full solution yet for $AdS_5 \times S^5$
- We are still lacking a solution of ordinary form factor axioms for $AdS_5 \times S^5$

Hexagon

- Somewhat miraculously exact solution exists for $AdS_5 \times S^5$
- Efficient for calculations in the perturbative regime!
  (wrapping appear at high loop orders)
- 'half wrappings'



DECOMPACTIFIED VERTEX



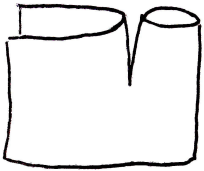
TWO 'HEXAGONS'

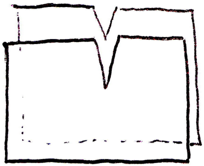
Decompactified string vertex

- Axioms solved for pp-wave
  $\longrightarrow$ reproduce the exact result
  (includes all wrappings w.r.t. size of the remaining closed string)
- Solutions exist for some relativistic integrable QFT's (like sinh-Gordon)
- No full solution yet for $AdS_5 \times S^5$
- We are still lacking a solution of ordinary form factor axioms for $AdS_5 \times S^5$

Hexagon

- Somewhat miraculously exact solution exists for $AdS_5 \times S^5$
- Efficient for calculations in the perturbative regime!
  (wrapping appear at high loop orders)
- 'half wrappings'



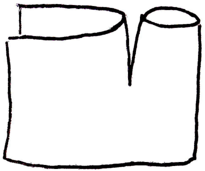


Decompactified string vertex

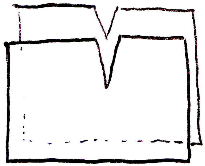
- Axioms solved for pp-wave
  $\longrightarrow$ reproduce the exact result
  (includes all wrappings w.r.t. size of the remaining closed string)
- Solutions exist for some relativistic integrable QFT's (like sinh-Gordon)
- No full solution yet for $AdS_5 \times S^5$
- We are still lacking a solution of ordinary form factor axioms for $AdS_5 \times S^5$

Hexagon

- Somewhat miraculously exact solution exists for $AdS_5 \times S^5$
- Efficient for calculations in the perturbative regime!
  (wrapping appear at high loop orders)
- 'half wrappings'



DECOMPACTIFIED VERTEX



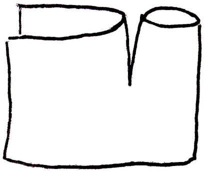
TWO 'HEXAGONS'

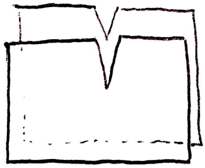
Decompactified string vertex

- Axioms solved for pp-wave
  $\longrightarrow$ reproduce the exact result
  (includes all wrappings w.r.t. size of the remaining closed string)
- Solutions exist for some relativistic integrable QFT's (like sinh-Gordon)
- No full solution yet for $AdS_5 \times S^5$
- We are still lacking a solution of ordinary form factor axioms for $AdS_5 \times S^5$

Hexagon

- Somewhat miraculously exact solution exists for $AdS_5 \times S^5$
- Efficient for calculations in the perturbative regime!
  (wrapping appear at high loop orders)
- 'half wrappings'



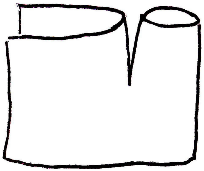


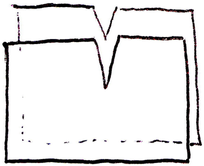
Decompactified string vertex

- Axioms solved for pp-wave
  $\longrightarrow$ reproduce the exact result
  (includes all wrappings w.r.t. size of the remaining closed string)
- Solutions exist for some relativistic integrable QFT's (like sinh-Gordon)
- No full solution yet for $AdS_5 \times S^5$
- We are still lacking a solution of ordinary form factor axioms for $AdS_5 \times S^5$

Hexagon

- Somewhat miraculously exact solution exists for $AdS_5 \times S^5$
- Efficient for calculations in the perturbative regime!
  (wrapping appear at high loop orders)
- 'half wrappings'





Decompactified string vertex

- Axioms solved for pp-wave
  $\longrightarrow$ reproduce the exact result
  (includes all wrappings w.r.t. size of the remaining closed string)
- Solutions exist for some relativistic integrable QFT's (like sinh-Gordon)
- No full solution yet for $AdS_5 \times S^5$
- We are still lacking a solution of ordinary form factor axioms for $AdS_5 \times S^5$

Hexagon

- Somewhat miraculously exact solution exists for $AdS_5 \times S^5$
- Efficient for calculations in the perturbative regime!
  (wrapping appear at high loop orders)
- 'half wrappings'

Power law corrections

Based on

1. experience with form factors — Pozsgay, Takacs

$$\langle \varnothing | \mathcal{O}(0) | \theta_1, \theta_2 \rangle_L = \frac{1}{\sqrt{\rho_2 \cdot S(\theta_1, \theta_2)}} \cdot \underbrace{f(\theta_1, \theta_2)}_{\infty-\text{volume form factor}}$$

   where θ_1, θ_2 are solutions of the Bethe Ansatz equations on a size L cylinder

2. direct OPE coefficient calculations

it seems that we only should:

1. Put in solutions of Bethe Ansatz equations as external particle momenta
2. include appropriate jacobian factors to adjust for standard normalization of states in finite volume

Power law corrections

Based on

1. experience with form factors — Pozsgay, Takacs

$$\langle \varnothing | \mathcal{O}(0) | \theta_1, \theta_2 \rangle_L = \frac{1}{\sqrt{\rho_2 \cdot S(\theta_1,\theta_2)}} \cdot \underbrace{f(\theta_1,\theta_2)}_{\infty-\textit{volume form factor}}$$

   where θ_1, θ_2 are solutions of the Bethe Ansatz equations on a size L cylinder

2. direct OPE coefficient calculations

it seems that we only should:

1. Put in solutions of Bethe Ansatz equations as external particle momenta
2. include appropriate jacobian factors to adjust for standard normalization of states in finite volume

Power law corrections

Based on

1. experience with form factors — Pozsgay, Takacs

$$\langle\varnothing|\mathcal{O}(0)|\theta_1,\theta_2\rangle_L = \frac{1}{\sqrt{\rho_2 \cdot S(\theta_1,\theta_2)}} \cdot \underbrace{f(\theta_1,\theta_2)}_{\infty-\text{volume form factor}}$$

   where θ_1, θ_2 are solutions of the Bethe Ansatz equations on a size L cylinder

2. direct OPE coefficient calculations

it seems that we only should:

1. Put in solutions of Bethe Ansatz equations as external particle momenta
2. include appropriate jacobian factors to adjust for standard normalization of states in finite volume

Power law corrections

Based on

1. experience with form factors — Pozsgay, Takacs

$$\langle \varnothing | \mathcal{O}(0) | \theta_1, \theta_2 \rangle_L = \frac{1}{\sqrt{\rho_2 \cdot S(\theta_1, \theta_2)}} \cdot \underbrace{f(\theta_1, \theta_2)}_{\infty-\textit{volume form factor}}$$

   where θ_1, θ_2 are solutions of the Bethe Ansatz equations on a size L cylinder

2. direct OPE coefficient calculations

it seems that we only should:

1. Put in solutions of Bethe Ansatz equations as external particle momenta
2. include appropriate jacobian factors to adjust for standard normalization of states in finite volume

Power law corrections

Based on

1. experience with form factors — Pozsgay, Takacs

$$\langle \varnothing | \mathcal{O}(0) | \theta_1, \theta_2 \rangle_L = \frac{1}{\sqrt{\rho_2 \cdot S(\theta_1, \theta_2)}} \cdot \underbrace{f(\theta_1, \theta_2)}_{\infty-\text{volume form factor}}$$

   where θ_1, θ_2 are solutions of the Bethe Ansatz equations on a size L cylinder

2. direct OPE coefficient calculations

it seems that we only should:

1. Put in solutions of Bethe Ansatz equations as external particle momenta
2. include appropriate jacobian factors to adjust for standard normalization of states in finite volume

Power law corrections

Based on

1. experience with form factors — Pozsgay, Takacs

$$\langle \varnothing | \mathcal{O}(0) | \theta_1, \theta_2 \rangle_L = \frac{1}{\sqrt{\rho_2 \cdot S(\theta_1, \theta_2)}} \cdot \underbrace{f(\theta_1, \theta_2)}_{\infty-\text{volume form factor}}$$

   where θ_1, θ_2 are solutions of the Bethe Ansatz equations on a size L cylinder

2. direct OPE coefficient calculations

it seems that we only should:

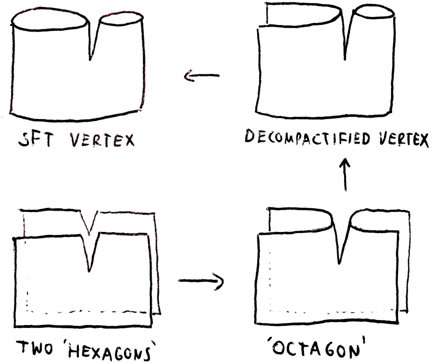
1. Put in solutions of Bethe Ansatz equations as external particle momenta
2. include appropriate jacobian factors to adjust for standard normalization of states in finite volume

Gluing back...

- Insert a summation over intermediate states on each edge Basso, Komatsu, Vieira
- At the wrapping order leads to formal divergent expressions Basso, Goncalves, Komatsu
- Especially subtle at higher wrapping orders

Question: Can we understand the gluing back and multiple wrapping in this case?

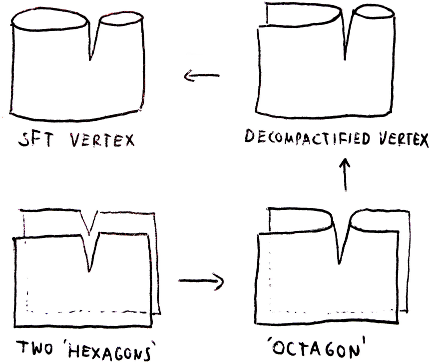
Gluing back...



- Insert a summation over intermediate states on each edge Basso, Komatsu, Vieira
- At the wrapping order leads to formal divergent expressions Basso, Goncalves, Komatsu
- Especially subtle at higher wrapping orders

Question: Can we understand the gluing back and multiple wrapping in this case?

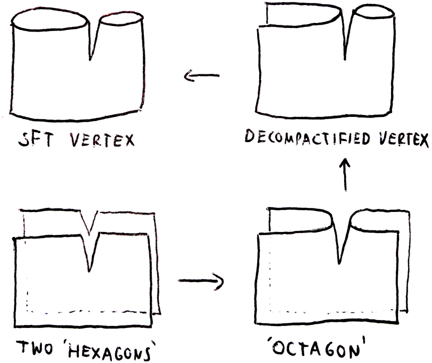
Gluing back...



- Insert a summation over intermediate states on each edge Basso, Komatsu, Vieira
- At the wrapping order leads to formal divergent expressions Basso, Goncalves, Komatsu
- Especially subtle at higher wrapping orders

Question: Can we understand the gluing back and multiple wrapping in this case?

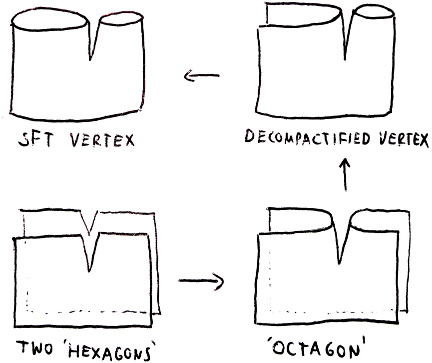
Gluing back...



- Insert a summation over intermediate states on each edge   Basso, Komatsu, Vieira
- At the wrapping order leads to formal divergent expressions   Basso, Goncalves, Komatsu
- Especially subtle at higher wrapping orders

Question: Can we understand the gluing back and multiple wrapping in this case?

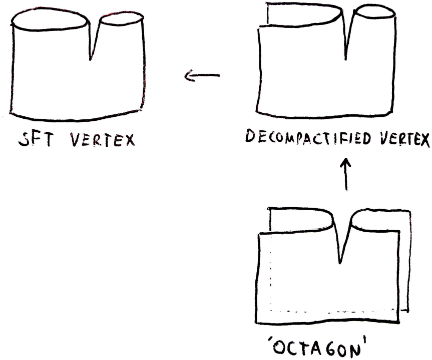
Gluing back...



- Insert a summation over intermediate states on each edge Basso, Komatsu, Vieira
- At the wrapping order leads to formal divergent expressions Basso, Goncalves, Komatsu
- Especially subtle at higher wrapping orders

Question: Can we understand the gluing back and multiple wrapping in this case?

Gluing back...



- Insert a summation over intermediate states on each edge Basso, Komatsu, Vieira
- At the wrapping order leads to formal divergent expressions Basso, Goncalves, Komatsu
- Especially subtle at higher wrapping orders

Question: Can we understand the gluing back and multiple wrapping in this case?

Look for guidance for other observables in relativistic setting

Standard relativistic observables

Ground state energy

- Can be obtained from the large R limit of the partition function

$$Z \sim e^{-RE_0(L)}$$

- For a free boson/fermion we have a simple formula

$$E_0(L) = \pm m \int_{-\infty}^{\infty} \frac{d\theta}{2\pi} \cosh\theta \log\left(1 \mp e^{-mL\cosh\theta}\right)$$

- Expanding the above formula in a power series in $e^{-mL\cosh\theta}$ gives multiple wrapping contributions to the ground state energy.
- Remarkably, the exact formula in the interacting case has the same form

$$E_0(L) = -m \int_{-\infty}^{\infty} \frac{d\theta}{2\pi} \cosh\theta \log\left(1 + e^{-\varepsilon(\theta)}\right)$$

where $\varepsilon(\theta)$ is a solution of the relevant TBA equation.

Standard relativistic observables

Ground state energy

- Can be obtained from the large R limit of the partition function

$$Z \sim e^{-RE_0(L)}$$

- For a free boson/fermion we have a simple formula

$$E_0(L) = \pm m \int_{-\infty}^{\infty} \frac{d\theta}{2\pi} \cosh\theta \log\left(1 \mp e^{-mL\cosh\theta}\right)$$

- Expanding the above formula in a power series in $e^{-mL\cosh\theta}$ gives multiple wrapping contributions to the ground state energy.
- Remarkably, the exact formula in the interacting case has the same form

$$E_0(L) = -m \int_{-\infty}^{\infty} \frac{d\theta}{2\pi} \cosh\theta \log\left(1 + e^{-\varepsilon(\theta)}\right)$$

where $\varepsilon(\theta)$ is a solution of the relevant TBA equation.

Ground state energy

- Can be obtained from the large R limit of the partition function

$$Z \sim e^{-RE_0(L)}$$

- For a free boson/fermion we have a simple formula

$$E_0(L) = \pm m \int_{-\infty}^{\infty} \frac{d\theta}{2\pi} \cosh\theta \log\left(1 \mp e^{-mL\cosh\theta}\right)$$

- Expanding the above formula in a power series in $e^{-mL\cosh\theta}$ gives multiple wrapping contributions to the ground state energy.
- Remarkably, the exact formula in the interacting case has the same form

$$E_0(L) = -m \int_{-\infty}^{\infty} \frac{d\theta}{2\pi} \cosh\theta \log\left(1 + e^{-\varepsilon(\theta)}\right)$$

where $\varepsilon(\theta)$ is a solution of the relevant TBA equation.

Standard relativistic observables

Ground state energy

- Can be obtained from the large R limit of the partition function

$$Z \sim e^{-RE_0(L)}$$

- For a free boson/fermion we have a simple formula

$$E_0(L) = \pm m \int_{-\infty}^{\infty} \frac{d\theta}{2\pi} \cosh\theta \log\left(1 \mp e^{-mL\cosh\theta}\right)$$

- Expanding the above formula in a power series in $e^{-mL\cosh\theta}$ gives multiple wrapping contributions to the ground state energy.
- Remarkably, the exact formula in the interacting case has the same form

$$E_0(L) = -m \int_{-\infty}^{\infty} \frac{d\theta}{2\pi} \cosh\theta \log\left(1 + e^{-\varepsilon(\theta)}\right)$$

where $\varepsilon(\theta)$ is a solution of the relevant TBA equation.

Standard relativistic observables

Ground state energy

- Can be obtained from the large R limit of the partition function

$$Z \sim e^{-RE_0(L)}$$

- For a free boson/fermion we have a simple formula

$$E_0(L) = \pm m \int_{-\infty}^{\infty} \frac{d\theta}{2\pi} \cosh\theta \log\left(1 \mp e^{-mL\cosh\theta}\right)$$

- Expanding the above formula in a power series in $e^{-mL\cosh\theta}$ gives multiple wrapping contributions to the ground state energy.
- Remarkably, the exact formula in the interacting case has the same form

$$E_0(L) = -m \int_{-\infty}^{\infty} \frac{d\theta}{2\pi} \cosh\theta \log\left(1 + e^{-\varepsilon(\theta)}\right)$$

where $\varepsilon(\theta)$ is a solution of the relevant TBA equation.

Standard relativistic observables

Ground state energy

- Can be obtained from the large R limit of the partition function

$$Z \sim e^{-RE_0(L)}$$

- For a free boson/fermion we have a simple formula

$$E_0(L) = \pm m \int_{-\infty}^{\infty} \frac{d\theta}{2\pi} \cosh\theta \log\left(1 \mp e^{-mL\cosh\theta}\right)$$

- Expanding the above formula in a power series in $e^{-mL\cosh\theta}$ gives multiple wrapping contributions to the ground state energy.
- Remarkably, the exact formula in the interacting case has the same form

$$E_0(L) = -m \int_{-\infty}^{\infty} \frac{d\theta}{2\pi} \cosh\theta \log\left(1 + e^{-\varepsilon(\theta)}\right)$$

where $\varepsilon(\theta)$ is a solution of the relevant TBA equation.

Standard relativistic observables

1-pt function (LeClair Mussardo formula)

- For a free boson/fermion

$$\langle \mathcal{O} \rangle_L = \sum_{n=0}^{\infty} \int \prod_{k=1}^{n} \frac{d\theta_k}{2\pi} \frac{1}{1 \mp e^{mL\cosh\theta_k}} F_n^c(\theta_1, \ldots, \theta_n)$$

where $F_n^c(\theta_1, \ldots, \theta_n)$ is the infinite volume (connected) diagonal form factor of the operator $\mathcal{O}$.

- This formula already has a form of summation over infinite set of states
- However the measure factor is nontrivial and distinct from the one for the ground state energy
- Remarkably enough the above formula again generalizes to the interacting case

$$\langle \mathcal{O} \rangle_L = \sum_{n=0}^{\infty} \int \prod_{k=1}^{n} \frac{d\theta_k}{2\pi} \frac{1}{1 + e^{\varepsilon(\theta_k)}} F_n^c(\theta_1, \ldots, \theta_n)$$

Standard relativistic observables

1-pt function (LeClair Mussardo formula)

- For a free boson/fermion

$$\langle \mathcal{O} \rangle_L = \sum_{n=0}^{\infty} \int \prod_{k=1}^{n} \frac{d\theta_k}{2\pi} \frac{1}{1 \mp e^{mL \cosh \theta_k}} F_n^c(\theta_1, \ldots, \theta_n)$$

where $F_n^c(\theta_1, \ldots, \theta_n)$ is the infinite volume (connected) diagonal form factor of the operator $\mathcal{O}$.

- This formula already has a form of summation over infinite set of states
- However the measure factor is nontrivial and distinct from the one for the ground state energy
- Remarkably enough the above formula again generalizes to the interacting case

$$\langle \mathcal{O} \rangle_L = \sum_{n=0}^{\infty} \int \prod_{k=1}^{n} \frac{d\theta_k}{2\pi} \frac{1}{1 + e^{\varepsilon(\theta_k)}} F_n^c(\theta_1, \ldots, \theta_n)$$

Standard relativistic observables

1-pt function (LeClair Mussardo formula)

- For a free boson/fermion

$$\langle \mathcal{O} \rangle_L = \sum_{n=0}^{\infty} \int \prod_{k=1}^{n} \frac{d\theta_k}{2\pi} \frac{1}{1 \mp e^{mL \cosh \theta_k}} F_n^c(\theta_1, \dots, \theta_n)$$

where $F_n^c(\theta_1, \dots, \theta_n)$ is the infinite volume (connected) diagonal form factor of the operator $\mathcal{O}$.

- This formula already has a form of summation over infinite set of states
- However the measure factor is nontrivial and distinct from the one for the ground state energy
- Remarkably enough the above formula again generalizes to the interacting case

$$\langle \mathcal{O} \rangle_L = \sum_{n=0}^{\infty} \int \prod_{k=1}^{n} \frac{d\theta_k}{2\pi} \frac{1}{1 + e^{\varepsilon(\theta_k)}} F_n^c(\theta_1, \dots, \theta_n)$$

Standard relativistic observables

1-pt function (LeClair Mussardo formula)

- For a free boson/fermion

$$\langle \mathcal{O} \rangle_L = \sum_{n=0}^{\infty} \int \prod_{k=1}^{n} \frac{d\theta_k}{2\pi} \frac{1}{1 \mp e^{mL \cosh \theta_k}} F_n^c(\theta_1, \ldots, \theta_n)$$

  where $F_n^c(\theta_1, \ldots, \theta_n)$ is the infinite volume (connected) diagonal form factor of the operator $\mathcal{O}$.

- This formula already has a form of summation over infinite set of states
- However the measure factor is nontrivial and distinct from the one for the ground state energy
- Remarkably enough the above formula again generalizes to the interacting case

$$\langle \mathcal{O} \rangle_L = \sum_{n=0}^{\infty} \int \prod_{k=1}^{n} \frac{d\theta_k}{2\pi} \frac{1}{1 + e^{\varepsilon(\theta_k)}} F_n^c(\theta_1, \ldots, \theta_n)$$

Standard relativistic observables

1-pt function (LeClair Mussardo formula)

- For a free boson/fermion

$$\langle \mathcal{O} \rangle_L = \sum_{n=0}^{\infty} \int \prod_{k=1}^{n} \frac{d\theta_k}{2\pi} \frac{1}{1 \mp e^{mL \cosh \theta_k}} F_n^c(\theta_1, \ldots, \theta_n)$$

  where $F_n^c(\theta_1, \ldots, \theta_n)$ is the infinite volume (connected) diagonal form factor of the operator $\mathcal{O}$.

- This formula already has a form of summation over infinite set of states
- However the measure factor is nontrivial and distinct from the one for the ground state energy
- Remarkably enough the above formula again generalizes to the interacting case

$$\langle \mathcal{O} \rangle_L = \sum_{n=0}^{\infty} \int \prod_{k=1}^{n} \frac{d\theta_k}{2\pi} \frac{1}{1 + e^{\varepsilon(\theta_k)}} F_n^c(\theta_1, \ldots, \theta_n)$$

Standard relativistic observables

1-pt function (LeClair Mussardo formula)

- For a free boson/fermion

$$\langle \mathcal{O} \rangle_L = \sum_{n=0}^{\infty} \int \prod_{k=1}^{n} \frac{d\theta_k}{2\pi} \frac{1}{1 \mp e^{mL \cosh \theta_k}} F_n^c(\theta_1, \ldots, \theta_n)$$

  where $F_n^c(\theta_1, \ldots, \theta_n)$ is the infinite volume (connected) diagonal form factor of the operator $\mathcal{O}$.
- This formula already has a form of summation over infinite set of states
- However the measure factor is nontrivial and distinct from the one for the ground state energy
- Remarkably enough the above formula again generalizes to the interacting case

$$\langle \mathcal{O} \rangle_L = \sum_{n=0}^{\infty} \int \prod_{k=1}^{n} \frac{d\theta_k}{2\pi} \frac{1}{1 + e^{\varepsilon(\theta_k)}} F_n^c(\theta_1, \ldots, \theta_n)$$

From octagon to the string vertex

Octagon

- Octagon with two particles on one edge(string):

$$O(\theta_1, \theta_2) = -\frac{1}{2} \frac{1}{\cosh \frac{\theta_1 - \theta_2}{2}}$$

Decompactified string vertex

- The *exact* (decompactified) pp-wave Neumann coefficient with one string of finite size L takes the form

$$N_L^{\infty}(\theta_1, \theta_2) = O(\theta_1, \theta_2) d_L(\theta_1) d_L(\theta_2)$$

- All wrapping corrections are contained in the functions $d_L(\theta)$

$$d_L(\theta) = \exp \left\{ \int_{-\infty}^{\infty} \frac{du}{2\pi} \frac{1}{\cosh(u-\theta)} \log(1 - e^{-mL \cosh u}) \right\}$$

From octagon to the string vertex

Octagon

- Octagon with two particles on one edge(string):

$$O(\theta_1, \theta_2) = -\frac{1}{2}\frac{1}{\cosh\frac{\theta_1-\theta_2}{2}}$$

Decompactified string vertex

- The *exact* (decompactified) pp-wave Neumann coefficient with one string of finite size L takes the form

$$N_L^{\infty}(\theta_1, \theta_2) = O(\theta_1, \theta_2) d_L(\theta_1) d_L(\theta_2)$$

- All wrapping corrections are contained in the functions $d_L(\theta)$

$$d_L(\theta) = \exp\left\{\int_{-\infty}^{\infty}\frac{du}{2\pi}\frac{1}{\cosh(u-\theta)}\log(1-e^{-mL\cosh u})\right\}$$

From octagon to the string vertex

Octagon

- Octagon with two particles on one edge(string):

$$O(\theta_1, \theta_2) = -\frac{1}{2} \frac{1}{\cosh \frac{\theta_1 - \theta_2}{2}}$$

Decompactified string vertex

- The *exact* (decompactified) pp-wave Neumann coefficient with one string of finite size L takes the form

$$N_L^\infty(\theta_1, \theta_2) = O(\theta_1, \theta_2) d_L(\theta_1) d_L(\theta_2)$$

- All wrapping corrections are contained in the functions $d_L(\theta)$

$$d_L(\theta) = \exp\left\{ \int_{-\infty}^{\infty} \frac{du}{2\pi} \frac{1}{\cosh(u - \theta)} \log(1 - e^{-mL \cosh u}) \right\}$$

From octagon to the string vertex

Octagon

- Octagon with two particles on one edge(string):

$$O(\theta_1, \theta_2) = -\frac{1}{2} \frac{1}{\cosh \frac{\theta_1 - \theta_2}{2}}$$

Decompactified string vertex

- The *exact* (decompactified) pp-wave Neumann coefficient with one string of finite size L takes the form

$$N_L^{\infty}(\theta_1, \theta_2) = O(\theta_1, \theta_2) d_L(\theta_1) d_L(\theta_2)$$

- All wrapping corrections are contained in the functions $d_L(\theta)$

$$d_L(\theta) = \exp\left\{ \int_{-\infty}^{\infty} \frac{du}{2\pi} \frac{1}{\cosh(u-\theta)} \log(1 - e^{-mL\cosh u}) \right\}$$

From octagon to the string vertex

Octagon

- Octagon with two particles on one edge(string):

$$O(\theta_1, \theta_2) = -\frac{1}{2} \frac{1}{\cosh \frac{\theta_1 - \theta_2}{2}}$$

Decompactified string vertex

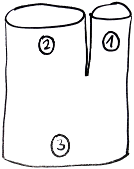
- The *exact* (decompactified) pp-wave Neumann coefficient with one string of finite size L takes the form

$$N_L^{\infty}(\theta_1, \theta_2) = O(\theta_1, \theta_2) d_L(\theta_1) d_L(\theta_2)$$

- All wrapping corrections are contained in the functions $d_L(\theta)$

$$d_L(\theta) = \exp\left\{ \int_{-\infty}^{\infty} \frac{du}{2\pi} \frac{1}{\cosh(u-\theta)} \log(1 - e^{-mL\cosh u}) \right\}$$

From octagon to the string vertex



Finite size string vertex

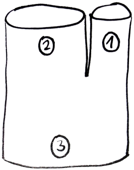
- Neumann coefficient with string lengths L_1,L_2 and $L_3 = L_1 + L_2$

$$N_{L_1}^{L_2}(\theta_1, \theta_2) = O(\theta_1, \theta_2) \cdot \frac{d_{L_1}(\theta_1) d_{L_2}(\theta_1)}{d_{L_3}(\theta_1)} \cdot \frac{d_{L_1}(\theta_2) d_{L_2}(\theta_2)}{d_{L_3}(\theta_2)}$$

- This can be rewritten in terms of the decompactified one $N_{L_1}^{\infty}(\theta_1, \theta_2)$ as:

$$N_{L_1}^{L_2}(\theta_1, \theta_2) = \underbrace{O(\theta_1, \theta_2) d_{L_1}(\theta_1) d_{L_1}(\theta_2)}_{\text{decompactified string vertex}} \cdot \frac{d_{L_2}(\theta_1)}{d_{L_1+L_2}(\theta_1)} \cdot \frac{d_{L_2}(\theta_2)}{d_{L_1+L_2}(\theta_2)}$$

From octagon to the string vertex



Finite size string vertex

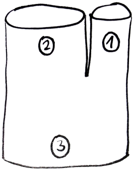
- Neumann coefficient with string lengths L_1,L_2 and $L_3 = L_1 + L_2$

$$N_{L_1}^{L_2}(\theta_1,\theta_2) = O(\theta_1,\theta_2)\cdot \frac{d_{L_1}(\theta_1)d_{L_2}(\theta_1)}{d_{L_3}(\theta_1)}\cdot\frac{d_{L_1}(\theta_2)d_{L_2}(\theta_2)}{d_{L_3}(\theta_2)}$$

- This can be rewritten in terms of the decompactified one $N_{L_1}^{\infty}(\theta_1,\theta_2)$ as:

$$N_{L_1}^{L_2}(\theta_1,\theta_2) = \underbrace{O(\theta_1,\theta_2)d_{L_1}(\theta_1)d_{L_1}(\theta_2)}_{\text{decompactified string vertex}}\cdot\frac{d_{L_2}(\theta_1)}{d_{L_1+L_2}(\theta_1)}\cdot\frac{d_{L_2}(\theta_2)}{d_{L_1+L_2}(\theta_2)}$$

From octagon to the string vertex



Finite size string vertex

- Neumann coefficient with string lengths L_1,L_2 and $L_3 = L_1 + L_2$

$$N_{L_1}^{L_2}(\theta_1, \theta_2) = O(\theta_1, \theta_2) \cdot \frac{d_{L_1}(\theta_1) d_{L_2}(\theta_1)}{d_{L_3}(\theta_1)} \cdot \frac{d_{L_1}(\theta_2) d_{L_2}(\theta_2)}{d_{L_3}(\theta_2)}$$

- This can be rewritten in terms of the decompactified one $N_{L_1}^{\infty}(\theta_1, \theta_2)$ as:

$$N_{L_1}^{L_2}(\theta_1, \theta_2) = \underbrace{O(\theta_1, \theta_2) d_{L_1}(\theta_1) d_{L_1}(\theta_2)}_{\textit{decompactified string vertex}} \cdot \frac{d_{L_2}(\theta_1)}{d_{L_1+L_2}(\theta_1)} \cdot \frac{d_{L_2}(\theta_2)}{d_{L_1+L_2}(\theta_2)}$$

Two ways of understanding these formulas:

1. Finite volume regularization in the mirror channel/cluster expansion

Dorey, Fioravanti, Tateo, Rim

2. $i\varepsilon$ prescription + careful handling of $\delta(0)$ divergences

LeClair, Lesage, Sachdev, Saleur

Concentrate on free massive boson (pp-wave in the SFT case)

Two ways of understanding these formulas:

1. Finite volume regularization in the mirror channel/cluster expansion

Dorey, Fioravanti, Tateo, Rim

2. $i\varepsilon$ prescription + careful handling of $\delta(0)$ divergences

LeClair, Lesage, Sachdev, Saleur

Concentrate on free massive boson (pp-wave in the SFT case)

Two ways of understanding these formulas:

1. Finite volume regularization in the mirror channel/cluster expansion

Dorey, Fioravanti, Tateo, Rim

2. $i\varepsilon$ prescription + careful handling of $\delta(0)$ divergences

LeClair, Lesage, Sachdev, Saleur

Concentrate on free massive boson (pp-wave in the SFT case)

Two ways of understanding these formulas:

1. Finite volume regularization in the mirror channel/cluster expansion

   Dorey, Fioravanti, Tateo, Rim

2. $i\varepsilon$ prescription + careful handling of $\delta(0)$ divergences

   LeClair, Lesage, Sachdev, Saleur

Concentrate on free massive boson (pp-wave in the SFT case)

Two ways of understanding these formulas:

1. Finite volume regularization in the mirror channel/cluster expansion

Dorey, Fioravanti, Tateo, Rim

2. $i\varepsilon$ prescription + careful handling of $\delta(0)$ divergences

LeClair, Lesage, Sachdev, Saleur

Concentrate on free massive boson (pp-wave in the SFT case)

Finite volume regularization in the mirror channel

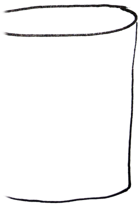
- We want to compute an observable for a cylinder of size L
- Compactify the mirror channel (vertical edge) to size R
- Sum over all multiparticle mirror state

$$1 + \sum_n e^{-E_n L} + \sum_{n_1 \geq n_2} e^{-E_{n_1} L - E_{n_2} L} + \dots$$

- Take the limit $R \to \infty$

$$\sum_n \to R \int_{-\infty}^{\infty} \frac{d\theta}{2\pi} m \cosh \theta$$

Finite volume regularization in the mirror channel



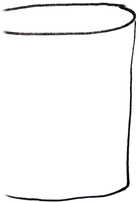
- We want to compute an observable for a cylinder of size L
- Compactify the mirror channel (vertical edge) to size R
- Sum over all multiparticle mirror state

$$1+\sum_n e^{-E_n L}+\sum_{n_1\geq n_2} e^{-E_{n_1}L-E_{n_2}L}+\dots$$

- Take the limit $R\to\infty$

$$\sum_n \to R\int_{-\infty}^{\infty}\frac{d\theta}{2\pi}m\cosh\theta$$

Finite volume regularization in the mirror channel



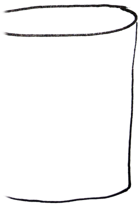
- We want to compute an observable for a cylinder of size L
- Compactify the mirror channel (vertical edge) to size R
- Sum over all multiparticle mirror state

$$1 + \sum_n e^{-E_n L} + \sum_{n_1 \geq n_2} e^{-E_{n_1} L - E_{n_2} L} + \ldots$$

- Take the limit $R \to \infty$

$$\sum_n \to R \int_{-\infty}^{\infty} \frac{d\theta}{2\pi} m \cosh\theta$$

Finite volume regularization in the mirror channel



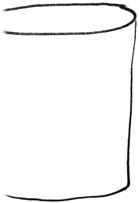
- We want to compute an observable for a cylinder of size L
- Compactify the mirror channel (vertical edge) to size R
- Sum over all multiparticle mirror state

$$1+\sum_{n} e^{-E_n L}+\sum_{n_1\geq n_2} e^{-E_{n_1}L-E_{n_2}L}+\dots$$

- Take the limit $R\to\infty$

$$\sum_n \to R\int_{-\infty}^{\infty}\frac{d\theta}{2\pi} m\cosh\theta$$

Finite volume regularization in the mirror channel



- We want to compute an observable for a cylinder of size L
- Compactify the mirror channel (vertical edge) to size R
- Sum over all multiparticle mirror state

$$1 + \sum_{n} e^{-E_n L} + \sum_{n_1 \geq n_2} e^{-E_{n_1} L - E_{n_2} L} + \dots$$

- Take the limit $R \to \infty$

$$\sum_{n} \to R \int_{-\infty}^{\infty} \frac{d\theta}{2\pi} m \cosh\theta$$

Finite volume regularization in the mirror channel

Subtlety

- Care must be taken about diagonal terms:

$$\sum_{n_1 \geq n_2} = \frac{1}{2} \sum_{n_1, n_2} + \frac{1}{2} \sum_{n_1 = n_2}$$

- The latter term survives in the large R limit!
- It can be thought of as a particle going twice around the cylinder

$$\sim e^{-2mL\cosh\theta}$$

- It is these terms which give rise to the $\log(1 - e^{-mL\cosh\theta})$ measure factor in the formula for ground state energy...
- The evaluation of observables sandwiched between such degenerate states is very nontrivial due to disconnected pieces...

Finite volume regularization in the mirror channel

Subtlety

- Care must be taken about diagonal terms:

$$\sum_{n_1 \geq n_2} = \frac{1}{2} \sum_{n_1, n_2} + \frac{1}{2} \sum_{n_1 = n_2}$$

- The latter term survives in the large R limit!
- It can be thought of as a particle going twice around the cylinder

$$\sim e^{-2mL \cosh \theta}$$

- It is these terms which give rise to the $\log(1 - e^{-mL \cosh \theta})$ measure factor in the formula for ground state energy...
- The evaluation of observables sandwiched between such degenerate states is very nontrivial due to disconnected pieces...

Finite volume regularization in the mirror channel

Subtlety

- Care must be taken about diagonal terms:

$$\sum_{n_1 \geq n_2} = \frac{1}{2} \sum_{n_1, n_2} + \frac{1}{2} \sum_{n_1 = n_2}$$

- The latter term survives in the large R limit!
- It can be thought of as a particle going twice around the cylinder

$$\sim e^{-2mL \cosh \theta}$$

- It is these terms which give rise to the $\log(1 - e^{-mL \cosh \theta})$ measure factor in the formula for ground state energy...
- The evaluation of observables sandwiched between such degenerate states is very nontrivial due to disconnected pieces...

Finite volume regularization in the mirror channel

Subtlety

- Care must be taken about diagonal terms:

$$\sum_{n_1 \geq n_2} = \frac{1}{2} \sum_{n_1, n_2} + \frac{1}{2} \sum_{n_1 = n_2}$$

- The latter term survives in the large R limit!
- It can be thought of as a particle going twice around the cylinder

$$\sim e^{-2mL \cosh \theta}$$

- It is these terms which give rise to the $\log(1 - e^{-mL \cosh \theta})$ measure factor in the formula for ground state energy...
- The evaluation of observables sandwiched between such degenerate states is very nontrivial due to disconnected pieces...

Finite volume regularization in the mirror channel

Subtlety

- Care must be taken about diagonal terms:

$$\sum_{n_1 \geq n_2} = \frac{1}{2} \sum_{n_1, n_2} + \frac{1}{2} \sum_{n_1 = n_2}$$

- The latter term survives in the large R limit!
- It can be thought of as a particle going twice around the cylinder

$$\sim e^{-2mL \cosh \theta}$$

- It is these terms which give rise to the $\log(1 - e^{-mL \cosh \theta})$ measure factor in the formula for ground state energy...
- The evaluation of observables sandwiched between such degenerate states is very nontrivial due to disconnected pieces...

Finite volume regularization in the mirror channel

Subtlety

- Care must be taken about diagonal terms:

$$\sum_{n_1 \geq n_2} = \frac{1}{2} \sum_{n_1, n_2} + \frac{1}{2} \sum_{n_1 = n_2}$$

- The latter term survives in the large R limit!
- It can be thought of as a particle going twice around the cylinder

$$\sim e^{-2mL \cosh \theta}$$

- It is these terms which give rise to the $\log(1 - e^{-mL \cosh \theta})$ measure factor in the formula for ground state energy...
- The evaluation of observables sandwiched between such degenerate states is very nontrivial due to disconnected pieces...

Expectation values

$$Z \langle \mathcal{X} \rangle_L = \langle \emptyset | \mathcal{X} | \emptyset \rangle_R + \sum_{n_1} \langle n_1 | \mathcal{X} | n_1 \rangle_R \, e^{-E_{n_1} L} +$$

$$+ \frac{1}{2} \sum_{n_1, n_2} \langle n_1 n_2 | \mathcal{X} | n_2 n_1 \rangle_R \, e^{-(E_{n_1} + E_{n_2}) L} + \frac{1}{2} \sum_{n_1} \langle n_1 n_1 | \mathcal{X} | n_1 n_1 \rangle_R \, e^{-2E_{n_1} L} + \ldots$$

- We need a prescription for the expectation values

$$\langle n_1 n_2 | \mathcal{X} | n_2 n_1 \rangle_R = \sum (\textit{measure factor}) \cdot (\infty\text{-volume quantity})$$

- For a local operator (LeClair-Mussardo formula) we have

$$\begin{aligned}
\langle n_1 | \mathcal{O} | n_1 \rangle_R &= \frac{F_1^c(\theta_1)}{RE_1} + F_0^c \\
\langle n_1 n_2 | \mathcal{O} | n_2 n_1 \rangle_R &= \frac{F_2^c(\theta_1, \theta_2)}{R^2 E_1 E_2} + \frac{F_1^c(\theta_1)}{RE_1} + \frac{F_1^c(\theta_2)}{RE_2} + F_0^c \\
\langle n_1 n_1 | \mathcal{O} | n_1 n_1 \rangle_R &= 0 + \mathbf{2} \cdot \frac{F_1^c(\theta_1)}{RE_1} + F_0^c
\end{aligned}$$

- The **2** leads to a change of $log(1 - e^{-mL\cosh\theta})$ into $1/(1 - e^{mL\cosh\theta})$ in the LeClair-Mussardo formula

Expectation values

$$Z\left\langle \mathcal{X} \right\rangle_L = \langle \emptyset | \mathcal{X} | \emptyset \rangle_R + \sum_{n_1} \langle n_1 | \mathcal{X} | n_1 \rangle_R \, e^{-E_{n_1} L} +$$

$$+ \frac{1}{2} \sum_{n_1, n_2} \langle n_1 n_2 | \mathcal{X} | n_2 n_1 \rangle_R \, e^{-(E_{n_1}+E_{n_2})L} + \frac{1}{2} \sum_{n_1} \langle n_1 n_1 | \mathcal{X} | n_1 n_1 \rangle_R \, e^{-2E_{n_1} L} + \dots$$

- We need a prescription for the expectation values

$$\langle n_1 n_2 | \mathcal{X} | n_2 n_1 \rangle_R = \sum (\textit{measure factor}) \cdot (\infty\text{-volume quantity})$$

- For a local operator (LeClair-Mussardo formula) we have

$$\begin{aligned}
\langle n_1 | \mathcal{O} | n_1 \rangle_R &= \frac{F_1^c(\theta_1)}{RE_1} + F_0^c \\
\langle n_1 n_2 | \mathcal{O} | n_2 n_1 \rangle_R &= \frac{F_2^c(\theta_1, \theta_2)}{R^2 E_1 E_2} + \frac{F_1^c(\theta_1)}{RE_1} + \frac{F_1^c(\theta_2)}{RE_2} + F_0^c \\
\langle n_1 n_1 | \mathcal{O} | n_1 n_1 \rangle_R &= 0 + \mathbf{2} \cdot \frac{F_1^c(\theta_1)}{RE_1} + F_0^c
\end{aligned}$$

- The **2** leads to a change of $log(1 - e^{-mL\cosh\theta})$ into $1/(1 - e^{mL\cosh\theta})$ in the LeClair-Mussardo formula

Expectation values

$$Z \langle \mathcal{X} \rangle_L = \langle \emptyset | \mathcal{X} | \emptyset \rangle_R + \sum_{n_1} \langle n_1 | \mathcal{X} | n_1 \rangle_R e^{-E_{n_1} L} +$$

$$+ \frac{1}{2} \sum_{n_1, n_2} \langle n_1 n_2 | \mathcal{X} | n_2 n_1 \rangle_R e^{-(E_{n_1} + E_{n_2}) L} + \frac{1}{2} \sum_{n_1} \langle n_1 n_1 | \mathcal{X} | n_1 n_1 \rangle_R e^{-2 E_{n_1} L} + \dots$$

- We need a prescription for the expectation values

$$\langle n_1 n_2 | \mathcal{X} | n_2 n_1 \rangle_R = \sum (\textit{measure factor}) \cdot (\infty\text{-volume quantity})$$

- For a local operator (LeClair-Mussardo formula) we have

$$\begin{aligned} \langle n_1 | \mathcal{O} | n_1 \rangle_R &= \frac{F_1^c(\theta_1)}{RE_1} + F_0^c \\ \langle n_1 n_2 | \mathcal{O} | n_2 n_1 \rangle_R &= \frac{F_2^c(\theta_1, \theta_2)}{R^2 E_1 E_2} + \frac{F_1^c(\theta_1)}{RE_1} + \frac{F_1^c(\theta_2)}{RE_2} + F_0^c \\ \langle n_1 n_1 | \mathcal{O} | n_1 n_1 \rangle_R &= 0 + \mathbf{2} \cdot \frac{F_1^c(\theta_1)}{RE_1} + F_0^c \end{aligned}$$

- The **2** leads to a change of $log(1 - e^{-mL\cosh\theta})$ into $1/(1 - e^{mL\cosh\theta})$ in the LeClair-Mussardo formula

Expectation values

$$Z \langle \mathcal{X} \rangle_L = \langle \emptyset | \mathcal{X} | \emptyset \rangle_R + \sum_{n_1} \langle n_1 | \mathcal{X} | n_1 \rangle_R \, e^{-E_{n_1} L} +$$

$$+ \frac{1}{2} \sum_{n_1, n_2} \langle n_1 n_2 | \mathcal{X} | n_2 n_1 \rangle_R \, e^{-(E_{n_1} + E_{n_2}) L} + \frac{1}{2} \sum_{n_1} \langle n_1 n_1 | \mathcal{X} | n_1 n_1 \rangle_R \, e^{-2 E_{n_1} L} + \dots$$

- We need a prescription for the expectation values

$$\langle n_1 n_2 | \mathcal{X} | n_2 n_1 \rangle_R = \sum (\textit{measure factor}) \cdot (\infty\text{-volume quantity})$$

- For a local operator (LeClair-Mussardo formula) we have

$$\langle n_1 | \mathcal{O} | n_1 \rangle_R = \frac{F_1^c(\theta_1)}{RE_1} + F_0^c$$

$$\langle n_1 n_2 | \mathcal{O} | n_2 n_1 \rangle_R = \frac{F_2^c(\theta_1, \theta_2)}{R^2 E_1 E_2} + \frac{F_1^c(\theta_1)}{RE_1} + \frac{F_1^c(\theta_2)}{RE_2} + F_0^c$$

$$\langle n_1 n_1 | \mathcal{O} | n_1 n_1 \rangle_R = 0 + \mathbf{2} \cdot \frac{F_1^c(\theta_1)}{RE_1} + F_0^c$$

- The **2** leads to a change of $log(1 - e^{-mL\cosh\theta})$ into $1/(1 - e^{mL\cosh\theta})$ in the LeClair-Mussardo formula

Expectation values

$$Z \langle \mathcal{X} \rangle_L = \langle \emptyset | \mathcal{X} | \emptyset \rangle_R + \sum_{n_1} \langle n_1 | \mathcal{X} | n_1 \rangle_R \, e^{-E_{n_1} L} +$$

$$+ \frac{1}{2} \sum_{n_1, n_2} \langle n_1 n_2 | \mathcal{X} | n_2 n_1 \rangle_R \, e^{-(E_{n_1}+E_{n_2})L} + \frac{1}{2} \sum_{n_1} \langle n_1 n_1 | \mathcal{X} | n_1 n_1 \rangle_R \, e^{-2E_{n_1} L} + \dots$$

- We need a prescription for the expectation values

$$\langle n_1 n_2 | \mathcal{X} | n_2 n_1 \rangle_R = \sum (\textit{measure factor}) \cdot (\infty\text{-volume quantity})$$

- For a local operator (LeClair-Mussardo formula) we have

$$\langle n_1 | \mathcal{O} | n_1 \rangle_R = \frac{F_1^c(\theta_1)}{RE_1} + F_0^c$$

$$\langle n_1 n_2 | \mathcal{O} | n_2 n_1 \rangle_R = \frac{F_2^c(\theta_1, \theta_2)}{R^2 E_1 E_2} + \frac{F_1^c(\theta_1)}{RE_1} + \frac{F_1^c(\theta_2)}{RE_2} + F_0^c$$

$$\langle n_1 n_1 | \mathcal{O} | n_1 n_1 \rangle_R = 0 + \mathbf{2} \cdot \frac{F_1^c(\theta_1)}{RE_1} + F_0^c$$

- The 2 leads to a change of $log(1 - e^{-mL\cosh\theta})$ into $1/(1 - e^{mL\cosh\theta})$ in the LeClair-Mussardo formula

Expectation values

$$Z \langle \mathcal{X} \rangle_L = \langle \emptyset | \mathcal{X} | \emptyset \rangle_R + \sum_{n_1} \langle n_1 | \mathcal{X} | n_1 \rangle_R e^{-E_{n_1} L} +$$

$$+ \frac{1}{2} \sum_{n_1, n_2} \langle n_1 n_2 | \mathcal{X} | n_2 n_1 \rangle_R e^{-(E_{n_1} + E_{n_2}) L} + \frac{1}{2} \sum_{n_1} \langle n_1 n_1 | \mathcal{X} | n_1 n_1 \rangle_R e^{-2 E_{n_1} L} + \dots$$

- We need a prescription for the expectation values

$$\langle n_1 n_2 | \mathcal{X} | n_2 n_1 \rangle_R = \sum (\textit{measure factor}) \cdot (\infty\text{-volume quantity})$$

- For a local operator (LeClair-Mussardo formula) we have

$$\langle n_1 | \mathcal{O} | n_1 \rangle_R = \frac{F_1^c(\theta_1)}{R E_1} + F_0^c$$

$$\langle n_1 n_2 | \mathcal{O} | n_2 n_1 \rangle_R = \frac{F_2^c(\theta_1, \theta_2)}{R^2 E_1 E_2} + \frac{F_1^c(\theta_1)}{R E_1} + \frac{F_1^c(\theta_2)}{R E_2} + F_0^c$$

$$\langle n_1 n_1 | \mathcal{O} | n_1 n_1 \rangle_R = 0 + \mathbf{2} \cdot \frac{F_1^c(\theta_1)}{R E_1} + F_0^c$$

- The **2** leads to a change of $log(1 - e^{-mL \cosh \theta})$ into $1/(1 - e^{mL \cosh \theta})$ in the LeClair-Mussardo formula

Question: How does the octagon and decompactified string vertex fit into this framework?

From the octagon to the decompactified vertex

Aim:

$$O(\theta_1, \theta_2) = -\frac{1}{2}\frac{1}{\cosh\frac{\theta_1-\theta_2}{2}} \quad \to \quad N_L^\infty(\theta_1,\theta_2) = O(\theta_1,\theta_2) d_L(\theta_1) d_L(\theta_2)$$

- We need to evaluate the octagon for two external particles and the set of auxillary particles on the mirror edges
- These will have rapidities θ_1, θ_2 and $u_i^{\pm} = u_i \pm i\frac{3\pi}{2}$
- Evaluate by Wick contractions
- Remarkably the connected part has a very simple *factorized* form e.g.

$$O^c(\theta_1,\theta_2,u_1^-,u_2^-,u_2^+,u_1^+) = O(\theta_1,\theta_2)\prod_{i=1}^{2}\left(\frac{-1}{\cosh(u_i-\theta_1)} + \frac{-1}{\cosh(u_i-\theta_2)}\right)$$

- The factorized form leads naturally to an exponentiation...

From the octagon to the decompactified vertex

Aim:

$$O(\theta_1, \theta_2) = -\frac{1}{2} \frac{1}{\cosh \frac{\theta_1 - \theta_2}{2}} \quad \rightarrow \quad N_L^\infty(\theta_1, \theta_2) = O(\theta_1, \theta_2) d_L(\theta_1) d_L(\theta_2)$$

- We need to evaluate the octagon for two external particles and the set of auxillary particles on the mirror edges
- These will have rapidities θ_1, θ_2 and $u_i^\pm = u_i \pm i\frac{3\pi}{2}$
- Evaluate by Wick contractions
- Remarkably the connected part has a very simple *factorized* form e.g.

$$O^c(\theta_1, \theta_2, u_1^-, u_2^-, u_2^+, u_1^+) = O(\theta_1, \theta_2) \prod_{i=1}^{2} \left(\frac{-1}{\cosh(u_i - \theta_1)} + \frac{-1}{\cosh(u_i - \theta_2)} \right)$$

- The factorized form leads naturally to an exponentiation...

From the octagon to the decompactified vertex

Aim:

$$O(\theta_1,\theta_2) = -\frac{1}{2}\frac{1}{\cosh\frac{\theta_1-\theta_2}{2}} \quad \to \quad N_L^\infty(\theta_1,\theta_2) = O(\theta_1,\theta_2)d_L(\theta_1)d_L(\theta_2)$$

- We need to evaluate the octagon for two external particles and the set of auxillary particles on the mirror edges
- These will have rapidities θ_1, θ_2 and $u_i^\pm = u_i \pm i\frac{3\pi}{2}$
- Evaluate by Wick contractions
- Remarkably the connected part has a very simple *factorized* form e.g.

$$O^c(\theta_1,\theta_2,u_1^-,u_2^-,u_2^+,u_1^+) = O(\theta_1,\theta_2)\prod_{i=1}^{2}\left(\frac{-1}{\cosh(u_i-\theta_1)}+\frac{-1}{\cosh(u_i-\theta_2)}\right)$$

- The factorized form leads naturally to an exponentiation...

From the octagon to the decompactified vertex

Aim:

$$O(\theta_1,\theta_2) = -\frac{1}{2}\frac{1}{\cosh\frac{\theta_1-\theta_2}{2}} \quad \to \quad N_L^\infty(\theta_1,\theta_2) = O(\theta_1,\theta_2) d_L(\theta_1) d_L(\theta_2)$$

- We need to evaluate the octagon for two external particles and the set of auxillary particles on the mirror edges
- These will have rapidities θ_1, θ_2 and $u_i^\pm = u_i \pm i\frac{3\pi}{2}$
- Evaluate by Wick contractions
- Remarkably the connected part has a very simple *factorized* form e.g.

$$O^c(\theta_1,\theta_2,u_1^-,u_2^-,u_2^+,u_1^+) = O(\theta_1,\theta_2)\prod_{i=1}^{2}\left(\frac{-1}{\cosh(u_i-\theta_1)} + \frac{-1}{\cosh(u_i-\theta_2)}\right)$$

- The factorized form leads naturally to an exponentiation...

From the octagon to the decompactified vertex

Aim:

$$O(\theta_1, \theta_2) = -\frac{1}{2} \frac{1}{\cosh \frac{\theta_1 - \theta_2}{2}} \quad \to \quad N_L^\infty(\theta_1, \theta_2) = O(\theta_1, \theta_2) d_L(\theta_1) d_L(\theta_2)$$

- We need to evaluate the octagon for two external particles and the set of auxillary particles on the mirror edges
- These will have rapidities θ_1, θ_2 and $u_i^\pm = u_i \pm i\frac{3\pi}{2}$
- Evaluate by Wick contractions
- Remarkably the connected part has a very simple *factorized* form e.g.

$$O^c(\theta_1, \theta_2, u_1^-, u_2^-, u_2^+, u_1^+) = O(\theta_1, \theta_2) \prod_{i=1}^{2} \left(\frac{-1}{\cosh(u_i - \theta_1)} + \frac{-1}{\cosh(u_i - \theta_2)} \right)$$

- The factorized form leads naturally to an exponentiation...

From the octagon to the decompactified vertex

Aim:

$$O(\theta_1, \theta_2) = -\frac{1}{2} \frac{1}{\cosh \frac{\theta_1 - \theta_2}{2}} \quad \rightarrow \quad N_L^{\infty}(\theta_1, \theta_2) = O(\theta_1, \theta_2) d_L(\theta_1) d_L(\theta_2)$$

- We need to evaluate the octagon for two external particles and the set of auxillary particles on the mirror edges
- These will have rapidities θ_1, θ_2 and $u_i^{\pm} = u_i \pm i\frac{3\pi}{2}$
- Evaluate by Wick contractions
- Remarkably the connected part has a very simple *factorized* form e.g.

$$O^c(\theta_1, \theta_2, u_1^-, u_2^-, u_2^+, u_1^+) = O(\theta_1, \theta_2) \prod_{i=1}^{2} \left(\frac{-1}{\cosh(u_i - \theta_1)} + \frac{-1}{\cosh(u_i - \theta_2)} \right)$$

- The factorized form leads naturally to an exponentiation...

From the octagon to the decompactified vertex

Aim:

$$\mathcal{O}(\theta_1,\theta_2) = -\frac{1}{2}\frac{1}{\cosh\frac{\theta_1-\theta_2}{2}} \quad \to \quad N_L^\infty(\theta_1,\theta_2) = \mathcal{O}(\theta_1,\theta_2)d_L(\theta_1)d_L(\theta_2)$$

- We need to evaluate the octagon for two external particles and the set of auxillary particles on the mirror edges
- These will have rapidities θ_1, θ_2 and $u_i^\pm = u_i \pm i\frac{3\pi}{2}$
- Evaluate by Wick contractions
- Remarkably the connected part has a very simple *factorized* form e.g.

$$\mathcal{O}^c(\theta_1,\theta_2,u_1^-,u_2^-,u_2^+,u_1^+) = \mathcal{O}(\theta_1,\theta_2)\prod_{i=1}^{2}\left(\frac{-1}{\cosh(u_i-\theta_1)}+\frac{-1}{\cosh(u_i-\theta_2)}\right)$$

- The factorized form leads naturally to an exponentiation...

From the octagon to the decompactified vertex

Aim:

$$\mathcal{O}(\theta_1, \theta_2) = -\frac{1}{2} \frac{1}{\cosh \frac{\theta_1 - \theta_2}{2}} \quad \rightarrow \quad N_L^\infty(\theta_1, \theta_2) = \mathcal{O}(\theta_1, \theta_2) d_L(\theta_1) d_L(\theta_2)$$

- We need to evaluate the octagon for two external particles and the set of auxillary particles on the mirror edges
- These will have rapidities θ_1, θ_2 and $u_i^\pm = u_i \pm i\frac{3\pi}{2}$
- Evaluate by Wick contractions
- Remarkably the connected part has a very simple *factorized* form e.g.

$$\mathcal{O}^c(\theta_1, \theta_2, u_1^-, u_2^-, u_2^+, u_1^+) = \mathcal{O}(\theta_1, \theta_2) \prod_{i=1}^{2} \left(\frac{-1}{\cosh(u_i - \theta_1)} + \frac{-1}{\cosh(u_i - \theta_2)} \right)$$

- The factorized form leads naturally to an exponentiation...

From the octagon to the decompactified vertex

- We assume expectation values without disconnected terms

$$\begin{aligned}\langle\emptyset|\,\mathcal{O}^{\theta_1,\theta_2}\,|\emptyset\rangle_R &= O(\theta_1,\theta_2)\\ \langle n_1|\,\mathcal{O}^{\theta_1,\theta_2}\,|n_1\rangle_R &= \frac{1}{RE_1}O^c(\theta_1,\theta_2,u_1^-,u_1^+)\\ \langle n_1 n_2|\,\mathcal{O}^{\theta_1,\theta_2}\,|n_2 n_1\rangle_R &= \frac{1}{R^2E_1E_2}O^c(\theta_1,\theta_2,u_1^-,u_2^-,u_2^+,u_1^+)\\ \langle n_1 n_1|\,\mathcal{O}^{\theta_1,\theta_2}\,|n_1 n_1\rangle_R &= \frac{1}{RE_1}O^c(\theta_1,\theta_2,u_1^-,u_1^+)\end{aligned}$$

- The above expressions lead to the exact answer for the decompactified string vertex with one string being of size L

$$\begin{aligned}N_L^\infty(\theta_1,\theta_2) &= O(\theta_1,\theta_2)\cdot\underbrace{e^{\int_{-\infty}^{\infty}\frac{du}{2\pi}\frac{1}{\cosh(u-\theta_1)}\log(1-e^{-mL\cosh u})}}_{d_L(\theta_1)}\cdot(\theta_1\to\theta_2)\\ &= O(\theta_1,\theta_2)d_L(\theta_1)d_L(\theta_2)\end{aligned}$$

From the octagon to the decompactified vertex

- We assume expectation values without disconnected terms

$$\begin{aligned}
\langle\emptyset|\,\mathcal{O}^{\theta_1,\theta_2}\,|\emptyset\rangle_R &= O(\theta_1,\theta_2)\\
\langle n_1|\,\mathcal{O}^{\theta_1,\theta_2}\,|n_1\rangle_R &= \frac{1}{RE_1}O^c(\theta_1,\theta_2,u_1^-,u_1^+)\\
\langle n_1 n_2|\,\mathcal{O}^{\theta_1,\theta_2}\,|n_2 n_1\rangle_R &= \frac{1}{R^2E_1E_2}O^c(\theta_1,\theta_2,u_1^-,u_2^-,u_2^+,u_1^+)\\
\langle n_1 n_1|\,\mathcal{O}^{\theta_1,\theta_2}\,|n_1 n_1\rangle_R &= \frac{1}{RE_1}O^c(\theta_1,\theta_2,u_1^-,u_1^+)
\end{aligned}$$

- The above expressions lead to the exact answer for the decompactified string vertex with one string being of size L

$$\begin{aligned}
N_L^{\infty}(\theta_1,\theta_2) &= O(\theta_1,\theta_2)\cdot\underbrace{e^{\int_{-\infty}^{\infty}\frac{du}{2\pi}\frac{1}{\cosh(u-\theta_1)}\log(1-e^{-mL\cosh u})}}_{d_L(\theta_1)}\cdot(\theta_1\to\theta_2)\\
&= O(\theta_1,\theta_2)d_L(\theta_1)d_L(\theta_2)
\end{aligned}$$

From the octagon to the decompactified vertex

- We assume expectation values without disconnected terms

$$\begin{aligned}
\langle\emptyset|\,\mathcal{O}^{\theta_1,\theta_2}\,|\emptyset\rangle_R &= O(\theta_1,\theta_2) \\
\langle n_1|\,\mathcal{O}^{\theta_1,\theta_2}\,|n_1\rangle_R &= \frac{1}{RE_1} O^c(\theta_1,\theta_2,u_1^-,u_1^+) \\
\langle n_1 n_2|\,\mathcal{O}^{\theta_1,\theta_2}\,|n_2 n_1\rangle_R &= \frac{1}{R^2 E_1 E_2} O^c(\theta_1,\theta_2,u_1^-,u_2^-,u_2^+,u_1^+) \\
\langle n_1 n_1|\,\mathcal{O}^{\theta_1,\theta_2}\,|n_1 n_1\rangle_R &= \frac{1}{RE_1} O^c(\theta_1,\theta_2,u_1^-,u_1^+)
\end{aligned}$$

- The above expressions lead to the exact answer for the decompactified string vertex with one string being of size L

$$\begin{aligned}
N_L^\infty(\theta_1,\theta_2) &= O(\theta_1,\theta_2)\cdot \underbrace{e^{\int_{-\infty}^{\infty}\frac{du}{2\pi}\frac{1}{\cosh(u-\theta_1)}\log(1-e^{-mL\cosh u})}}_{d_L(\theta_1)}\cdot(\theta_1\to\theta_2) \\
&= O(\theta_1,\theta_2)d_L(\theta_1)d_L(\theta_2)
\end{aligned}$$

From the decompactified to the full string vertex

- Now perform Wick contractions using

$$N_{L_1}^{\infty}(\theta_1, \theta_2) = O(\theta_1, \theta_2) d_{L_1}(\theta_1) d_{L_1}(\theta_2)$$

- Similar factorization holds..
- New ingredient:

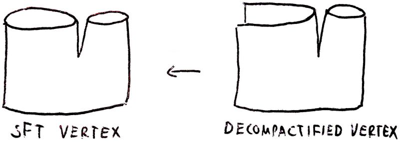
$$d_{L_1}(u^+) d_{L_1}(u^-) = \frac{mL_1}{2\pi^2} \left(1 - e^{-mL_1 \cosh u}\right)$$

- We make now a modified assumption

$$\langle n_1 n_1 | \mathcal{N}_{L_1}^{\theta_1, \theta_2} | n_1 n_1 \rangle_R = \frac{1}{RE_1} O(\theta_1, \theta_2) \left(\frac{-1}{\cosh(u_1 - \theta_1)} + \frac{-1}{\cosh(u_1 - \theta_2)} \right) \times$$

$$\times d_{2L_1}(u_1^+) d_{2L_1}(u_1^-)$$

From the decompactified to the full string vertex



- Now perform Wick contractions using

$$N^{\infty}_{L_1}(\theta_1,\theta_2) = O(\theta_1,\theta_2) d_{L_1}(\theta_1) d_{L_1}(\theta_2)$$

- Similar factorization holds..
- New ingredient:

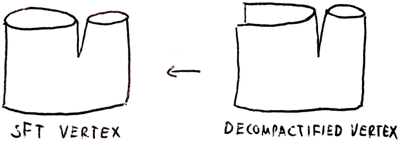
$$d_{L_1}(u^+) d_{L_1}(u^-) = \frac{mL_1}{2\pi^2}\left(1 - e^{-mL_1 \cosh u}\right)$$

- We make now a modified assumption

$$\langle n_1 n_1 | \mathcal{N}^{\theta_1,\theta_2}_{L_1} | n_1 n_1 \rangle_R = \frac{1}{RE_1} O(\theta_1,\theta_2) \left(\frac{-1}{\cosh(u_1-\theta_1)} + \frac{-1}{\cosh(u_1-\theta_2)} \right) \times$$

$$\times d_{2L_1}(u_1^+) d_{2L_1}(u_1^-)$$

From the decompactified to the full string vertex



- Now perform Wick contractions using

$$N^{\infty}_{L_1}(\theta_1,\theta_2) = O(\theta_1,\theta_2) d_{L_1}(\theta_1) d_{L_1}(\theta_2)$$

- Similar factorization holds..
- New ingredient:

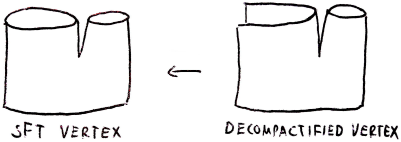
$$d_{L_1}(u^+) d_{L_1}(u^-) = \frac{mL_1}{2\pi^2}\left(1 - e^{-mL_1 \cosh u}\right)$$

- We make now a modified assumption

$$\langle n_1 n_1 | \mathcal{N}^{\theta_1,\theta_2}_{L_1} | n_1 n_1 \rangle_R = \frac{1}{RE_1} O(\theta_1,\theta_2)\left(\frac{-1}{\cosh(u_1-\theta_1)} + \frac{-1}{\cosh(u_1-\theta_2)}\right) \times$$

$$\times d_{2L_1}(u_1^+) d_{2L_1}(u_1^-)$$

From the decompactified to the full string vertex



- Now perform Wick contractions using

$$N^{\infty}_{L_1}(\theta_1, \theta_2) = O(\theta_1, \theta_2) d_{L_1}(\theta_1) d_{L_1}(\theta_2)$$

- Similar factorization holds..
- New ingredient:

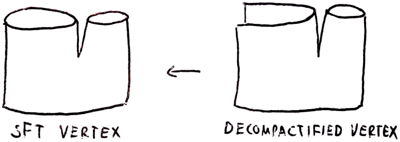
$$d_{L_1}(u^+) d_{L_1}(u^-) = \frac{mL_1}{2\pi^2} \left(1 - e^{-mL_1 \cosh u}\right)$$

- We make now a modified assumption

$$\langle n_1 n_1 | \mathcal{N}^{\theta_1, \theta_2}_{L_1} | n_1 n_1 \rangle_R = \frac{1}{RE_1} O(\theta_1, \theta_2) \left(\frac{-1}{\cosh(u_1 - \theta_1)} + \frac{-1}{\cosh(u_1 - \theta_2)} \right) \times$$

$$\times d_{2L_1}(u_1^+) d_{2L_1}(u_1^-)$$

From the decompactified to the full string vertex



- Now perform Wick contractions using

$$N_{L_1}^{\infty}(\theta_1, \theta_2) = O(\theta_1, \theta_2) d_{L_1}(\theta_1) d_{L_1}(\theta_2)$$

- Similar factorization holds..
- New ingredient:

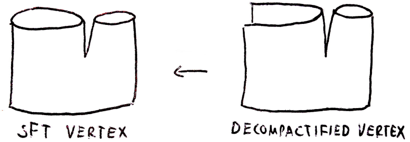
$$d_{L_1}(u^+) d_{L_1}(u^-) = \frac{mL_1}{2\pi^2} \left(1 - e^{-mL_1 \cosh u}\right)$$

- We make now a modified assumption

$$\langle n_1 n_1 | \mathcal{N}_{L_1}^{\theta_1, \theta_2} | n_1 n_1 \rangle_R = \frac{1}{RE_1} O(\theta_1, \theta_2) \left(\frac{-1}{\cosh(u_1 - \theta_1)} + \frac{-1}{\cosh(u_1 - \theta_2)} \right) \times$$

$$\times d_{2L_1}(u_1^+) d_{2L_1}(u_1^-)$$

From the decompactified to the full string vertex



- Now perform Wick contractions using

$$N_{L_1}^{\infty}(\theta_1, \theta_2) = O(\theta_1, \theta_2) d_{L_1}(\theta_1) d_{L_1}(\theta_2)$$

- Similar factorization holds..
- New ingredient:

$$d_{L_1}(u^+) d_{L_1}(u^-) = \frac{mL_1}{2\pi^2} \left(1 - e^{-mL_1 \cosh u}\right)$$

- We make now a modified assumption

$$\langle n_1 n_1 | \mathcal{N}_{L_1}^{\theta_1, \theta_2} | n_1 n_1 \rangle_R = \frac{1}{RE_1} O(\theta_1, \theta_2) \left(\frac{-1}{\cosh(u_1 - \theta_1)} + \frac{-1}{\cosh(u_1 - \theta_2)} \right) \times$$

$$\times d_{\mathbf{2}L_1}(u_1^+) d_{\mathbf{2}L_1}(u_1^-)$$

From the decompactified to the full string vertex

- Repeating the summation over multiple wrapping states lead to the full finite size string vertex Neumann coefficient

$$N_{L_1}^{L_2}(\theta_1,\theta_2) = O(\theta_1,\theta_2)\cdot\frac{d_{L_1}(\theta_1)d_{L_2}(\theta_1)}{d_{L_3}(\theta_1)}\cdot\frac{d_{L_1}(\theta_2)d_{L_2}(\theta_2)}{d_{L_3}(\theta_2)}$$

Some loose ends

1. We have not derived the formulas for the (asymptotic) finite volume expectation values (note hoewver that even in the Le Clair-Mussardo case such expressions are conjectural)
2. However it is far from trivial that such a simple choice exists which reproduces the exact finite volume expressions
3. We are currently investigating the $i\varepsilon$ prescription calculation which seems to be more predictive... work in progress

From the decompactified to the full string vertex

- Repeating the summation over multiple wrapping states lead to the full finite size string vertex Neumann coefficient

$$N_{L_1}^{L_2}(\theta_1, \theta_2) = O(\theta_1, \theta_2) \cdot \frac{d_{L_1}(\theta_1) d_{L_2}(\theta_1)}{d_{L_3}(\theta_1)} \cdot \frac{d_{L_1}(\theta_2) d_{L_2}(\theta_2)}{d_{L_3}(\theta_2)}$$

Some loose ends

1. We have not derived the formulas for the (asymptotic) finite volume expectation values (note hoewver that even in the Le Clair-Mussardo case such expressions are conjectural)
2. However it is far from trivial that such a simple choice exists which reproduces the exact finite volume expressions
3. We are currently investigating the $i\varepsilon$ prescription calculation which seems to be more predictive... work in progress

From the decompactified to the full string vertex

- Repeating the summation over multiple wrapping states lead to the full finite size string vertex Neumann coefficient

$$N_{L_1}^{L_2}(\theta_1,\theta_2) = O(\theta_1,\theta_2)\cdot\frac{d_{L_1}(\theta_1)d_{L_2}(\theta_1)}{d_{L_3}(\theta_1)}\cdot\frac{d_{L_1}(\theta_2)d_{L_2}(\theta_2)}{d_{L_3}(\theta_2)}$$

Some loose ends

1. We have not derived the formulas for the (asymptotic) finite volume expectation values (note hoewver that even in the Le Clair-Mussardo case such expressions are conjectural)
2. However it is far from trivial that such a simple choice exists which reproduces the exact finite volume expressions
3. We are currently investigating the $i\varepsilon$ prescription calculation which seems to be more predictive... work in progress

From the decompactified to the full string vertex

- Repeating the summation over multiple wrapping states lead to the full finite size string vertex Neumann coefficient

$$N_{L_1}^{L_2}(\theta_1, \theta_2) = O(\theta_1, \theta_2) \cdot \frac{d_{L_1}(\theta_1) d_{L_2}(\theta_1)}{d_{L_3}(\theta_1)} \cdot \frac{d_{L_1}(\theta_2) d_{L_2}(\theta_2)}{d_{L_3}(\theta_2)}$$

Some loose ends

1. We have not derived the formulas for the (asymptotic) finite volume expectation values (note hoewver that even in the Le Clair-Mussardo case such expressions are conjectural)
2. However it is far from trivial that such a simple choice exists which reproduces the exact finite volume expressions
3. We are currently investigating the $i\varepsilon$ prescription calculation which seems to be more predictive... work in progress

From the decompactified to the full string vertex

- Repeating the summation over multiple wrapping states lead to the full finite size string vertex Neumann coefficient

$$N_{L_1}^{L_2}(\theta_1, \theta_2) = O(\theta_1, \theta_2) \cdot \frac{d_{L_1}(\theta_1) d_{L_2}(\theta_1)}{d_{L_3}(\theta_1)} \cdot \frac{d_{L_1}(\theta_2) d_{L_2}(\theta_2)}{d_{L_3}(\theta_2)}$$

Some loose ends

1. We have not derived the formulas for the (asymptotic) finite volume expectation values (note hoewver that even in the Le Clair-Mussardo case such expressions are conjectural)
2. However it is far from trivial that such a simple choice exists which reproduces the exact finite volume expressions
3. We are currently investigating the $i\varepsilon$ prescription calculation which seems to be more predictive... work in progress

From the decompactified to the full string vertex

- Repeating the summation over multiple wrapping states lead to the full finite size string vertex Neumann coefficient

$$N_{L_1}^{L_2}(\theta_1, \theta_2) = O(\theta_1, \theta_2) \cdot \frac{d_{L_1}(\theta_1) d_{L_2}(\theta_1)}{d_{L_3}(\theta_1)} \cdot \frac{d_{L_1}(\theta_2) d_{L_2}(\theta_2)}{d_{L_3}(\theta_2)}$$

Some loose ends

1. We have not derived the formulas for the (asymptotic) finite volume expectation values (note hoewver that even in the Le Clair-Mussardo case such expressions are conjectural)
2. However it is far from trivial that such a simple choice exists which reproduces the exact finite volume expressions
3. We are currently investigating the $i\varepsilon$ prescription calculation which seems to be more predictive... work in progress

From the decompactified to the full string vertex

- Repeating the summation over multiple wrapping states lead to the full finite size string vertex Neumann coefficient

$$N_{L_1}^{L_2}(\theta_1, \theta_2) = O(\theta_1, \theta_2) \cdot \frac{d_{L_1}(\theta_1) d_{L_2}(\theta_1)}{d_{L_3}(\theta_1)} \cdot \frac{d_{L_1}(\theta_2) d_{L_2}(\theta_2)}{d_{L_3}(\theta_2)}$$

Some loose ends

1. We have not derived the formulas for the (asymptotic) finite volume expectation values (note hoewver that even in the Le Clair-Mussardo case such expressions are conjectural)
2. However it is far from trivial that such a simple choice exists which reproduces the exact finite volume expressions
3. We are currently investigating the $i\varepsilon$ prescription calculation which seems to be more predictive... work in progress

Conclusions

- The structure of multiple wrapping is surprisingly subtle even for free boson theories!
- This is especially so in the case of nontrivial topologies – octagon, decompactified vertex and the full finite volume vertex
- Still many open questions remain...
- Solution of form factor axioms for $AdS_5 \times S^5$..
- Resumming the hexagons into an octagon

  see Basso, Goncalves, Komatsu for hexagon+single wrapping in AdS

Conclusions

- The structure of multiple wrapping is surprisingly subtle even for free boson theories!
- This is especially so in the case of nontrivial topologies – octagon, decompactified vertex and the full finite volume vertex
- Still many open questions remain...
- Solution of form factor axioms for $AdS_5 \times S^5$..
- Resumming the hexagons into an octagon

  see Basso, Goncalves, Komatsu for hexagon+single wrapping in AdS

Conclusions

- The structure of multiple wrapping is surprisingly subtle even for free boson theories!
- This is especially so in the case of nontrivial topologies – octagon, decompactified vertex and the full finite volume vertex
- Still many open questions remain...
- Solution of form factor axioms for $AdS_5 \times S^5$..
- Resumming the hexagons into an octagon

  see Basso, Goncalves, Komatsu for hexagon+single wrapping in AdS

Conclusions

- The structure of multiple wrapping is surprisingly subtle even for free boson theories!
- This is especially so in the case of nontrivial topologies – octagon, decompactified vertex and the full finite volume vertex
- Still many open questions remain...
- Solution of form factor axioms for $AdS_5 \times S^5$..
- Resumming the hexagons into an octagon

  see Basso, Goncalves, Komatsu for hexagon+single wrapping in AdS

Conclusions

- The structure of multiple wrapping is surprisingly subtle even for free boson theories!
- This is especially so in the case of nontrivial topologies – octagon, decompactified vertex and the full finite volume vertex
- Still many open questions remain...
- Solution of form factor axioms for $AdS_5 \times S^5$..
- Resumming the hexagons into an octagon

  see Basso, Goncalves, Komatsu for hexagon+single wrapping in AdS

Conclusions

- The structure of multiple wrapping is surprisingly subtle even for free boson theories!
- This is especially so in the case of nontrivial topologies – octagon, decompactified vertex and the full finite volume vertex
- Still many open questions remain...
- Solution of form factor axioms for $AdS_5 \times S^5$..
- Resumming the hexagons into an octagon

  see Basso, Goncalves, Komatsu for hexagon+single wrapping in AdS