Dissipative Hydrodynamics in Superspace

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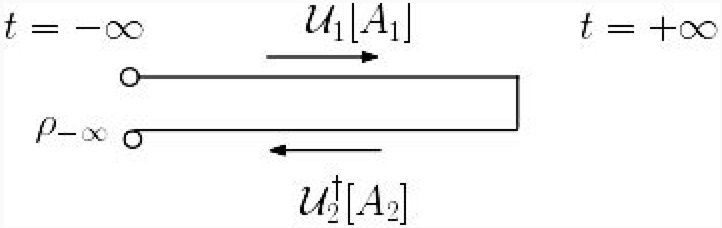
Dissipative hydrodynamics in superspace

We want to understand **hydrodynamics from first principles**.

Hydrodynamics is a **thermal** system **out-of-equilibrium** ➡ the **Schwinger-Keldysh partition function** is the natural formalism



This formalism allows to systematically account for **all** the universal, time-ordered, thermal correlators (retarded, advanced, symmetric,...)

$$Z[A_1,A_2] = \mathrm{Tr}\left(\mathcal{U}_1[A_1]\rho_{-\infty}\mathcal{U}_2^\dagger[A_2]\right) = \int \mathcal{D}\phi_1\mathcal{D}\phi_2 e^{iS_1[\phi_1;A_1]-iS_2[\phi_2;A_2]}$$

$$= \int \mathcal{D}\xi_1\,\mathcal{D}\xi_2\, e^{iS_{eff}[\xi_1,\xi_2;A_1,A_2]}$$

IR degrees of freedom

low-energy Schwinger-Keldysh effective action

[S. Grozdanov, J. Polonyi (2013)],

[M. Harder, P. Kovtun, A. Ritz (2015)],

[M. Crossley, P. Glorioso, H. Liu (2015-2017)],

[K. Jensen, N. P. F., A. Yarom (2017)],

[F. Haehl, R. Loganayagam, M. Rangamani (2015-2017)].

The symmetries of the Schwinger-Keldysh effective action will be implemented using **superspace techniques.** Our construction complements existing formulations.

The Schwinger-Keldysh partition function

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The **symmetries** of the Schwinger-Keldysh partition function can be deduced from its microscopic representation

$$Z[A_1, A_2] = \mathrm{Tr}\big(\mathcal{U}[A_1]\rho_{-\infty}\mathcal{U}^\dagger[A_2]\big)$$

1) Topological Schwinger-Keldysh symmetry:

$$Z[A_1 = A_2] = \mathrm{Tr}\big(\rho_{-\infty}\big)$$

$$\left.\frac{\delta Z}{\delta A_r \ldots \delta A_r}\right|_{A_a=0} = \langle \mathcal{O}_a \ldots \mathcal{O}_a \rangle = 0$$

$$A_r = \frac{1}{2}(A_1 + A_2)\ , \quad A_a = A_1 - A_2$$

$$\int d\sigma\,(A_1\mathcal{O}_1 - A_2\mathcal{O}_2) = \int d\sigma\,(A_r\mathcal{O}_a + A_a\mathcal{O}_r)$$

2) Reality condition and positivity:

$$Z[A_1, A_2]^* = Z[A_2^*, A_1^*]$$

$$\mathrm{Im}S_{eff} \geq 0$$

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For **thermal states** there is yet another symmetry of the Schwinger-Keldysh partition function

$\rho_{-\infty} = e^{-b\mathcal{H}}$ ➡ as a translation operator in imaginary time $t \to t - ib$

3) KMS symmetry (for thermal states) + CPT:

$$Z[A_1(t_1), A_2(t_2)] = Z[A_1(-t_1), A_2(-t_2 - ib)]$$

3 bis) Topological KMS symmetry $$\left.\frac{\delta Z}{\delta \tilde{A}_r \dots \delta \tilde{A}_r}\right|_{\tilde{A}_a=0} = \langle \tilde{\mathcal{O}}_a \dots \tilde{\mathcal{O}}_a \rangle = 0$$

$$\tilde{A}_r(t) = \frac{1}{2}\left(A_1(t) + A_2(t - ib)\right) \qquad \tilde{A}_a(t) = A_1(t) - A_2(t - ib)$$

$$\int d\sigma \left(\tilde{A}_r \tilde{\mathcal{O}}_a + \tilde{A}_a \tilde{\mathcal{O}}_r\right)$$

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The **IR degrees of freedom** are maps between target spaces and a worldvolume manifold.

4) Symmetries are doubled

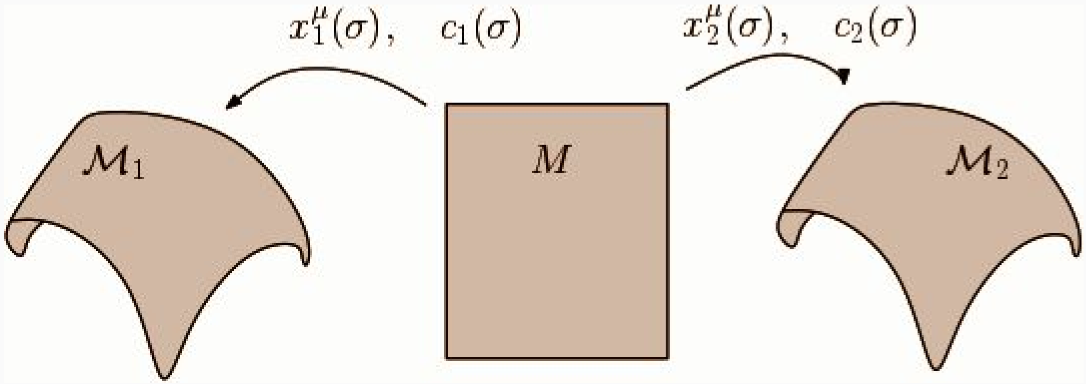
- Each of the two target spaces is endowed with a set of symmetries
- A **worldvolume** manifold is convenient to compare source/operators

$$g_{1\,ij}(\sigma) = \partial_i x_1^\mu \partial_j x_1^\nu g_{1\,\mu\nu}(x_1(\sigma))$$

$$B_{1\,i}(\sigma) = \partial_i x_1^\mu B_{1\,\mu}(x_1(\sigma)) + \partial_i c_1(\sigma)$$

$$B_{r\,i} = \frac{1}{2}\left(B_{1\,i} + B_{2\,i}\right)$$

$$B_{a\,i} = B_{1\,i} - B_{2\,i}$$



Promote the maps to dynamical fields, the equations of motion are related to the Ward identities

$$\frac{\delta S_{eff}}{\delta c_1} = 0\,, \qquad \frac{\delta S_{eff}}{\delta c_2} = 0 \qquad \Rightarrow \qquad \nabla_\mu J_1^\mu = 0\,, \quad \nabla_\mu J_2^\mu = 0$$

The superspace implementation of the symmetries

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A **topological sector** can be implemented using a **BRST-type symmetry**, in the same way as for (Witten-type) topological QFTs

1. There exists a scalar, Grassmannian, nilpotent charge Q
2. The action S and observables are Q-closed
3. The operator belonging to the topological sector is Q-exact

e.g. $$\frac{\delta Z}{\delta g_{\mu\nu}} \sim \langle T^{\mu\nu} \rangle = \int \mathcal{D}\phi \, T^{\mu\nu} e^{iS} = \int \mathcal{D}\phi \, \delta_Q \left(V^{\mu\nu} e^{iS} \right) = 0$$

The Schwinger-Keldysh partition function has 2 topological sectors for $\mathcal{O}_a$ and $\tilde{\mathcal{O}}_a$ hence we may use 2 Grassmannian charges: Q_{SK} and $\bar{Q}_{KMS}$

$$[Q_{SK}, \mathcal{O}_a] = 0 \qquad \{Q_{SK}, \mathcal{G}\mathcal{O}_g\} = -\mathcal{A}\mathcal{O}_a \qquad [Q_{SK}, \mathcal{R}\mathcal{O}_r] = \bar{\mathcal{G}}O_{\bar{g}} \qquad \{Q_{SK}, \mathcal{O}_{\bar{g}}\} = 0$$

$$[\bar{Q}_{KMS}, \tilde{\mathcal{O}}_a] = 0 \qquad \{\bar{Q}_{KMS}, \tilde{\bar{\mathcal{G}}}\tilde{\mathcal{O}}_{\bar{g}}\} = \tilde{\mathcal{A}}\tilde{\mathcal{O}}_a \qquad [\bar{Q}_{KMS}, \tilde{\mathcal{R}}\tilde{\mathcal{O}}_r] = \tilde{\mathcal{G}}\tilde{\mathcal{O}}_g \qquad \{\bar{Q}_{KMS}, \tilde{\mathcal{O}}_g\} = 0$$

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The action of the nilpotent charges can be geometrized in superspace

$$\mathbb{O} = \mathcal{R}\mathcal{O}_r + \theta\bar{\mathcal{G}}\mathcal{O}_{\bar{g}} + \bar{\theta}\mathcal{G}\mathcal{O}_g + \bar{\theta}\theta\mathcal{A}\mathcal{O}_a$$

Covariant generalization of the time-translation generator of the initial thermal state

$$\delta_{Q_{SK}}\mathbb{O} = \frac{\partial}{\partial\theta}\mathbb{O} \qquad \delta_{\bar{Q}_{KMS}}\mathbb{O} = \left(\frac{\partial}{\partial\bar{\theta}} + i\delta_\beta\theta\right)\mathbb{O}$$

$$\delta_\beta = \delta_\beta[\beta^i, \Lambda_\beta]$$

$$D_{\bar{\theta}}\mathbb{O} = \frac{\partial}{\partial\bar{\theta}}\mathbb{O} \qquad D_\theta\mathbb{O} = \left(\frac{\partial}{\partial\theta} - i\delta_\beta\bar{\theta}\right)\mathbb{O}$$

[G. Parisi, N. Sourlas (1979)].

The **most general low-energy Schwinger-Keldysh effective action** is a functional of superfields, bosonic derivatives and superderivatives in superspace

The reality condition (2)

$$S_{eff} = \int d\sigma\, d\theta\, d\bar{\theta} L(\mathbb{O}, \partial, iD_\theta, D_{\bar{\theta}}) + (\text{KMS conjugate})$$

Full KMS symmetry

The topological Schwinger-Keldysh (1) and KMS (3bis) symmetries (3)

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Let us for example consider a probe U(1) gauge field:

$$\mathbb{B}_i = \mathcal{R} B_{r\,i} + (\text{ghosts}) + \bar{\theta}\theta \mathcal{A} B_{a\,i} \qquad \mathcal{A}\mathcal{R}^{-1} = \frac{1}{2}\coth\left(\frac{i\delta_\beta}{2}\right) i\delta_\beta$$

The most general action up to quadratic order in field expansion:

$$S_{eff} = \frac{1}{2}\int \frac{d^d k}{(2\pi)^d} d\theta d\bar{\theta} \Big(\mathbb{B}_i(k) F^{ij}(-ik,\beta)\mathbb{B}_j(-k) + iD_\theta \mathbb{B}_i(k)\sigma^{ij}(-ik,\,\beta) D_{\bar{\theta}}\mathbb{B}_j(-k) + \widetilde{\mathbb{B}}_i(k) F^{ij}(ik,\beta)\widetilde{\mathbb{B}}_j(-k) - i\widetilde{D}_{\bar{\theta}}\widetilde{\mathbb{B}}_i(k)\sigma^{ij}(ik,\,\beta)\widetilde{D}_\theta \widetilde{\mathbb{B}}_j(-k)\Big)$$

Fluctuation-dissipation relations are also satisfied

$$iG^{ij}_{sym} = \frac{1}{2}\coth\left(\frac{b\omega}{2}\right)\left(G^{ij}_{ret} - G^{ij}_{adv}\right)$$

Conclusions

Dissipative hydrodynamics in superspace

We used the symmetries of the microscopic Schwinger-Keldysh partition function to constrain the form of effective field theories for thermal states using superspace.

- Compared to previous literature:
  - Our arguments for the topological Schwinger-Keldysh and KMS symmetries are given a priori.
  - We impose 2 supercharges only.
  - We implement the full KMS symmetry which accounts for the fluctuation-dissipation relations.
  - We work beyond the classical statistical limit.

- Outlook:
  - A complete effective action?
  - Entropy current?
  - Generalization to out-of-time ordered correlators (important for detecting chaos)
  - AdS/CFT embedding? Where would be the ghosts?
  - Applications to turbulence, cosmology,...

Thank you!