BINARY BLACK HOLE MERGER: THE THEORY

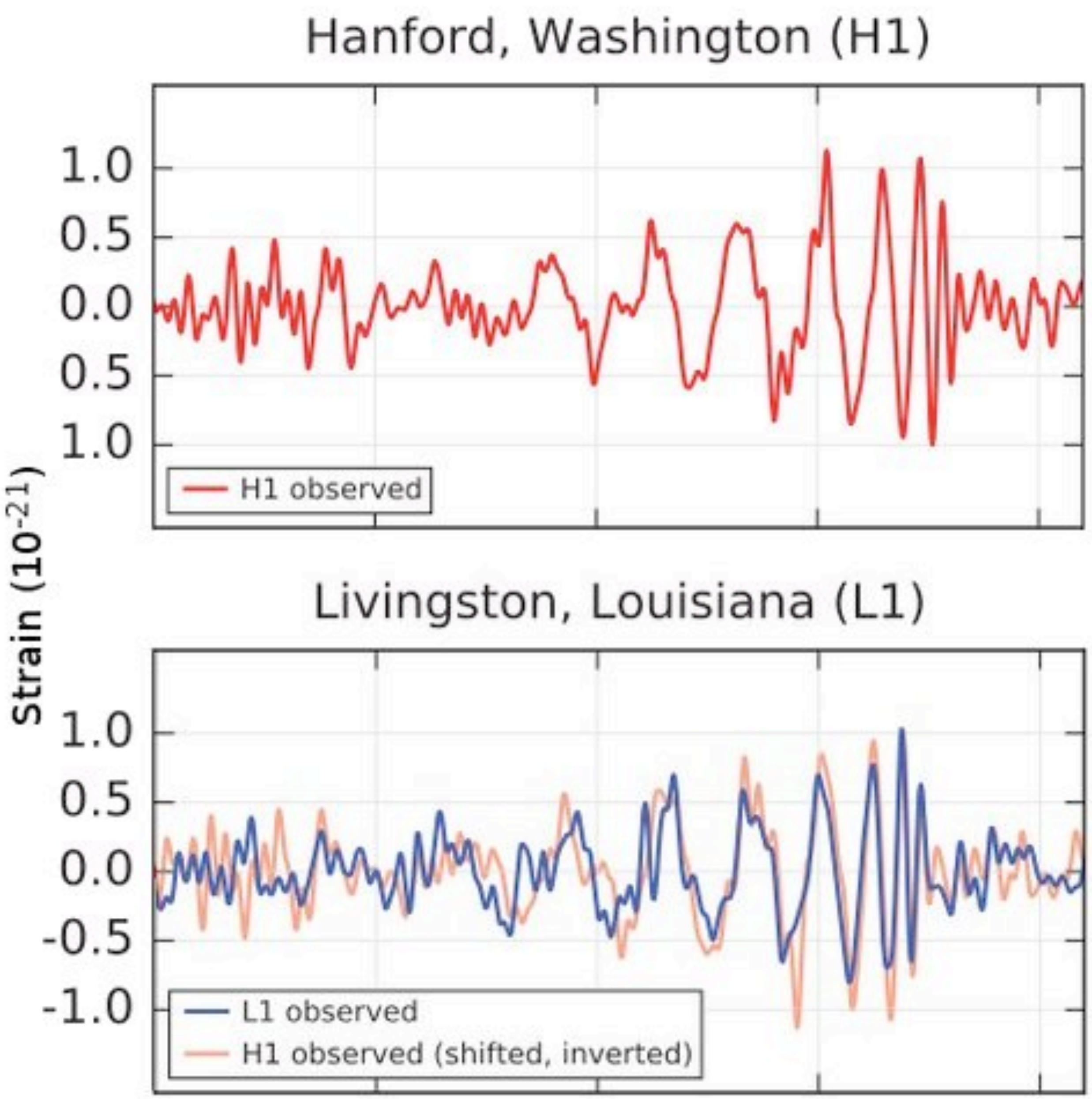
INTERFACING NUMERICAL AND ANALYTICAL RELATIVITY

Alessandro Nagar
Centro Fermi, INFN, Sezione di Torino &
Institut des Hautes Etudes Scientifiques (IHES)
[nagar@ihes.fr]

The IHES effective-one-body (EOB) code: https://eob.ihes.fr
T. Damour, AN, S. Bernuzzi, D. Bini...

A. Nagar, 21 February 2017

GW150914



$$\text{strain} = \frac{\delta L}{L}$$

GW150914 parameters:

$$\boxed{\begin{aligned} m_1 &= 35.7 M_\odot \\ m_2 &= 29.1 M_\odot \\ M_f &= 61.8 M_\odot \end{aligned}}$$

$$a_1 \equiv S_1/(m_1^2) = 0.31^{+0.48}_{-0.28}$$

$$a_2 \equiv S_2/(m_2^2) = 0.46^{+0.48}_{-0.42}$$

$$a_f \equiv \frac{J_f}{M_f^2} = 0.67$$

$$q \equiv \frac{m_1}{m_2} = 1.27$$

Symmetric mass ratio

$$\boxed{\nu \equiv \frac{m_1 m_2}{(m_1 + m_2)^2} = 0.2466}$$

THE THEORY...

Is needed to compute waveform templates for characterizing the source (GWs were detected...but WHAT was detected?)

Theory is needed to study the 2-body problem in General Relativity (dynamics & gravitational wave emission)

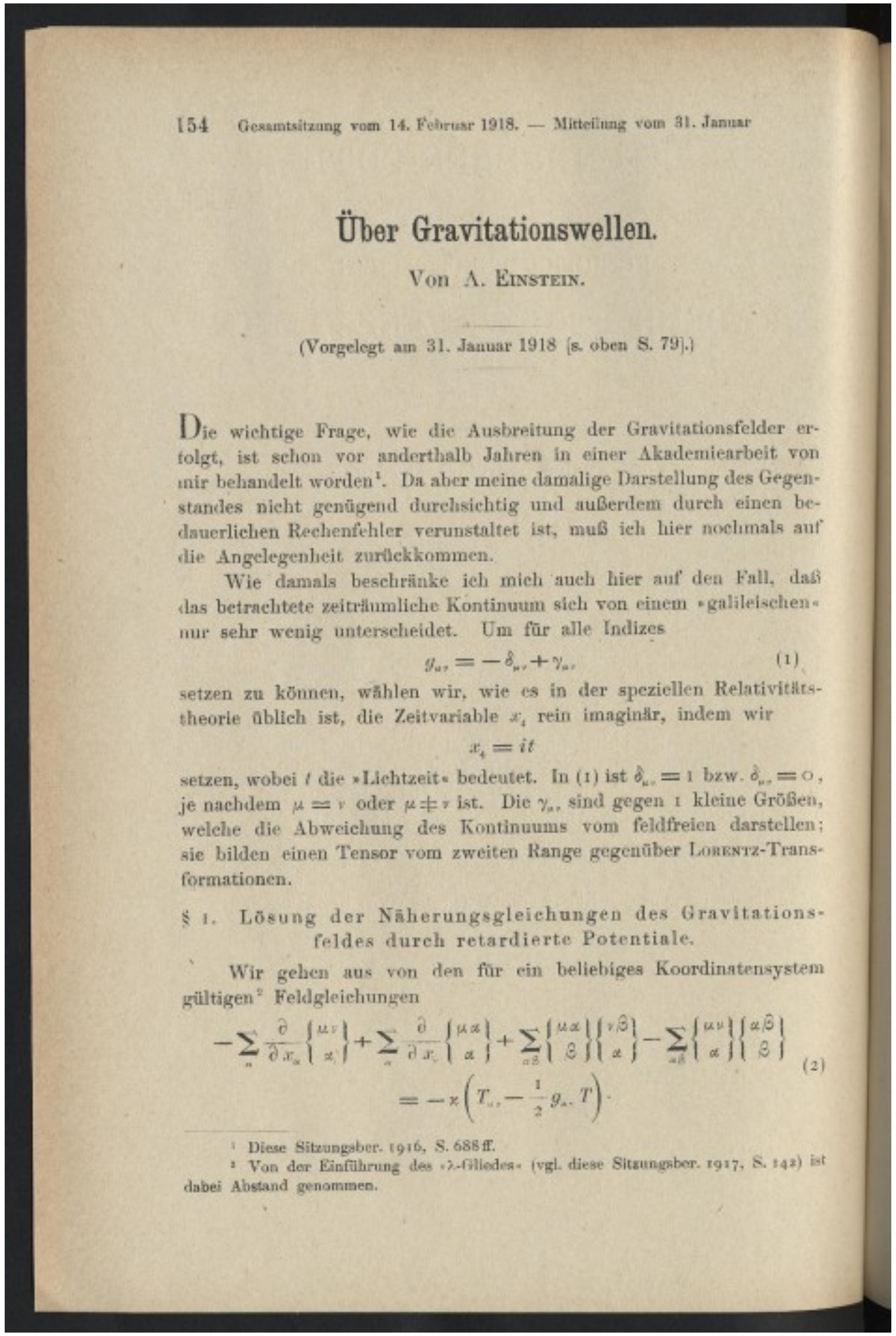
Theory: SYNERGY between

Analytical and Numerical General Relativity

(AR/NR)

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

UBER GRAVITATIONSWELLEN (EINSTEIN, 1918)



154 Gesamtsitzung vom 14. Februar 1918. — Mitteilung vom 31. Januar

Über Gravitationswellen.

Von A. Einstein.

(Vorgelegt am 31. Januar 1918 [s. oben S. 79].)

Die wichtige Frage, wie die Ausbreitung der Gravitationsfelder erfolgt, ist schon vor anderthalb Jahren in einer Akademiearbeit von mir behandelt worden[1]. Da aber meine damalige Darstellung des Gegenstandes nicht genügend durchsichtig und außerdem durch einen bedauerlichen Rechenfehler verunstaltet ist, muß ich hier nochmals auf die Angelegenheit zurückkommen.

Wie damals beschränke ich mich auch hier auf den Fall, daß das betrachtete zeiträumliche Kontinuum sich von einem »galileischen« nur sehr wenig unterscheidet. Um für alle Indizes

$$g_{\mu\nu} = -\delta_{\mu\nu} + \gamma_{\mu\nu} \qquad (1)$$

setzen zu können, wählen wir, wie es in der speziellen Relativitätstheorie üblich ist, die Zeitvariable x_4 rein imaginär, indem wir

$$x_4 = it$$

setzen, wobei t die »Lichtzeit« bedeutet. In (1) ist $\delta_{\mu\nu} = 1$ bzw. $\delta_{\mu\nu} = 0$, je nachdem $\mu = \nu$ oder $\mu \neq \nu$ ist. Die $\gamma_{\mu\nu}$ sind gegen 1 kleine Größen, welche die Abweichung des Kontinuums vom feldfreien darstellen; sie bilden einen Tensor vom zweiten Range gegenüber Lorentz-Transformationen.

§ 1. Lösung der Näherungsgleichungen des Gravitationsfeldes durch retardierte Potentiale.

Wir gehen aus von den für ein beliebiges Koordinatensystem gültigen[2] Feldgleichungen

$$-\sum_{\alpha} \frac{\partial}{\partial x_\alpha} \begin{Bmatrix} \mu\nu \\ \alpha \end{Bmatrix} + \sum_{\alpha} \frac{\partial}{\partial x_\nu} \begin{Bmatrix} \mu\alpha \\ \alpha \end{Bmatrix} + \sum_{\alpha\beta} \begin{Bmatrix} \mu\alpha \\ \beta \end{Bmatrix} \begin{Bmatrix} \nu\beta \\ \alpha \end{Bmatrix} - \sum_{\alpha\beta} \begin{Bmatrix} \mu\nu \\ \alpha \end{Bmatrix} \begin{Bmatrix} \alpha\beta \\ \beta \end{Bmatrix} = -\varkappa \left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T \right). \qquad (2)$$

[1] Diese Sitzungsber. 1916, S. 688 ff.

[2] Von der Einführung des »λ-Gliedes« (vgl. diese Sitzungsber. 1917, S. 142) ist dabei Abstand genommen.

$$g_{ij} = \delta_{ij} + h_{ij}$$

h_{ij} is transverse and traceless and propagates at the speed of light

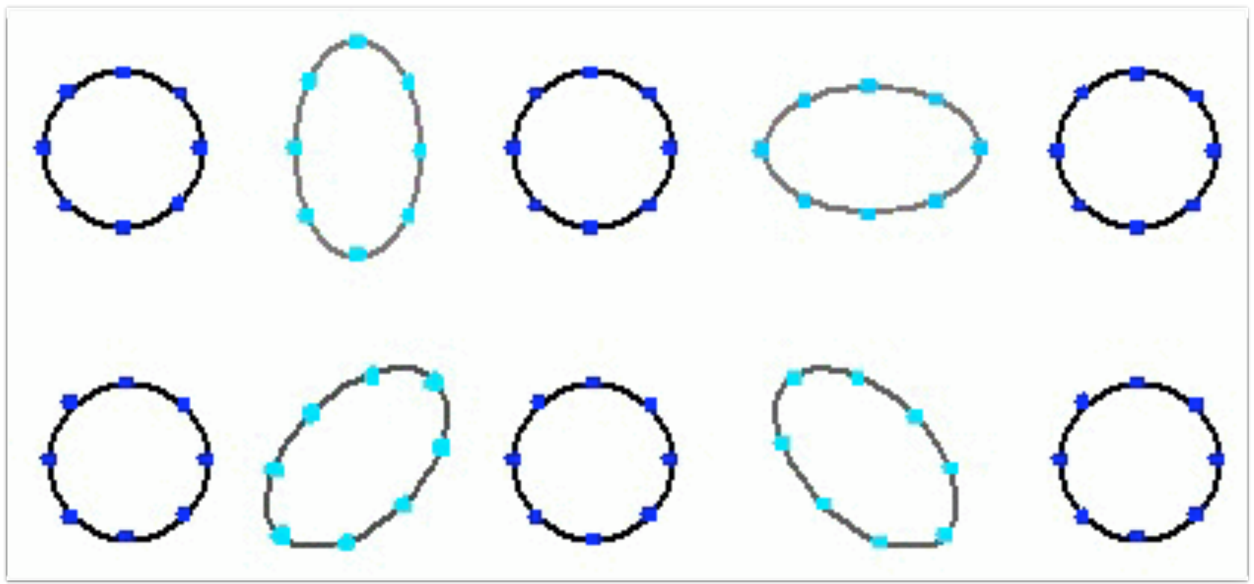
lunedì 20 febbraio 17

GRAVITATIONAL WAVES: TWO HELICITY STATES $s = \pm 2$

Massless, two helicity states,

i.e., two transverse-traceless (TT) tensor polarizations propagating at $v = c$

$$h_{ij} = h_+(x_i x_j - y_i y_j) + h_\times(x_i y_j + y_1 x_j)$$



h_+

$h_\times$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h$$

$$\partial_\rho \partial^\rho \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu}$$

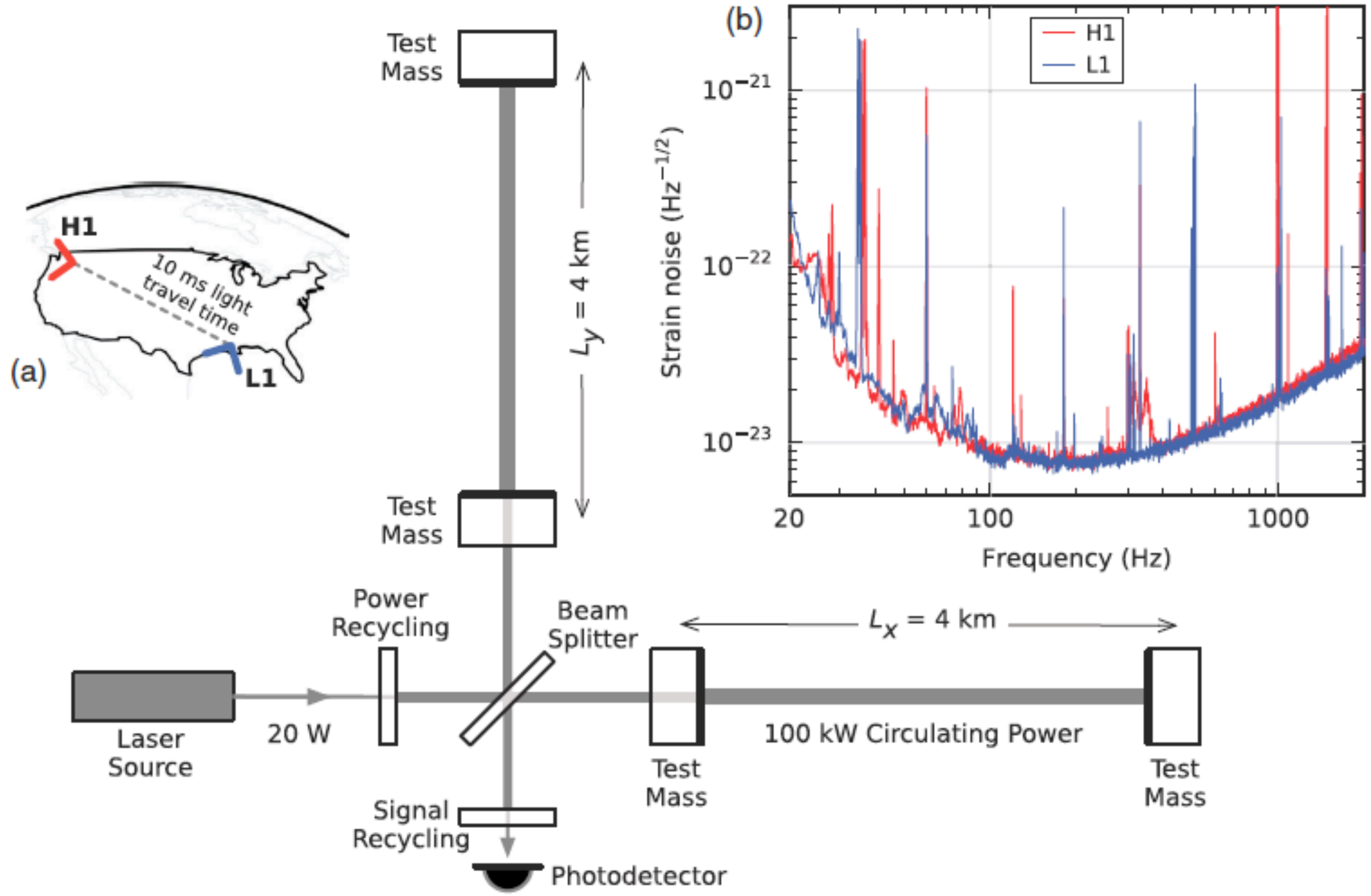
GRAVITATIONAL WAVES: PIONEERING THEIR DETECTION

Joseph Weber (1919-2000)

General Relativity and Gravitational Waves
(Interscience Publishers, NY, 1961)

$$\frac{\delta L}{L} \approx h_{ij} n^i n^j$$

LASER INTERFEROMETER GW DETECTORS



HOW TO DETECT & MEASURE: MATCHED FILTERING!

To extract/do parameter estimation of the GW signal from detector's output (lost in broadband noise $S_n(f)$)

$$\langle output | h_{\rm template} \rangle = \int \frac{df}{S_n(f)} o(f) h^*_{\rm template}(f)$$

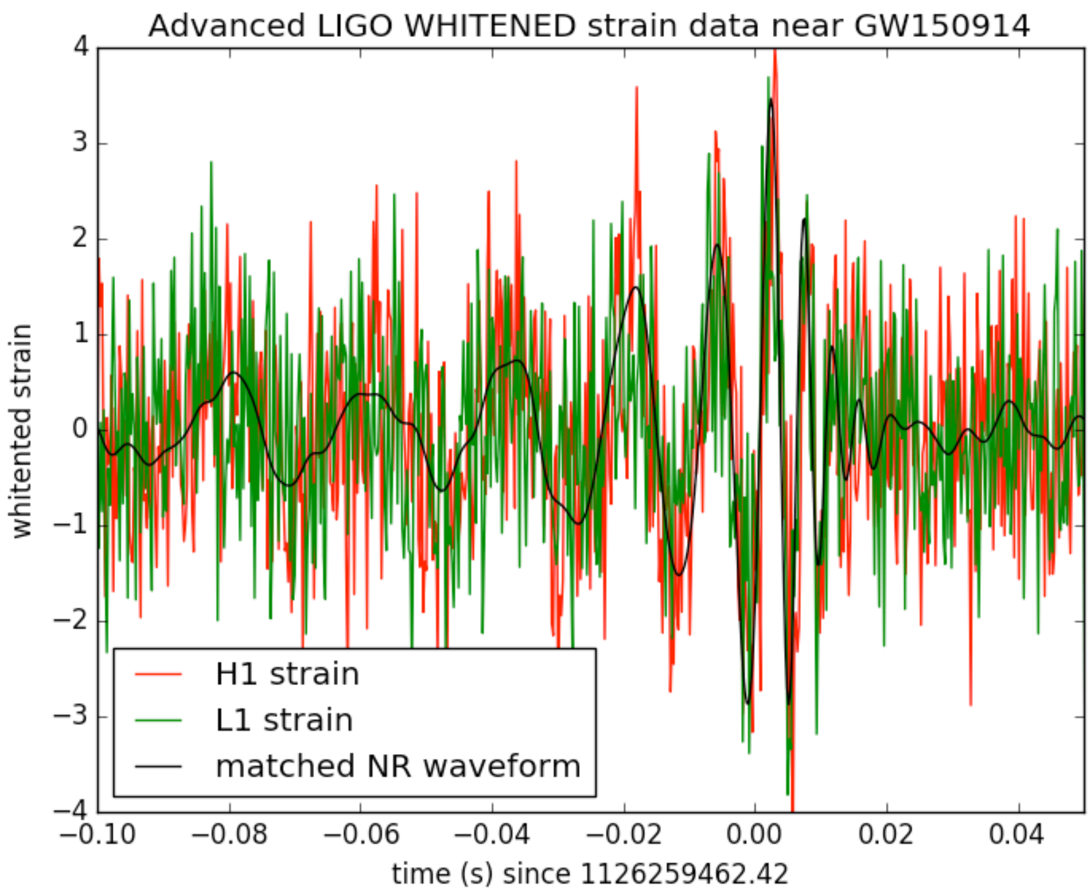
Detector's output

Template of expected GW signal

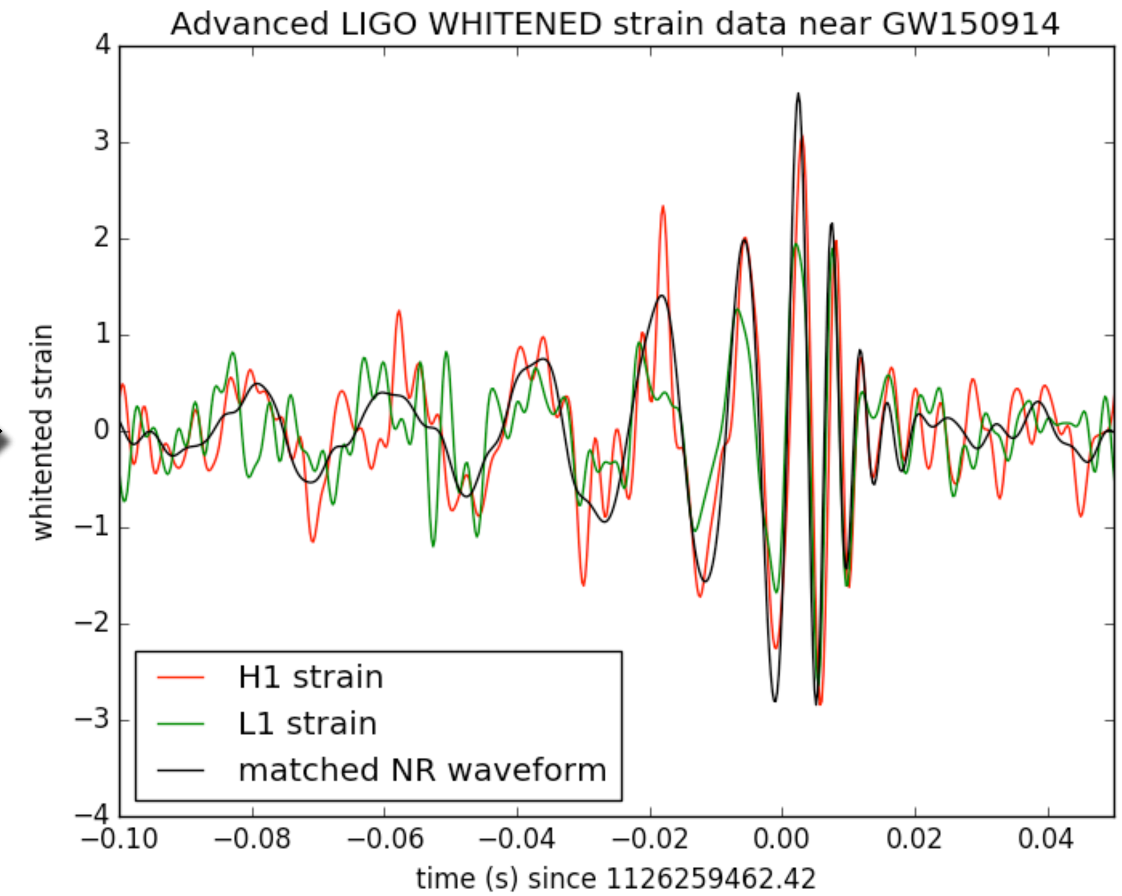
Need waveform templates!

GW150914

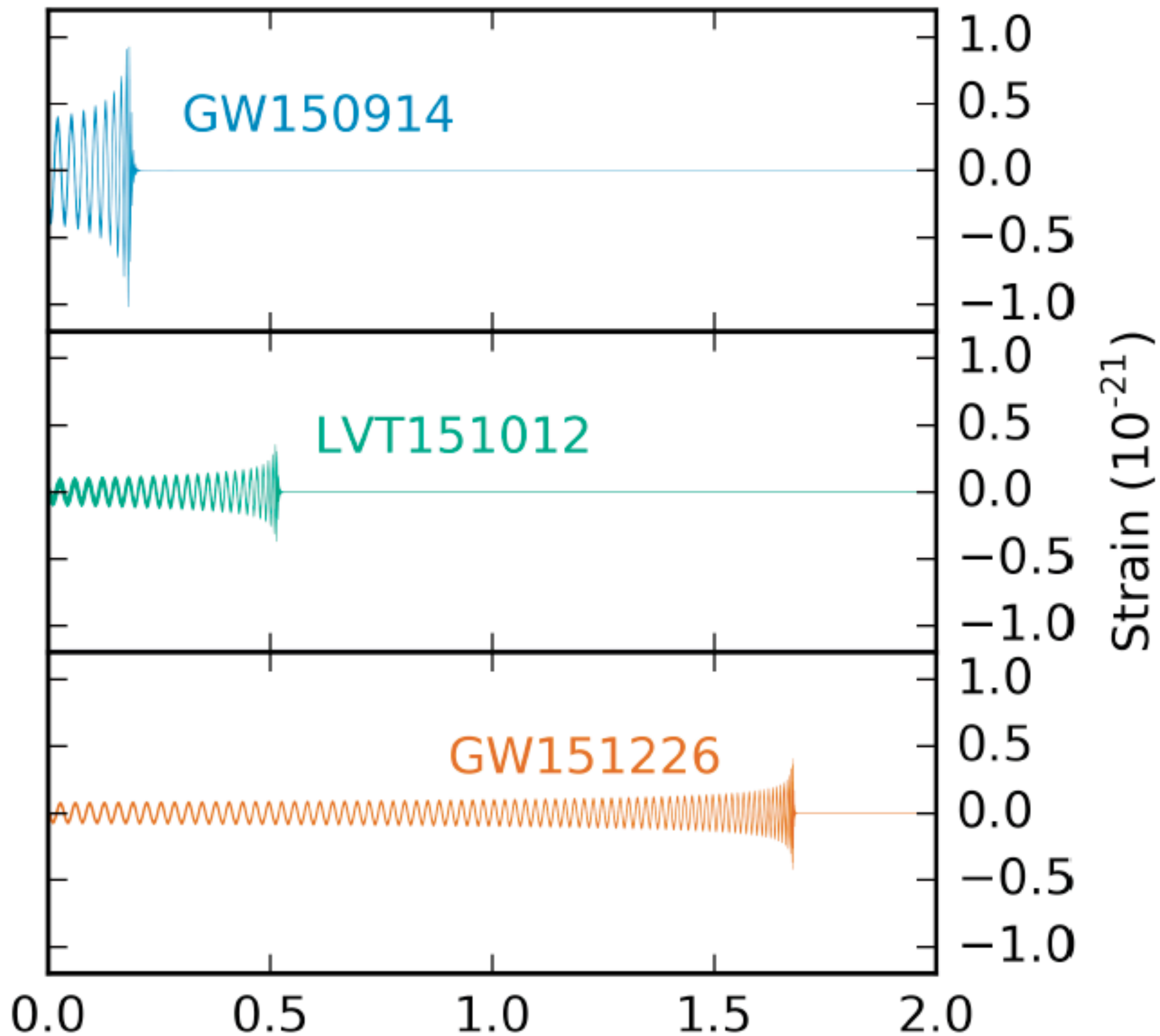
was so loud that it could be seen with the naked eye...



pass-band filtering



OBSERVED GRAVITATIONAL WAVE SIGNALS



from 30 Hz (sec)

$29M_\odot + 35M_\odot$

$23M_\odot + 13M_\odot$

$14M_\odot + 7.5M_\odot$

BH radii: $\simeq 20 - 100\,\mathrm{km}$

BINARY SYSTEMS: NEWTONIAN PRELIMINARIES

Newtonian binary systems in circular orbits: Kepler's law

$$GM = \Omega^2 R^3$$

$$\frac{v^2}{c^2} = \frac{GM}{c^2 R} = \left(\frac{GM\Omega}{c^3}\right)^{2/3}$$

$$M = m_1 + m_2$$

Einstein (1918) quadrupole formula: GW luminosity (energy flux)

$$P_{\rm gw} = \frac{dE_{\rm gw}}{dt} = \frac{32}{5}\frac{c^5}{G}\nu^2 x^5$$

$$x = \left(\frac{v}{c}\right)^2$$

$$\nu = \frac{\mu}{M} = \frac{m_1 m_2}{M^2}$$

GWS FROM COMPACT BINARIES: BASICS

$$E^{\rm orbital} = E^{\rm kinetic} + E^{\rm potential} = -\frac{1}{2}\frac{m_1 m_2}{R} = -\frac{1}{2}\mu x$$

Balance argument

$$\frac{dE^{\rm orbital}}{dt} = P_{GW} = \frac{dE_{\rm GW}}{dt}$$

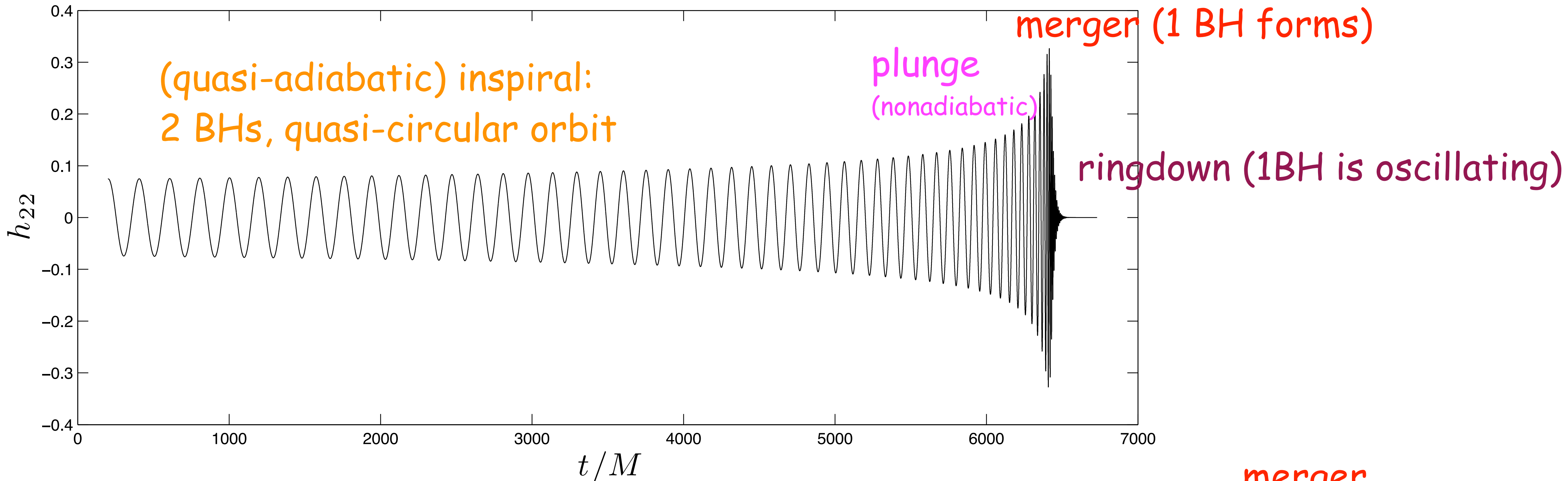
$$\omega_{22}^{\rm GW} = 2\pi f_{22}^{\rm GW} = 2\Omega^{\rm orbital}$$

$$\boxed{f_{GW}^{22} = \frac{1}{\pi}\left(\frac{5}{256\nu}\right)^{3/8}\left(\frac{1}{t - t_{\rm coalescence}}\right)^{3/8}}$$

MONOTONICALLY GROWING FREQUENCY: CHIRP!

BBHS: WAVEFORM OVERWIEV

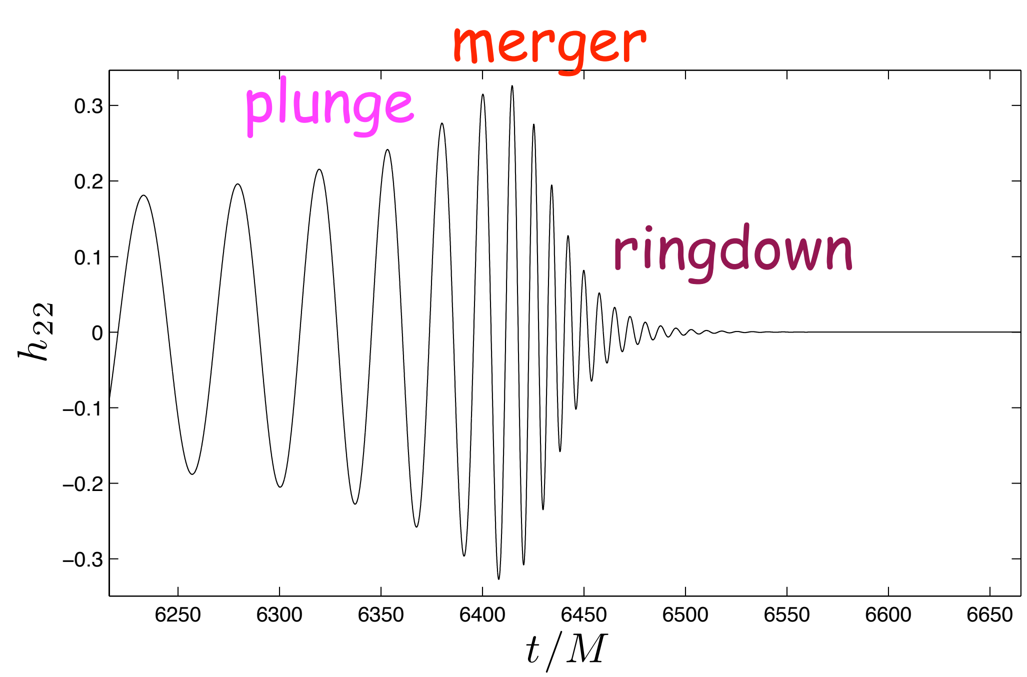
$$h_+ - i h_\times = \frac{1}{r}\sum_{\ell m} h_{\ell m}\, {}_{-2}Y_{\ell m}(\theta,\phi) \qquad h\left(m_1, m_2, \vec{S}_1, \vec{S}_2\right)$$



e.g: equal-mass BBH, aligned-spins

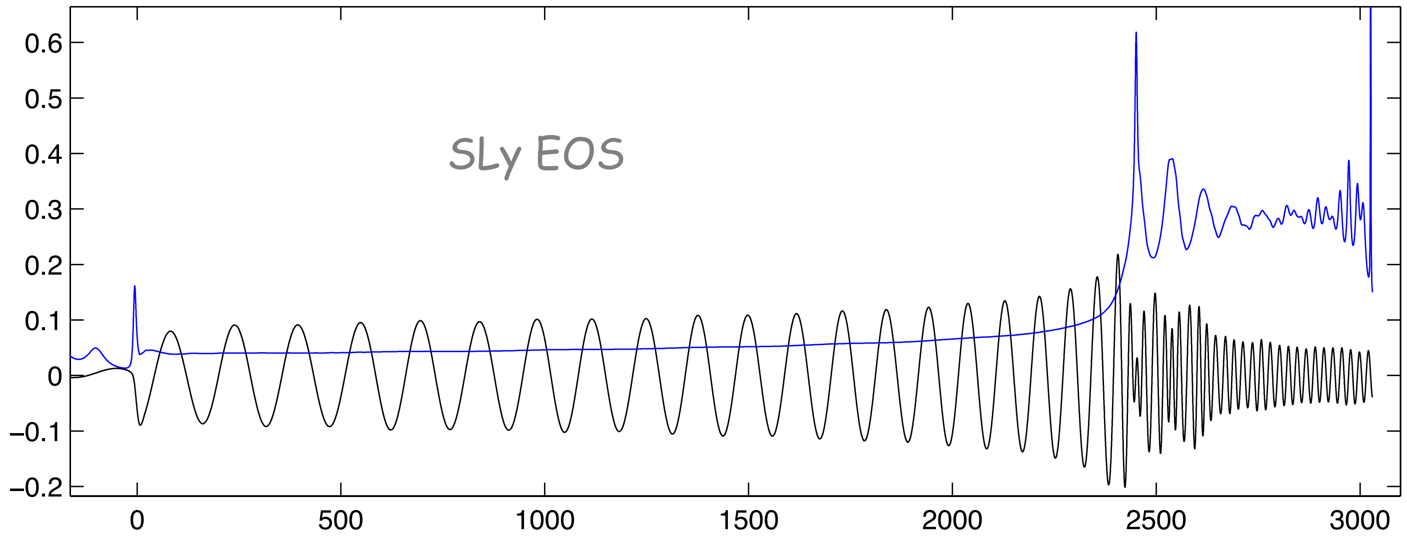
$$\chi_1 = \chi_2 = +0.98$$

- SXS (Simulating eXtreme Spacetimes) collaboration
- www.blackholes.org
- Free catalog of waveforms (downloadable)



lunedì 20 febbraio 17

BINARY NEUTRON STARS (BNS)?



All BNS need is Love!

$$q = 1 \qquad M = 2.7 M_\odot$$

- Tidal effects
- Love numbers (tidal "polarization" constants)
- EOS dependence & "universality"
- EOB/NR for BNS

See:

Damour&Nagar, PRD 2009

Damour&Nagar, PRD 2010

Damour,Nagar et al., PRL 2011

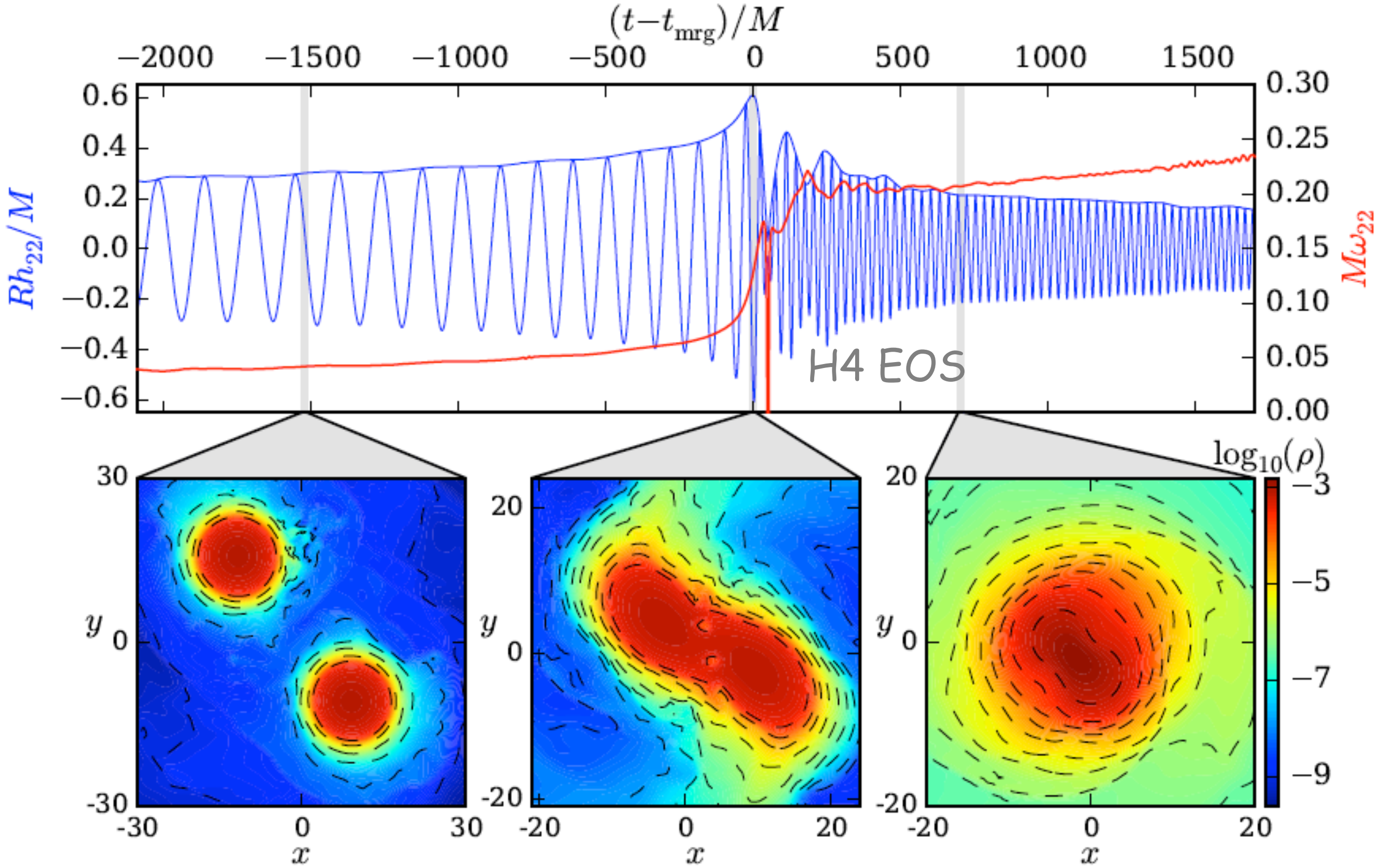
Bini,Damour&Faye, PRD2012

Bini&Damour, PRD 2014

Bernuzzi, Nagar, et al, PRL 2014

Bernuzzi, Nagar, Dietrich, PRL 2015

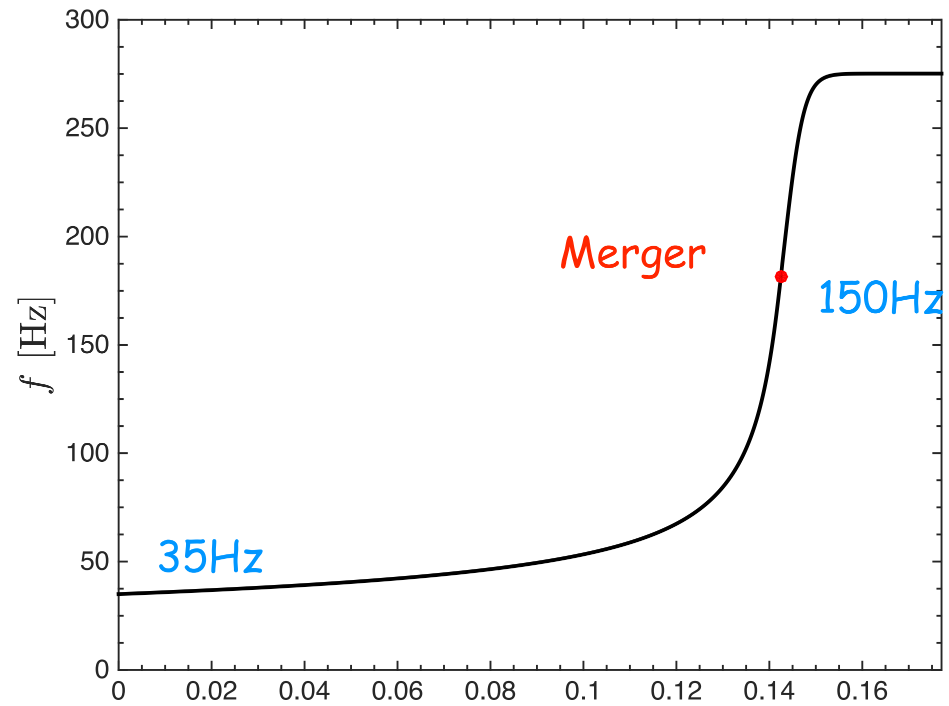
Bernuzzi, Nagar, Dietrich & Damour,PRL, 2015



FAST CHIRP: COULD GW150914 BE A BNS?

The merger occurs at frequencies too low to be a "standard" BNS

GW frequency grows fro 35Hz to 150Hz around peak (factor 4) over the observed 8GWs cycles



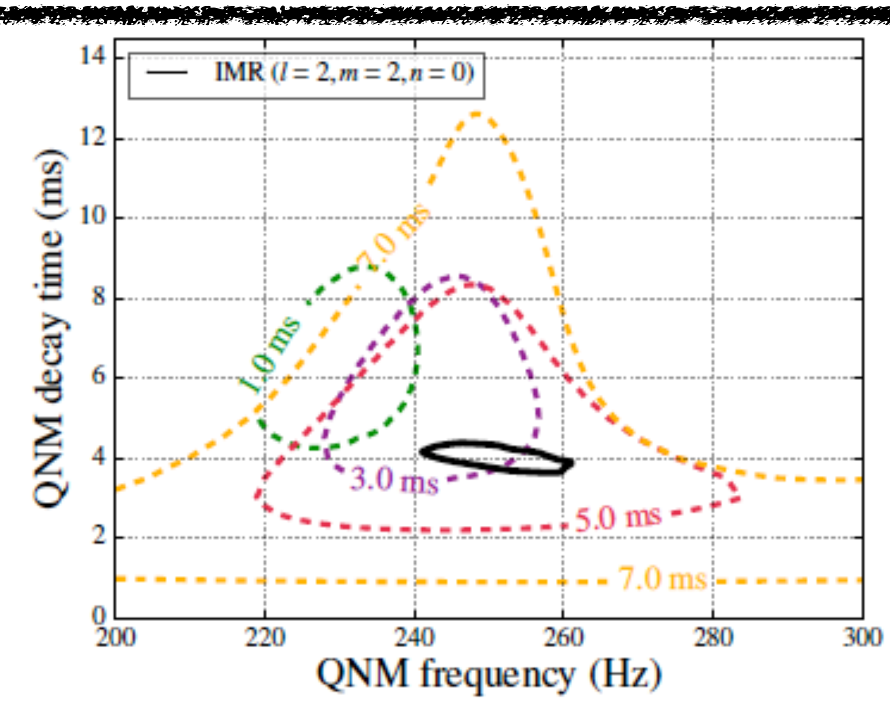


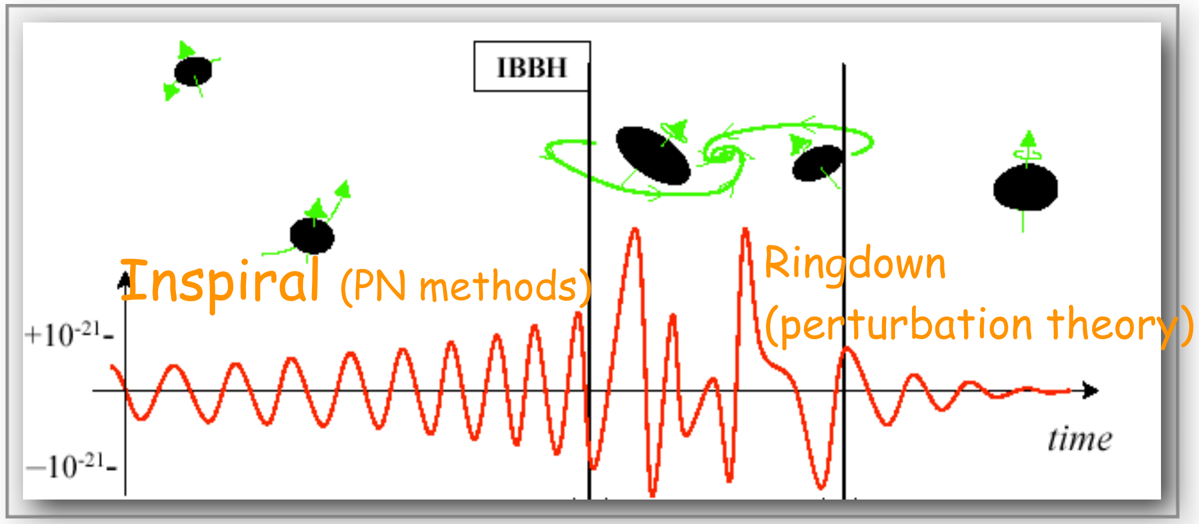
FIG. 4. We show the posterior 90% confidence regions from Bayesian parameter estimation for a damped-sinusoid model, assuming different start-times $t_0 = t_M + 1, 3, 5, 7$ ms, labeled by offset from the merger time t_M of the most-probable waveform from GW150914. The black solid line shows contours of 90% confidence region for the frequency f_0 and decay time τ of the $\ell = 2$, $m = 2$ and $n = 0$ (i.e., the least damped) QNM obtained from the inspiral-merger-ringdown waveform for the entire detector's bandwidth.

But the final answer is that consistency was found between inspiral and ringdown!

Proposal for improved analysis with "more" ringdown: Del Pozzo&Nagar, arXiv:1606.03952

TEMPLATES FOR GWS FROM BBH COALESCENCE

Brady, Craighton & Thorne, 1998

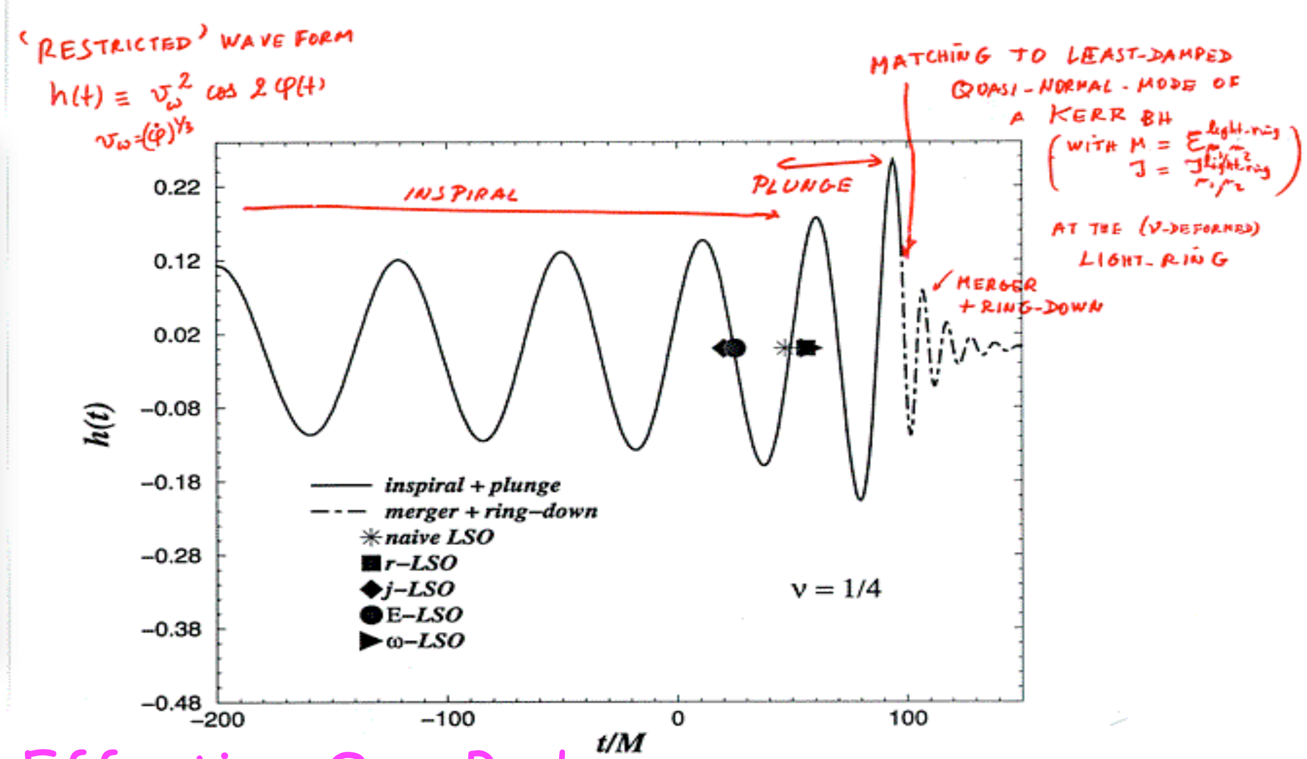


Inspiral (PN methods)

Ringdown (perturbation theory)

Merger:

Numerical Relativity

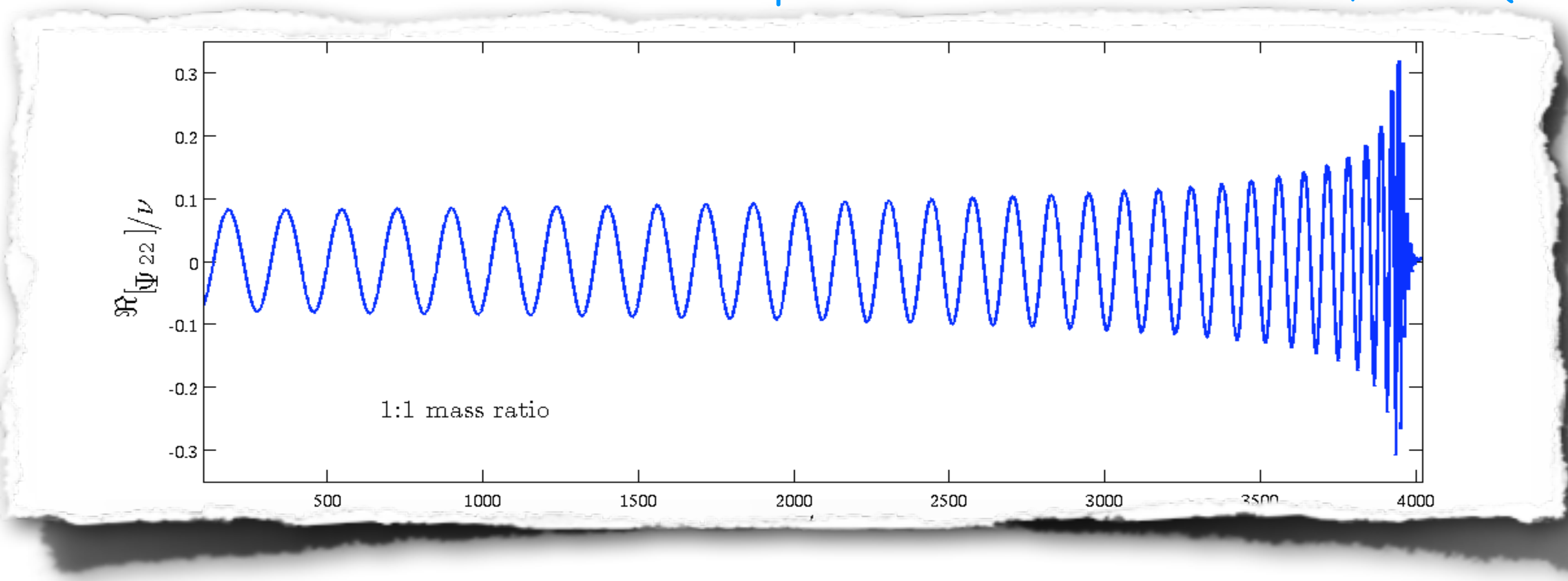


Effective-One-Body (Buonanno & Damour (2000)

PN-resummation (Damour,Iyer, Sathyaprakash (1998)

Numerical Relativity: >= 2005 (F. Pretorius, Campanelli et al., Baker et al.)

Most accurate data: Caltech-Cornell spectral code: M. Scheel et al., 2008 (SXS collaboration)



Spectral code

Extrapolation (radius & resolution)

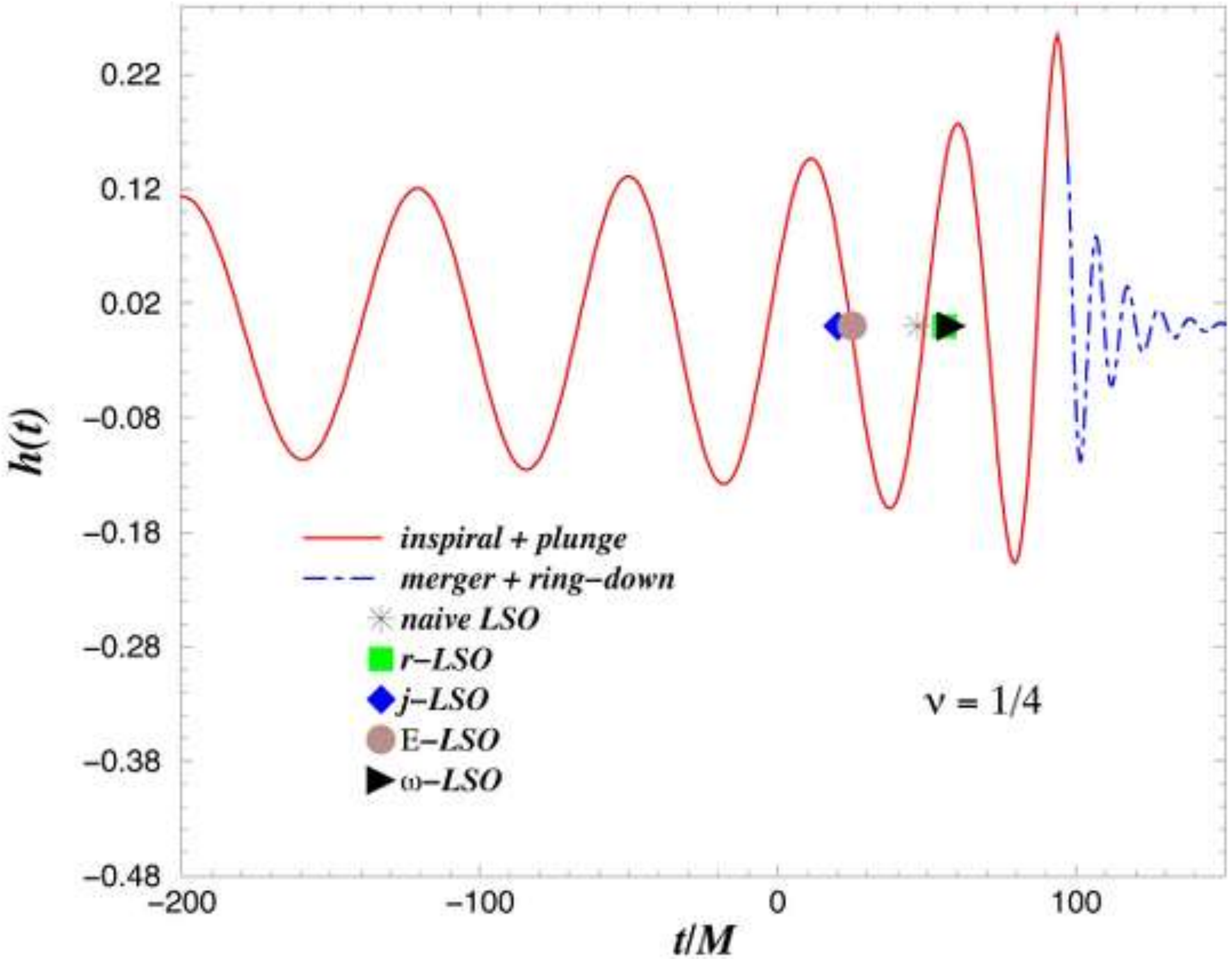
Phase error:

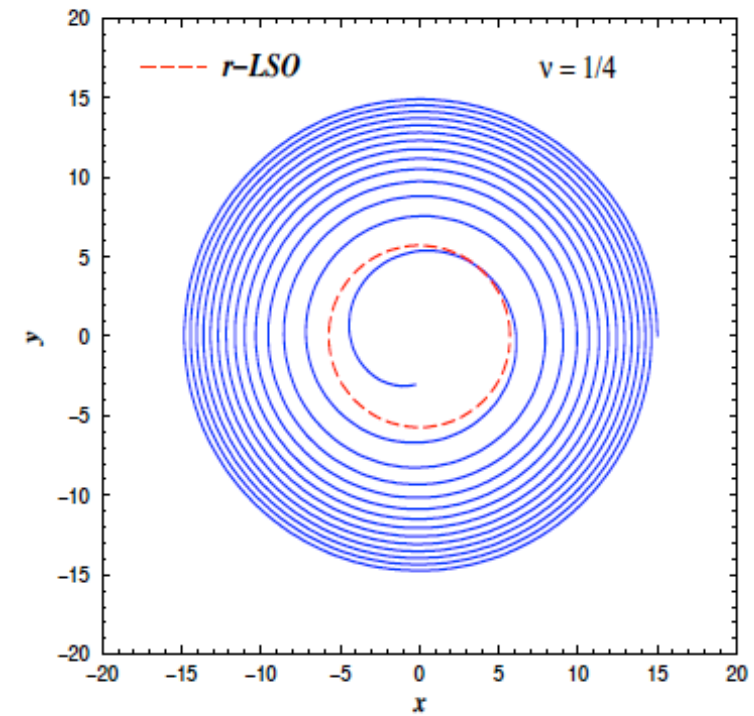
< 0.02 rad (inspiral)

<0.1 rad (ringdown)

EFFECTIVE ONE BODY (EOB): 2000

Numerical Relativity was not working (yet...)
EOB formalism was predictive, qualitatively and semi-quantitatively correct (10%)





- Blurred transition from inspiral to plunge
- Final black-hole mass
- Final black hole spin
- Complete waveform

A. Buonanno & T. Damour, PRD 59 (1999) 084006
A. Buonanno & T. Damour, PRD 62 (2000) 064015

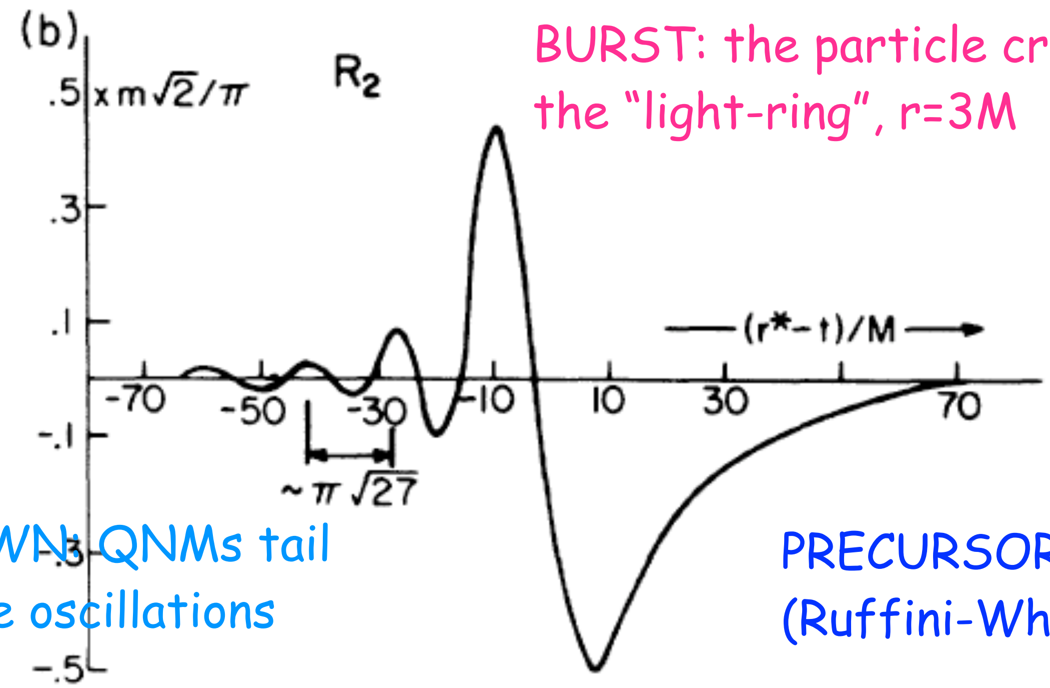
$$\nu = \frac{m_1 m_2}{(m_1+m_2)^2} = \frac{\mu}{M}$$

> 2005: Developing EOB & interfacing with NR
2 groups did (and are doing) it

- A.Buonanno et al. (AEI)
- T.Damour & AN + (>2005)

PRECURSOR-BURST-RINGDOWN STRUCTURE :1972

Davis, Ruffini & Tiomno: radial plunge of a test-particle onto a Schwarzschild black hole (Regge-Wheeler-Zerilli BH perturbation theory)



BURST: the particle crosses the "light-ring", r=3M

RINGDOWN: QNMs tail
Spacetime oscillations

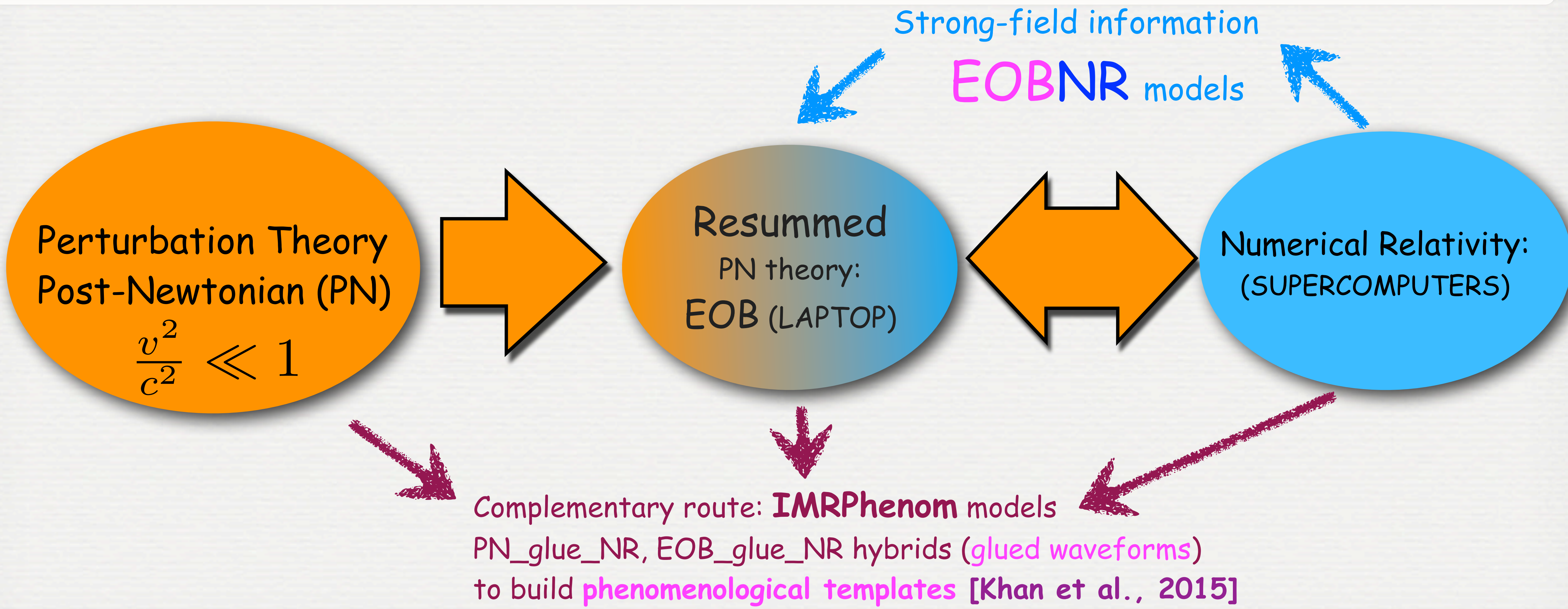
PRECURSOR: Quadrupole formula
(Ruffini-Wheeler approximation)

IMPORTANCE OF AN ANALYTICAL FORMALISM

- **Theoretical**: physical understanding of the coalescence process, especially in complicated situations (e.g., precessing spins).

- **Practical**: need many thousands of accurate GWs templates for detection and data analysis. Need analytical templates: $h\left(m_1, m_2, \vec{S}_1, \vec{S}_2\right)$

- **Solution**: synergy between analytical & numerical relativity



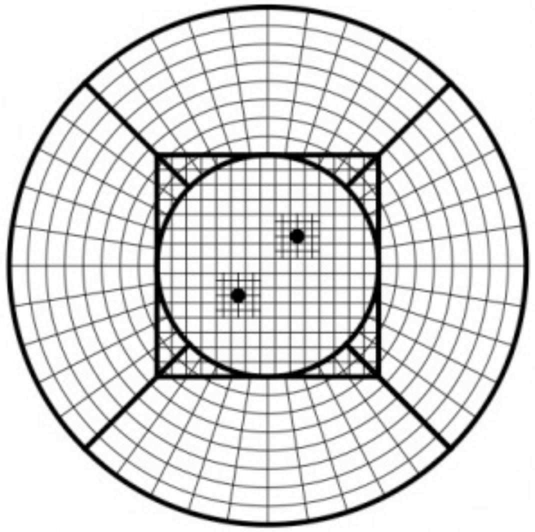
BBH & BNS COALESCENCE: NUMERICAL RELATIVITY

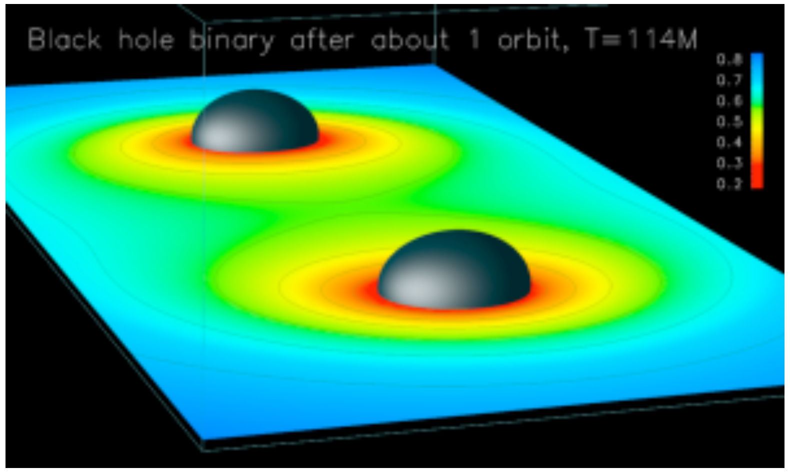
Numerical relativity is complicated & computationally expensive:

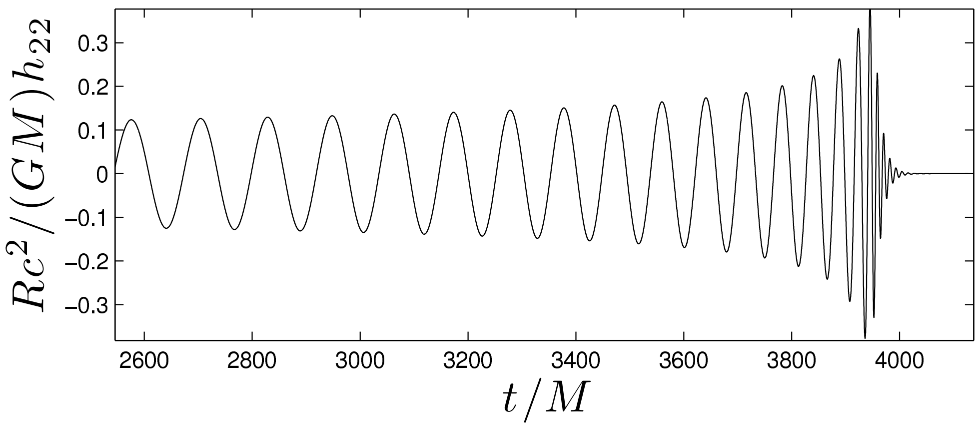
- Formulation of Einstein equations (BSSN, harmonic, Z4c,...)
- Setting up initial data (solution of the constraints)
- Gauge choice
- Numerical approach (finite-differencing (FD, e.g. Llama) vs spectral (SpEC,SXS))
- High-order FD operators
- Treatment of BH singularity (excision vs punctures)
- Wave extraction problem on finite-size grids (Cauchy-Characteristic vs extrapolation)
- Huge computational resources (mass-ratios 1:10; spin)
- Adaptive-mesh-refinement
- Error budget (convergence rates are far from clean...)
- For BNS: further complications due to GR-Hydrodynamics for matter
- Months of running/analysis to get one accurate waveform....

Multi-patch grid structure

(Llama FD code, Pollney & Reisswig)







lunedì 20 febbraio 17

A catalog of 171 high-quality binary black-hole simulations for gravitational-wave astronomy [PRL 111 (2013) 241104]

Abdul H. Mroué,[1] Mark A. Scheel,[2] Béla Szilágyi,[2] Harald P. Pfeiffer,[1] Michael Boyle,[3] Daniel A. Hemberger,[3] Lawrence E. Kidder,[3] Geoffrey Lovelace,[4,2] Sergei Ossokine,[1,5] Nicholas W. Taylor,[2] Anıl Zenginoğlu,[2] Luisa T. Buchman,[2] Tony Chu,[1] Evan Foley,[4] Matthew Giesler,[4] Robert Owen,[6] and Saul A. Teukolsky[3]

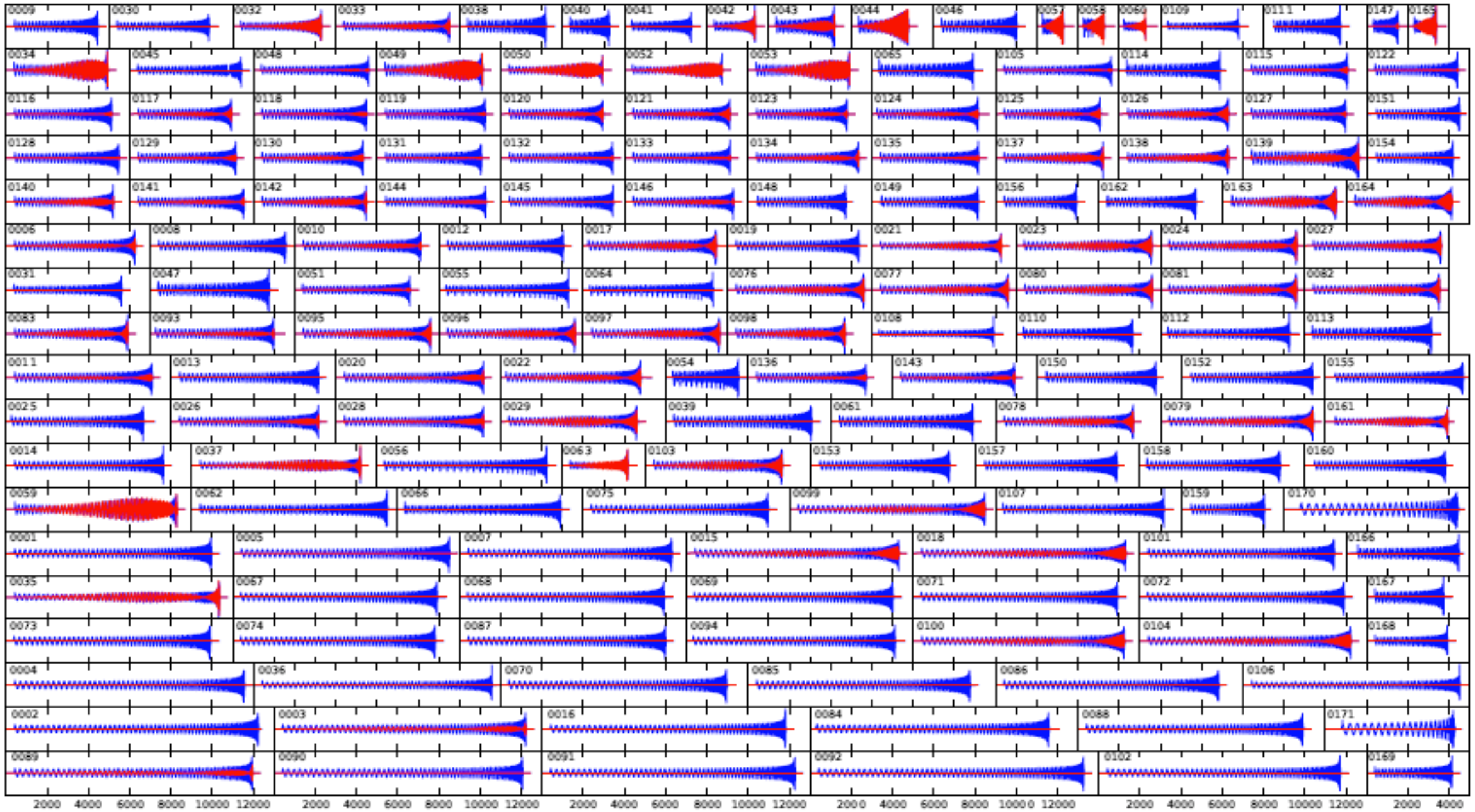


FIG. 3: Waveforms from all simulations in the catalog. Shown here are h_+ (blue) and $h_\times$ (red) in a sky direction parallel to the initial orbital plane of each simulation. All plots have the same horizontal scale, with each tick representing a time interval of $2000M$, where M is the total mass.

• www.blackholes.org

But (at least) 250.000 templates were used...

ANALYTICALLY: MOTION AND GW IN GR

Hamiltonian: conservative part of the dynamics

Radiation reaction: mechanical energy/angular momentum goes away in GWs and backreacts on the system.

The (closed) orbit CIRCULARIZES and SHRiNks with time

Waveform

General Relativity is NONLINEAR!

Post-Newtonian (PN) approximation: expansion in $\frac{v^2}{c^2}$

PROBLEM OF MOTION IN GENERAL RELATIVITY

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}(x) \ , \ h_{\mu\nu} \ll 1$$

$$h_{00} \sim h_{ij} \sim \frac{v^2}{c^2} \ , \ h_{0i} \sim \frac{v^3}{c^3} \ , \ \partial_0 h \sim \frac{v}{c}\partial_i h$$

Approximation methods

- post-Minkowskian (Einstein 1916)
- post-Newtonian (Droste 1916)
- Matching of asymptotic expansions: body zone/near zone/wave zone
- Numerical Relativity

One-chart versus Multi-chart approaches

Coupling between Einstein field equations and equations of motion

Strongly self-gravitating bodies: neutron stars or black holes

$$h_{\mu\nu}(x) \sim 1$$

Skeletonized: $T_{\mu\nu}$ point-masses ? delta-functions in GR

Multipolar Expansion

QFT-like calculations

Need to go to very high-orders of approximation

Use a "cocktail": PM, PN, MPM, MAE, EFT,an. reg., dim. reg.,...

POST-NEWTONIAN HAMILTONIAN (C.O.M)

$$\widehat{H}^{\rm NR}_{\rm real}(\mathbf{q},\mathbf{p}) = \widehat{H}_{\rm N}(\mathbf{q},\mathbf{p}) + \widehat{H}_{\rm 1PN}(\mathbf{q},\mathbf{p}) + \widehat{H}_{\rm 2PN}(\mathbf{q},\mathbf{p}) + \widehat{H}_{\rm 3PN}(\mathbf{q},\mathbf{p})\,, \qquad (4.27)$$

where

$$\widehat{H}_{\rm N}(\mathbf{q},\mathbf{p}) = \frac{\mathbf{p}^2}{2} - \frac{1}{q}, \qquad (4.28a)$$

Newton (0PN)

$$\widehat{H}_{\rm 1PN}(\mathbf{q},\mathbf{p}) = \frac{1}{8}(3\nu-1)(\mathbf{p}^2)^2 - \frac{1}{2}\left[(3+\nu)\mathbf{p}^2 + \nu(\mathbf{n}\cdot\mathbf{p})^2\right]\frac{1}{q} + \frac{1}{2q^2}, \qquad (4.28b)$$

(1PN, 1938) - [Einstein-Infeld-Hoffman]

$$\widehat{H}_{\rm 2PN}(\mathbf{q},\mathbf{p}) = \frac{1}{16}\left(1-5\nu+5\nu^2\right)(\mathbf{p}^2)^3 + \frac{1}{8}\left[\left(5-20\nu-3\nu^2\right)(\mathbf{p}^2)^2 - 2\nu^2(\mathbf{n}\cdot\mathbf{p})^2\mathbf{p}^2 - 3\nu^2(\mathbf{n}\cdot\mathbf{p})^4\right]\frac{1}{q}$$
$$+ \frac{1}{2}\left[(5+8\nu)\mathbf{p}^2 + 3\nu(\mathbf{n}\cdot\mathbf{p})^2\right]\frac{1}{q^2} - \frac{1}{4}(1+3\nu)\frac{1}{q^3}, \qquad (4.28c)$$

(2PN, 1982/83) - [Damour-Deruelle]

$$\widehat{H}_{\rm 3PN}(\mathbf{q},\mathbf{p}) = \frac{1}{128}\left(-5+35\nu-70\nu^2+35\nu^3\right)(\mathbf{p}^2)^4$$
$$+ \frac{1}{16}\left[\left(-7+42\nu-53\nu^2-5\nu^3\right)(\mathbf{p}^2)^3 + (2-3\nu)\nu^2(\mathbf{n}\cdot\mathbf{p})^2(\mathbf{p}^2)^2 + 3(1-\nu)\nu^2(\mathbf{n}\cdot\mathbf{p})^4\mathbf{p}^2 - 5\nu^3(\mathbf{n}\cdot\mathbf{p})^6\right]\frac{1}{q}$$
$$+ \left[\frac{1}{16}\left(-27+136\nu+109\nu^2\right)(\mathbf{p}^2)^2 + \frac{1}{16}(17+30\nu)\nu(\mathbf{n}\cdot\mathbf{p})^2\mathbf{p}^2 + \frac{1}{12}(5+43\nu)\nu(\mathbf{n}\cdot\mathbf{p})^4\right]\frac{1}{q^2}$$
$$+ \left\{\left[-\frac{25}{8} + \left(\frac{1}{64}\pi^2 - \frac{335}{48}\right)\nu - \frac{23}{8}\nu^2\right]\mathbf{p}^2 + \left(-\frac{85}{16} - \frac{3}{64}\pi^2 - \frac{7}{4}\nu\right)\nu(\mathbf{n}\cdot\mathbf{p})^2\right\}\frac{1}{q^3}$$
$$+ \left[\frac{1}{8} + \left(\frac{109}{12} - \frac{21}{32}\pi^2 + \omega_{\rm static}\right)\nu\right]\frac{1}{q^4}. \qquad (4.28d)$$

(3PN, 2000) - [Damour, Jaranowski,Schaefer]

...and 4PN too, [Damour, Jaranowski&Schaefer 2014/2015] - 4 loop calculation

$$\mathbf{q} = \mathbf{q}_1 - \mathbf{q}_2$$

$$\mathbf{p} = \mathbf{p}_1 = -\mathbf{p}_2$$

PN-EXPANDED (CIRCULAR) ENERGY FLUX (3.5PN)

$$\frac{dE}{dt} = -\mathcal{L}$$

balance equation

Mechanical loss GW luminosity

$$\mathcal{L} = \underbrace{\frac{32c^5}{5G}\nu^2 x^5}_{\text{Newtonian quadrupole formula}}\left\{1 + \left(-\frac{1247}{336} - \frac{35}{12}\nu\right)x + 4\pi x^{3/2} + \left(-\frac{44711}{9072} + \frac{9271}{504}\nu + \frac{65}{18}\nu^2\right)x^2 \right.$$
$$+ \left(-\frac{8191}{672} - \frac{583}{24}\nu\right)\pi x^{5/2}$$
$$+ \left[\frac{6643739519}{69854400} + \frac{16}{3}\pi^2 - \frac{1712}{105}C - \frac{856}{105}\ln(16\,x)\right.$$
$$\left. + \left(-\frac{134543}{7776} + \frac{41}{48}\pi^2\right)\nu - \frac{94403}{3024}\nu^2 - \frac{775}{324}\nu^3\right]x^3$$
$$\left. + \left(-\frac{16285}{504} + \frac{214745}{1728}\nu + \frac{193385}{3024}\nu^2\right)\pi x^{7/2} + \mathcal{O}\left(\frac{1}{c^8}\right)\right\}.$$

$$C = \gamma_E = 0.5772156649...$$

TAYLOR-EXPANDED (CIRCULAR) 3PN WAVEFORM

Blanchet, Iyer&Joguet, 02; Blanchet, Damour, Iyer&Esposito-Farese, 04; Kidder07; Blanchet et al.,08

$$
\begin{aligned}
h^{22} = & -8\sqrt{\frac{\pi}{5}}\frac{G\nu m}{c^2 R}e^{-2i\phi}x\Big\{1 - x\Big(\frac{107}{42} - \frac{55}{42}\nu\Big) + x^{3/2}\Big[2\pi + 6i\ln\Big(\frac{x}{x_0}\Big)\Big] - x^2\Big(\frac{2173}{1512} + \frac{1069}{216}\nu - \frac{2047}{1512}\nu^2\Big)\\
& - x^{5/2}\Big[\Big(\frac{107}{21} - \frac{34}{21}\nu\Big)\pi + 24i\nu + \Big(\frac{107i}{7} - \frac{34i}{7}\nu\Big)\ln\Big(\frac{x}{x_0}\Big)\Big]\\
& + x^3\Big[\frac{27\,027\,409}{646\,800} - \frac{856}{105}\gamma_E + \frac{2}{3}\pi^2 - \frac{1712}{105}\ln 2 - \frac{428}{105}\ln x\\
& - 18\Big[\ln\Big(\frac{x}{x_0}\Big)\Big]^2 - \Big(\frac{278\,185}{33\,264} - \frac{41}{96}\pi^2\Big)\nu - \frac{20\,261}{2772}\nu^2 + \frac{114\,635}{99\,792}\nu^3 + \frac{428i}{105}\pi + 12i\pi\ln\Big(\frac{x}{x_0}\Big)\Big] + O(\epsilon^{7/2})\Big\},
\end{aligned}
$$

$$x = (M\Omega)^{2/3} \sim v^2/c^2$$

$$M = m_1 + m_2$$

$$\nu = \frac{m_1 m_2}{M^2}$$

EFFECTIVE-ONE-BODY (EOB)

approach to the general relativistic two-body problem

(Buonanno-Damour 99, 00, Damour-Jaranowski-Schäfer 00, Damour 01, Damour-Nagar 07, Damour-Iyer-Nagar 08)

key ideas:

(1) Replace two-body dynamics (m_1, m_2) by dynamics of a particle $(\mu \equiv m_1 m_2/(m_1+m_2))$ in an effective metric $g_{\mu\nu}^{\text{eff}}(u)$, with

$$u \equiv GM/c^2R\,, \quad M \equiv m_1 + m_2$$

(2) Systematically use RESUMMATION of PN expressions (both $g_{\mu\nu}^{\text{eff}}$ and $\mathcal{F}_{RR}$) based on various physical requirements

(3) Require continuous deformation w.r.t. $\nu \equiv \mu/M \equiv m_1 m_2/(m_1+m_2)^2$ in the interval $0 \leq \nu \leq \frac{1}{4}$

STRUCTURE OF THE EOB FORMALISM

PN dynamics
(DD81,D82,DJS01,IF03,BDIF 04)

PN rad losses
WW76, BDIWW95, BDEFI 05

PN waveform
BD89, B95&05,ABIQ04,

BH perturbations
RW57, Z70, Z72

Resummed (BD99)

Resummed (DIS98)

Resummed (DN07,DIN08)

QNMs spectrum
$\sigma_N = \alpha_N + i\omega_N$

EOB Hamiltonian
$H_{\rm EOB}$

EOB Rad. Reac. force
$\hat{\mathcal{F}}_\varphi$

Factorized waveform

Matching at merger time

$$h_{\ell m} = h^{(N,\epsilon)}_{\ell m}\hat{h}^{(\epsilon)}_{\ell m}$$
$$\hat{h}^{(\epsilon)}_{\ell m} = \hat{S}^{(\epsilon)}_{eff} T_{\ell m} e^{i\delta_{\ell m}}\rho^{\ell}_{\ell m}$$

$$\frac{dr}{dt} = \left(\frac{A}{B}\right)^{1/2}\frac{\partial \hat{H}_{\rm EOB}}{\partial p_{r_*}},$$
$$\frac{dp_{r_*}}{dt} = -\left(\frac{A}{B}\right)^{1/2}\frac{\partial \hat{H}_{\rm EOB}}{\partial r},$$
$$\Omega \equiv \frac{d\varphi}{dt} = \frac{\partial \hat{H}_{\rm EOB}}{\partial p_\varphi},$$
$$\frac{dp_\varphi}{dt} = \hat{\mathcal{F}}_\varphi .$$

BNS: tides
(Love numbers)

$$h^{\rm ringdown}_{\ell m}(t) = \sum_N C^+_N e^{-\sigma^+_N (t-t_m)}$$

Phenomenological fit to NR postmerger phase

EOB waveform

$$h^{\rm EOB}_{\ell m} = \theta(t_m - t) h^{\rm insplunge}_{\ell m}(t) + \theta(t - t_m) h^{\rm ringdown}_{\ell m}(t)$$

TWO-BODY/EOB "CORRESPONDENCE":

THINK QUANTUM-MECHNICALLY (J.A. WHEELER)

Real 2-body system
(in the c.o.m. frame)
(m_1, m_2)

1:1 map

An effective particle
in some effective metric

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

$$g_{\mu\nu}^{\mathrm{eff}}$$

$$\mathcal{E} = f(E)$$

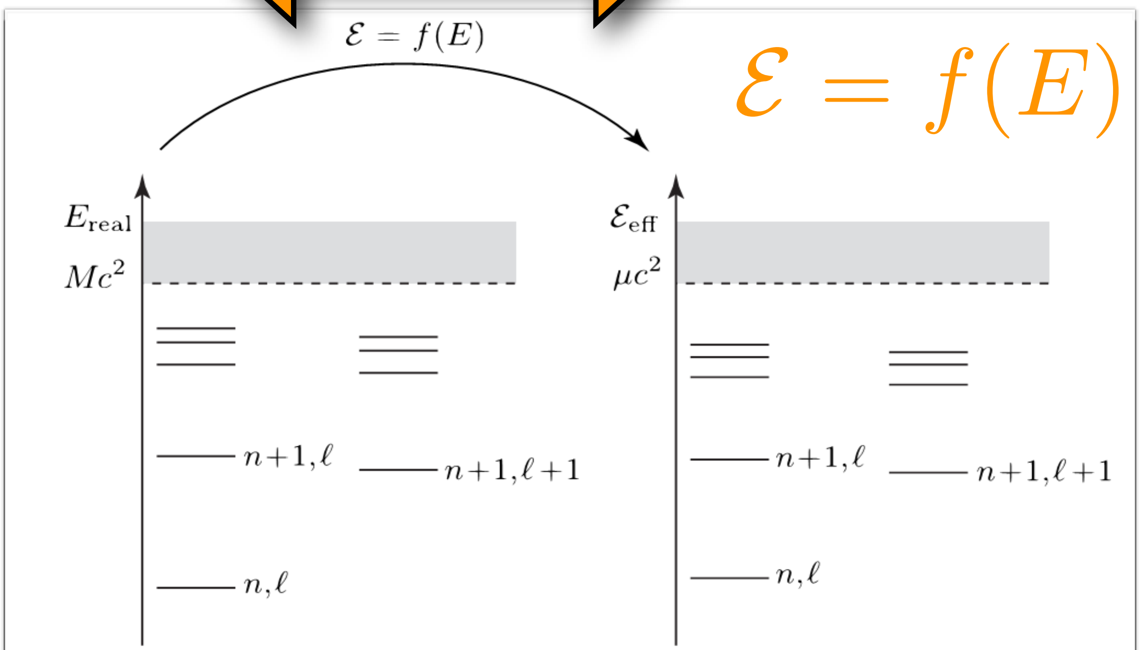


Figure 1: Sketch of the correspondence between the quantized energy levels of the real and effective conservative dynamics. n denotes the 'principal quantum

$$\mu^2 + g_{\mathrm{eff}}^{\mu\nu} \frac{\partial S_{\mathrm{eff}}}{\partial x^\mu} \frac{\partial S_{\mathrm{eff}}}{\partial x^\nu} + \mathcal{O}(p^4) = 0$$

Sommerfeld's
"Old Quantum Mechanics"
(action-angle variables &
Delaunay Hamiltonian)

$$\begin{aligned} J &= \ell\hbar = \frac{1}{2\pi}\oint p_\varphi d\varphi \\ N &= n\hbar = I_r + J \\ I_r &= \frac{1}{2\pi}\oint p_r dr \end{aligned}$$

$$H^{\mathrm{classical}}(q,p) \longrightarrow H^{\mathrm{classical}}(I_a) \longrightarrow \boxed{E^{\mathrm{quantum}}(I_a = n_a h) = f^{-1}[\mathcal{E}_{\mathrm{eff}}^{\mathrm{quantum}}(I_a^{\mathrm{eff}} = n_a h)]}$$

THE 2-BODY HAMILTONIAN [2PN]

The 2-body Hamiltonian at 2PN (c.o.m. frame)

$$H_{2\mathrm{PN}}^{\mathrm{relative}}(\mathbf{q},\mathbf{p}) = H_0(\mathbf{q},\mathbf{p}) + \frac{1}{c^2}H_2(\mathbf{q},\mathbf{p}) + \frac{1}{c^4}H_4(\mathbf{q},\mathbf{p})$$

The Newtonian limit

$$H_0(\mathbf{q},\mathbf{p}) = \frac{\mathbf{p}^2}{2\mu} + \frac{GM\mu}{|\mathbf{q}|}$$

4 additional terms at 1PN
7 additional terms at 2PN
11 additional terms at 3PN

Rewrite the c.o.m. (reduced, non-relativistic) energy using action variables

Obtain the 2PN "quantum" energy levels: Delaunay Hamiltonian [Damour-Schaefer 1988]

$$E_{2\mathrm{PN}}^{\mathrm{NR}} = -\frac{1}{2}\mu\frac{\alpha^2}{n^2}\left[1 + \frac{\alpha^2}{c^2}\left(\frac{c_{11}}{n\ell} + \frac{c_{20}}{n^2}\right) + \frac{\alpha^4}{c^4}\left(\frac{c_{13}}{n\ell^3} + \frac{c_{22}}{n^2\ell^2} + \frac{c_{31}}{n^3\ell} + \frac{c_{40}}{n^4}\right)\right]$$

Balmer formula!

$\alpha = (GM\mu)/\hbar$

$N = n\hbar$

$e \equiv \mu$

$Ze \equiv GM$

$$E_n = -\frac{\mu}{2}\frac{e^4}{\hbar^2 n^2}$$

$$E_{2\mathrm{PN}}^{\mathrm{relativistic}}(n,\ell) = Mc^2 + E_{2\mathrm{PN}}^{\mathrm{NR}}(n,\ell)$$

THE EOB ENERGY MAP

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

Real 2-body system
(PN-expanded Hamiltonian
in the c.o.m. frame)

$$(m_1, m_2)$$

1:1 map

An effective particle
in some effective metric

$$g_{\mu\nu}^{\text{eff}}$$

Simple energy map:

$$\mathcal{E}_{\text{eff}} = \frac{E_{\text{real}}^2 - m_1^2 - m_2^2}{2M}$$

EOB Hamiltonian:

$$H_{\text{EOB}} = M\sqrt{1 + 2\nu\left(\hat{H}_{\text{eff}} - 1\right)}$$

$$M = m_1 + m_2$$

$$\nu = \frac{\mu}{M}$$

$$\hat{H}_{\text{eff}} = \frac{H_{\text{eff}}}{\mu}$$

EXPLICIT FORM OF THE EOB HAMILTONIAN

EOB Hamiltonian

$$H_{\rm EOB} = M\sqrt{1+2\nu\left(\hat{H}_{\rm eff}-1\right)}$$

All functions are a ν-dependent deformation of the Schwarzschild ones

$$A(r) = 1 - 2u + 2\nu u^3 + a_4 \nu u^4$$

$$a_4 = \frac{94}{3} - \frac{41}{32}\pi^2 \simeq 18.6879027$$

$$A(r)B(r) = 1 - 6\nu u^2 + 2(3\nu - 26)\nu u^3$$

$$u = GM/(c^2 R)$$

Simple effective Hamiltonian:

$$\hat{H}_{\rm eff} \equiv \sqrt{p_{r_*}^2 + A(r)\left(1 + \frac{p_\varphi^2}{r^2} + z_3\frac{p_{r_*}^4}{r^2}\right)}$$

$$p_{r_*} = \left(\frac{A}{B}\right)^{1/2} p_r$$

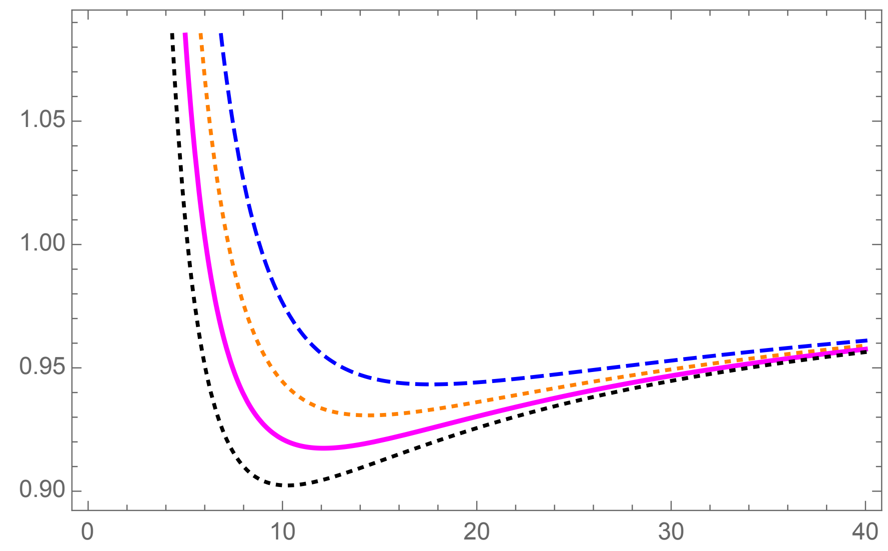
Crucial EOB radial potential

Contribution at 3PN

EFFECTIVE POTENTIALS

Newtonian gravity (any mass ratio):
circular orbits are always stable. No plunge.

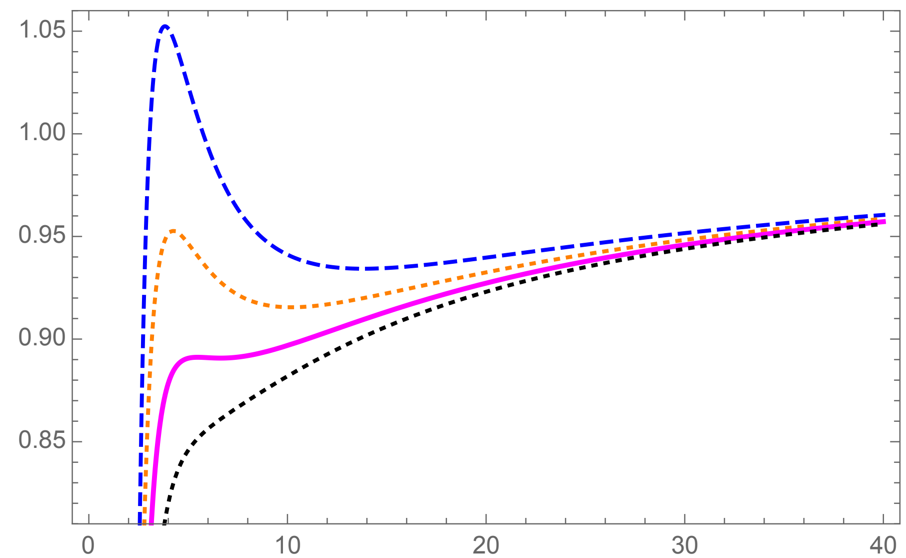
$$W^{\rm eff}_{\rm Newt} = 1 - \frac{2}{r} + \frac{p_\varphi^2}{r^2}$$



Test-body on Schwarzschild black hole:

last stable orbit (LSO) at **r=6M**; plunge

$$W^{\rm eff}_{\rm Schwarzschild} = \left(1 - \frac{2}{r}\right)\left(1 + \frac{p_\varphi^2}{r^2}\right)$$



EOB, Black-hole binary, any mass ratio:

last stable orbit (LSO) at **r<6M** plunge

$$W^{\rm eff}_{\rm EOB} = A(r;\nu)\left(1 + \frac{p_\varphi^2}{r^2}\right)$$

ν **-deformation of the Schwarzschild case!**

lunedì 20 febbraio 17

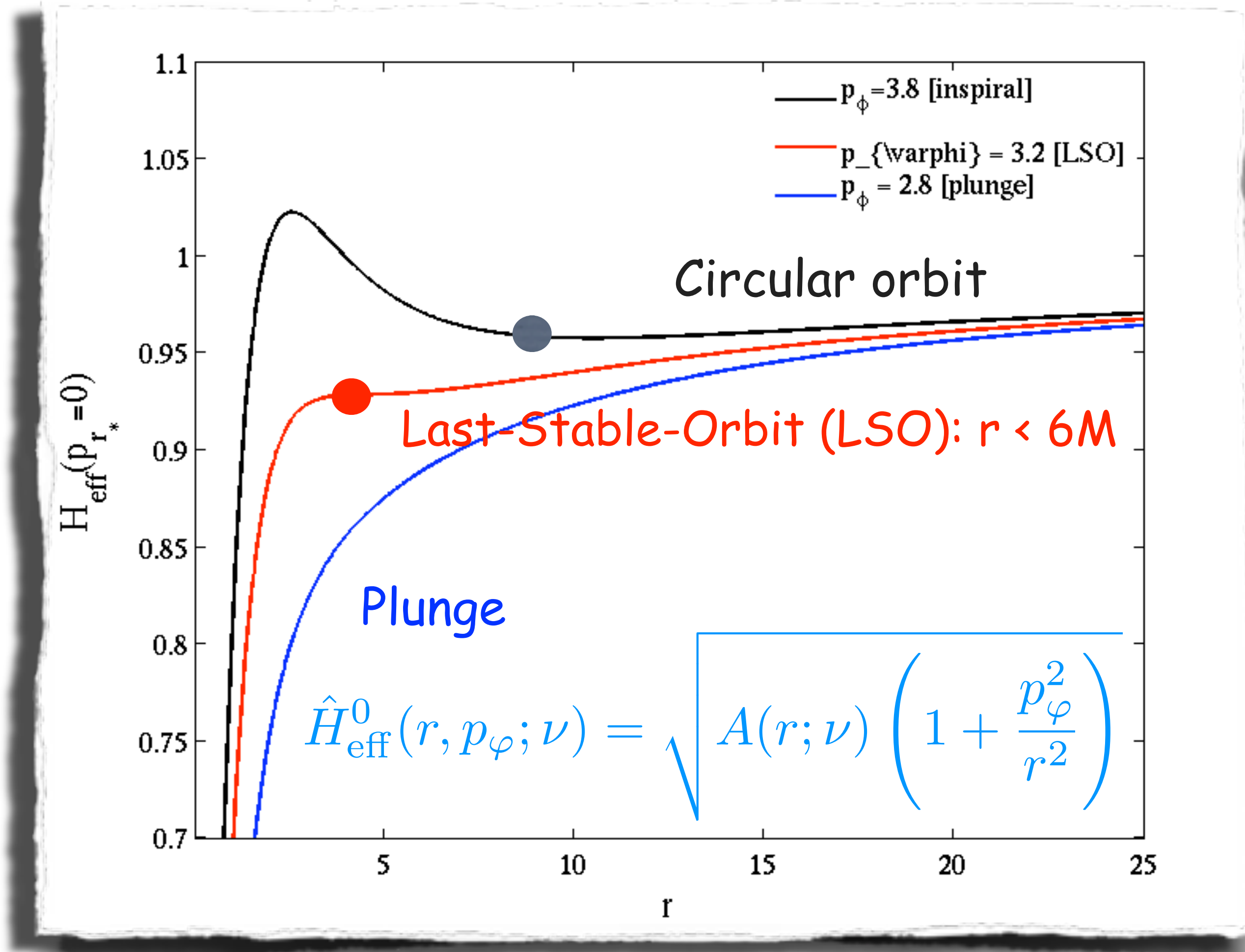
HAMILTON'S EQUATIONS & RADIATION REACTION

$$\dot{r} = \left(\frac{A}{B}\right)^{1/2} \frac{\partial \hat{H}_{\rm EOB}}{\partial p_{r_*}}$$

$$\dot{\varphi} = \frac{\partial \hat{H}_{\rm EOB}}{\partial p_\varphi} \equiv \Omega$$

$$\dot{p}_{r_*} = -\left(\frac{A}{B}\right)^{1/2} \frac{\partial \hat{H}_{\rm EOB}}{\partial r} + \hat{\mathcal{F}}_{r_*}$$

$$\dot{p}_\varphi = \hat{\mathcal{F}}_\varphi$$



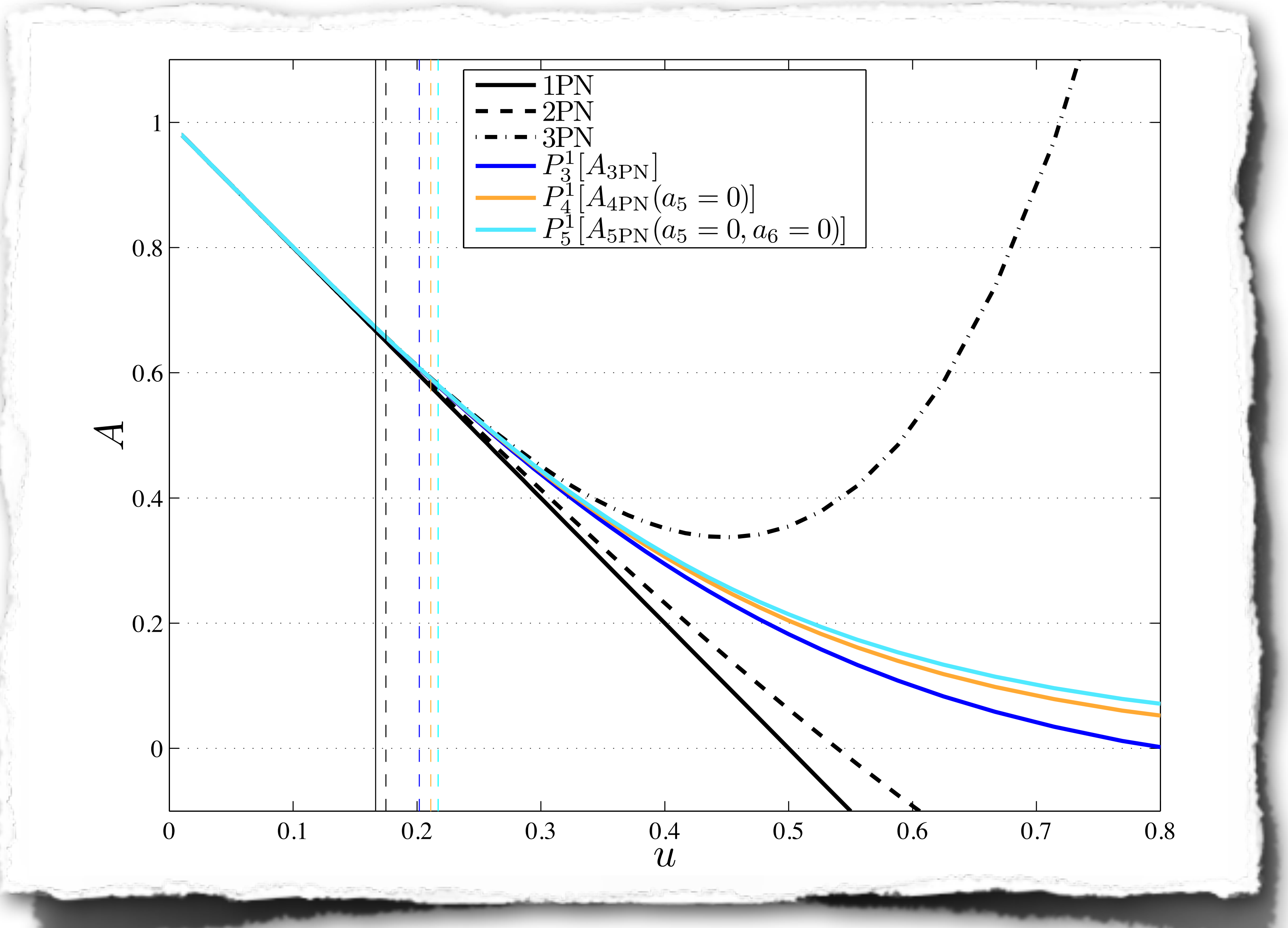
- The system must radiate angular momentum
- How?Use PN-based (Taylor-expanded) radiation reaction force (ang-mom flux)
- Need flux resummation

$$\hat{\mathcal{F}}_\varphi^{\rm Taylor} = -\frac{32}{5}\nu\Omega^5 r_\Omega^4 \hat{F}^{\rm Taylor}(v_\varphi)$$

Plus horizon contribution [AN&Akcay2012]

Resummation multipole by multipole
(Damour&Nagar 2007,
Damour, Iyer & Nagar 2008,
Damour & Nagar, 2009)

USE OF PADE APPROXIMANTS



- Continuity with Schwarzschild metric: A(r) needs to have a zero
- Simple (possible) prescription: use a Padé representation of the potential

$$A(r) = P^1_3[A^{3\mathrm{PN}}(r)] = \frac{1 + n_1 u}{1 + d_1 u + d_2 u^2 + d_3 u^3}$$

MULTIPOLAR WAVEFORM RESUMMATION

Resummation of the waveform (and flux) multipole by multipole (CRUCIAL!)

[Damour&Nagar 2007, Damour, Iyer, Nagar 2008]

Next-to-quasi-circular correction

$$h_{\ell m} \equiv \underbrace{h^{(N,\epsilon)}_{\ell m}}\,\underbrace{\hat{h}^{(\epsilon)}_{\ell m}}_{\text{PN-correction}}\,\underbrace{\hat{h}^{\rm NQC}_{\ell m}}$$

Newtonian × PN × NQC

$$\hat{h}^{(\epsilon)}_{\ell m} = \hat{S}^{(\epsilon)}_{\rm eff}\, T_{\ell m}\, e^{i\delta_{\ell m}}\, \rho^{\ell}_{\ell m}$$

Remnant phase and modulus corrections: "improved" PN series

The "Tail factor"

$$T_{\ell m} = \frac{\Gamma(\ell + 1 - 2i\hat{\hat{k}})}{\Gamma(\ell+1)} e^{\pi \hat{\hat{k}}} e^{2i\hat{\hat{k}}\ln(2kr_0)}$$

Resums an infinite number of leading logarithms in tail effects (hereditary contributions)

Effective source:

EOB (effective) energy (even-parity modes)

EOB angular momentum (odd-parity modes)

4PN analytically complete + 5PN logarithmic term in the A(u) function:

[Damour 2009, Blanchet et al. 2010, Barack, Damour & Sago 2010, Le Tiec et al. 2011, Barausse et al. 2011,Akcay et al. 2012, Bini& Damour2013, DamourJaranowski&Schaefer 2014].

$$A_{5\mathrm{PN}}^{\mathrm{Taylor}} = 1 - 2u + 2\nu u^3 + \left(\frac{94}{3} - \frac{41}{32}\pi^2\right)\nu u^4 + \nu[a_5^c(\nu) + a_5^{\ln}\ln u]u^5 + \nu[a_6^c(\nu) + a_6^{\ln}\ln u]u^6$$

1PN 2PN 3PN 4PN 5PN

$$a_5^{\log} = \frac{64}{5}$$

$$a_5^c = a_{5_0}^c + \nu a_{5_1}^c$$

$$a_{5_0}^c = -\frac{4237}{60} + \frac{2275}{512}\pi^2 + \frac{256}{5}\log(2) + \frac{128}{5}\gamma$$

$$a_{5_1}^c = -\frac{221}{6} + \frac{41}{32}\pi^2$$

4PN fully known ANALYTICALLY!

$$a_6^{\log} = -\frac{7004}{105} - \frac{144}{5}\nu$$

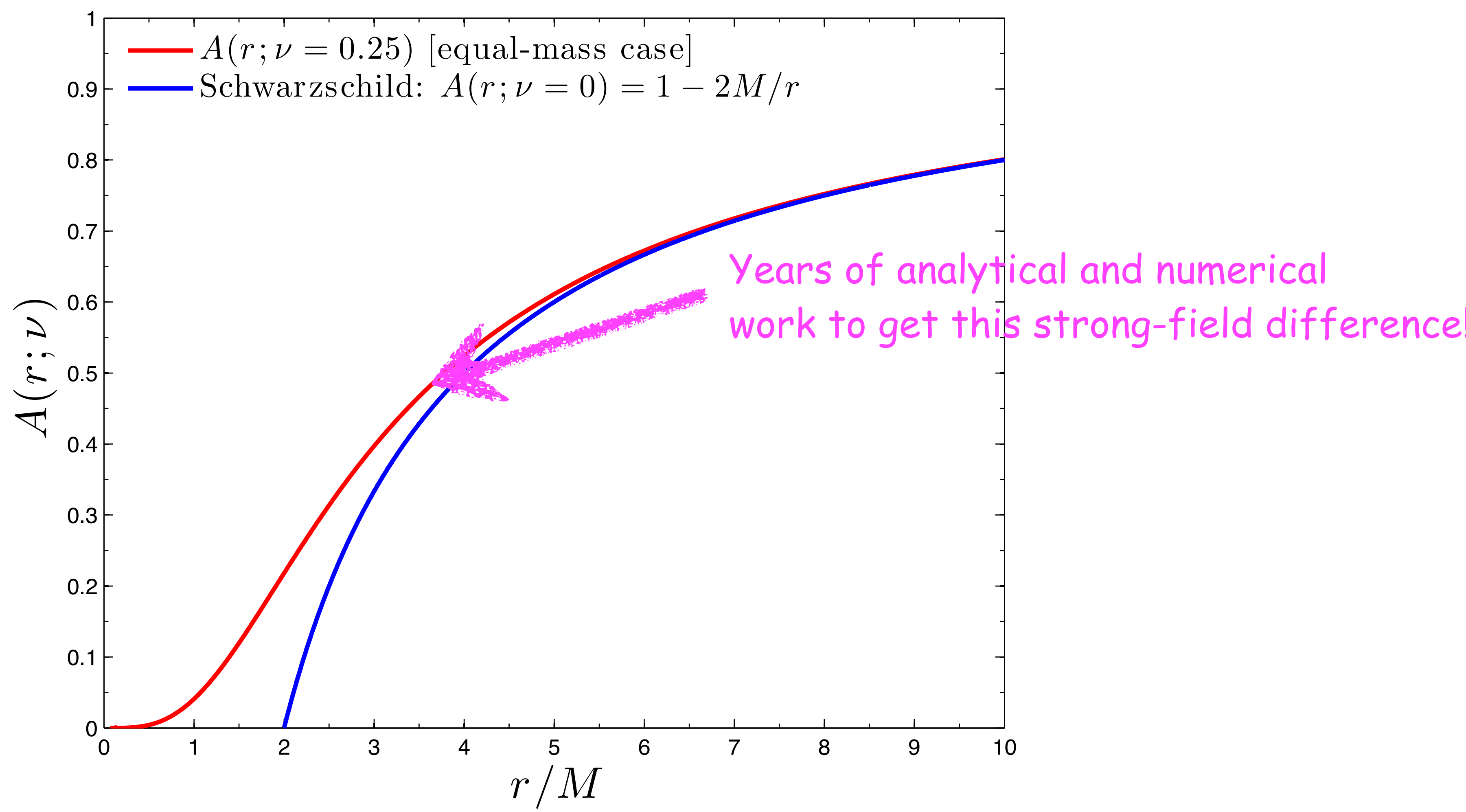
5PN logarithmic term (analytically known)

NEED ONE "effective" 5PN parameter from NR waveform data: $a_6^c(\nu)$

State-of-the-art EOB potential (5PN-resummed):

$$A(u;\nu,a_6^c) = P_5^1[A_{5\mathrm{PN}}^{\mathrm{Taylor}}(u;\nu,a_6^c)]$$

THE EOB[NR] POTENTIAL



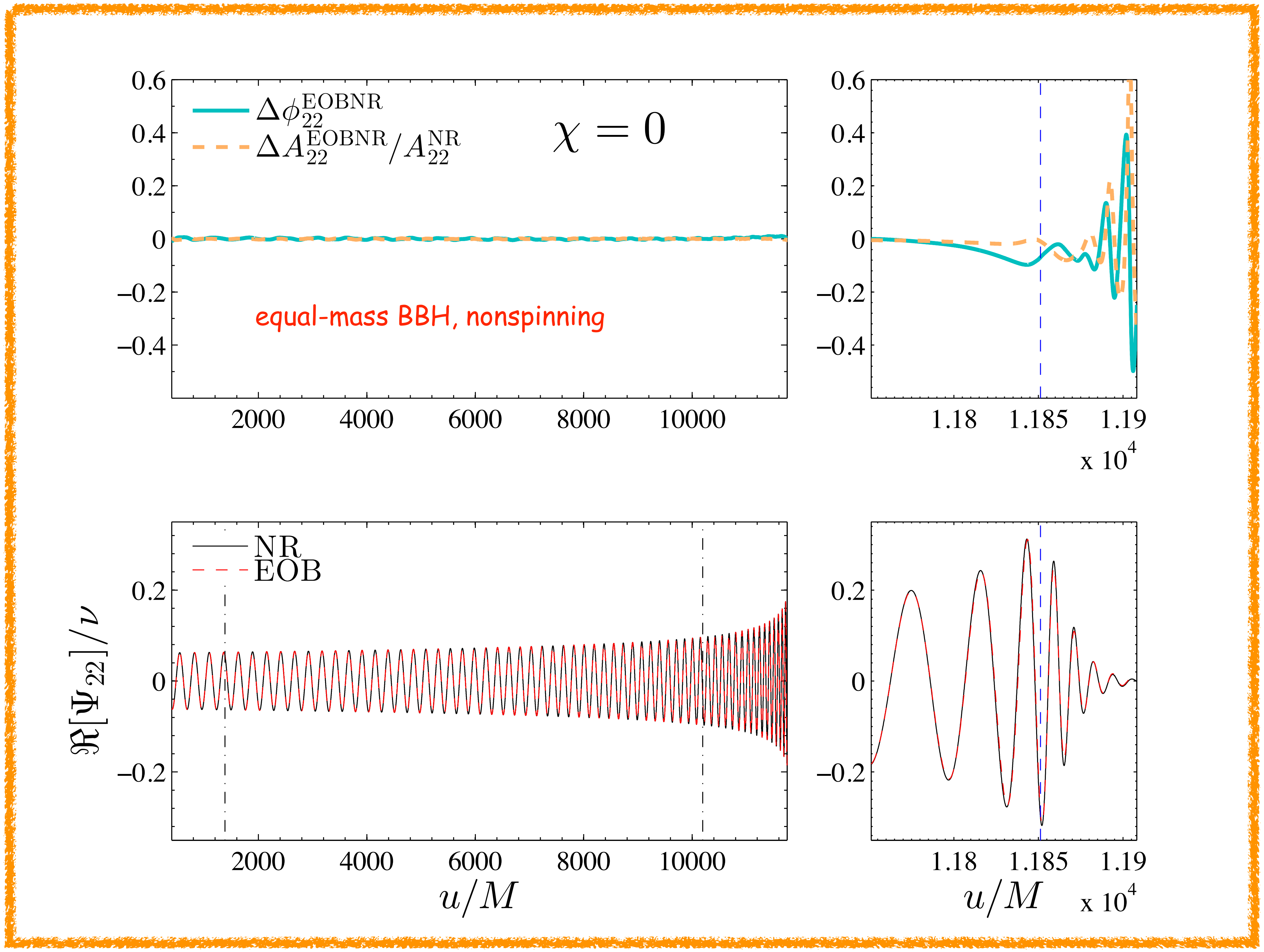
From EOB/NR-fitting: $a_6^c(\nu) = 3097.3\nu^2 - 1330.6\nu + 81.3804$

TAKE AWAY:

BBH system is more bound, smaller "separation" and higher frequencies!

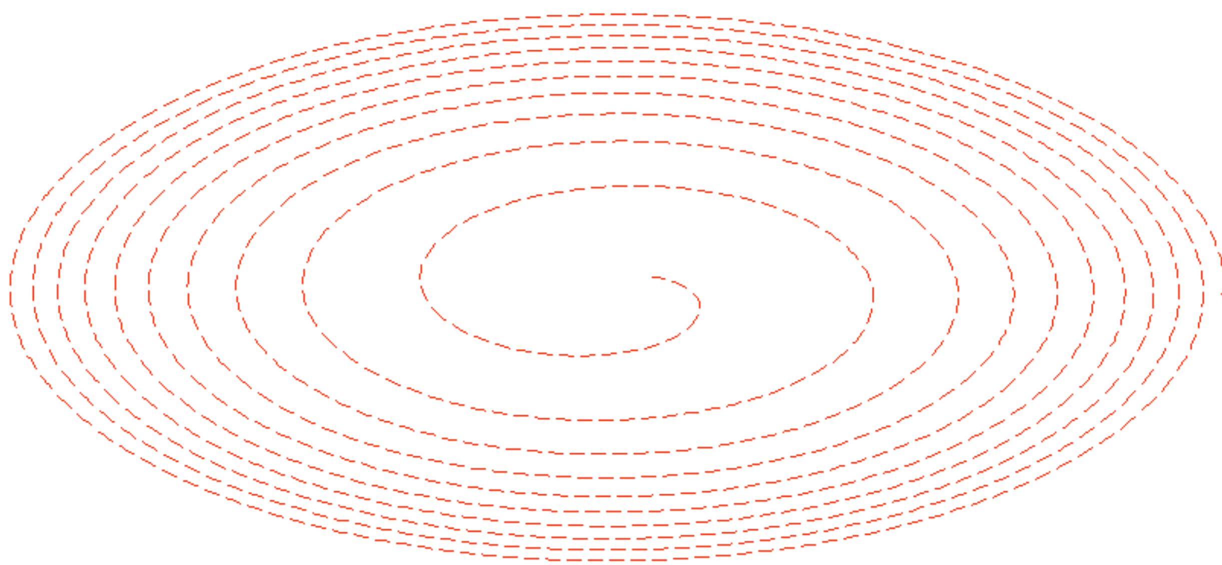
NDRP, arXiv:1506.08457

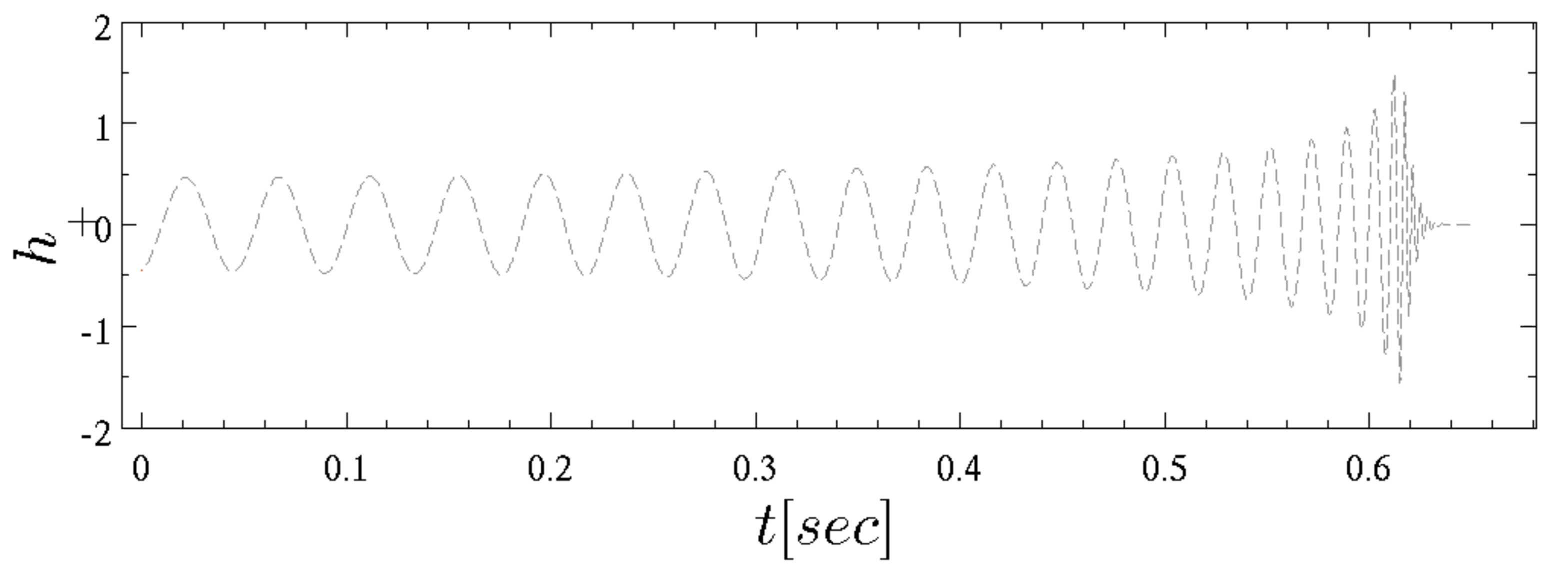
RESULTS: EOBNR/NR WAVEFORMS (NO SPIN)



Nagar, Damour, Reisswig & Pollney, PRD 93 (2016), 04404

equal-mass case





2
1
0
-1
-2
0
0.1
0.2
0.3
0.4
0.5
0.6
h_+
$t[sec]$

HIGHER MODES (NO SPIN)

- Unpublished, but free to download at eob.ihes.fr (Matlab code)
- Check unfaithfulness vs NR surrogate

(G. Pratten & AN, 2016 in preparation)

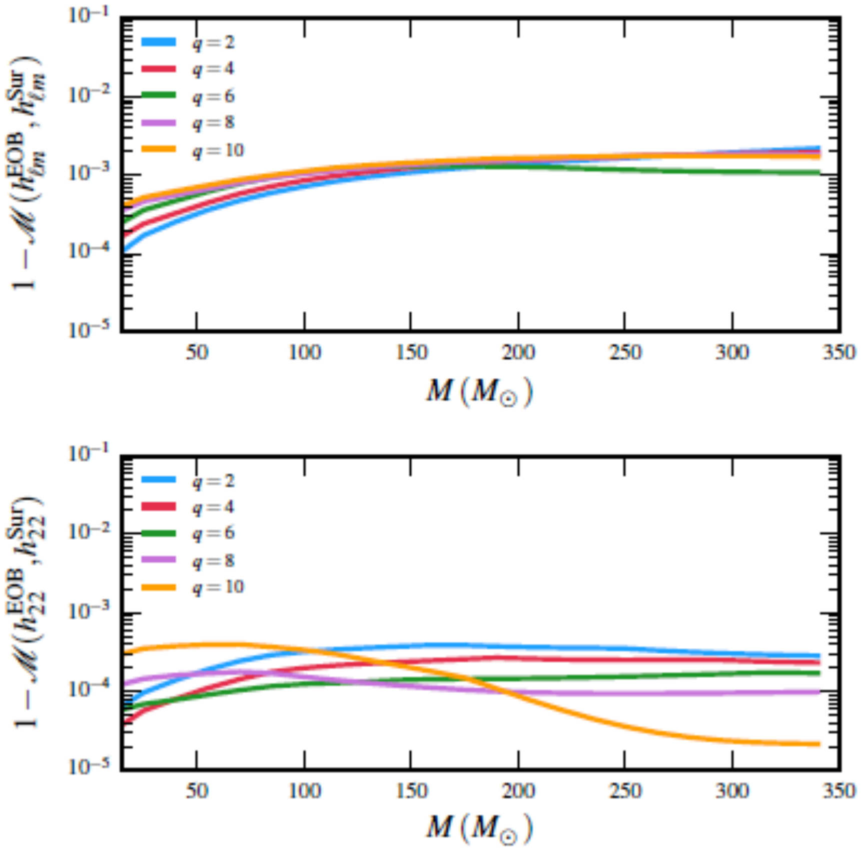
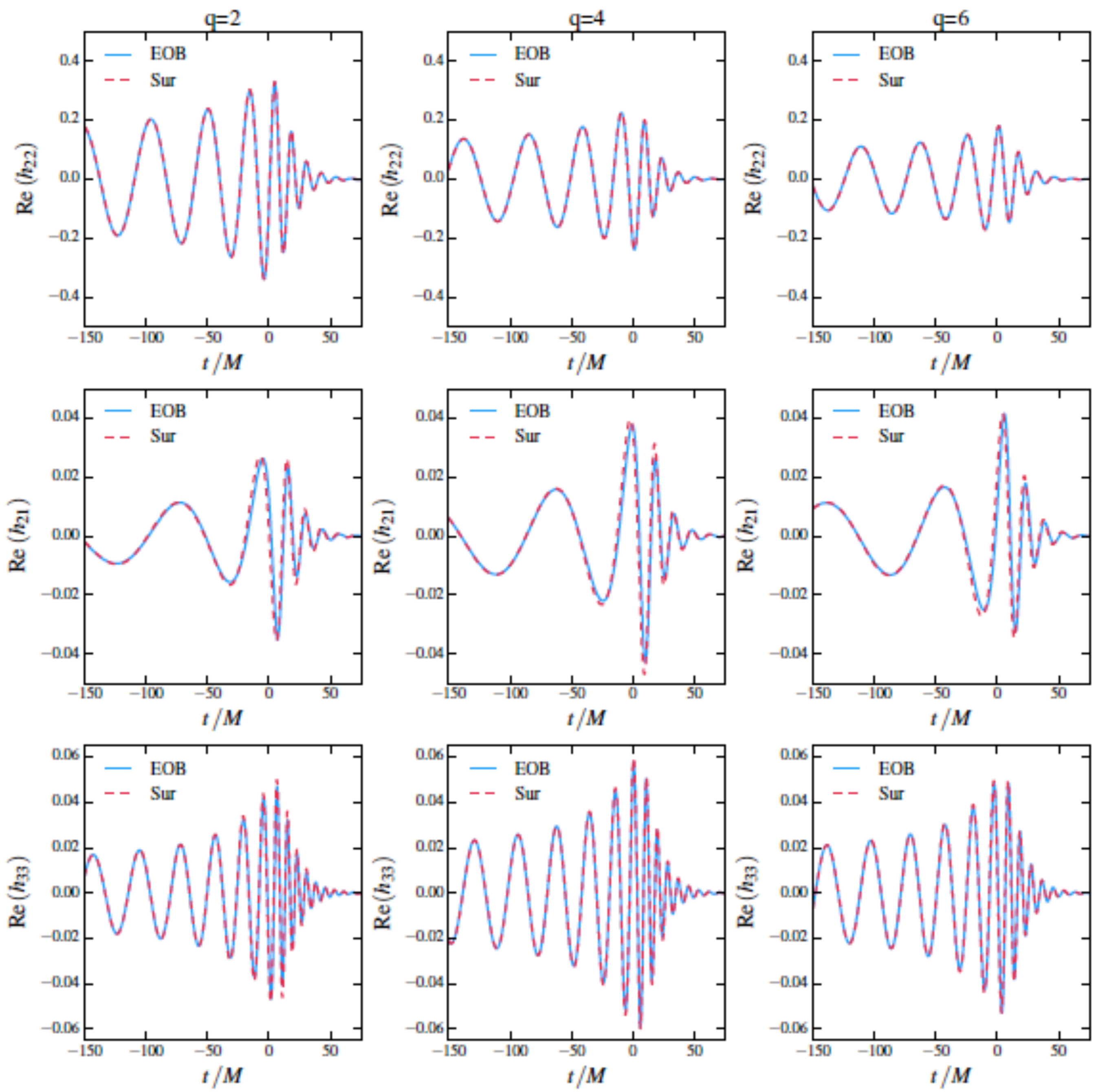


FIG. 4. Unfaithfulness for the EOB waveforms against surrogate waveforms as a function of the total mass of the system M for mass ratios $q = 2, 4, 6, 8$ and 10 assuming $\theta = \phi = \pi/3$. The top plot shows a comparison of mutlimodal waveforms constructed from (22), (21) and (33). The bottom plot shows a comparison for waveforms constructed from just the (22) modes.



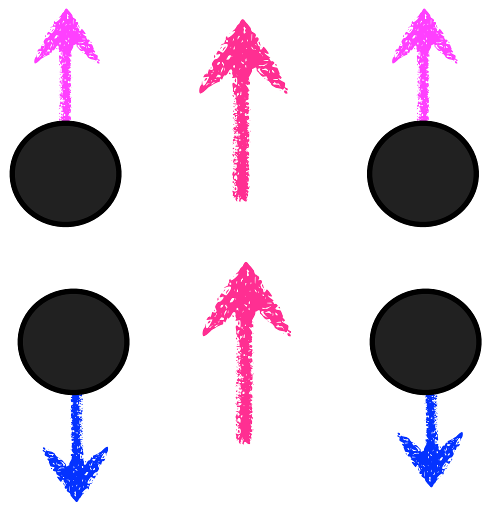
SPINNING BBHS

Spin-orbit & spin-spin couplings

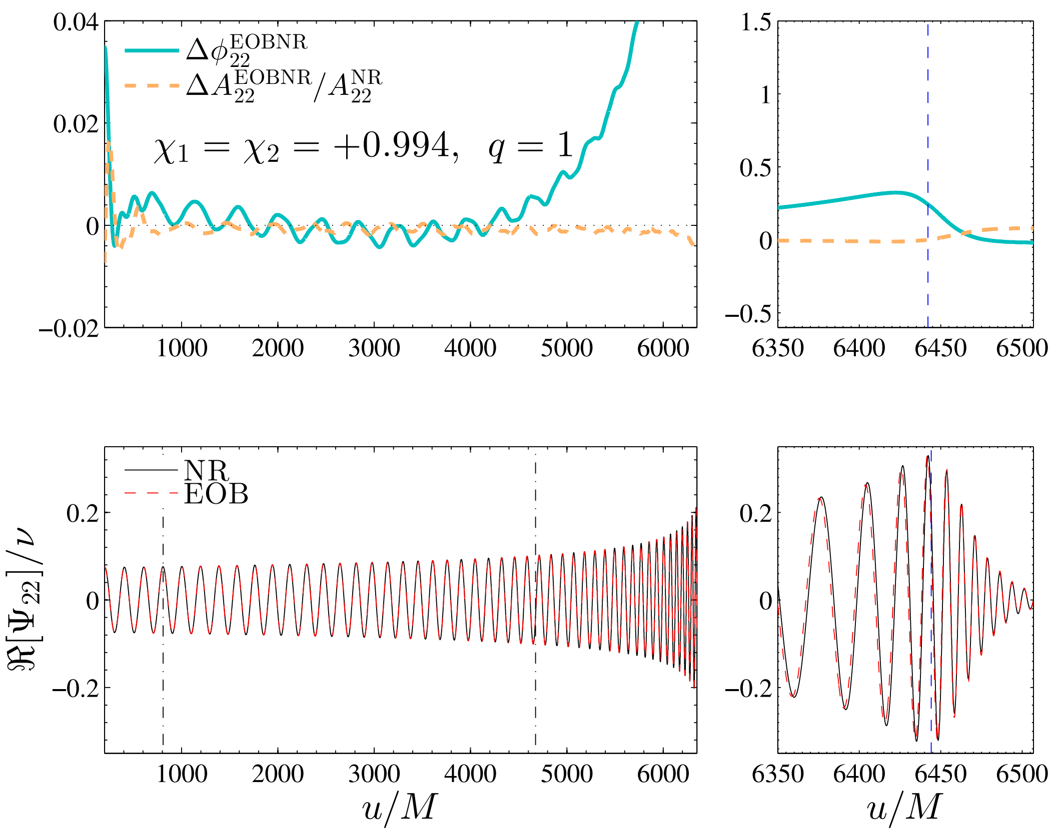
(i) Spins aligned with L: repulsive (slower) L-o-n-g-e-r INSPIRAL

(ii) Spins anti-aligned with L: attractive (faster) Shorter INSPIRAL

(iii) Misaligned spins: precession of the orbital plane (waveform modulation)



$$\chi_{1,2} = \frac{c\,\mathbf{S}_{1,2}}{Gm_{1,2}^2}$$



EOB/NR agreement: sophisticated (though rather simple) model for spin-aligned binaries

Damour&Nagar, PRD90 (2014), 024054 (Hamiltonian)
Damour&Nagar, PRD90 (2014), 044018 (Ringdown)
Nagar,Damour, Reisswig & Pollney, PRD 93 (2016), 044046

AEI model, SEOBNRv4, Bohe et al., arXiv:1611.03703v1 (PRD in press)

SO & SS EFFECTS IN EOB HAMILTONIAN

New way of combining available knowledge within some Hamiltonian
[Damour&Nagar, PRD 2014]

$$\hat{H}_{\rm eff} = \frac{g_S^{\rm eff}}{r^3}\mathbf{L}\cdot\mathbf{S} + \frac{g_{S^*}^{\rm eff}}{r^3}\mathbf{L}\cdot\mathbf{S}^* + \sqrt{A(1+\gamma^{ij}p_i p_j + Q_4(p))}$$

with the structure

$$g_S^{\rm eff} = 2 + \nu(\text{PN corrections}) + (\text{spin})^2\text{corrections}$$

$$g_{S^*}^{\rm eff} = \left(\frac{3}{2} + \text{test mass coupling}\right) + \nu(\text{PN corrections}) + (\text{spin})^2\text{corrections}$$

$$A = 1 - \frac{2}{r} + \nu(\text{PN corrections}) + (\text{spin})^2\text{corrections}$$

$$\gamma^{ij} = \gamma^{ij}_{\rm Kerr} + \nu(\text{PN corrections}) + \dots$$

$$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 = M^2(X_1^2\chi_1 + X_2^2\chi_2)$$

$$X_i = m_i/M$$

$$\mathbf{S}^* = \frac{m_2}{m1}\mathbf{S}_1 + \frac{m_1}{m_2}\mathbf{S}_2 = M^2\nu(\chi_1+\chi_2)$$

$$-1 \le \chi_i \le 1$$

THE TWO TYPES OF SPIN-ORBIT COUPLINGS

$$\hat{H}^{\rm eff}_{\rm SO} = G_S \mathbf{L}\cdot\mathbf{S} + G_{S*}\mathbf{L}\cdot\mathbf{S}^* \qquad G_S = \frac{1}{r^3} g_S^{\rm eff}, \quad G_{S*} = \frac{1}{r^3} g_{S*}^{\rm eff}$$

In the Kerr limit, only S-type gyro-gravitomagnetic ratio enters:

$$g_S^{\rm eff} = 2\frac{r^2}{r^2 + a^2\left[(1-\cos^2\theta)\left(1+\frac{2}{r}\right)+2\cos^2\theta\right]+\frac{a^4}{r^2}\cos^2\theta} = 2 + \mathcal{O}[({\rm spin})^2]$$

PN calculations yield (in some spin gauge)[DJS08, Hartung&Steinhoff11,Nagar11,Barausse&Buonanno11]

“Effective” NNNLO SO-coupling

$$g_S^{\rm eff} = 2 + \frac{1}{c^2}\left\{-\frac{1}{r}\frac{5}{8}\nu - \frac{33}{8}(\mathbf{n}\cdot\mathbf{p})^2\right\}$$
$$+\frac{1}{c^4}\left\{-\frac{1}{r^2}\left(\frac{51}{4}\nu + \frac{\nu^2}{8}\right) + \frac{1}{r}\left(-\frac{21}{2}\nu + \frac{23}{8}\nu^2\right)(\mathbf{n}\cdot\mathbf{p})^2 + \frac{5}{8}\nu(1+7\nu)(\mathbf{n}\cdot\mathbf{p})^4\right\}, \quad +\frac{1}{c^6}\frac{\nu c_3}{r^3}$$

$$g_{S^*}^{\rm eff} = \frac{3}{2} + \frac{1}{c^2}\left\{-\frac{1}{r}\left(\frac{9}{8}+\frac{3}{4}\nu\right) - \left(\frac{9}{4}\nu + \frac{15}{8}\right)(\mathbf{n}\cdot\mathbf{p})^2\right\}$$
$$+\frac{1}{c^4}\left\{-\frac{1}{r^2}\left(\frac{27}{16}+\frac{39}{4}\nu+\frac{3}{16}\nu^2\right) + \frac{1}{r}\left(\frac{69}{16} - \frac{9}{4}\nu + \frac{57}{16}\nu^2\right)(\mathbf{n}\cdot\mathbf{p})^2 + \left(\frac{35}{16}+\frac{5}{2}\nu+\frac{45}{16}\nu^2\right)(\mathbf{n}\cdot\mathbf{p})^4\right\} + \frac{1}{c^6}\frac{\nu c_3}{r^3}$$

The NR-informed effective parameter makes the spin-orbit coupling stronger or weaker with respect to the simple analytical prediction

40 NR SXS Datasets (public in the fall of 2013 and used before for SEOBNRv2)

TABLE I: EOB/NR phasing comparison. The columns report: the number of the dataset; the name of the configuration in the SXS catalog; the mass ratio $q = m_1/m_2$; the symmetric mass ratio ν; the dimensionless spins χ_1 and χ_2; the phase difference $\Delta\phi^{\rm EOBNR} \equiv \phi^{\rm EOB} - \phi^{\rm NR}$ computed at NR merger; the NR phase uncertainty at NR merger $\delta\phi^{\rm NR}_{\rm mrg}$ (when available) measured taking the difference between the two highest resolution levels (see text); the maximum value of the unfaithfulness $\bar{F} \equiv 1 - F$ as per Eq. (22). The $\Delta\phi^{\rm EOBNR}$'s in brackets for $\chi_1 = \chi_2 > +0.85$ were obtained using Eq. (21) for $\Delta t^{\rm NQC}(\chi)$.

#	Name	N orbits	q	ν	χ_1	χ_2	$\Delta\phi^{\rm EOBNR}_{\rm mrg}$ [rad]	$\delta\phi^{\rm NR}_{\rm mrg}$ [rad]	max($\bar{F}$)
1	SXS:BBH:none	14	1	0.25	0.0	0.0	−0.016	...	0.00087
2	SXS:BBH:0066	28	1	0.25	0.0	0.0	+0.010	...	0.00068
3	SXS:BBH:0002	32.42	1	0.25	0.0	0.0	+0.073	0.066	0.00101
4	SXS:BBH:0007	29.09	1.5	0.24	0	0	+0.05	0.018	0.00201
5	SXS:BBH:0169	15.68	2	$0.\bar{2}$	0	0	−0.15	0.02	0.00045
6	SXS:BBH:0030	18.22	3	0.1875	0	0	−0.074	0.087	0.00035
7	SXS:BBH:0167	15.59	4	0.16	0	0	−0.059	0.52	0.00035
8	SXS:BBH:0056	28.81	5	$0.13\bar{8}$	0	0	−0.089	0.44	0.00038
9	SXS:BBH:0166	21.56	6	0.1224	0	0	−0.198	...	0.00037
10	SXS:BBH:0063	25.83	8	0.0987	0	0	−0.453	1.01	0.00292
11	SXS:BBH:0185	24.91	9.98911	0.0827	0	0	−0.0051	0.376	0.00066
12	SXS:BBH:0004	30.19	1	0.25	−0.50	0.0	−0.017	0.068	0.00403
13	SXS:BBH:0005	30.19	1	0.25	+0.50	0.0	+0.08	0.28	0.00052
14	SXS:BBH:0156	12.42	1	0.25	−0.95	−0.95	+0.32	2.17	0.00058
15	SXS:BBH:0159	12.67	1	0.25	−0.90	−0.90	+0.06	0.38	0.00047
16	SXS:BBH:0154	13.24	1	0.25	−0.80	−0.80	+0.11	...	0.00044
17	SXS:BBH:0151	14.48	1	0.25	−0.60	−0.60	−0.049	0.14	0.00042
18	SXS:BBH:0148	15.49	1	0.25	−0.44	−0.44	+0.14	0.72	0.00043
19	SXS:BBH:0149	17.12	1	0.25	−0.20	−0.20	+0.45	0.90	0.00085
20	SXS:BBH:0150	19.82	1	0.25	+0.20	+0.20	+0.94	0.99	0.00275
21	SXS:BBH:0152	22.64	1	0.25	+0.60	+0.60	+0.01	0.36	0.00068
22	SXS:BBH:0155	24.09	1	0.25	+0.80	+0.80	−0.39	0.26	0.00110
23	SXS:BBH:0153	24.49	1	0.25	+0.85	+0.85	+0.06	...	0.00059
24	SXS:BBH:0160	24.83	1	0.25	+0.90	+0.90	+0.41 (+0.41)	0.80	0.00117
25	SXS:BBH:0157	25.15	1	0.25	+0.95	+0.95	+0.37 (+0.83)	1.18	0.00295
26	SXS:BBH:0158	25.27	1	0.25	+0.97	+0.97	+0.37 (+0.49)	1.26	0.00325
27	SXS:BBH:0172	25.35	1	0.25	+0.98	+0.98	+0.99 (+0.46)	2.02	0.00422
28	SXS:BBH:0177	25.40	1	0.25	+0.99	+0.99	+0.22 (+0.48)	0.40	0.00507
29	SXS:BBH:0178	25.43	1	0.25	+0.994	+0.994	+0.24 (+0.23)	−0.53	0.00506
30	SXS:BBH:0013	23.75	1.5	0.24	+0.5	0	+0.31	...	0.00058
31	SXS:BBH:0014	22.63	1.5	0.24	−0.5	0	−0.15	0.15	0.00046
32	SXS:BBH:0162	18.61	2	$0.\bar{2}$	+0.6	0	−0.20	0.71	0.00027
33	SXS:BBH:0036	31.72	3	0.1875	−0.5	0	+0.08	0.065	0.00040
34	SXS:BBH:0031	21.89	3	0.1875	+0.5	0	+0.12	0.034	0.00023
35	SXS:BBH:0047	22.72	3	0.1875	+0.5	+0.5	−0.034	...	0.00030
36	SXS:BBH:0046	14.39	3	0.1875	−0.5	−0.5	+0.36	...	0.00054
37	SXS:BBH:0110	24.24	5	$0.13\bar{8}$	+0.5	0	+0.24	...	0.00016
38	SXS:BBH:0060	23.17	5	$0.13\bar{8}$	−0.5	0	+0.21	0.8	0.00034
39	SXS:BBH:0064	19.16	8	0.0987	−0.5	0	+0.026	0.8	0.00042
40	SXS:BBH:0065	33.97	8	0.0987	+0.5	0	+1.33	−3.0	0.00040

Several equal-mass, equal-spin data

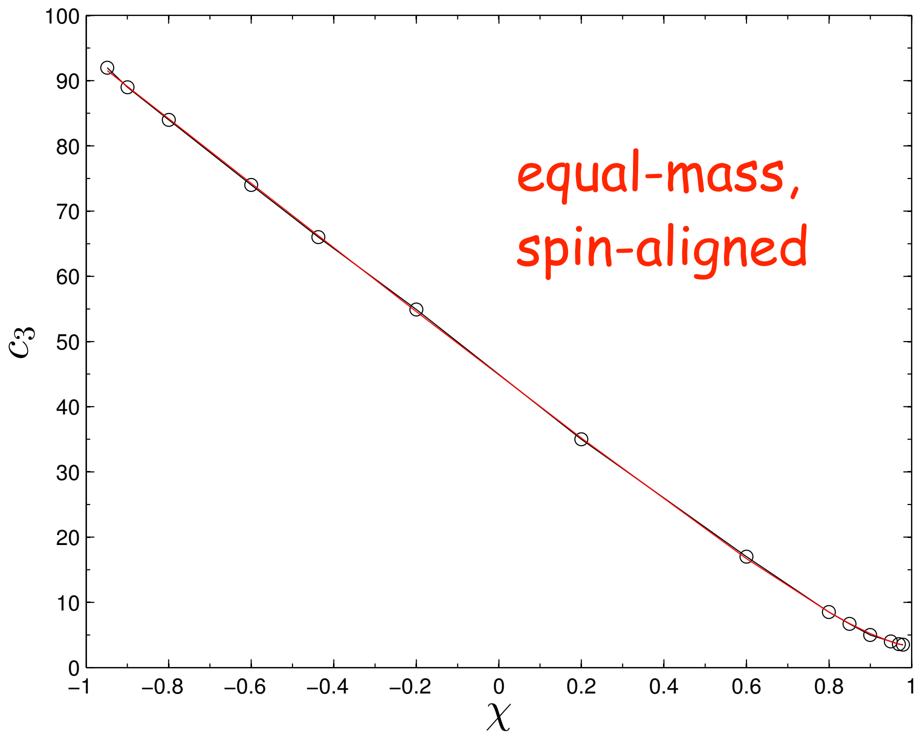
Just a few unequal-mass, unequal-spin data

SPIN-ORBIT NR INFORMATION

Procedure:

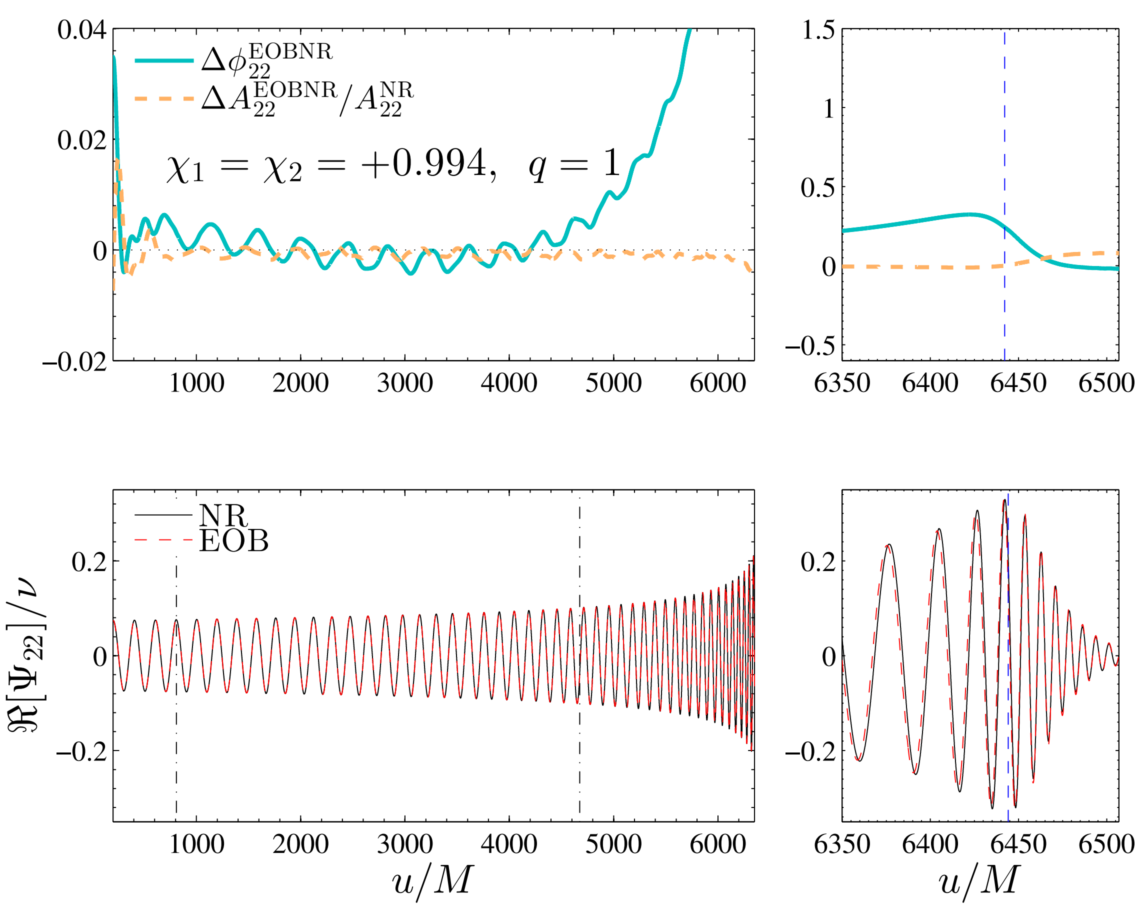
(i) align waveforms in the early inspiral;

(ii) tune the parameter to have phase difference compatible with the NR uncertainty



Quasi-linear function of the spins

$$c_3(\tilde{a}_1, \tilde{a}_2, \nu) = p_0 \frac{1 + n_1(\tilde{a}_1 + \tilde{a}_2) + n_2(\tilde{a}_1 + \tilde{a}_2)^2}{1 + d_1(\tilde{a}_1 + \tilde{a}_2)}$$
$$+ (p_1\nu + p_2\nu^2 + p_2\nu^3)(\tilde{a}_1 + \tilde{a}_2)\sqrt{1 - 4\nu}$$
$$+ p_4(\tilde{a}_1 - \tilde{a}_2)\nu^2,$$



+ interpolating fits for NQC functioning point, ringdown coefficients etc. (Achille's heel...still ok..)

$$\tilde{a}_{1,2} = X_{1,2}\chi_{1,2}$$

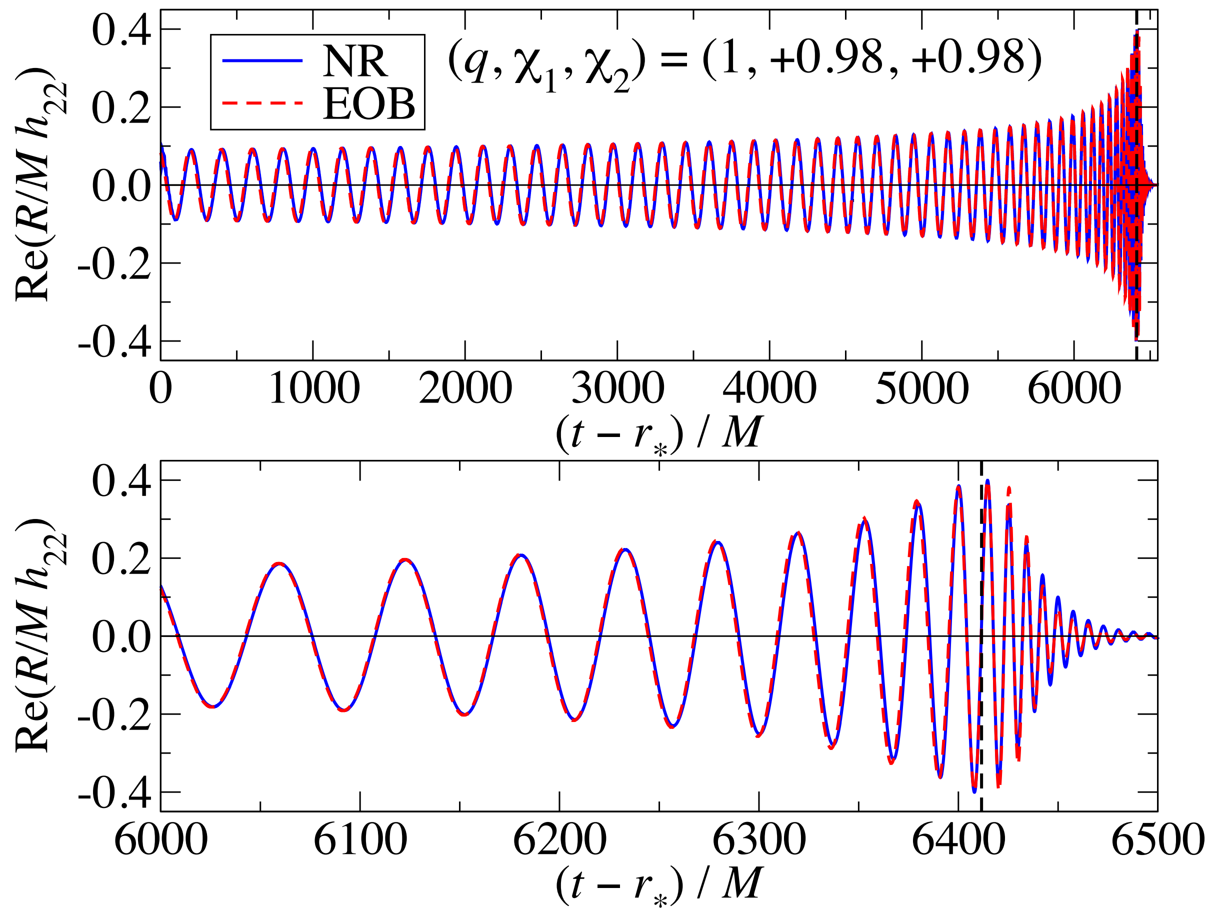
$$X_{1,2} \equiv \frac{m_{1,2}}{M}$$

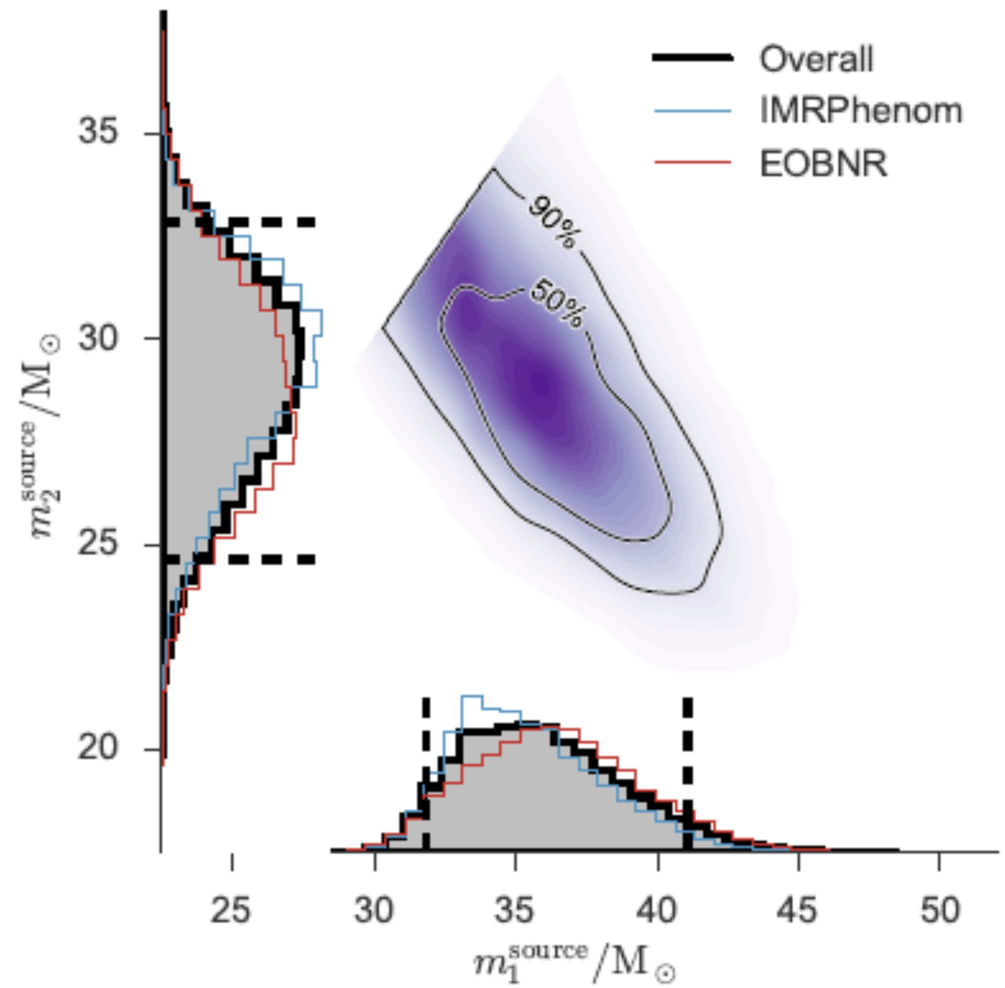
EOBNR MODEL USED FOR GW150914

Different EOB Hamiltonian [Barausse & Buonanno11, Taracchini et al.12]

SEOBNRv2: Taracchini, Buonanno et al., PRD 89, 061502 (R), 2014

SEOBNRv2_ROM_DoubleSpin: M. Puerrer, CQG 31, 195010 (2014)





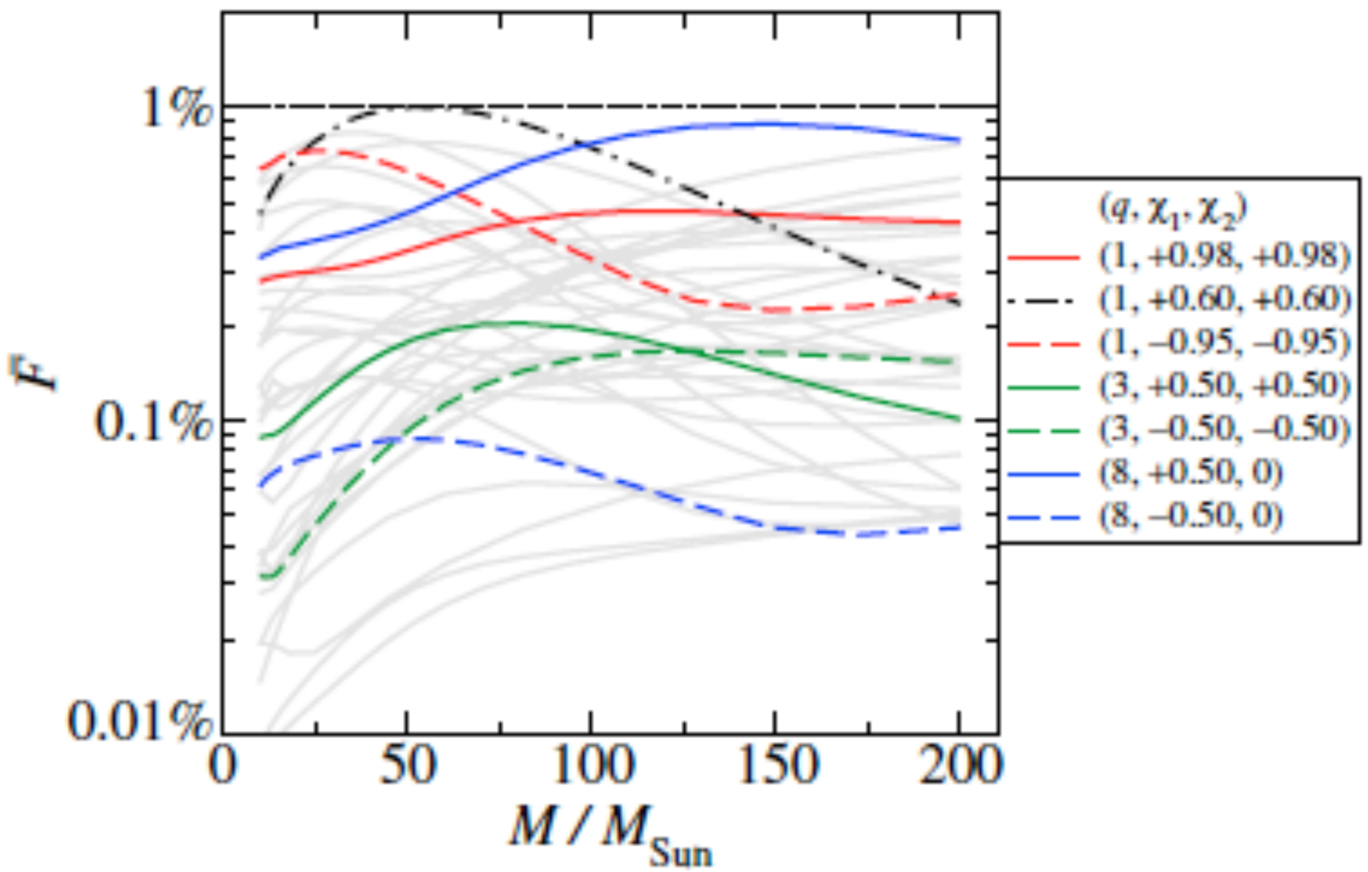
Effectively used to get the masses:
SEOBNRv2_ROM_DoubleSpin
IMRPhenom (Khan et al., 2015)

just AFTER, the best choices
were cross checked with NR simulations!

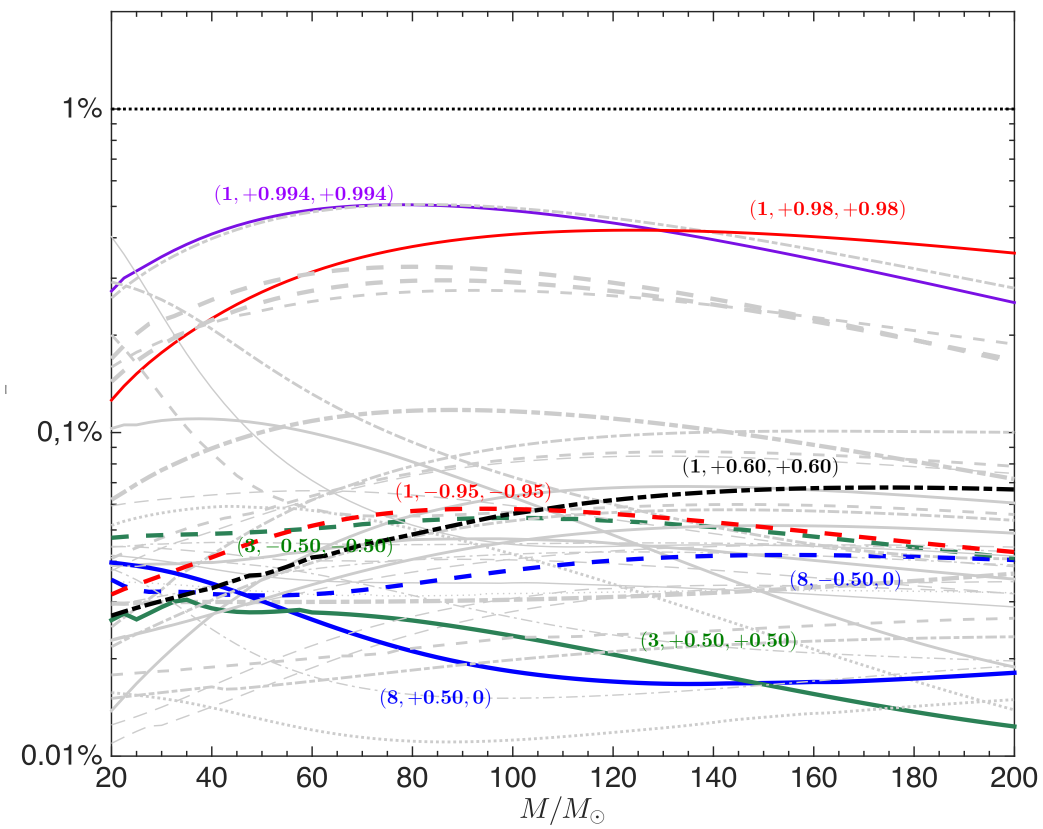
IHES EOBNR MODEL

Best existing EOBNR model WAS NOT used for parameter estimation:
EOB/EOBNR UNFAITHFULNESS (40 NR SXS dataset)

SEOBNRv2



IHESEOB_spin

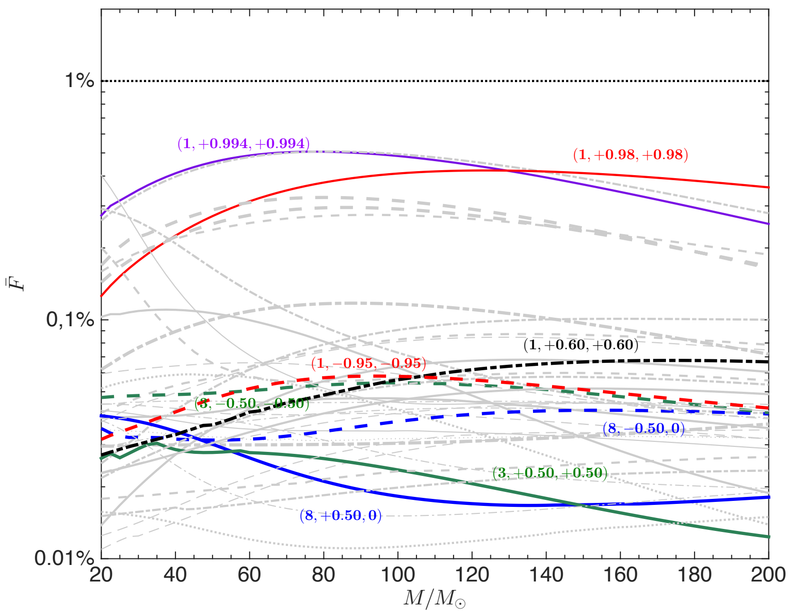


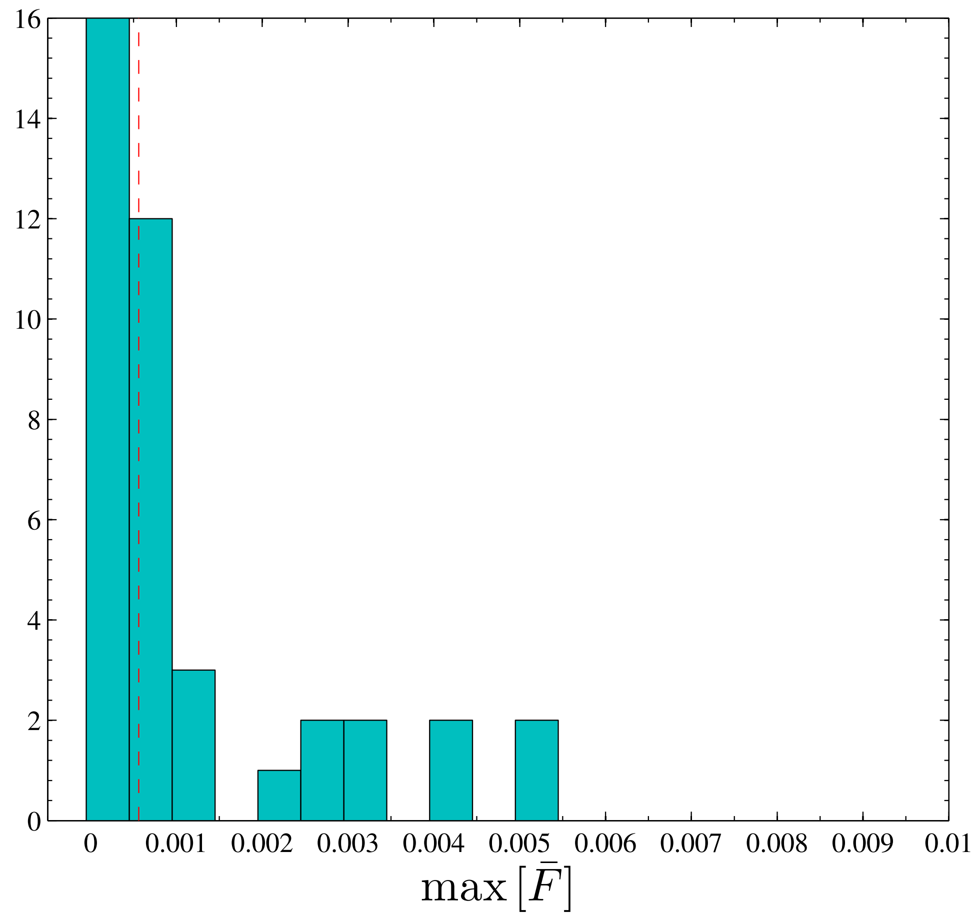
$$\bar{F} \equiv 1 - \max_{t_0,\phi_0} \frac{\langle h_{22}^{\rm EOB}, h_{22}^{\rm NR}\rangle}{||h_{22}^{\rm EOB}||\,||h_{22}^{\rm NR}||}$$

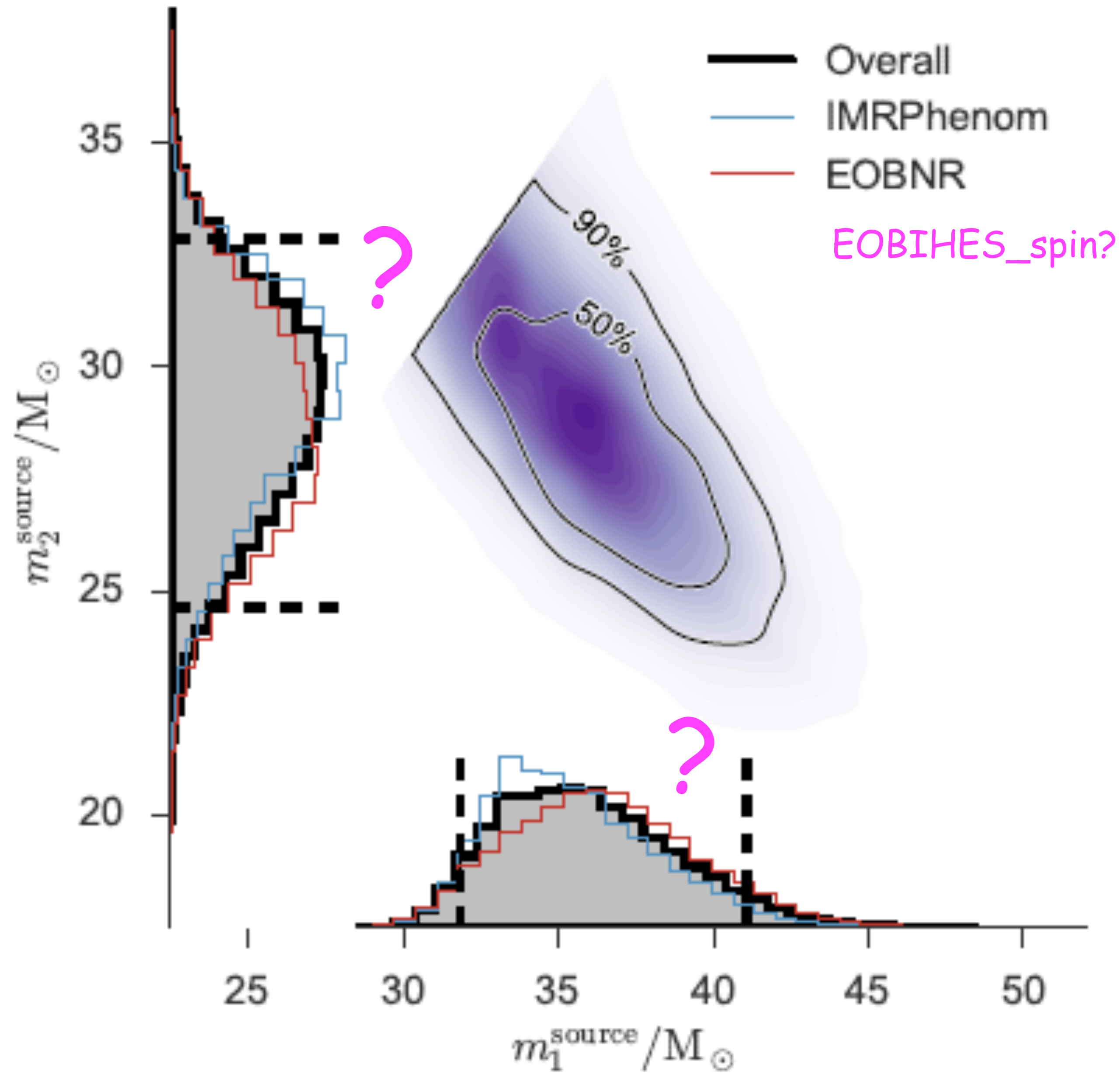
$$\langle h_1, h_2\rangle \equiv 4\Re \int_{f_{\rm min}}^{\infty} \tilde{h}_1(f)\tilde{h}_2^*(f)/S_n(f)\, df$$

Nagar,Damour, Reisswig & Pollney, PRD 93 (2016), 044046

FIRST QUESTION: MEASURING PARAMETERS

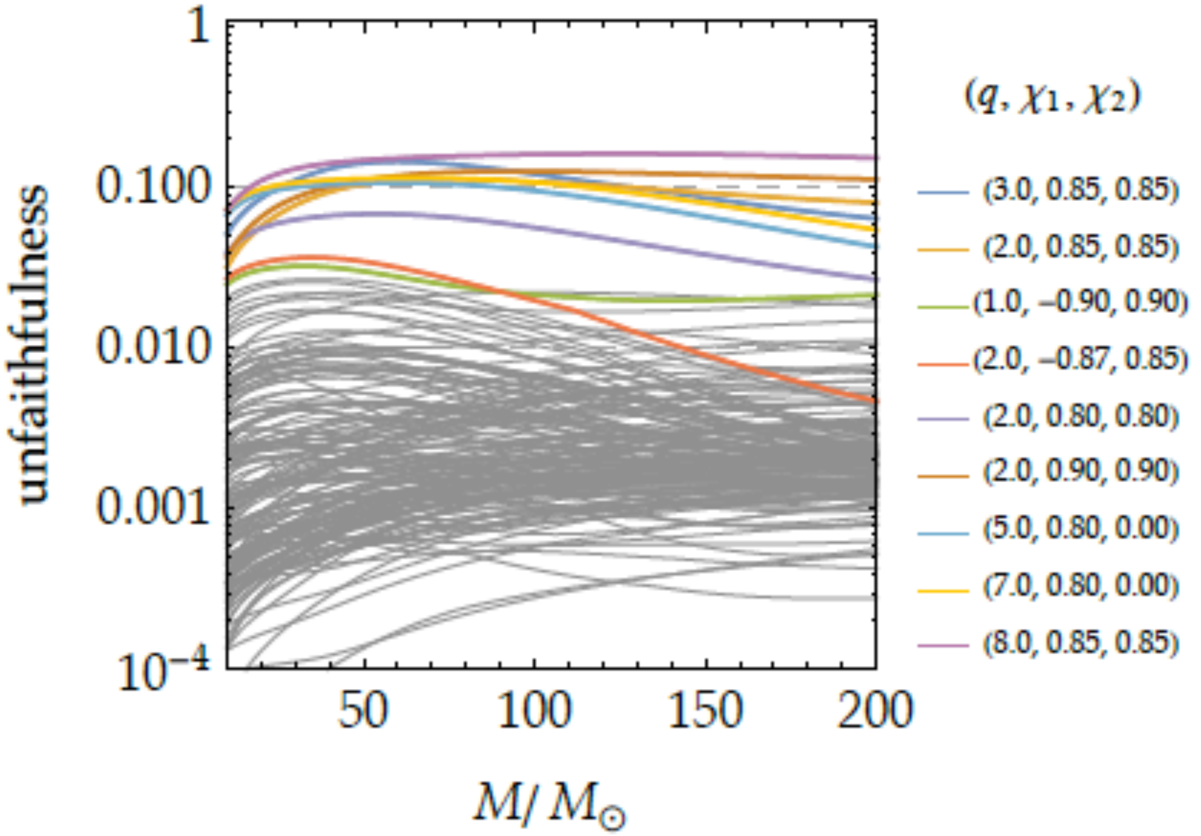




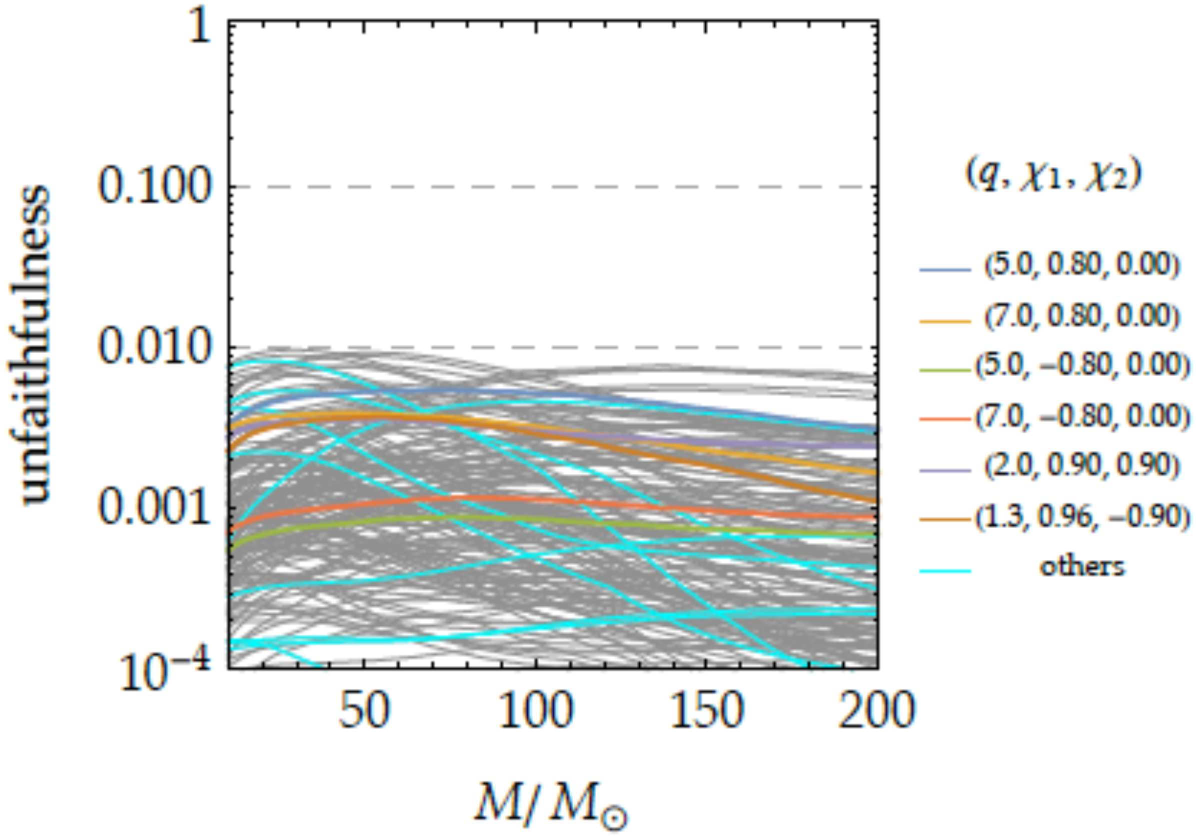


ROBUSTNESS?

SEOBNRv2



SEOBNRv4



grey: below 3%

AEI model: Bohe et al. arXiv: 1611.03703v1

4 parameters

Strong recalibration of the state-of-the-art SEOBNRv2 model (used for O1) to have it faithful towards a set of 141 NR simulations (about 100 new ones)

$$d_{SO} = +147.481449\chi^3\nu^2 - 568.651115\chi^3\nu + 66.198703\chi^3 - 343.313058\chi^2\nu + 2495.293427\chi\nu^2 - 44.532373\,,$$

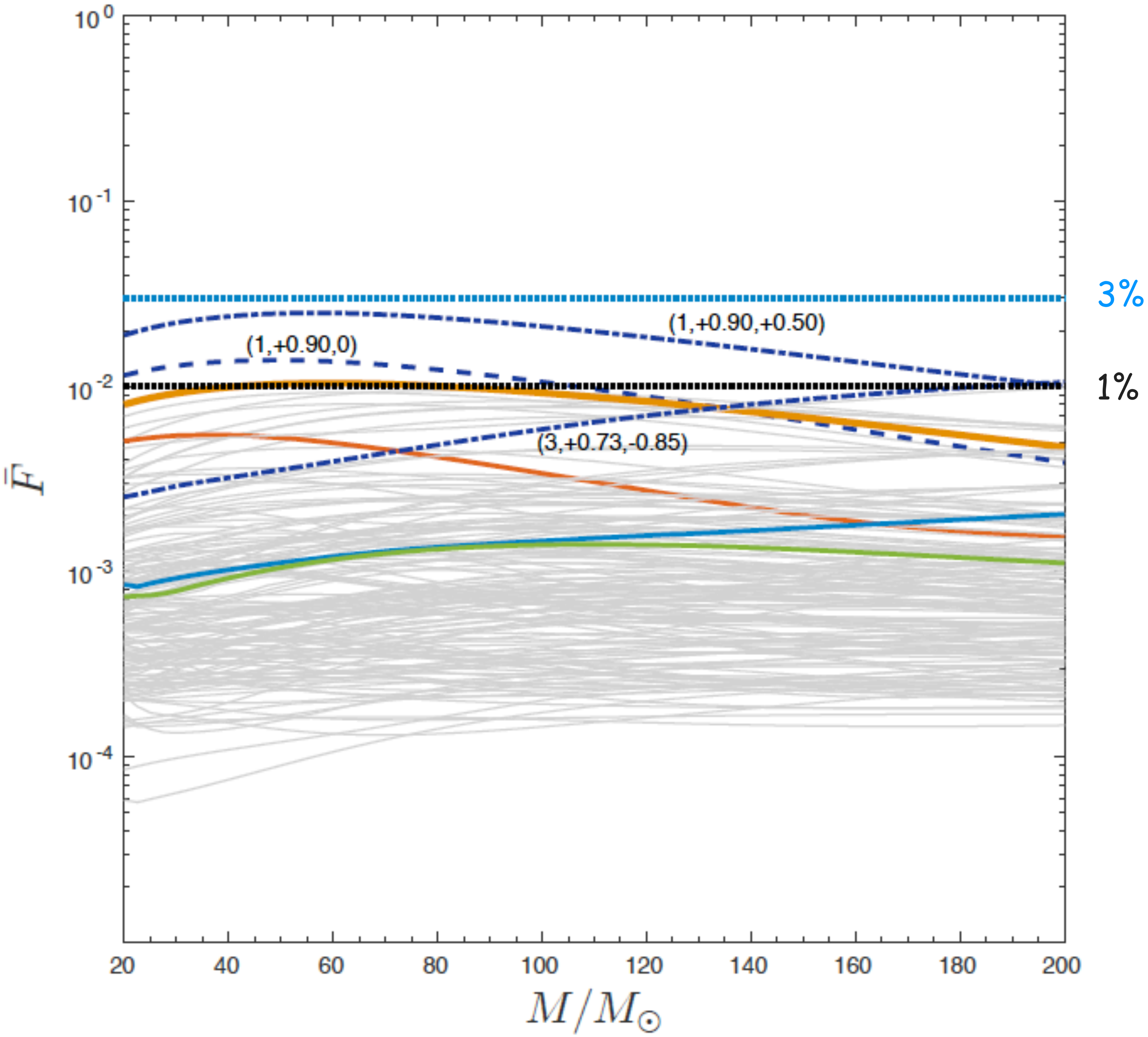
$$d_{SS} = +528.511252\chi^3\nu^2 - 41.000256\chi^3\nu + 1161.780126\chi^2\nu^3 - 326.324859\chi^2\nu^2 + 37.196389\chi\nu + 706.958312\nu^3 - 36.027203\nu + 6.068071\,,$$

More NR simulations seem essential to “calibrate & improve” the AEI EOBNR model

BUT THIS IS NOT GENERAL...

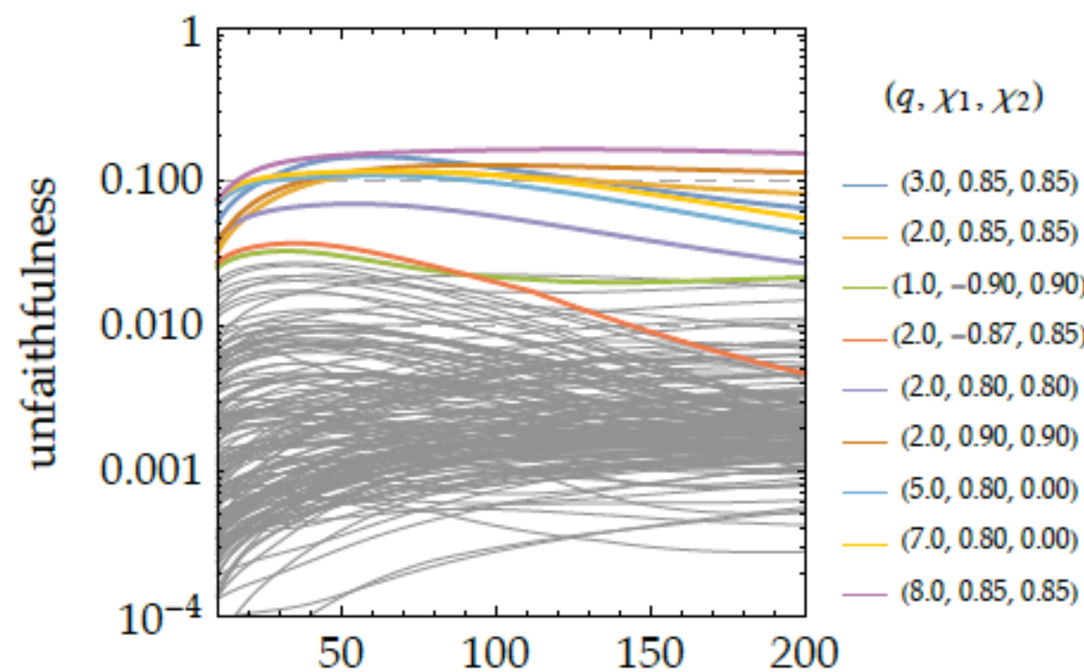
October 31st: 93 NR datasets released publicly. These are those used to calibrate SEOBNRv4 (+ others non public)
First use them to cross-check our model.

Interpolating NR fits for NQC point & ringdown. Previous NR data plus (5,-0.90,0)



Our EOBNR model is very robust and consistent ALSO outside the "information" domain over 93 new waveforms. Three outliers above 1% (but always below 3%).

Better performance than SEOBNRv2 with no need of further NR information

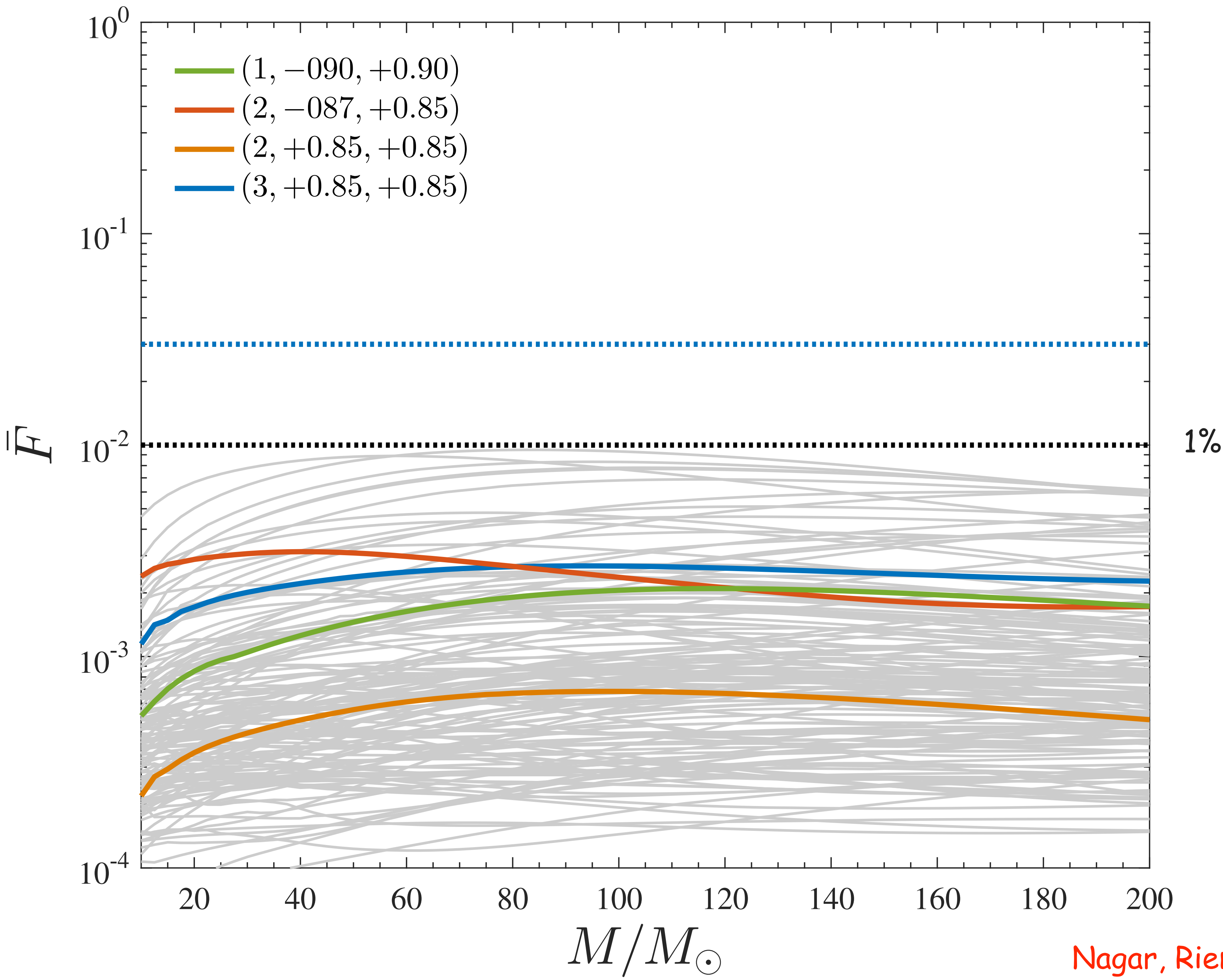


MINIMAL RECALIBRATION

Best value of the c3 parameter for the three outliers. Check phase agreement in the time-domain to be within the NR error bar. New fit to the best values to determine new values of the parameters of the unequal-mass sector.

Recalibration with 3 more NR datasets; 90 datasets as a cross/check.

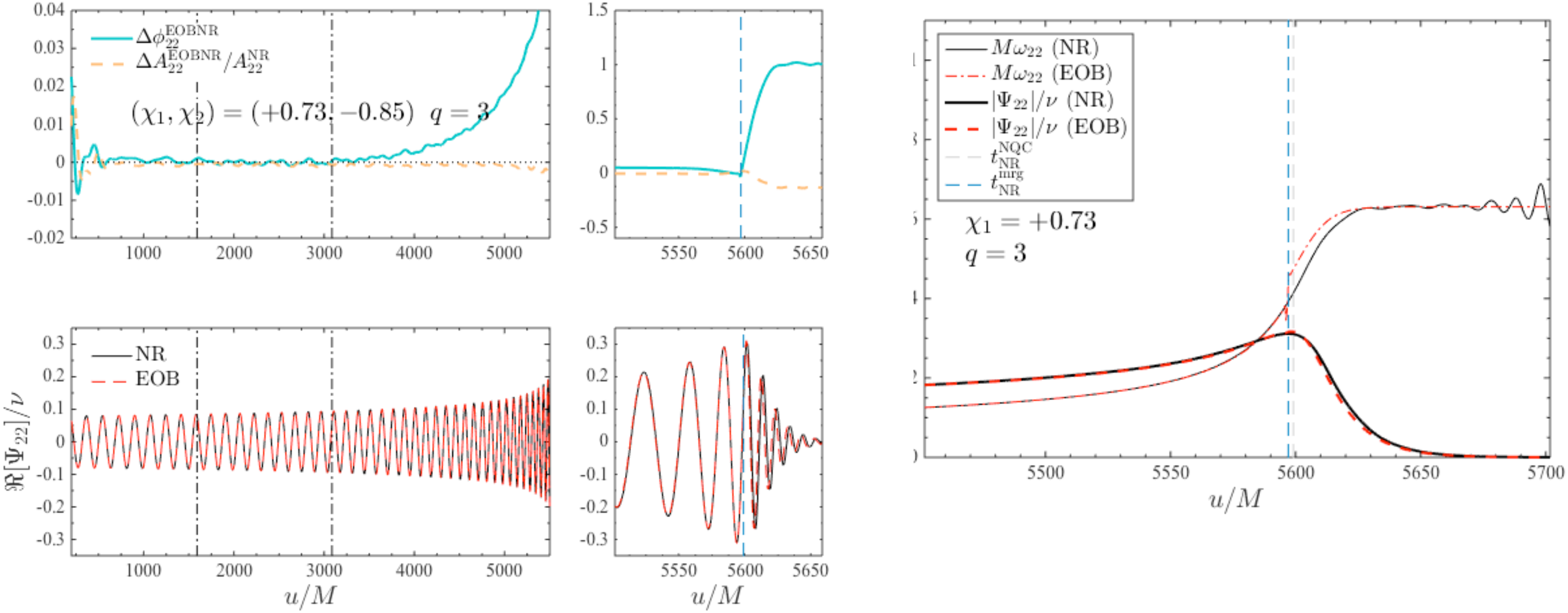
Done by hand, no need of sophisticated mechanisms/algorithms. IMPROVABLE: NQC & RINGDOWN FITS USING MORE NR DATA



(3,+0.85,+0.85)
(1,+0.90,-0.90)
(2,-0.87,+0.85)
(2,+0.85,+0.85)

Nagar, Riemenschneider & Pratten, 2017, in prep.

WHAT TO IMPROVE?



More NR data sets to be included both in the NQC-functioning-point fit as well as in the postmerger fit (see Del Pozzo & Nagar, arXiv:1606.03952). This is an easily solvable problem (in progress).

It is reasonable to aim at 0.1% level unfaithfulness. This is easily at reach of the model. More precise “calibration” and/or improved theoretical structures.

PRECESSION

Different EOB Hamiltonian [Barausse & Buonanno11, Taracchini et al.12]

SEOBNRv3: Taracchini, Buonanno et al., PRD 89, 061502 (R), 2014

Babak, Taracchini & Buonanno, 2016

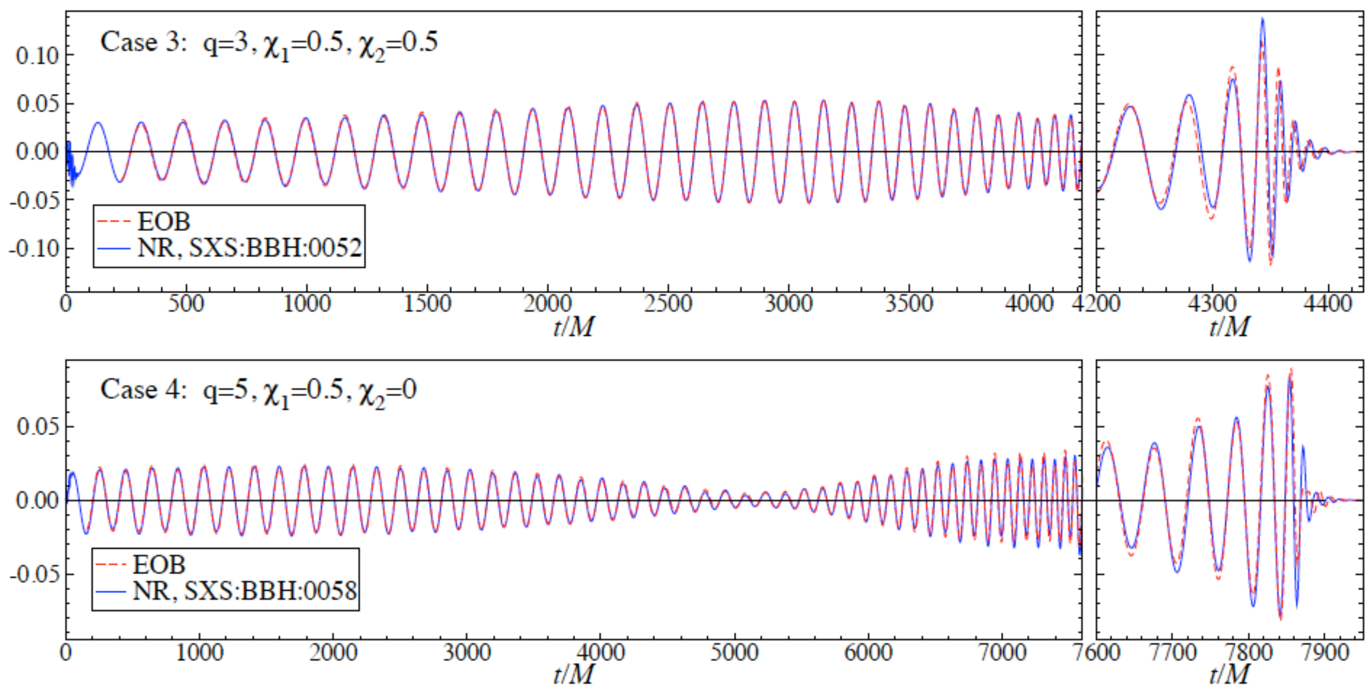


FIG. 9: We show for cases 3 and 4 of Table I the GW polarization h_+, containing contributions from $\ell = 2$ modes, that propagates along a direction $\hat{N}$ specified by spherical coordinates $\theta = \pi/3$ and $\phi = \pi/2$ associated with the inertial source frame $\{e_1^S, e_2^S, e_3^S\}$. The EOB waveforms start at the after-junk-radiation times of $t = 230M$ and $t = 160M$, respectively.

Good EOBNR/NR agreement.
The method works

Slow: analysis is time-consuming

Improvements in the implementation are needed

PhenomP: P. Schmidt et al. 2012/2014

Phenomenological Precessing model that takes into account precession effects at leading order by “twisting” nonprecessing waveforms.

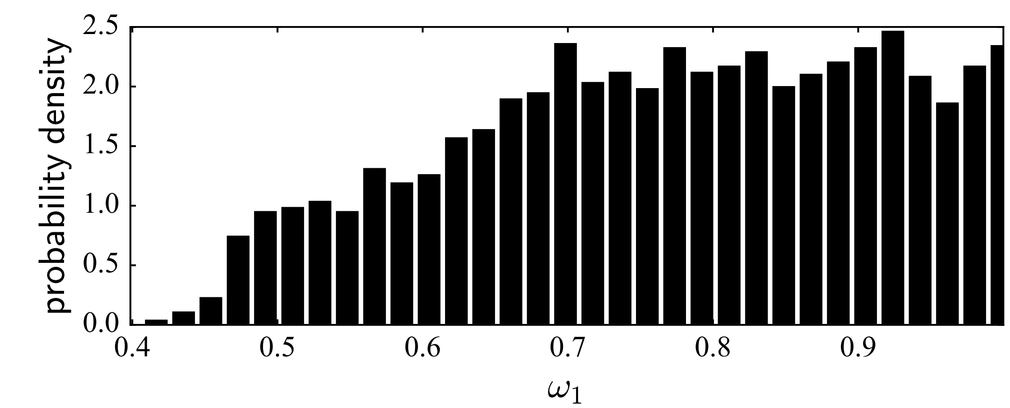
Conclusion: no precession could be really seen.

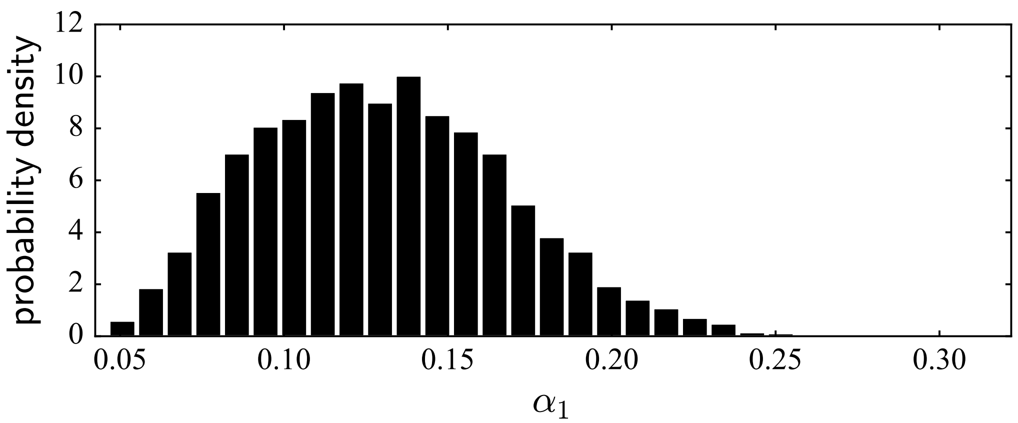
POSTMERGER DESCRIPTION

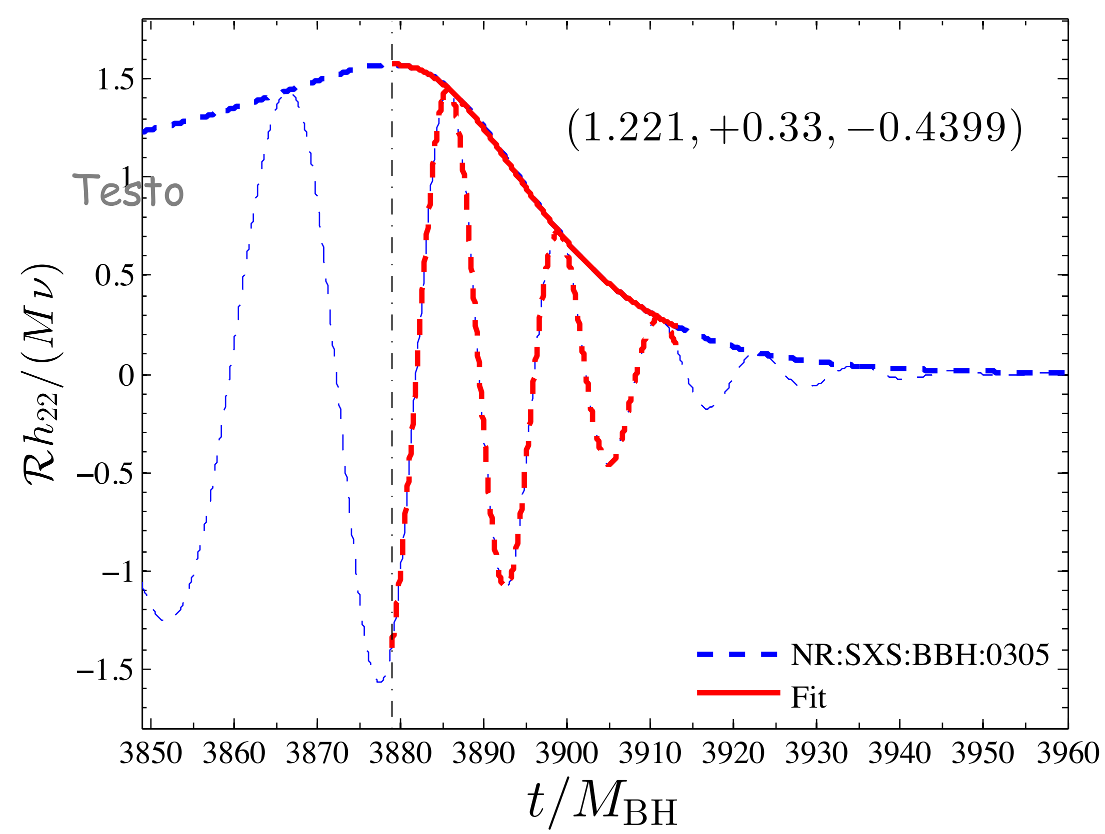
Damour&AN, PRD 2014: motivated because the “standard” QNMs attachment is far from trivial for high-spins
Originally conceived for EOB; useful also as a stand-alone postmerger template
Del Pozzo & AN, arXiv: 1606.03952

ANALYTIC TEMPLATE for the FULL POSTMERGER signal coming from a suitable fit of NR data.

$$\sigma_1 = \alpha_1 + i\omega_1$$







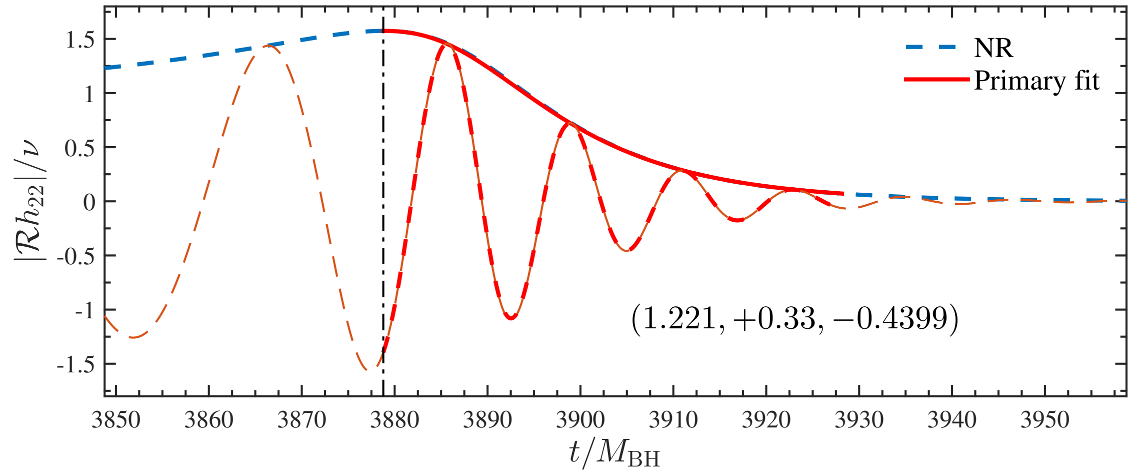
EFFECTIVE FIT

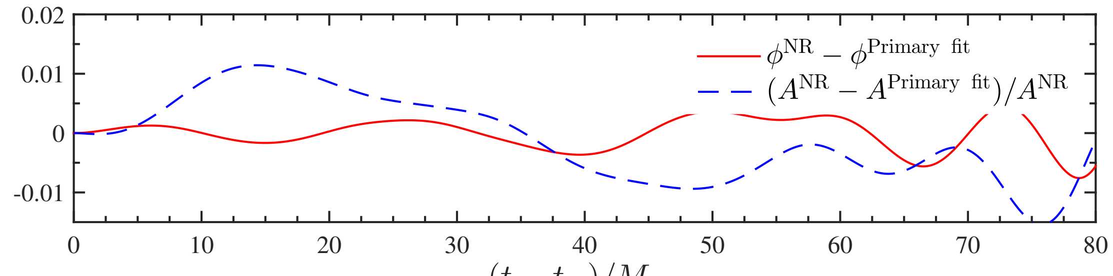
Damour&AN 2014

Factorize the fundamental QNM, fit what remains

$$h(\tau) = e^{\sigma_1 \tau - i\phi_0} \bar{h}(\tau)$$

$$\bar{h}(\tau) \equiv A_{\bar{h}} e^{i\phi_{\bar{h}}(\tau)}.$$





Good performance of primary fits (modulo details...)

$$A_{\bar{h}}(\tau) = c_1^A \tanh(c_2^A \tau + c_3^A) + c_4^A,$$

$$\phi_{\bar{h}}(\tau) = -c_1^\phi \ln\left(\frac{1 + c_3^\phi e^{-c_2^\phi \tau} + c_4^\phi e^{-2c_2^\phi \tau}}{1 + c_3^\phi + c_4^\phi}\right)$$

Do this for various SXS dataset and then build up a (simple-minded) interpolating fit

$$c_2^A = \frac{1}{2}\alpha_{21}, \qquad \alpha_{21} = \alpha_2 - \alpha_1$$

$$c_4^A = \hat{A}_{22}^{\rm mrg} - c_1^A \tanh(c_3^A),$$

$$c_1^A = \hat{A}_{22}^{\rm mrg}\, \alpha_1 \frac{\cosh^2(c_3^A)}{c_2^A},$$

$$c_1^\phi = \Delta\omega \frac{1 + c_3^\phi + c_4^\phi}{c_2^\phi(c_3^\phi + 2c_4^\phi)}, \qquad \Delta\omega \equiv \omega_1 - M_{\rm BH}\omega_{22}^{\rm mrg}$$

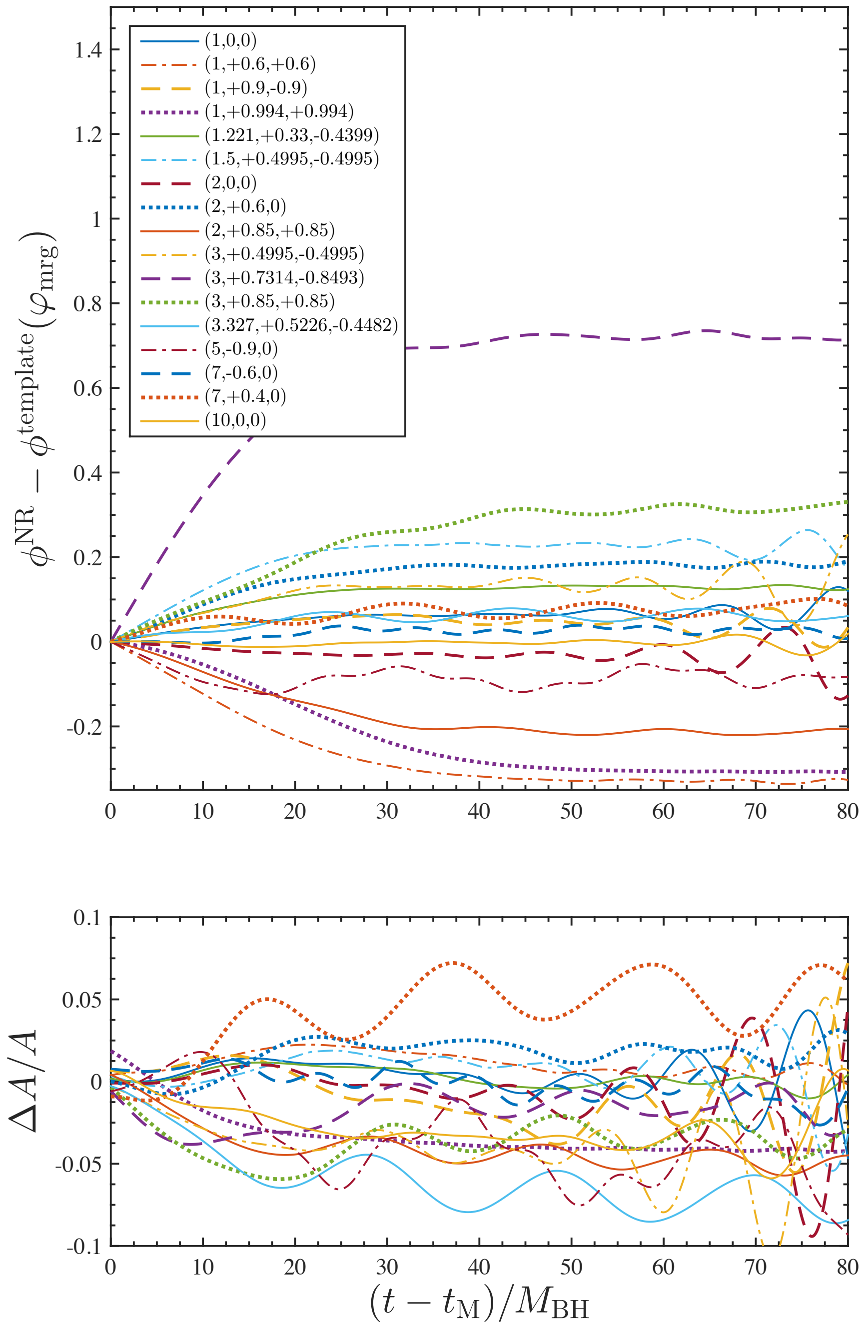
$$c_2^\phi = \alpha_{21},$$

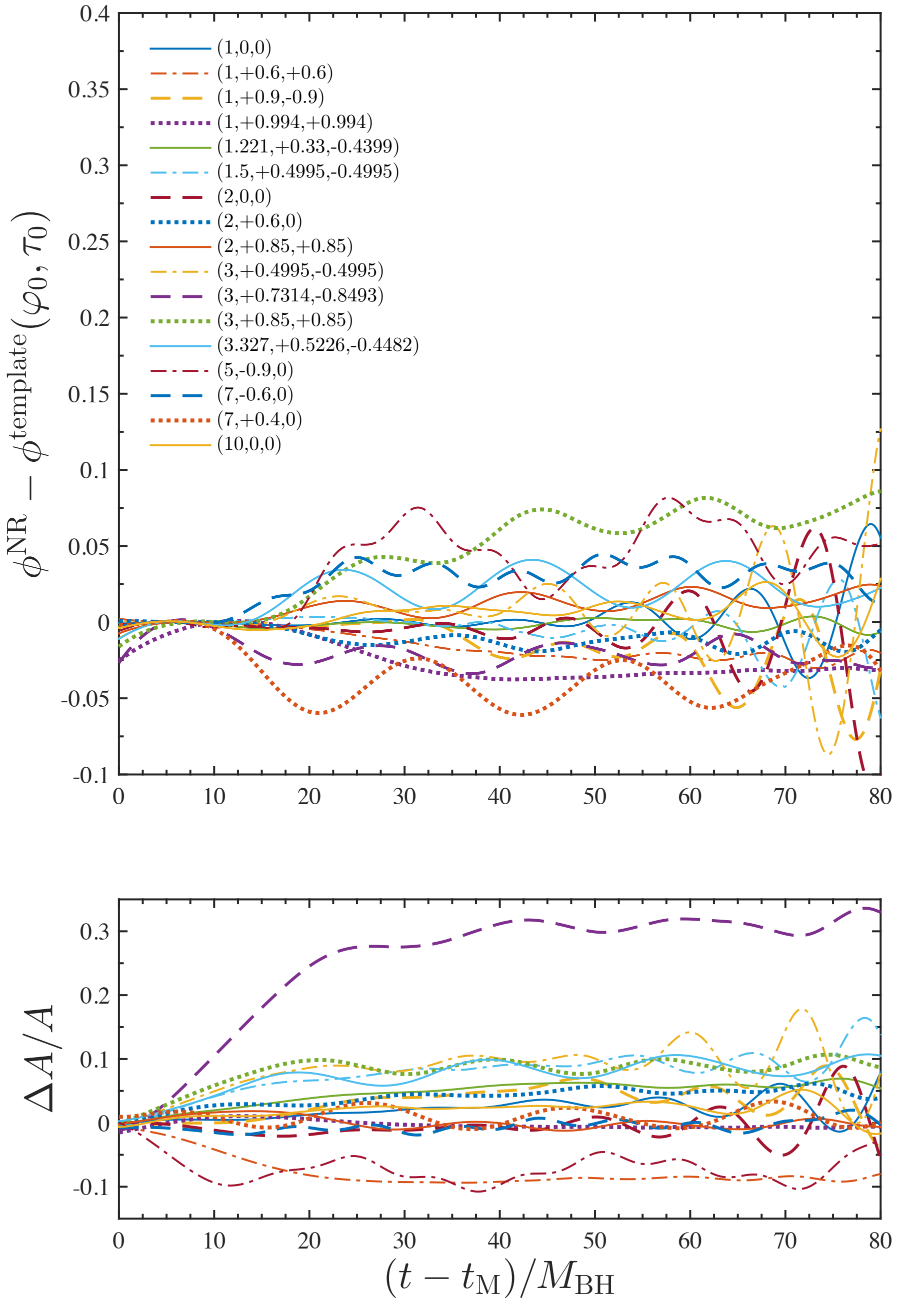
Black-list:

(1) the structure due to m<0 modes is not included (yet)
(2) large-mass ratios/high spin: amplitude problems
(3) problems are extreme for high-spin EMRL waves
(4) more flexible fit-template needed
(5) improve/check over all datasets (SXS & BAM for large mass-ratios & consistency with EMRL)

lunedì 20 febbraio 17

TESTS





Phase alignment@mrg

Time&phase shift alignment (as template)

WAVEFORM RECONSTRUCTION

SNR=10

SNR=20

SNR=50

SNR=100

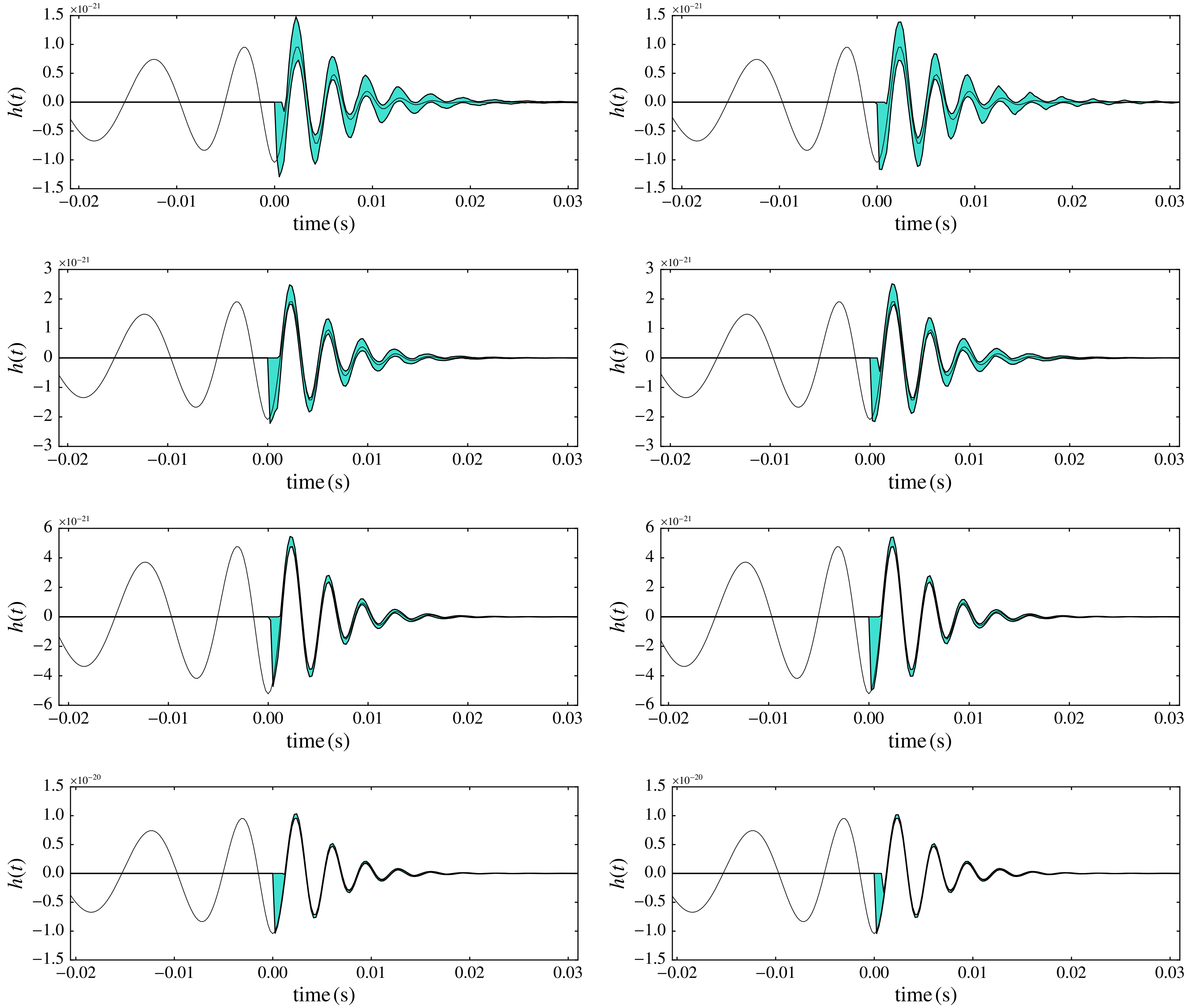
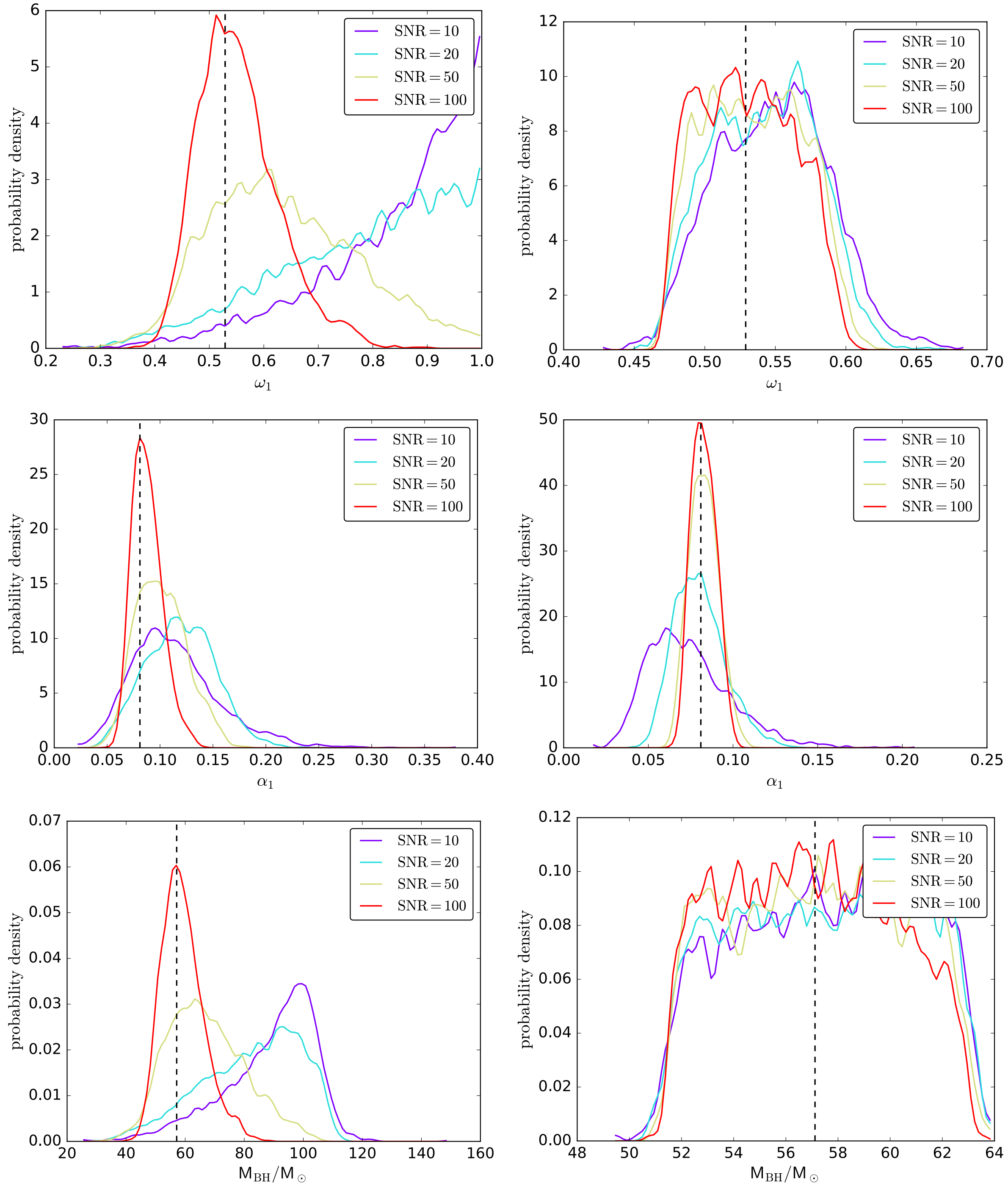


FIG. 4: From top to bottom right panels: GE case reconstructed post-merger waveform and corresponding 90% confidence region for SXS:BBH:0305 with post-merger SNR = 10, 20, 50 and 100. On the left hand side CO reconstructed post-merger waveform and corresponding 90% confidence region for SXS:BBH:0305 with post-merger SNR = 10, 20, 50 and 100. In all cases, the post-merger waveform is reconstructed very accurately, with uncertainty decreasing as the post-merger SNR increases.

MEASURING

GW150914-like signal

No priors on individual masses



Priors on individual masses

TABLE II: Dataset of the SXS catalog used for the cross-validation of the template waveform, see Fig. 3. The last two columns list fundamental QNMs frequency inferred from NR data and measured with the post-merger template, after adding to the NR waveform some Gaussian noise. For all waveforms, we fixed the post-merger SNR = 10. The uncertainty on the measured quantities corresponds to the 90% credible regions. The datasets marked with an * were used in the construction of the template

ID	q	ν	$S_1/(m_1)^2$	$S_2/(m_2)^2$	$M_{\rm BH}/M$	$J_{\rm BH}/M_{\rm BH}^2$	$\sigma_1^{\rm NR}$	$\sigma_1^{\rm measured}$
*	1	0.25	0	0	0.95161	0.6864	$0.0813 + i0.527$	$0.07^{+0.02}_{-0.01} + i0.52^{+0.06}_{-0.06}$
SXS:BBH:0152*	1	0.25	+0.60	+0.60	0.9269	0.8578	$0.0706 + i0.629$	$0.06^{+0.02}_{-0.02} + i0.64^{+0.06}_{-0.07}$
SXS:BBH:0211	1	0.25	+0.90	−0.90	0.9511	0.6835	$0.081 + i0.525$	$0.06^{+0.02}_{-0.02} + i0.50^{+0.05}_{-0.06}$
SXS:BBH:0178*	1	0.25	+0.994	+0.994	0.8867	0.9499	$0.053 + i0.746$	$0.08^{+0.03}_{-0.02} + i0.74^{+0.08}_{-0.07}$
SXS:BBH:0305	1.221	0.2475	+0.3300	−0.4399	0.9520	0.6921	$0.081 + i0.529$	$0.07^{+0.05}_{-0.03} + i0.55^{+0.06}_{-0.06}$
SXS:BBH:0025	1.5	0.2400	+0.4995	−0.4995	0.9504	0.7384	$0.079 + i0.550$	$0.08^{+0.04}_{-0.03} + i0.56^{+0.06}_{-0.07}$
SXS:BBH:0184	2	$0.\bar{2}$	0	0	0.9612	0.6234	$0.083 + i0.502$	$0.28^{+0.20}_{-0.22} + i0.53^{+0.41}_{-0.39}$
SXS:BBH:0162	2	$0.\bar{2}$	+0.6000	0	0.9461	0.8082	$0.075 + i0.591$	$0.08^{+0.04}_{-0.03} + i0.56^{+0.08}_{-0.07}$
SXS:BBH:0257	2	$0.\bar{2}$	+0.85	+0.85	0.9199	0.9175	$0.062 + i0.694$	$0.07^{+0.03}_{-0.02} + i0.67^{+0.07}_{-0.08}$
SXS:BBH:0045	3	0.1875	+0.4995	−0.4995	0.9628	0.7410	$0.079 + i0.552$	$0.21^{+0.26}_{-0.18} + i0.59^{+0.36}_{-0.45}$
SXS:BBH:0292	3	0.1875	+0.7314	−0.8493	0.9560	0.8266	$0.073 + i0.604$	$0.08^{+0.03}_{-0.02} + i0.58^{+0.07}_{-0.07}$
SXS:BBH:0293	3	0.1875	+0.85	+0.85	0.9142	0.9362	$0.062 + i0.689$	$0.07^{+0.03}_{-0.02} + i0.67^{+0.07}_{-0.07}$
SXS:BBH:0317	3.327	0.1777	0.5226	−0.4482	0.9642	0.7462	$0.078 + i0.554$	$0.06^{+0.02}_{-0.02} + i0.55^{+0.05}_{-0.06}$
SXS:BBH:0208*	5	$0.13\bar{8}$	−0.90	0	0.98822	−0.12817	$0.089 + i0.359$	$0.11^{+0.02}_{-0.02} + i0.40^{+0.04}_{-0.04}$
SXS:BBH:0203	7	0.1094	+0.40	0	0.9836	0.6056	$0.083 + i0.495$	$0.07^{+0.02}_{-0.01} + i0.48^{+0.06}_{-0.04}$
SXS:BBH:0207	7	0.1094	−0.60	0	0.9909	−0.0769	$0.089 + i0.364$	$0.08^{+0.02}_{-0.01} + i0.35^{+0.04}_{-0.04}$
SXS:BBH:0064*	8	0.0987	−0.50	0	0.9922	−0.0526	$0.089 + i0.367$	$0.09^{+0.12}_{-0.05} + i0.46^{+0.11}_{-0.08}$
SXS:BBH:0185	9.990	0.0827	0	0	0.9917	0.2608	$0.087 + i0.412$	$0.12^{+0.04}_{-0.03} + i0.42^{+0.07}_{-0.06}$

OUTLOOK

1. NR/EOB (IMRPhenom is also EOB based) is the way to go. NON resummed templates are useless. Same for BNS up to merger

2. EOB_IHES_spin: Analytical freedom: only two flexibility parameters that are extracted from NR data as simple (separate) functions of symmetric mass ratio and spin magnitude

3. Compatibility (within NR errors) between such EOBNR model and state-of-the art NR data over mass ratio and spin (+precession using SEOBNRv3 exists)

4. Improvements needed: best templates, were NOT used for analyses (though this is irrelevant now).This will be done in the next future on the Virgo/INFN side

CONCLUSION

The wave has passed....

...and we were (reasonably) prepared!

Though more work to improve modelization further is needed!

Matlab EOB code (working for BNS too...), free download: https://eob.ihes.fr.
More infos: https://gravitational_waves.ihes.fr/

MEASURING THE SCATTERING ANGLE IN NR

Damour, Guercilena, Hinder, Hopper, Nagar and Rezzolla, PRD **89**, 081503 (R), 2014

Comparing EOB/PN/NR

Analytics: rely only on conservative dynamics using NR losses

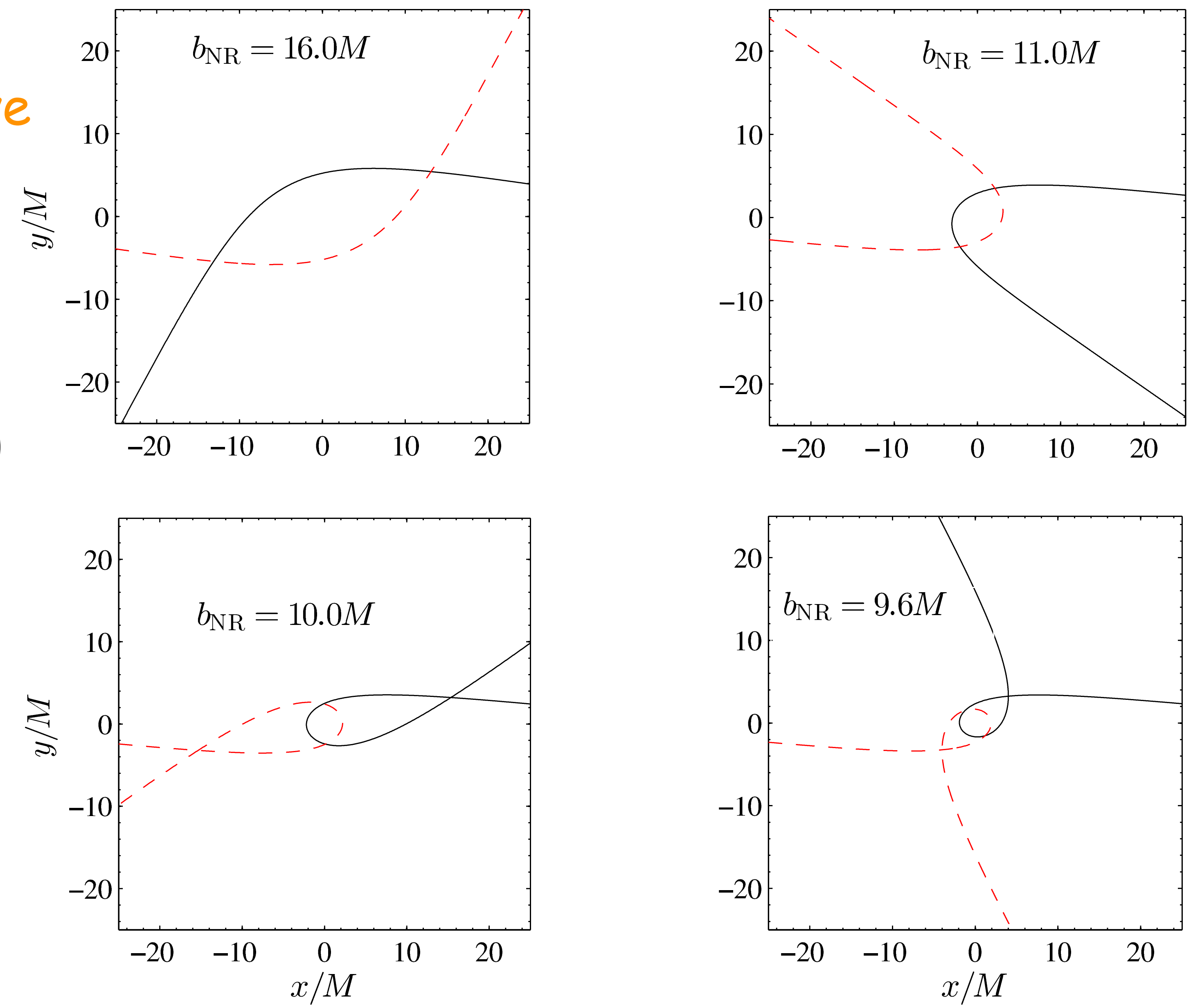
$$\chi^{\rm AR} = \chi^{\rm conservative}(\bar{E}, \bar{J})$$

$$\bar{E} \equiv (E_{\rm in} + E_{\rm out})/2$$

$$\bar{J} \equiv (J_{\rm in} + J_{\rm out})/2$$

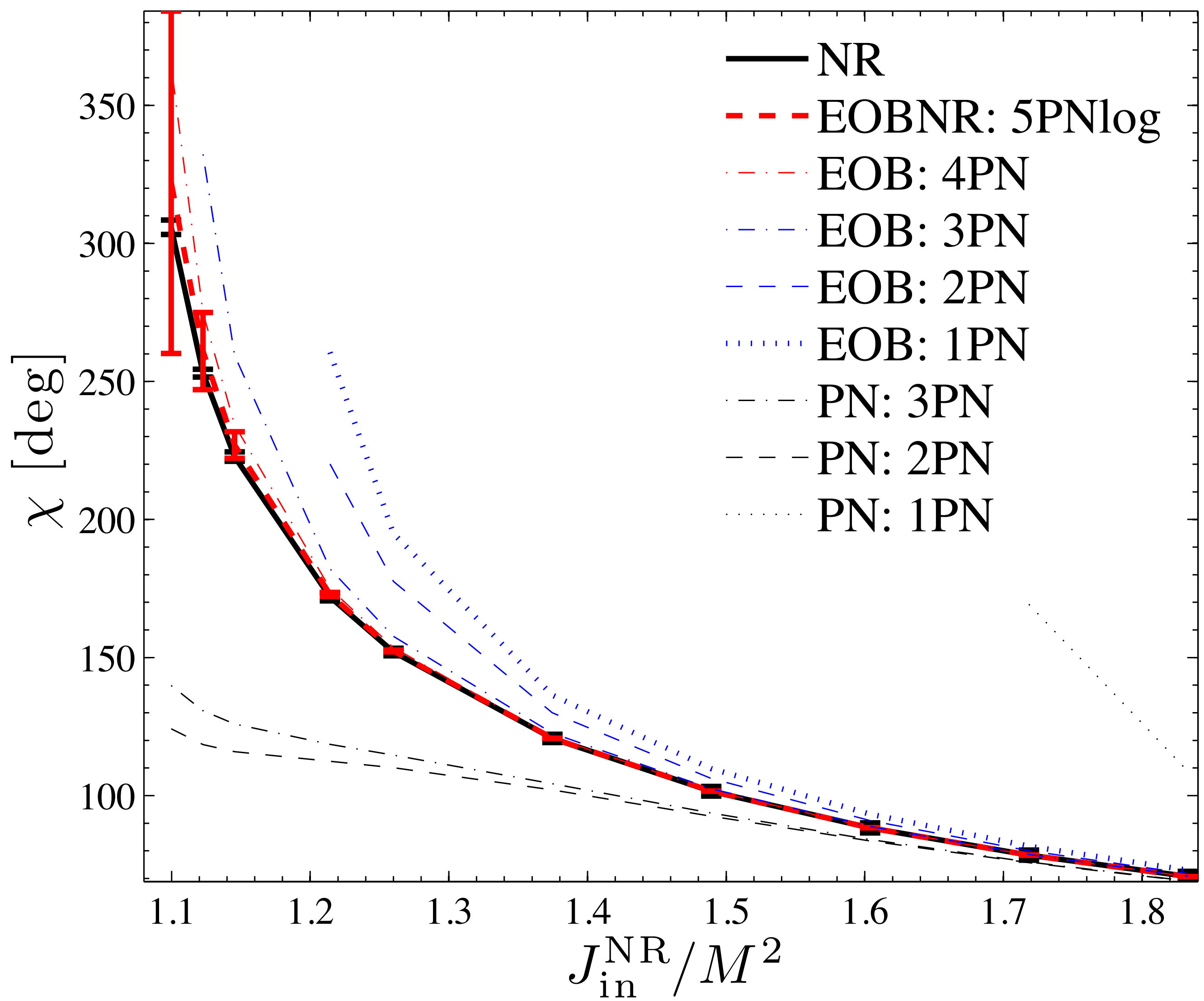
$$E_{\rm out} = E_{\rm in} - \Delta E^{\rm GW}_{\rm NR}$$

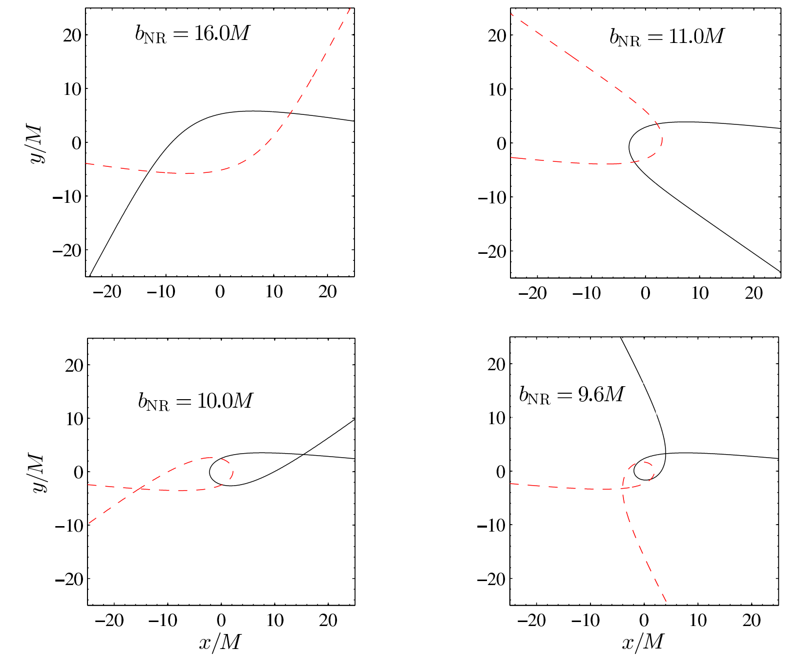
$$J_{\rm out} = J_{\rm in} - \Delta J^{\rm GW}_{\rm NR}$$

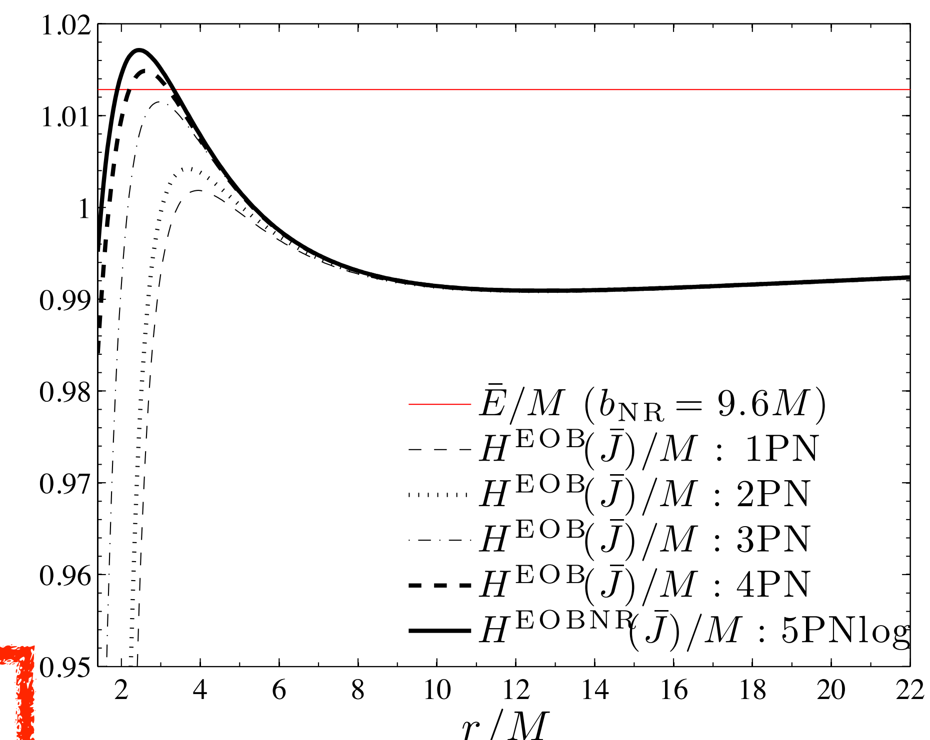


STRONG FIELD: EOB/NR SCATTERING ANGLE

Damour, Guercilena, Hinder, Hopper,
Nagar and Rezzolla, PRD **89**, 081503 (R), 2014







NR uncertainties on scattering angles are still large to firmly distinguish one A function to the other.

Damour&Nagar, wip