Black Holes and Thermoelectric Transport

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In seeking applications of holography to real materials, thermoelectric conductivities are important observables

$$\begin{pmatrix} J^i \\ Q^i \end{pmatrix} = \begin{pmatrix} \sigma^{ij} & T\alpha^{ij} \\ T\bar{\alpha}^{ij} & T\bar{\kappa}^{ij} \end{pmatrix} \begin{pmatrix} E_j \\ \zeta_j \end{pmatrix}$$

$$\zeta \leftrightarrow -(\nabla T)/T$$

The DC conductivity is particularly interesting.

Naively an IR observable, but depends on UV physics

e.g. CFT on Minkowski space:

Translational invariance $\Rightarrow$ momentum conserved $\Rightarrow$ $\sigma_{DC} = \infty$

More precisely, $Re[\sigma(\omega)] \sim \delta(\omega)$

General framework for dissipating momentum:

Holographic Lattices

CFT with a deformation by an operator that **explicitly** breaks translation invariance.

[Horowitz, Santos, Tong] [.........]

- For example, consider D=4 black holes with bulk scalar field

$$\phi(r, x^i) \rightarrow \frac{\phi_s(x^i)}{r} + \frac{v(x^i)}{r^2} + \dots \qquad r \rightarrow \infty$$

Corresponds to a deformation of the CFT:

$$L_{CFT} \rightarrow L_{CFT} + \int dx \phi_s(x) \mathcal{O}(x)$$

Holographic lattices are interesting

The lattice deformation can lead to:

- 'Coherent metals' aka Drude physics - smoothed out $\delta(\omega)$

  [Hartnoll,Hoffman][…]

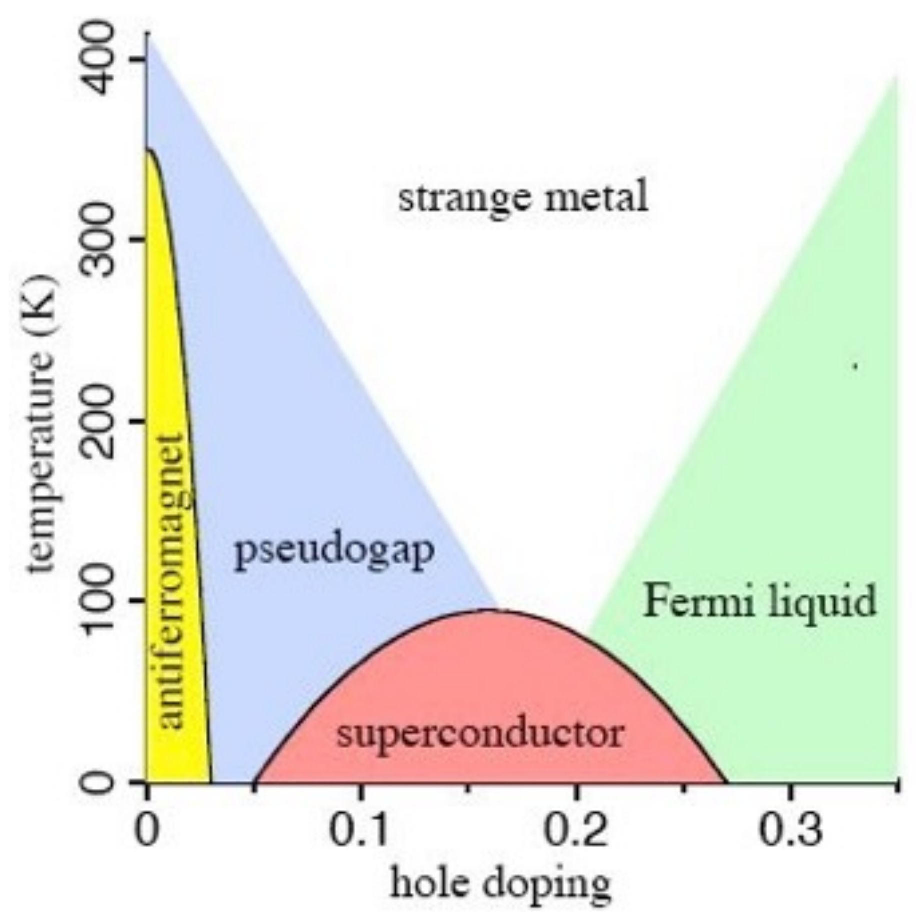
  Arises when momentum is nearly conserved.

- Novel 'incoherent' metals

  [Donos,JPG][Gouteraux][…..]

- Insulators and M-I transitions

  [Donos,Hartnoll][Donos,JPG][……]



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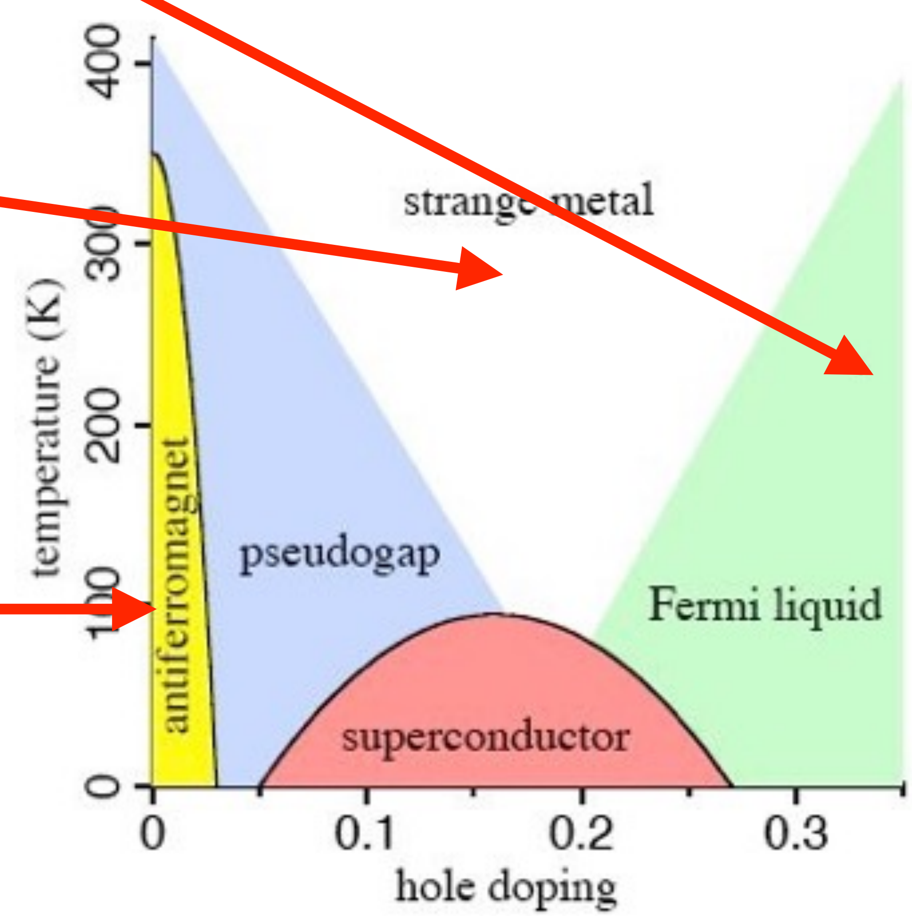

Plan of talk

- Universal result in holography

DC conductivity matrix is universally obtained exactly by solving generalised linearised Navier-Stokes equations for an incompressible auxiliary fluid on the curved black hole horizon

[Donos,JPG]

Hydrostatic fluid equations give exact statements about correlation functions of CFT

Exact version of old membrane paradigm for black holes

- Various comments…

Consider hydrodynamic limit of holographic lattices: can extract local currents. Use this to show that thermal backflow can occur

DC Conductivity and Navier-Stokes

Illustrate with D=4 Einstein-Maxwell Theory

$$S = \int d^4x\sqrt{-g}\Big[R + 6 - \frac{1}{4}F^2 + \dots\Big]$$

Holographic lattice: static, planar black hole that preserve time reversal invariance T

- Behaviour at AdS boundary $r \to \infty$

$$ds^2 \to r^{-2}dr^2 + r^2\left[g^{\infty}_{tt}(x)dt^2 + g^{\infty}_{ij}(x)dx^i dx^j\right]$$

$$A \to A^{\infty}_t(x)dt$$

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i.e. periodic sources for boundary T^{tt}, T^{ij} and J^t

- Setup: Perturb the black hole by DC sources $E_i, \ \zeta_i$

$$\delta(ds^2) = \delta g_{\mu\nu} dx^\mu dx^\nu + 2t g_{tt} \zeta_i dt dx^i$$

$$\delta A = \delta a_\mu dx^\mu - tE_i dx^i + tA_t \zeta_i dx^i$$

- Behaviour at AdS boundary:

  The only sources are $E_i \quad \zeta_i \qquad \zeta \leftrightarrow -(\nabla T)/T$

- Behaviour at the horizon: perturbation is regular

- Naively: now solve full bulk perturbation. However…

- Result 1: Zero modes of the electric and heat currents at the horizon are equal the zero modes of those in the CFT

Extends early work of [Kovtun,Son,Starinets] [Iqbal, Liu]

Illustrate with electric currents:

Bulk equations of motion $\nabla_\mu F^{\mu\nu} = 0$

Define the bulk electric current density as $J^i = \sqrt{-g}F^{ir}$

$$\partial_i J^i = 0\,, \qquad \partial_r J^i = \partial_j \left(\sqrt{-g}F^{ji}\right)$$

At AdS boundary, $J^i(x)|_\infty$, is local current of dual CFT

Observe zero mode $\bar{J}^i = \int d^2x J^i$ is independent of radius

$$\bar{J}^i|_\infty = \bar{J}^i_0$$

This, in itself, is NOT a membrane paradigm

• Result 2: Obtain local currents $J^i(x)|_0$, $Q^i(x)|_0$ on horizon as functions of E_i , ζ_i and hence DC conductivity of CFT

Use Hamiltonian decomposition of equations of motion with respect to the radial coordinate:

$$\mathcal{H} = N\,H + N_\mu\,H^\mu + \Phi\,C\,,$$

Evaluate constraints at the "stretched horizon"

Find a decoupled sector for a subset of the perturbation which forms a closed set of equations.
Moreover, they give $J^i|_0$ and $Q^i|_0$ [Donos,JPG]

This IS a membrane paradigm

Define

$$(v^i, p, w) \quad \leftrightarrow \quad (\delta g_{it}^{(0)}, \delta g_{rt}^{(0)}, \delta a_t^{(0)})$$

$$Q_0^i = 4\pi T\sqrt{h}v^i$$

$$J_0^i = \sqrt{h}[a_t^{(0)}v^i + h^{ij}(E_j + \partial_j w)]$$

Find:

$$\partial_i Q_0^i = 0$$

$$\partial_i J_0^i = 0$$

$$-2\nabla^i\nabla_{(i}v_{j)} = (4\pi T\,\zeta_j - \nabla_j\,p) + a_t^{(0)}(E_j + \nabla_j w)$$

Linear, time-independent, forced Navier-Stokes equations for a charged, incompressible fluid on the curved black hole horizon

Comments I

- The formalism can be generalised from Einstein-Maxwell to any theory of gravity in holography

  E.g. Scalar fields ϕ give extra viscous terms appear in the Stokes equations $\nabla_j\phi^{(0)}\nabla_i\phi^{(0)}v^i$ [Banks,Donos,JPG]

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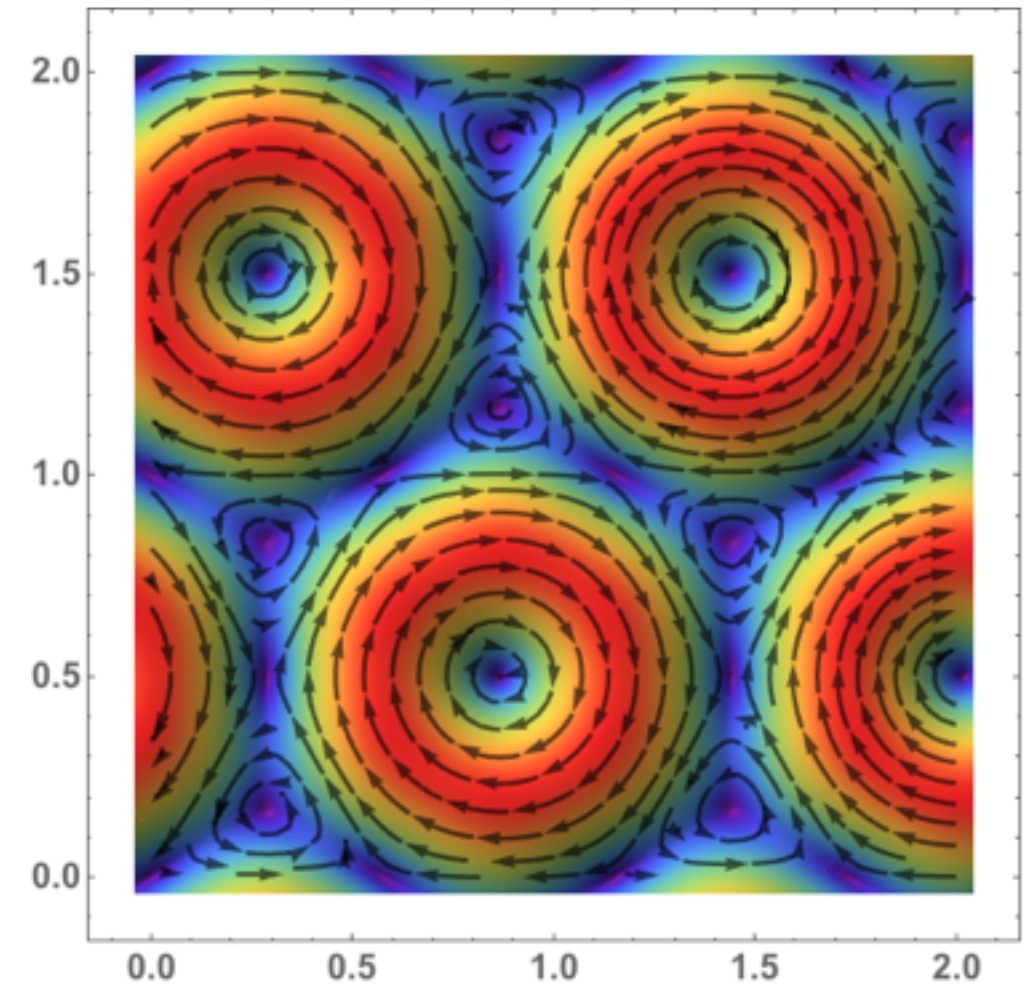
- Break T-reversal invariance [Donos,JPG,Griffin,Melgar]

Allow for equilibrium holographic lattices with magnetic fields but also local magnetisation currents and heat magnetisation currents

$$J_\infty^{(B)i} = \partial_j M^{(B)ij}$$

$$Q_\infty^{(B)i} = \partial_j M_T^{(B)ij}$$



Find: extra Lorentz and Coriolis terms in Navier-Stokes eqs

Comments II

[Donos,JPG,Griffin,Melgar]

Generalise to higher derivative theories of gravity

- Result 1: Zero modes of the electric and heat currents at the horizon are equal the zero modes of those in the CFT

  Introduce DC sources in a time independent way

  Obtain result by doing a KK reduction on the time direction

- Result 2: Obtain local currents $J^i(x)|_0$, $Q^i(x)|_0$ on horizon as functions of E_i , ζ_i and hence DC conductivity of CFT

For the special case of Gauss-Bonnet gravity showed this is still valid and obtain generalised higher derivative Navier-Stokes

For Gauss-Bonnet gravity in D dimensions

$$\nabla_i(\delta^i_j - 4\tilde{\alpha}G^i_j)v^j = 0$$

$$-2\nabla^i\left(S^{kl}_{ij}\nabla_k v_l\right) = \left(\delta^i_j - 4\tilde{\alpha}G^i_j\right)\left(4\pi T\zeta_i - \nabla_i p\right)$$

where

$$S^{kl}_{ij} = [1 - \tilde{\alpha}2(D-4)(D-1)]\,\delta_i^{(k}\delta_j^{l)} + \tilde{\alpha}\left[2h_{ij}R^{kl} + 4\delta^{(k}_{(i}R^{l)}_{j)} + 4R_i{}^{(k}{}_j{}^{l)}\right]$$

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The $\tilde{\alpha}$ corrected shear viscosity

[Brigante,Liu,Myers,Shenker,Yaida]

Comments III: Two interesting perturbative limits

$$\mathcal{L} = \mathcal{L}_{CFT} + h(x)\mathcal{O}(x)$$

Take lattice to be periodic with $x \sim x + 2\pi L$

Typical amplitude h_0

Largest wave number $k_{max} = n/L$

- a) Perturbative lattice: $h_0/T^{d-\Delta} << 1$

  Coherent metals with Drude peaks

  Can prove a generalised Wiedemann-Franz law

- b) Hydrodynamic limit: $k_{max}/T << 1$

  Fluid-gravity also valid in this limit

b) Hydrodynamic limit $k_{max}/T << 1$

Holographic lattices with source for $T^{\mu\nu}$ only

Equivalent to CFT on curved manifold with metric $h_{\mu\nu}$

e.g consider

$$ds^2 = -dt^2 + h_{ij}(x)dx^i dx^j$$

Two simplifications in hydro limit: [Banks,Donos,JPG,Griffin,Melgar]

- Black hole horizon metric is the UV metric h_{ij}
- Solve Navier-Stokes equations using h_{ij} gives local heat currents $Q^i(x)$ of CFT

Can also obtain this result from fluid gravity.
In fact this result is valid for non-holographic CFTs too

Thermal backflow for CFTs

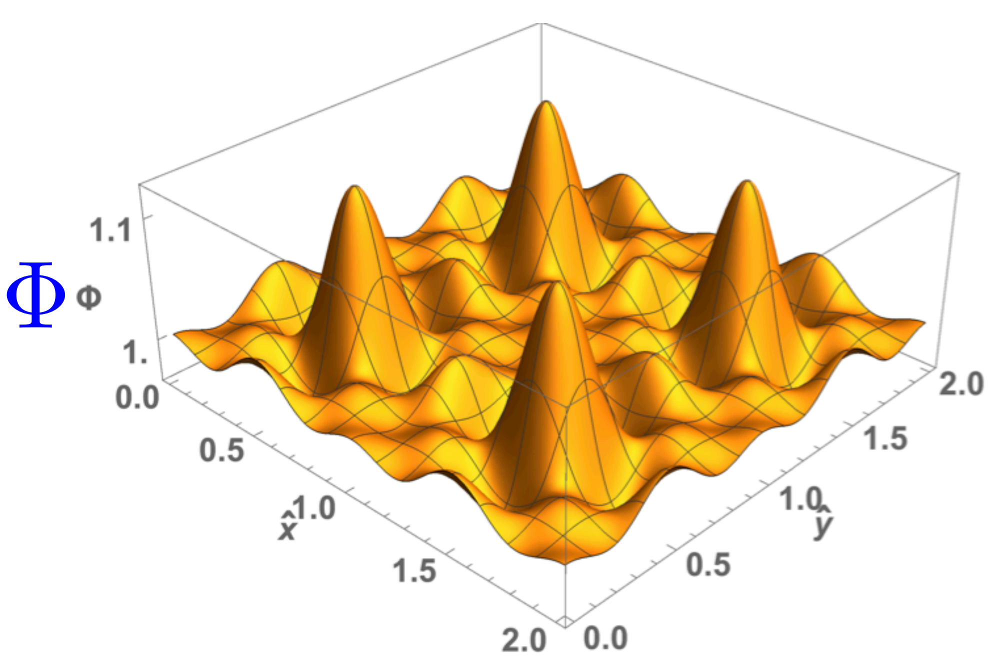
Example: CFT on metric $ds^2 = -dt^2 + \Phi(x)dx^i dx^i$

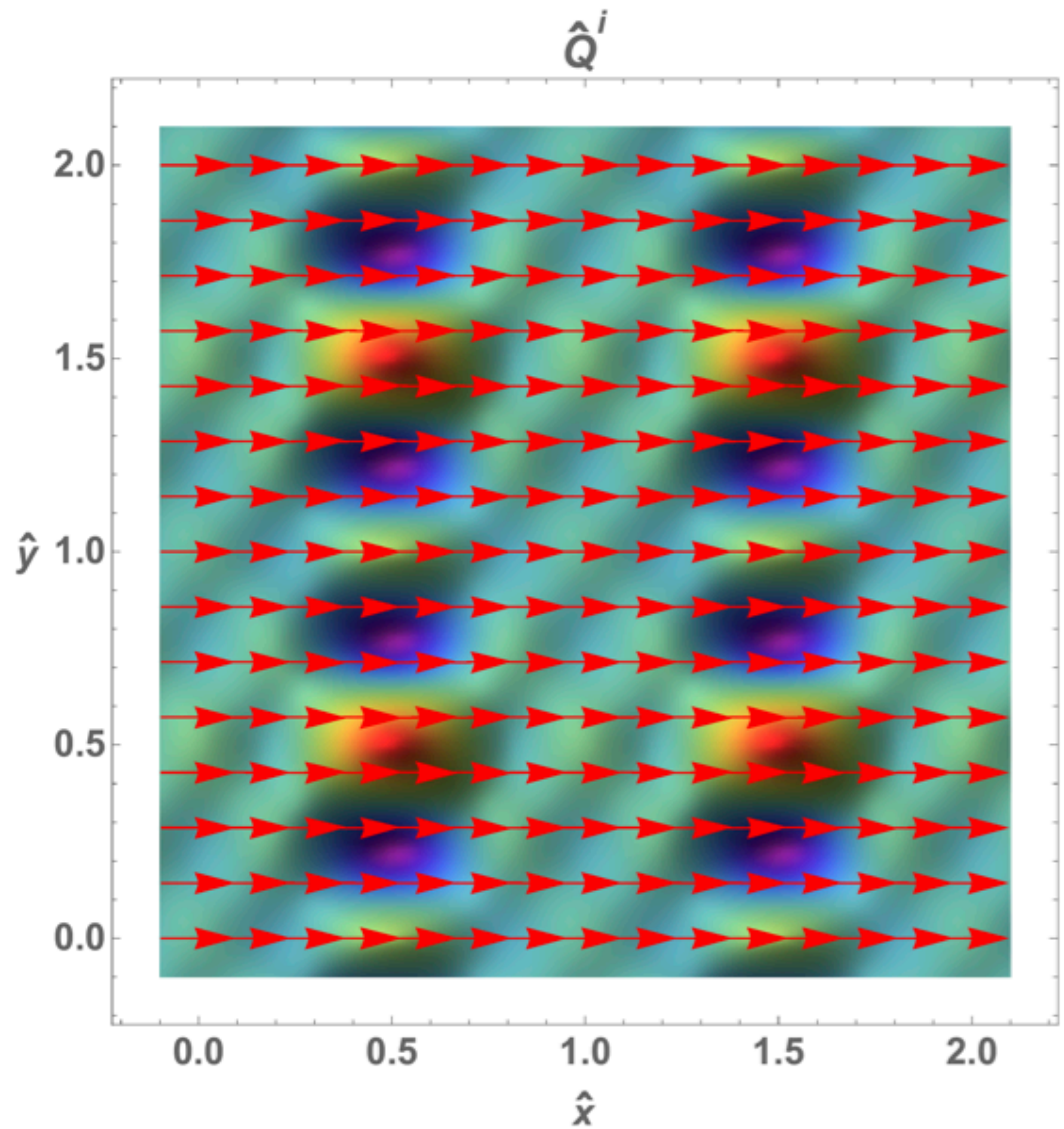
Equivalent to CFT with periodic isotropic spatial strain

Conformally equivalent to spatially varying temperature

$$ds^2 = -\Phi(x)^{-1}dt^2 + dx^i dx^i$$

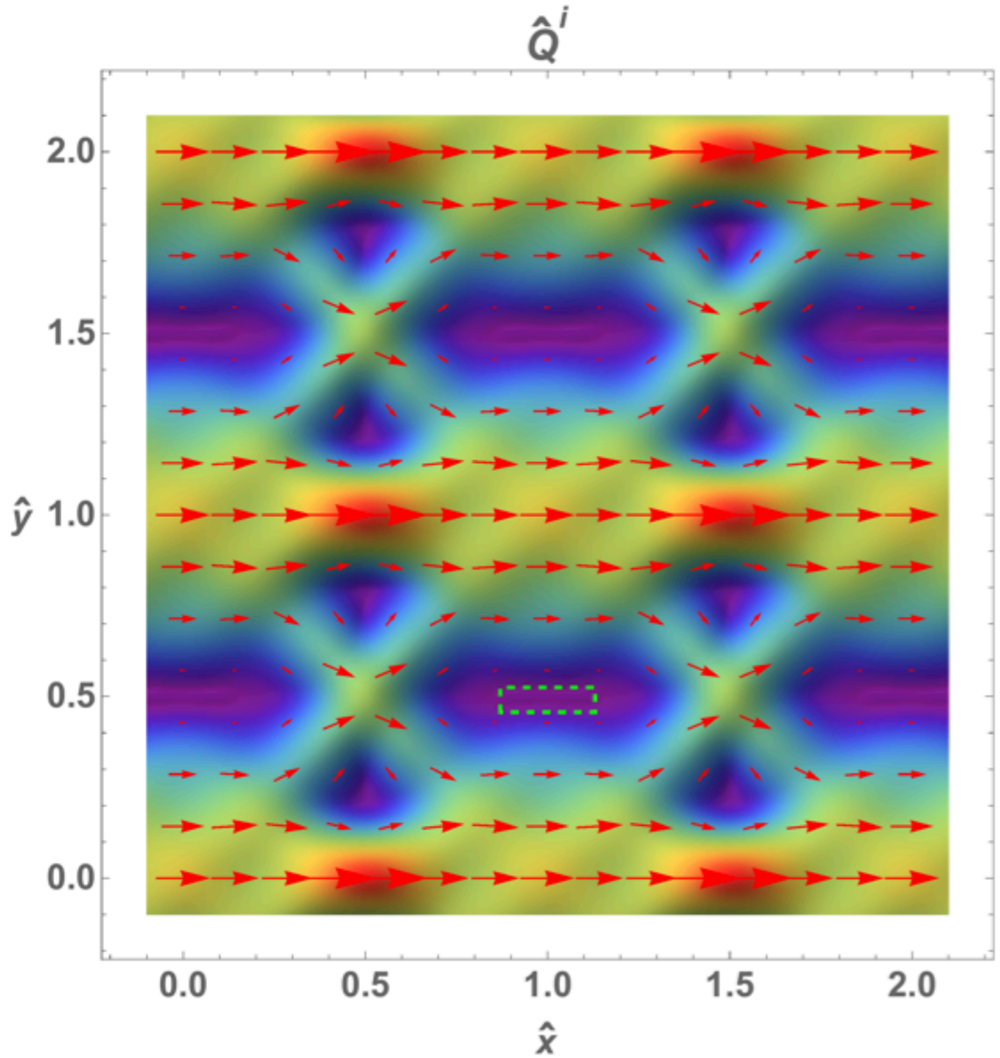
Small amplitude deformation Φ





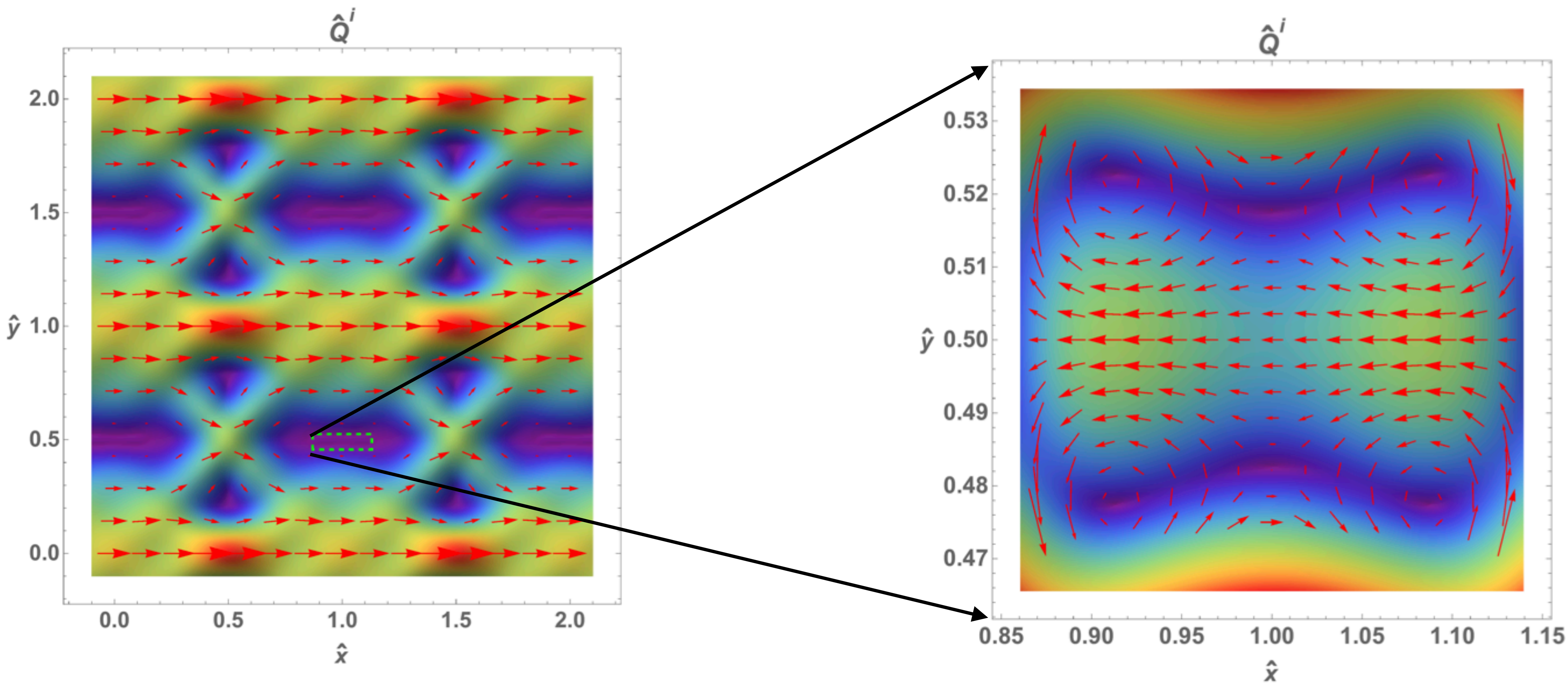
"Ohmic" response

Increase amplitude of Φ



Increase amplitude of Φ

Thermal back flow!



Perhaps observable in graphene?

c.f. electric backflow discussed by

[Levitov,Falkovich] [Bandurin et al] [Moll,Kushwaha,Nandi,Schmidt,MacKenzie]

Final Comments

- Solve Navier-Stokes on black hole horizons

  Gives, universally, DC thermoelectric conductivities for all T

  Exact version of membrane paradigm

- Valid for higher derivative gravity

- Solve the Navier-Stokes equations perturbatively:

  a) Perturbative lattices - small amplitude

  b) Hydrodynamic limit - long wavelength.

  Can connect to fluid gravity and hence relate our membrane paradigm to fluid gravity

  General DC result very obscure in fluid-gravity

  Predict thermal backflow: observable?