Loops in AdS from CFT

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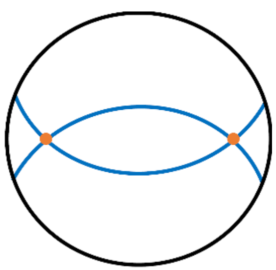

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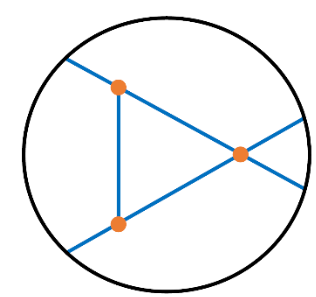


with Alday, Bissi, and Perlmutter

Outline

1) Introduction and motivations
2) The conformal bootstrap and its 1/N expansion
3) Loop diagrams in AdS from the conformal bootstrap at order $1/N^4$

1) Introduction and motivations

Gauge/gravity duality

- 20 years ago a strong/weak coupling duality was discovered between gauge theories, and gravitational theories on space-times with specific asymptotics.
- Easy computations on one side teach us about other side at strong coupling:
- Classical string theories = large N gauge theories. In some cases classical gravity theories = specific large N gauge theories, interesting by themselves or as toy models

Gauge/gravity duality

- Perturbative large N gauge theories = classical string theories at high curvatures (in some cases high-spin gravities).
- Finite N gauge theories = quantum gravity (at large or small curvature).
- 1/N expansion in gauge theories = perturbative expansion of quantum gravity.
- Our goal : use this in CFTs to compute loop diagrams of gravity / field theory, on anti-de Sitter space (for simplicity).

Why compute loops in AdS ?

- Loop diagrams useful in AdS/CFT in order to go to higher orders in 1/N in general gauge theories. (In general stringy but sometimes reduce to effective field theory.)
- Example of quantum field theory in curved space-times with boundaries. Direct evaluation of loop diagrams by Feynman diagrams is very complicated, even in this maximally symmetric space (and even at tree level).

Why compute loops in AdS ?

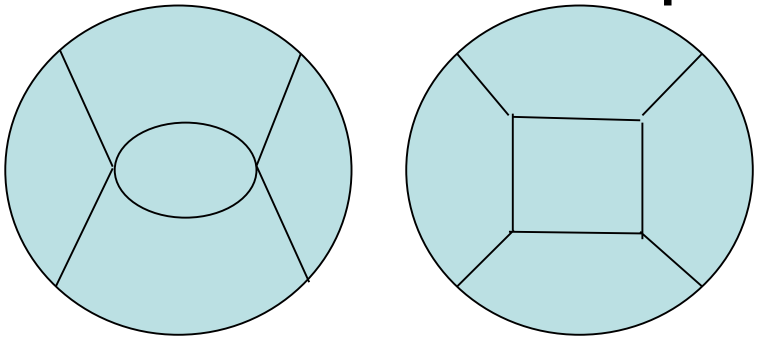
- Loops in AdS technically complicated but conceptually simple – determined by tree-level action up to finite number of renormalization conditions (=changes in tree-level couplings). (Also true in string theory.)
- Implies that leading order in 1/N expansion determines all higher orders !
- Is this true ?
- Can we use the dual field theory to simplify computation of loop diagrams in AdS ?

How to do this ?

- Perturbative large N gauge theories = classical theories at high curvatures, non-local at AdS scale = not what we want.
- So need to use non-perturbative methods.
- We will use 1/N expansion of conformal bootstrap = a general way to constrain correlation functions in CFTs from the "crossing equation". We will see that the resulting constraints are enough to compute loop diagrams in AdS.

What will we compute ?

- In general : many "single-trace operators" mapping to particles in AdS (including string states), many loop diagrams contribute to a given correlator.
- Simplify : use "toy CFTs" with a small number of "single-trace operators". Useful since AdS diagrams obey crossing even when no full dual CFT. First step.



2) The conformal bootstrap and its 1/N expansion

Conformal bootstrap

- CFT : primary operators $O_i(x)$ and OPE coefficients c_{ijk}. Consistency of the OPE in $<O_1(x_1)O_2(x_2)O_3(x_3)O_4(x_4)>$ requires that

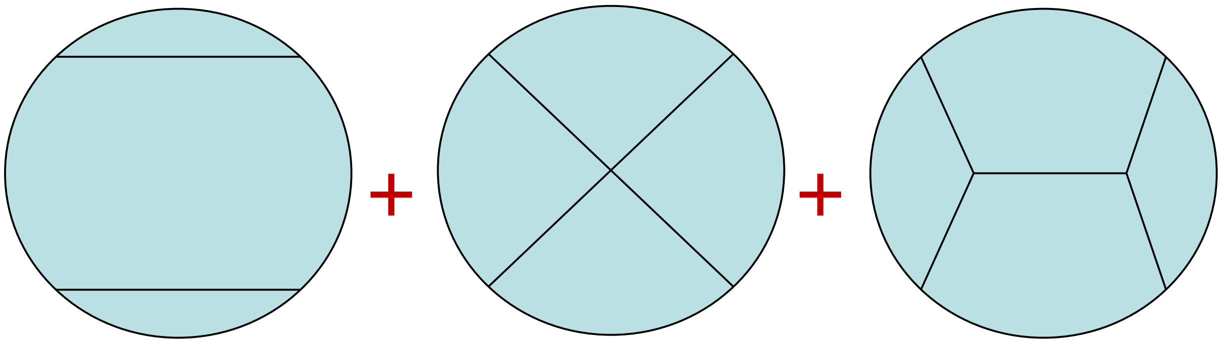
$$\sum_k c_{12k}c_{34k}\, G_k(u,v) \approx \sum_k c_{14k}c_{23k}\, G_k(v,u)$$

for conformal cross-ratios u,v and "(super)conformal blocks" $G_k(v,u)$ depending on operator dimensions, spins.

- "Crossing equation" necessary but not sufficient for consistent CFT.

Holographic bootstrap

- QG on AdS_{d+1} is a CFT_d → automatically obeys crossing. When weakly coupled can expand correlators in "Witten diagrams":

<OOOO>=  + + +

- Each diagram (summed over channels) obeys crossing, though not full CFT. Can use for any weakly coupled field theory on AdS (e.g. Φ^4); truncation.

Holographic bootstrap

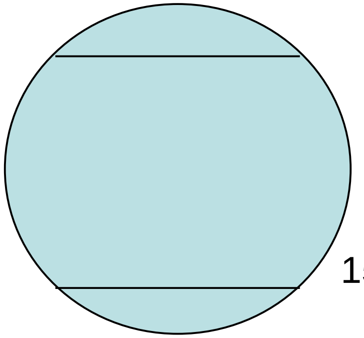
- Weakly coupled field theories on AdS are dual to (generalized) "large N CFTs". Each particle in AdS is dual to a "single-trace operator" O_i, of dimension Δ_i fixed by m_i
- The full list of bulk states = operators in the "CFT" (with finite dimension at large N) is the multi-particle (multi-trace) operators: "double-trace" $[O_i O_j]$, etc.
- Denote schematically bulk coupling as $1/N$. Then c_{OiOjOk} = 3-point coupling = $O(1/N)$, $c_{OiOj[OiOj]}=O(1)$, $\Delta_{[OiOj]}= \Delta_i+\Delta_j+O(1/N^2)$, …

The large N crossing equation

- A field theory like Φ^4 or Φ^3 on AdS_{d+1} gives a solution to crossing, order by order in 1/N.
- Consider the large N expansion of <OOOO> with "single-trace" O dual to Φ, of dimension Δ. Can analyze analytically.
- Can expand crossing equation $\sum_k c_{OOk} c_{OOk}\, G_k(u,v) \approx \sum_k c_{OOk} c_{OOk}\, G_k(v,u)$ order by order in 1/N.
- Each operator starts contributing at some order, and can expand its c_{OOk} and dimension.

The tree-level crossing equation

- O(1) : Only contributions to crossing $\sum_k c_{OOk} c_{OOk} \, G_k(u,v) \approx \sum_k c_{OOk} c_{OOk} \, G_k(v,u)$ are from the identity operator and from $[OO]_{n,l} \sim O\Box^n d^l O$ operators, with their leading order dimension $\Delta_{[OO]} = 2\Delta + 2n + l$ appearing in the conformal blocks.
- In AdS have only the disconnected diagram (+ permutations) at this order :
- These diagrams give the correct contribution at O(1).

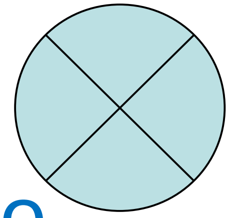


The tree-level crossing equation

- $O(1/N^2)$: Have contributions to crossing $\sum_k c_{OOk} c_{OOk} G_k(u,v) \approx \sum_k c_{OOk} c_{OOk} G_k(v,u)$ from k=some single-trace operator O', and from corrections to the leading order contribution of k=[OO].
- Have two types of corrections : to $c_{OO[OO]} = 1 + \frac{c_1}{N^2} + \ldots$, and to $\Delta_{[OO]} = 2\Delta + \frac{\Delta_1}{N^2} + \frac{\Delta_2}{N^4} \ldots$
- Latter appears in $G_k(u,v)$ which has terms going as u^{Δ_k} (times integer powers). So get $u^{\Delta_{[OO]}} = u^{2\Delta}\left(1 + \frac{\Delta_1}{N^2}\log(u) + \frac{1}{2}\left(\frac{\Delta_1}{N^2}\right)^2 log^2(u) + \cdots\right)$

The tree-level crossing equation

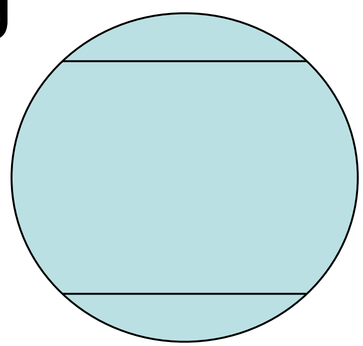
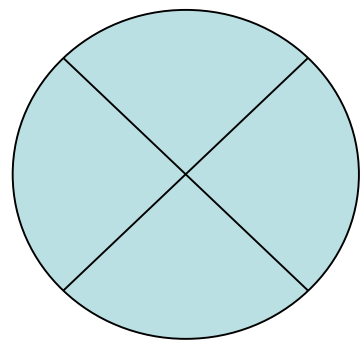
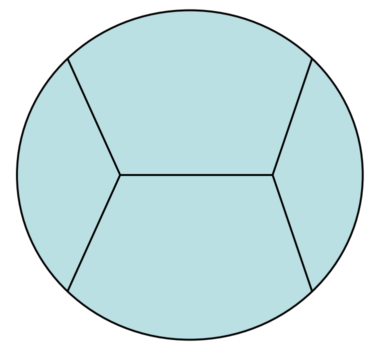
- Consider for simplicity a bulk theory with action Φ^4 (with derivative couplings). Then only [OO] operators appear, crossing
  $\sum_k c_{OOk} c_{OOk} G_k(u,v) \approx \sum_k c_{OOk} c_{OOk} G_k(v,u)$
  is a linear equation for c_1 and Δ_1, and can find all solutions by comparing $u^n v^m(\log(u))(\log(v))$ terms on both sides.
- In AdS come just from X diagram:



- Heemskerk et al : solutions in one-to-one correspondence with possible Φ^4 couplings.

Summary of holographic bootstrap

- This is general story at $O(1/N^2)$=tree-level.
- Any field theory in AdS with some particle content gives a solution to crossing, by summing

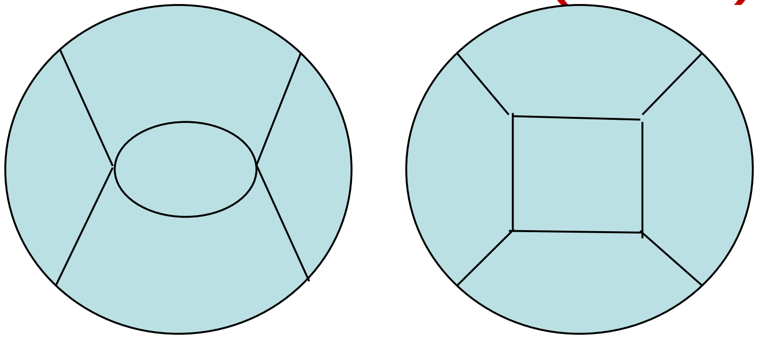
<OOOO>=  +  + 

- Every solution to crossing with corresponding operator content corresponds to a field theory on AdS.

3) Loop diagrams from the conformal bootstrap at order $1/N^4$

One-loop or O(1/N^4)

- In a QFT (or effective field theory like SUGRA), tree-level action determines also loop amplitudes, up to a finite number of coupling constants ("renormalization conditions"). Should be true also in AdS.
- One-loop bulk diagrams not yet computed, except in ϕ^4 (Penedones, Fitzpatrick+Kaplan). Contribute to <OOOO> at O(1/N^4) :



One-loop or $O(1/N^4)$

- So can we take a solution to crossing at $O(1/N^2)$ = a tree-level bulk theory, and use only crossing to compute a solution at $O(1/N^4)$ = one-loop diagrams ?
- At least in some cases : yes ! (Up to freedom in changing bulk couplings.)
- More precisely, can do it if only [OO] appears at $O(1/N^4)$. If also [O’O’] appear, may need input from additional 4-point functions <OOO’O’> at order $O(1/N^2)$.

One-loop or $O(1/N^4)$

- Claim: can use crossing to compute one-loop diagrams in AdS in theories like Φ^4 or Φ^3 or 5d $\mathcal{N}=8$ SUGRA on AdS_5, just from tree-level <OOOO>. Position/Mellin space.
- In theories like $\mathcal{N}=4$ SYM, even at very strong coupling (=10d SUGRA), need more input from $\langle O_m O_m O_n O_n \rangle$, but then should be able to compute (say) $\langle O_2 O_2 O_2 O_2 \rangle$ at $O(1/N^4)$ from leading large N answers. (Up to local bulk couplings.) Operator mixing.

Details

- Can compute in position / Mellin space.
- Simplest when only 4-point couplings in AdS at tree level – Δ_1 non-zero only for finite number of operators.
- $\log^2(u)$ only comes from

$$u^{\Delta[OO]} = u^{2\Delta}\left(1 + \frac{\Delta_1}{N^2}\log(u) + \frac{1}{2}\left(\frac{\Delta_1}{N^2}\right)^2 \log^2(u) + \cdots\right)$$

- So $\log^2(v)$ determined by crossing and tree-level values of Δ_1.

Details

- In expansion of amplitude $\sum_{n,l} c_{OO[OO]_{n,l}} c_{OO[OO]_{n,l}} G_{[OO]_{n,l}}(u,v)$ each term has no $\log^2(v)$ – comes from divergence in sum over l.
- Using this can compute $(\Delta_2)_{n,l}$ order by order in $1/l$. Moreover expansion in $1/l$ converges – can get full amplitude at order $1/N^4$.
- Have divergences precisely where expected from bulk (renormalization).

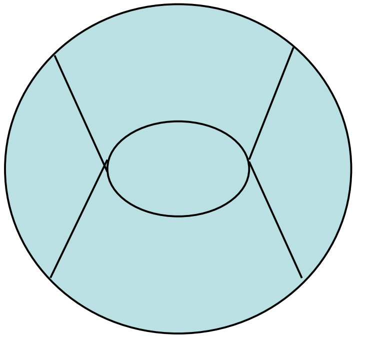
Details

- Can do same computation in Mellin space :
- $G(u,v) \sim \int_{-i\infty}^{i\infty} ds dt\, M(s,t) u^{t/2}\, v^{(2\Delta-s-t)/2}$
$$\Gamma^2\left(\frac{2\Delta-t}{2}\right)\Gamma^2\left(\frac{2\Delta-s}{2}\right)\Gamma^2\left(\frac{s+t-2\Delta}{2}\right)$$
- Advantages and disadvantages.
- Correlators much simpler there – single poles in s, t, 4Δ-s-t for intermediate single-trace states. Poles for double-trace states start appearing just at $O(1/N^4)$, residue fixed by $O(1/N^2)$ results.

Results

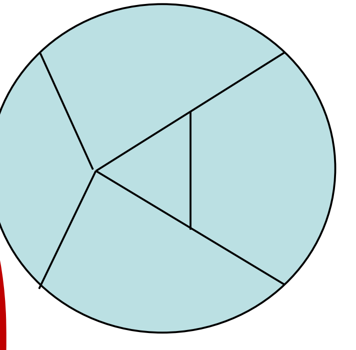
- In Mellin space reproduce Φ^4 result :

$$M(s,t) \propto \sum_{m=0}^{\infty} \frac{R_m}{t-(4+2m)} + \text{crossed}$$



where $R_m = -\frac{9(3m+4)4^m(m+1)!^2}{(2m+3)!}$

- We also computed a new diagram in $\lambda_3\phi^3 + \lambda_4\phi^4$ theory: in d=4

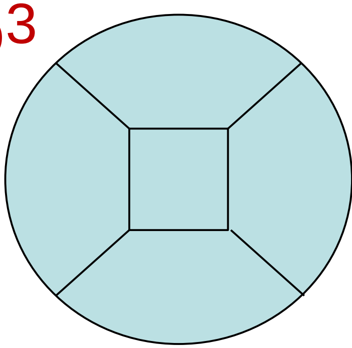


$$M(s,t) = \lambda_3^2\lambda_4 \left(\frac{40\,{}_3F_2(1,1,2-\frac{t}{2};\frac{5}{2},3-\frac{t}{2};1)}{t-4} + \frac{56\,{}_3F_2(2,2,3-\frac{t}{2};\frac{7}{2},4-\frac{t}{2};1)}{5(t-6)} \right)$$

Future directions

- Many more examples (technically difficult but conceptually clear) :

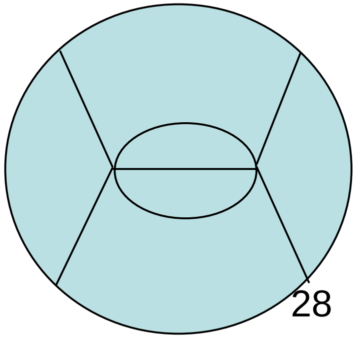
1) Single-trace operators in OPE, e.g. Φ^3
2) Diagrams with multiple fields
3) Loops with gravitons
4) Mixings of double-trace operators (needed in $\mathcal{N}$=4 SYM)



- Can we formulate AdS Feynman rules directly in Mellin space ?
- Beyond 1/N ?

Higher-loops

- In principle can extend to higher loops = higher orders in $1/N$, but new bulk couplings appear, and, related to this, also higher-trace operators appear in OPE, so need extra tree-level information (like 5-point functions) to get full answer. In some cases 2-loop doable.



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