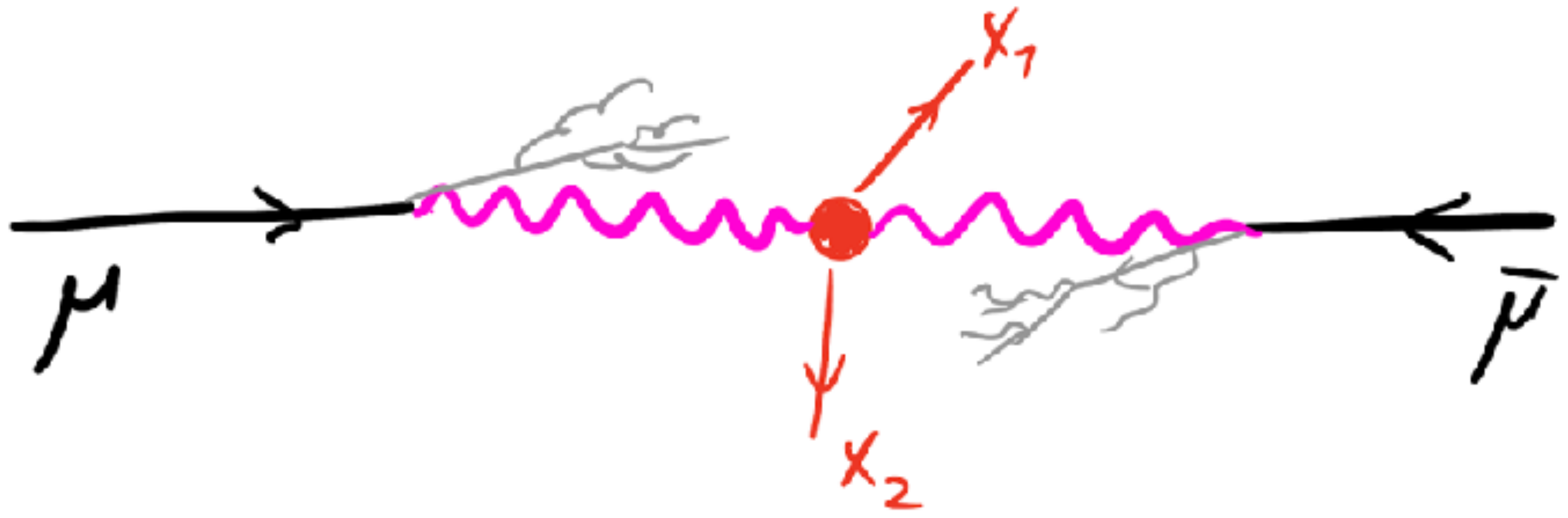
EW PDFs and their impact at Muon Colliders



David Marzocca

INFN Trieste

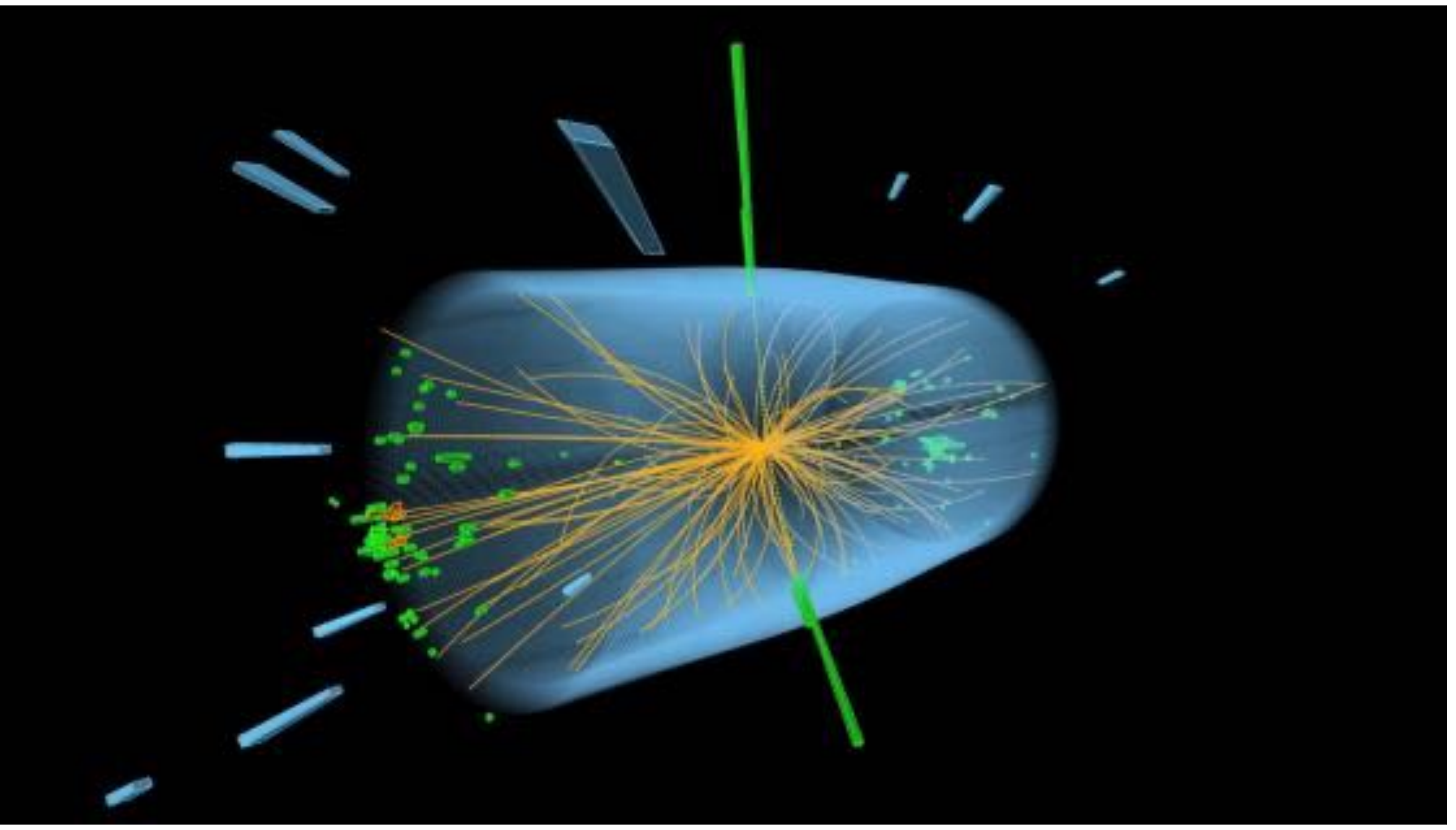
Based on:
- **Francesco Garosi**, D.M., **Sokratis Trifinopoulos** *JHEP 09 (2023) 107* [2303.16964]
- D.M. and **Alfredo Stanzione** [2408.13191]
- **F. Garosi**, **R. Capdevilla**, D.M. and **B. Stechauner** [2410.21383]

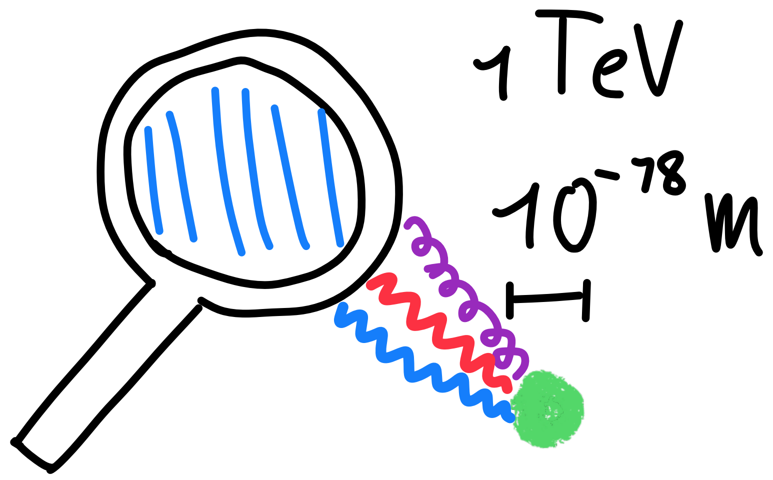
LePDF

Source + Downloads available at
https://github.com/DavidMarzocca/LePDF

Milano - 16/12/2024

Beyond the atto-scale





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The **incredible results** produced by its experiments have been possible also thanks to the
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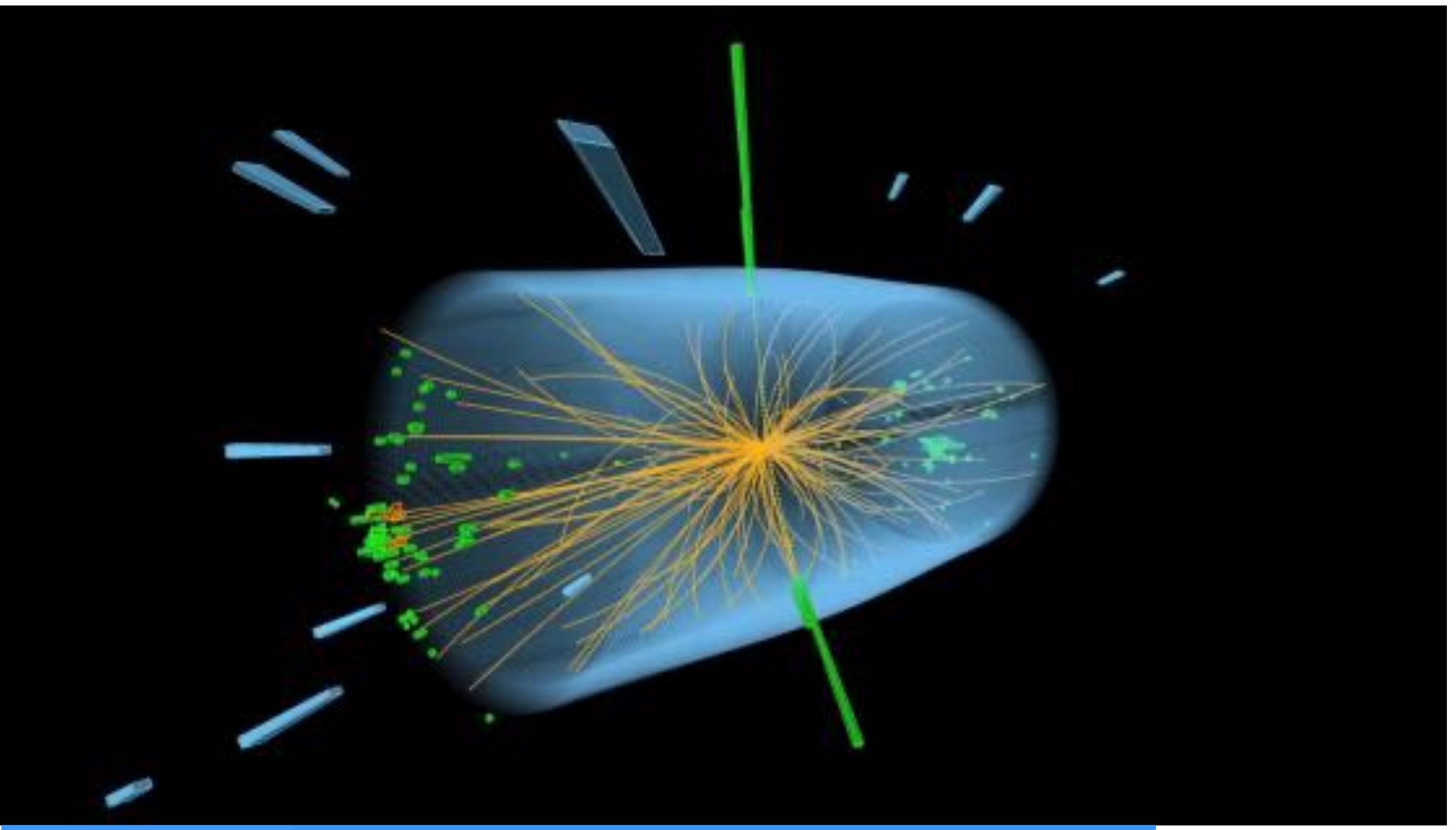

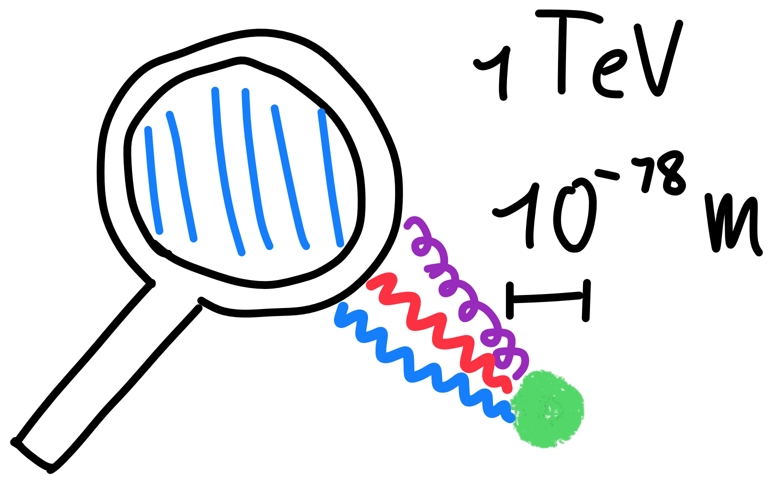

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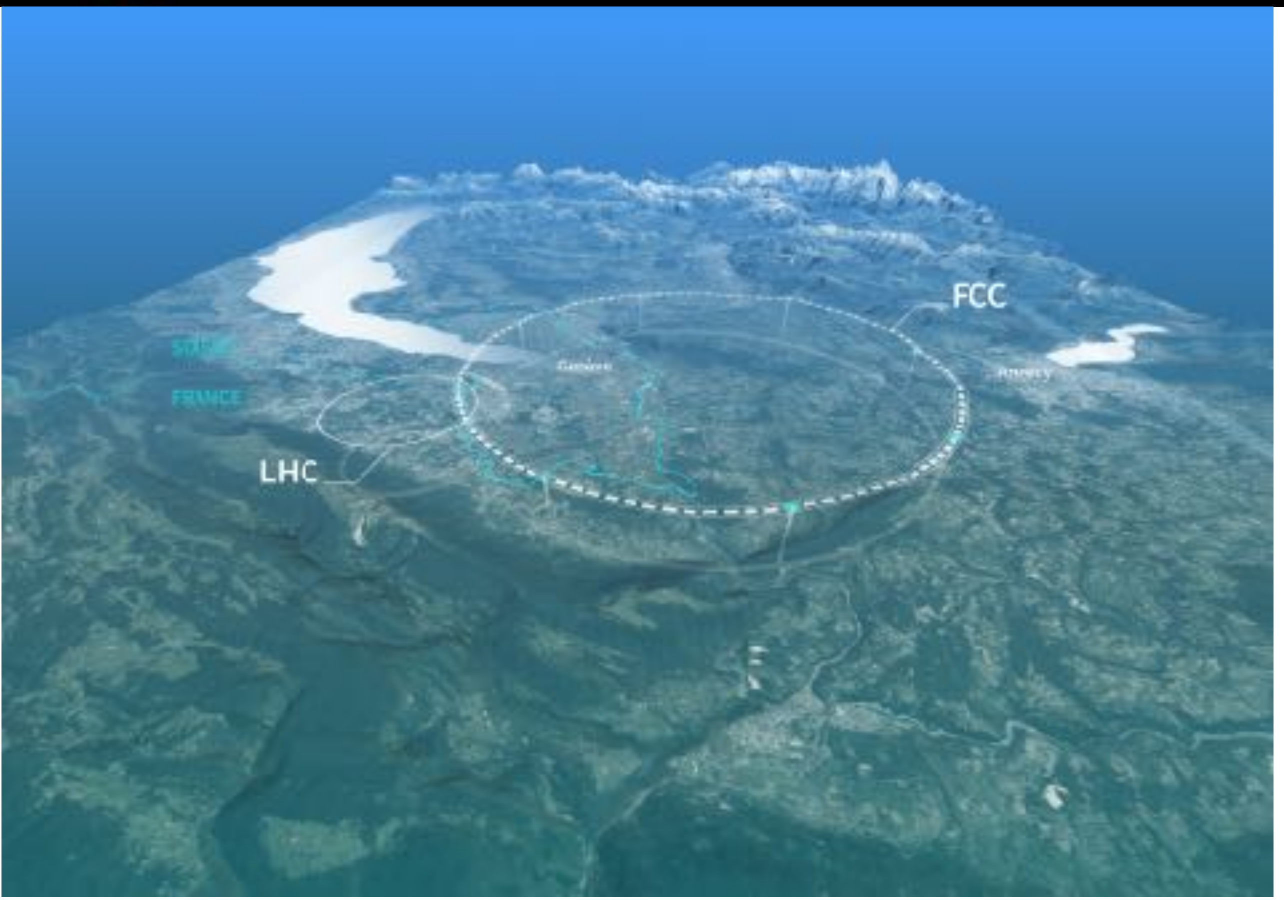

The **next step** in the exploration of physics at the smallest distances will be the **10 TeV regime**.

> ***Indirectly*** from EW and flavour measurements in high-intensity experiments (FCC-ee, Belle-II, LFV, etc)

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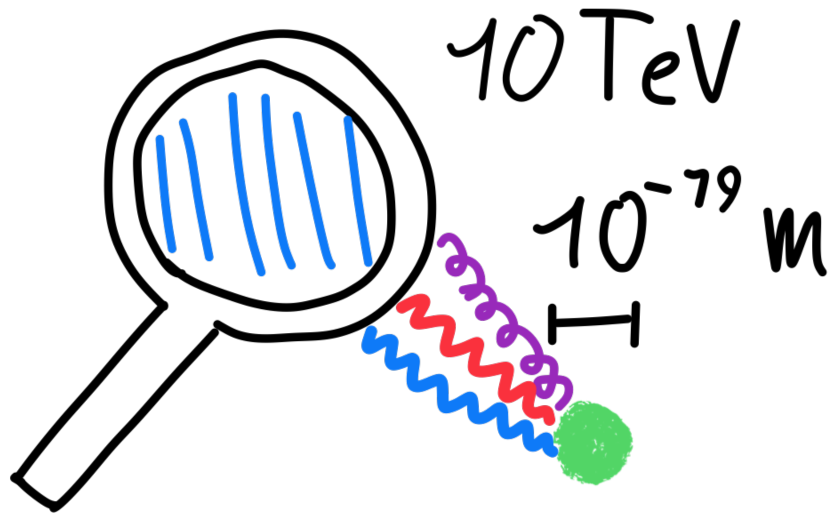

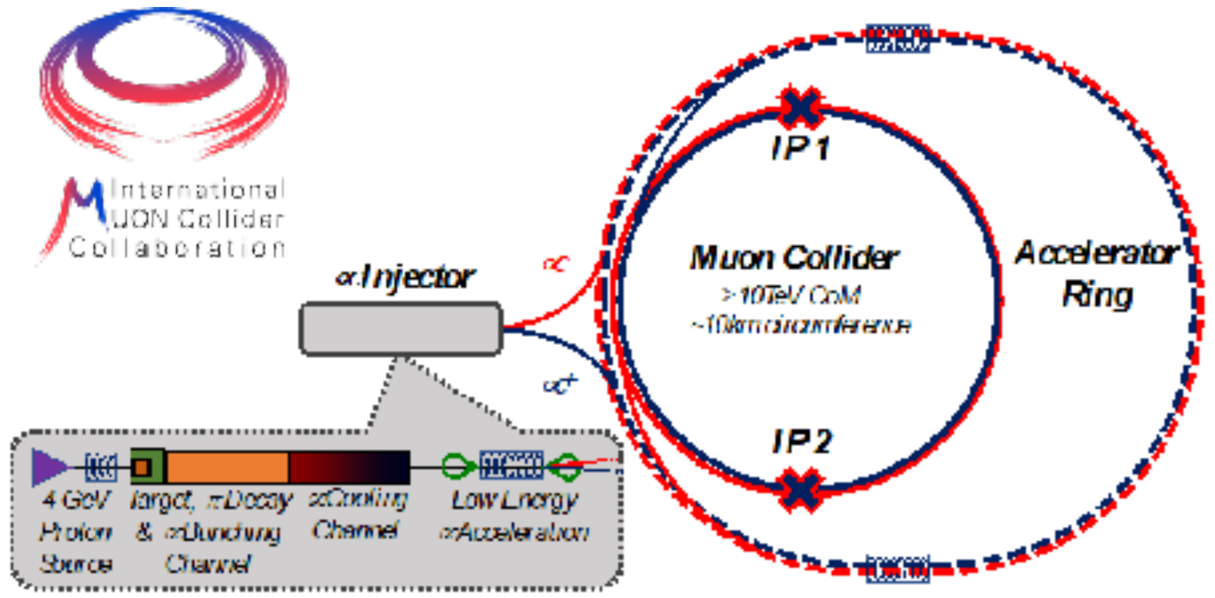

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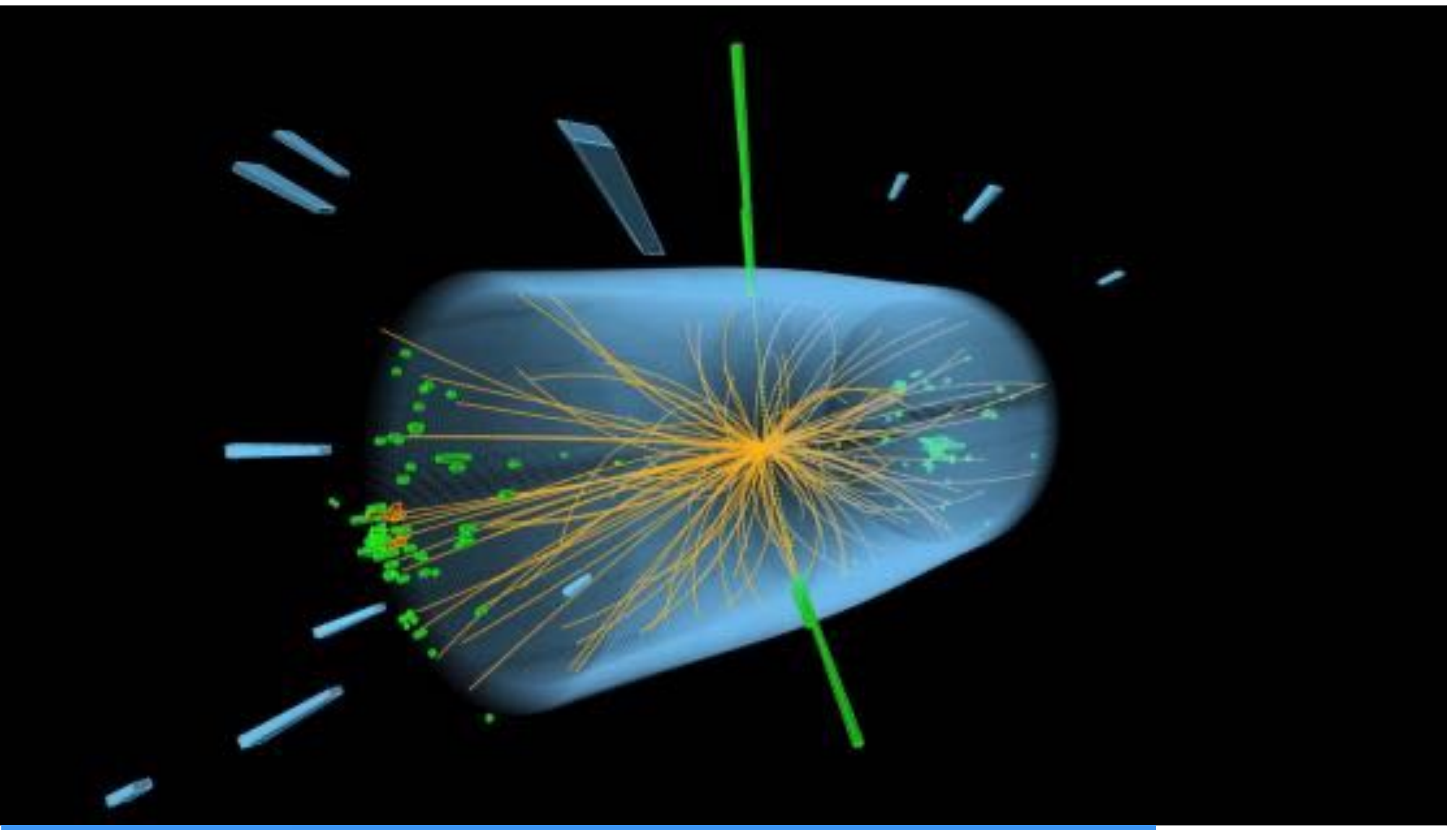

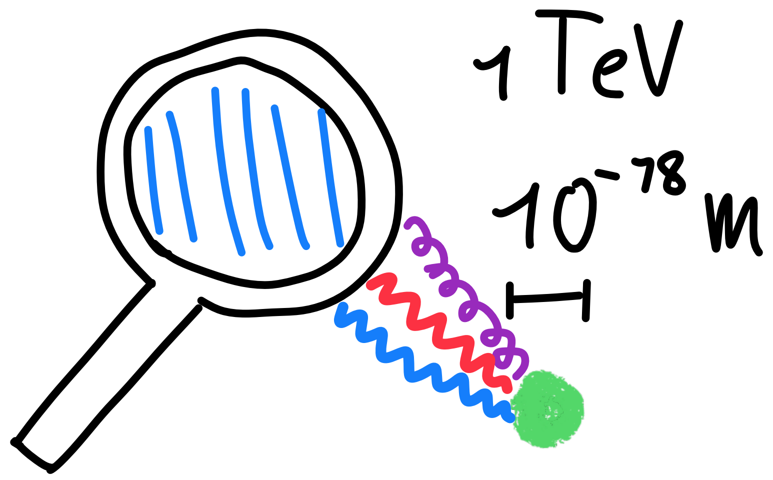

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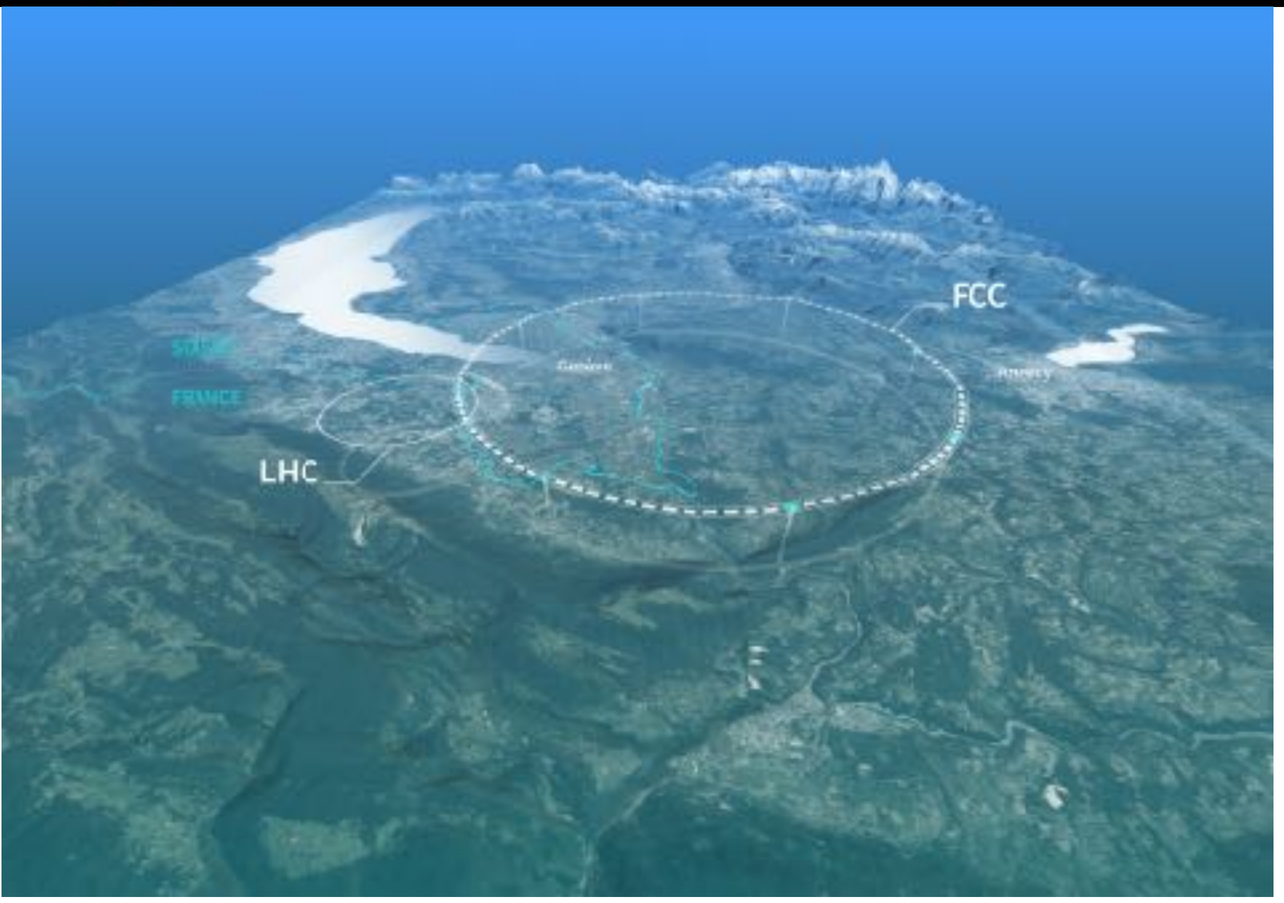

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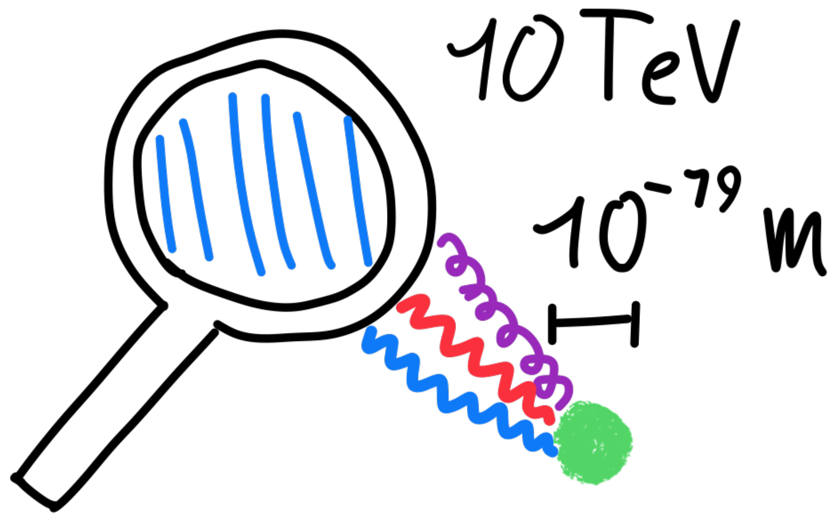
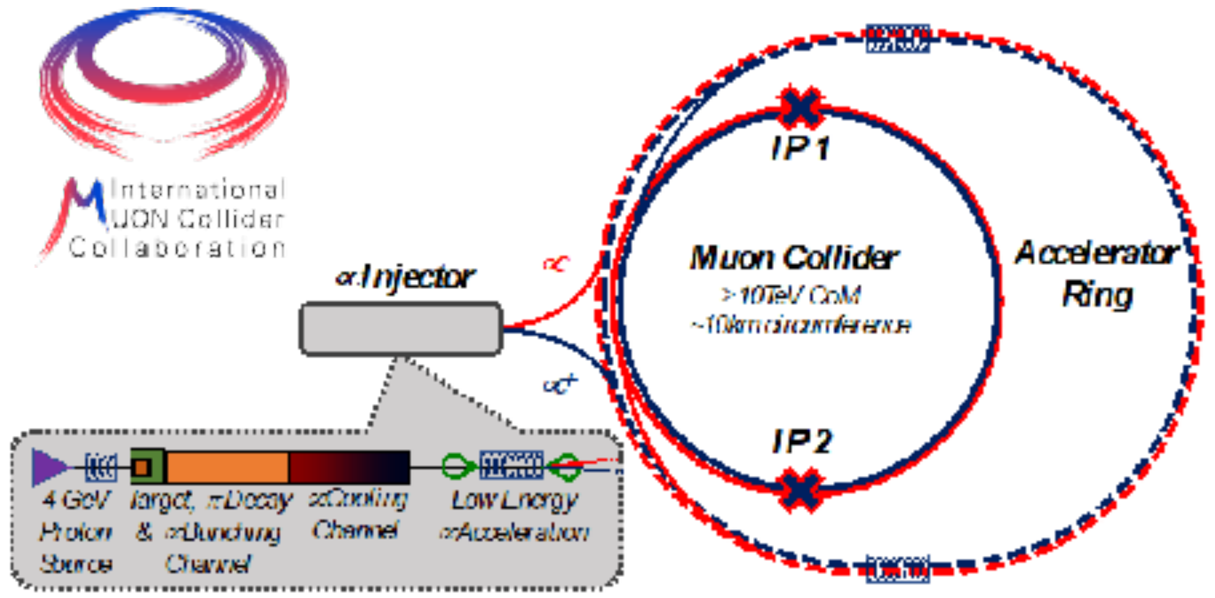
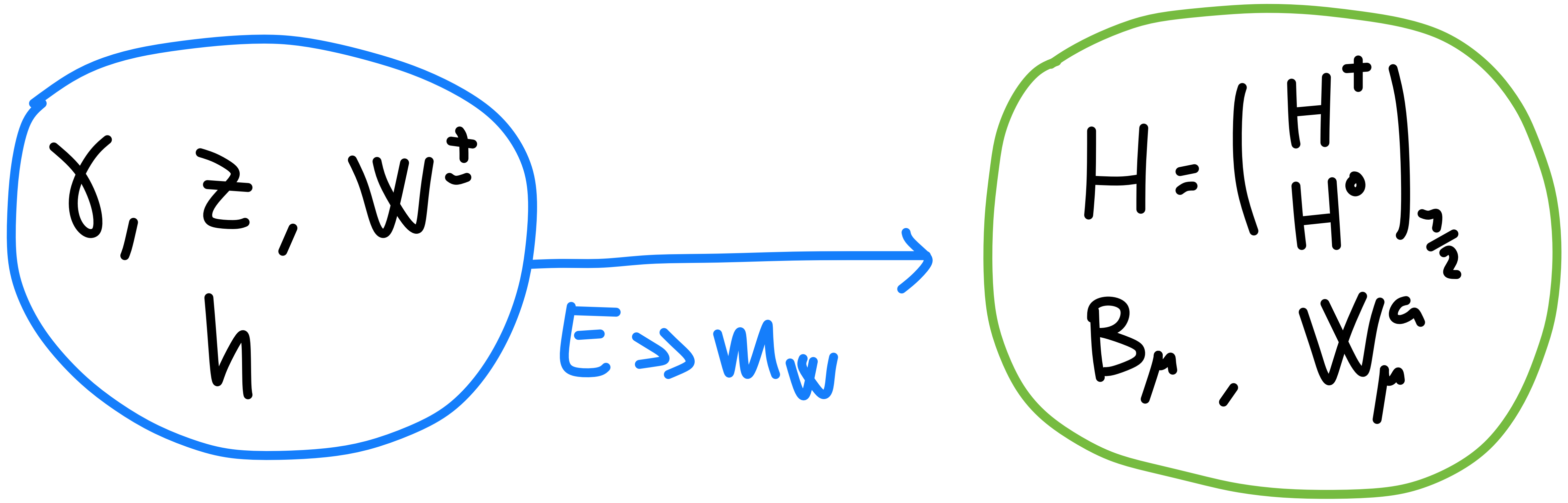




This energy regime is **exciting**, not only for the **possibility of uncovering New Physics**, but also because it contains **Standard Model phenomena never observed before**.

Electroweak interactions @ multi-TeV

EW symmetry-breaking effects (e.g. due to EW masses) diminish at large energies: **EW symmetry restoration**.



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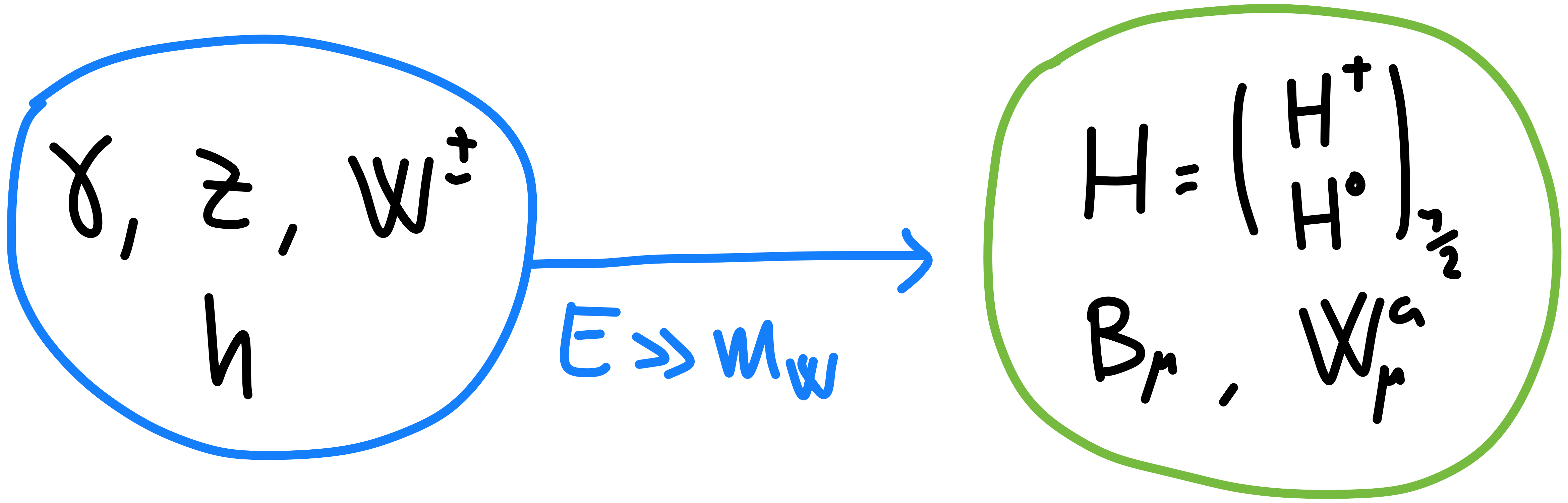

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proton or muon beams, distinguish W vs. Z vs. γ, etc..

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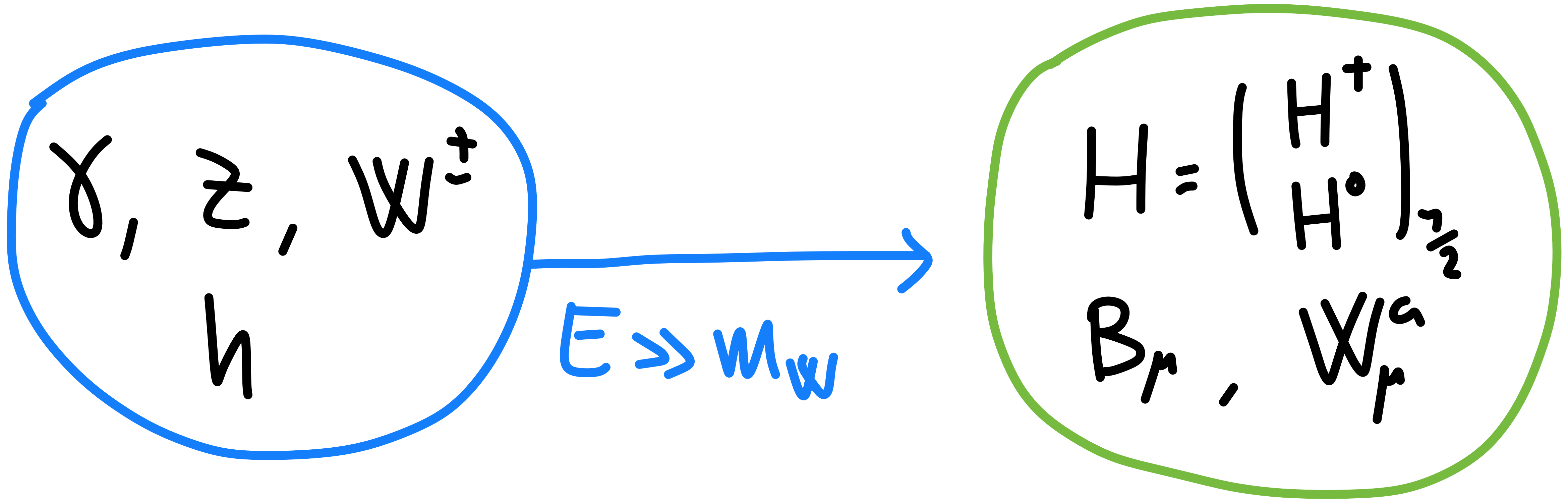

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New "exotic" SM effects related to this will be extremely important to be studied and understood in detail, both theoretically and experimentally:

WW scattering unitarization, **EW radiation**, **EW PDFs**, **EW jets**, etc..

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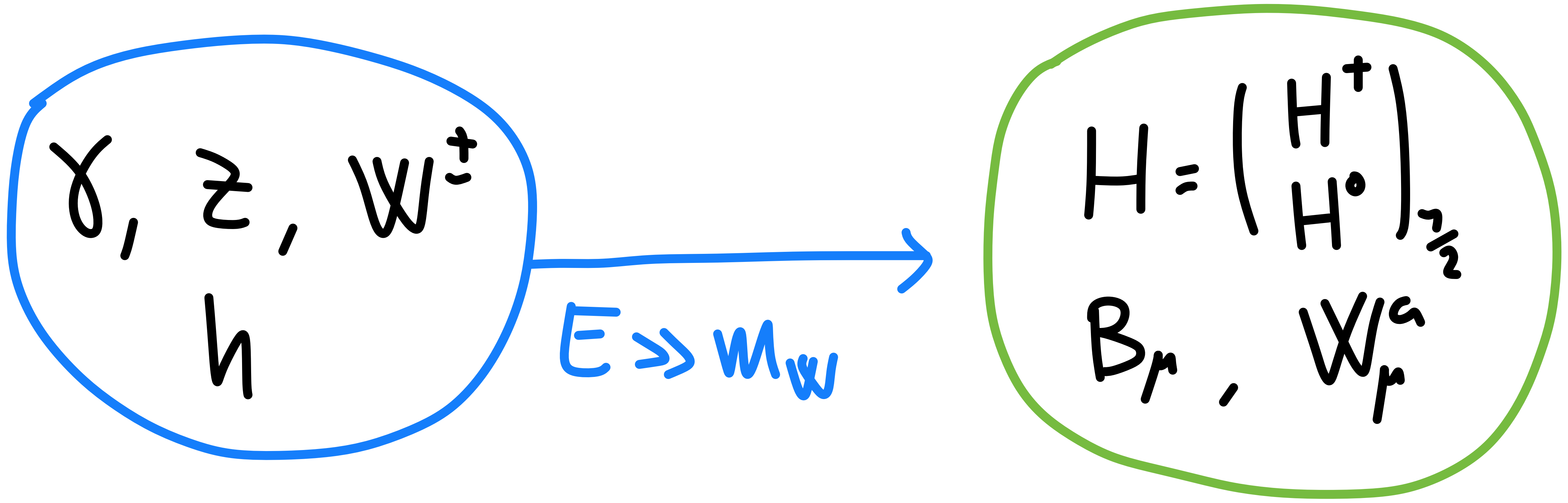

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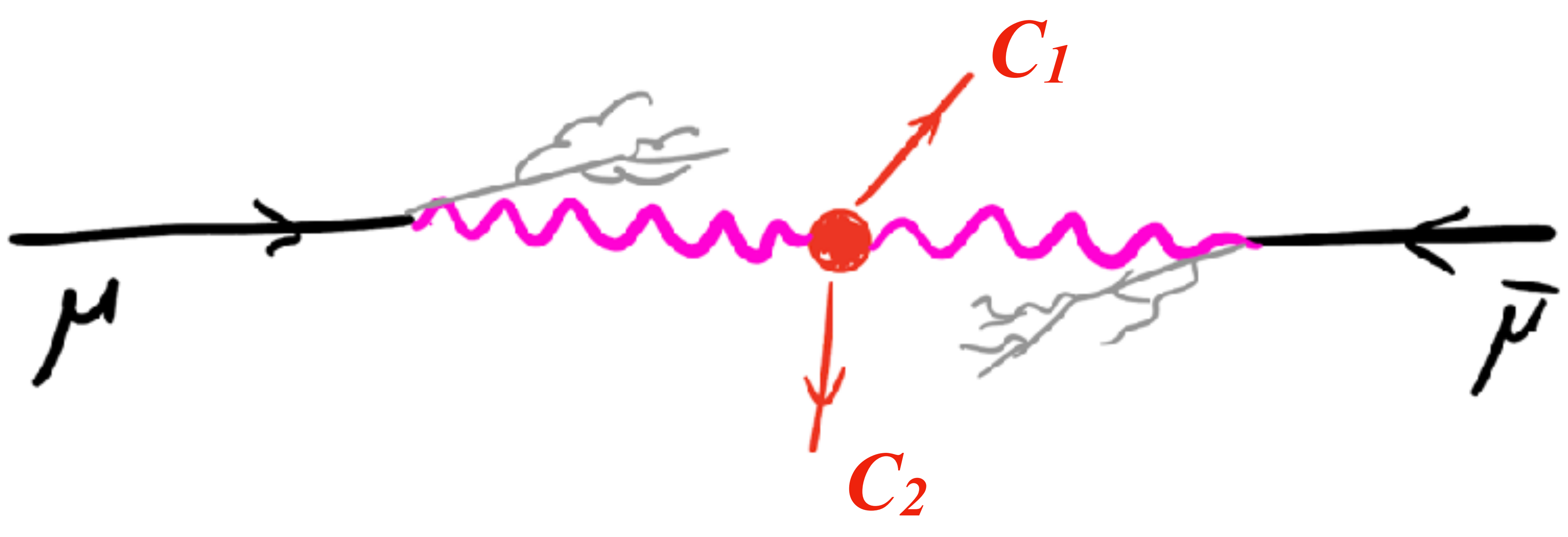
LHC: QCD era → **FCC-hh, MuC: EW era**

Muon Colliders are the ideal environment to study this physics with high precision!

Electroweak interactions @ multi-TeV

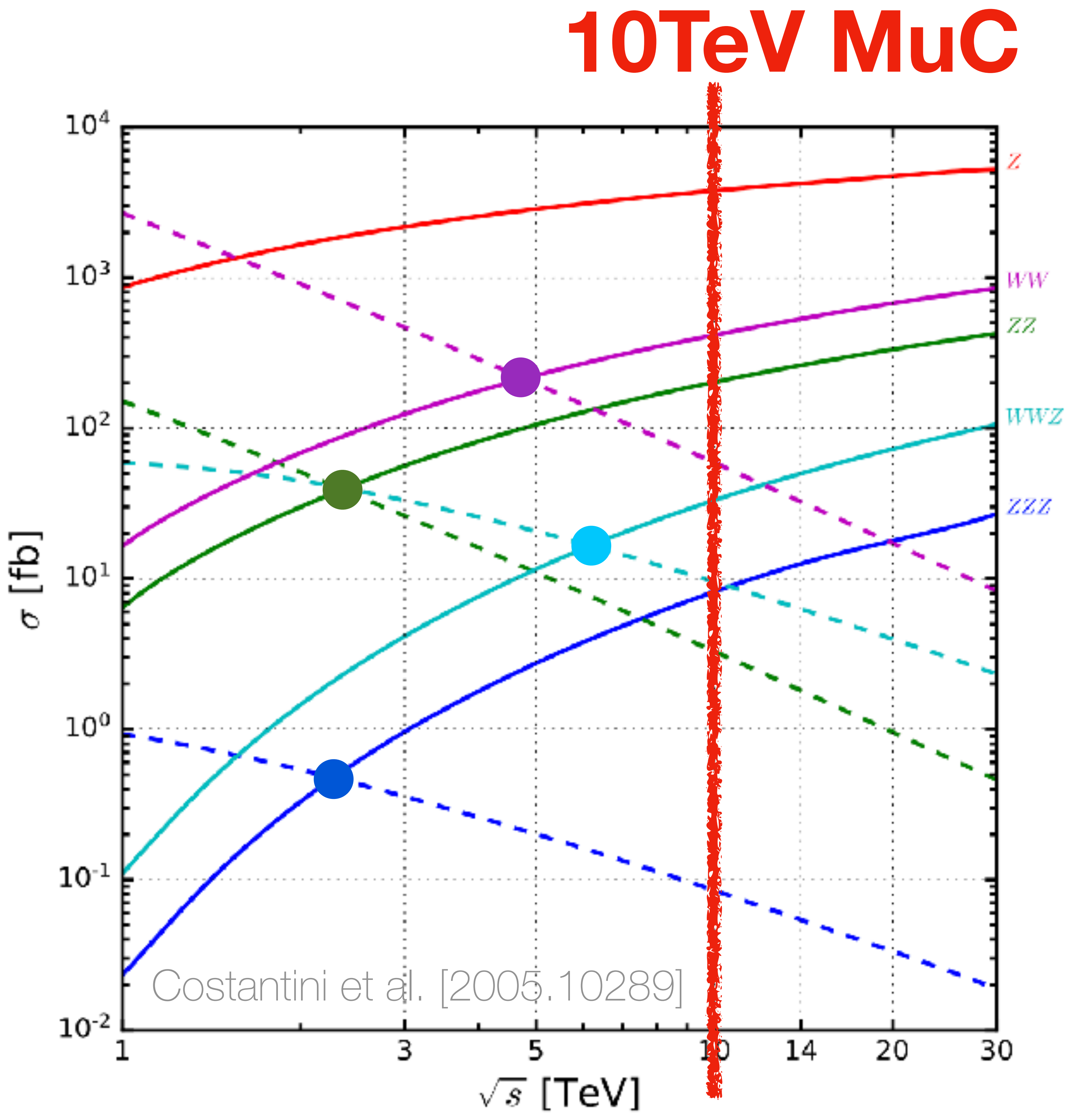
EW radiation (W and Z bosons) becomes as important as QED.

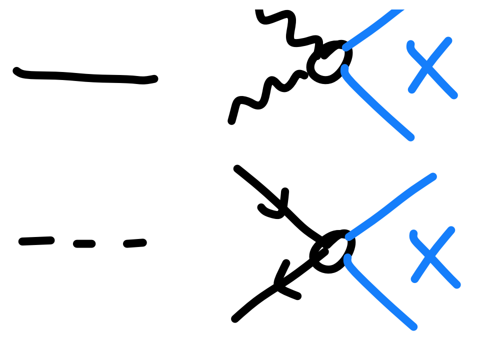


For processes **well above threshold**, the **contribution from collinear virtual bosons** emitted from the muons can become **dominant**.

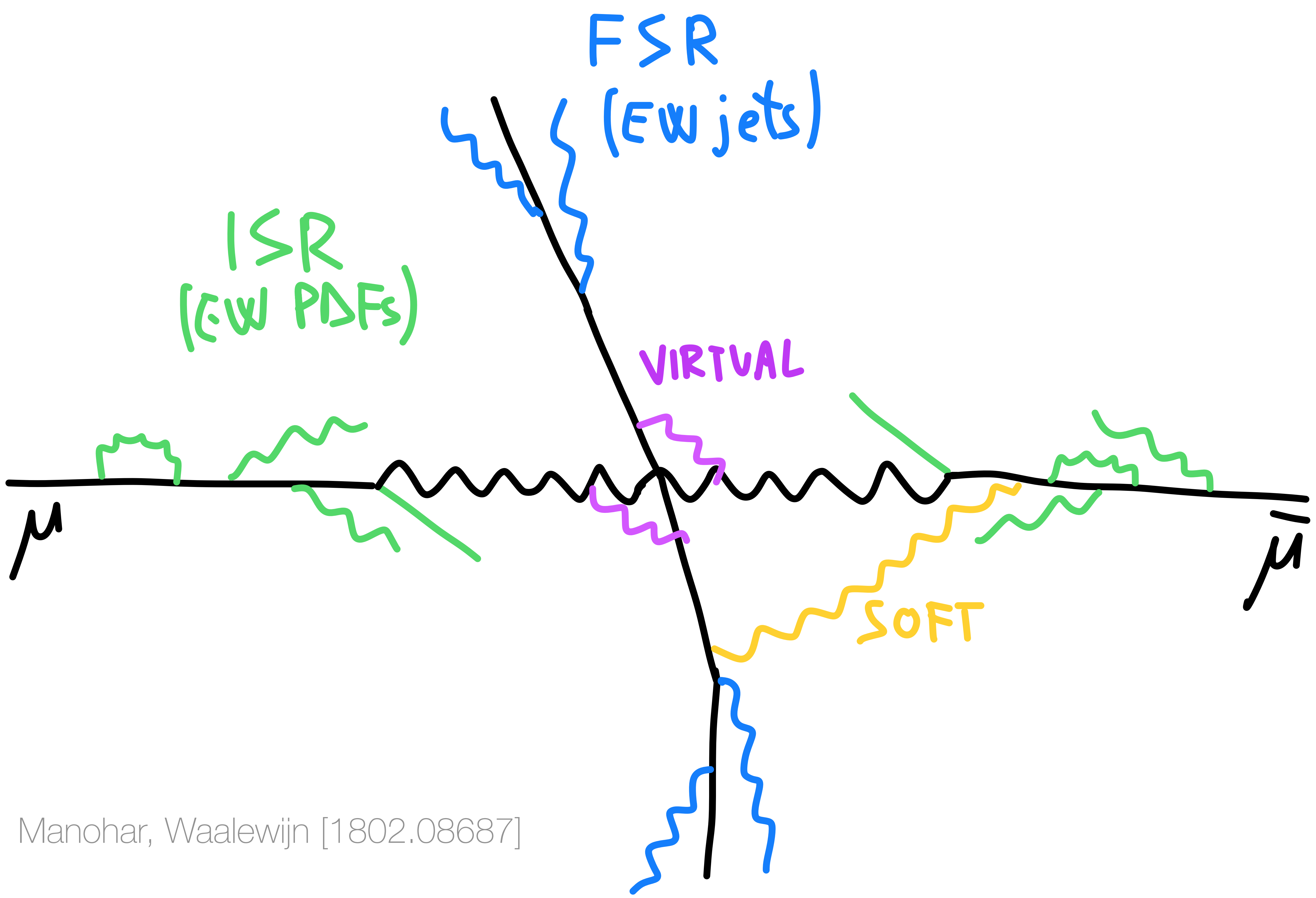
"The muon colliders are gauge boson colliders"

[Muon smasher's guide]





EW radiation



Manohar, Waalewijn [1802.08687]

Largest effects due to the **Sudakov double logarithms**: EW corrections (both virtual and real) that grow as:

$$\delta\sigma_{EW} \propto \alpha_2 \left(\log \frac{E^2}{m_W^2}\right)^2$$

They can give **O(1) effects at multi-TeV** scales

EWDL appear in:

- initial state radiation (**ISR**) M. Ciafaloni, P. Ciafaloni, Comelli [hep-ph/0001142, hep-ph/0103315]
- **virtual** corrections Denner, Pozzorini [hep-ph/0010201], Pozzorini [hep-ph/0201077]
- final state radiation (**FSR**)
- **soft radiation** between initial and final states

Manohar, Waalewijn [1802.08687]

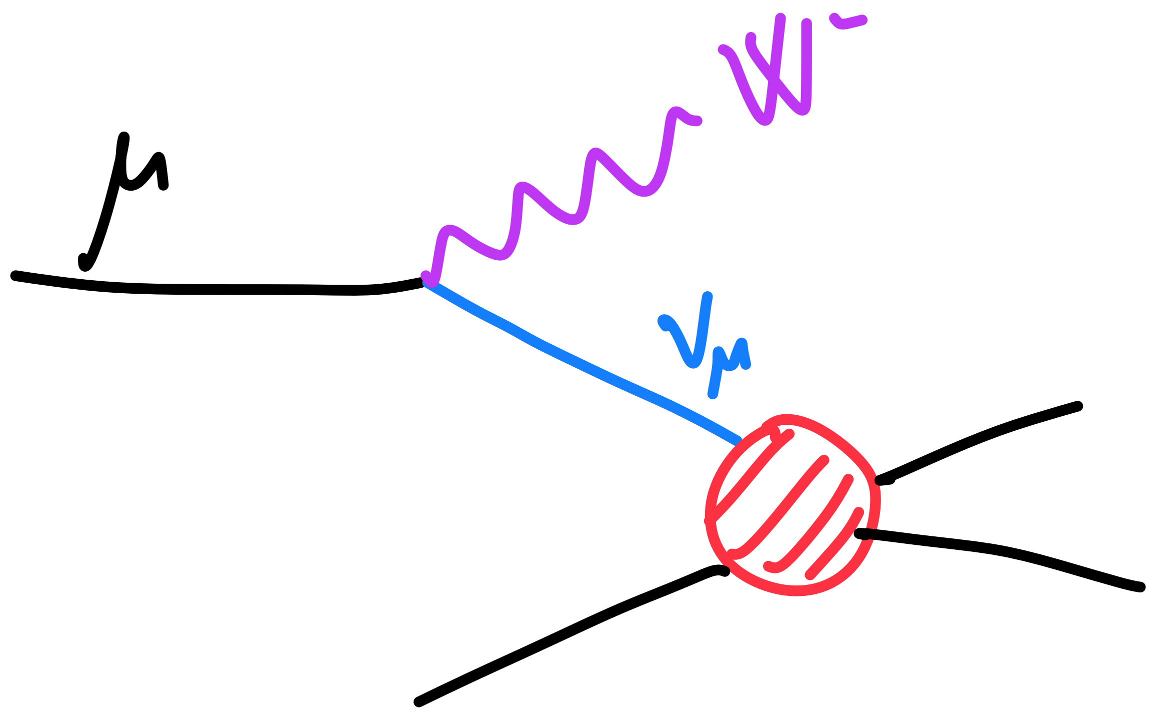
I will focus on the **EWDL arising from ISR**, which can be resummed with the PDF formalism by integrating **EW DGLAP equations**.

EW Sudakov double-logs

(EWDL)

IR divergences associated to W radiation do not cancel

In **ISR**:



$$\propto f_{\mu_L} \left(\log \frac{E^2}{M_W^2} \right)^2$$

collinear * soft
log

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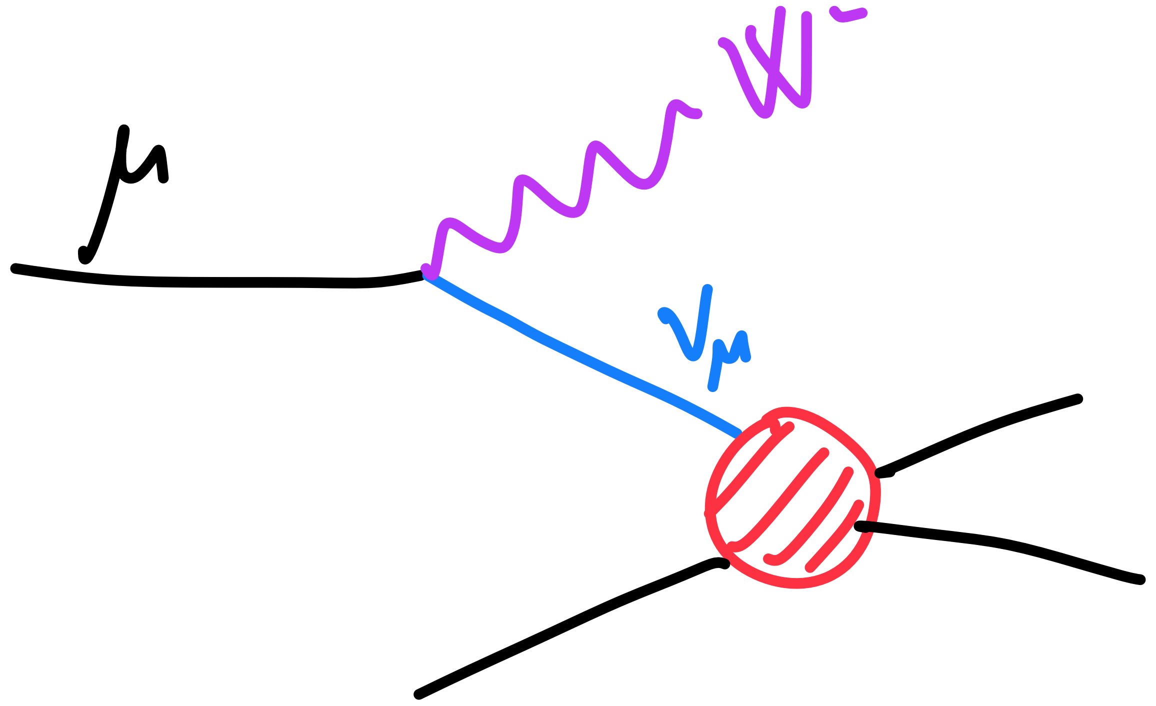

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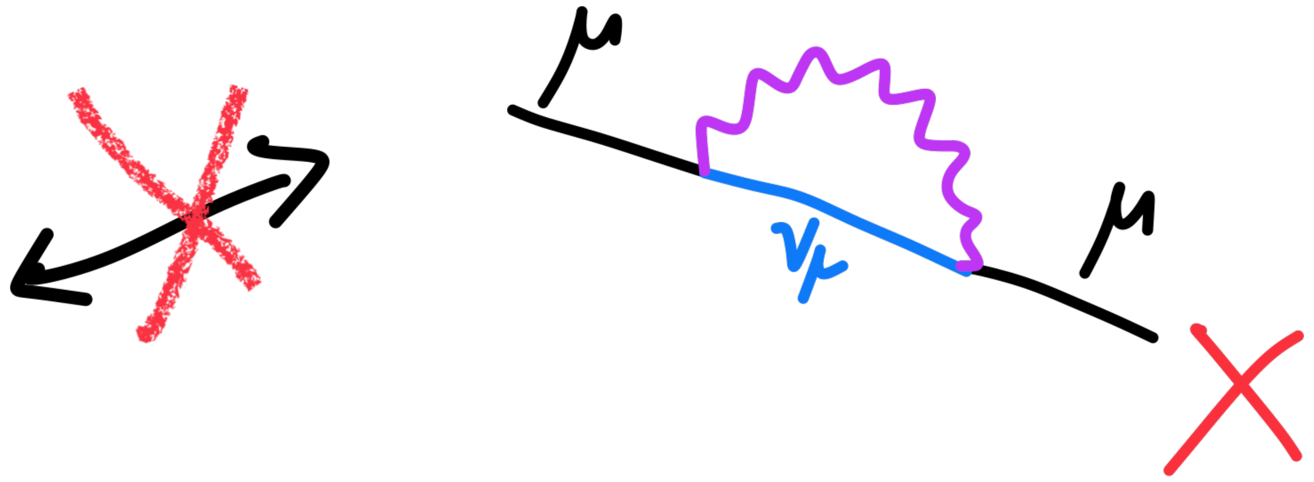

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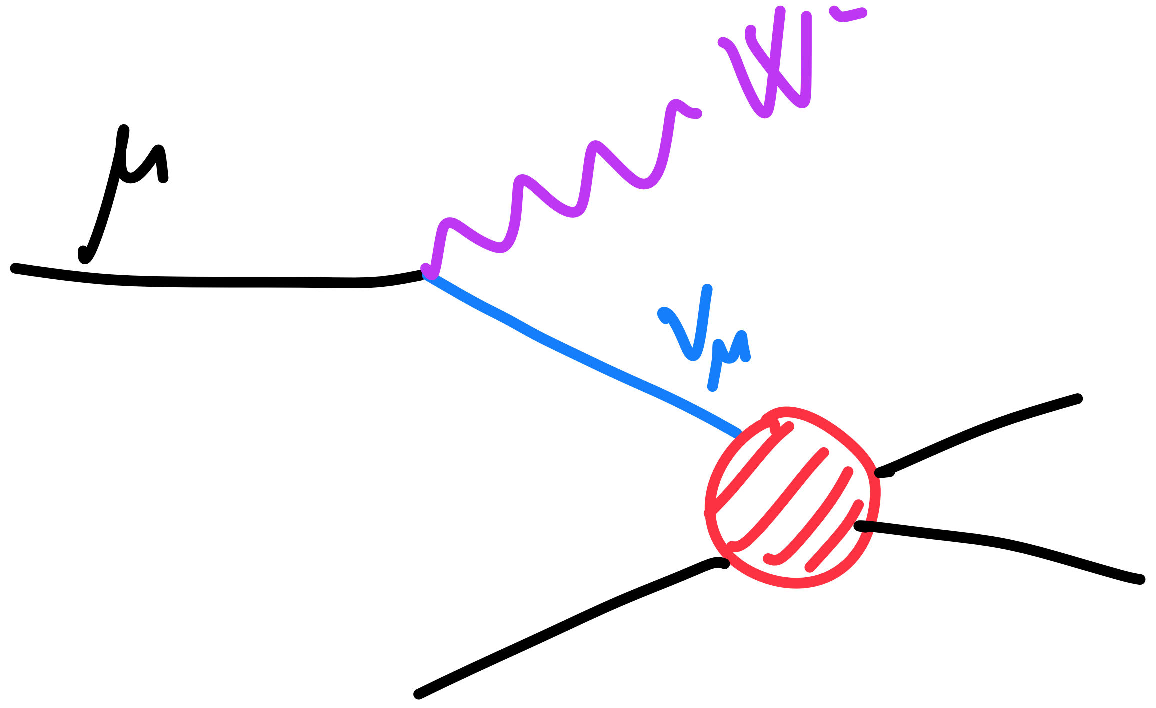

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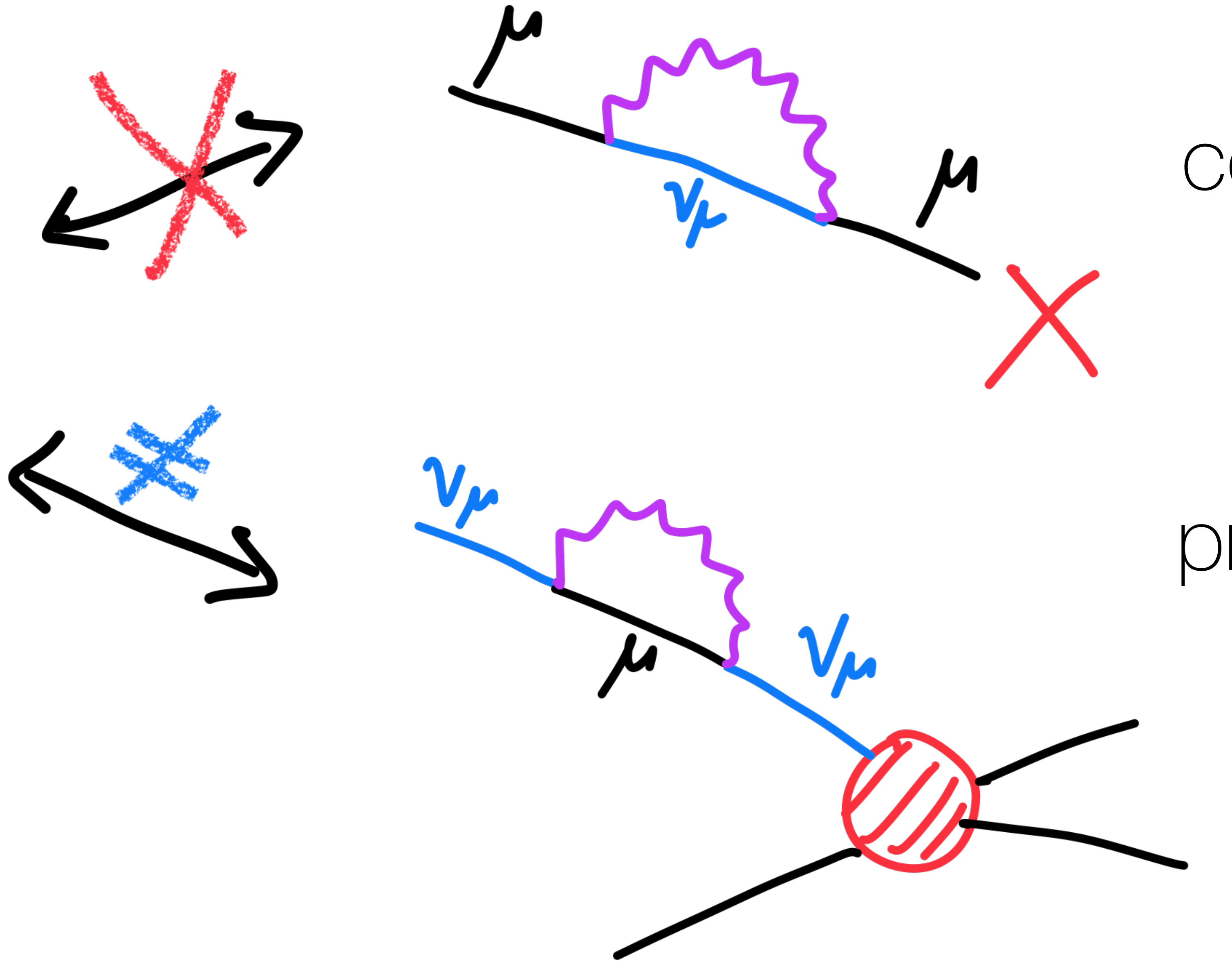

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The allowed one is proportional to the neutrino PDF: different.

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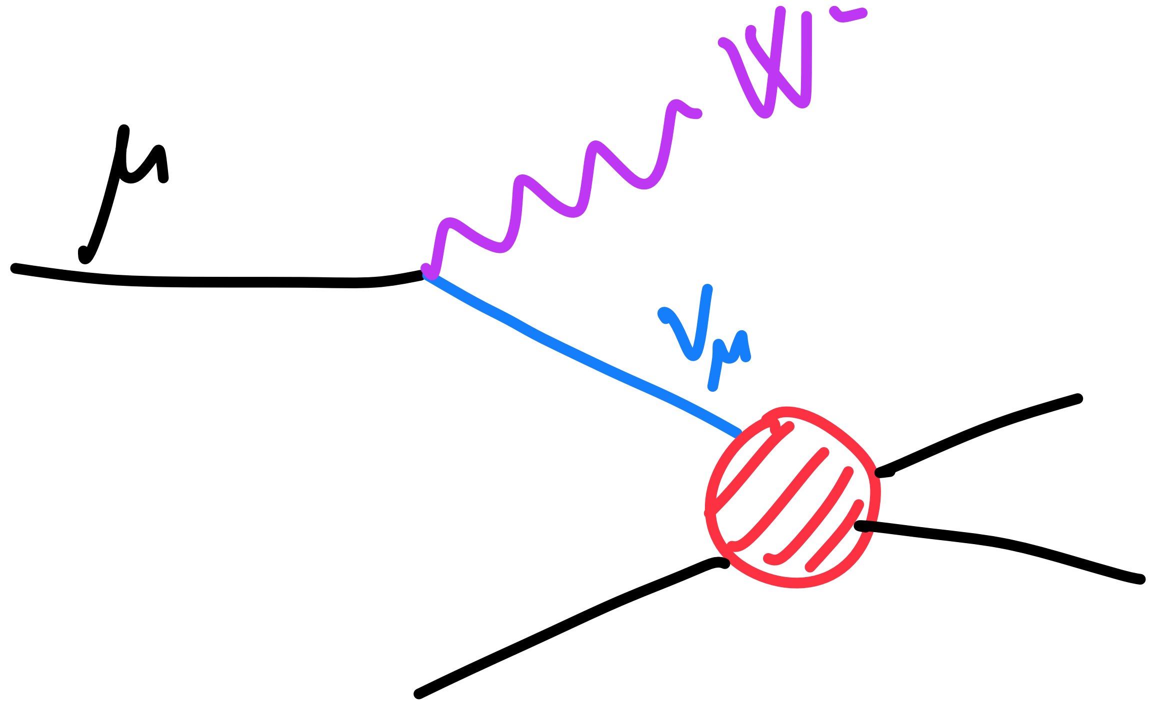

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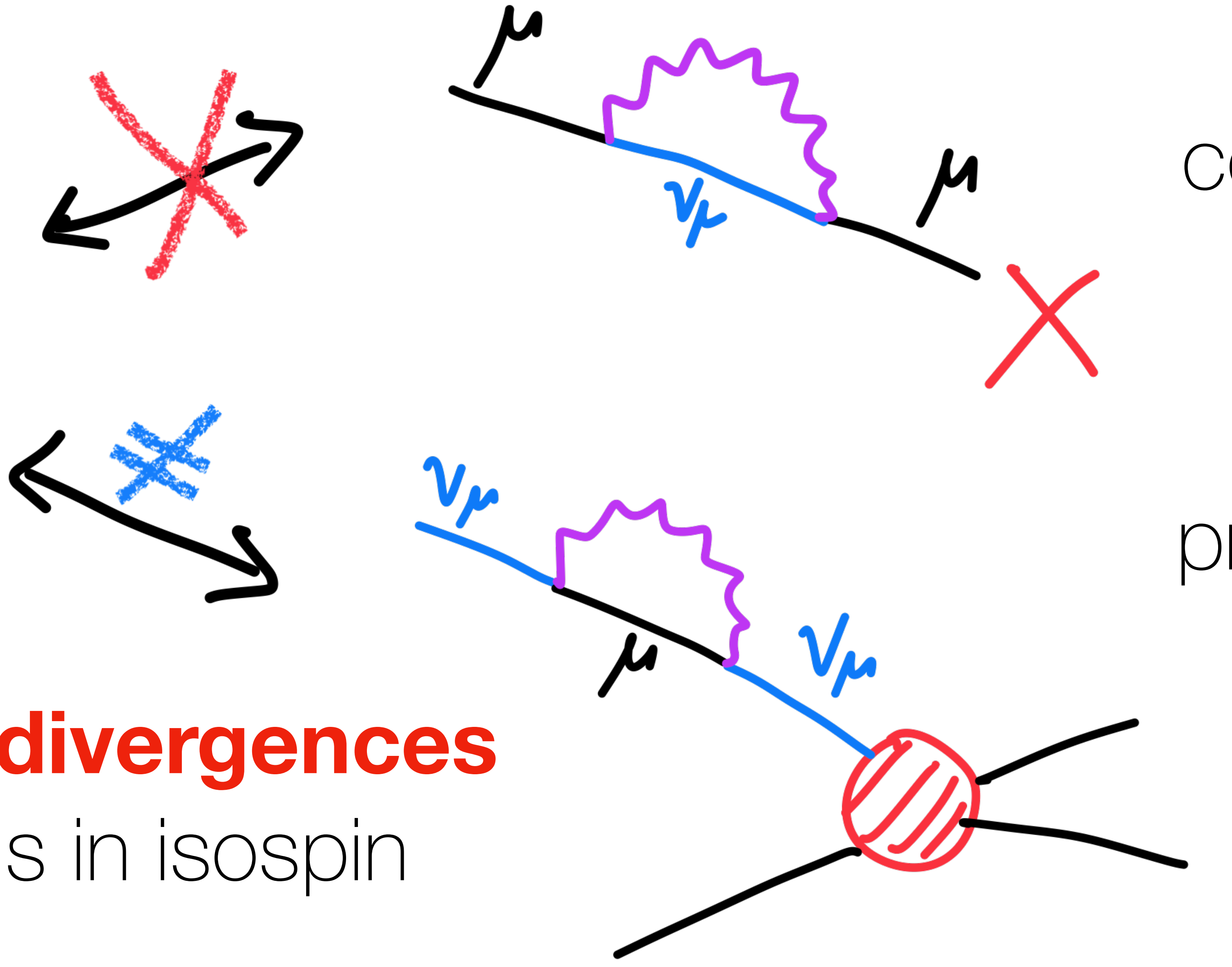

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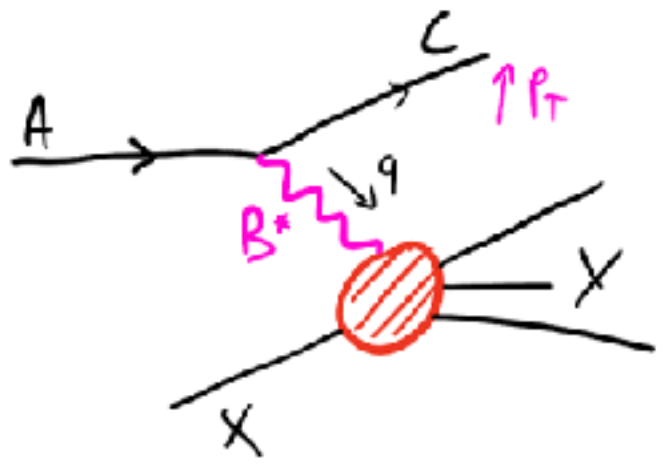
They arise as a **non-cancellation of the IR soft divergences** ($z \to 1$) between real emission and virtual corrections in isospin flipping transitions (e.g. $\mu_L \leftrightarrow \nu_\mu$) with $W^\pm$ emission.

P. Ciafaloni, Comelli [hep-ph/9809321], Fadin et al. [hep-ph/9910338], M. Ciafaloni, P. Ciafaloni, Comelli [hep-ph/0001142, hep-ph/0103315]
see also Denner, Pozzorini [hep-ph/0010201], Pozzorini [hep-ph/0201077], Manohar [1409.1918], Pagani, Zaro [2110.03714], …
Manohar, Waalewijn [1802.08687], Chen, Glioti, Rattazzi, Ricci, Wulzer [2202.10509]

Violation of the Bloch-Nordsieck theorem also for inclusive processes, due to the **initial state being EW non-singlet**.

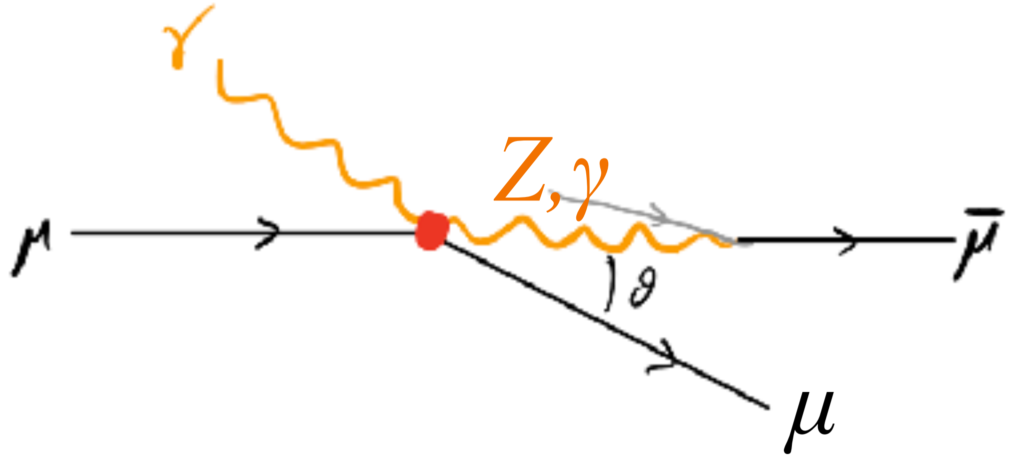
Outline

In this talk I will focus on effects related to
EW Parton Distribution Functions
and their **applications to Muon Collider processes**.

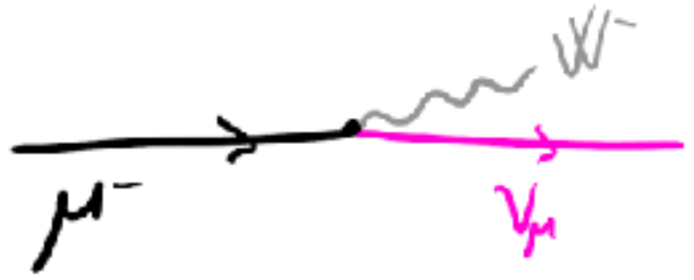


- Introduction on **PDFs for lepton colliders** and EW PDFs.

Pheno

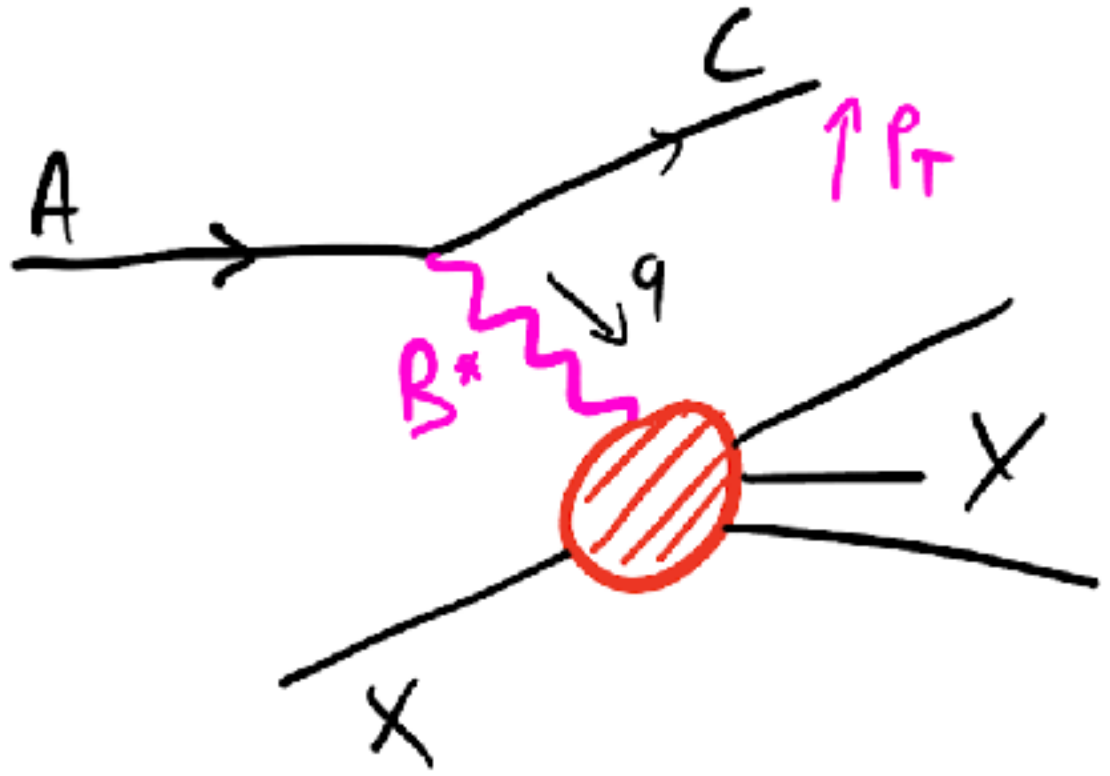


- Effects of the **mixed Z/γ** PDF at Muon Colliders



- Impact of the **muon-neutrino PDF** at Muon Colliders

Collinear Radiation and PDFs



The emission of radiation from the initial state, followed by a hard scattering process, gives rise to a **collinear logarithm**

$$\sigma \propto \int_{\mu_{IR}}^{E_{hard}} \frac{dp_T^2}{p_T^2} \sim \log \frac{E_{hard}^2}{\mu_{IR}^2}$$

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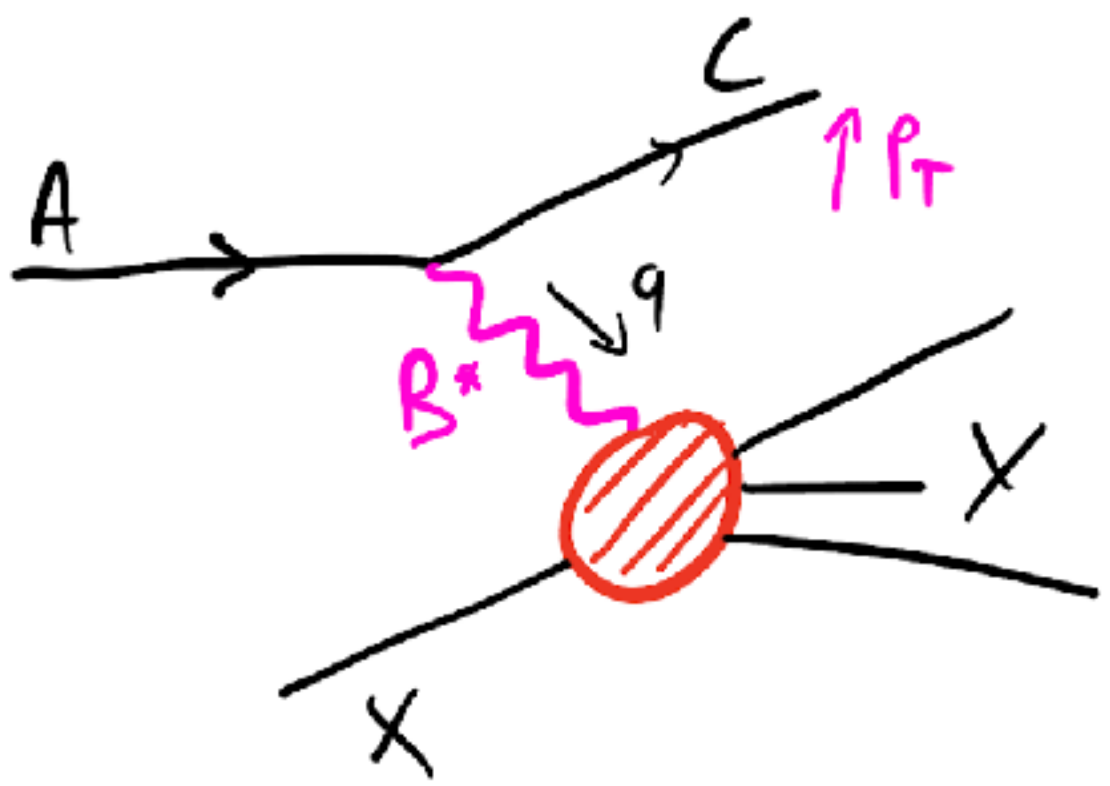

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Collinear Factorization: The amplitudes for collinear splitting and hard scattering can be factorised if $p_T \ll E_{hard}$. [see e.g. Cuomo, Vecchi, Wulzer 1911.12366, …]

$$i\mathcal{M}(AX \to CY) = \sum_B i\mathcal{M}^{\rm split}(A \to CB^*)\frac{i}{Q^2}i\mathcal{M}^{\rm hard}(BX \to Y)\,(1 + \mathcal{O}(\delta_{m,\perp}))$$

$$\delta_\perp = |\mathbf{k}_\perp|/E \ll 1$$
$$\delta_m = m/E \ll 1$$

missing power corrections

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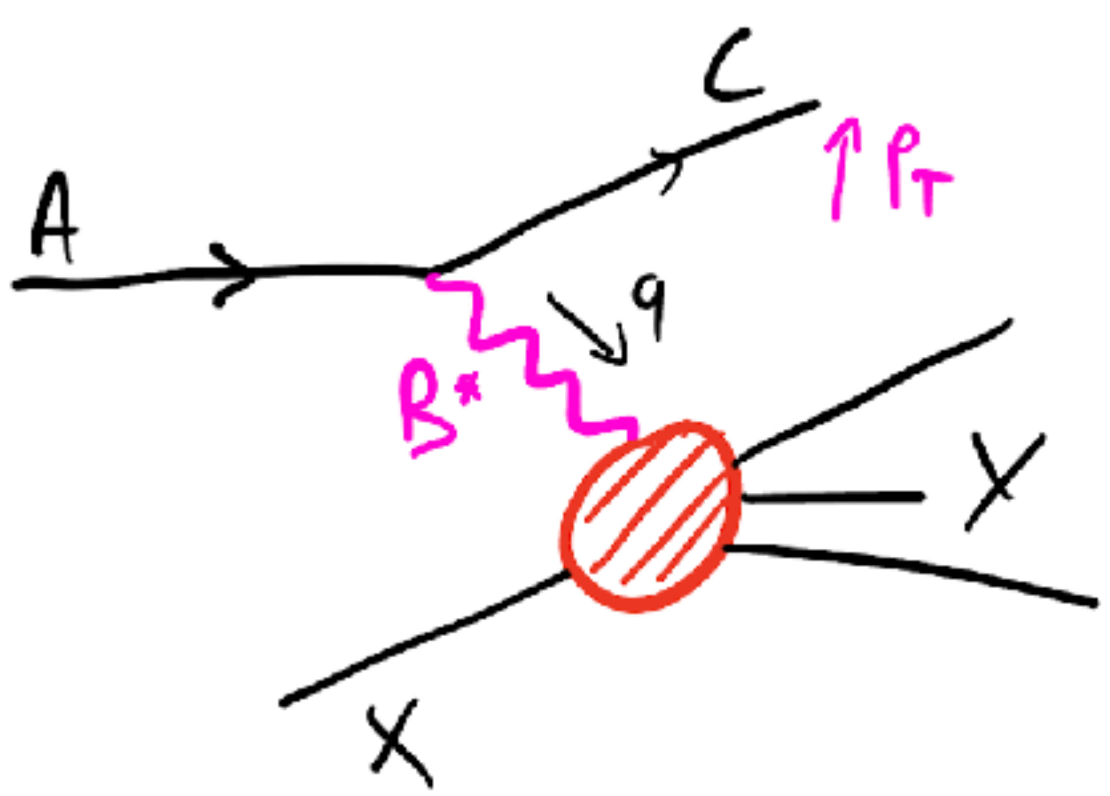

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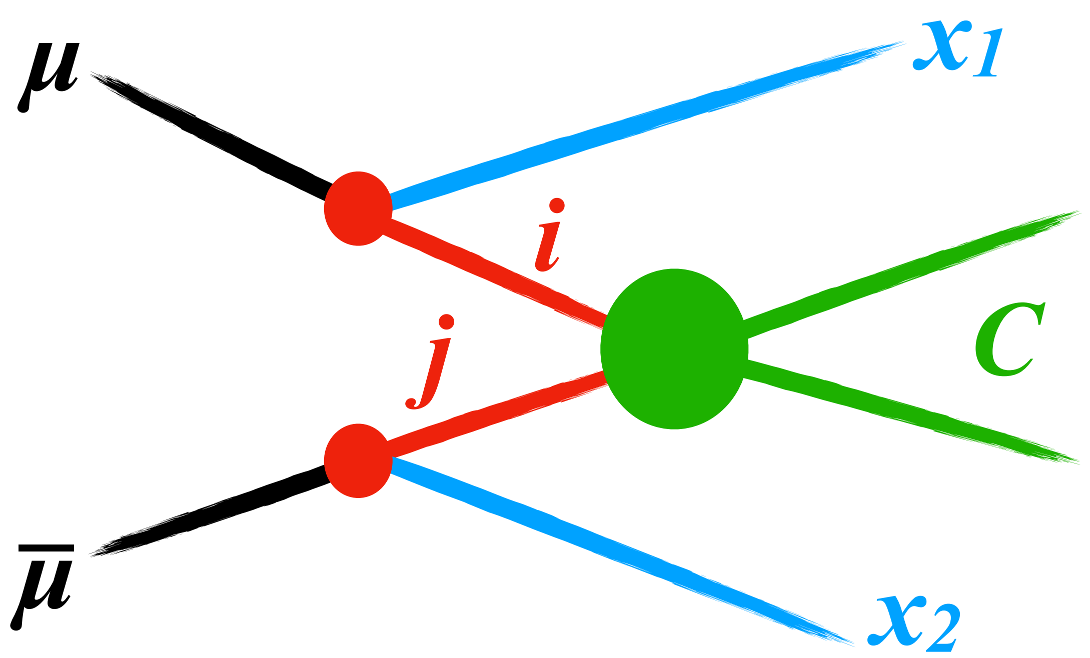

This allows to describe the hard process, inclusive over collinear radiation, in terms of **generalised Parton Distribution Functions**, like for proton colliders:

$$\sigma(\mu\bar{\mu} \to C + X) = \int_0^1 dx_1 \int_0^1 dx_2 \sum_{ij} f_i(x_1,\mu)\, f_j(x_2,\mu)\, \hat{\sigma}(ij \to C)(\hat{s})$$

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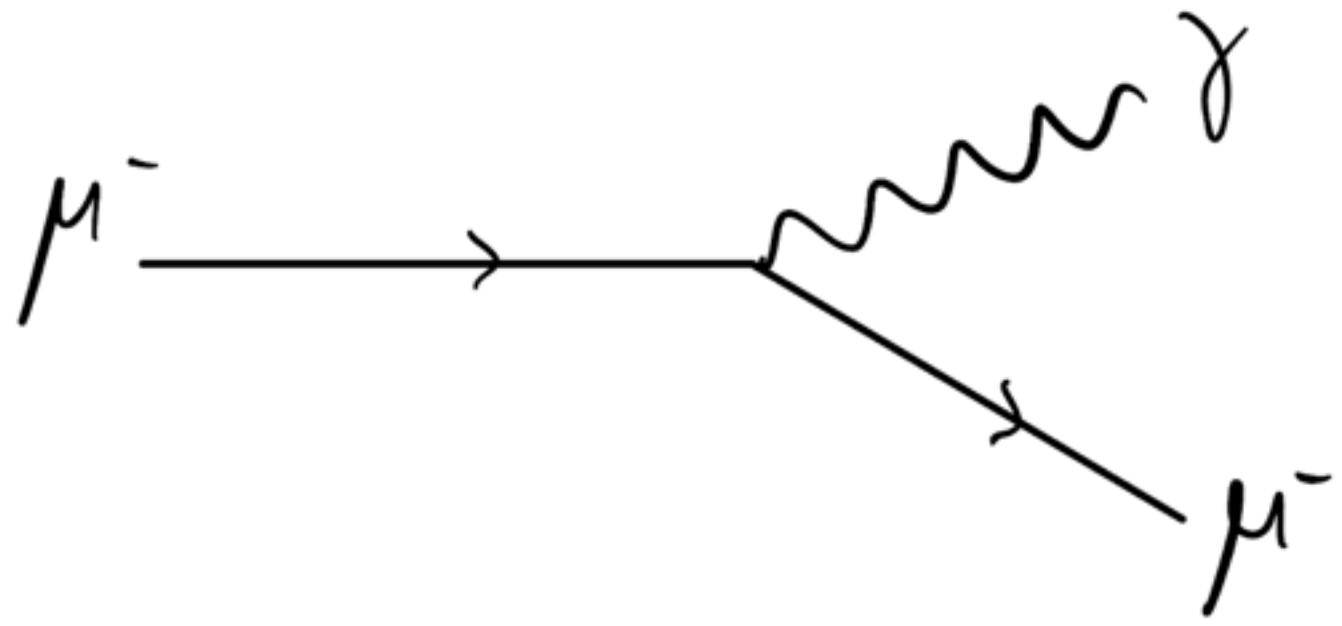
The case of collinear **photon emission** from an electron gives the ***Equilvalent Photon Approximation***



$$f_\gamma^{\mathrm{EPA}}(x) = \frac{\alpha}{2\pi} P_{\gamma e}(x) \log \frac{E^2}{m_e^2}$$

Fermi ('24) Weizsacker, Williams ('34) Landau, Lifschitz ('34)

LO Splitting function

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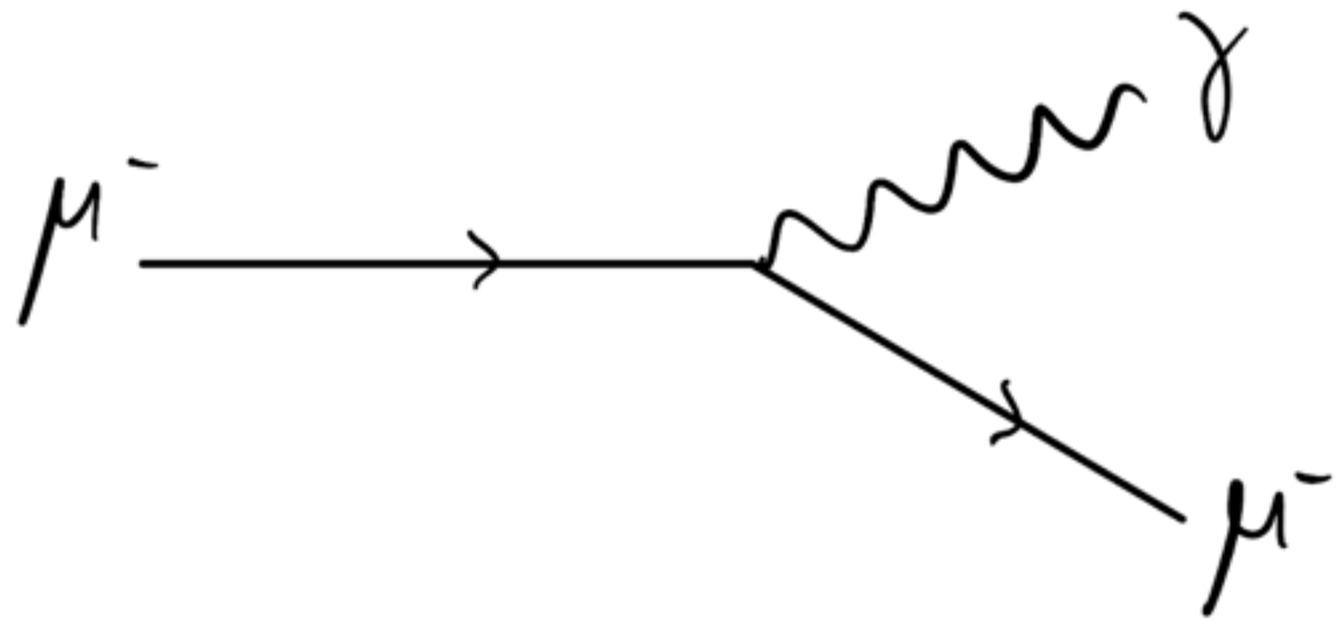

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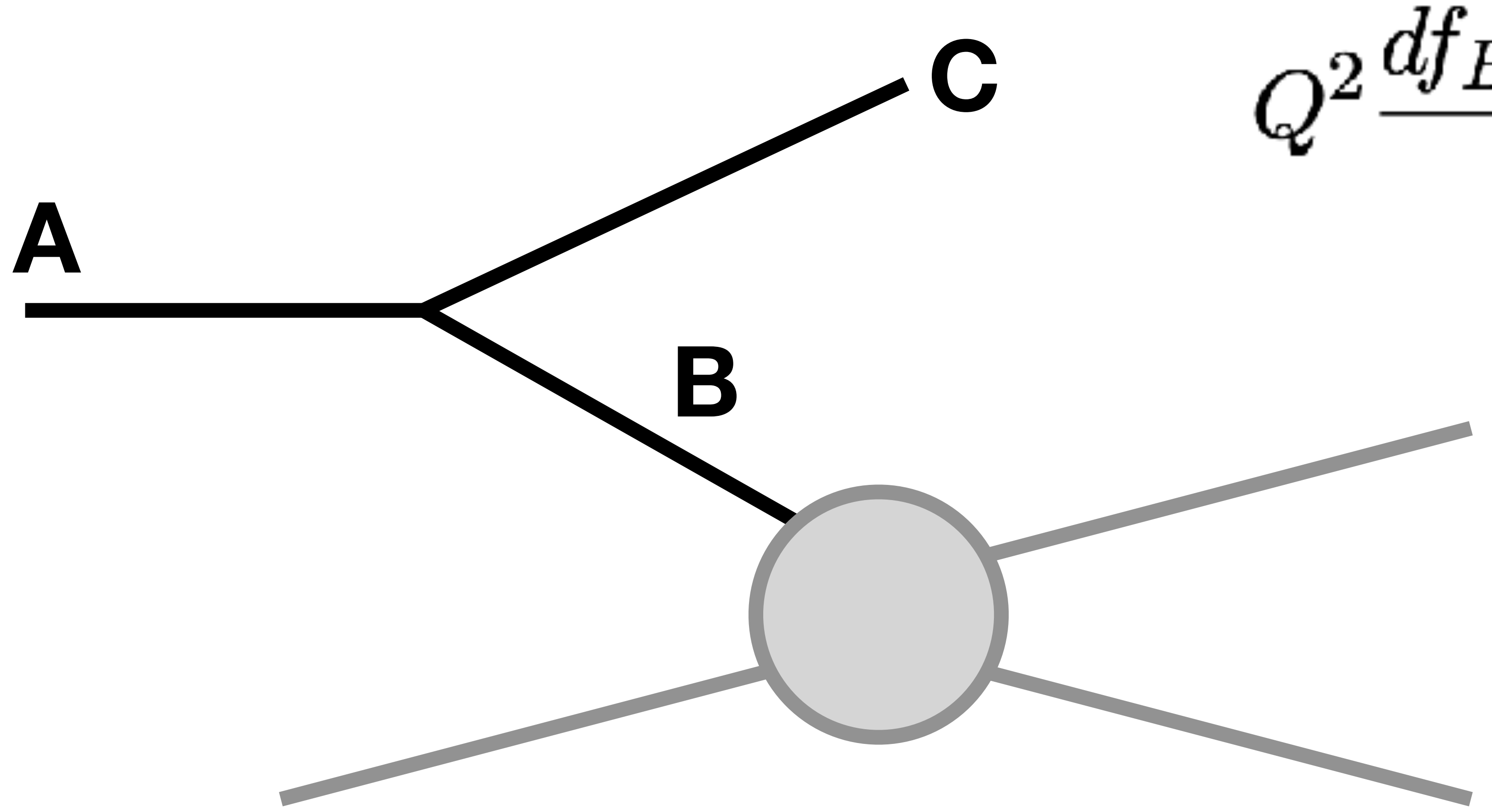

$$Q^2 \frac{df_B(x,Q^2)}{dQ^2} = P_B^v\, f_B(x,Q^2) + \sum_{A,C} \frac{\alpha_{ABC}(Q)}{2\pi} \int_x^1 \frac{dz}{z} P_{BA}^C(z)\, f_A\left(\frac{x}{z}, Q^2\right)$$

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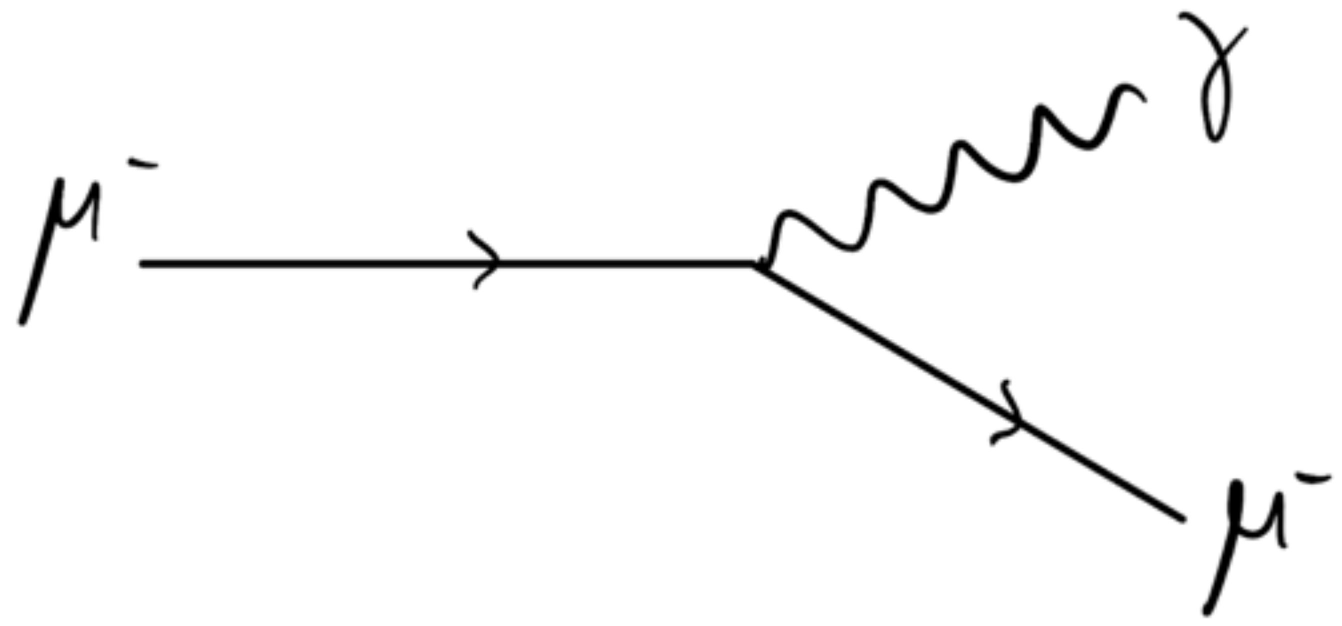

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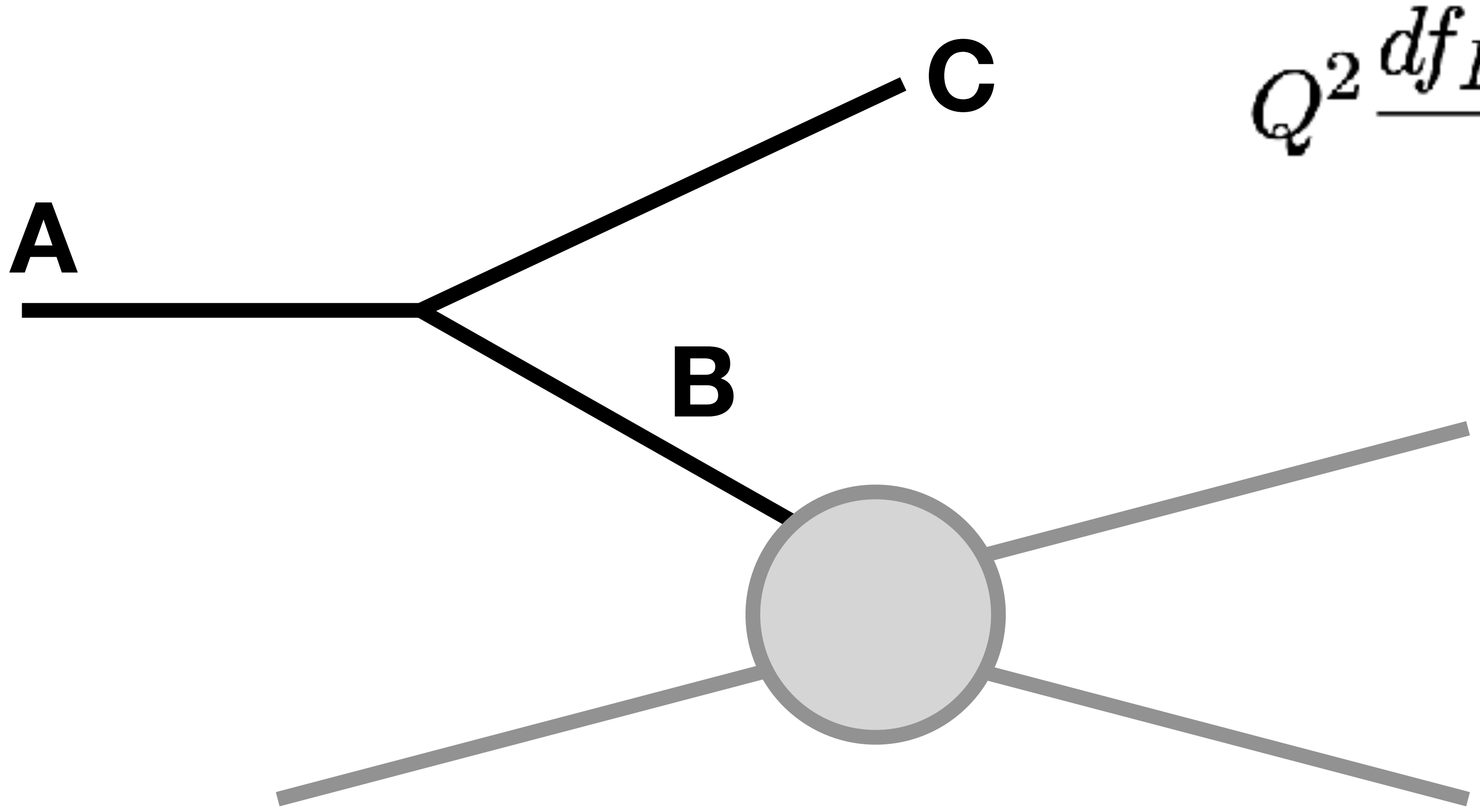

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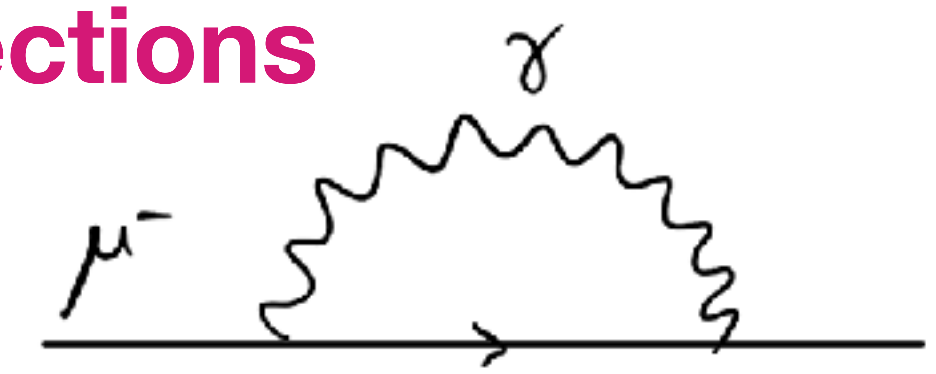
Virtual corrections



They cancel the **IR divergence** $(z \to 1)$ of **real soft emissions**.

Evolution below the EW scale

The boundary condition for the DGLAP equations is set by $f_\mu(x, m_\mu) = \delta(1\text{-}x) + O(\alpha)$

NLO corrections in Frixione [1909.03886]

DGLAP equations for a lepton can be solved from first principles (perturbative):

resummation of all the Leading Logarithms (**LL**): $(\alpha \log Q^2 / m_\mu{}^2)^n$.

For **QED:** $\alpha \log \ll 1$, so **fixed-order approximation** can be sufficient for accessible scales.

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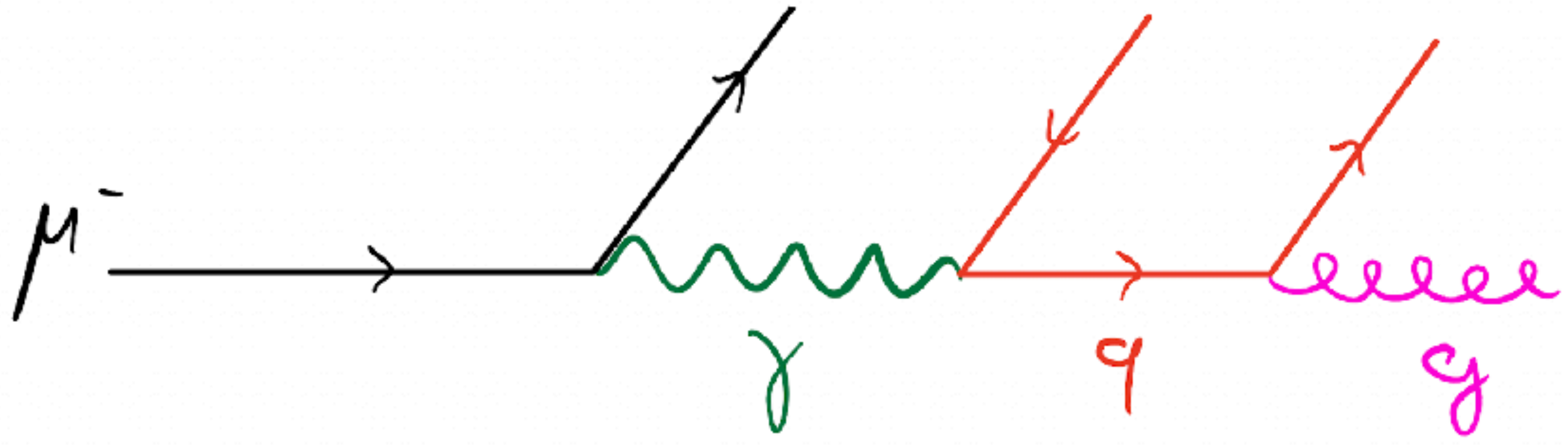
However, for **collinear QCD radiation** (quark & gluon content of a lepton) **resummation is required**.



Evolution below the EW scale

The boundary condition for the DGLAP equations is set by $f_\mu(x, m_\mu) = \delta(1\text{-}x) + O(\alpha)$

NLO corrections in Frixione [1909.03886]

DGLAP equations for a lepton can be solved from first principles (perturbative):

resummation of all the Leading Logarithms (**LL**): $(\alpha \log Q^2 / m_\mu^2)^n$.

For **QED:** $\alpha \log \ll 1$, so **fixed-order approximation** can be sufficient for accessible scales.

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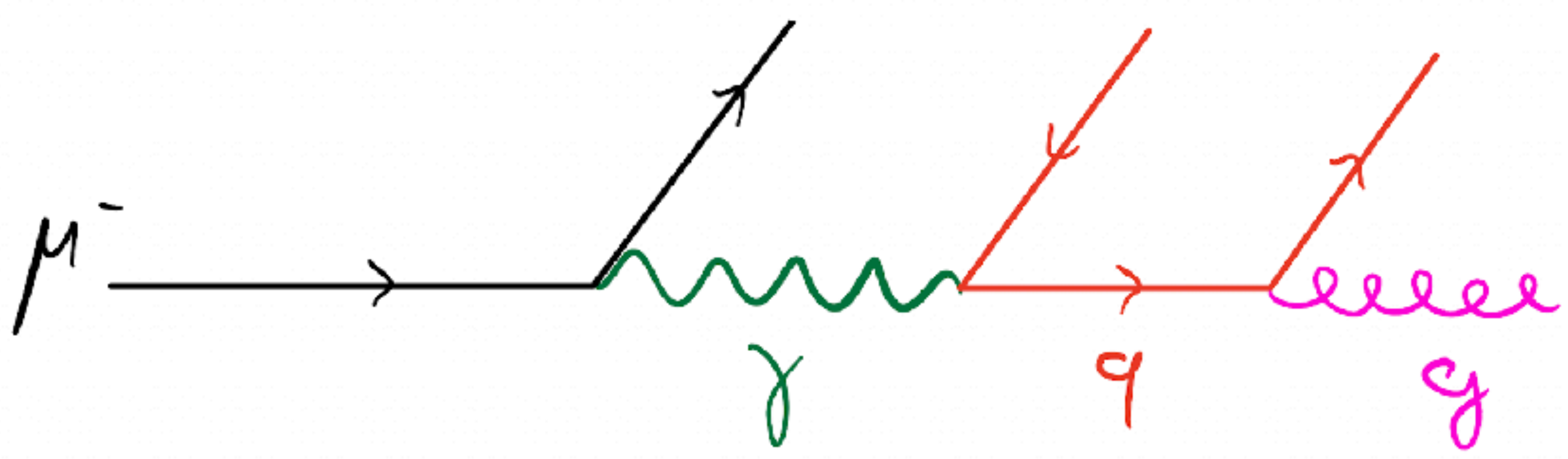
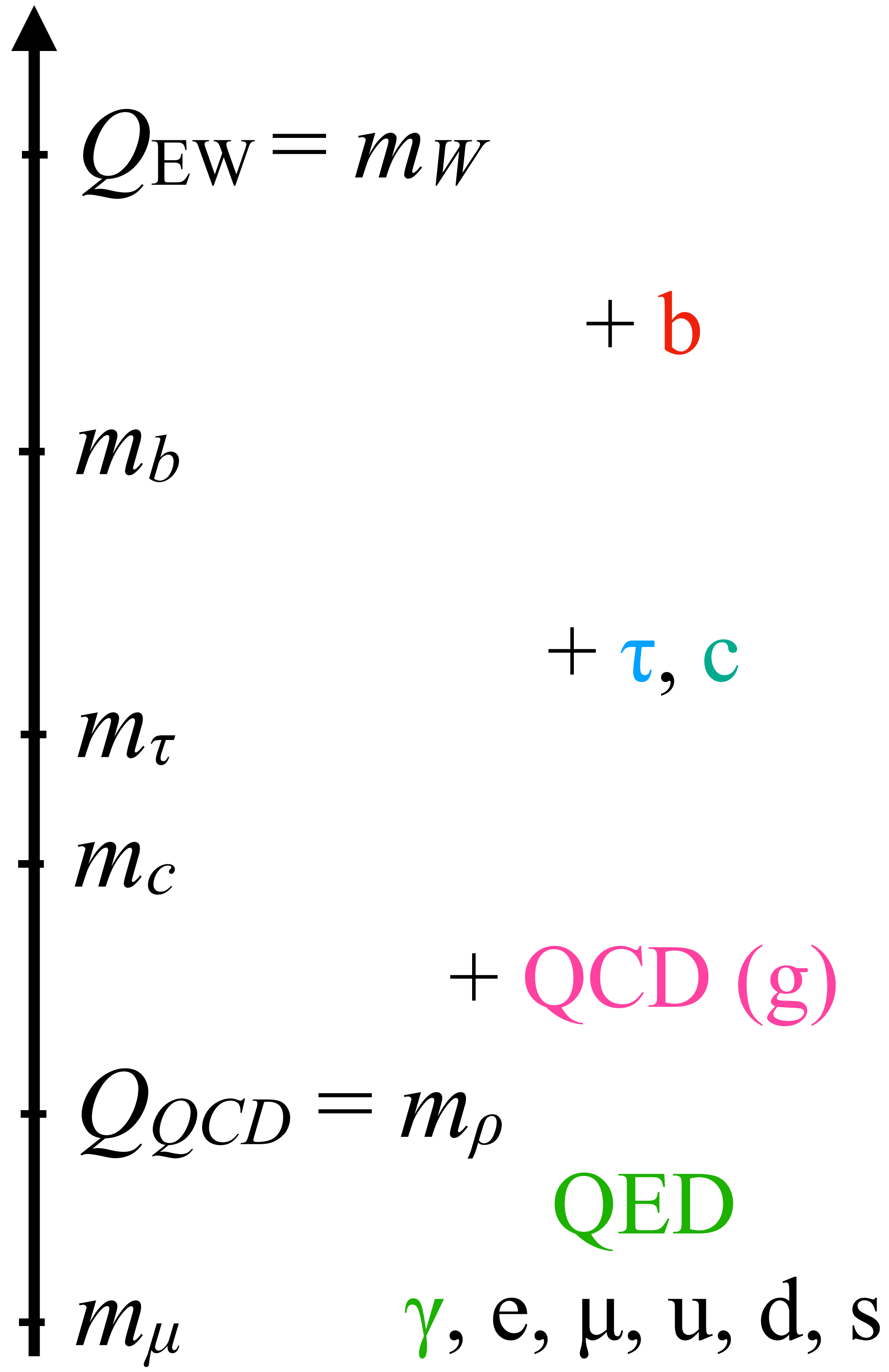


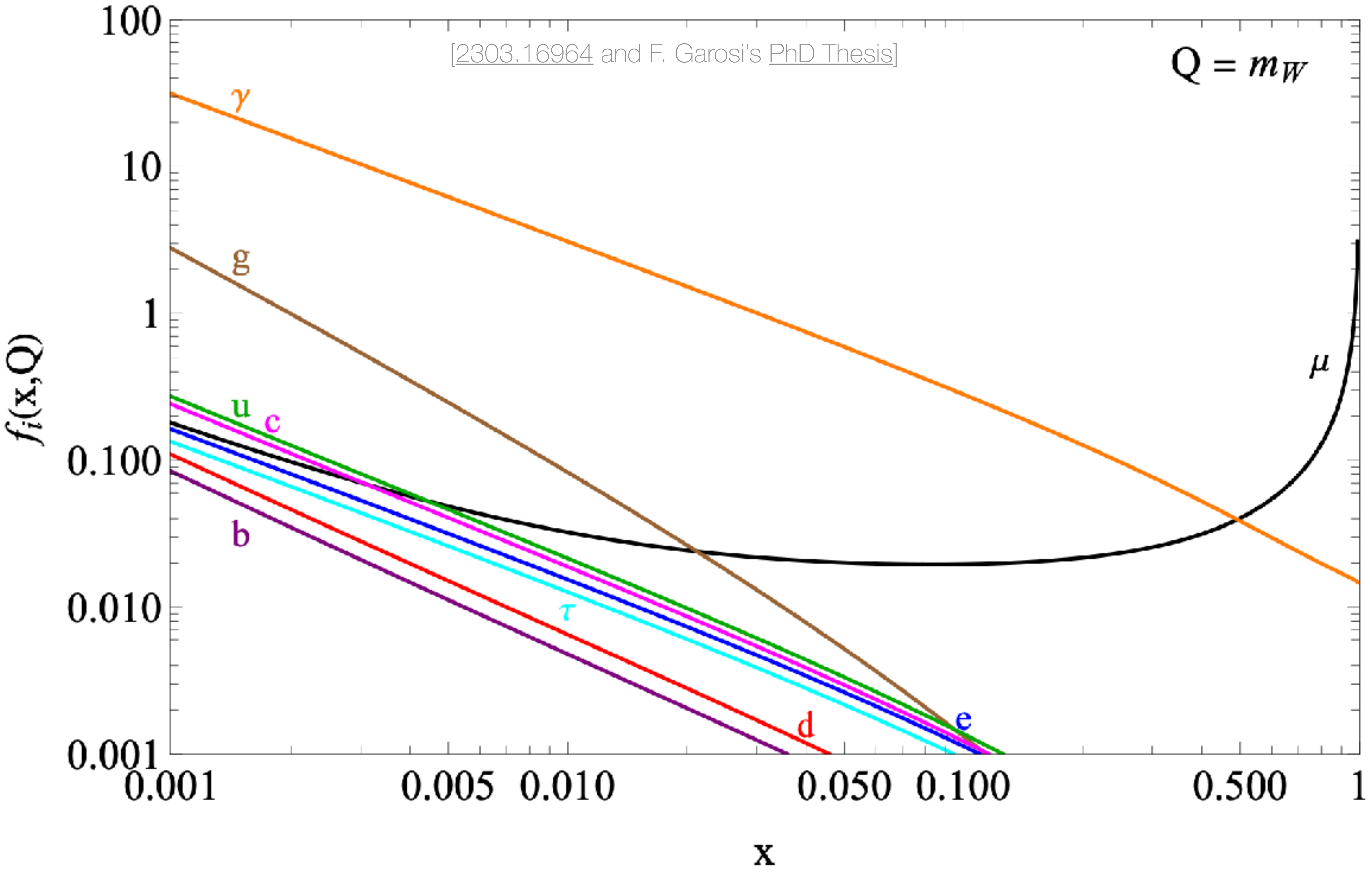
Treatment of **QCD**: gluons only active above Q_{QCD}.

Drees, Godbole [hep-ph/9403229]
Han, Ma, Xie [2103.09844]

For a different approach see e.g. Frixione, Stagnitto [2309.07516]



Reaching the EW scale



Due to CP and flavour symmetry:

$$f_e = f_{\bar{e}} = f_{\bar{\mu}}\ , \qquad f_c = f_{\bar{c}}\ ,$$
$$f_\tau = f_{\bar{\tau}}\ , \qquad f_d = f_{\bar{d}} = f_s = f_{\bar{s}}\ ,$$
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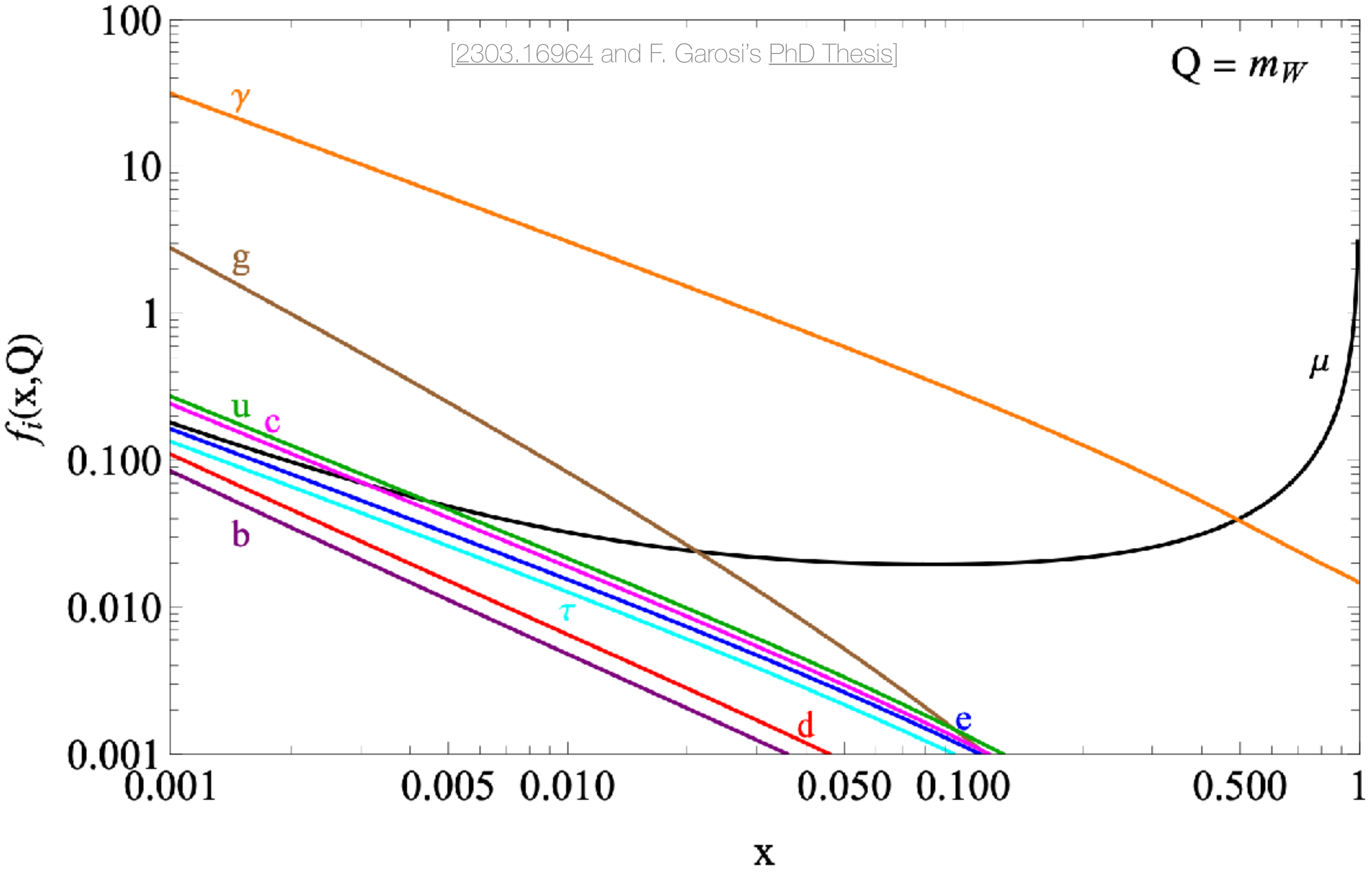
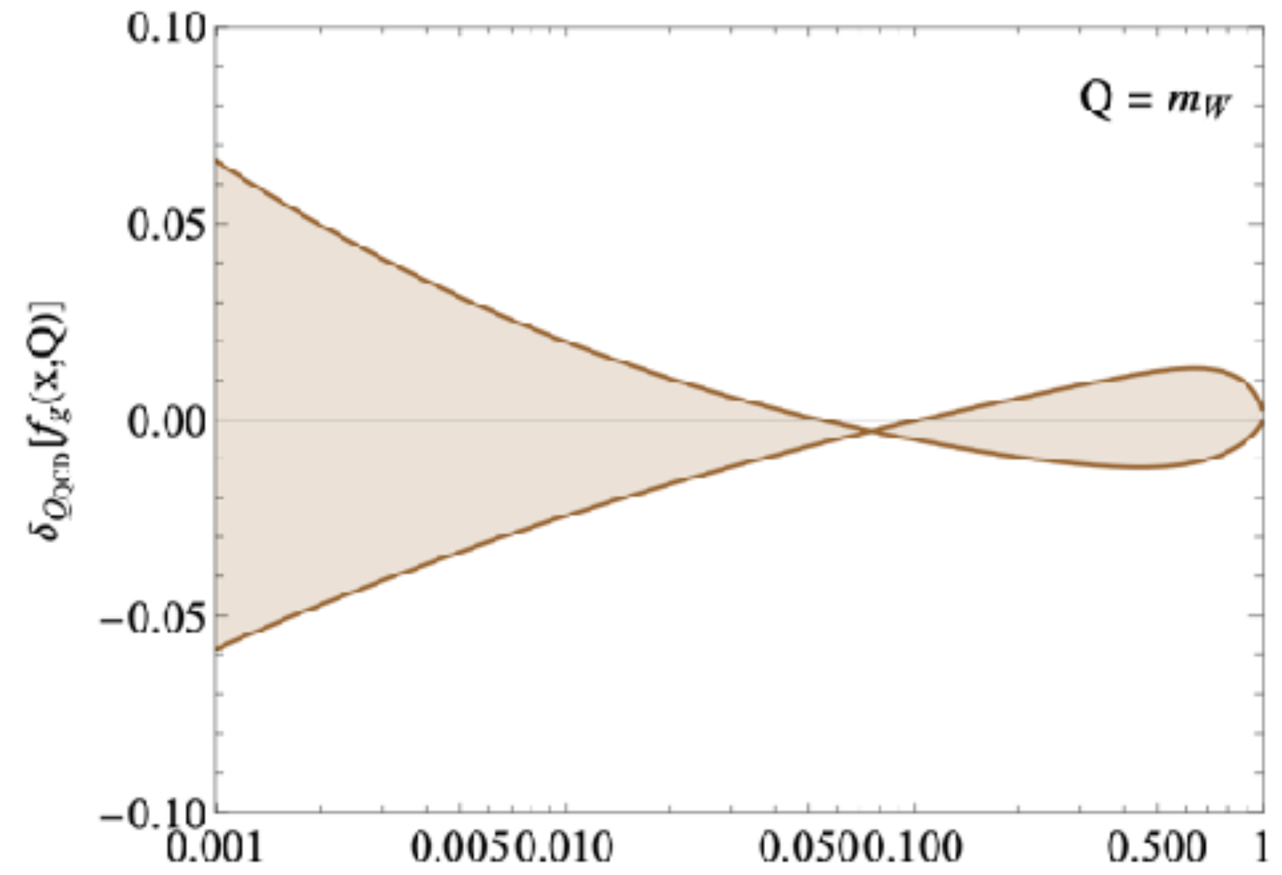
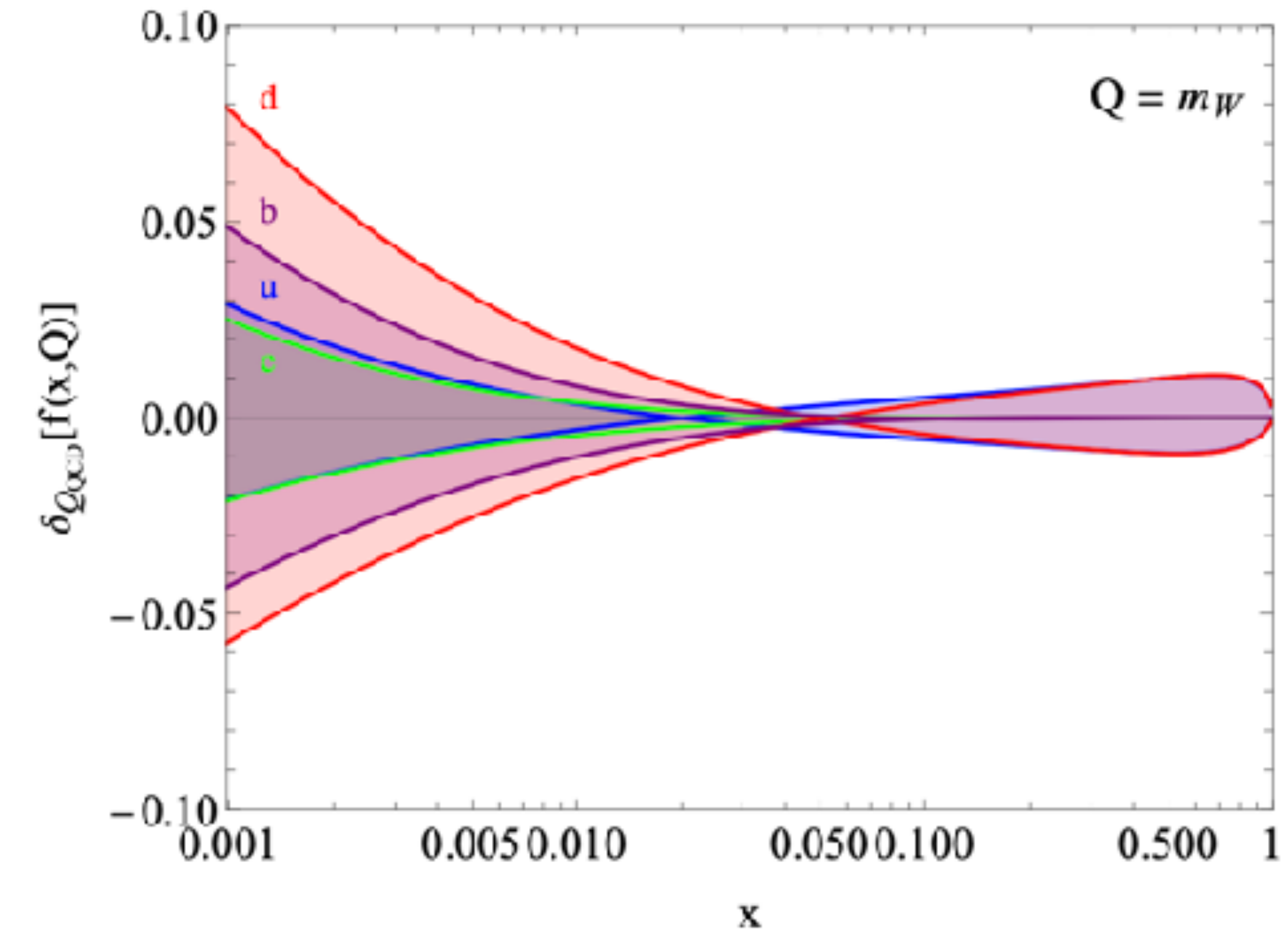


Changing the QCD scale in

$$Q_{QCD} = [0.5 - 1]\ \text{GeV}$$





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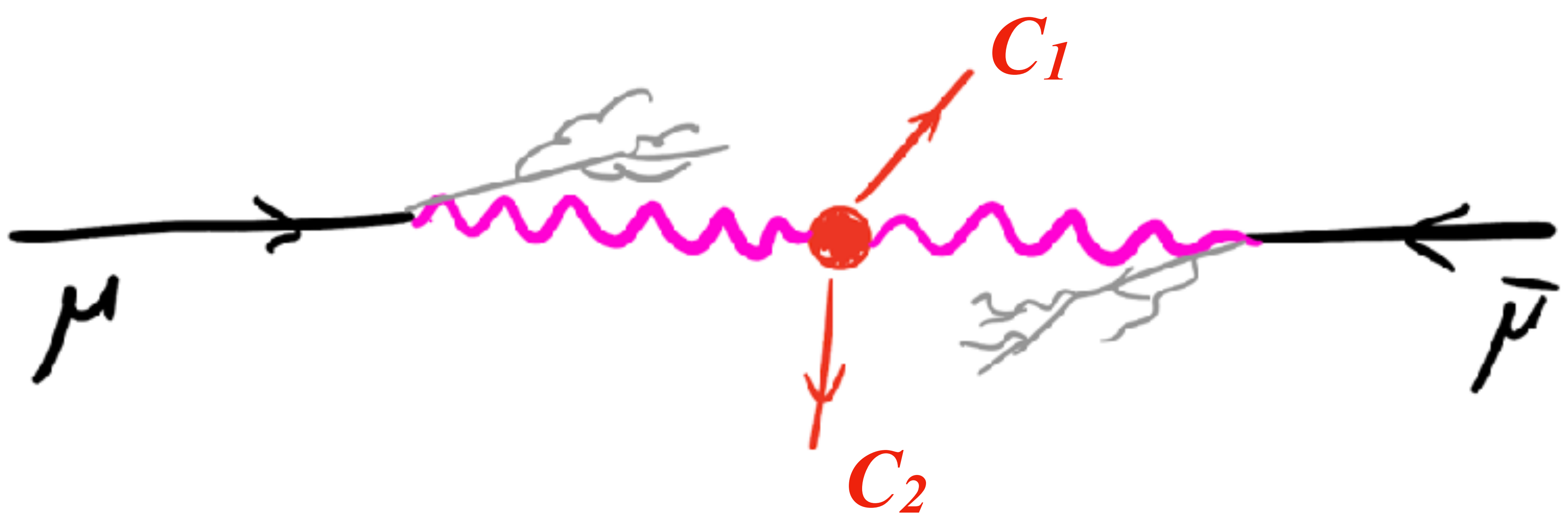
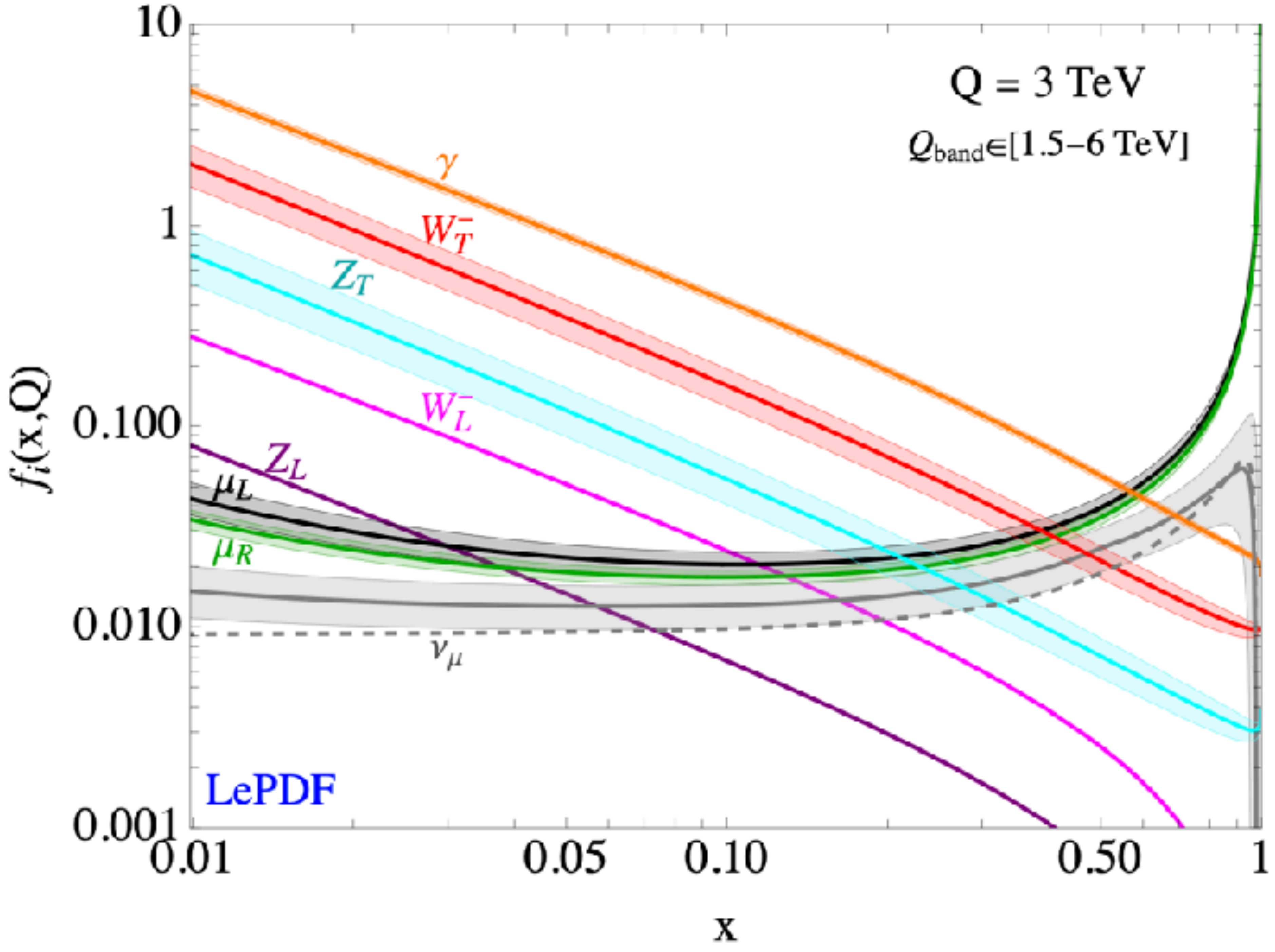
For **leptons** and the **photon**, relative variations are **smaller than 10^{-5}**.

Above the EW scale

EW radiation (W and Z bosons) becomes as important as QED.



$$\int \sigma_{EW} \propto \alpha_2 \left(\log \frac{E^2}{m_W^2}\right)^2$$

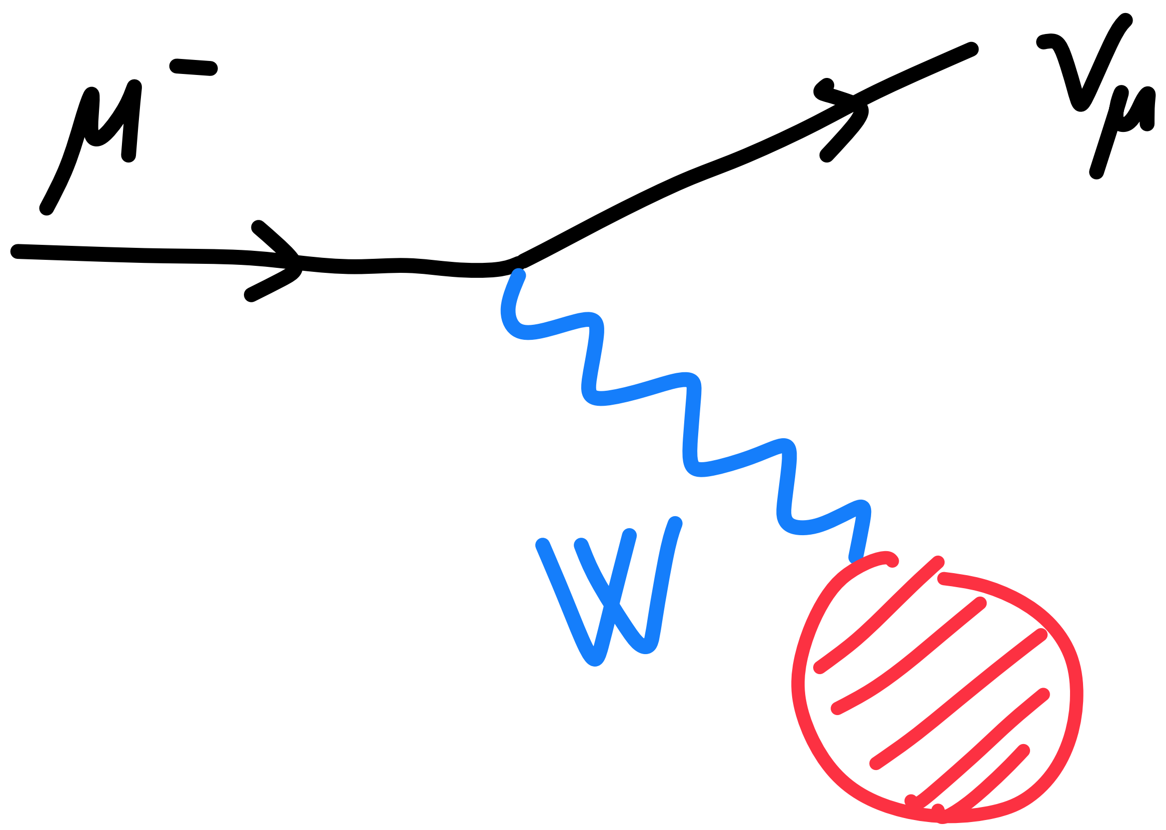


Effective Vector Boson Approximation

At energies **above the EW scale**, **collinear emission of EW gauge bosons** can be described at **LO** with the **Effective Vector Boson Approximation**

Kane, Repko, Rolnik; Dawson; Chanowitz, Gaillard '84, See also Borel et al. [1202.1904], Costantini et al. [2005.10289] Ruiz et al. [2111.02442], etc…

Can be derived by solving the DGLAP equations at fixed order α.



$$f^{(\alpha)}_{W^-_\pm}(x,Q^2) = \frac{\alpha_2}{8\pi} P^f_{V_\pm f_L}(x) \left(\log \frac{Q^2 + (1-x)m_W^2}{m_\mu^2 + (1-x)m_W^2} - \frac{Q^2}{Q^2 + (1-x)m_W^2} \right)$$

$$f^{(\alpha)}_{W^-_L}(x,Q^2) = \frac{\alpha_2}{4\pi} \frac{1-x}{x} \frac{Q^2}{Q^2 + (1-x)m_W^2}$$

(similar expressions also for Z_T, Z_L, Z/γ)

This is now implemented in `MadGraph5_aMC@NLO`

[Ruiz, Costantini, Maltoni, Mattelaer 2111.02442]

Do we need SM/EW PDFs?

The W, Z **PDFs** are suppressed compared to the photon only by a factor ~ 3 at O(few) TeV.
They induce the **dominant contribution in a large class of processes** (vector boson collider).

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Why not just EVA?

★ For **QCD (gluon and quarks) DGLAP resummation is required** since α_s is large at small scales.

★ The expected **relative corrections to the LO EVA** result are proportional to (*Sudakov double logs*)

$$\alpha_2 \left(\log \frac{Q^2}{M_W^2} \right)^2 \sim 1$$

for $Q \sim 1.5$ TeV.
still sizeable at lower Q.

For precise vector boson PDFs at the TeV scale it is important to re-sum the EW double logs.

→ **PDF approach**

M. Ciafaloni, P. Ciafaloni, Comelli [hep-ph/0111109]
Bauer, Ferland, Webber [1703.08562]

EW PDFs

All SM interactions and fields must be considered and several new effects must be taken into account:

- **PDFs become polarised**, since EW interactions are chiral. Bauer, Webber [1808.08831]

- At high energies **EW Sudakov double logarithms** are generated. P. Ciafaloni, Comelli [hep-ph/0007096, hep-ph/0001142, hep-ph/0505047], Bauer, Webber [1703.08562, 1808.08831], Chen, Han, Tweedie [1611.00788],

- **Neutral bosons interfere with each other: Z/γ and h/Z_L PDFs mix**. P. Ciafaloni, Comelli [hep-ph/0007096, hep-ph/0505047] Chen, Han, Tweedie [1611.00788]

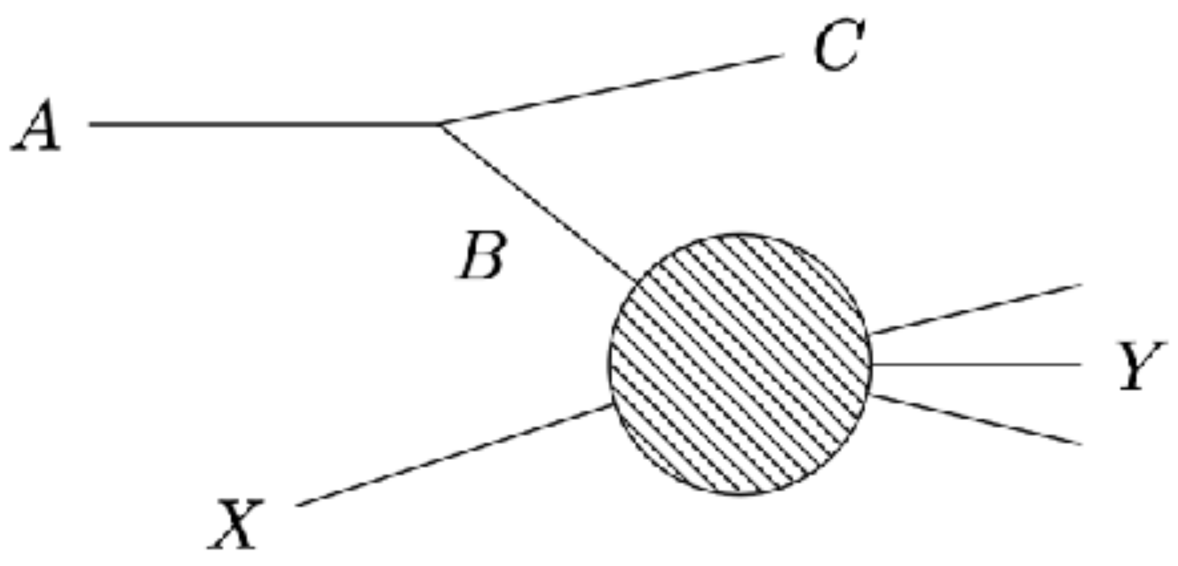
- **Mass effects** of partons with EW masses (W, Z, h, t) become relevant and some remain so even at multi-TeV scale.

- EW symmetry is broken. Another set of splitting functions, proportional to v^2 instead of p_T^2, arise: **ultra-collinear splitting functions**. Chen, Han, Tweedie [1611.00788]

PDFs of a muon

The **DGLAP equations** describe the evolution of the PDFs

M. Ciafaloni, P. Ciafaloni, D. Comelli hep-ph/0111109, hep-ph/0505047]



$$Q^2 \frac{df_B(x,Q^2)}{dQ^2} = P_B^v \, f_B(x,Q^2) + \sum_{A,C} \frac{\alpha_{ABC}}{2\pi} \widetilde{P}_{BA}^C \otimes f_A + \frac{v^2}{16\pi^2 Q^2} \sum_{A,C} \widetilde{U}_{BA}^C \otimes f_A$$

Virtual corrections

Real emission

ultra-collinear terms (EWSB)

Chen, Han, Tweedie [1611.00788]

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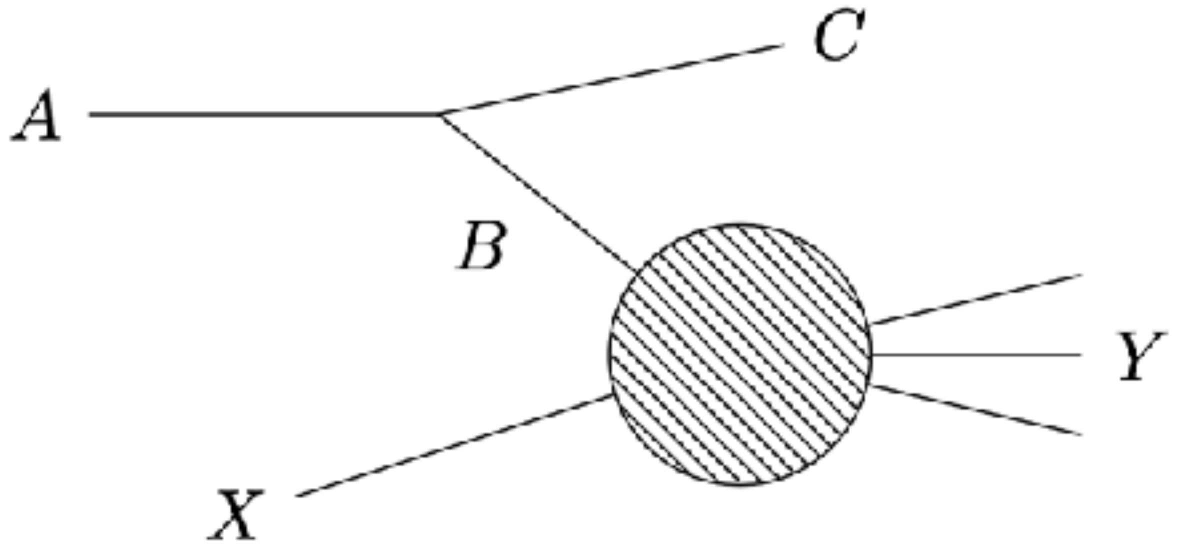

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Virtual corrections

Real emission

ultra-collinear terms (EWSB)

Chen, Han, Tweedie [1611.00788]

After identifying PDFs which are identical because of flavour symmetry, we remain with **54 independent PDFs**.

$$f_{\bar{e}_L} = f_{\bar{\mu}_L}\,, \quad f_{\bar{e}_R} = f_{\bar{\mu}_R}\,, \quad f_{\bar{\nu}_e} = f_{\bar{\nu}_\mu}\,,$$
$$f_{d_R} = f_{s_R}\,, \quad f_{\bar{d}_R} = f_{\bar{s}_R}\,.$$

Leptons	$e_{L,R}$	$\mu_{L,R}$	$\tau_{L,R}$	ν_e	ν_μ	ν_τ	$\bar{\ell}_{L,R}$	$\bar{\tau}_{L,R}$	$\bar{\nu}_\ell$	$\bar{\nu}_\tau$
Quarks	$u_{L,R}$	$d_{L,R}$	$c_{L,R}$	s_L	$t_{L,R}$	$b_{L,R}$	+ h.c.			
Gauge Bosons	$\gamma_\pm$	$Z_\pm$	$Z/\gamma_\pm$	$W^\pm_\pm$	$g_\pm$					
Scalars	h	Z_L	h/Z_L	$W^\pm_L$						

Ultra-collinear emissions

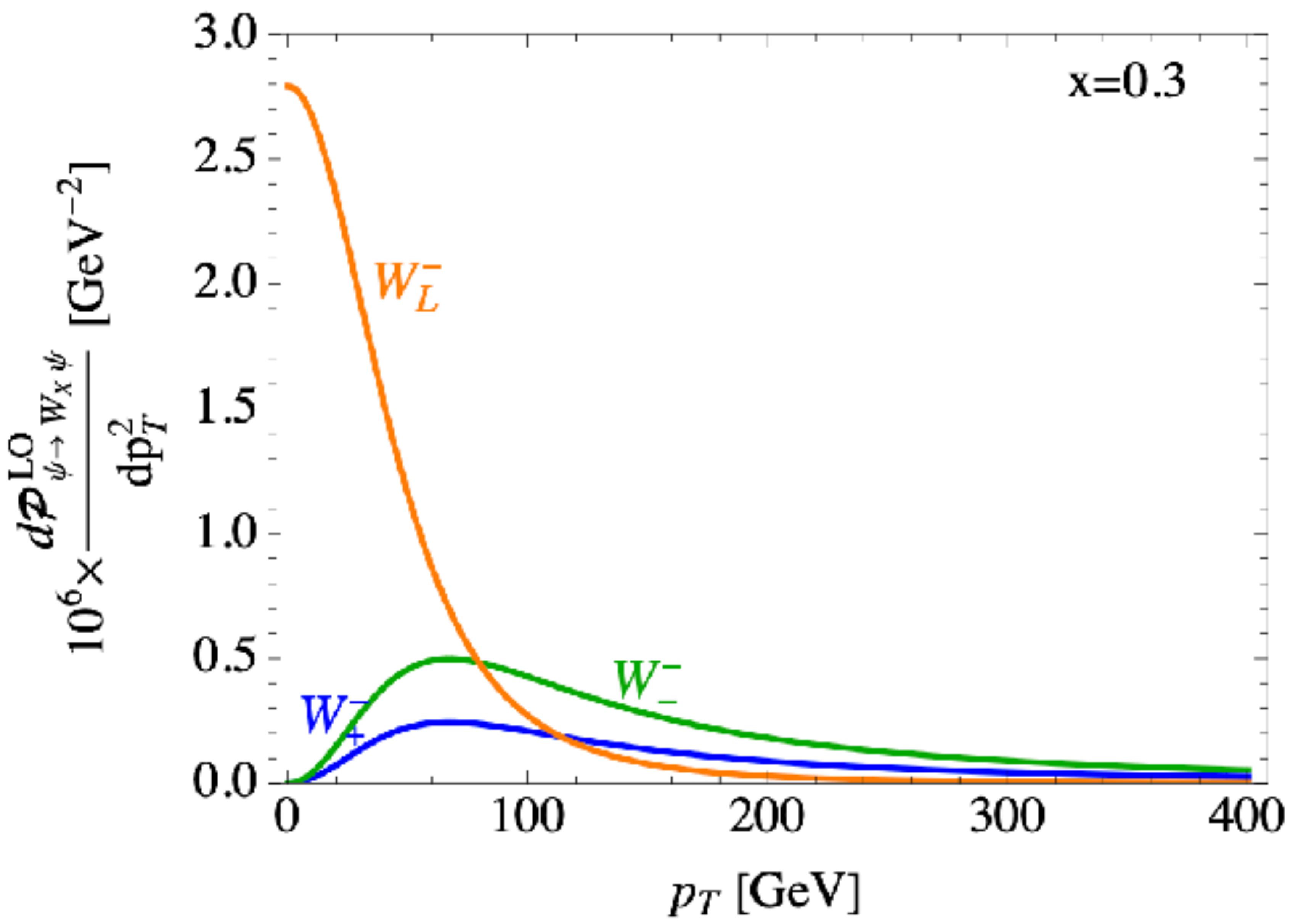
Upon EWSB, further **splittings proportional to v^2** are generated.
They generalise the EWA splitting $f \to W_L f'$ Chen, Han, Tweedie [1611.00788]

ultra-collinear terms (EWSB)

$$Q^2 \frac{df_B(x,Q^2)}{dQ^2} = P_B^v f_B(x,Q^2) + \sum_{A,C} \frac{\alpha_{ABC}}{2\pi} \widetilde{P}_{BA}^C \otimes f_A + \frac{v^2}{16\pi^2 Q^2} \sum_{A,C} \widetilde{U}_{BA}^C \otimes f_A$$

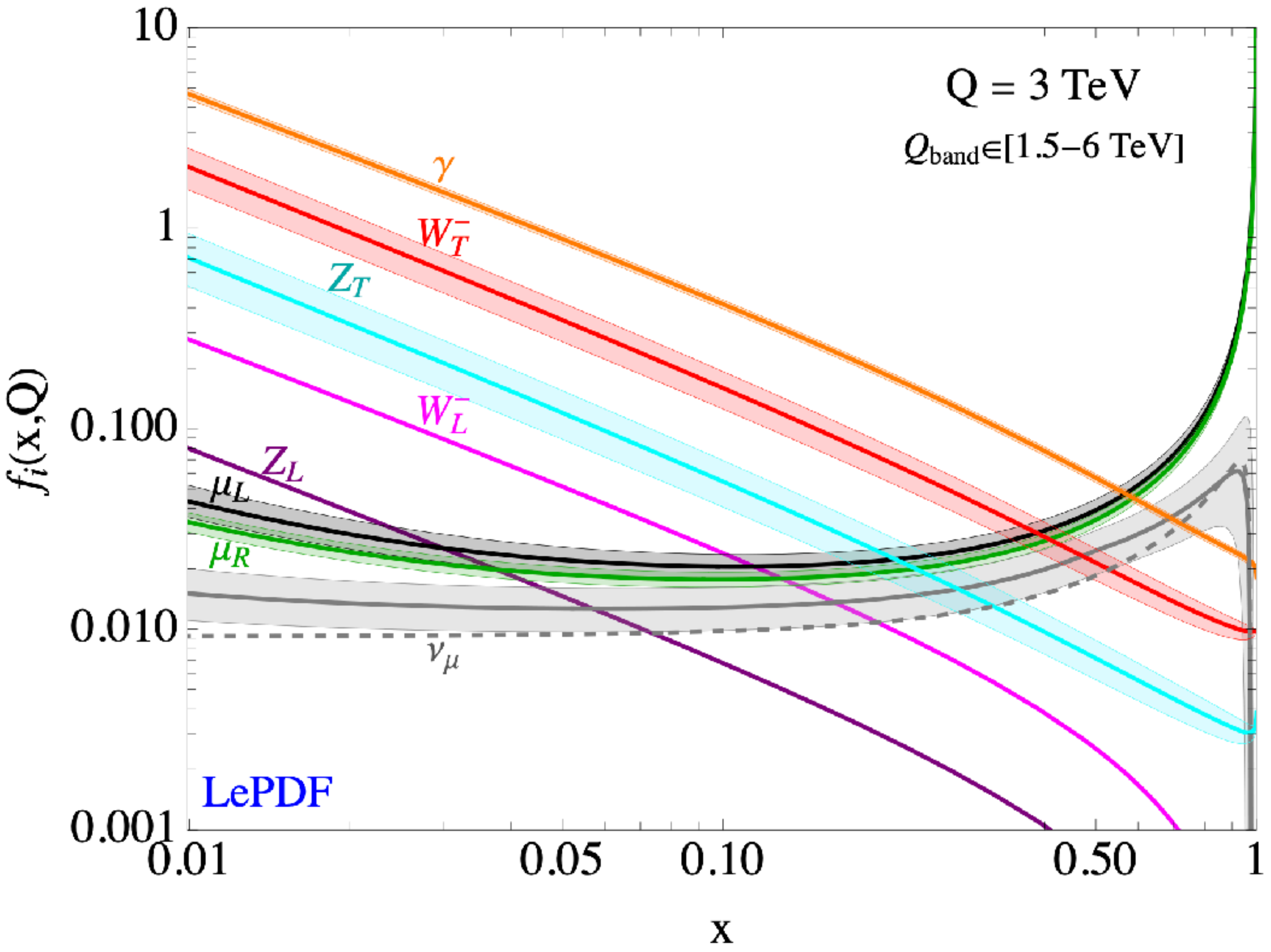
$$f_{W_\pm^-}^{(\alpha)}(x,Q^2) = \int_{m_\mu^2}^{Q^2} dp_T^2 \frac{1}{2} \frac{d\mathcal{P}_{\psi \to W_T \psi}}{dp_T^2}(x,p_T^2)$$

The extra Q^2 factor in the denumerator removes the logarithmic increase with scale: UC-terms contributions are **dominated from emissions with p_T below m_W**. They become constant at large scales.



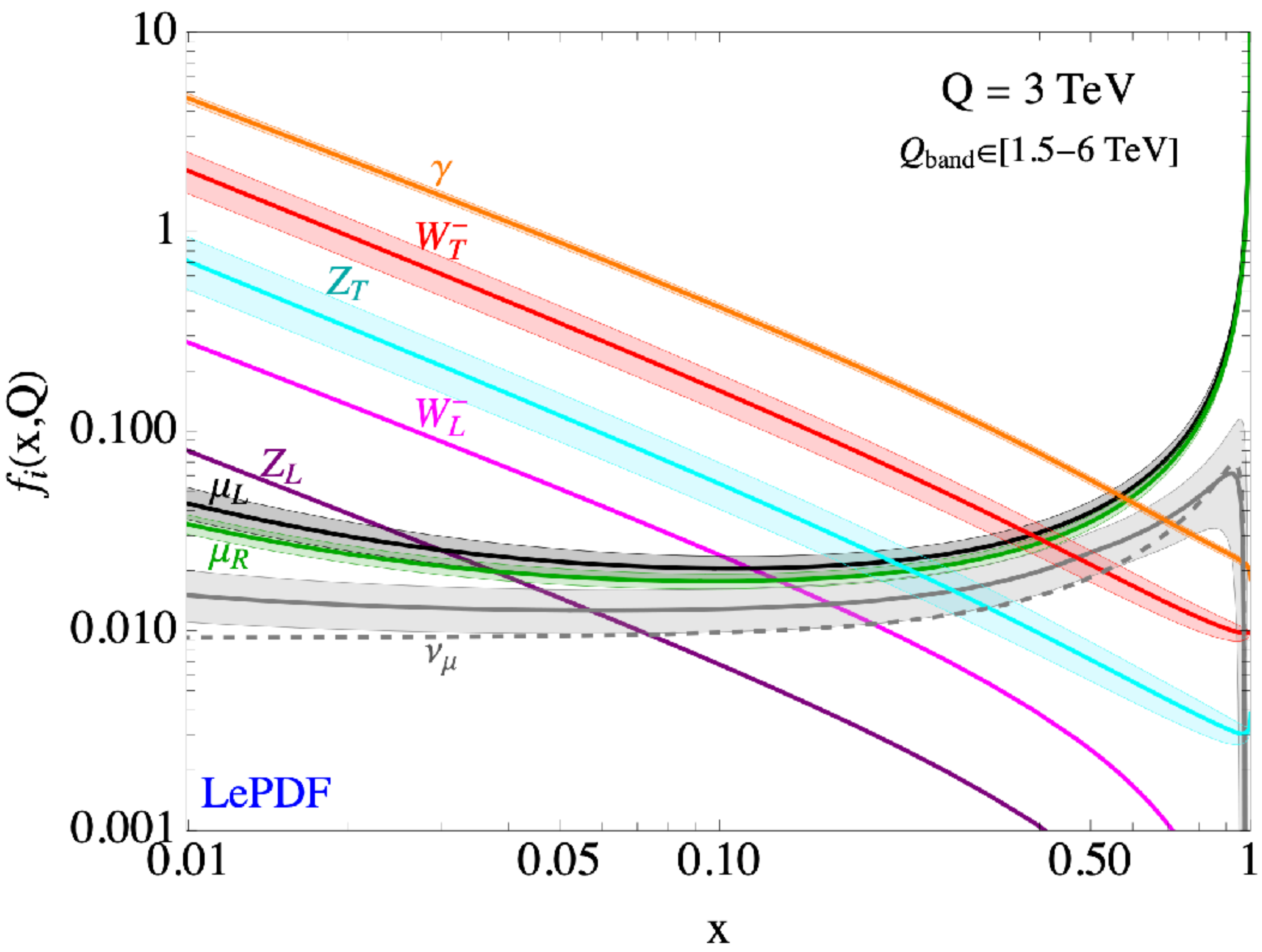
[we match at Q=m_W with the value obtained via the LO EWA result at that same scale.]

LePDF



- Sizeable PDFs of EW gauge bosons
- Large muon-neutrino PDF for $x \gtrsim 0.5$

LePDF



- Sizeable PDFs of EW gauge bosons
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We show **scale uncertainties** by varying the factorisation scale by a factor of 2.

QED: $\log \frac{Q^2}{m_\mu^2} \to 7\%$ @ 3TeV

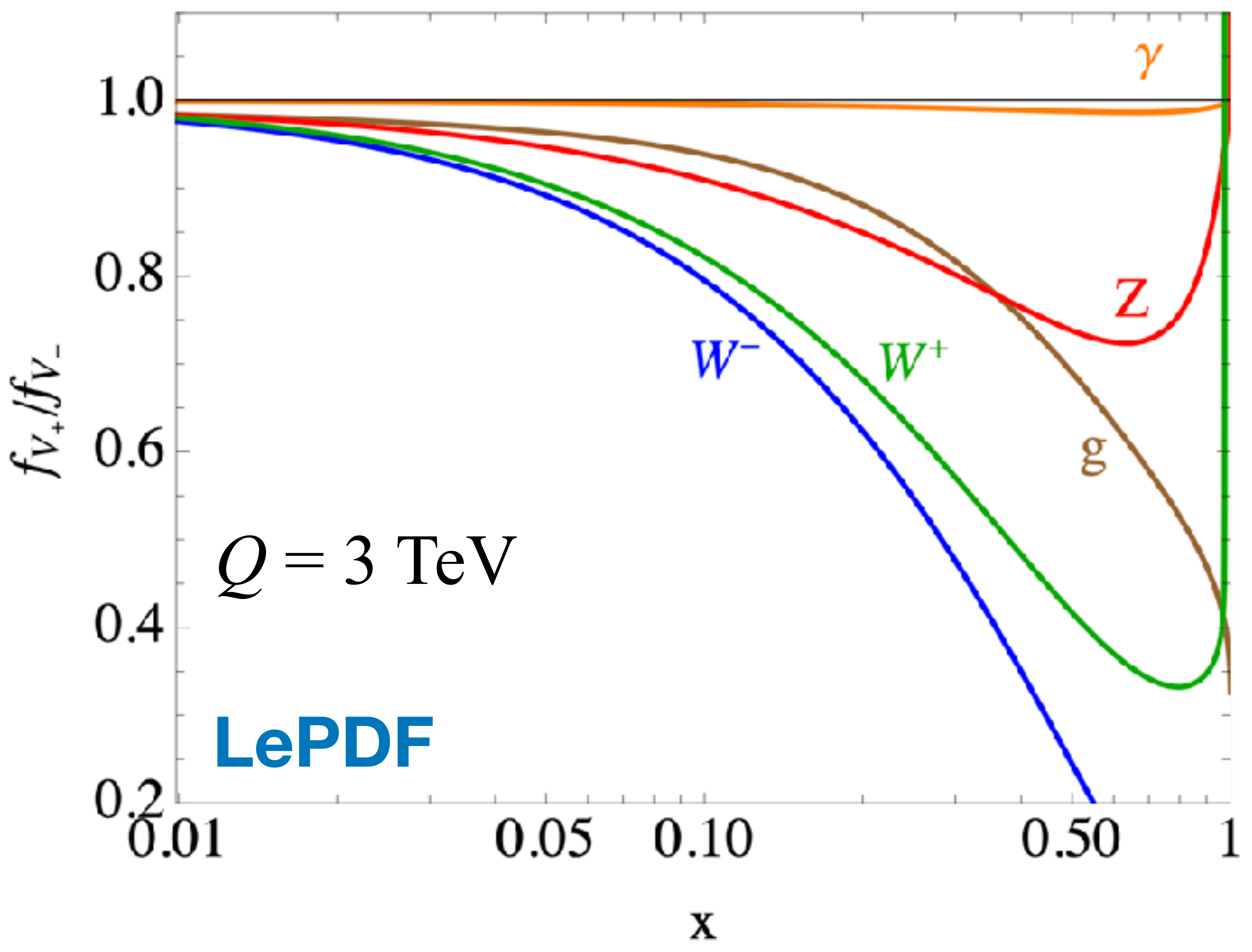
EW: $\log \frac{Q^2}{m_W^2} \to 24\%$ @ 3TeV

Theory improvements are required to reduce these uncertainties down to the percent level.

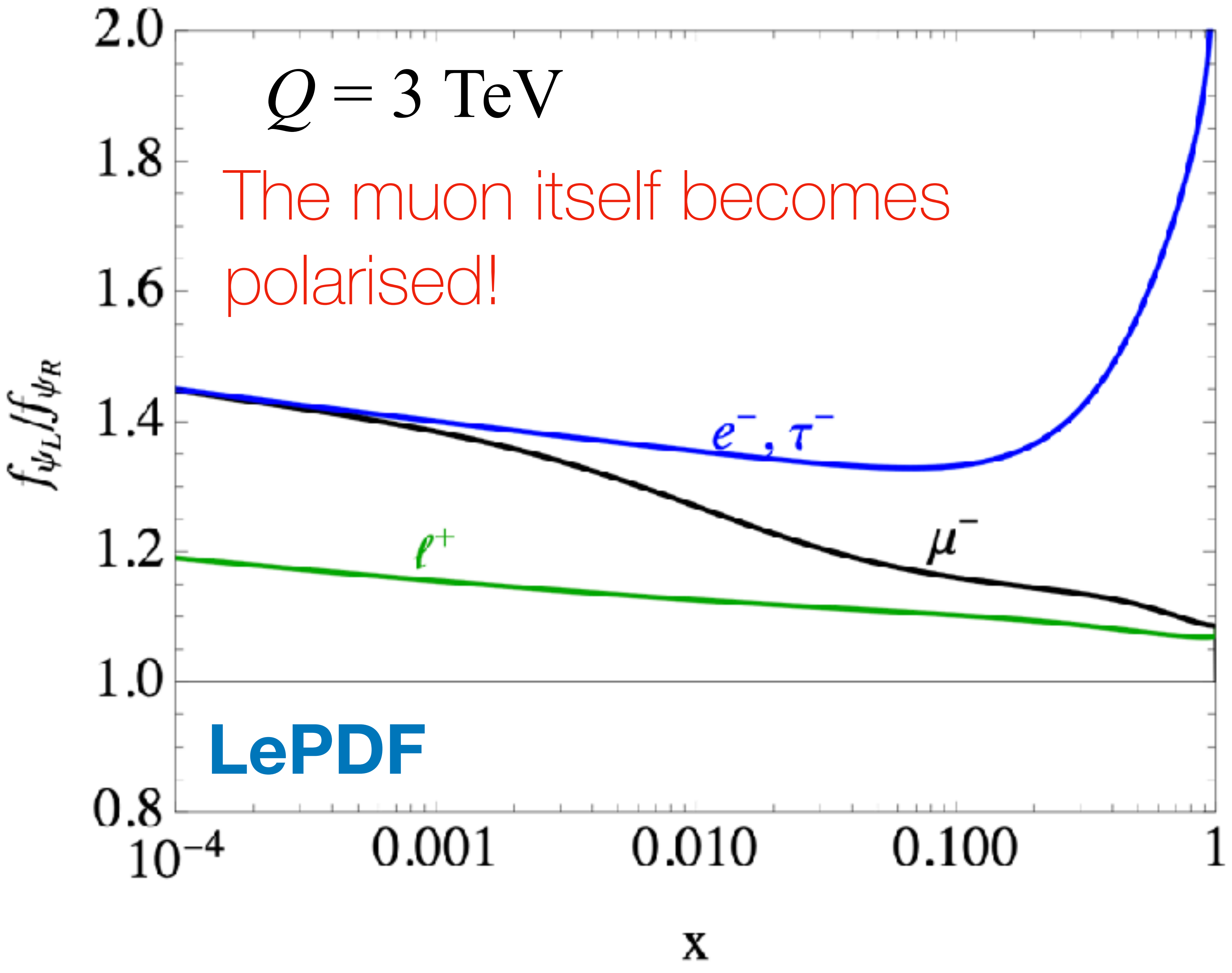
Polarisation

Since EW interactions are chiral, PDFs become polarised. Bauer, Webber [1808.08831]

Vectors polarisation: V_+ / V_-



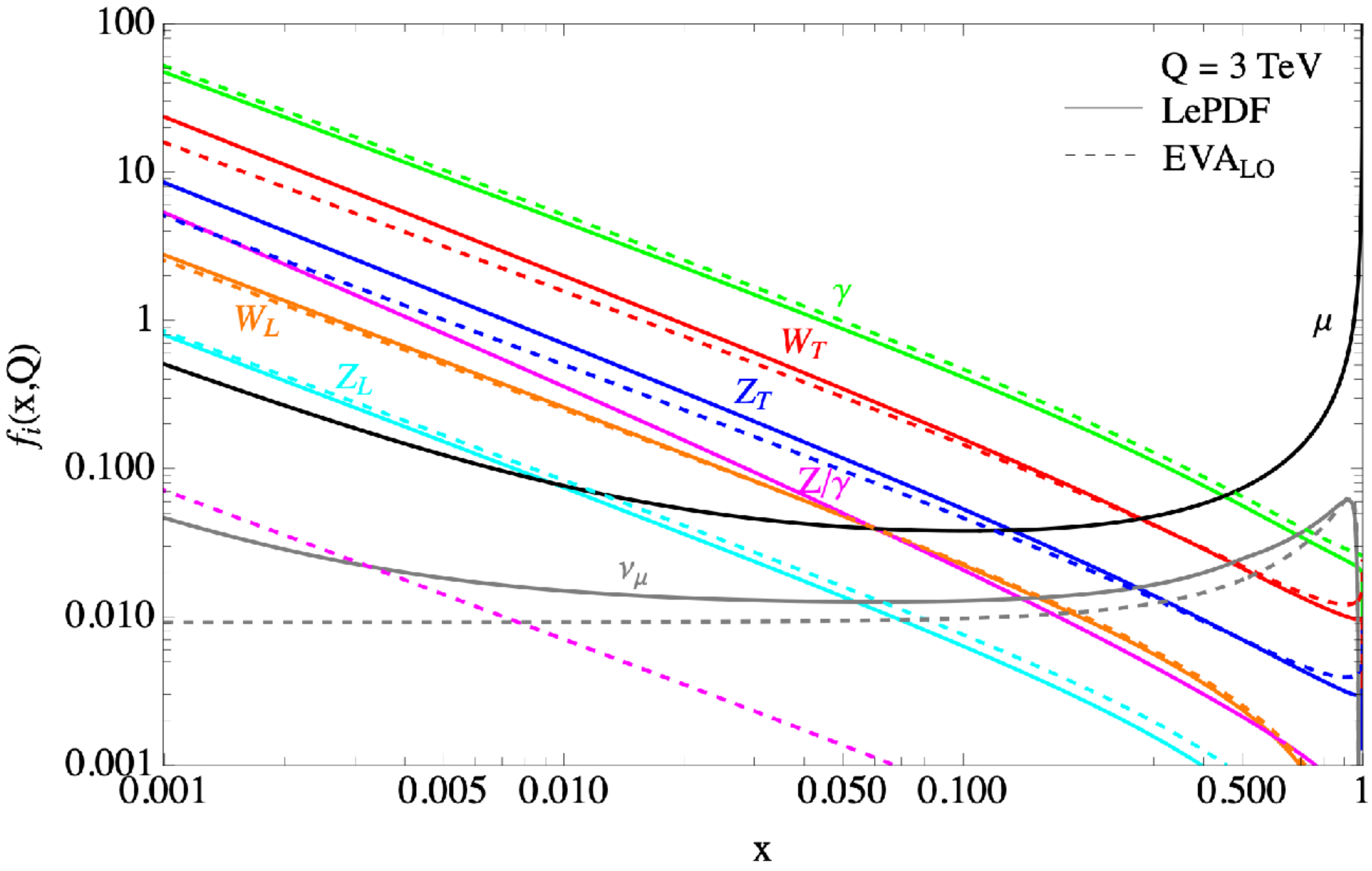
Fermions polarisation: ψ_L / ψ_R



O(1) polarisation effects! (except for photon PDF)

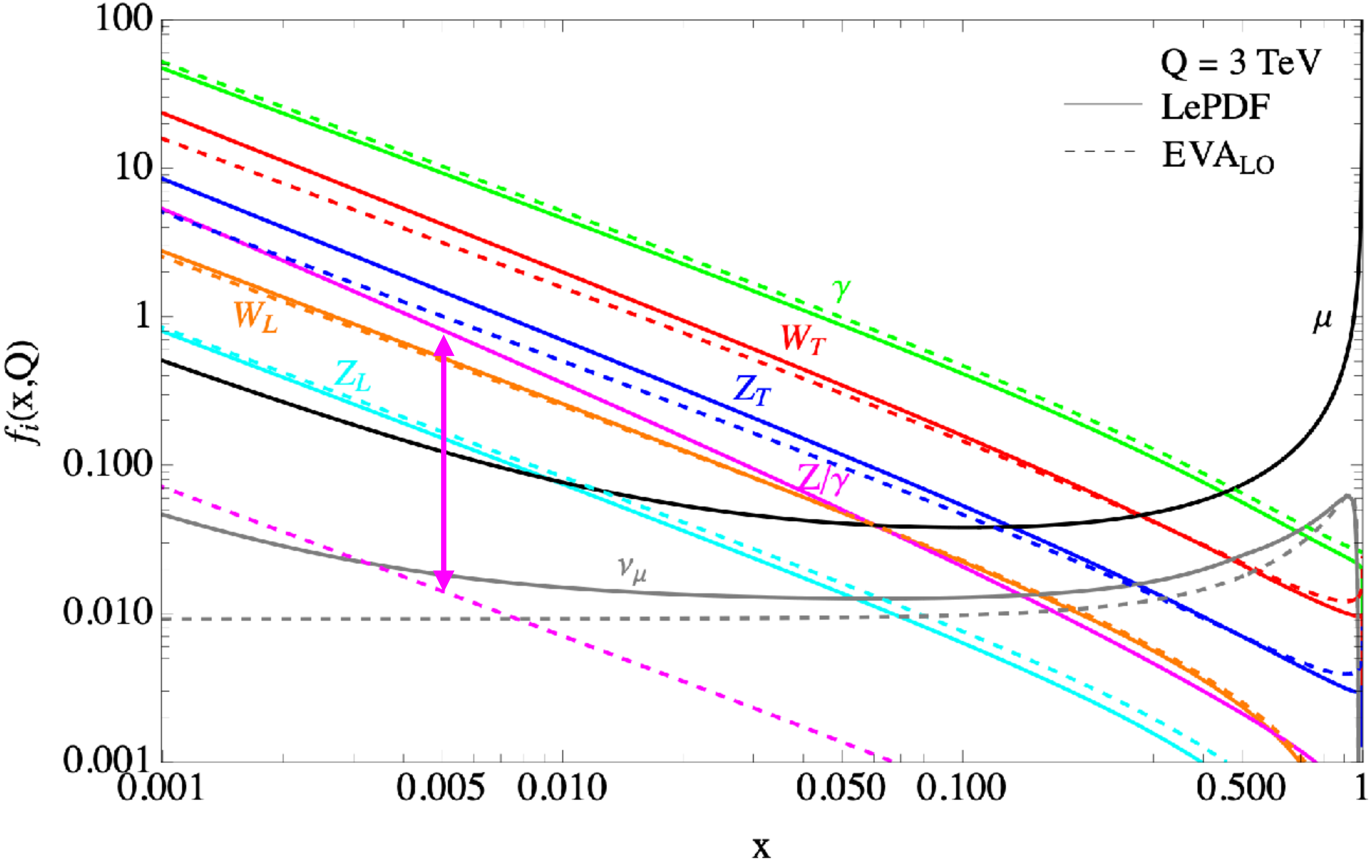
E.g. in case of W^- PDF, coupled to μ_L, the PDF for RH W's goes to zero for $x \to 1$ faster than LH W's, since $P_{V+f_L}(z) = (1-z)/z$ while $P_{V-f_L}(z) = 1/z$.

LePDF vs. EVA


Q = 3 TeV
LePDF
EVA_{LO}
γ
W_T
Z_T
W_L
Z_L
Z/γ
μ
ν_μ
$f_i(x,Q)$
x
100
10
1
0.100
0.010
0.001
0.001
0.005
0.010
0.050
0.100
0.500
1

LePDF vs. EVA

$$f^{(\alpha)}_{Z/\gamma\pm}(x,Q^2) = -\frac{\sqrt{\alpha_\gamma \alpha_2}}{2\pi c_W}\left(P^f_{V_\pm f_L}(x) Q^Z_{\mu_L} + P^f_{V_\pm f_R}(x) Q^Z_{\mu_R}\right)\log\frac{Q^2+(1-x)m_Z^2}{m_\mu^2+(1-x)m_Z^2}$$



★

The **EVA Z/γ PDF is off by ~10^2**, Will focus on this in a few slides.

LePDF vs. EVA

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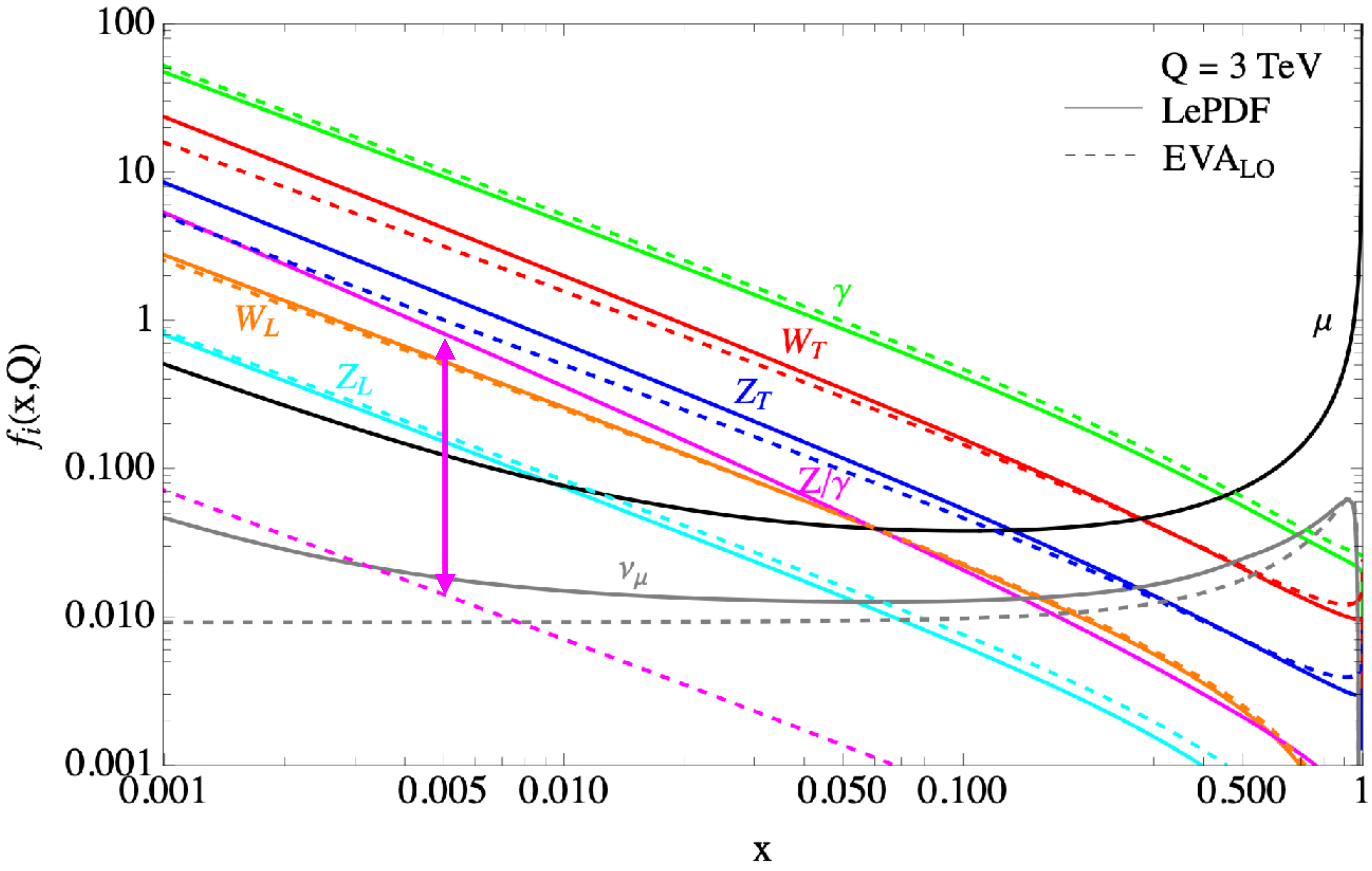


★ The **EVA Z/γ PDF is off by ~10^2**, Will focus on this in a few slides.

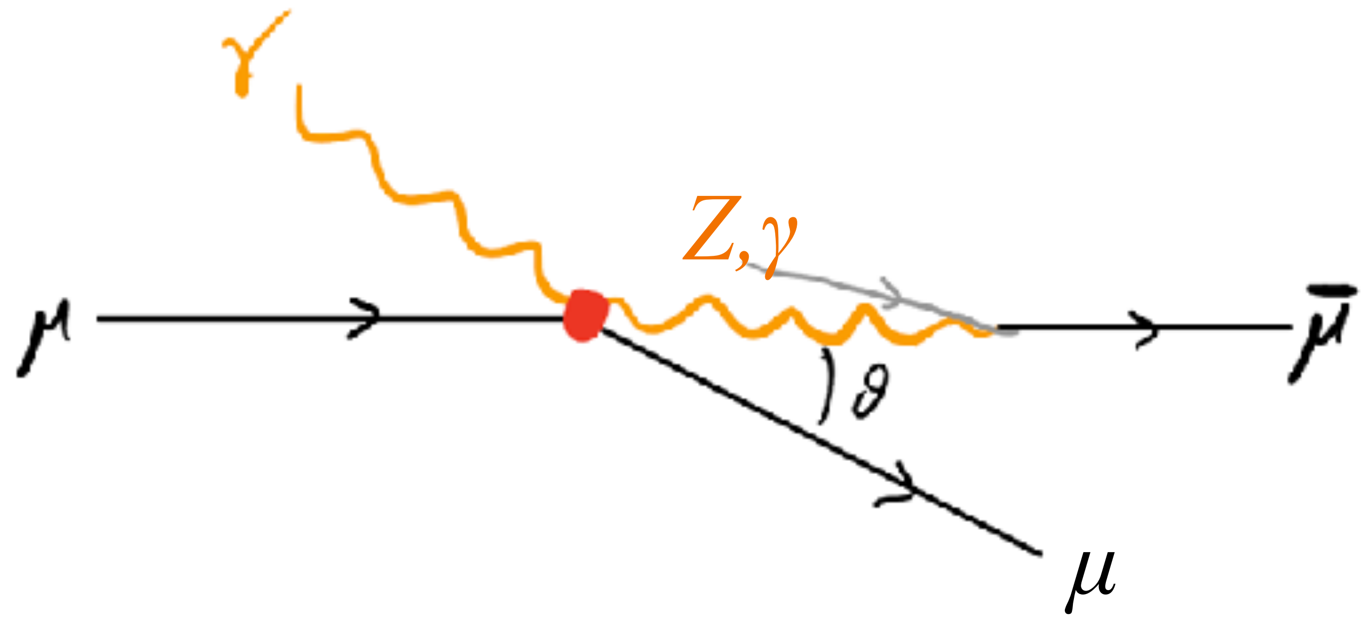
★ We can also see a **sizeable deviation** (in this log-log plot) for the **W_T** and **Z_T** PDF.

Mostly due to the double-log arising at $O(\alpha^2)$ from VVV interactions.

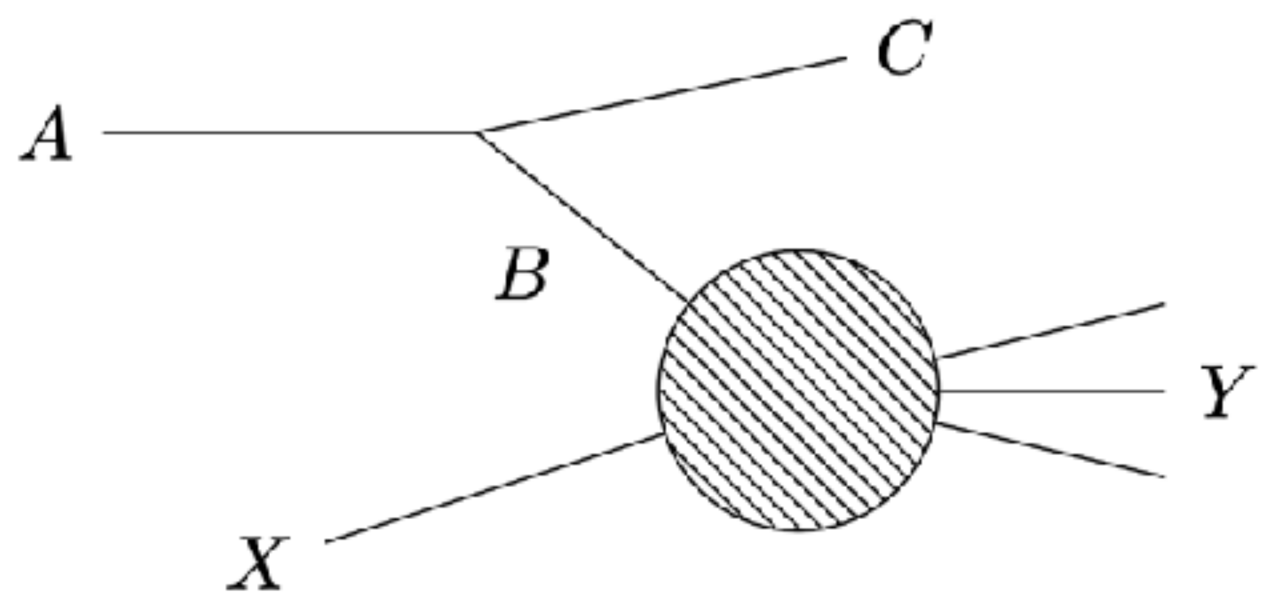
More details in [2303.16964]

Pheno of EW PDF effects (1) Mixed Z/γ PDF

[D.M. and **A. Stanzione** 2408.13191]



Photon - Z mixing PDF



Factorisation takes place at the amplitude level:

$$i\mathcal{M}(AX \to CY) = \sum_B i\mathcal{M}^{\rm split}(A \to CB^*)\frac{i}{Q^2}i\mathcal{M}^{\rm hard}(BX \to Y)\,(1+\mathcal{O}(\delta_{m,\perp}))$$

[Cuomo, Vecchi, Wulzer 1911.12366, …]

$$\delta_\perp = |\mathbf{k}_\perp|/E \ll 1$$
$$\delta_m = m/E \ll 1$$

Photon - Z mixing PDF

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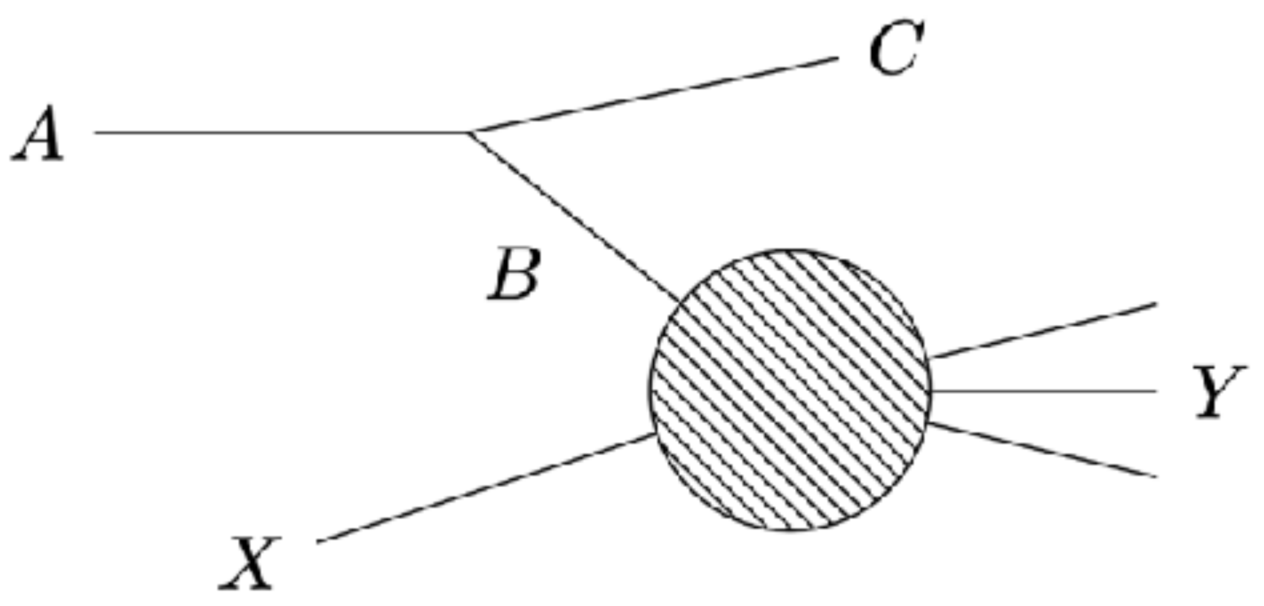
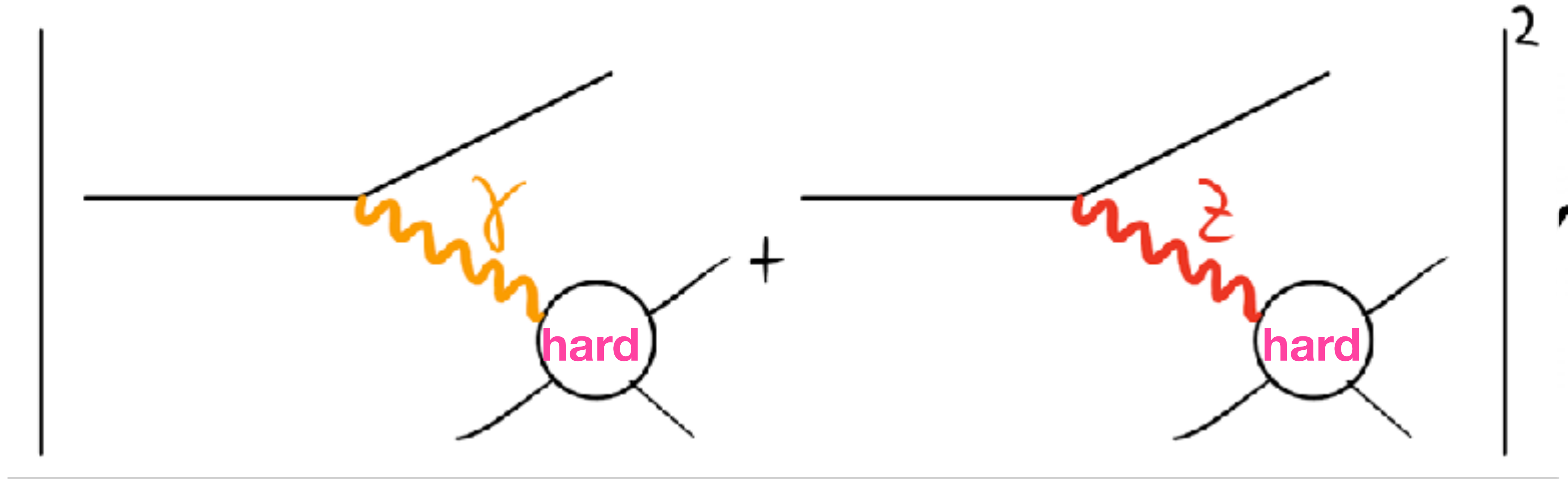


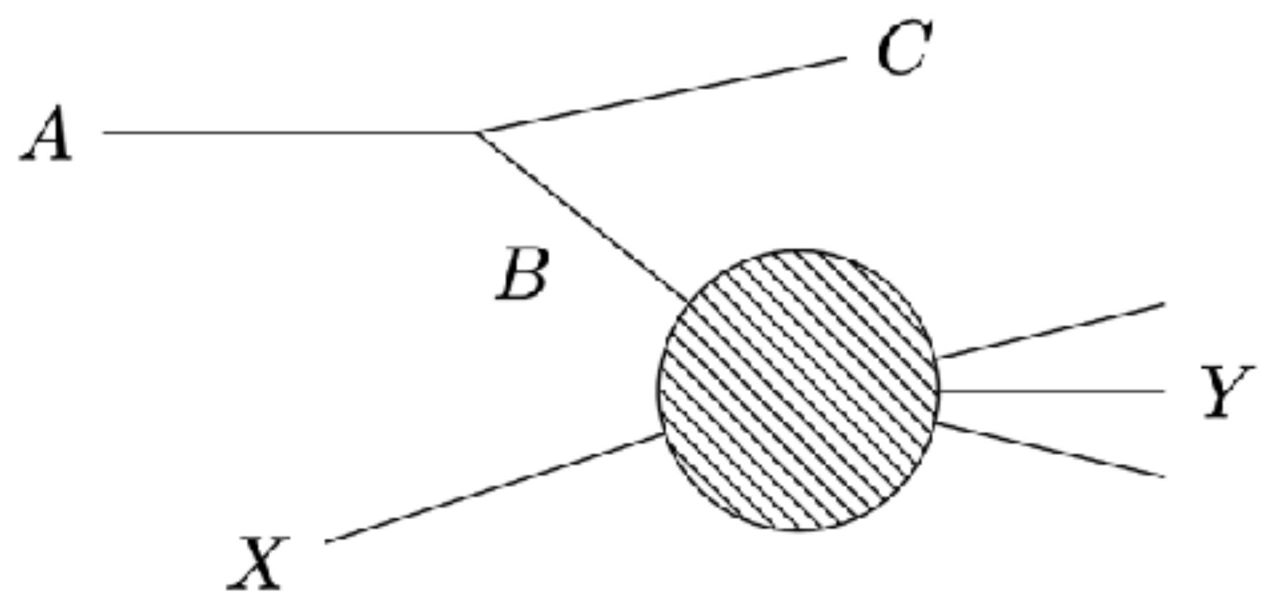
If two different states B and B' can enter in the same splitting and hard **processes**, they **can interfere:**

In the SM this can happen between: Z_T **and** γ Z_L and H



$$\sim \sum_{i,j=\gamma,Z} \mathcal{M}_i^{\text{split}}\, \mathcal{M}_i^{\text{hard}}\, \mathcal{M}_j^{\text{split}\,*}\, \mathcal{M}_j^{\text{hard}\,*}$$

Photon - Z mixing PDF



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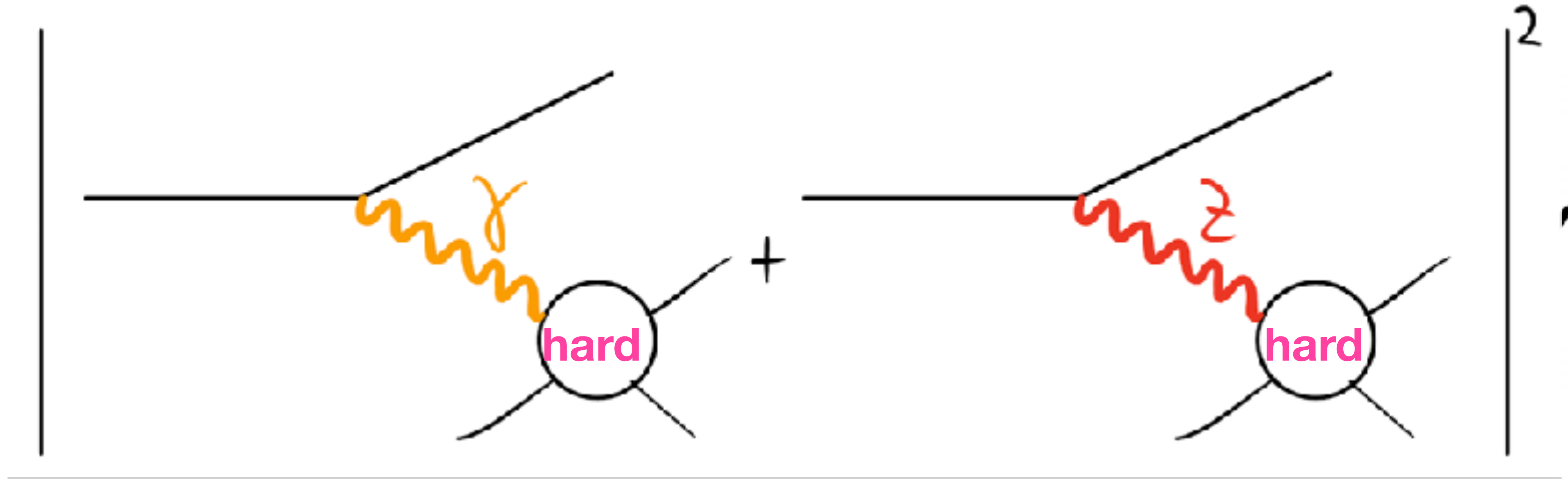
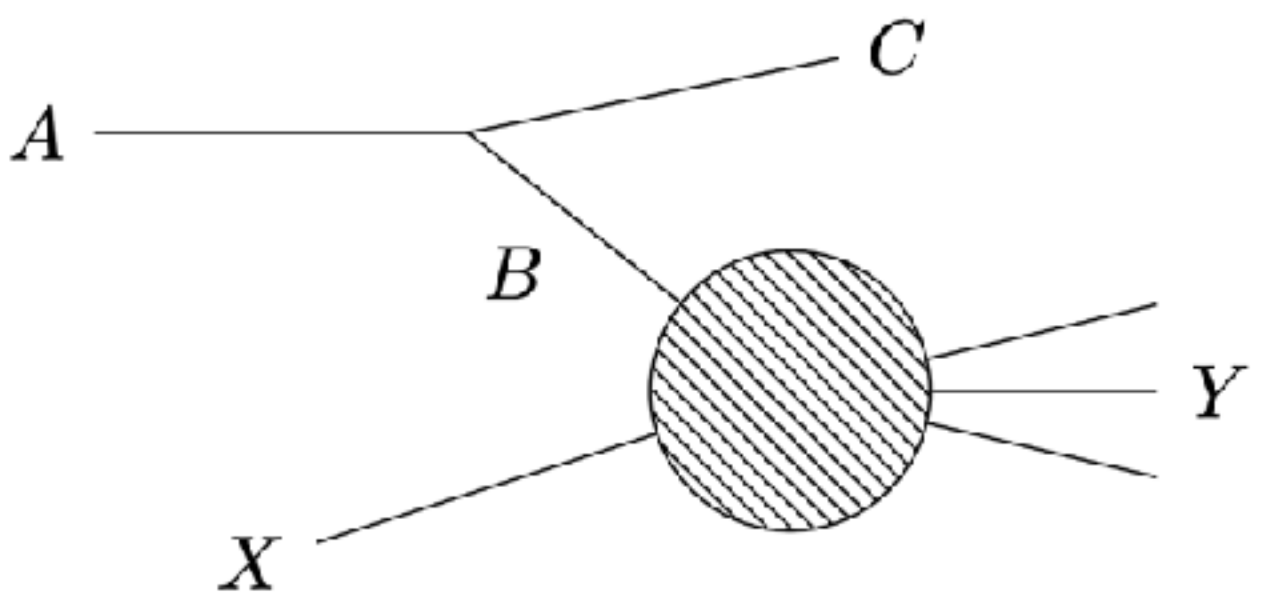


$$\sim \sum_{i,j=\gamma,Z} \mathcal{M}_i^{\text{split}} \mathcal{M}_i^{\text{hard}} \mathcal{M}_j^{\text{split}\,*} \mathcal{M}_j^{\text{hard}\,*} \sim \sum_{i,j=\gamma,Z} d\rho_{ij}^{\text{split}}\, d\rho_{ji}^{\text{hard}}$$

Photon - Z mixing PDF



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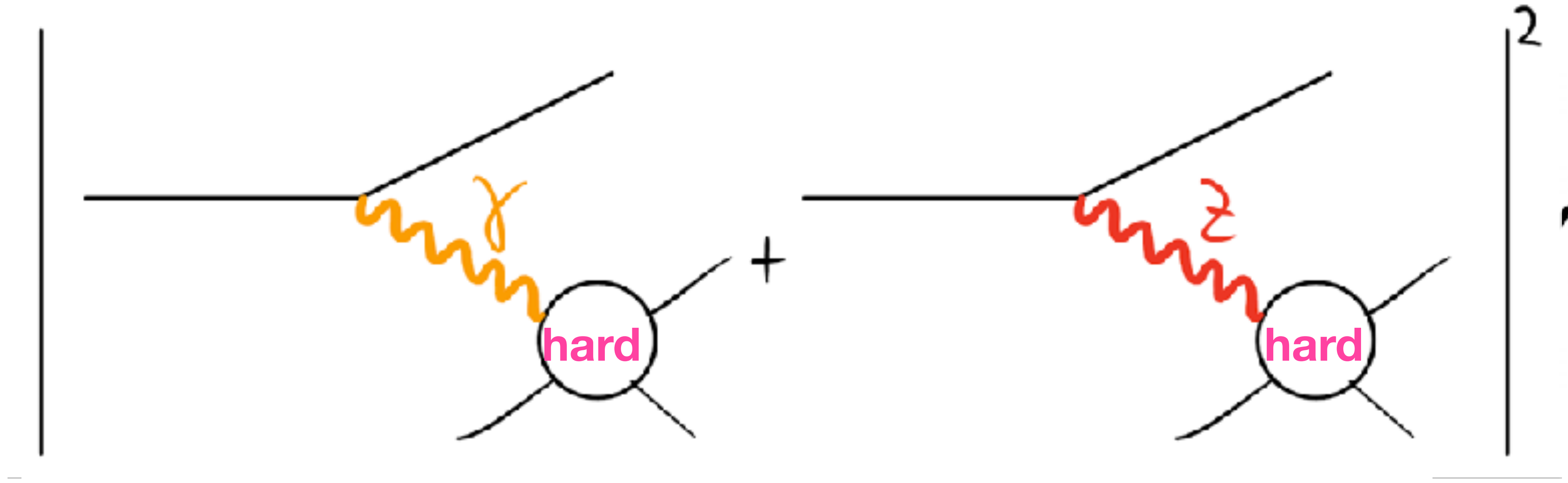


$$\sim \sum_{i,j=\gamma,Z} \mathcal{M}_i^{\text{split}}\,\mathcal{M}_i^{\text{hard}}\,\mathcal{M}_j^{\text{split}\,*}\,\mathcal{M}_j^{\text{hard}\,*} \sim \sum_{i,j=\gamma,Z} d\rho_{ij}^{\text{split}}\,d\rho_{ji}^{\text{hard}}$$

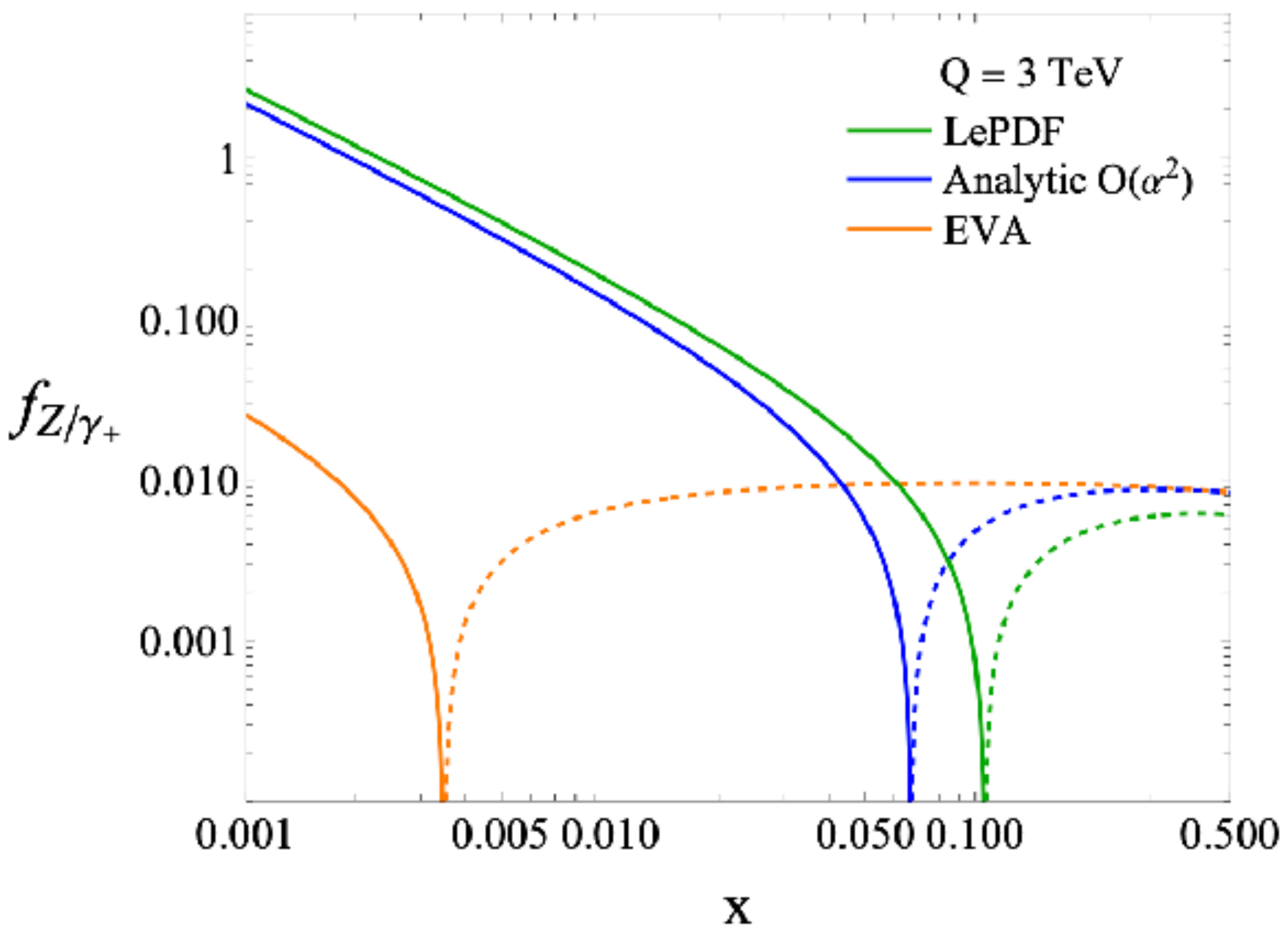
$$d\sigma \sim \mathrm{Tr}\left[\underbrace{\begin{pmatrix} f_\gamma & f_{Z\gamma} \\ f_{Z\gamma}^* & f_Z \end{pmatrix}}_{d\rho^{\text{split}}} \cdot \underbrace{\begin{pmatrix} |\mathcal{M}_\gamma^h|^2 & \mathcal{M}_Z^h\mathcal{M}_\gamma^{h*} \\ \mathcal{M}_\gamma^h\mathcal{M}_Z^{h*} & |\mathcal{M}_Z^h|^2 \end{pmatrix}}_{d\rho^{\text{hard}}}\right]$$ up to $\mathrm{O}(k_T^2/E^2, m^2/E^2)$

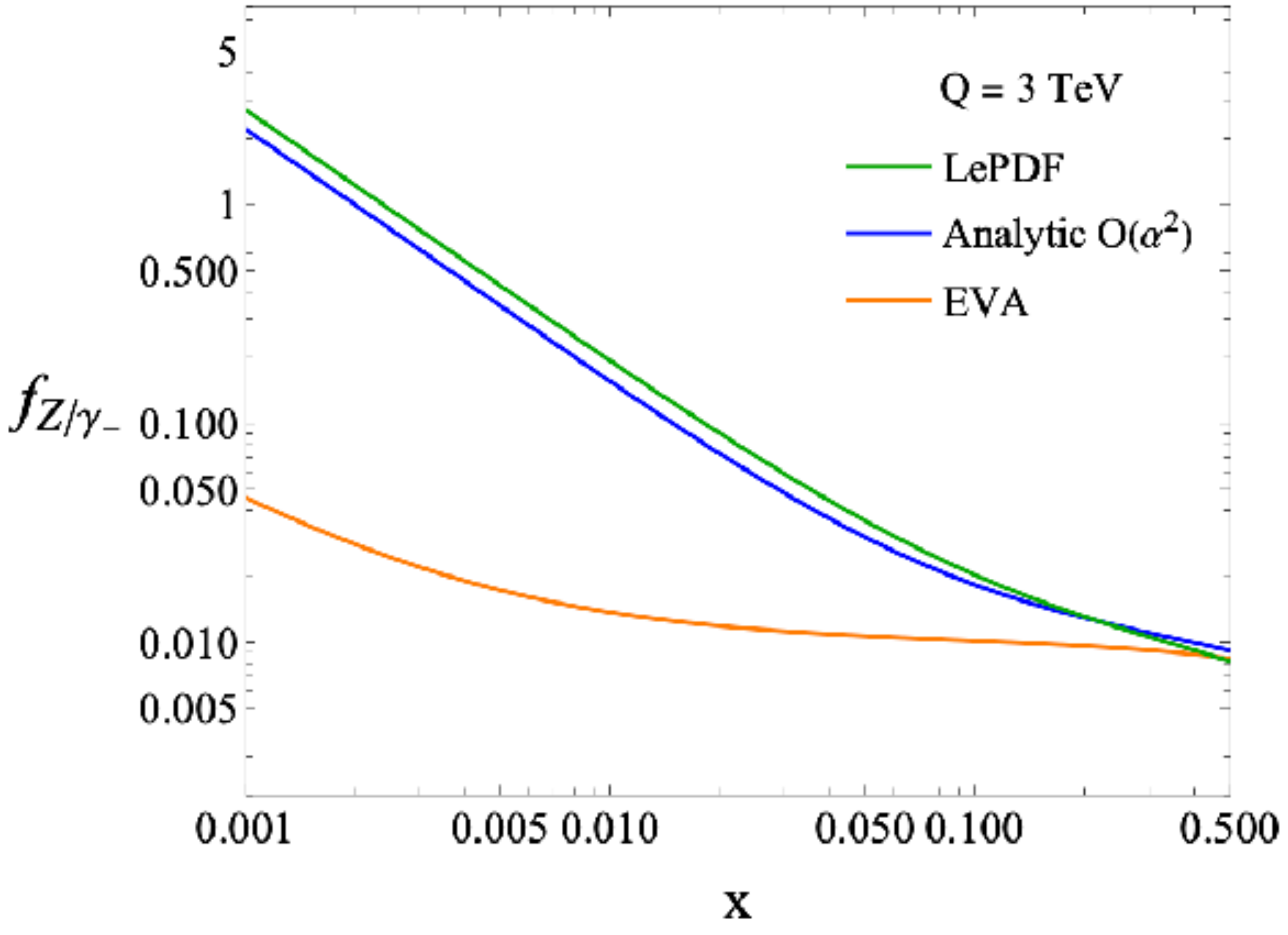
To describe the interference in the splitting one introduces the **mixed Z/γ PDF**.
(Similarly also for Z_L and H)

P. Ciafaloni, Comelli [hep-ph/0007096, hep-ph/0505047]
Chen, Han, Tweedie [1611.00788]

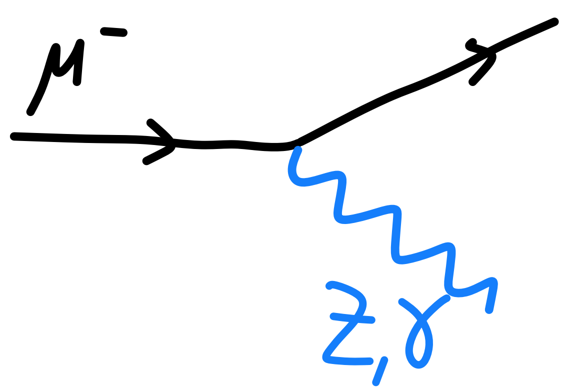
The *different virtuality* due to the different masses is an effect of $O(\delta_m^2)$.

Comparison with EVA





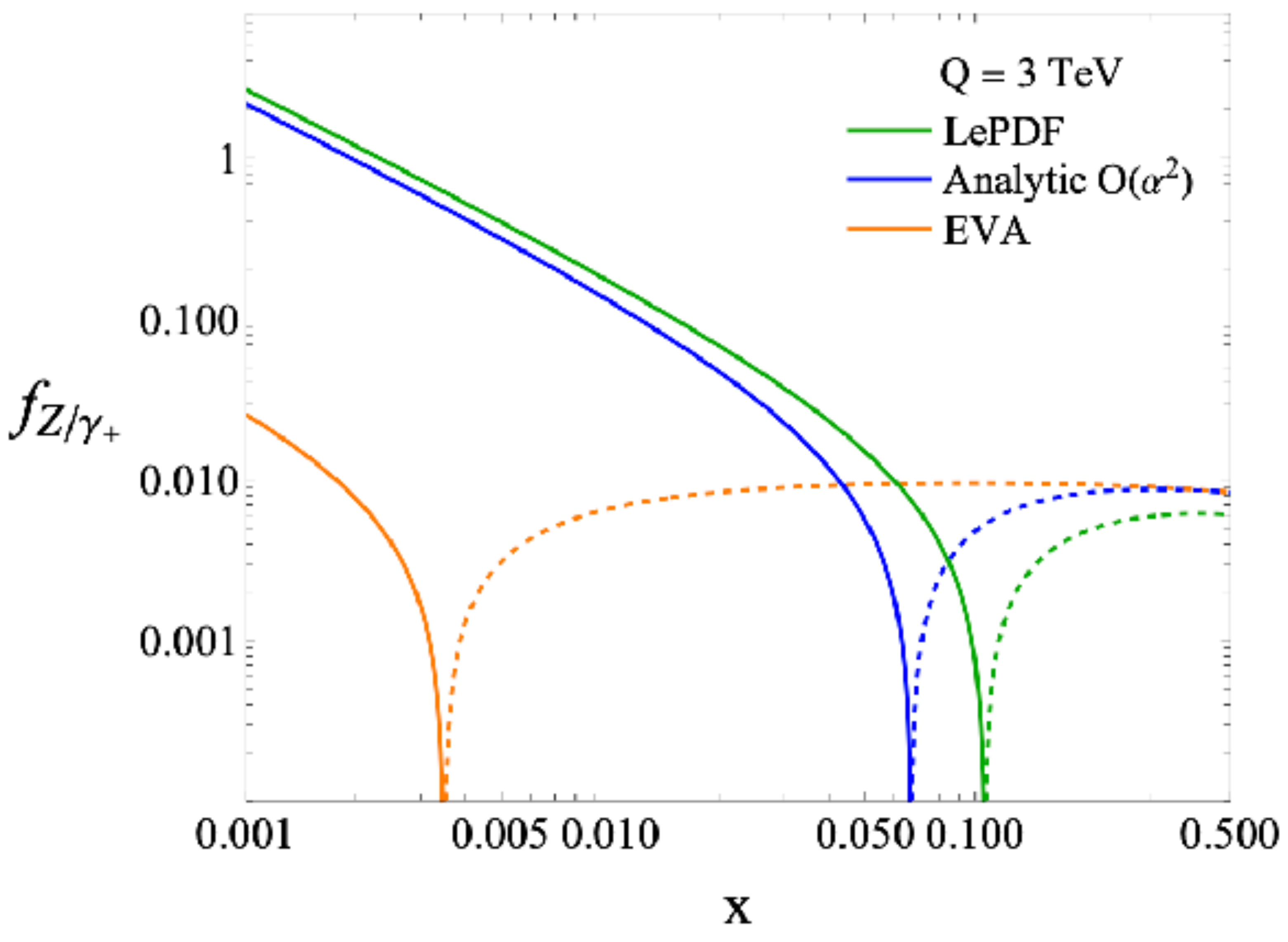
Solving iteratively the DGLAP equations at O(α) one can derive the **LO EVA for the Z/γ PDF**:

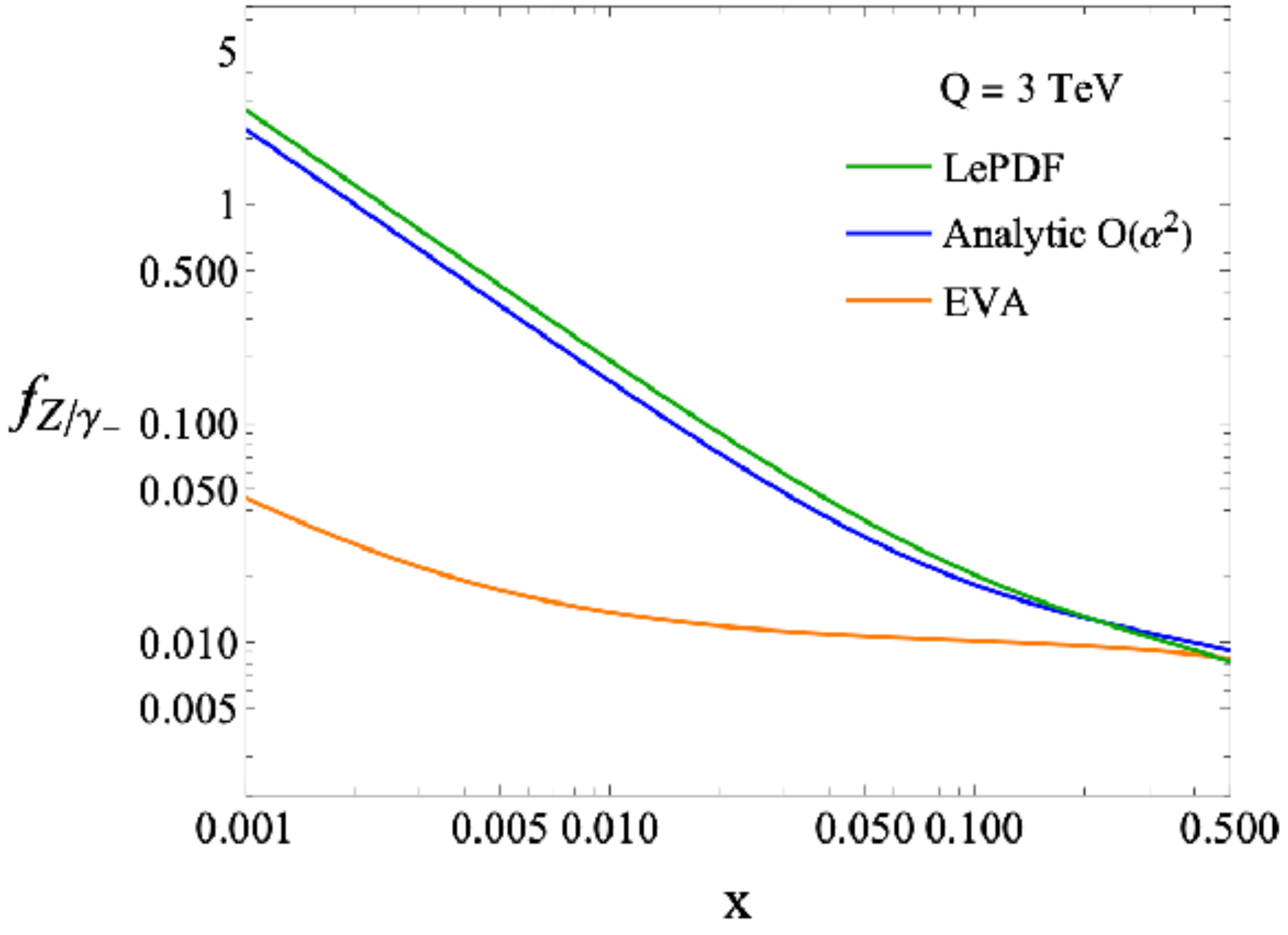


$$f^{(\alpha)}_{Z/\gamma} \approx -\frac{\alpha_{\gamma Z}}{2\pi}\left(P_{V f_L}(x)\, Q^Z_{\mu_L} + P_{V f_R}(x)\, Q^Z_{\mu_R}\right)\log\frac{Q^2}{m_Z^2}$$

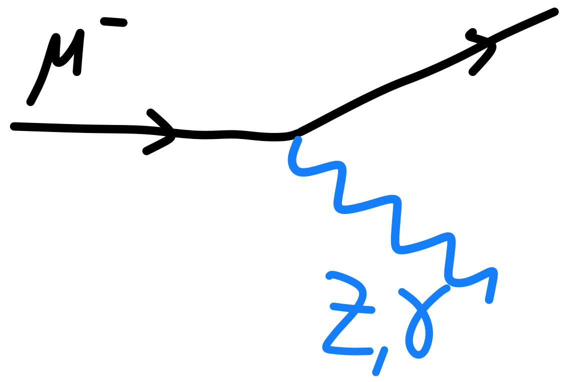
$$\overset{x\ll 1}{\approx} -\frac{\alpha_{\gamma Z}}{2\pi}\,\frac{1}{x}\left(Q^Z_{\mu_L} + Q^Z_{\mu_R}\right)\log\frac{Q^2}{m_Z^2}$$

Comparison with EVA





Solving iteratively the DGLAP equations at O(α) one can derive the **LO EVA for the Z/γ PDF**:



$$f^{(\alpha)}_{Z/\gamma} \approx -\frac{\alpha_{\gamma Z}}{2\pi}\left(P_{Vf_L}(x)\, Q^Z_{\mu_L} + P_{Vf_R}(x)\, Q^Z_{\mu_R}\right)\log\frac{Q^2}{M_Z^2}$$

$$\overset{x\ll 1}{\approx} -\frac{\alpha_{\gamma Z}}{2\pi}\,\frac{1}{x}\left(Q^Z_{\mu_L} + Q^Z_{\mu_R}\right)\log\frac{Q^2}{M_Z^2}$$

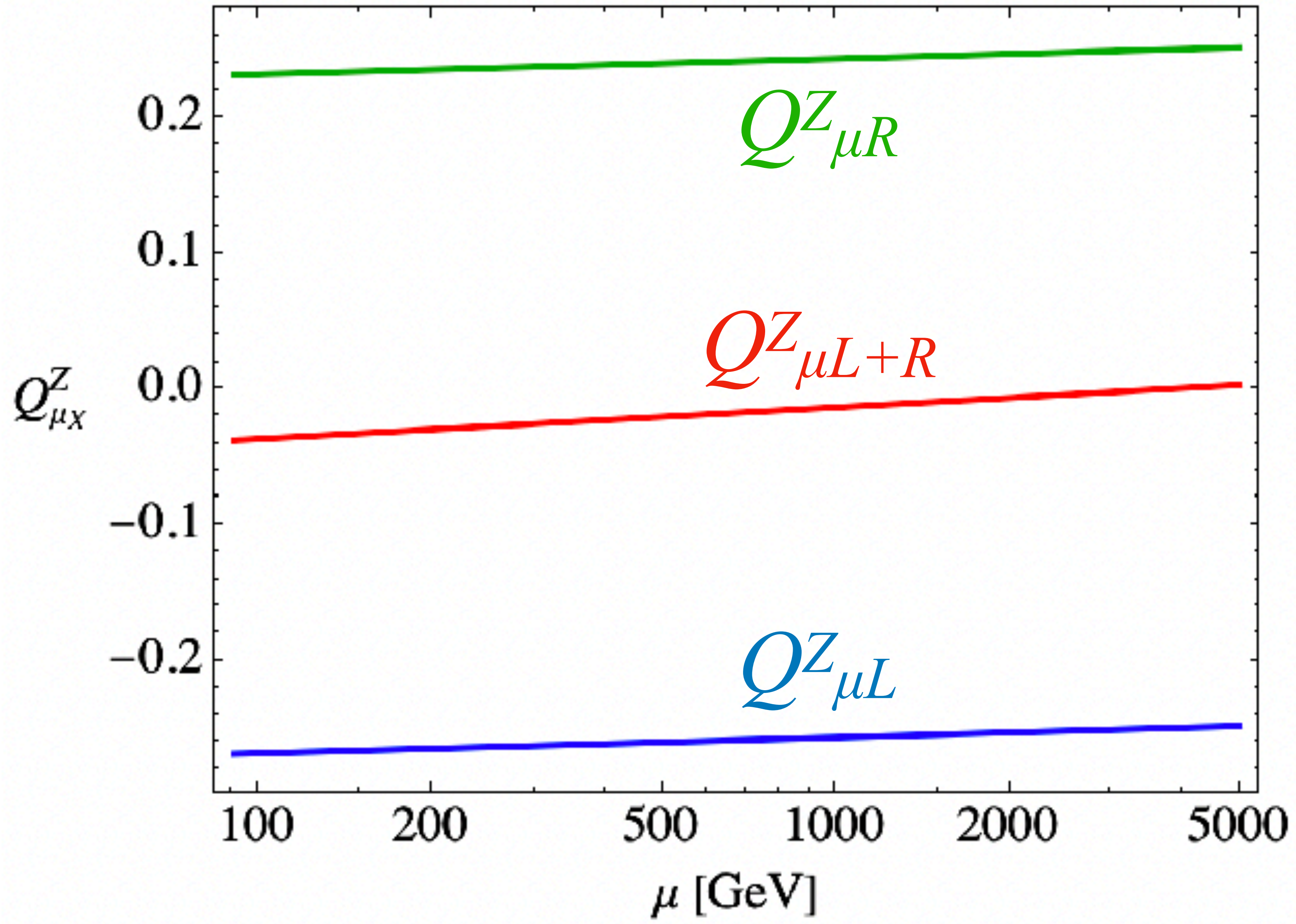
proportional to the **vector-like μ coupling to the Z boson**:

$$Q^Z_{\mu_{L+R}} = -\frac{1}{2} + s_W^2 + s_W^2 = -\frac{1}{2} + 2 s_W^2 \ll 1$$

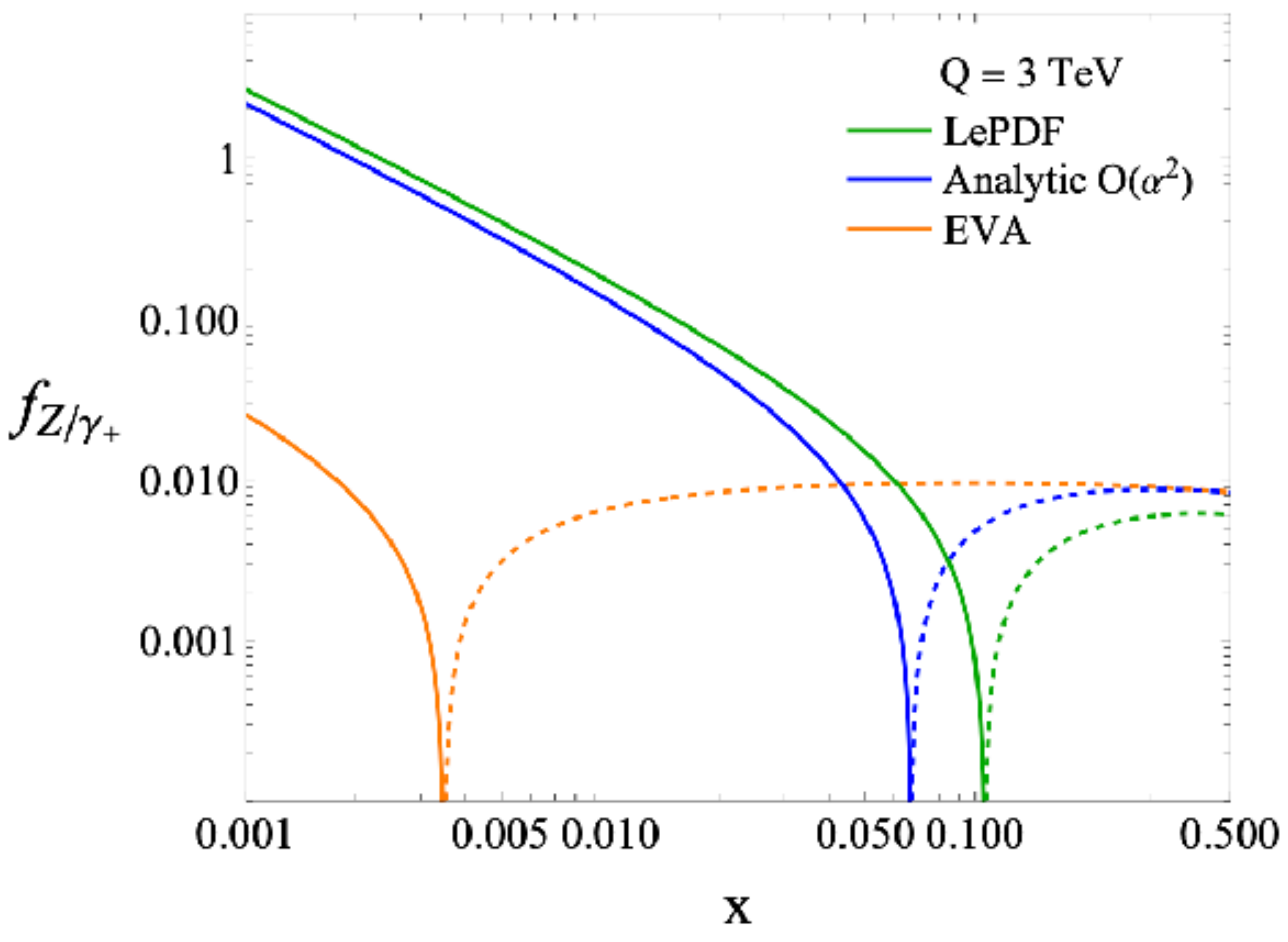
ACCIDENTAL SUPPRESSION !

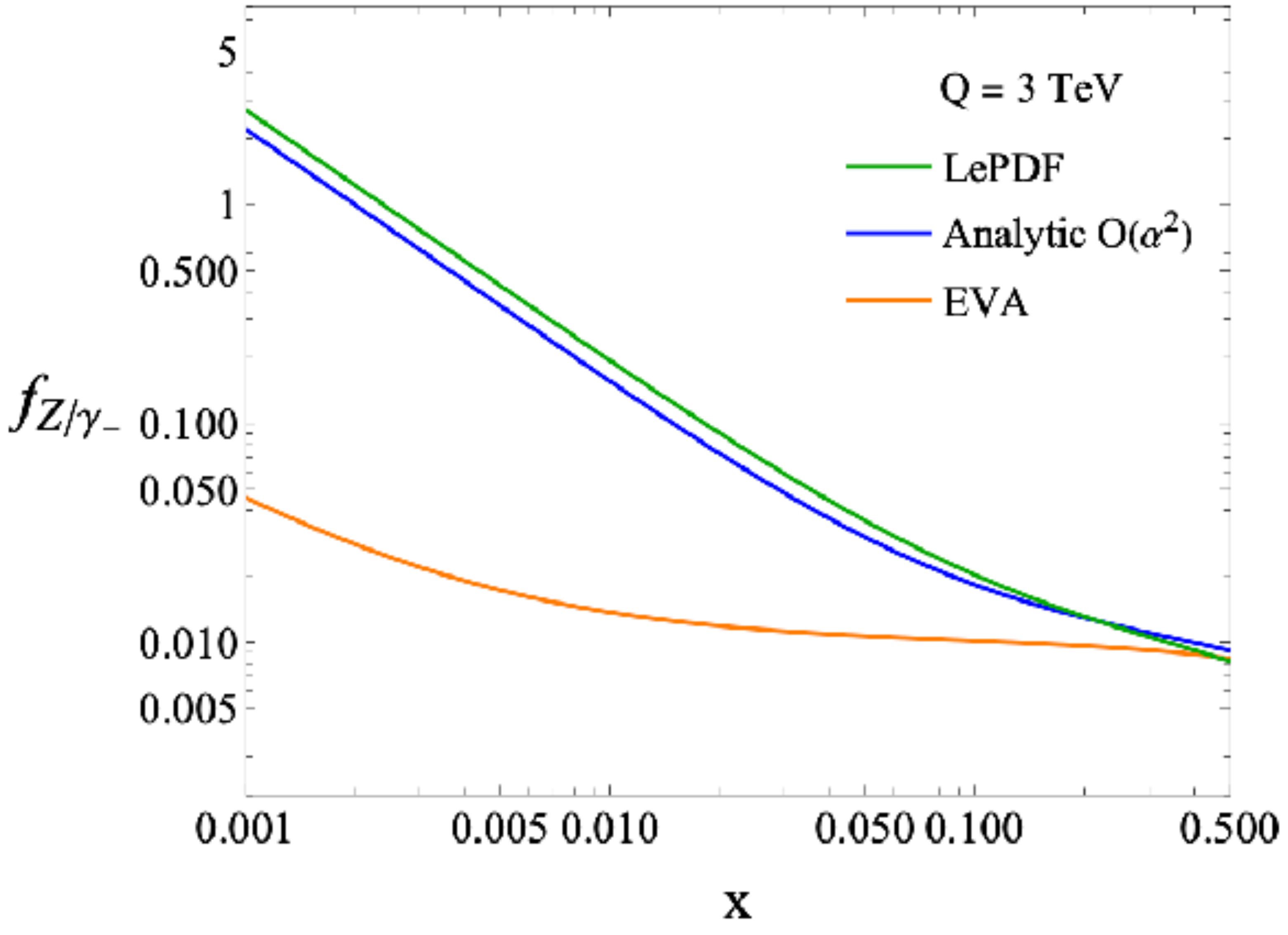
- possible because **at LO, LH and RH *μ* PDFs are equal**.

In the **full result** a O(1) polarisation arises, which lifts the cancellation.
Also, at O(α²) other contributions become dominant, due to Sudakov logs.



Extending EVA to O(α²)



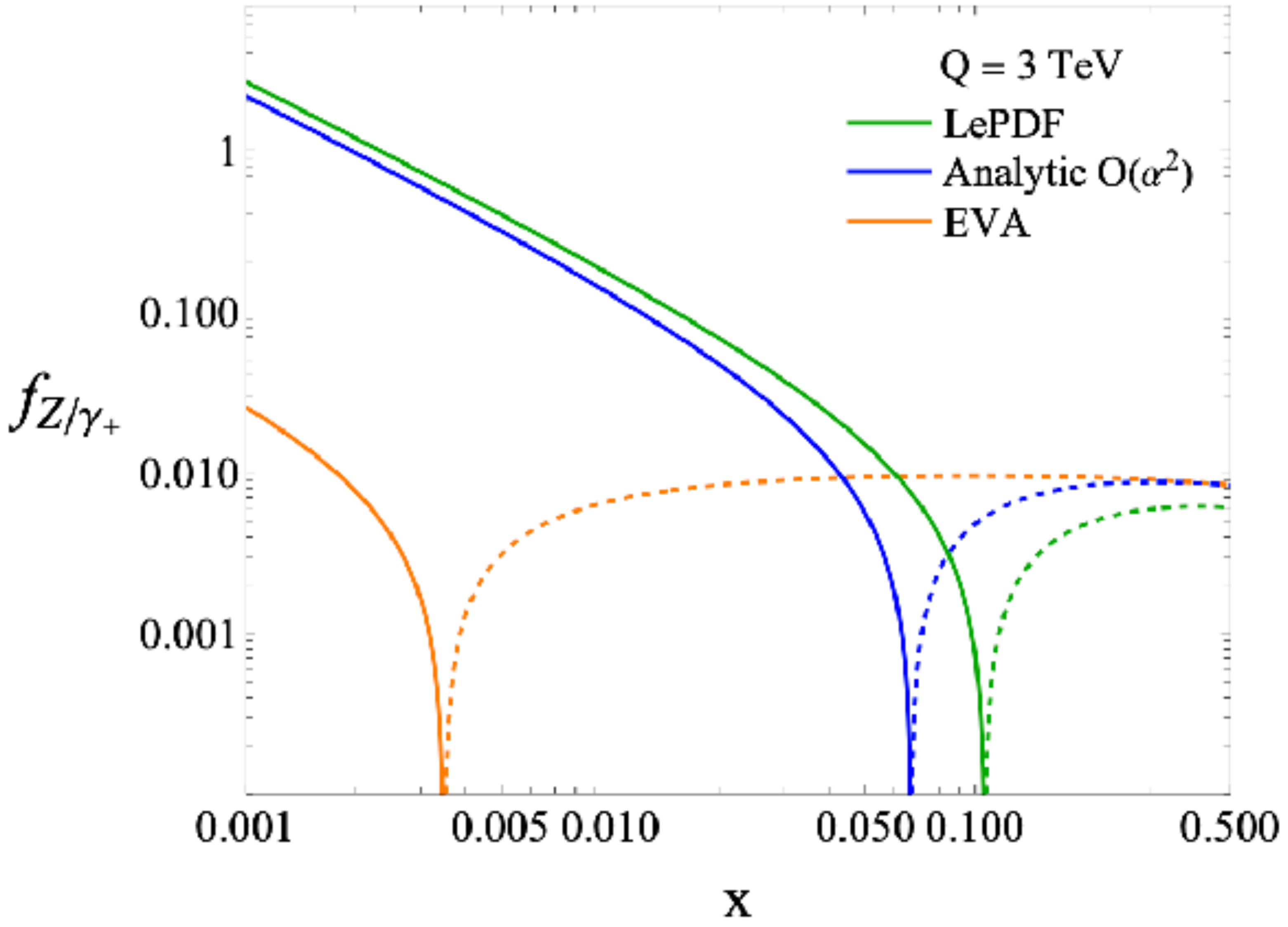


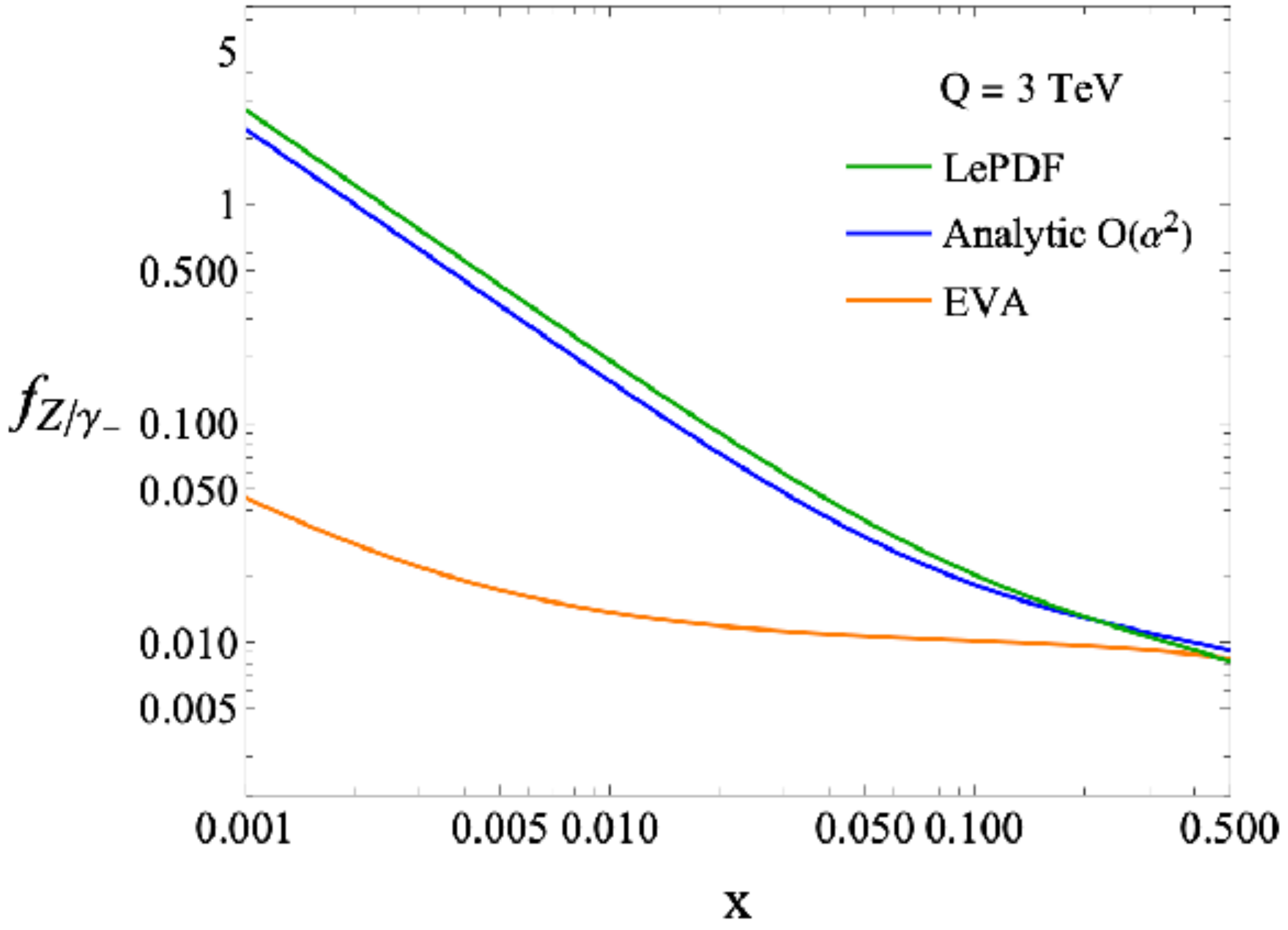
We can go **one order higher** by using the O(α) EVA expressions in the RHS of the DGLAP equation:

$$\frac{df^{(\alpha^2)}_{Z/\gamma_+}(x,Q^2)}{dt} = \frac{\alpha_{\gamma 2}(t)}{2\pi} 2c_W P^V_{V_+V_\pm}\otimes f^{(\alpha)}_{W^-_\pm} + \frac{\alpha_{\gamma 2}(t)}{2\pi}\frac{c_{2W}(t)}{c_W(t)} P^h_{V_+h}\otimes f^{(\alpha)}_{W^-_L} + \frac{\alpha_{\gamma 2}(t)}{2\pi}\frac{2}{c_W(t)}\sum_f Q_f\left[Q^Z_{f_L}P^f_{V_+f_L}\otimes f^{(\alpha)}_{f_L} + Q^Z_{f_R}P^f_{V_-f_L}\otimes f^{(\alpha)}_{f_R}\right]$$

$$t = \log(Q^2/m_\mu^2)$$

Extending EVA to O(α²)





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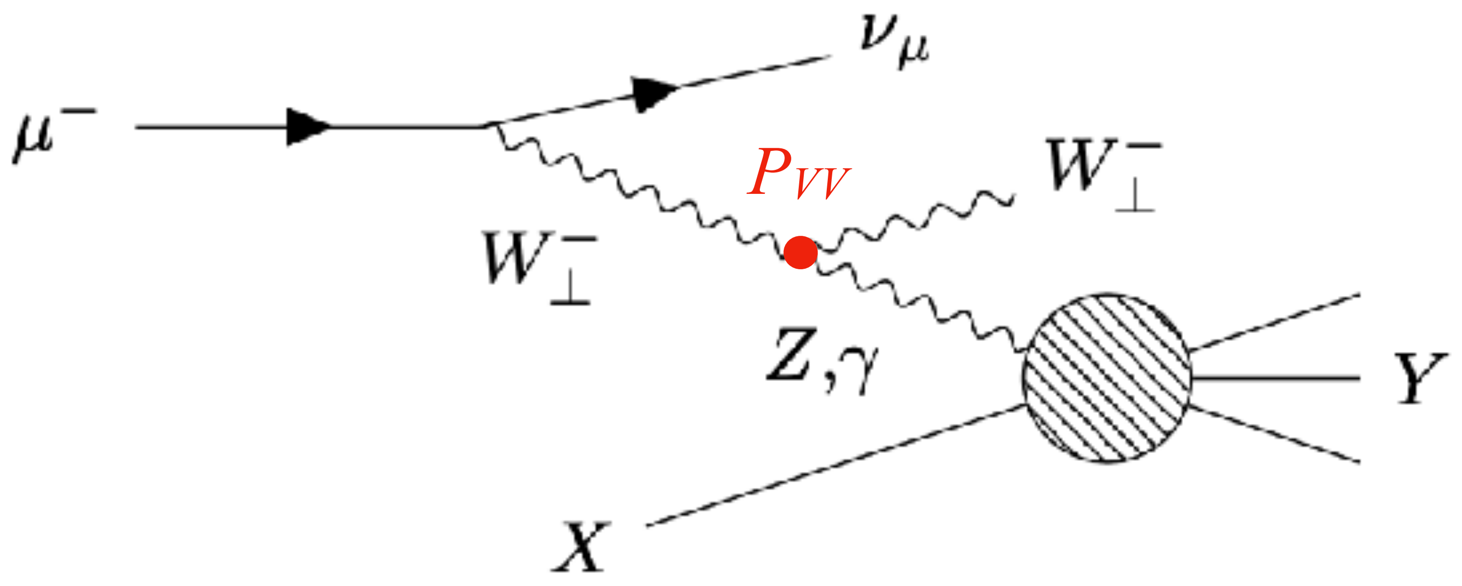
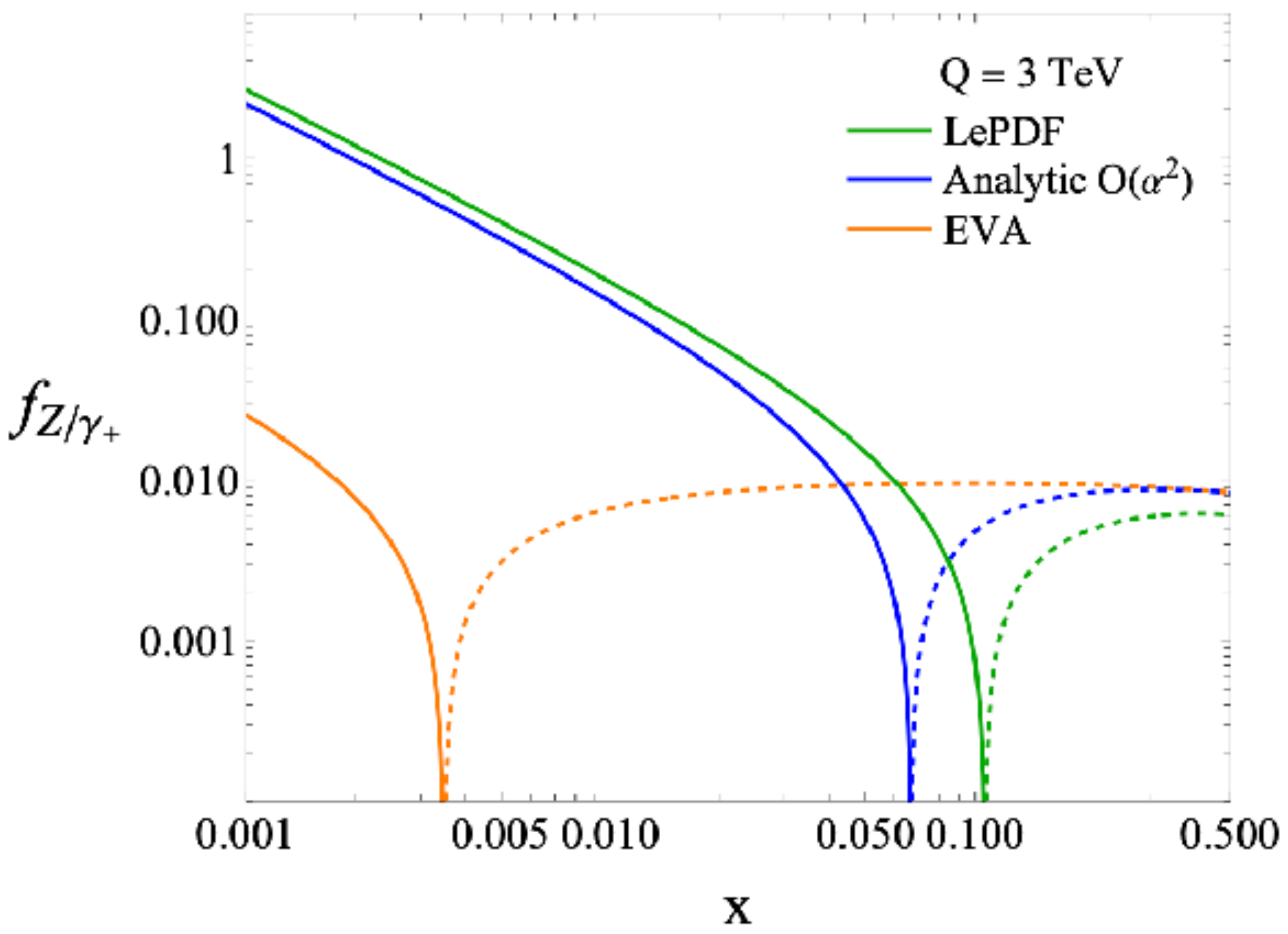
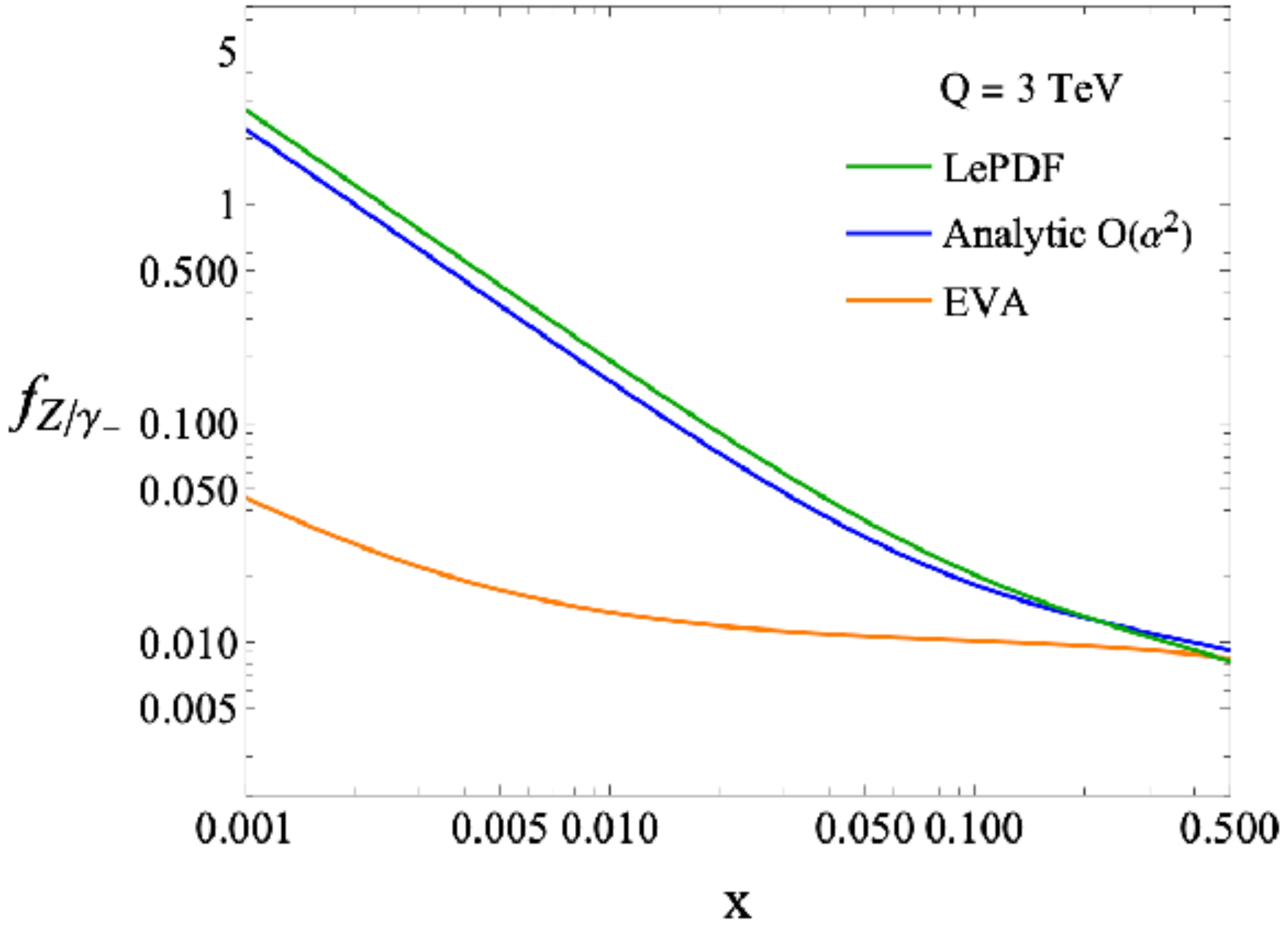
Let us focus on the first term, where $\quad f^{(\alpha)}_{W^-_\pm}(x,Q^2) \approx \frac{\alpha_2}{8\pi} P^f_{V_\pm f_L}(x) \log\frac{Q^2}{m_Z^2}$

Corresponds to a **double-emission**



Extending EVA to O(α²)





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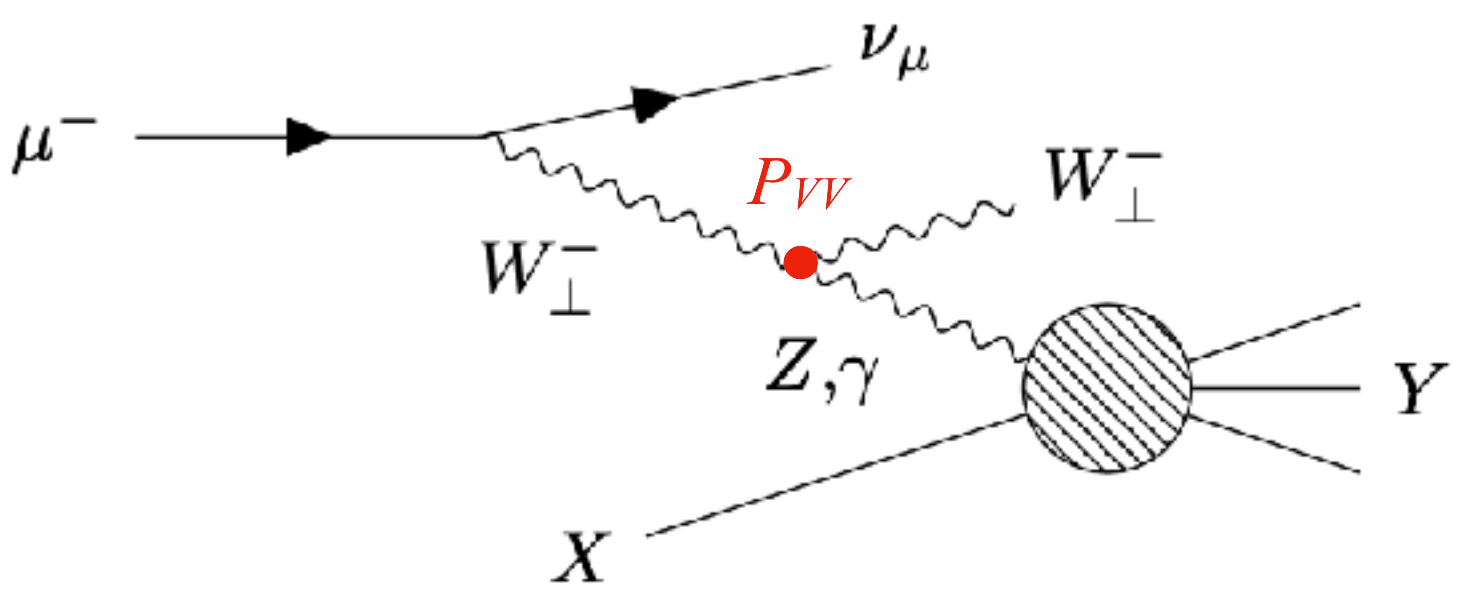


The result for that term is:

$$f^{(\alpha^2)P_{VV}}_{Z/\gamma_+}(x,Q) = \frac{\alpha_2\alpha_{\gamma 2}}{96\pi^2 x}(t-t_Z)^2\, 2c_W(x-1)^2\cdot\left[(t-t_Z)+J(x)\right]$$

$$f^{(\alpha^2)P_{VV}}_{Z/\gamma_-}(x,Q) = \frac{\alpha_2\alpha_{\gamma 2}}{96\pi^2 x}(t-t_Z)^2\, 8\cdot\left[(t-t_Z)+K(x)\right],$$

$J(x)$ and $K(x)$ are O(1) functions of x.

The full O(α²) expression gives a much more accurate approximation to the numerical result.

A **Sudakov double-log** appears:

$$\alpha^2(t-t_Z)^3 = \alpha^2\log^3(Q^2/m_Z^2)$$

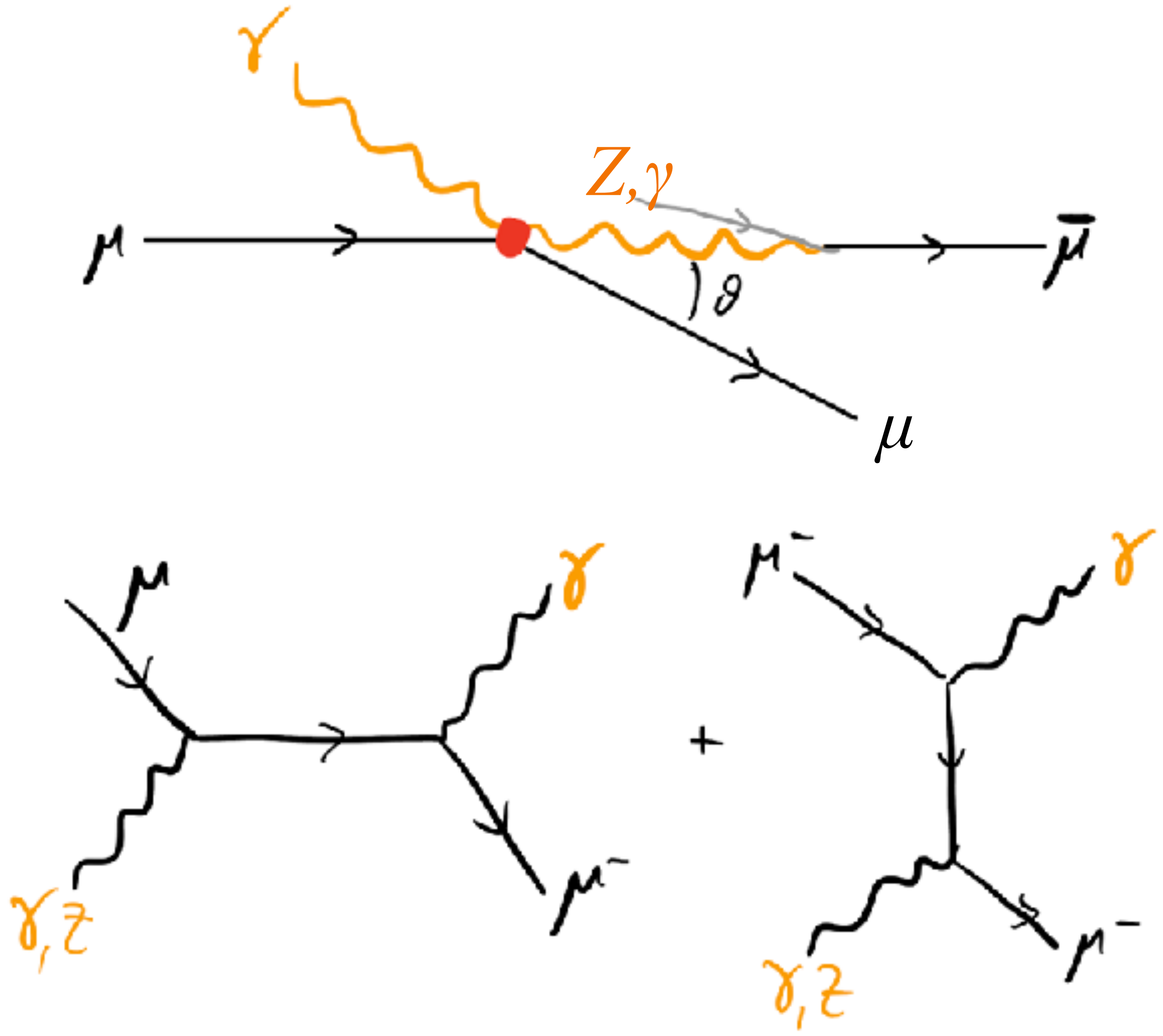
Applications

1) Is this mixed PDF observable in some process?

2) What is the impact it has on SM and BSM cross sections?

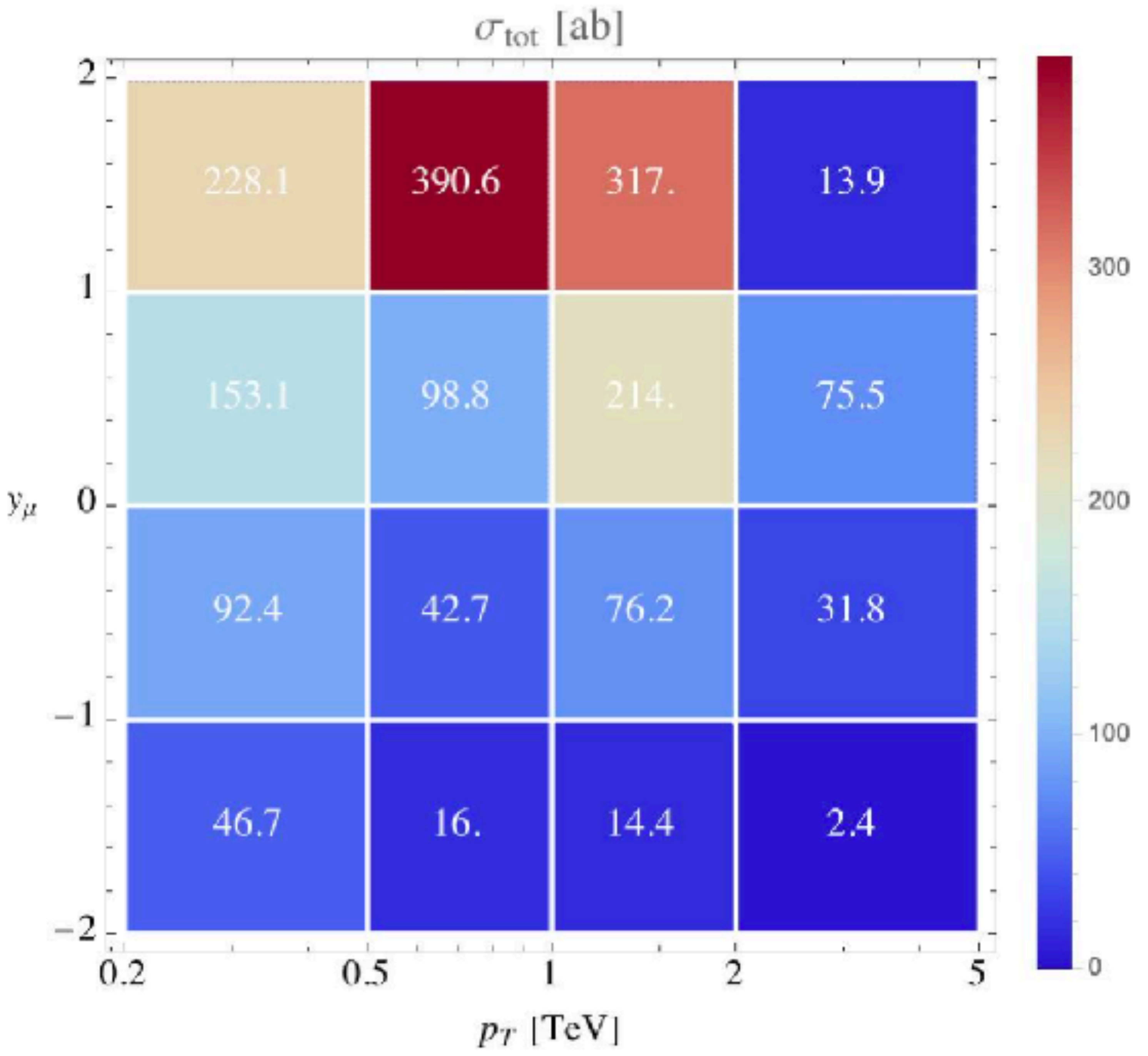
Compton Scattering @ MuC

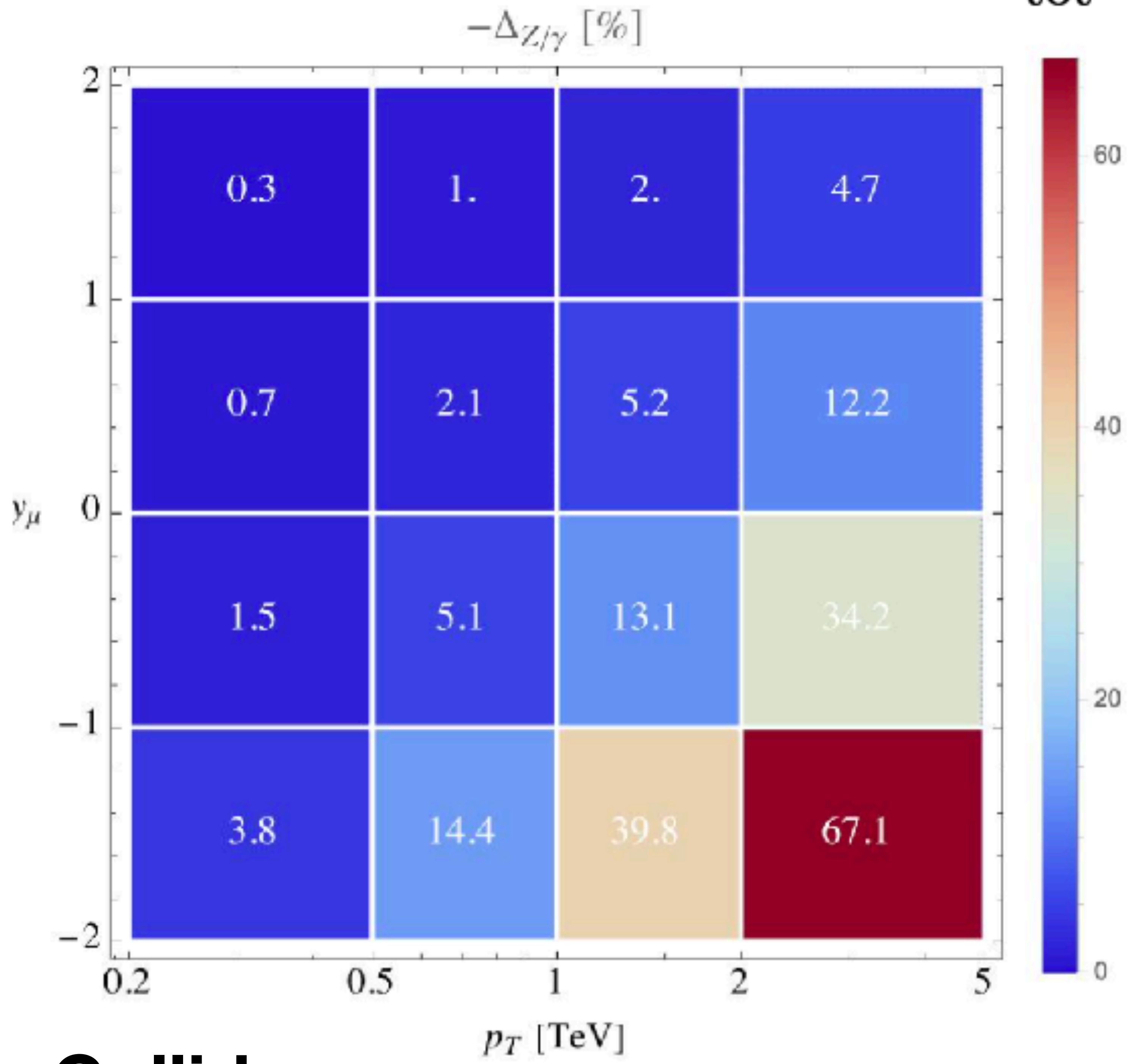
No large new physics effect is expected in this process, since muon couplings to photon and Z boson are well tested. It is thus **perfect to study this EW SM effect**.



Cross section in **bins** of **muon rapidity and p_T**

$$\Delta_{Z/\gamma} \equiv \frac{\sigma_{Z/\gamma}}{\sigma_{\text{tot}}}$$





@ 10 TeV Muon Collider

We also include the background from $\nu_\mu W^- \to \mu\,\gamma$, its contribution is however marginal.

To what precision could we measure it?

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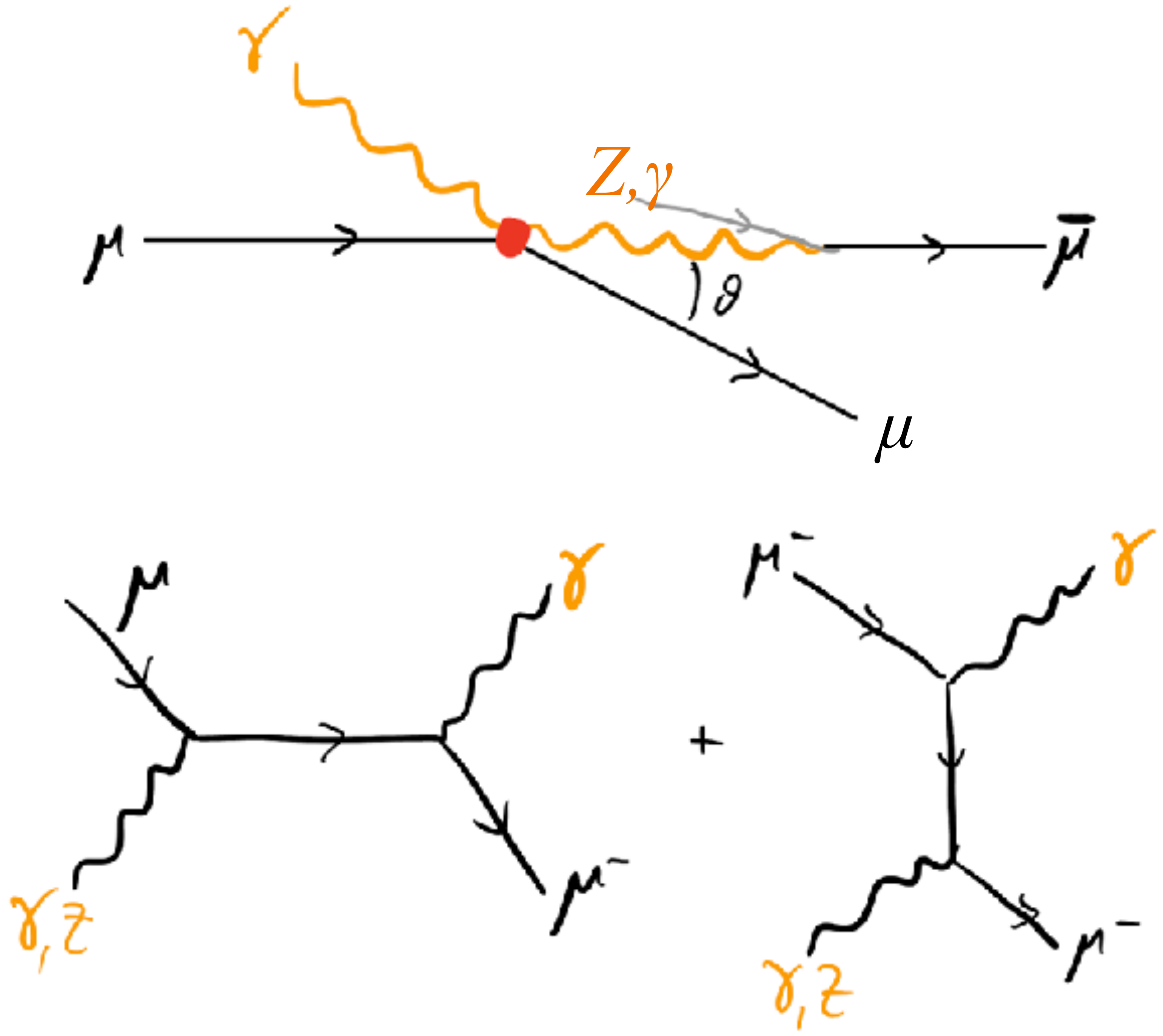

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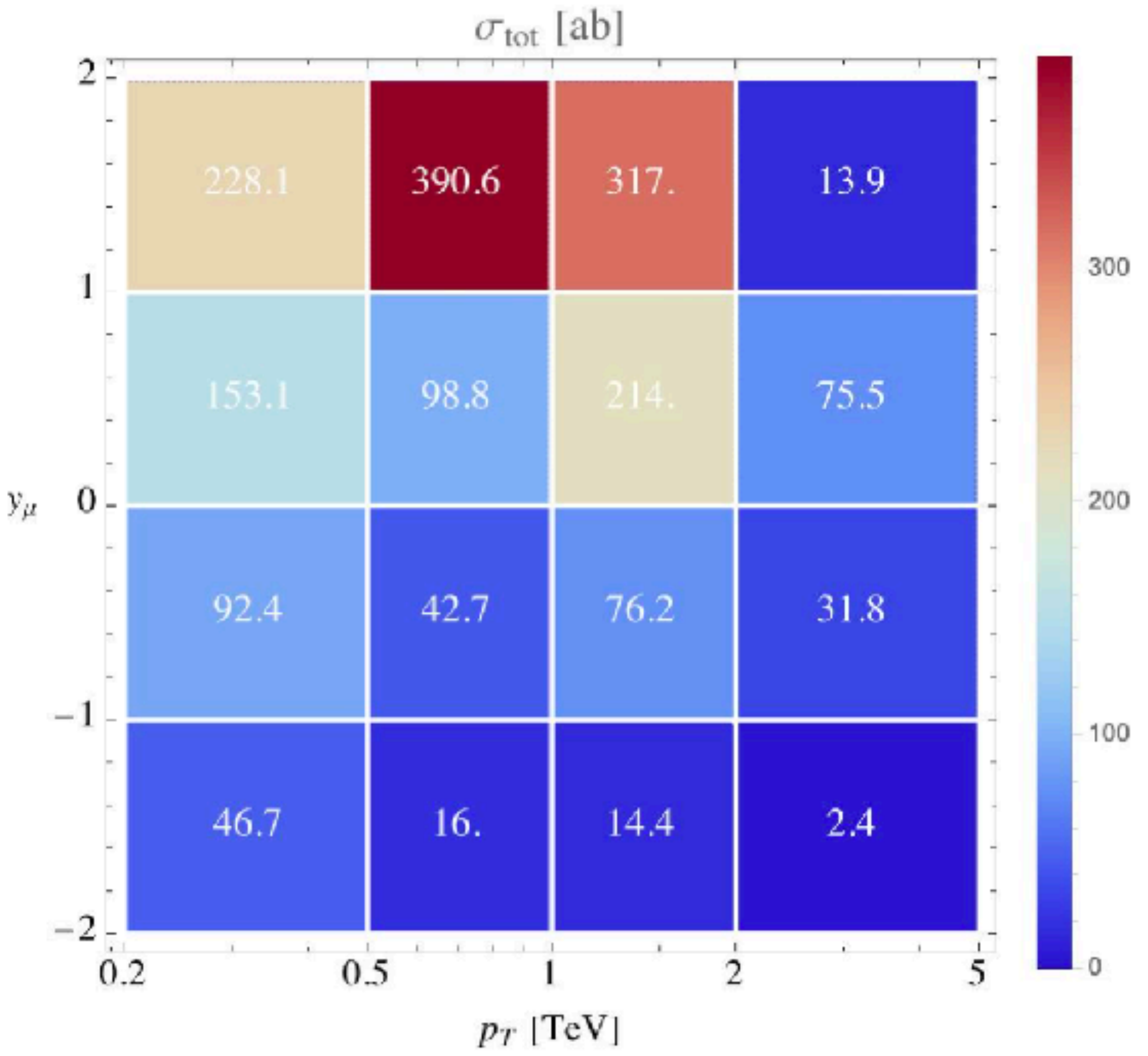
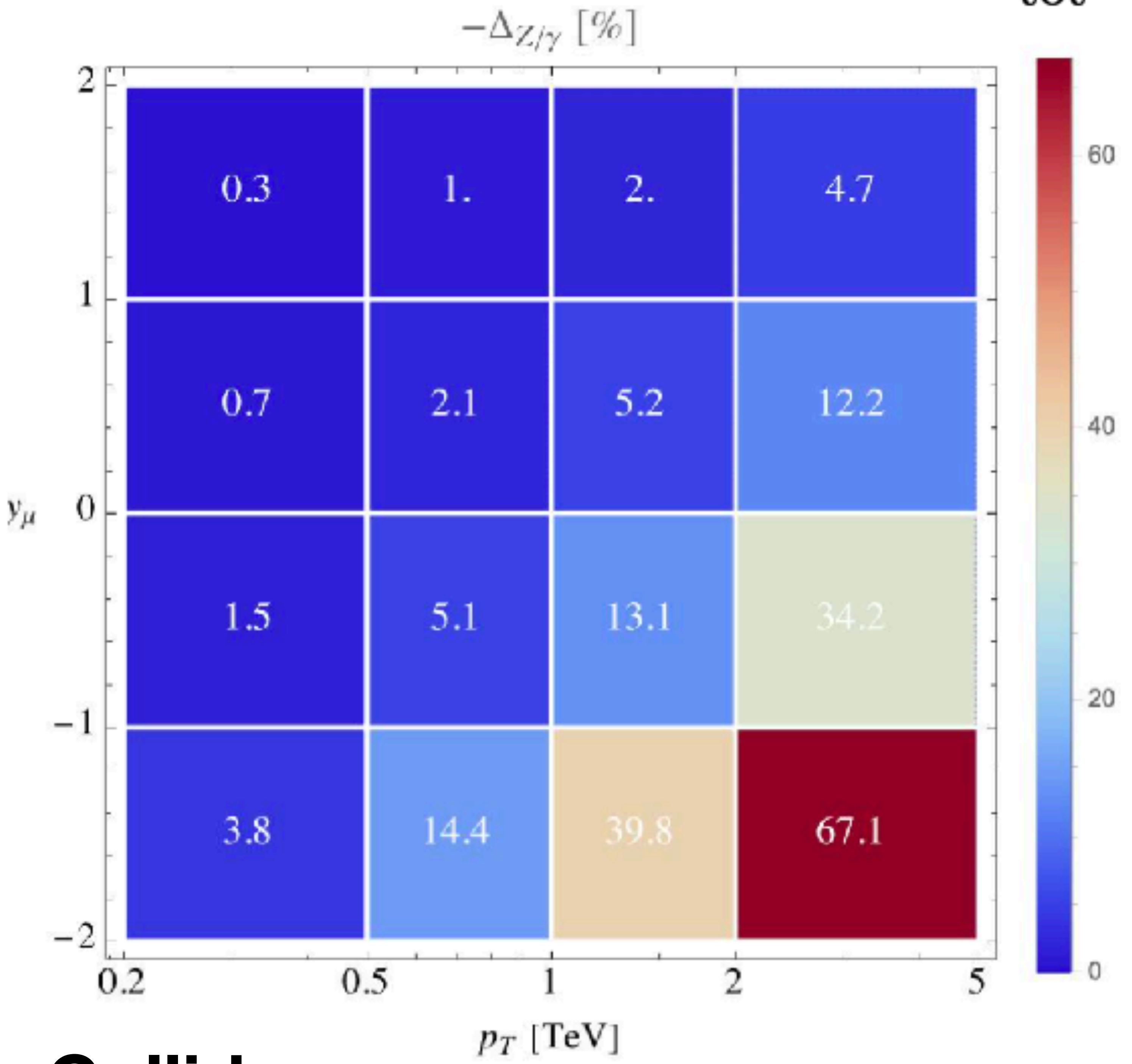




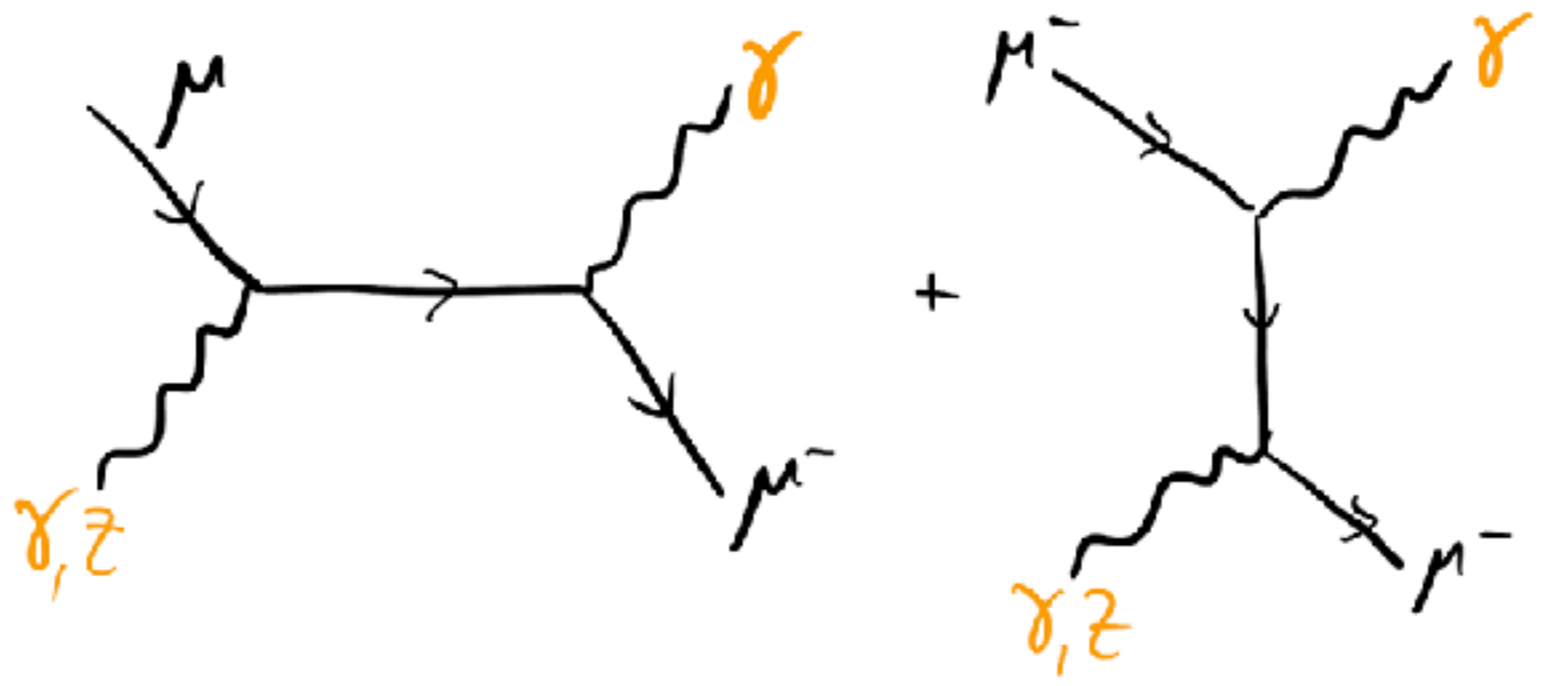
@ 10 TeV Muon Collider

We also include the background from $\nu_\mu W^- \to \mu\,\gamma$, its contribution is however marginal.

The **mixed $Z\gamma$ PDF** can **contribute from few % up to ~ 70%,** depending on the phase space region.

To what precision could we measure it?

Compton Scattering @ MuC



We estimate the precision with which we can measure this effect, over the null hypothesis that it is zero, with a simple χ^2 test:

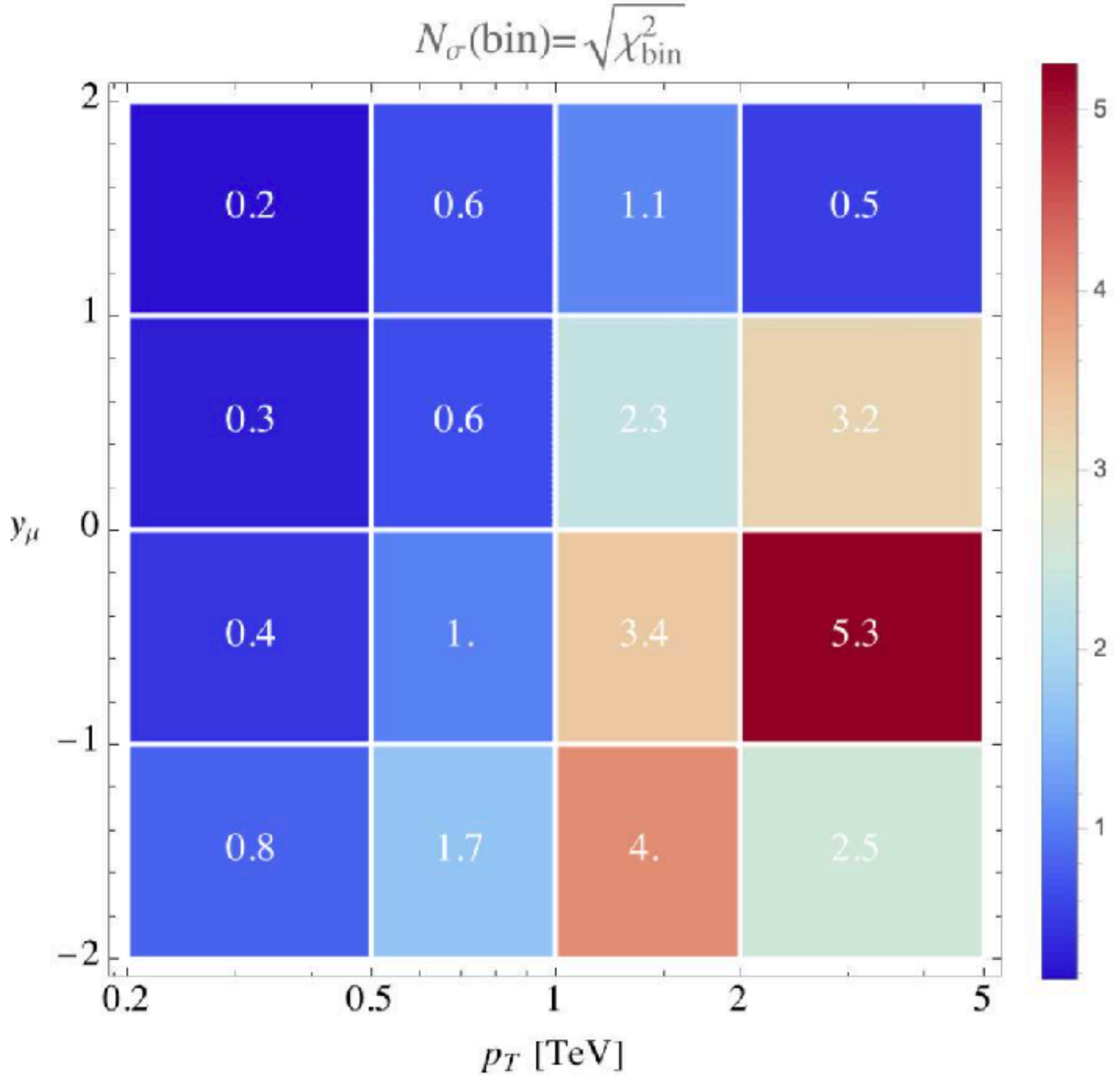
$$N_\sigma(\text{bin}) \equiv \sqrt{\chi^2_{\text{bin}}} \approx \left(\mathcal{L} \frac{(\sigma_{\text{tot}} - \hat{\sigma})^2}{\hat{\sigma}} \right)^{1/2}$$

where $\hat{\sigma} = \sigma_{\text{tot}} - \sigma_{Z/\gamma}$

$$\mathcal{L} = 10\,\text{ab}^{-1}$$

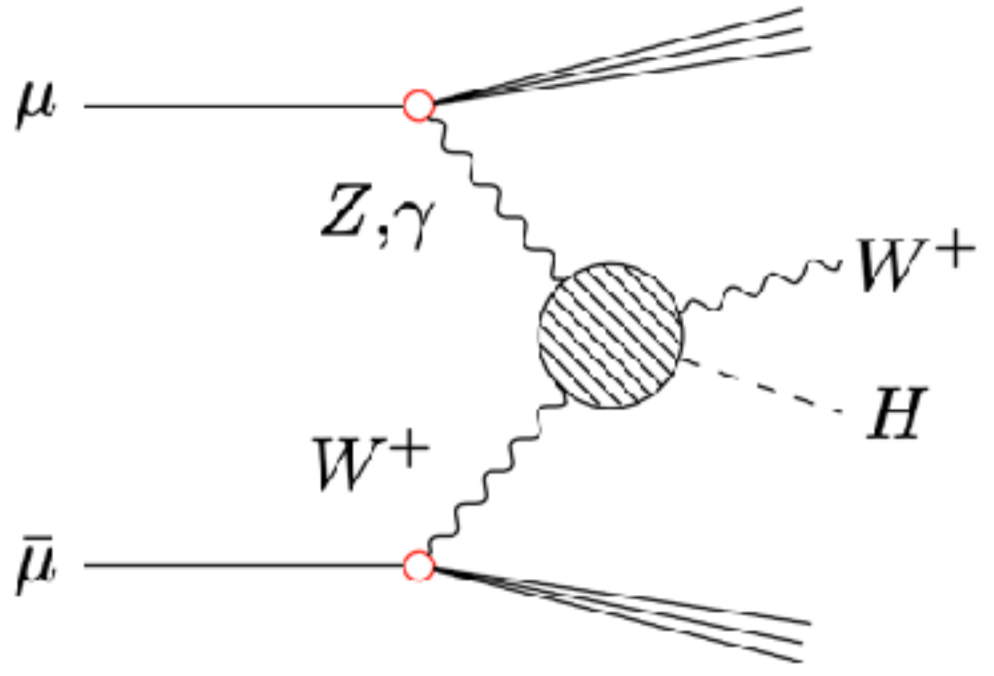
Statistical uncertainties of few % in the most sensitive bins: we neglect systematics.

The effect due to the Z/γ PDF can potentially be observed with more than 5σ precision at a future 10TeV MuC.



Impact in Higgs physics

Consider **associated W H production at a MuC**



The **mixed Z/γ PDF gives a contribution. How big?**

$$\frac{d^3\sigma_{\rm tot}}{dy_3 dy_4 dm} = f_1(x_1) f_2(x_2) \frac{m^3}{2s} \frac{1}{\cosh^2 y_*} \frac{d\sigma_{\rm H}}{dt}(1\,2 \to 3\,4)$$

We impose cuts: $|y_W| < 2\,,\quad |y_H| < 2\,,\quad m > 0.5\,\mathrm{TeV}$

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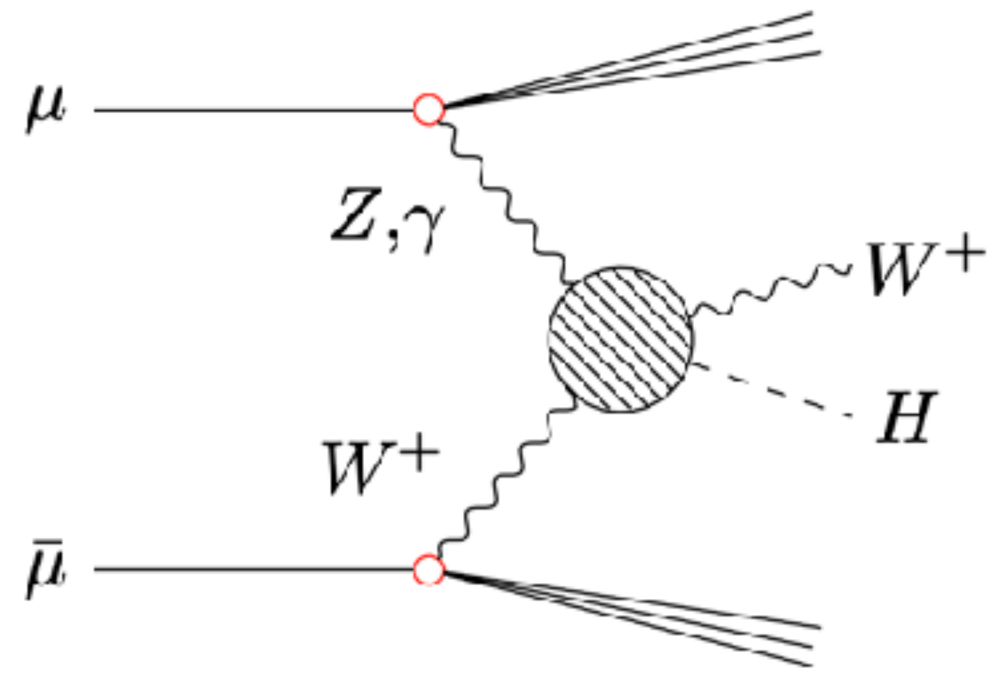

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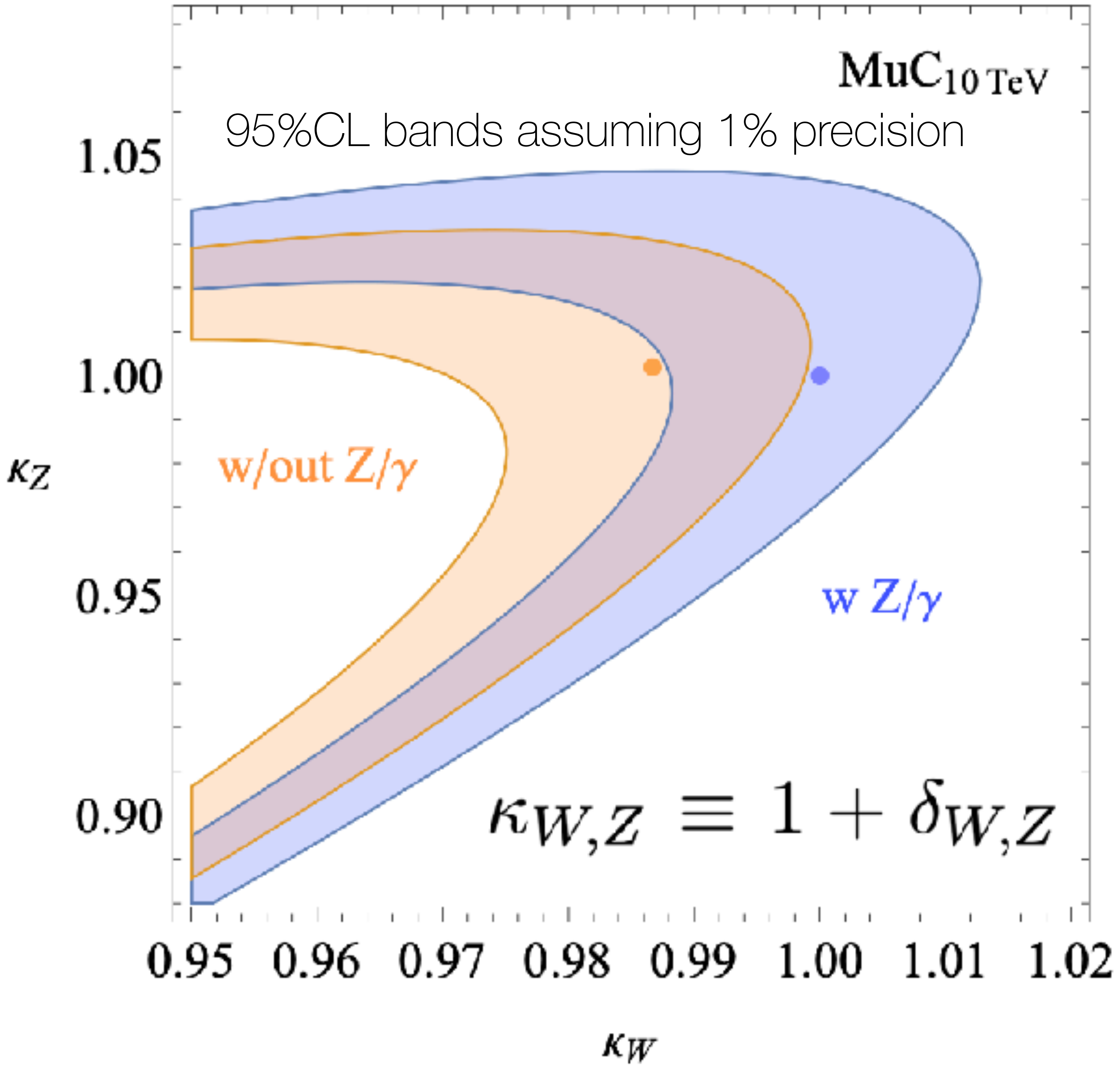
$$\sigma^{10\,\text{TeV}}_{\text{no-}Z/\gamma}\,[\text{fb}] = 6.81 + 15.58\,\delta_W - 1.96\,\delta_Z + 135.7\delta_W^2 - 255.8\delta_W\delta_Z + 126.9\delta_Z^2\,,$$
$$\sigma^{10\,\text{TeV}}_{\text{tot}}\,[\text{fb}] = 6.63 + 15.25\,\delta_W - 1.99\,\delta_Z + 135.6\delta_W^2 - 255.9\delta_W\delta_Z + 126.9\delta_Z^2\,,$$

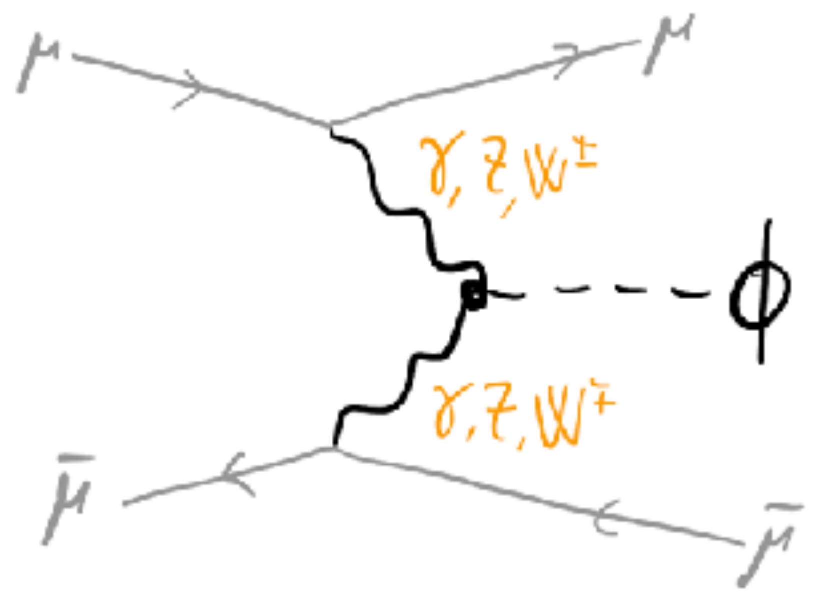


It **modifies the SM cross section by 3%**, to be compared with an expected precision in this channel of about 1% (value used in the plot).

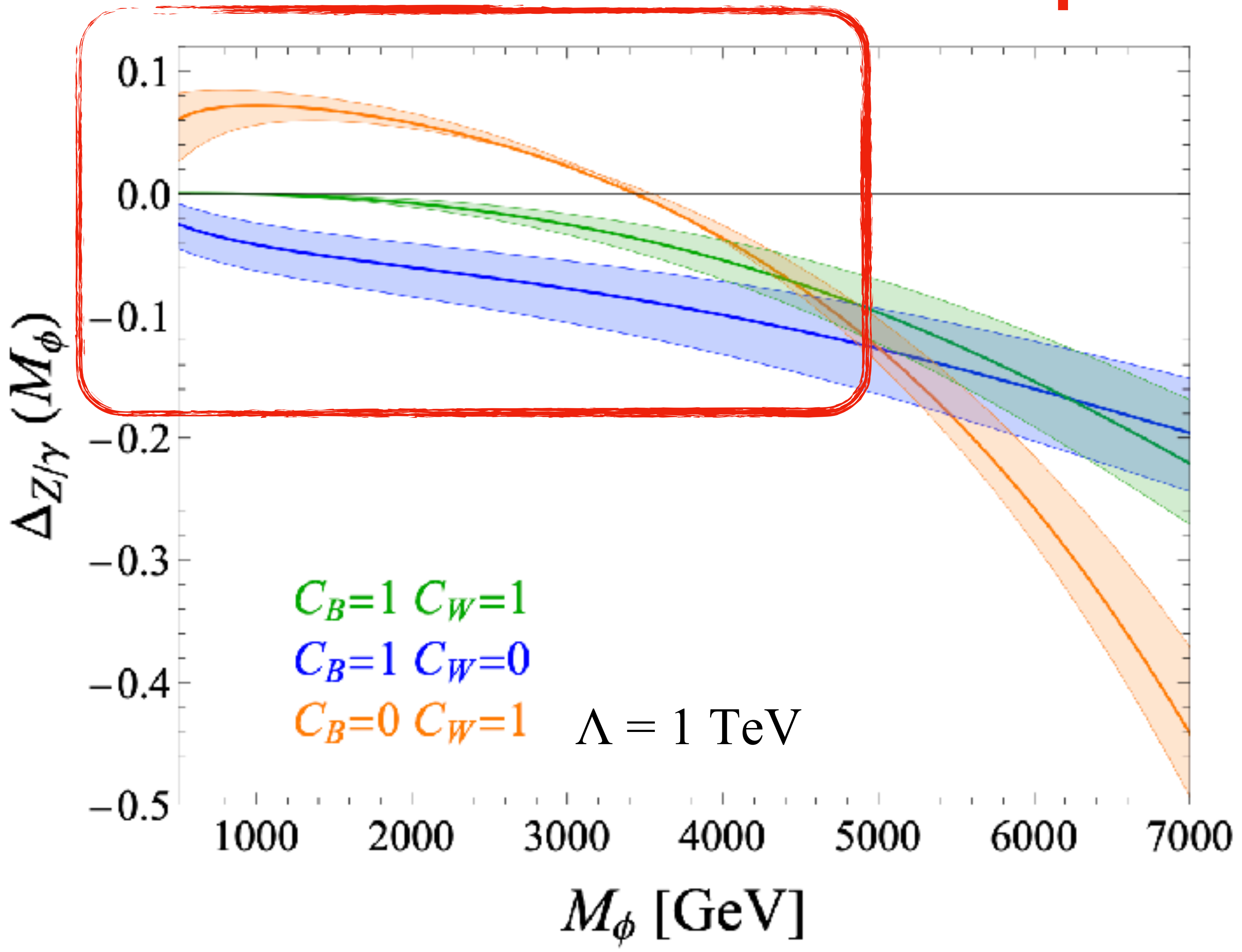
$$\sigma^{10\,\text{TeV}}_{\text{no-}Z/\gamma}\,[\text{fb}] = 135.70\,\kappa_W^2 + 126.93\,\kappa_Z^2 - 255.82\,\kappa_W\kappa_Z$$
$$\delta\sigma^{10\,\text{TeV}}_{Z/\gamma}\,[\text{fb}] = -0.15\,\kappa_W^2 - 0.030\,\kappa_W\kappa_Z\,,$$

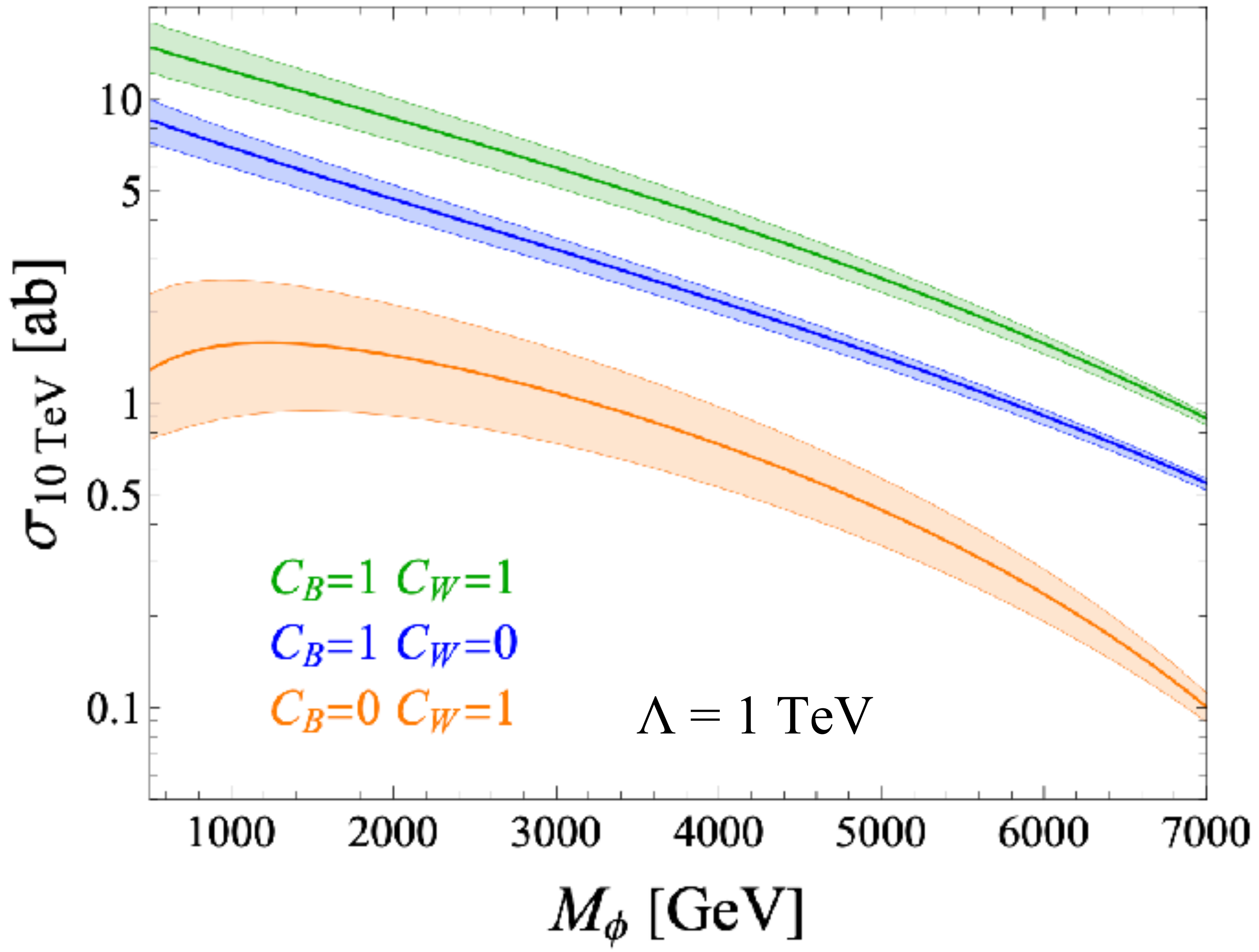
Single- ALP production @ MuC

$$\mathcal{L}_{\phi VV} = \frac{C_W}{\Lambda}\phi W^a_{\mu\nu}\widetilde{W}^{\mu\nu,a} + \frac{C_B}{\Lambda}\phi B_{\mu\nu}\widetilde{B}^{\mu\nu}$$



This ALP can be produced at muon colliders by (transverse) vector boson fusion.
What is the **impact** of the **mixed $Z\gamma$ PDF**?





~ 10% effect in the interesting mass region!

Z/γ at fixed order?

Can we describe **accurately enough** this contribution from a **fixed-order** calculation?

Z/γ at fixed order?

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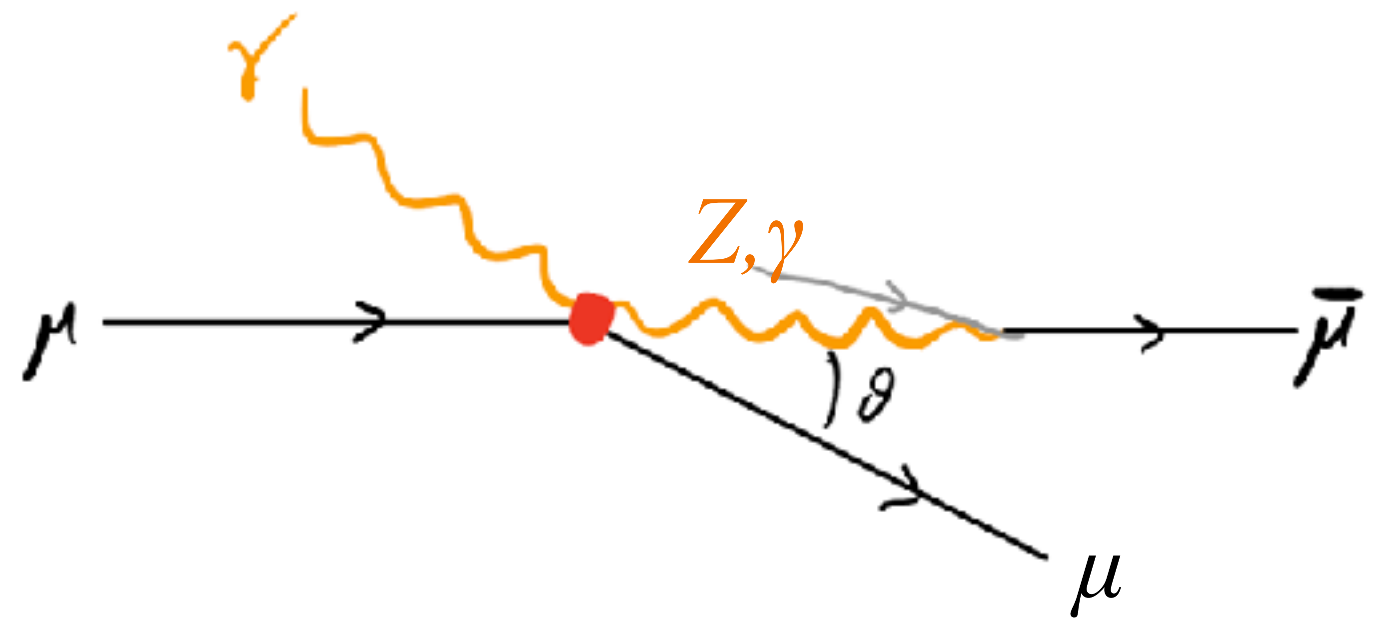
Consider **Compton Scattering**



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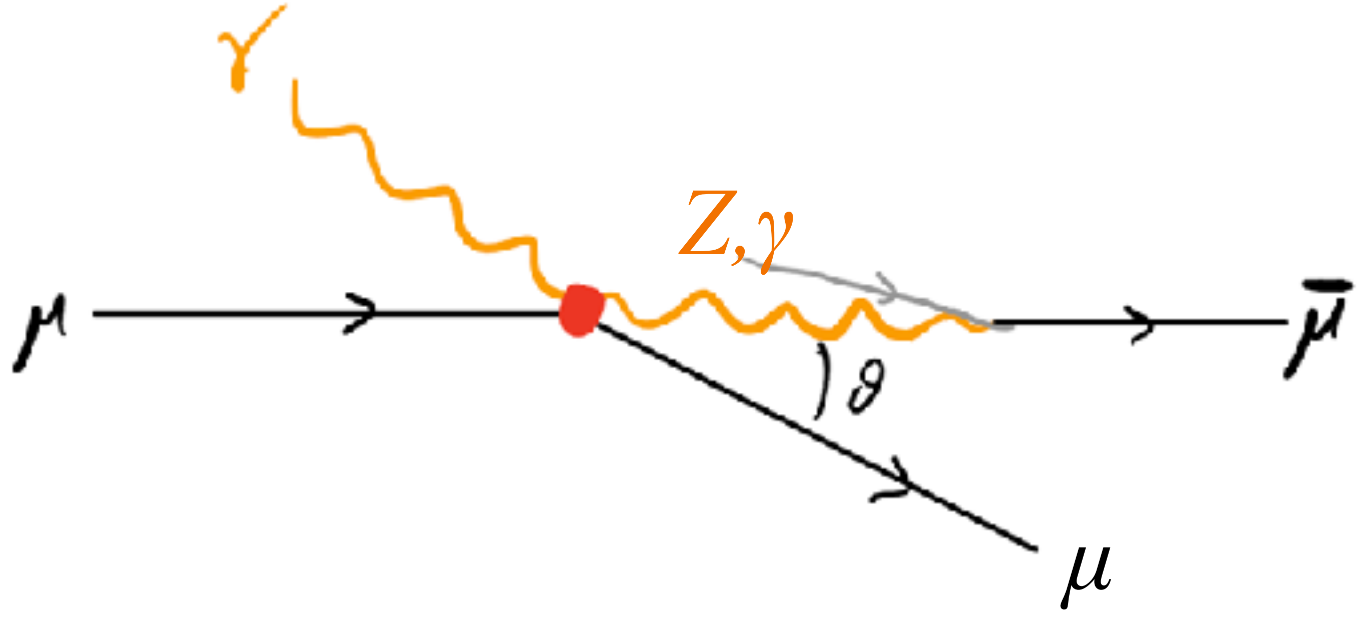
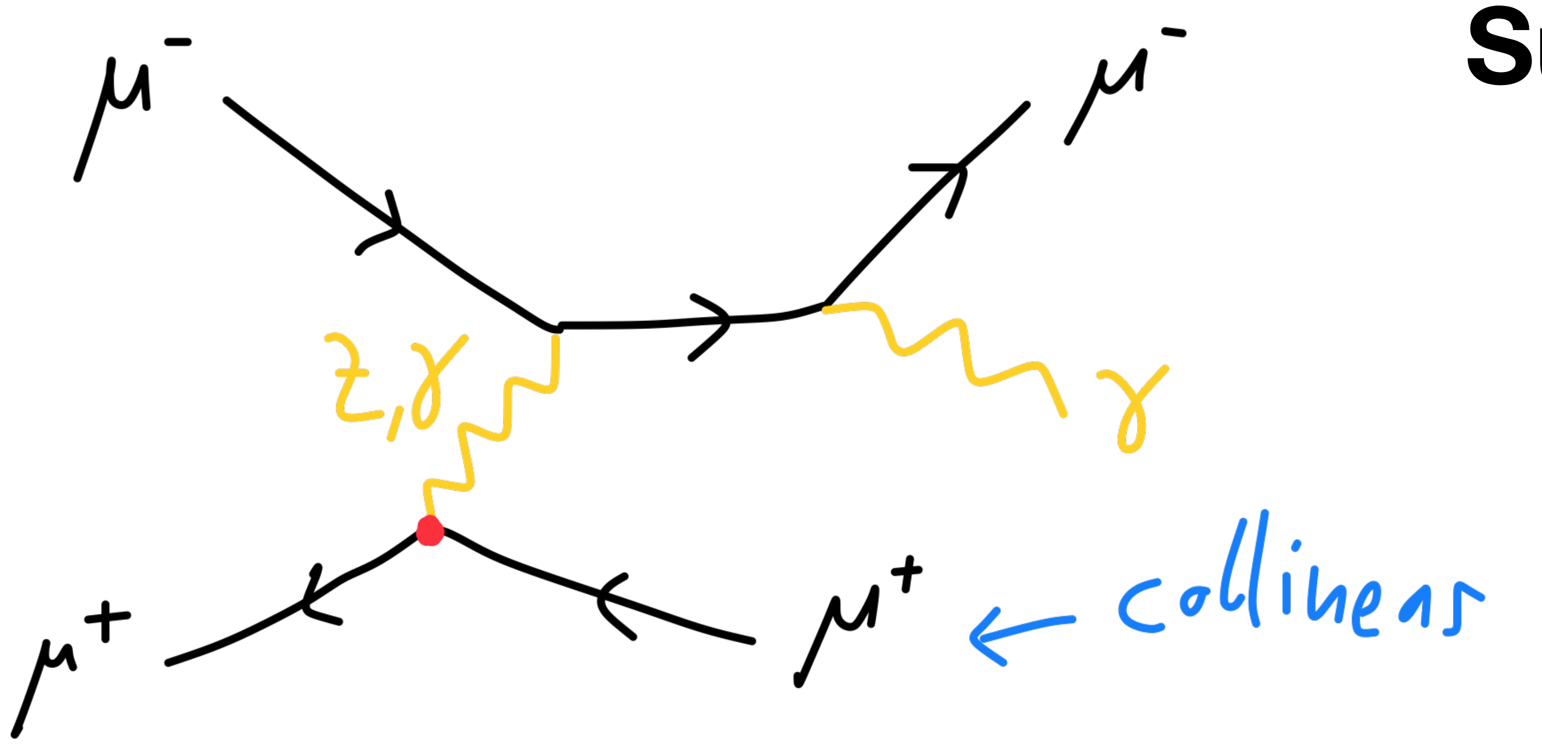


At **LO**:



Suppressed by the vector coupling of muon to Z:

$$Q^{Z}_{\mu_{L+R}} = -\frac{1}{2} + 2 s_W^2 \ll 1$$

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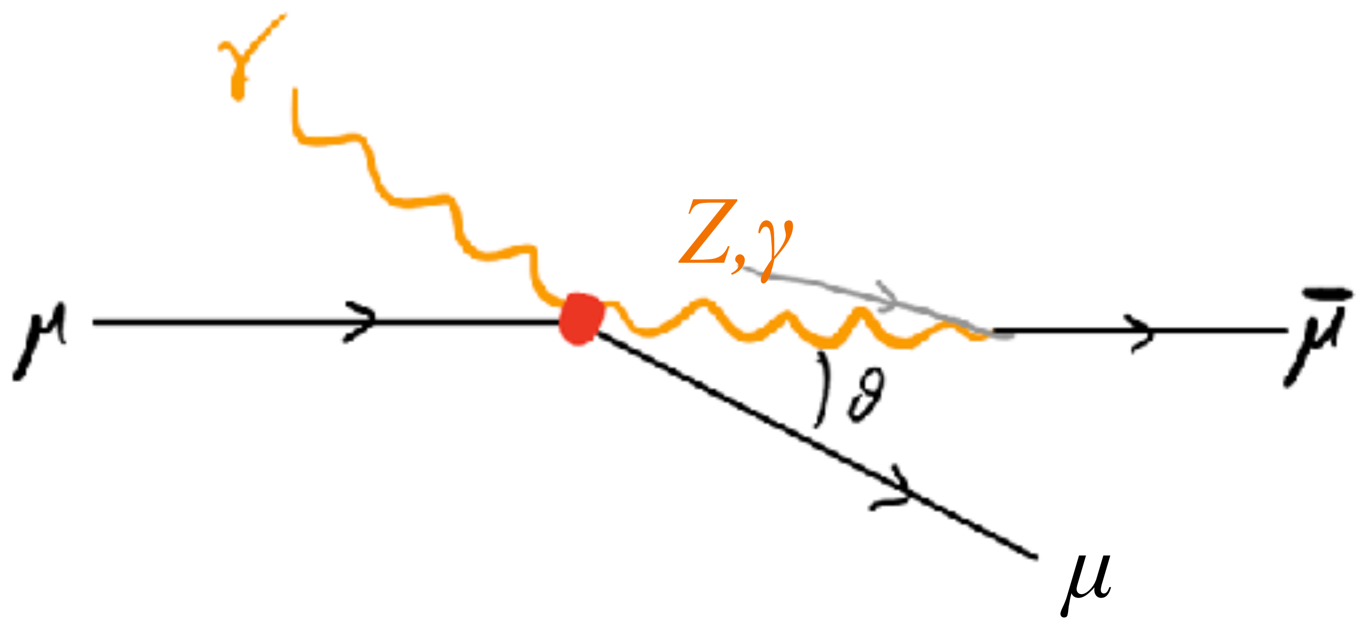

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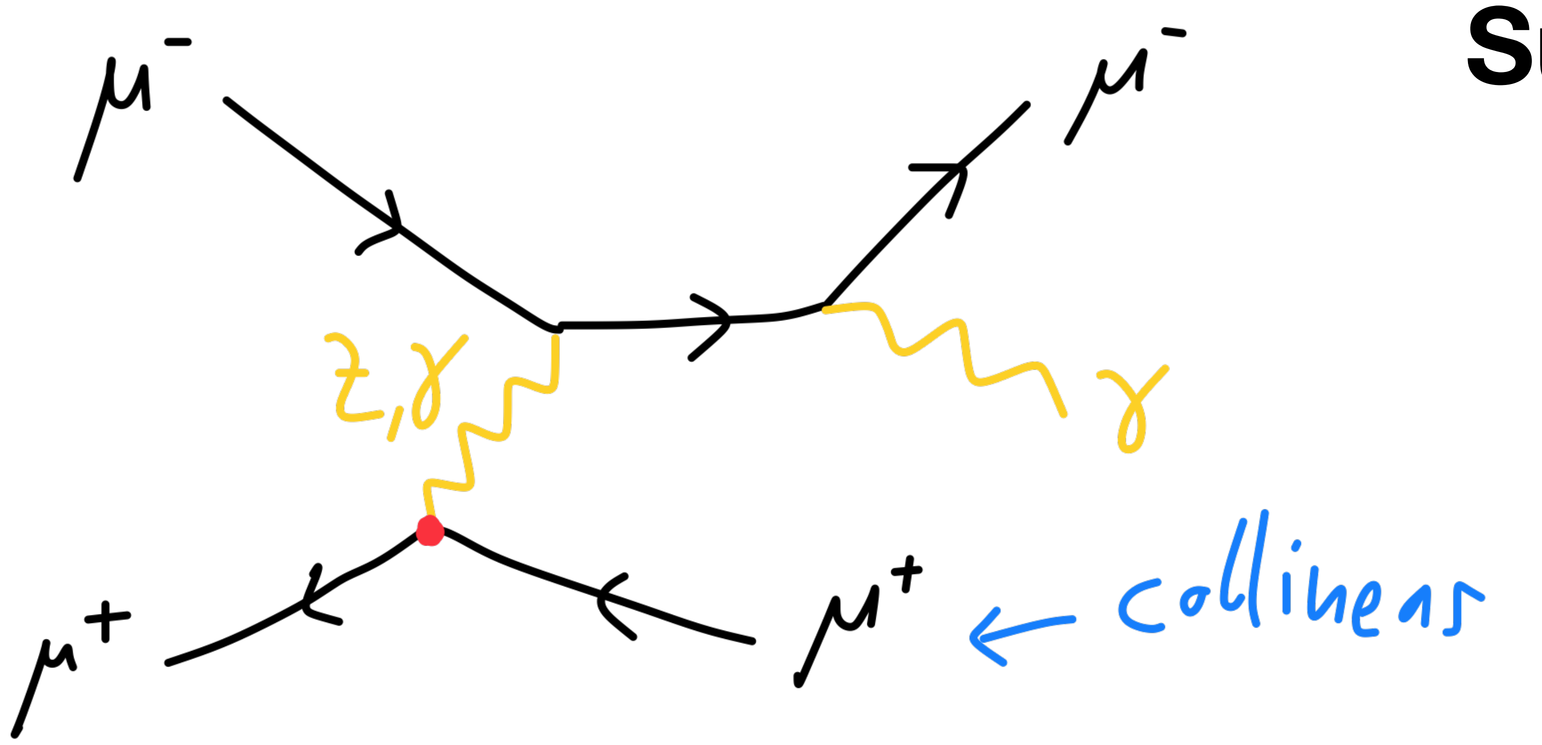

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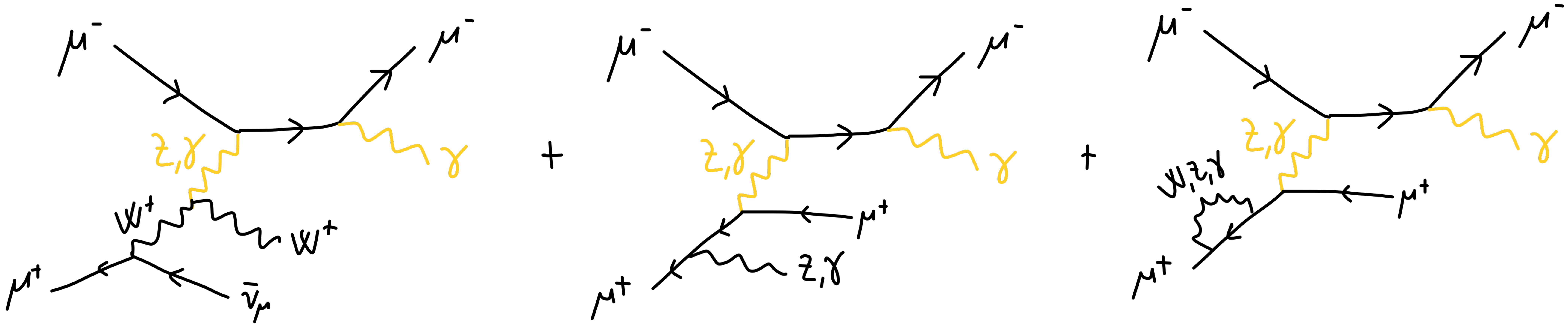
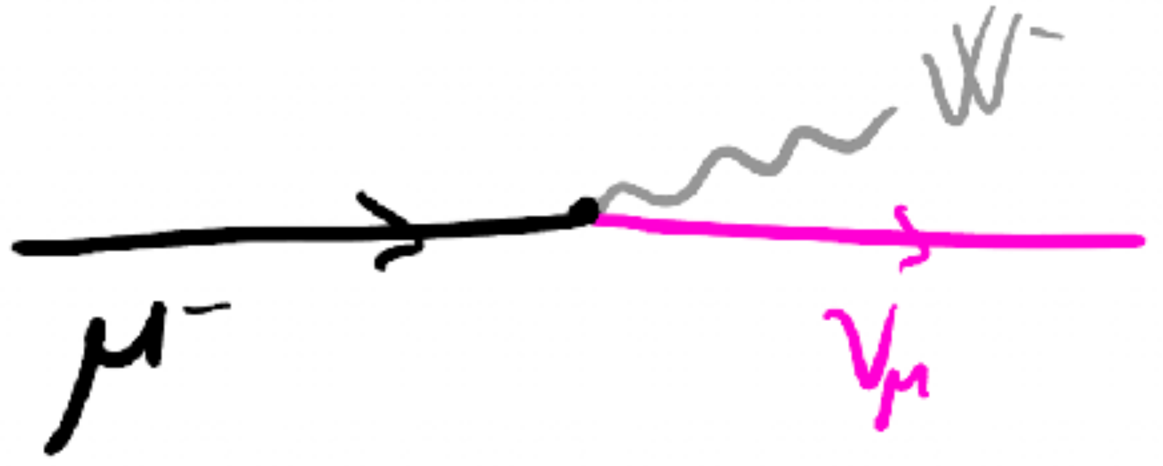
Need to consider at least one more splitting to recover the correct value of the mixed Z/γ contribution



Much more complicated to evaluate. PDFs allow to resume all these and do a simpler computation.

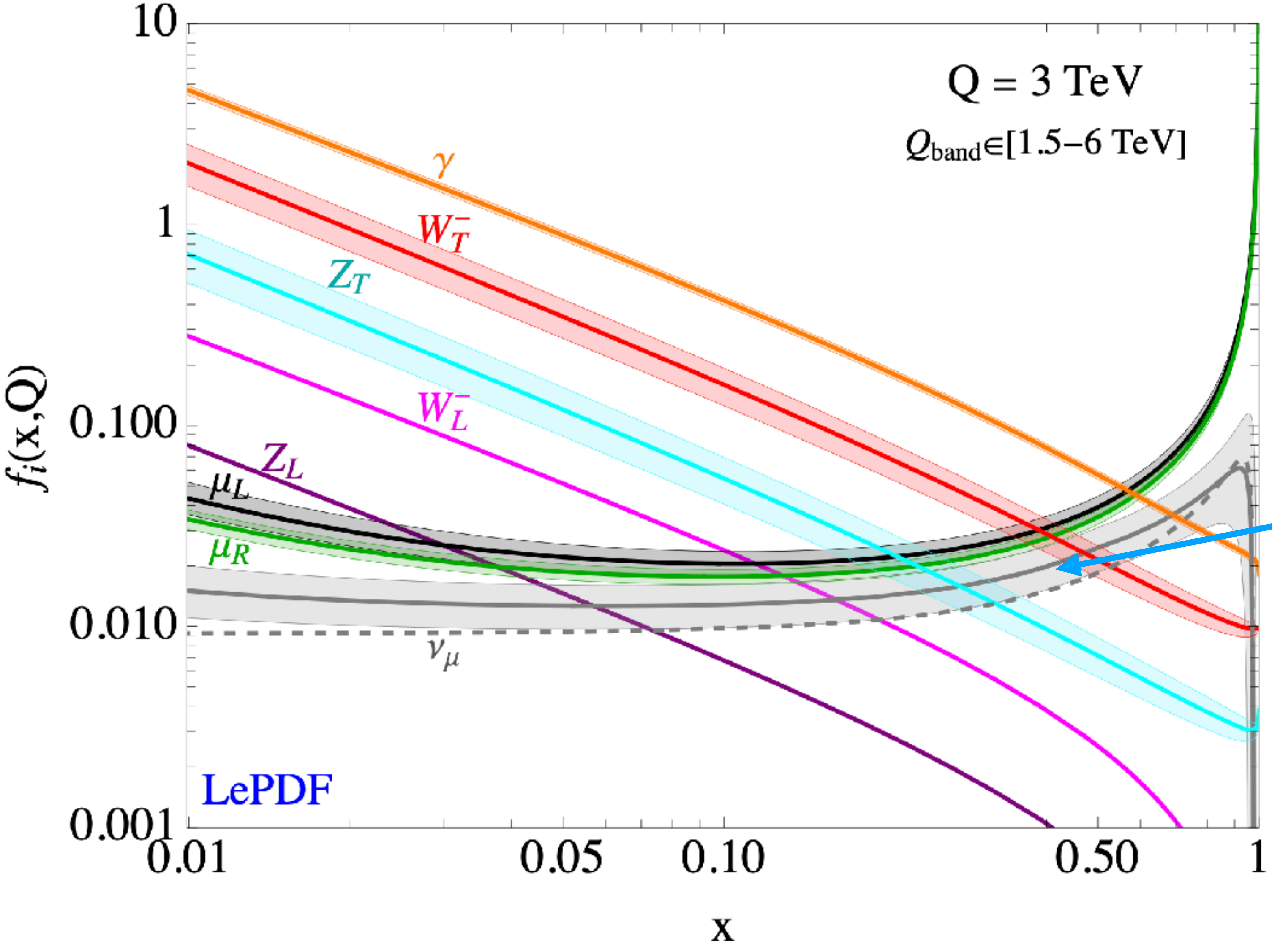
Pheno of EW PDF effects
(2)
Muon neutrino PDF

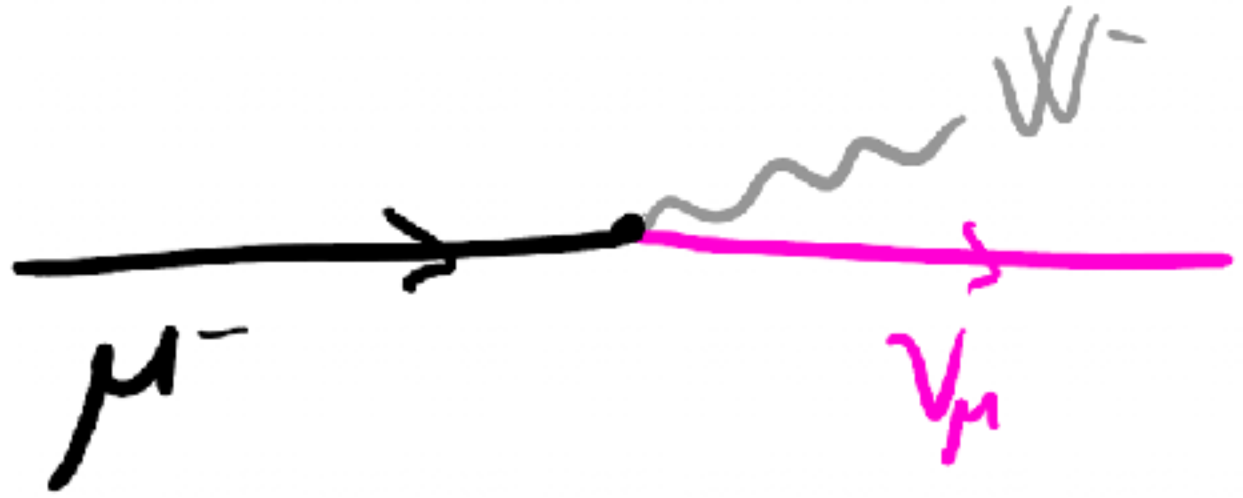
[F. Garosi, R. Capdevilla, D.M. and B. Stechauner 2410.21383]



Muon Neutrino PDF

Emission of **collinear W^- from the muon** generates a **muon neutrino content inside of the muon**.





Particularly **large at x ≳ 0.3** due to the IR divergence of the $\mu \to W \nu_\mu$ splitting

Muon Neutrino PDF from LePDF

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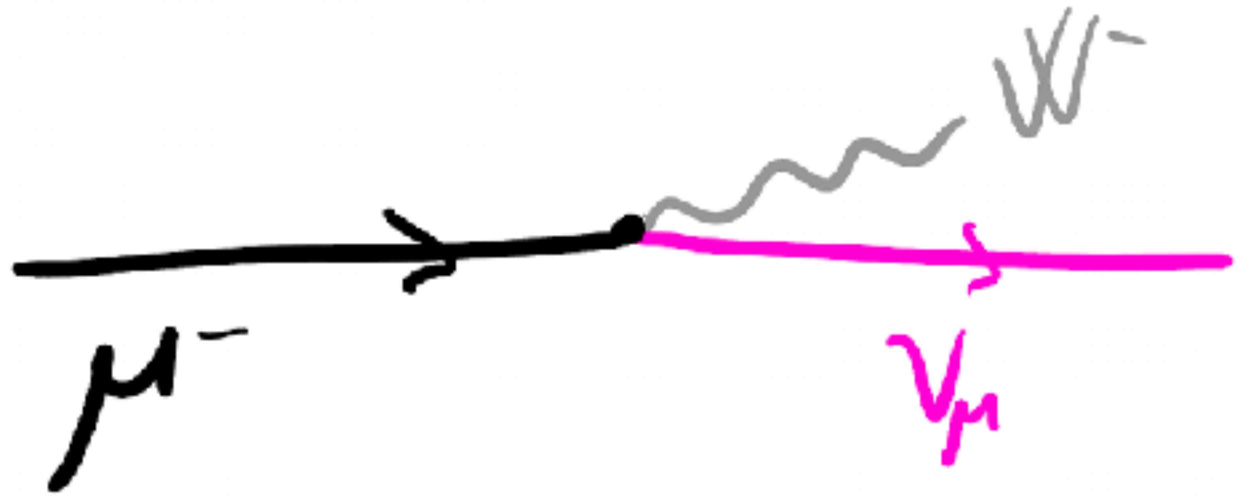

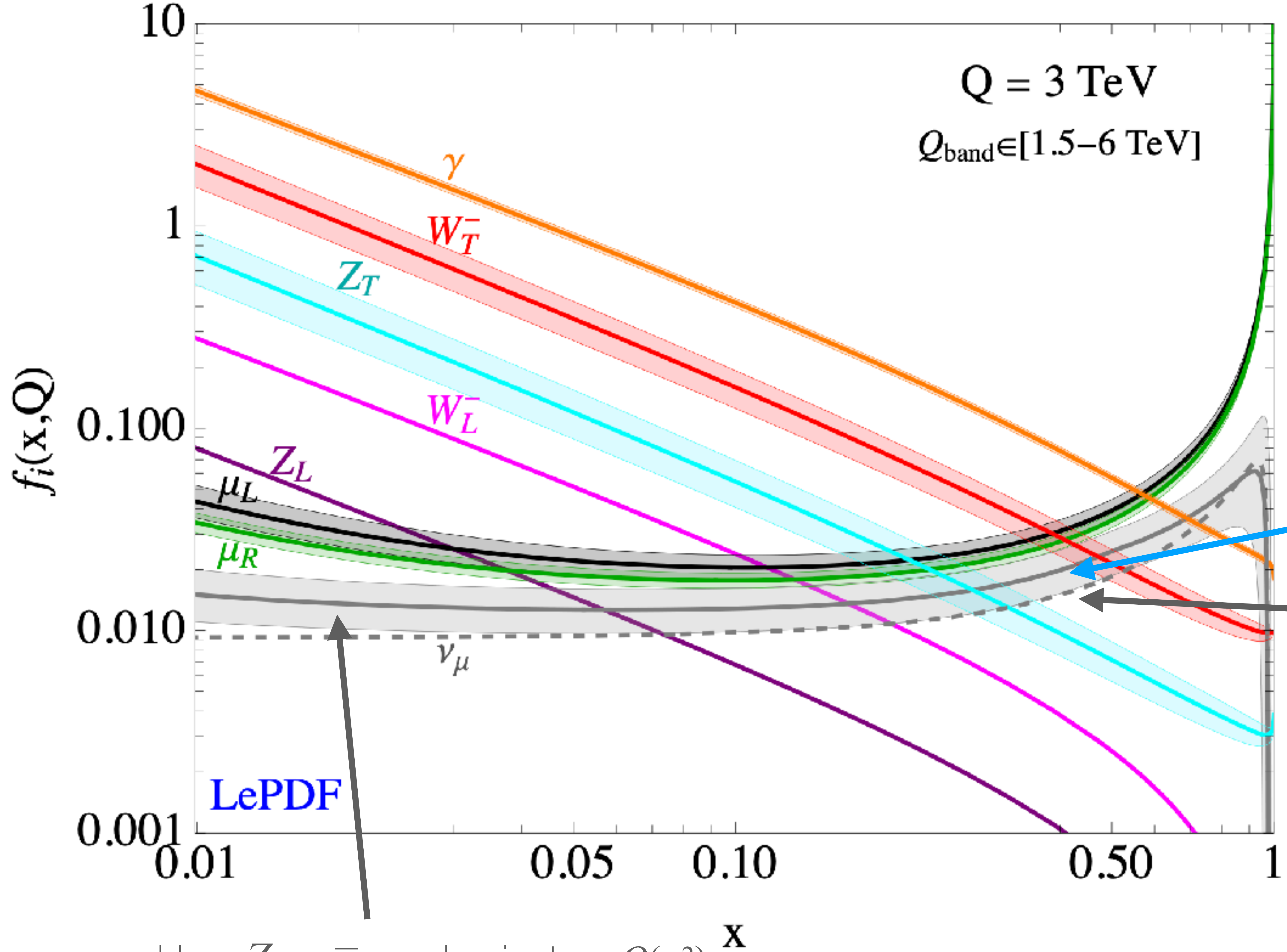

Here $Z \to \overline{\nu}_\mu\, \nu_\mu$ dominates: $O(\alpha^2)$

Particularly **large at x ≳ 0.3** due to the IR divergence of the $\mu \to W\, \nu_\mu$ splitting

Muon Neutrino PDF from LePDF

We can compute the ν_μ **PDF at $O(\alpha)$** (as for EVA)

$$f^{(\alpha)}_{\nu_\mu}(x,Q^2) = \frac{\alpha_2(Q)}{8\pi}\,\theta\left(Q^2 - \frac{m_W^2}{(1-x)^2}\right)\left[\frac{1+x^2}{1-x}\left(\log\frac{Q^2+xm_W^2}{m_W^2} + \right.\right.$$
$$\left. + \log\frac{(1-x)^2}{1+x(1-x)^2} + \frac{xm_W^2}{Q^2+xm_W^2} + \frac{1}{1+x(1-x)^2} - 1\right)$$
$$\left. + \frac{2x^2(1-x)^2}{(1-x)(1+x(1-x)^2)}\,\frac{Q^2-m_W^2}{Q^2+xm_W^2}\right],$$

Muon Neutrino PDF

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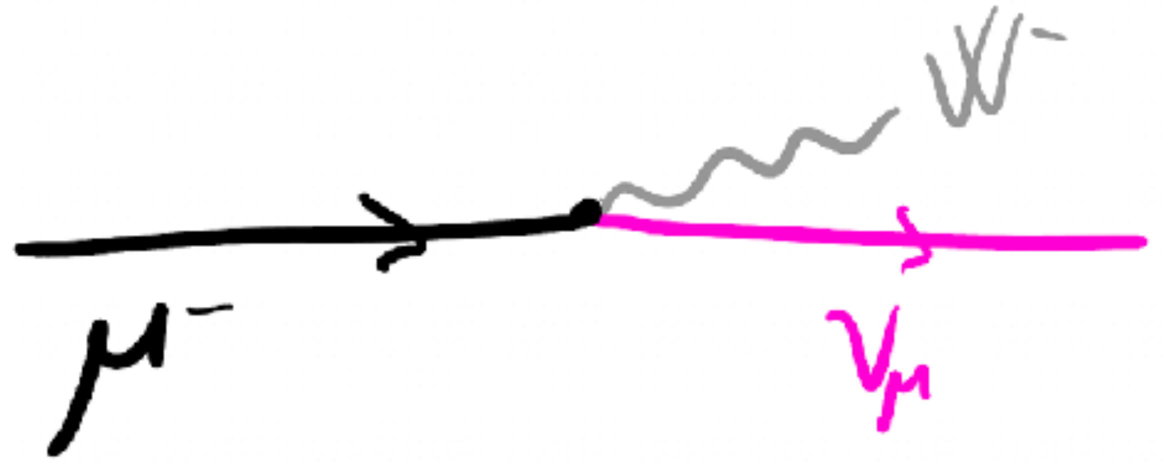
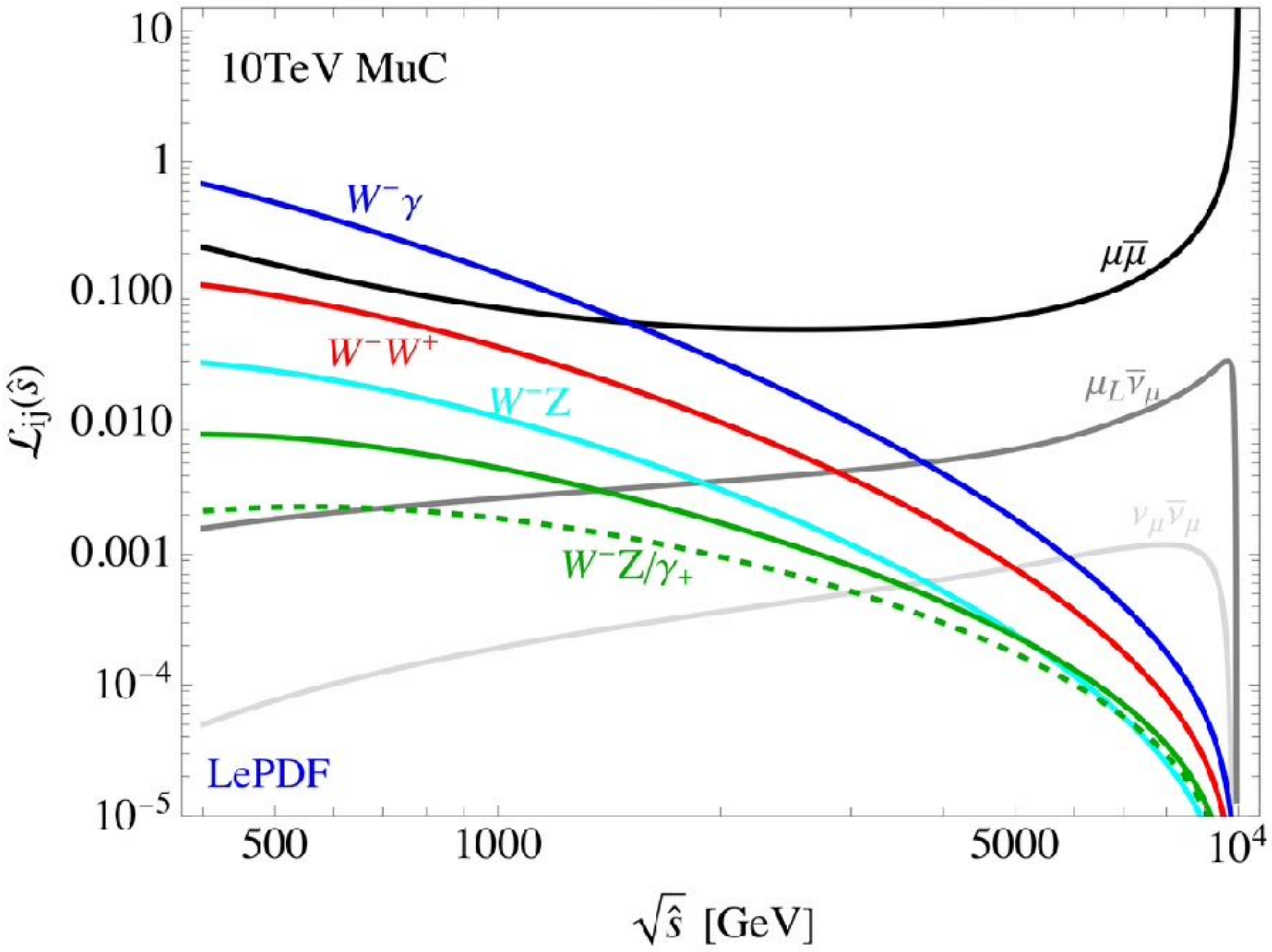
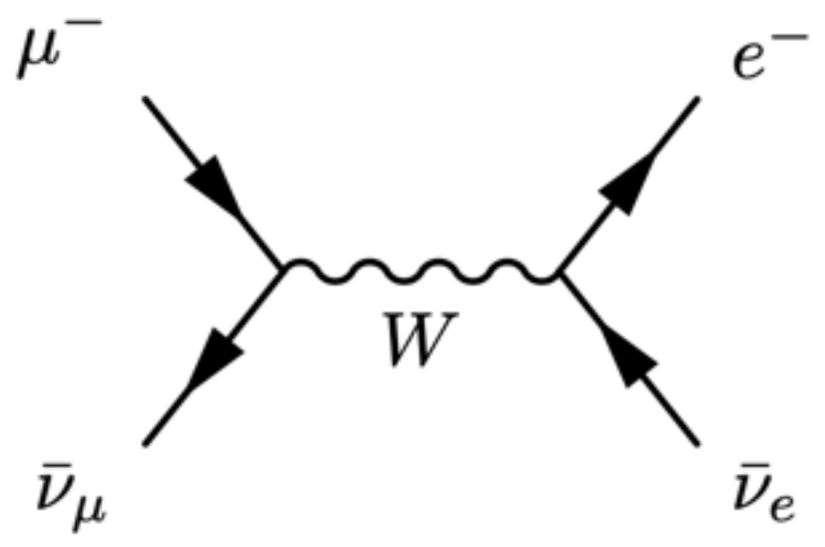


Also in terms of parton luminosities, it is clear that **the contribution from the neutrino PDF will be important in the high-energy tail of EW processes**.

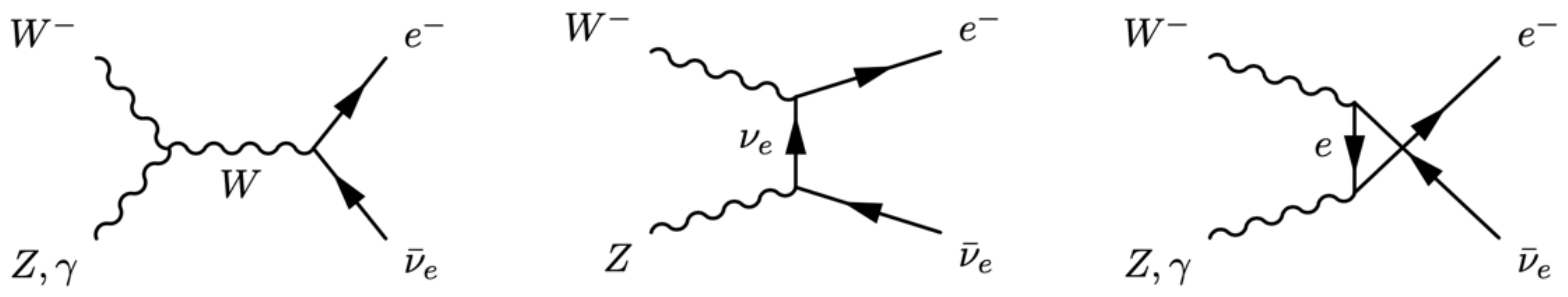
$$\mathcal{L}_{ij}(\hat{s}, s_0) = \int_0^1 \frac{dz}{z} f_{i;\mu}\left(z, \frac{\hat{s}}{4}\right) f_{j;\bar{\mu}}\left(\frac{\hat{s}}{z s_0}, \frac{\hat{s}}{4}\right)$$



Observing f_{ν_μ} in $e^- \nu_e$ production



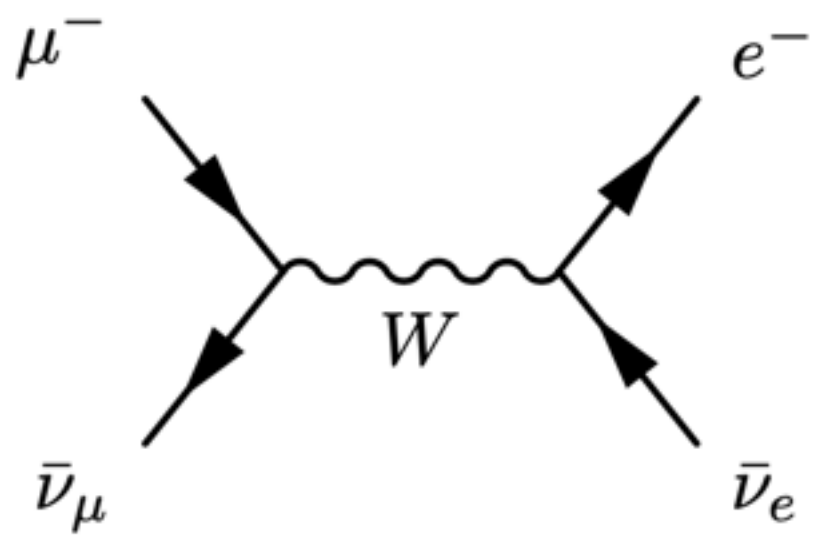
Our “signal”



Our “background”

$\hat{\sigma}_{sig}, \hat{\sigma}_{bkg} \sim \frac{\alpha_2}{\hat{s}}$

Observing f_{ν_μ} in $e^- \nu_e$ production



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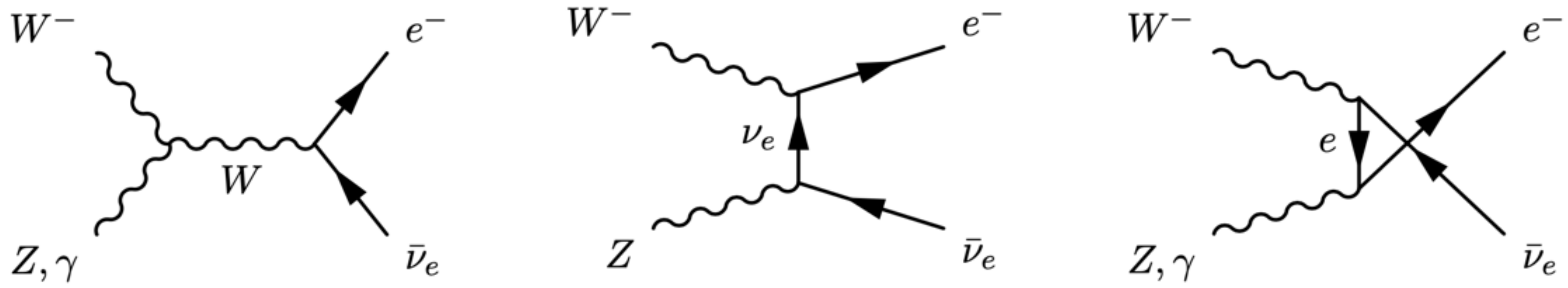


Our "background"

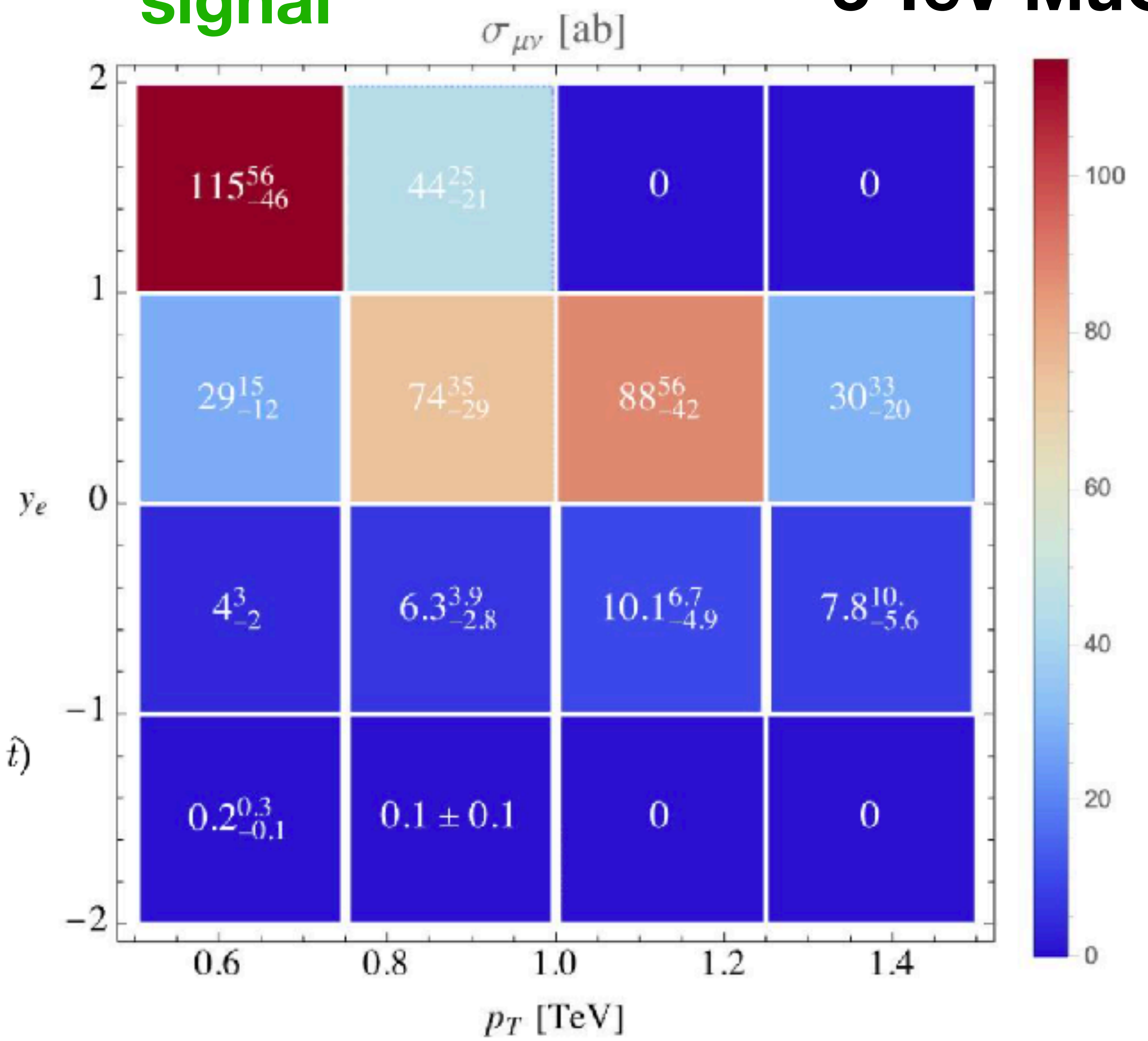
$\hat{\sigma}_{sig}, \hat{\sigma}_{bkg} \sim \frac{\alpha_2}{\hat{s}}$

We compute **xsec in bins of electron rapidity and p_T**, for both signal and background:

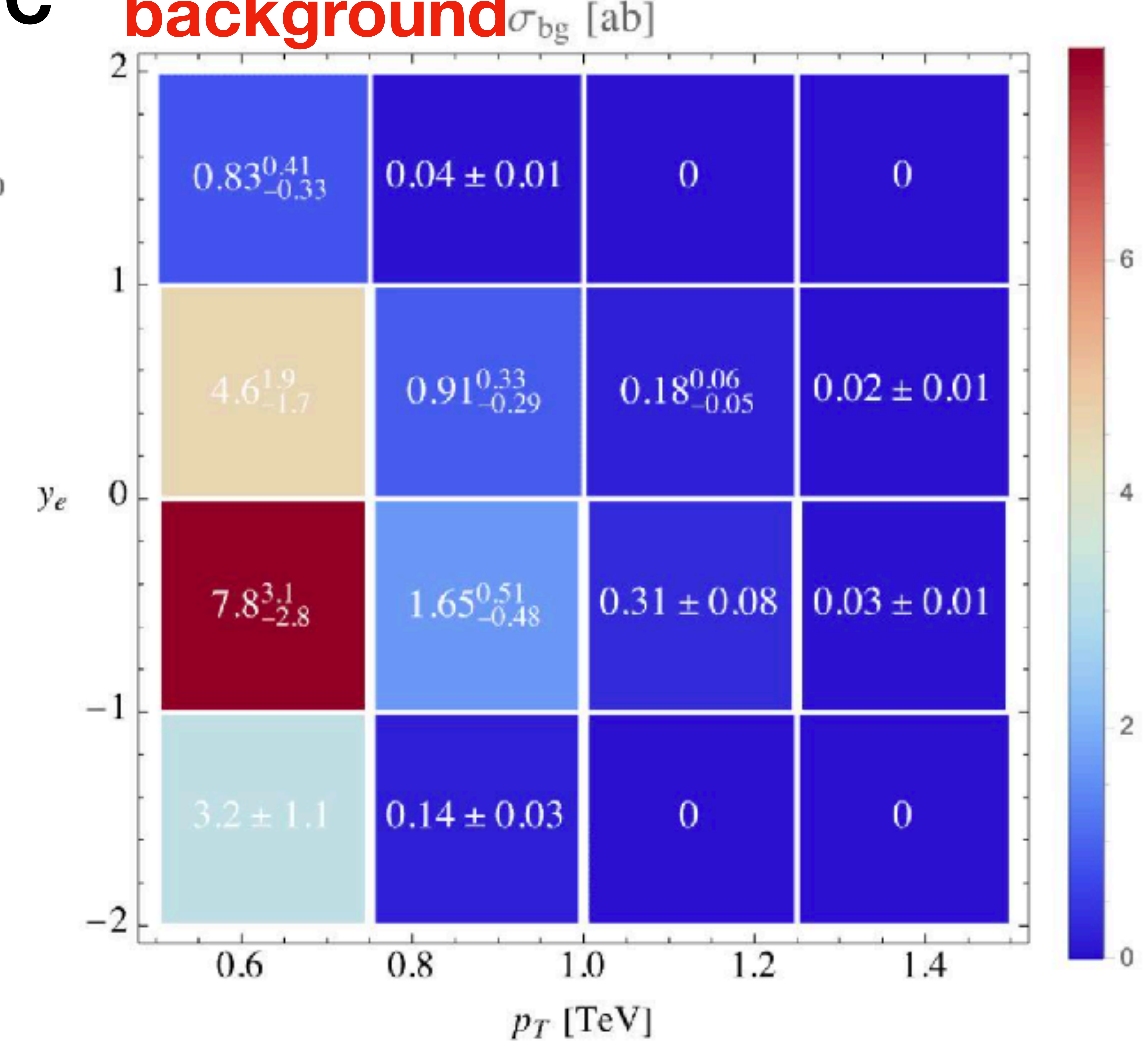
$$\frac{d^3\sigma(\mu\bar{\mu} \to e^-\bar{\nu}_e + X)}{dy_e dy_\nu dp_T} = \sum_{i,j} f_i^{\mu}(x_1) f_j^{\bar{\mu}}(x_2) \left(\frac{2p_T\hat{s}}{s_0}\right) \frac{d\hat{\sigma}}{d\hat{t}}(ij \to e^-\bar{\nu}_e)(\hat{s},\hat{t})$$

signal

3 TeV MuC



background



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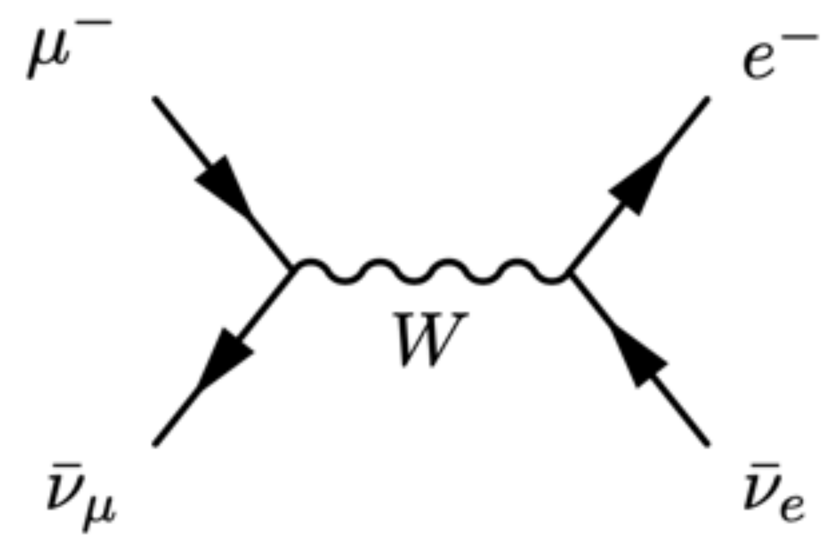
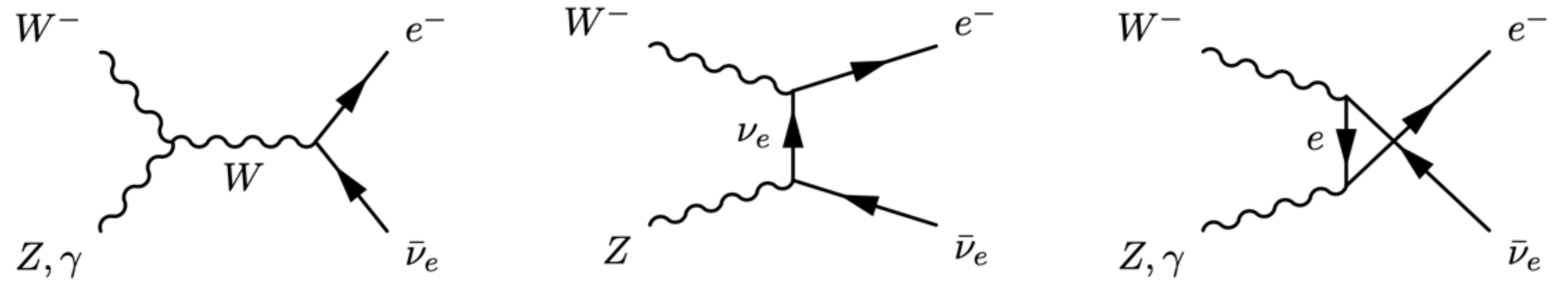


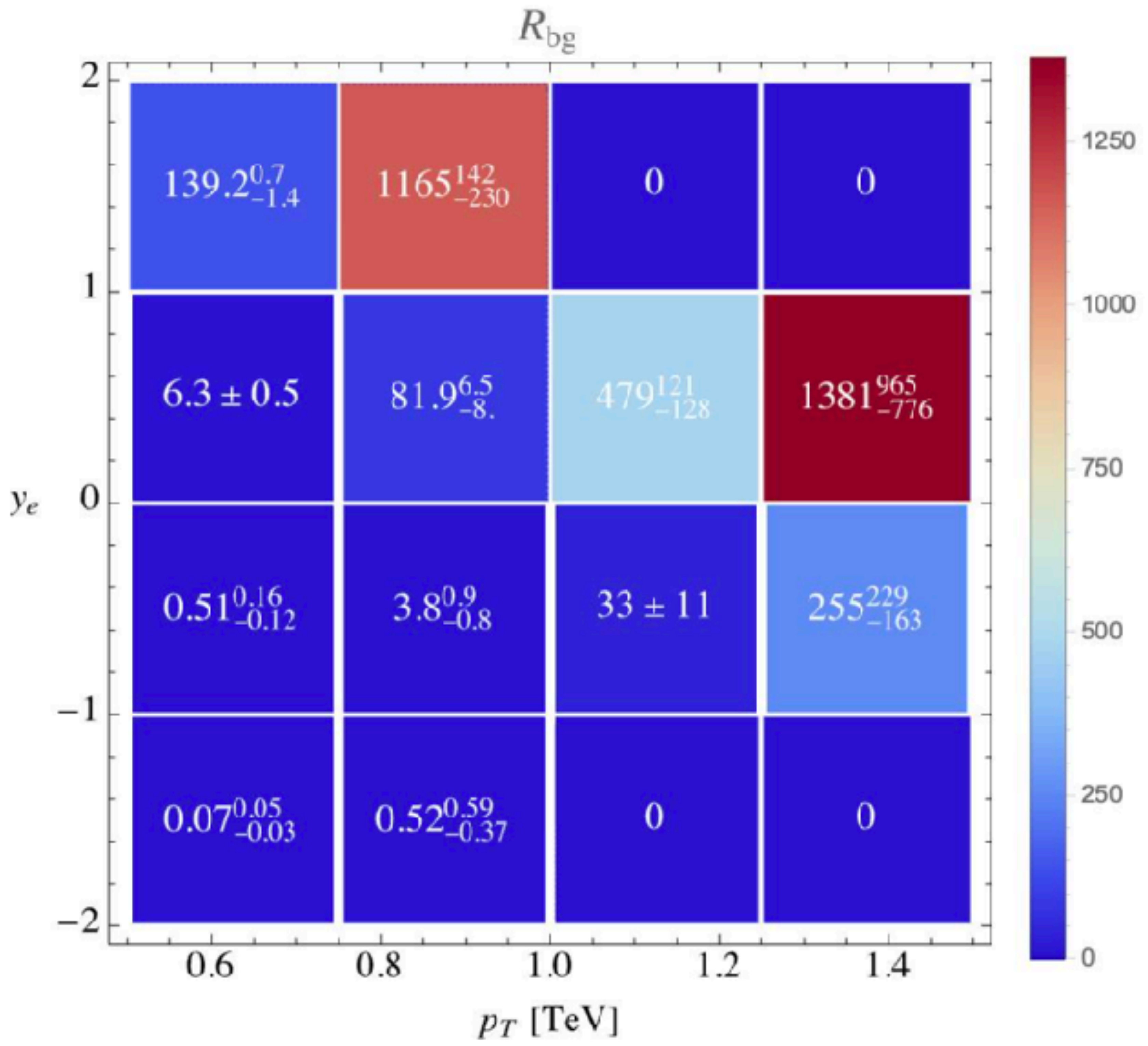
Our "signal"



Our "background"

We define the **signal/background** ratio:

$$R_{\rm bg}^{e\nu} = \frac{\sigma(\mu^-\bar{\nu}_\mu \to e\bar{\nu}_e)}{\sigma_{\rm bg}^{e\nu}}$$

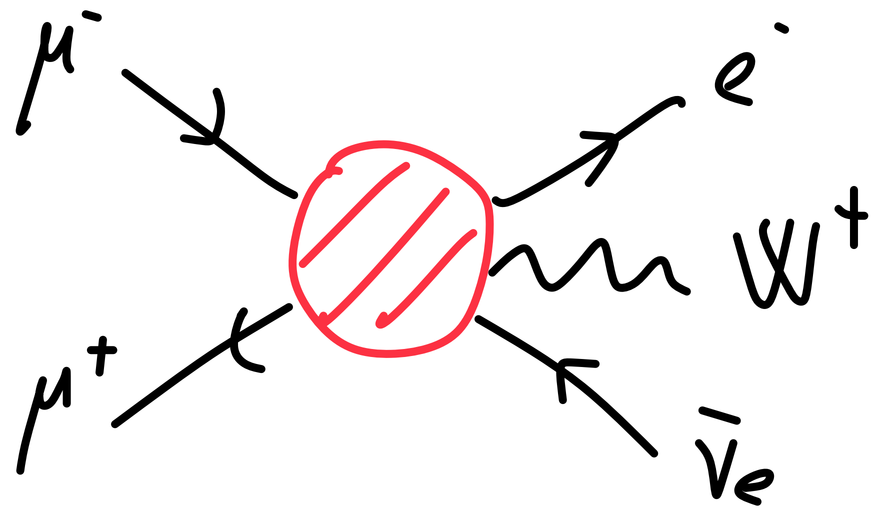


Clearly, the **contribution from neutrino PDF is very large and dominates** for **forward electrons** and increases with p_T.

Comparison with MadGraph at LO

How well does the PDF computation agree with the fixed-order result?
We use MadGraph5 to generate at LO the process:

$$\mu^- \mu^+ \to e^- \bar{\nu}_e W^+$$



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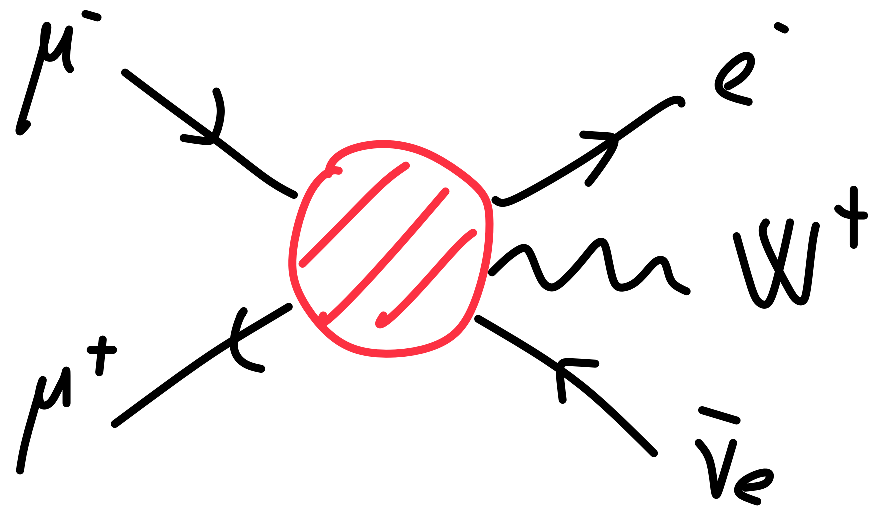
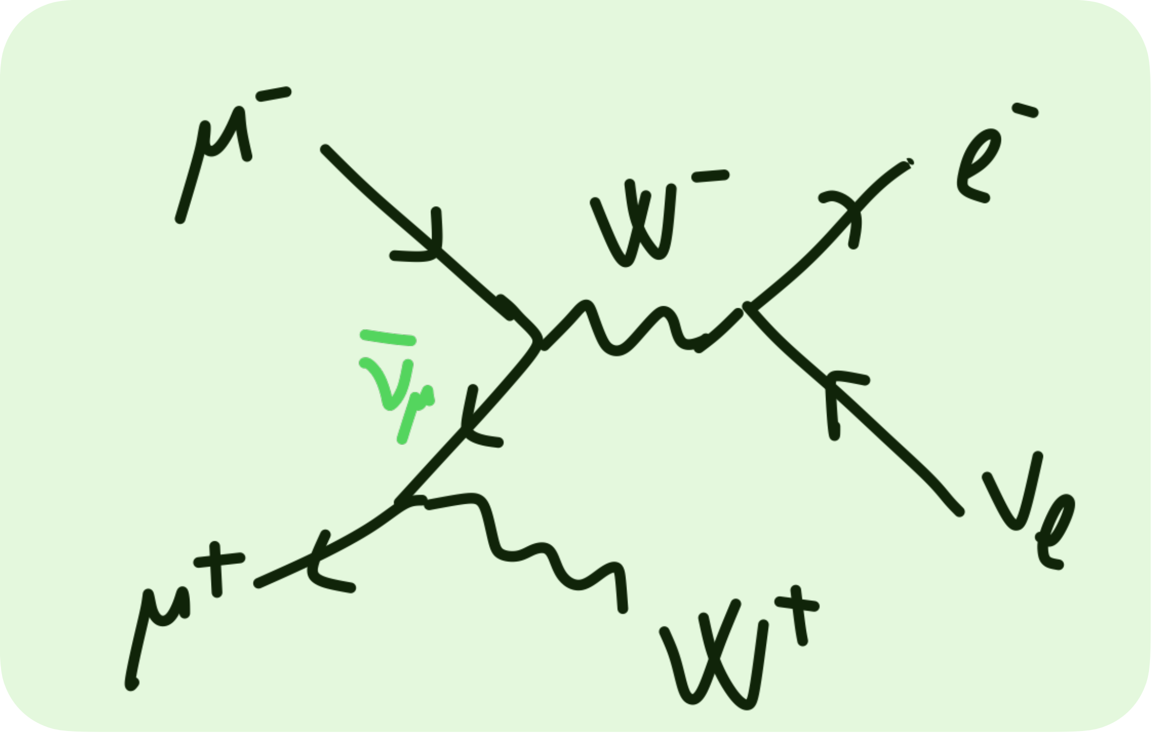
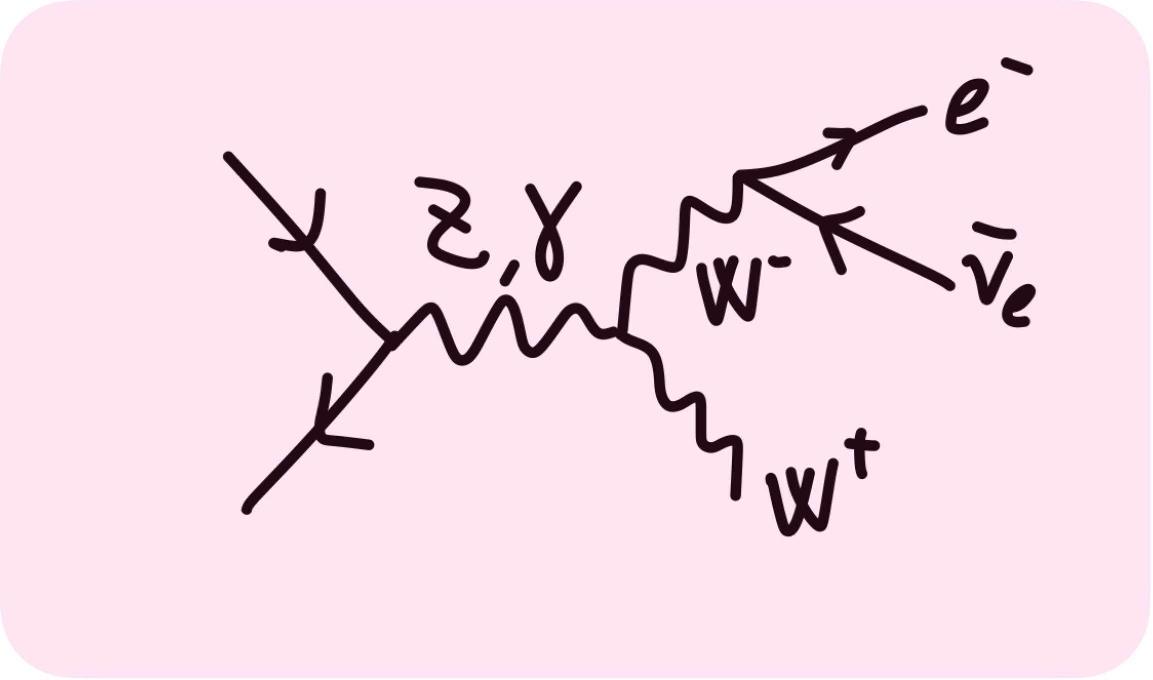
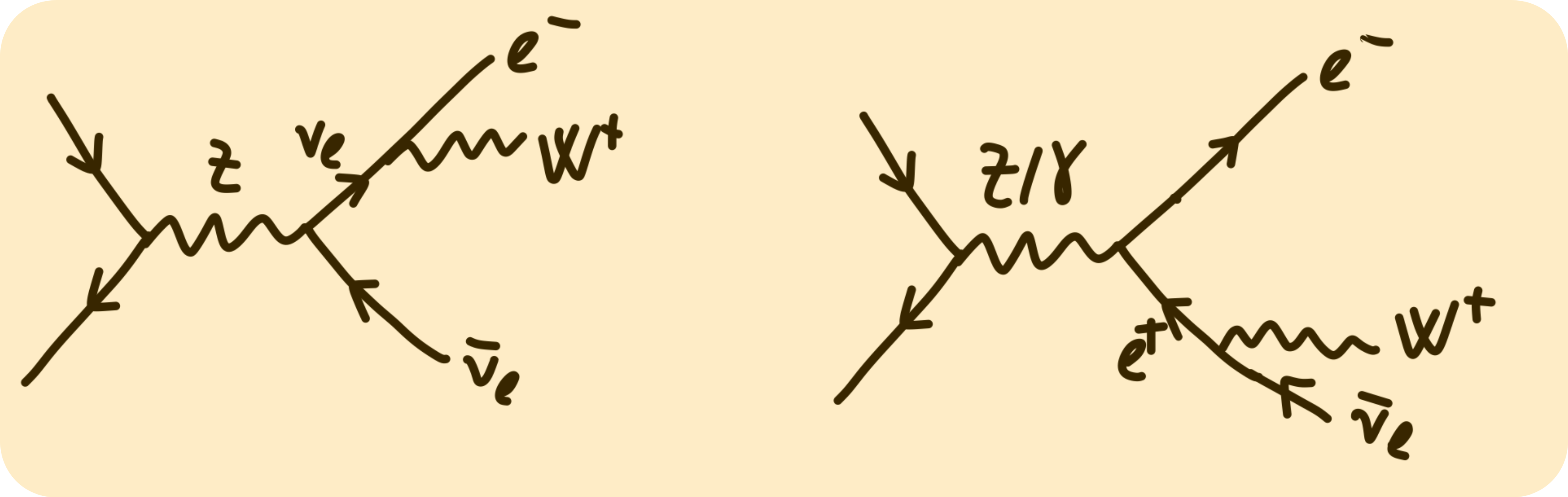


We are interested in the region with **collinear W boson from ISR**, however the full process has:





W W* production



Neutral-current dilepton + W-FSR

For this exercise we neglect the fact that the neutrino momentum cannot be reconstructed.

Comparison with MadGraph at LO

For the comparison, we apply the same cuts on the hard final states:

$$|y_e| < 2\ , \quad p_T^e > 1\,\text{TeV}\ , \quad p_T^\nu > 1\,\text{TeV}\ , \quad M(e,\nu_e) > 500\,\text{GeV}$$

> In the region of **large e+ν_e invariant mass**, the **WW* channel is negligible**.

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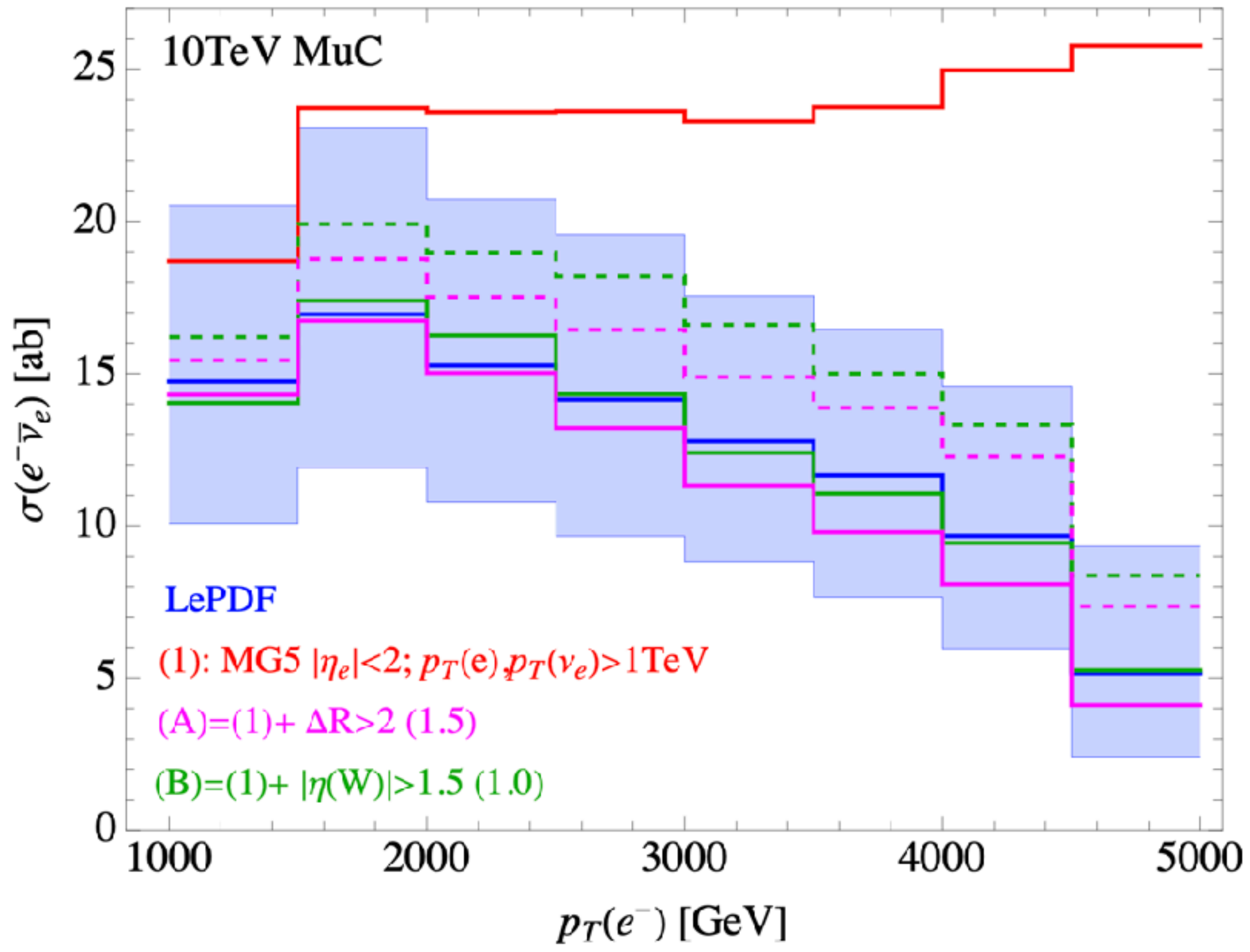

We see a large discrepancy, which grows at large pT.
By inspection, this is due to **central Ws**, emitted as **FSR**.

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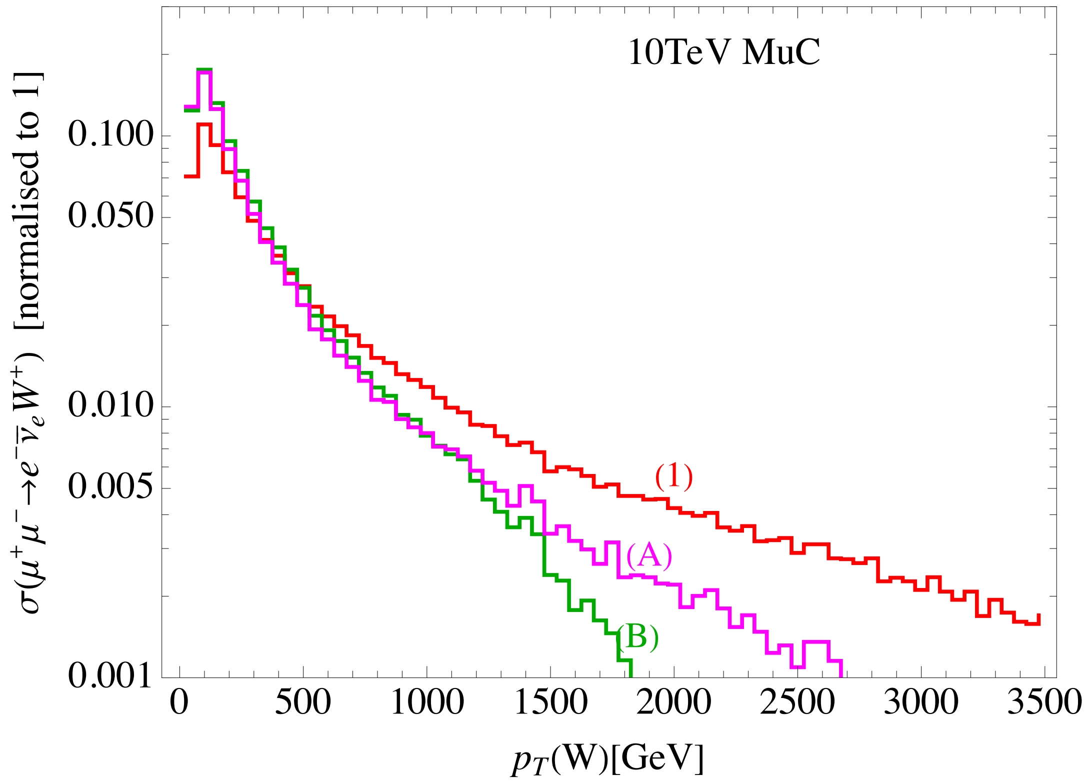

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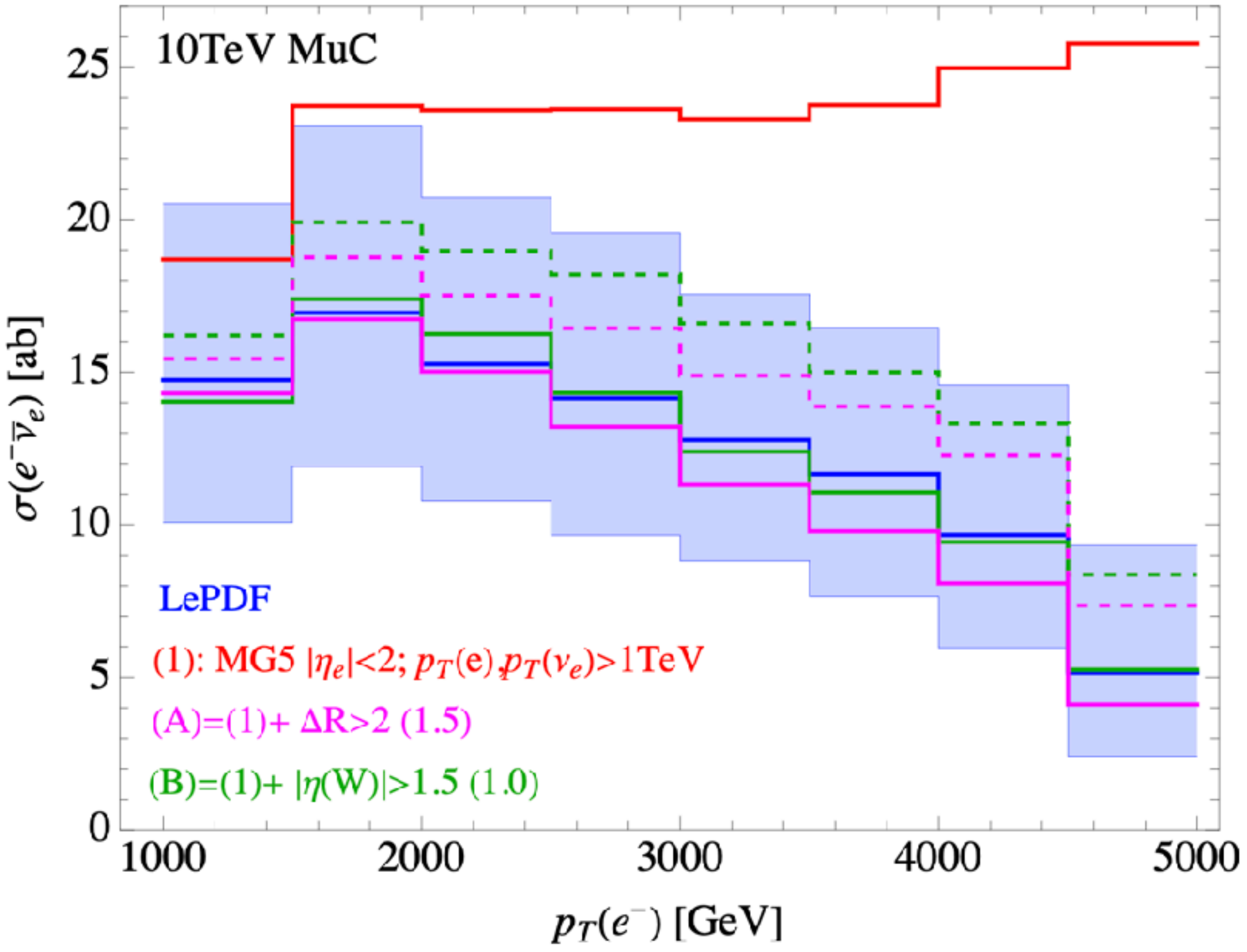

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We addresses in **two possible ways**:

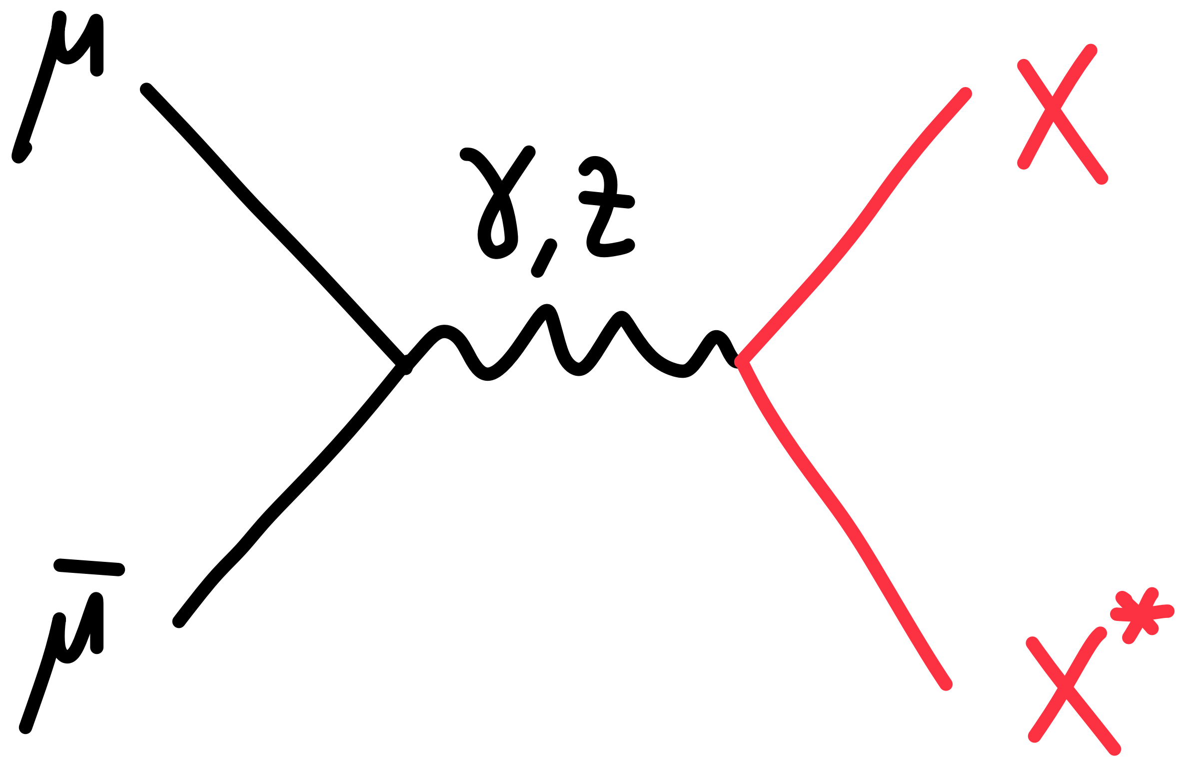
A $\Delta R(i,j) > 2$ or 1.5 > Isolated electron and neutrino

B $|\eta_W| > 1.5$ or 1.0 > Collinear W

These cuts are **successful in selecting the collinear W emission** and give results compatible with PDFs, however are **not inclusive on the emitted radiation**.

Charged-current pair production of NP

Typical **pair-production** of heavy states at a MuC proceeds in **neutral-current**:



The MuC **reach on $\mathbf{M_X}$** is approximately $E_{MuC}/2$.

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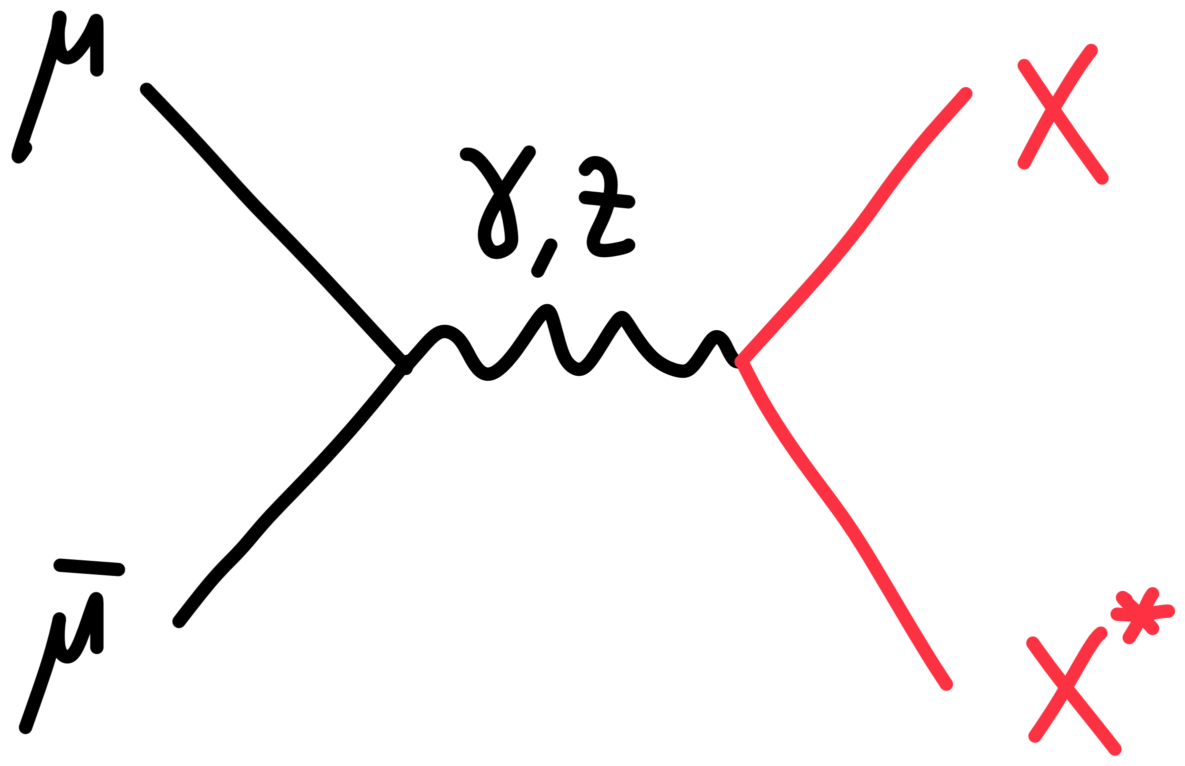

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We also want to explore the **SU(2) structure** of the new state, for instance if it is a **doublet**:

$$X = \begin{pmatrix} X_+ \\ X_- \end{pmatrix}_{Y_X}$$

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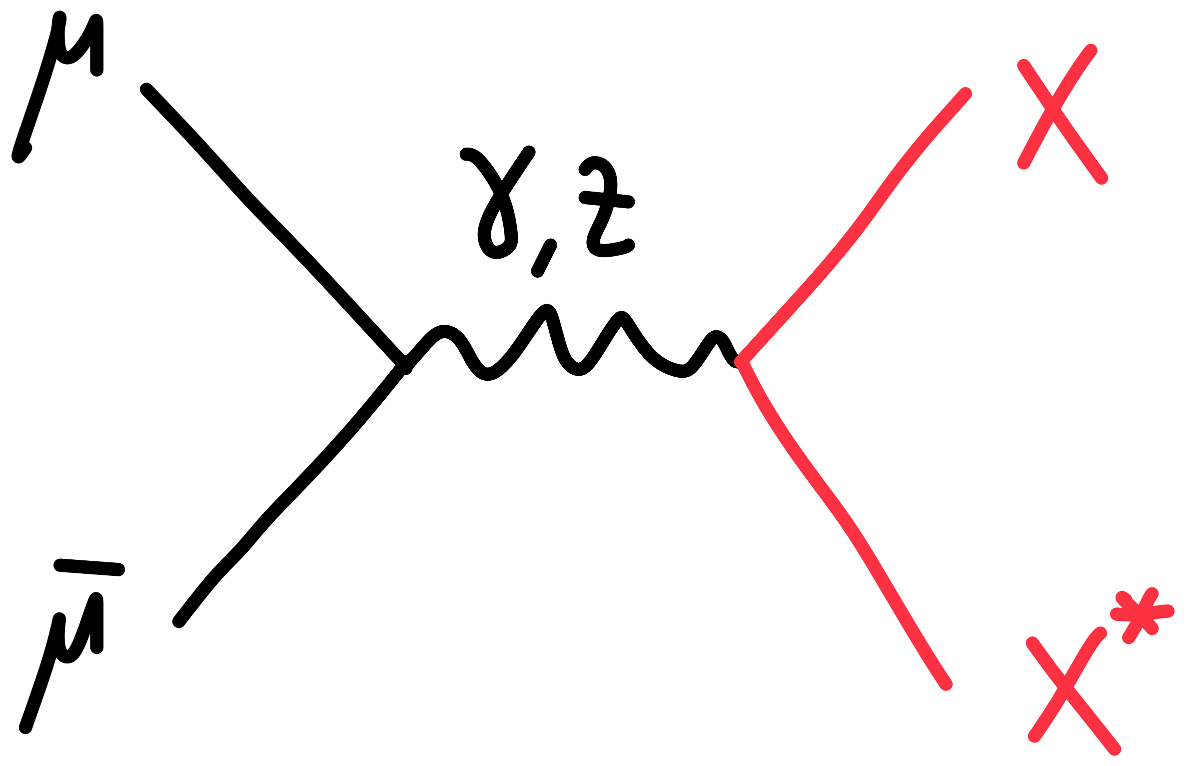

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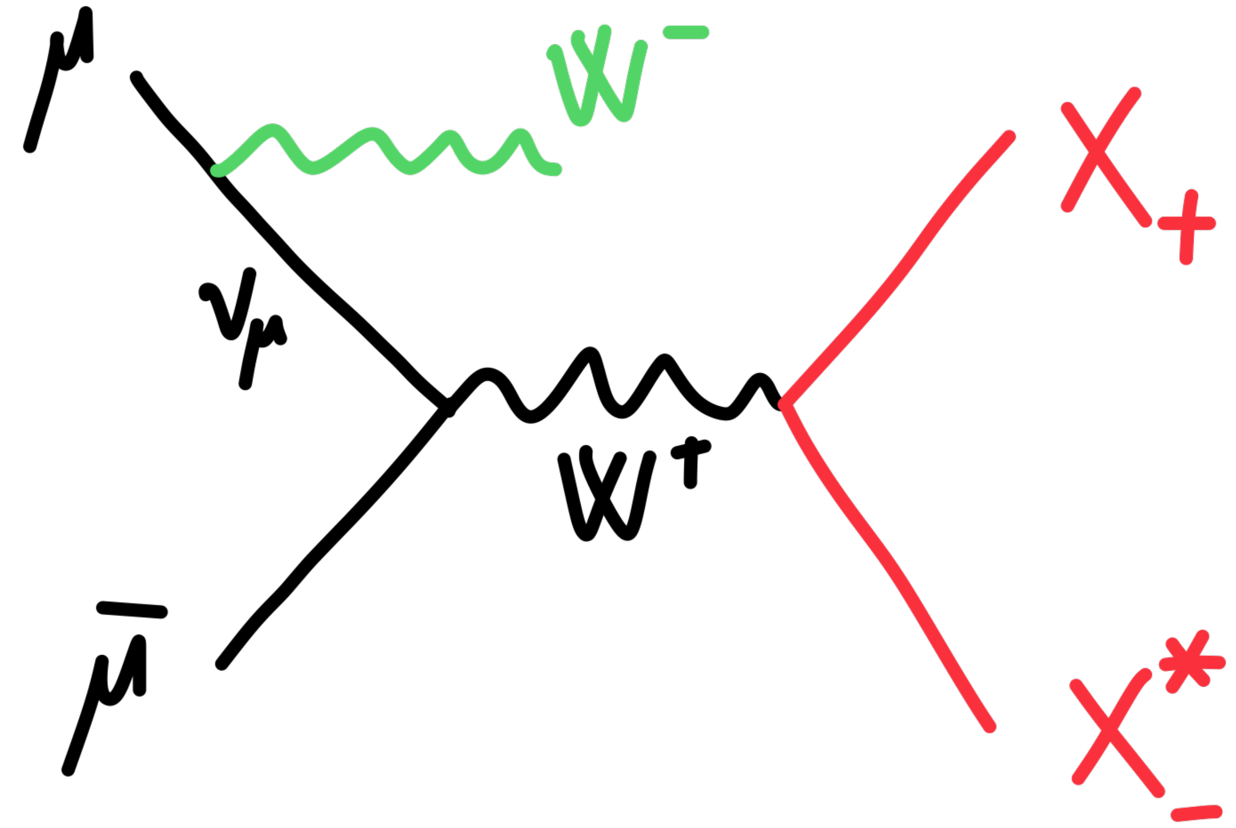
What is the **MuC reach in charged-current?**



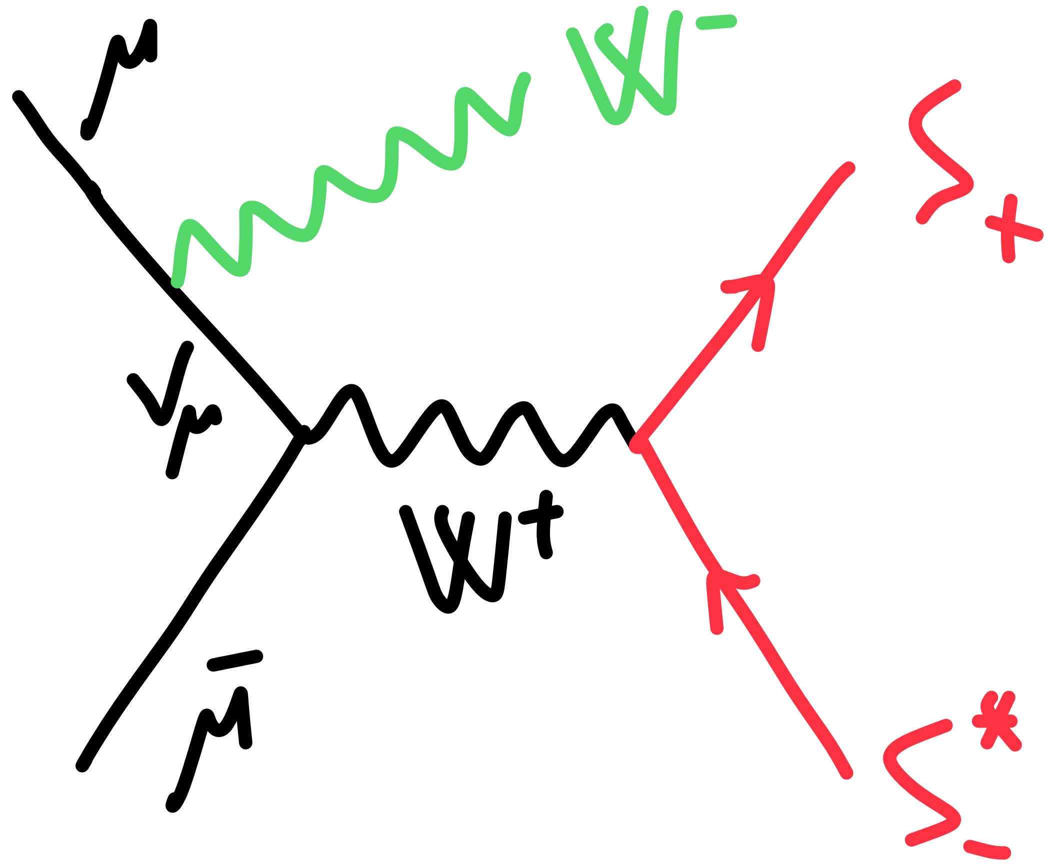
Can we use the **neutrino PDF** to describe it?

Charged-current pair production of NP

As example, we take a **heavy scalar doublet** $S = \begin{pmatrix} S_+ \\ S_- \end{pmatrix}$

(for concreteness we fix **Y=1/6**, so **Q(S_+)=2/3** and **Q(S_-)=-1/3**)

The **CC pair-production** proceeds via **collinear W emission from ISR**:



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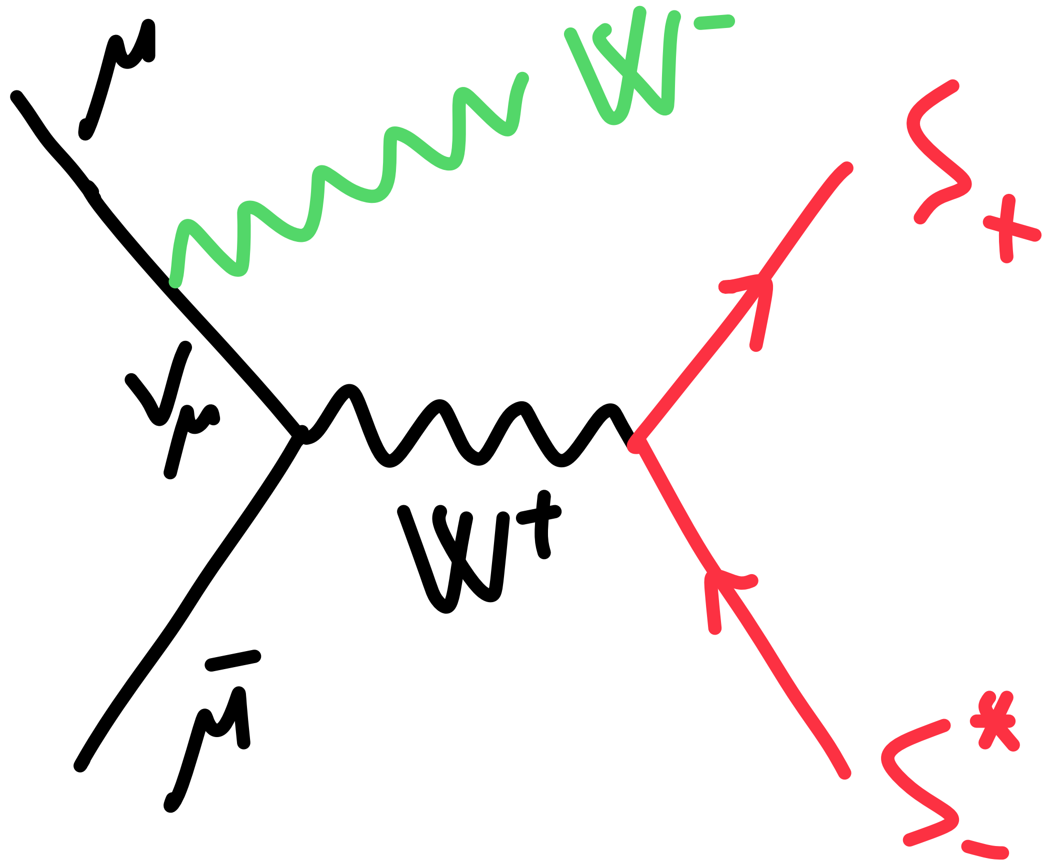
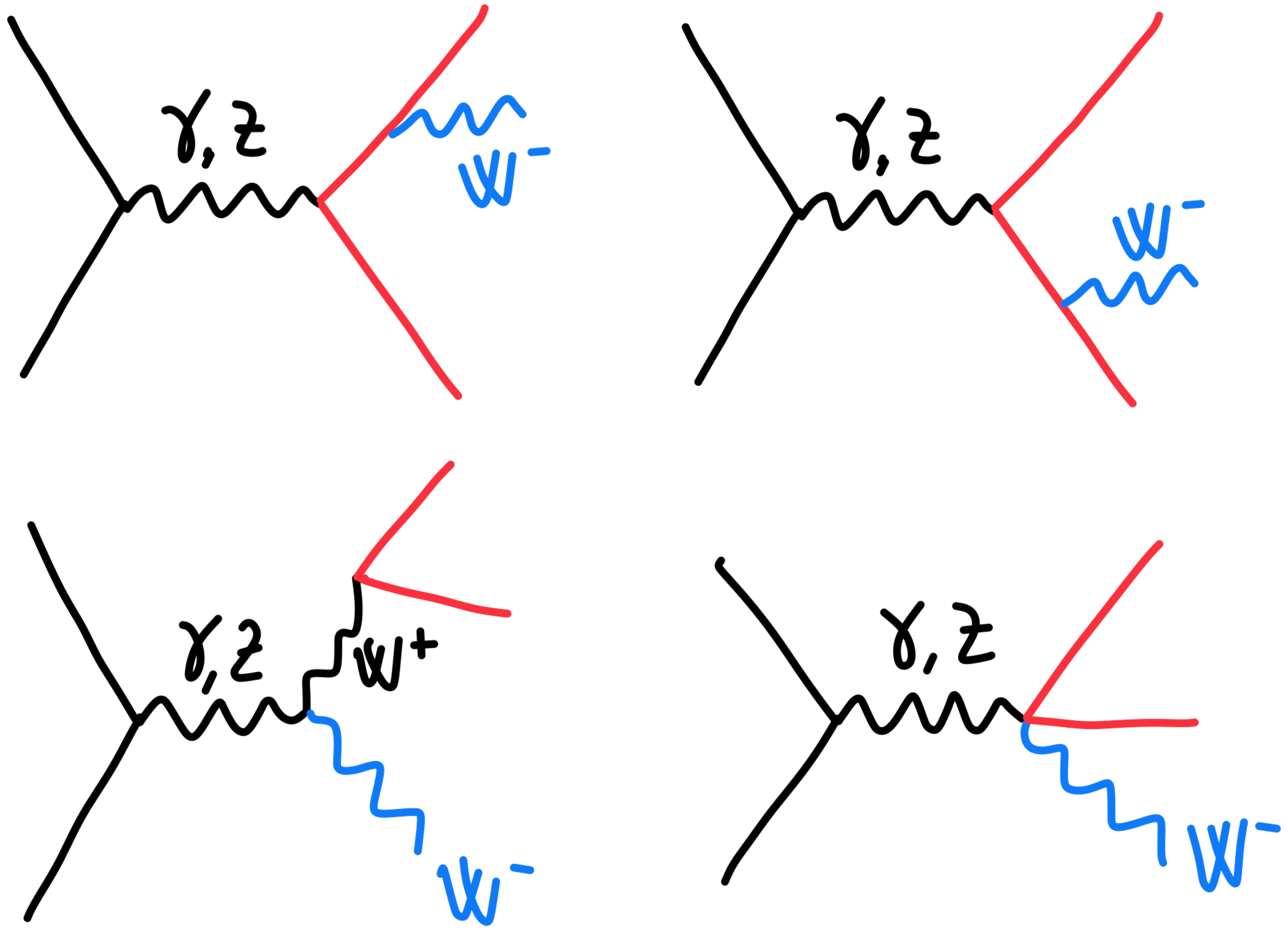


At LO, however, there are also other contributions to the same final state:



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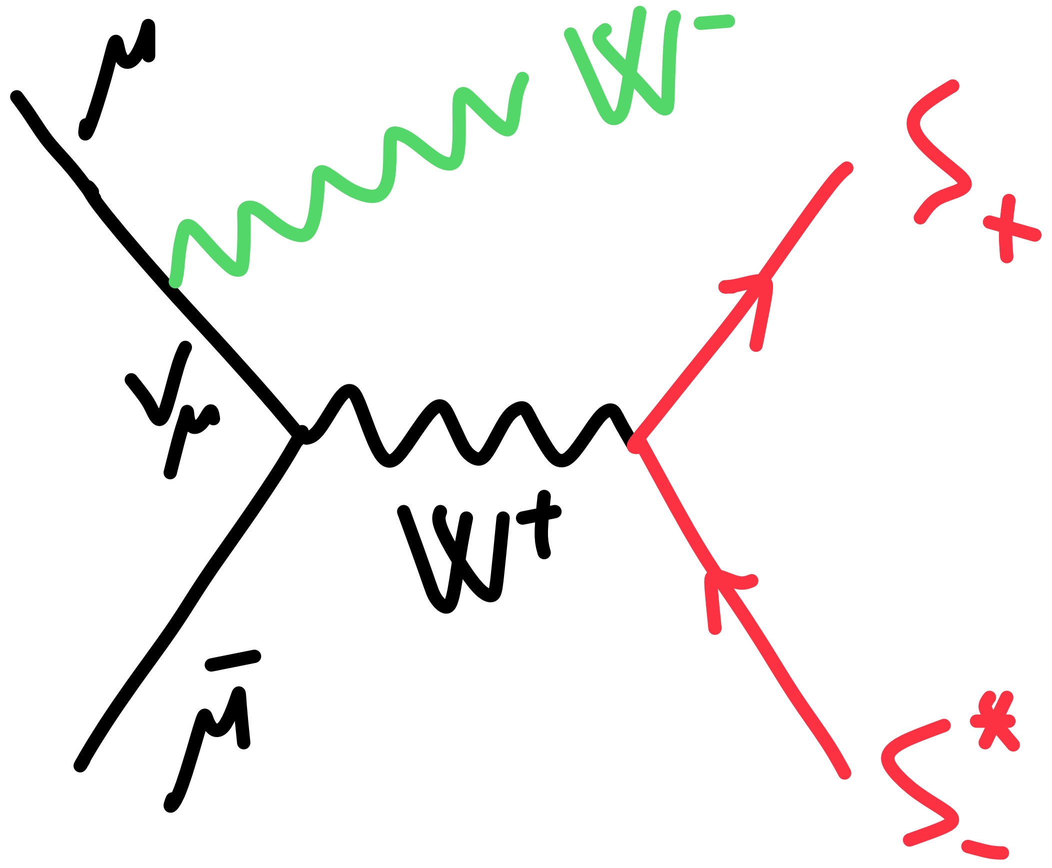

If the ISR one dominates, the **W** should be **forward and with small p_T**.

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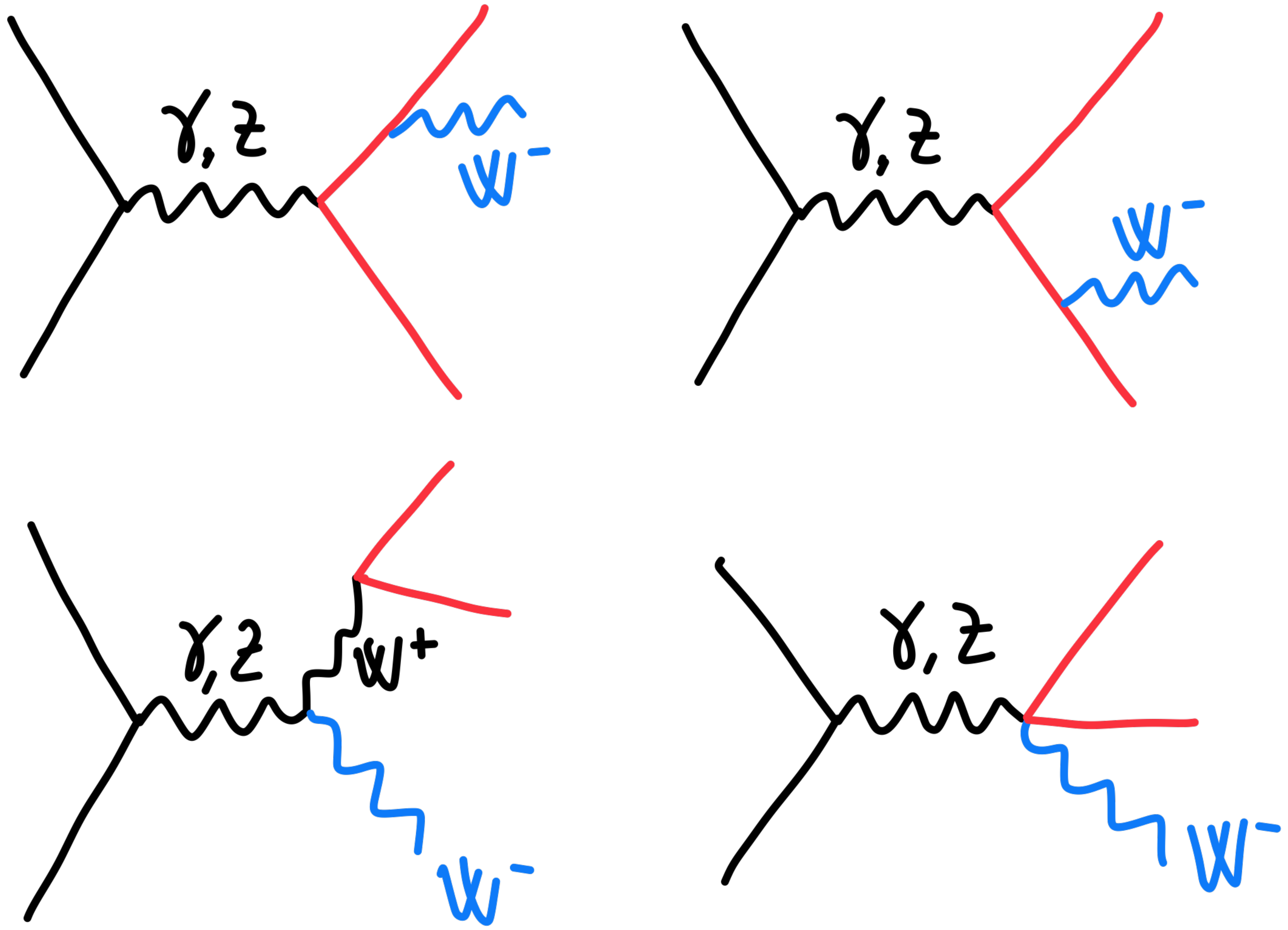
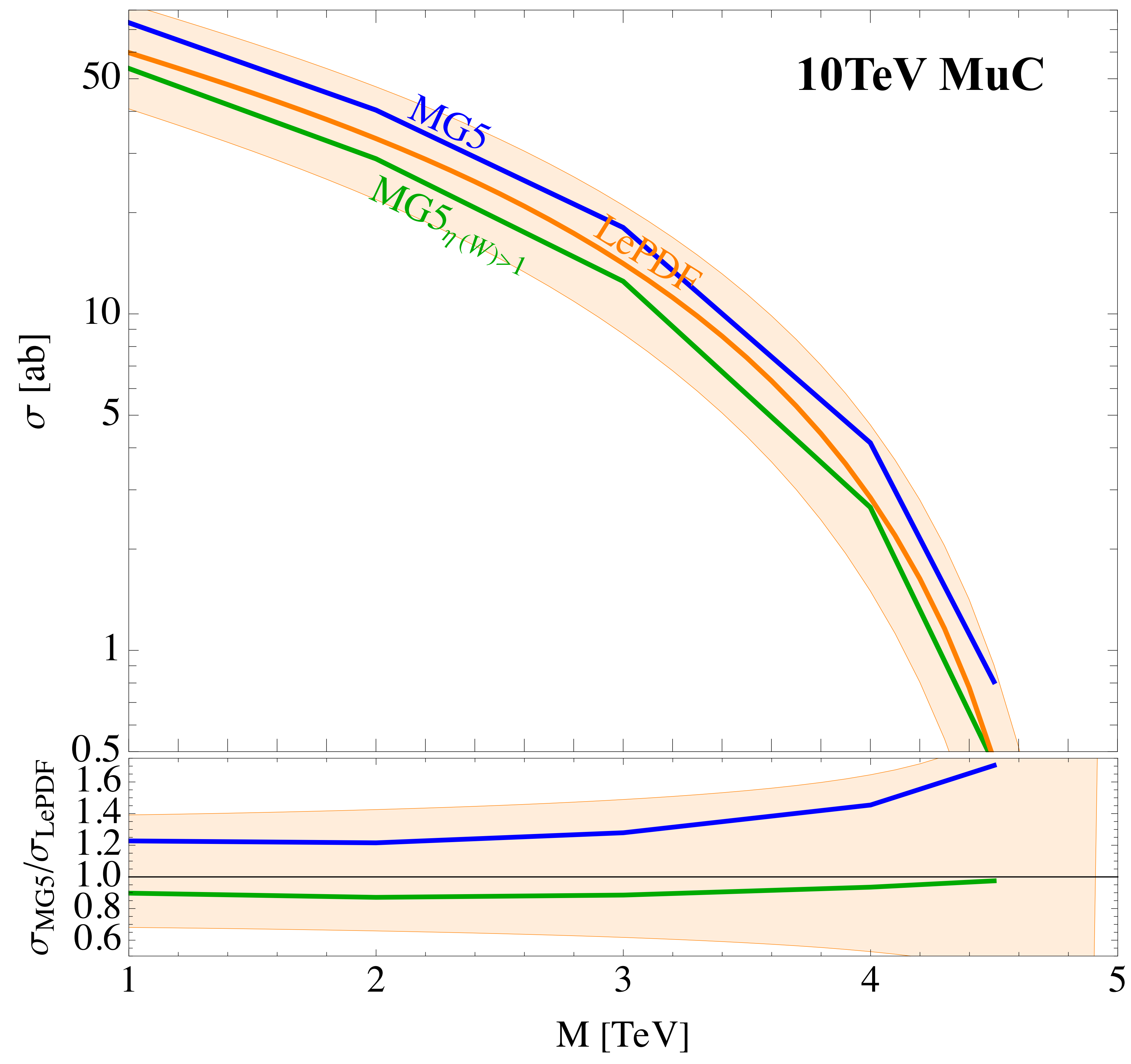
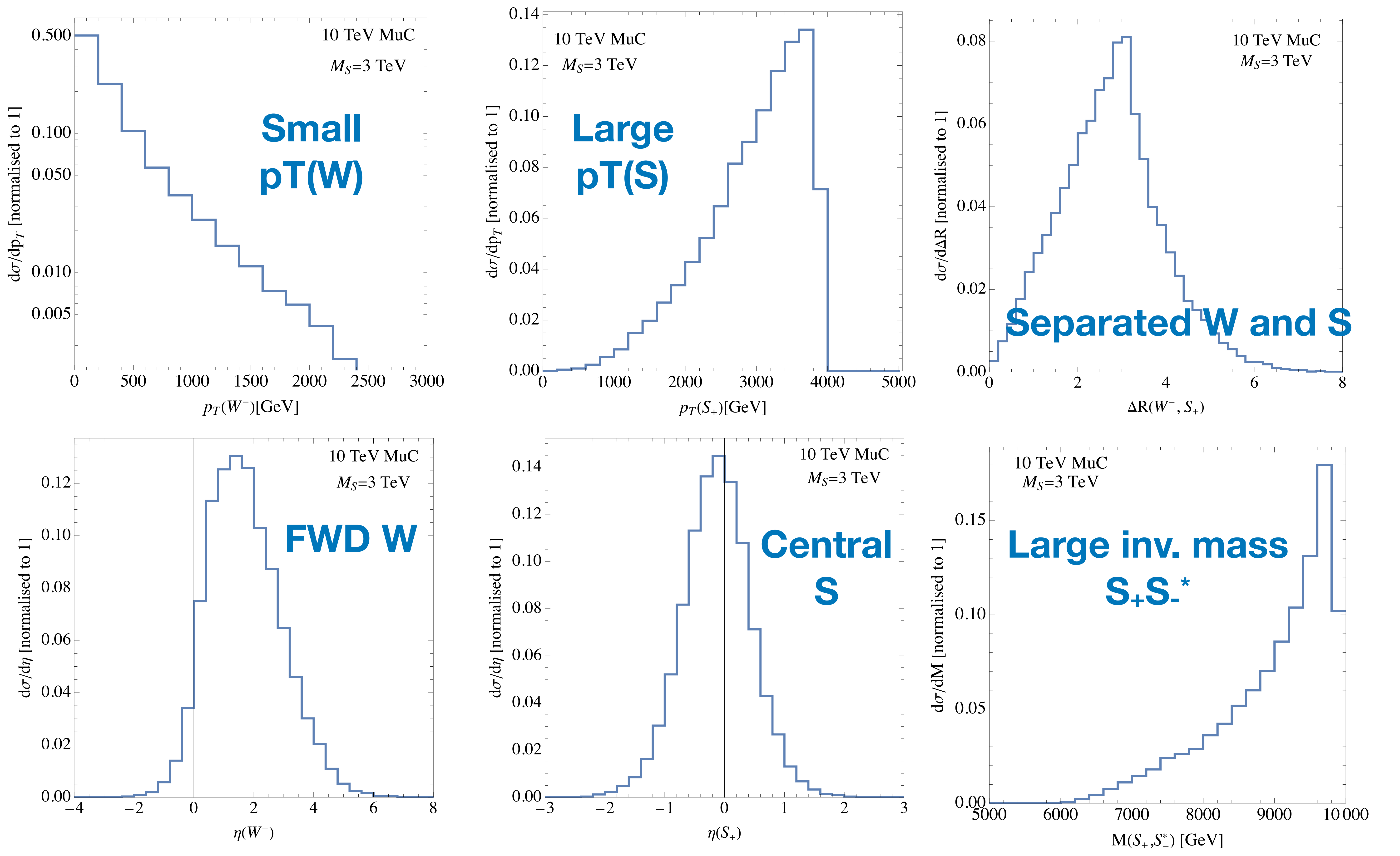


Charged-current pair production of NP

We compute the total cross section with **LePDF** and with **MadGraph5 at LO**.



Distribution of events for **M_S = 3 TeV**.



Compatible with W mostly from ISR.

We also impose a **cut η(W)>1** to select forward Ws.

Outlook

For **future high-energy colliders**, **EW corrections will be large and are going to play a crucial role**. EW symmetry becomes effectively restored and a plethora of new effects are expected to appear.

EW PDFs allow to resum large logarithms appearing in **EW ISR** emission: an ingredient towards a **full understanding of EW radiation** effects, **necessary to reach 1% precision**.

Open questions remain:

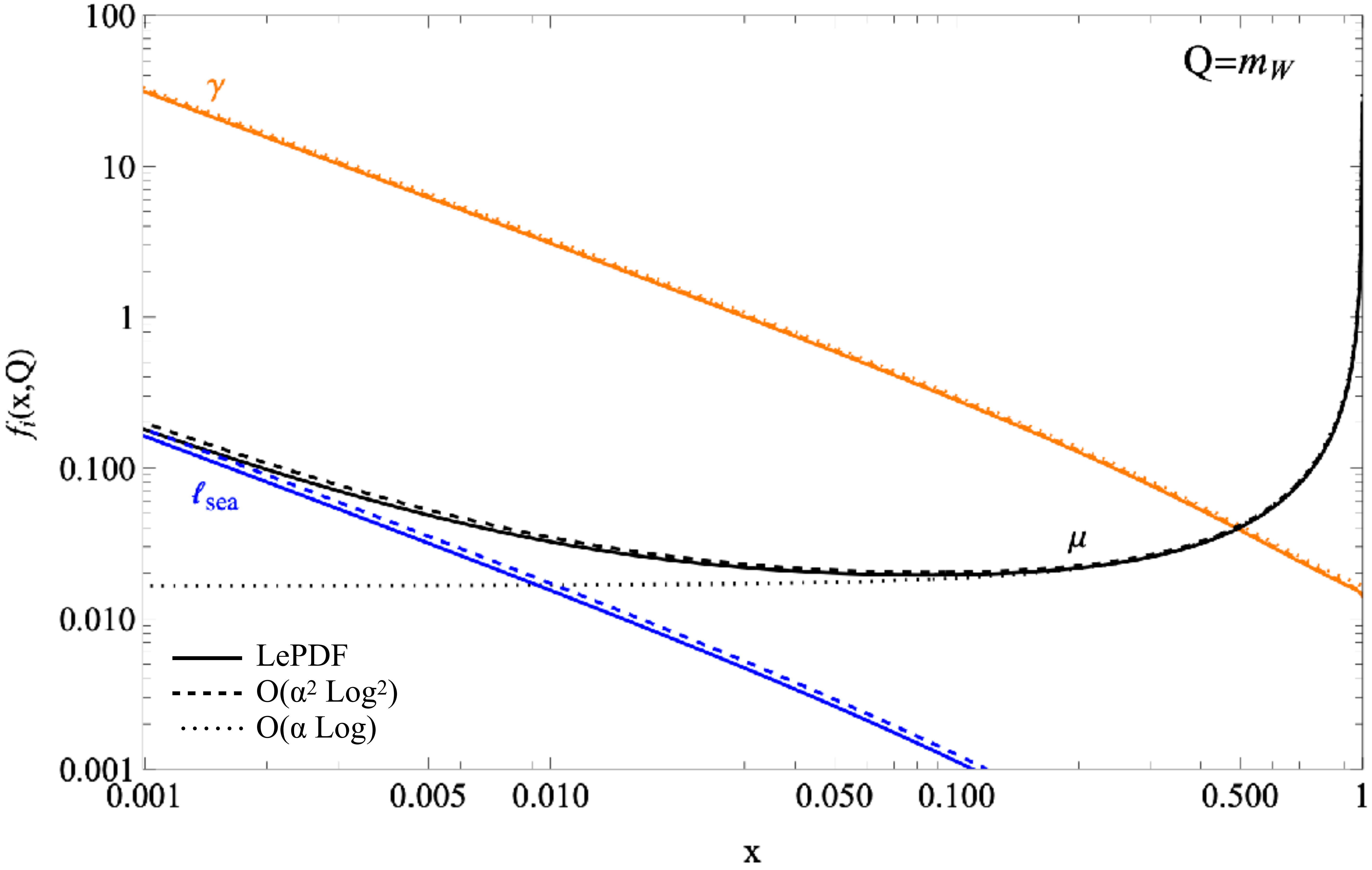
- Fixed-order computations allow to describe the emitted radiation, but do not resum the large logs. Which is more important? Matched results?
- For the 10 TeV MuC, are QED+QCD PDFs sufficient, with EW radiation treated at fixed order?
- Double logs appear also in virtual corrections and FSR, are they all equally important? Can they be resummed separately? Will we need to define observables in terms of "EW jets"?

Muon Colliders would usher us into the "**EW era**", the same theory progress that was required to make the most out of LHC data will be required in order to precisely predict observables at those energies.

Thank you!

Backup

Analytical PDFs - QED



We can derive analytical approximations for the resummed PDFs by solving the DGLAP equations iteratively order-by-order:

Including up to O(α^2 Log^2)

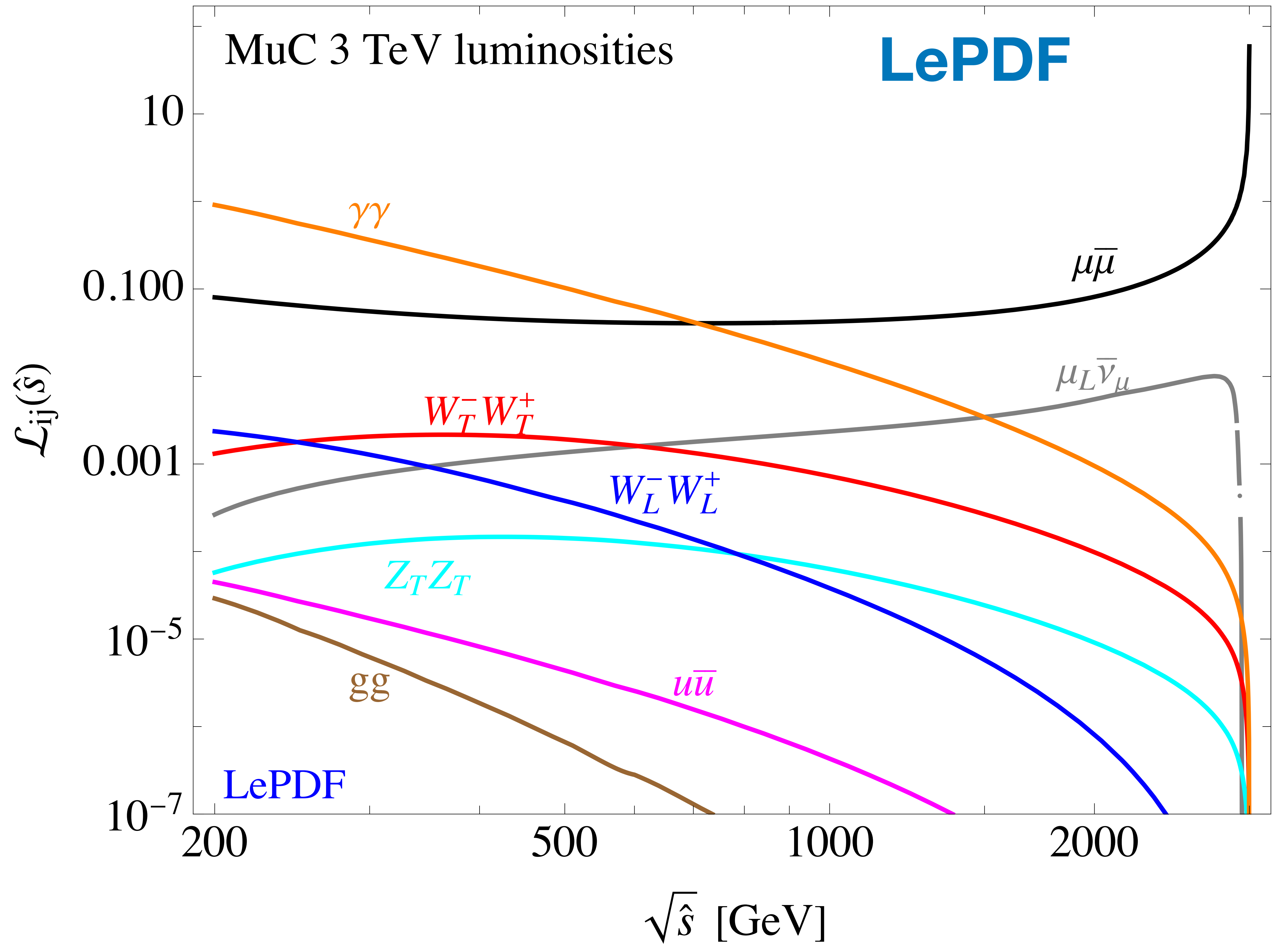
$$f_\mu^{(\alpha^2)}(x,t) = \delta(1-x) + \frac{\alpha_\gamma}{2\pi}\, t \left(\frac{3}{2}\delta(1-x) + P_{ff}^V(x)\right)$$
$$+ \frac{1}{2}\left(\frac{\alpha_\gamma}{2\pi}\, t\right)^2 \left[\frac{9}{4}\delta(1-x) + 3P_{ff}^V(x) + I_{fVVf}(x) + I_{ffff}(x)\right] ,$$
$$f_{\ell_{sea}}^{(\alpha^2)}(x,t) = \frac{1}{2}\left(\frac{\alpha_\gamma}{2\pi}\, t\right)^2 I_{fVVf}(x) ,$$
$$f_\gamma^{(\alpha^2)}(x,t) = \frac{\alpha_\gamma}{2\pi}\, t\, P_{Vf}^f(x) + \frac{1}{2}\left(\frac{\alpha_\gamma}{2\pi}\, t\right)^2 \left[\left(\frac{3}{2} - \frac{2}{3}N_f^{\mathrm{QED}}\right) P_{Vf}^f(x) + I_{Vfff}(x)\right]$$

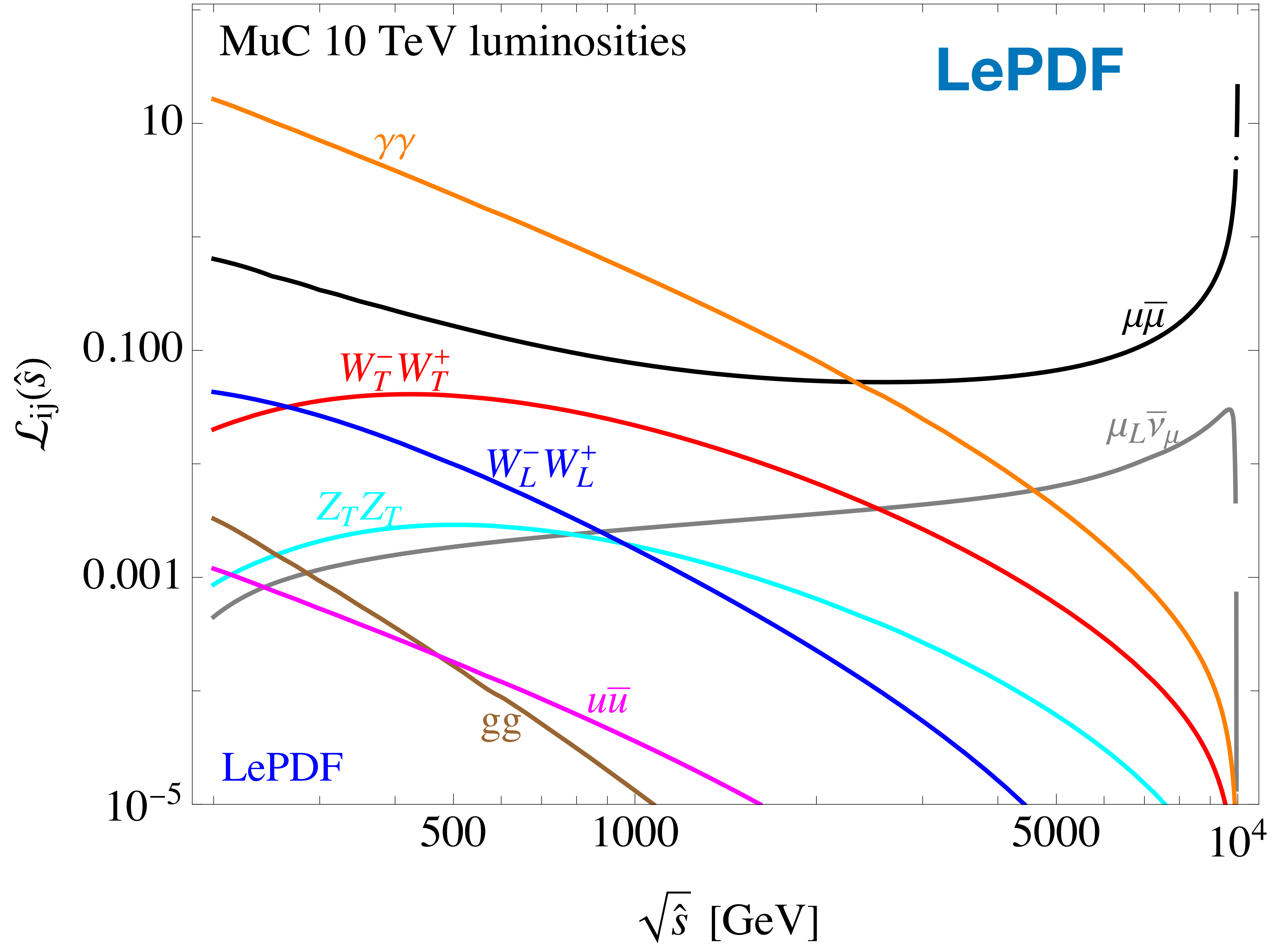
$$t \equiv \log\left(Q^2/m_{\ell_v}^2\right) \qquad I_{ABBC}(x) \equiv \int_x^1 \tfrac{dz}{z} P_{AB}^X(z) P_{BC}^Y(x/z)$$

MuC Luminosities

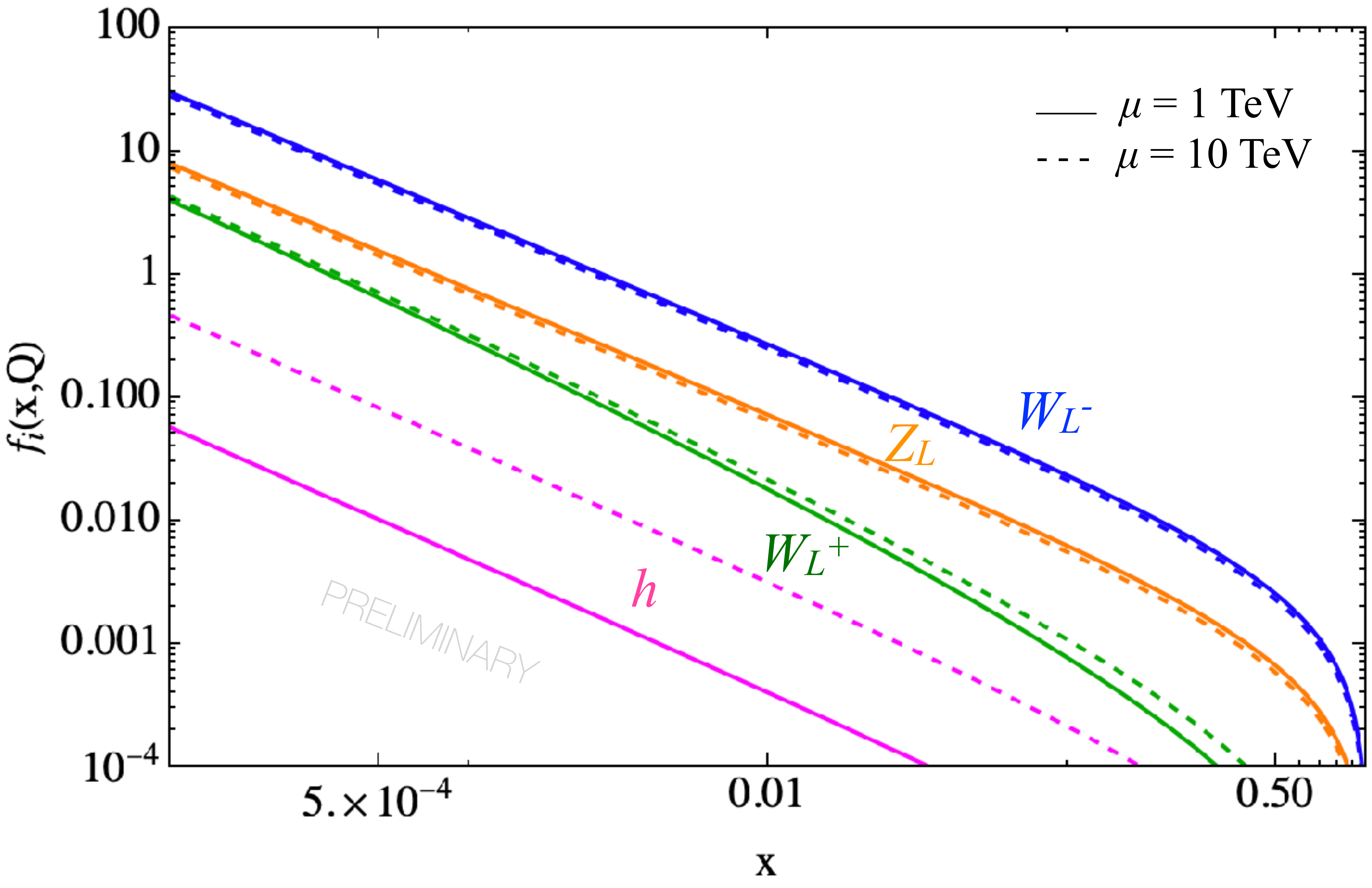
Some examples of **parton luminosities** for muon colliders.

$$\mathcal{L}_{ij}(\hat{s}) = \int_{\hat{s}/s_0}^{1} dx \frac{1}{x} f_i^{(\mu)}\left(x, \frac{\sqrt{\hat{s}}}{2}\right) f_j^{(\bar{\mu})}\left(\frac{\hat{s}}{x s_0}, \frac{\sqrt{\hat{s}}}{2}\right)$$





Scalars



PDFs of longitudinal gauge bosons are dominated by ultra-collinear contributions from the muon (and muon neutrino, for the W^+), which do not scale.

The Higgs instead has no coupling to massless fermions, so its PDF has no large ultra-collinear contributions.

Top quark PDF

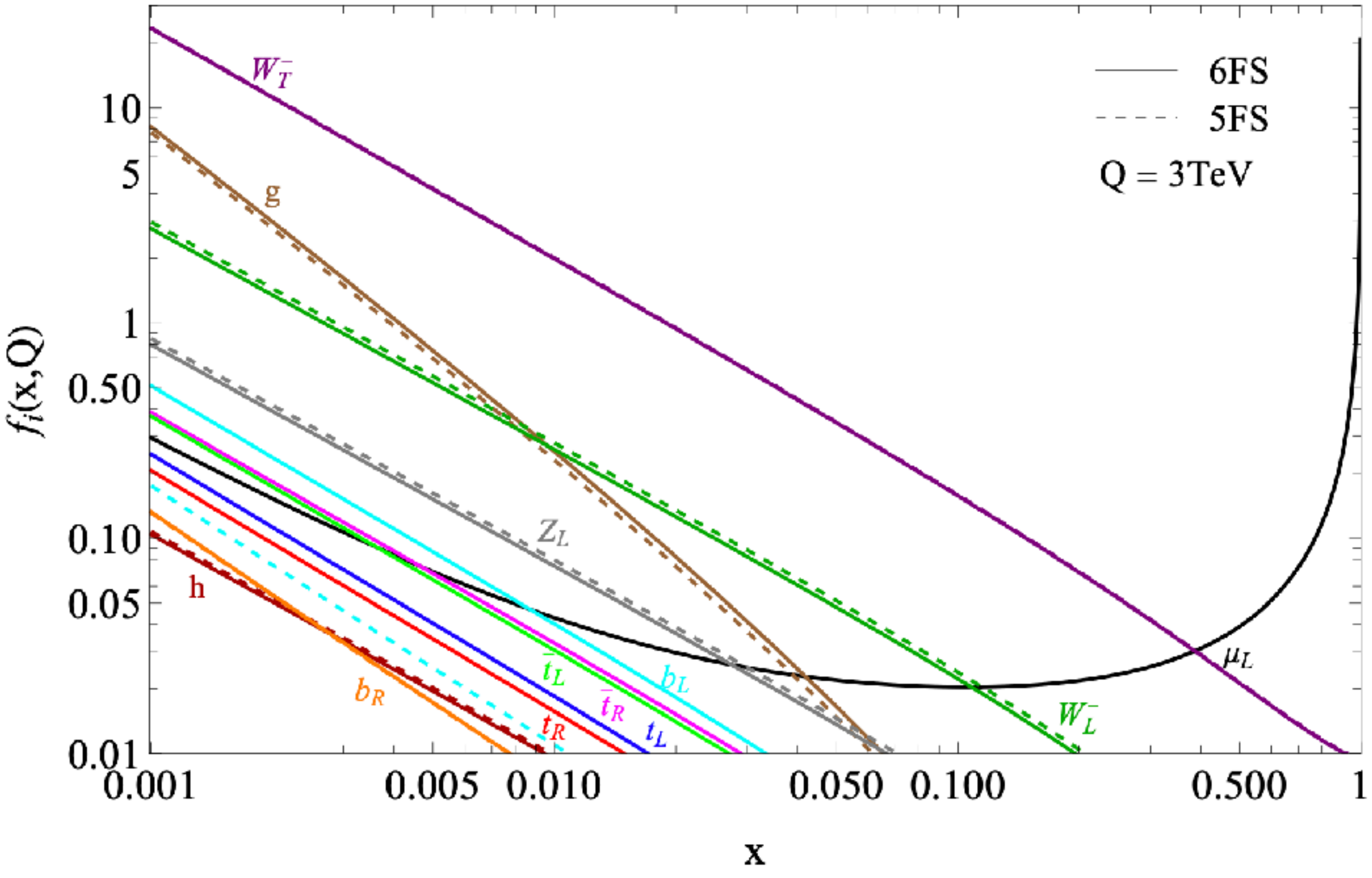
For hard scattering energies $E \gg m_t$, terms with $\log E/m_t$ due to collinear emission of top quarks can arise.

These can be resummed by including the **top quark PDF** within the DGLAP evolution, in a **6FS**.

Barnett, Haber, Soper '88; Olness, Tung '88

Whether or not this is useful depends on the process under consideration.

Dawson, Ismail, Low [1405.6211]
Han, Sayre, Westhoff [1411.2588]



We provide two version of the codes: **5FS** and **6FS**.
In the 6FS we keep **finite top quark mass** effects, like we do for other heavy SM states.

EW Sudakov double logs from ISR

In case of collinear W emission they can be implemented (and resummed) at he **Double Log** level equations by putting an **explicit IR cutoff z_{max}** $= 1 - Q_{EW}/Q$ $\quad (Q_{EW} = m_W)$

M. Ciafaloni, P. Ciafaloni, Comelli [hep-ph/0111109]
Bauer, Ferland, Webber [1703.08562]
see Manohar, Waalewijn [1802.08687] for a different approach

$$\frac{\alpha_{ABC}(Q)}{2\pi}\int_x^1 \frac{dz}{z} P_{BA}^C(z) f_A\left(\frac{x}{z}, Q^2\right) \quad \to \quad \frac{\alpha_{ABC}(Q)}{2\pi}\int_x^{z_{\max}^{ABC}(Q)} \frac{dz}{z} P_{BA}^C(z) f_A\left(\frac{x}{z}, Q^2\right)$$

This modifies also the **virtual corrections** as:

$$P_A^v(Q) \supset -\sum_{B,C} \frac{\alpha_{ABC}(Q)}{2\pi}\int_0^{z_{\max}^{ABC}(Q)} dz\, z\, P_{BA}^C(z)$$

The non-cancellation of the z_{max} dependence between emission and virtual corrections generates the double logs.

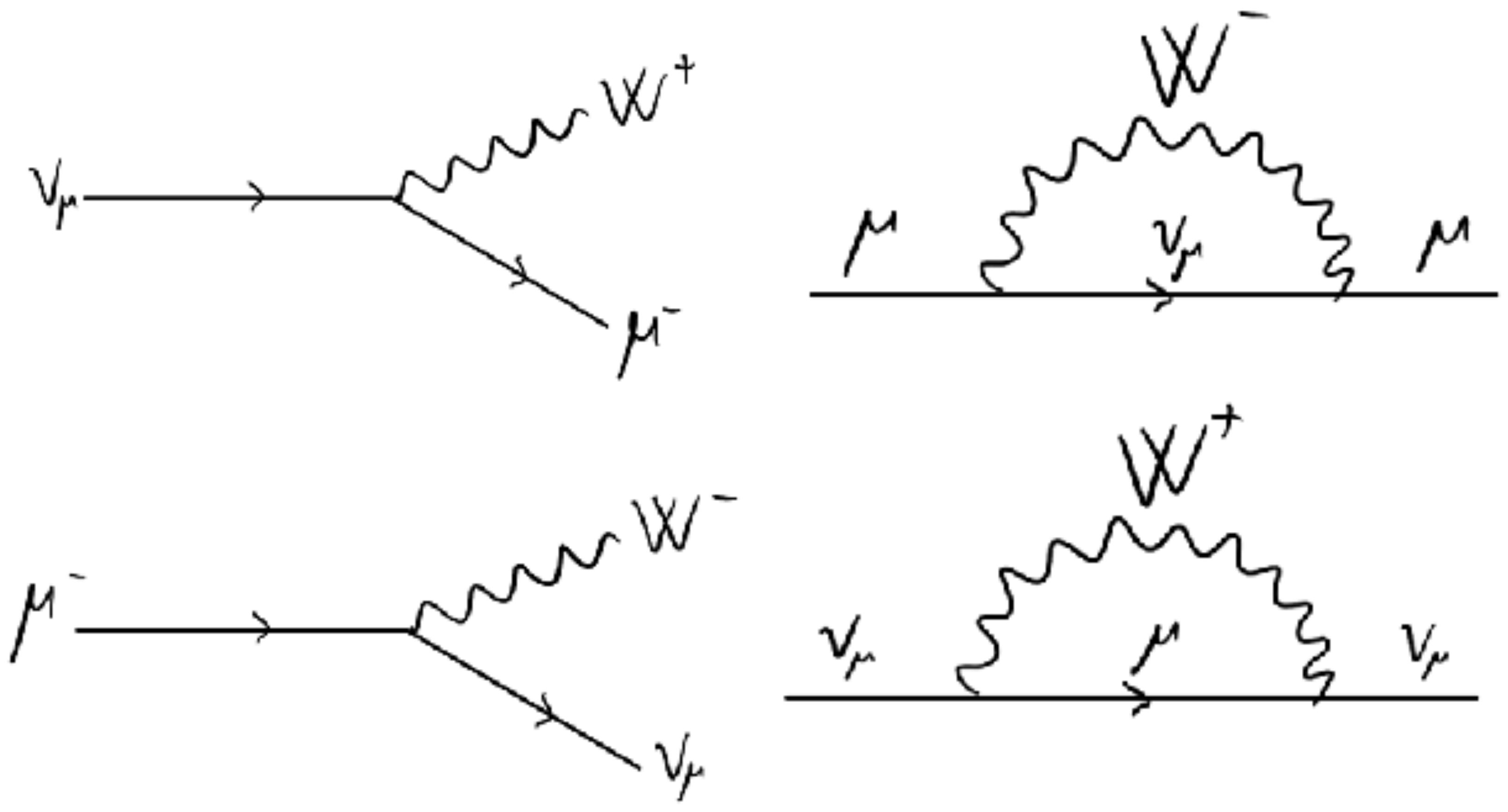
This happens if $\quad P_{BA}^C,\, U_{BA}^C \propto \frac{1}{1-z}$ and $A \neq B$ $\quad$ otherwise we set $z_{max}=1$ and use the +-distribution.

EW Sudakov double logs from ISR

For illustration, let us consider the **muon** and **neutrino** DGLAP equations and only **interactions with transverse W±**

$$\frac{df_{\mu_L}}{d\log\mu^2} = \frac{\alpha_2}{2\pi}\frac{1}{2}\int_0^{z_{\max}(\mu)} dz P_{ff,G}(z)\left(\frac{1}{z}f_{\nu_\mu}\left(\frac{x}{z},\mu\right) - z f_{\mu_L}(x,\mu)\right) + \text{ IR-finite terms}$$

$$\frac{df_{\nu_\mu}}{d\log\mu^2} = \frac{\alpha_2}{2\pi}\frac{1}{2}\int_0^{z_{\max}(\mu)} dz P_{ff,G}(z)\left(\frac{1}{z}f_{\mu_L}\left(\frac{x}{z},\mu\right) - z f_{\nu_\mu}(x,\mu)\right) + \text{ IR-finite terms}$$



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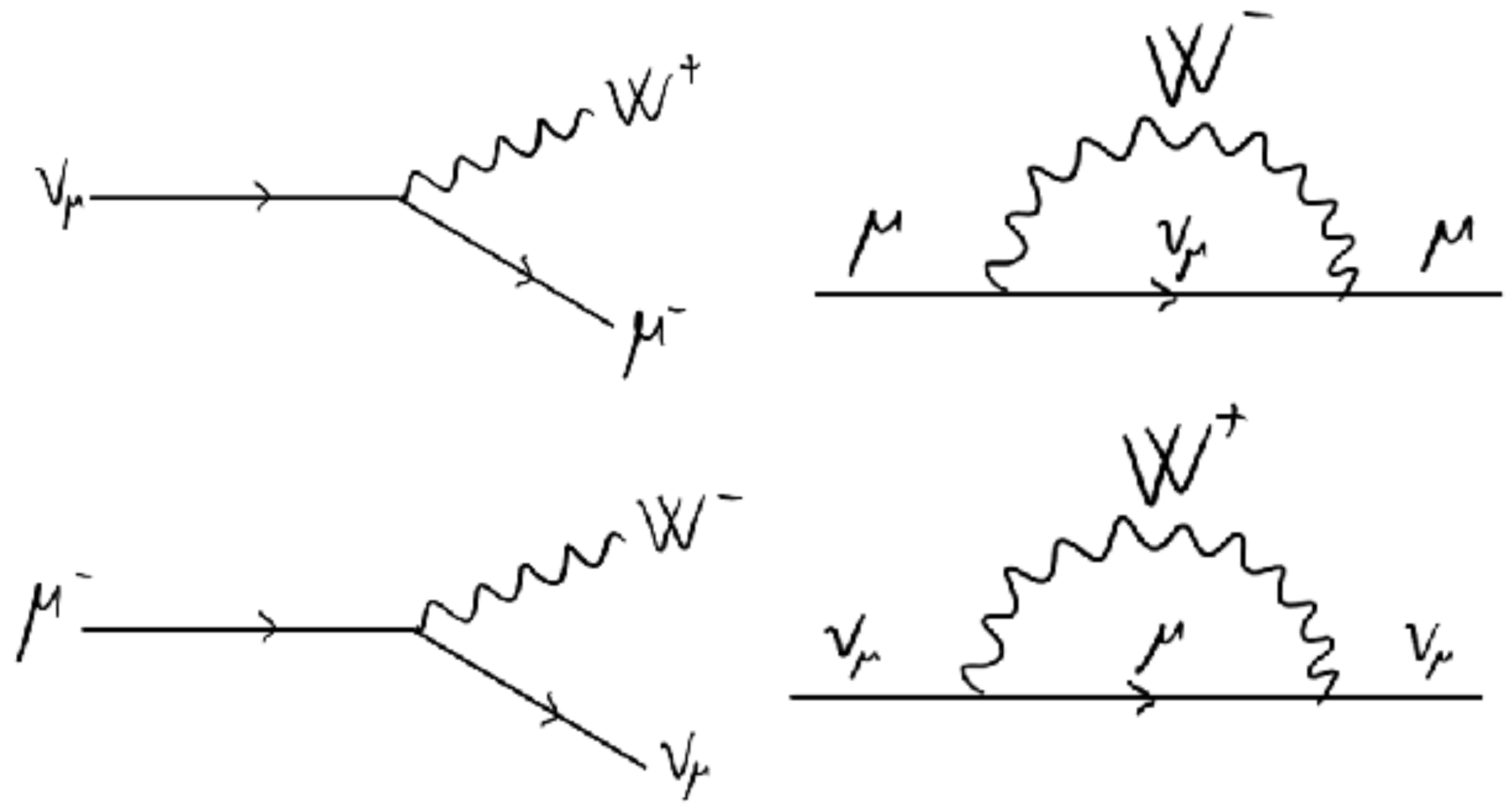


We are interested in the IR divergent terms, take z ➞ 1 for all regular terms inside the integrand:

$$\frac{df_{\mu_L}}{d\log\mu^2} \approx -\frac{\alpha_2}{4\pi}\Delta f_{L_2}(x)\int_0^{z_{\max}(\mu)} dz\frac{2}{1-z} + \ldots \approx -\frac{\alpha_2(\mu)}{4\pi}\log\frac{\mu^2}{\mu_{\rm EW}^2}\Delta f_{L_2}(x) + \ldots\ ,$$

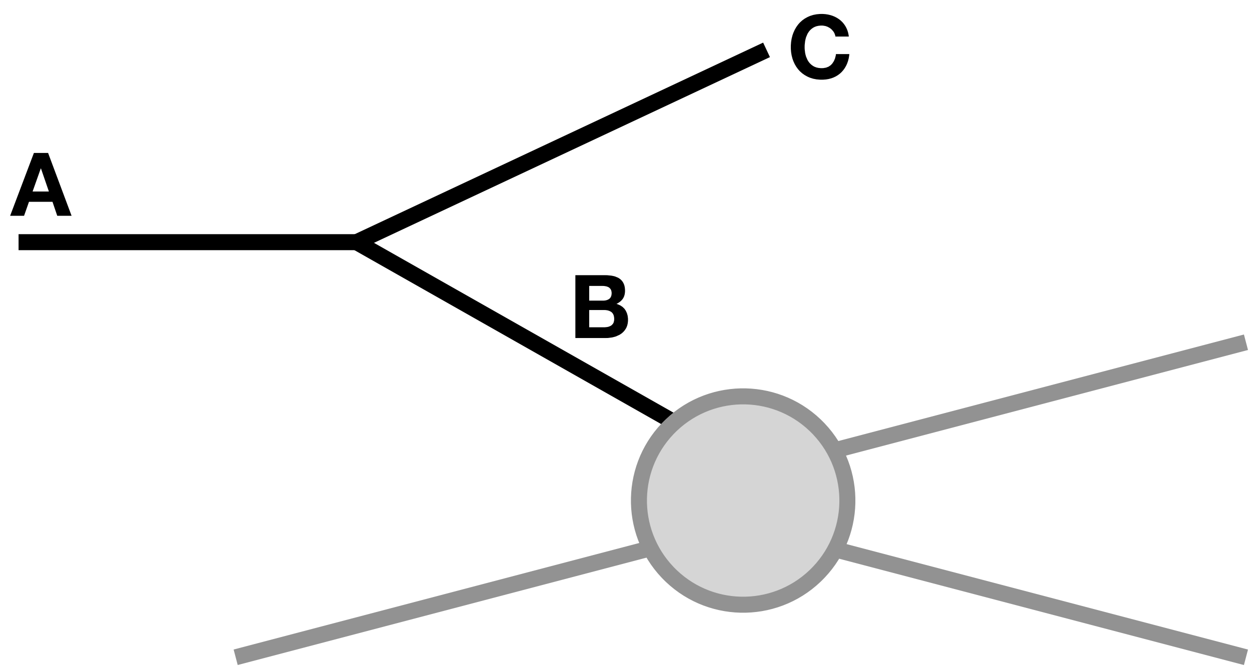
$$\frac{df_{\nu_\mu}}{d\log\mu^2} \approx \frac{\alpha_2}{4\pi}\Delta f_{L_2}(x)\int_0^{z_{\max}(\mu)} dz\frac{2}{1-z} + \ldots \approx \frac{\alpha_2(\mu)}{4\pi}\log\frac{\mu^2}{\mu_{\rm EW}^2}\Delta f_{L_2}(x) + \ldots\ ,$$

$$\Delta f_{L_2}(x) \equiv f_{\mu_L}(x) - f_{\nu_\mu}(x)$$

Upon integration in log μ^2 one gets the double log: it is negative for the muon and positive for the neutrino, tends to restore $SU(2)_L$ invariance at high scales and vanishes when the two become equal.

It is **not present for Z and γ** interactions with fermions, since in the RHS the same fermion PDF enters.

Mass effects



1) Kinematical effects of emitted real radiation

The particle C is emitted on-shell: its energy is bounded to be $E_C = (z\text{-}x)\ E > m_C$

$$z \geq x + \frac{m_C}{E}$$

In the limit where collinear factorisation is valid, $E \gg p_T, m$, **we can neglect this effect**.

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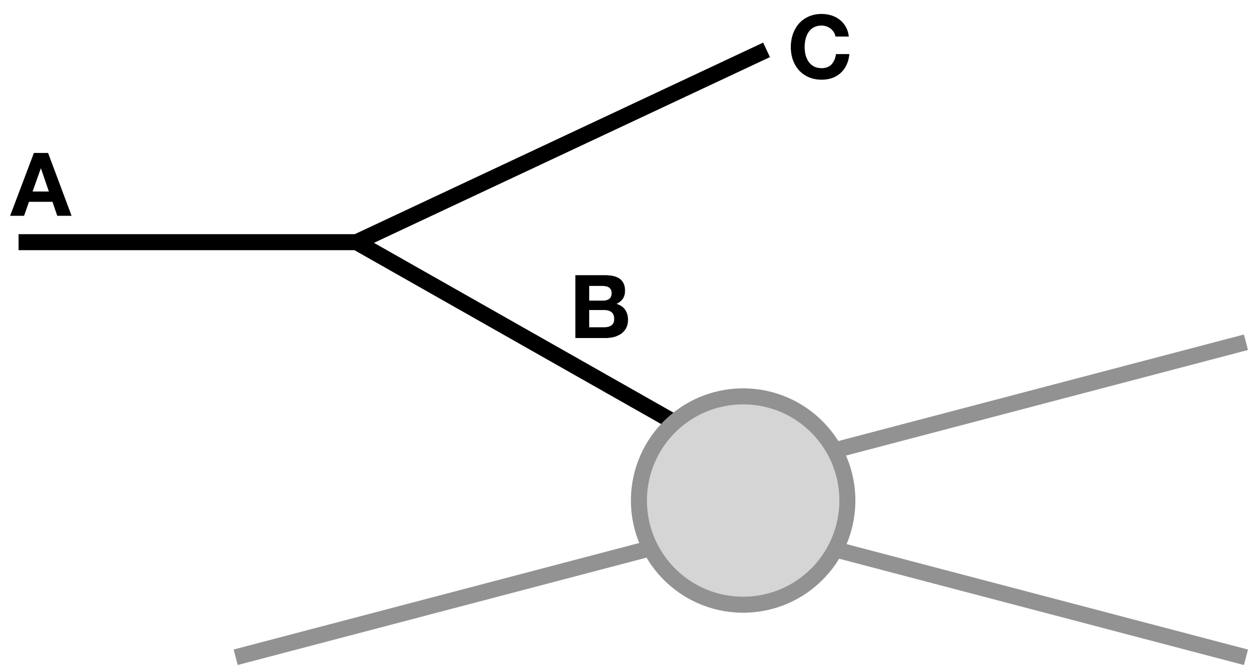

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2) Propagator effects

The mass modifies the propagator of the off-shell parton which then enters the hard scattering:

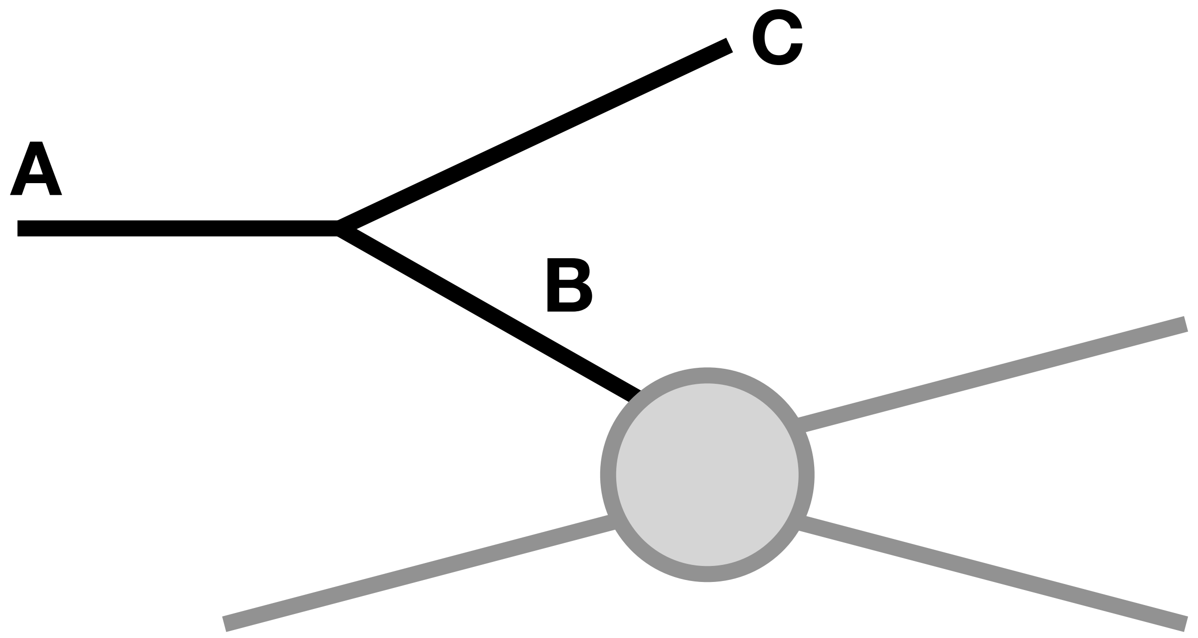
$$\tilde{p}_T^2 \equiv \bar{z}(m_B^2 - q^2) = p_T^2 + z m_C^2 + \bar{z} m_B^2 - z\bar{z} m_A^2 + \mathcal{O}\left(\frac{m^2}{E^2}, \frac{p_T^2}{E^2}\right)$$

This can be implemented by a rescaling of the massless splitting functions:

$$P_{BA}^C(z) \quad \rightarrow \quad \tilde{P}_{BA}^C(z, p_T^2) = \left(\frac{p_T^2}{\tilde{p}_T^2}\right)^2 P_{BA}^C(z)$$

Chen, Han, Tweedie [1611.00788]

Mass effect



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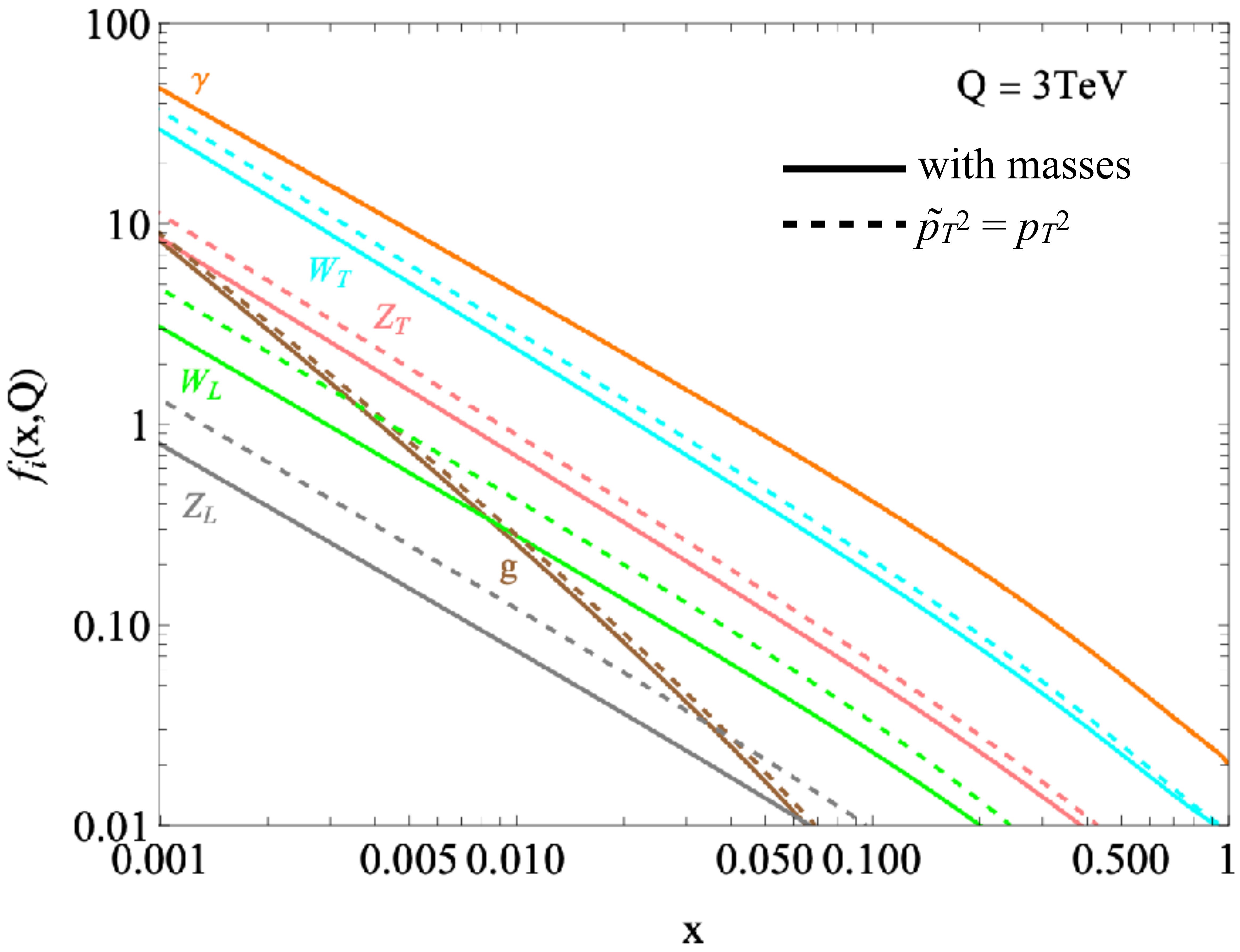


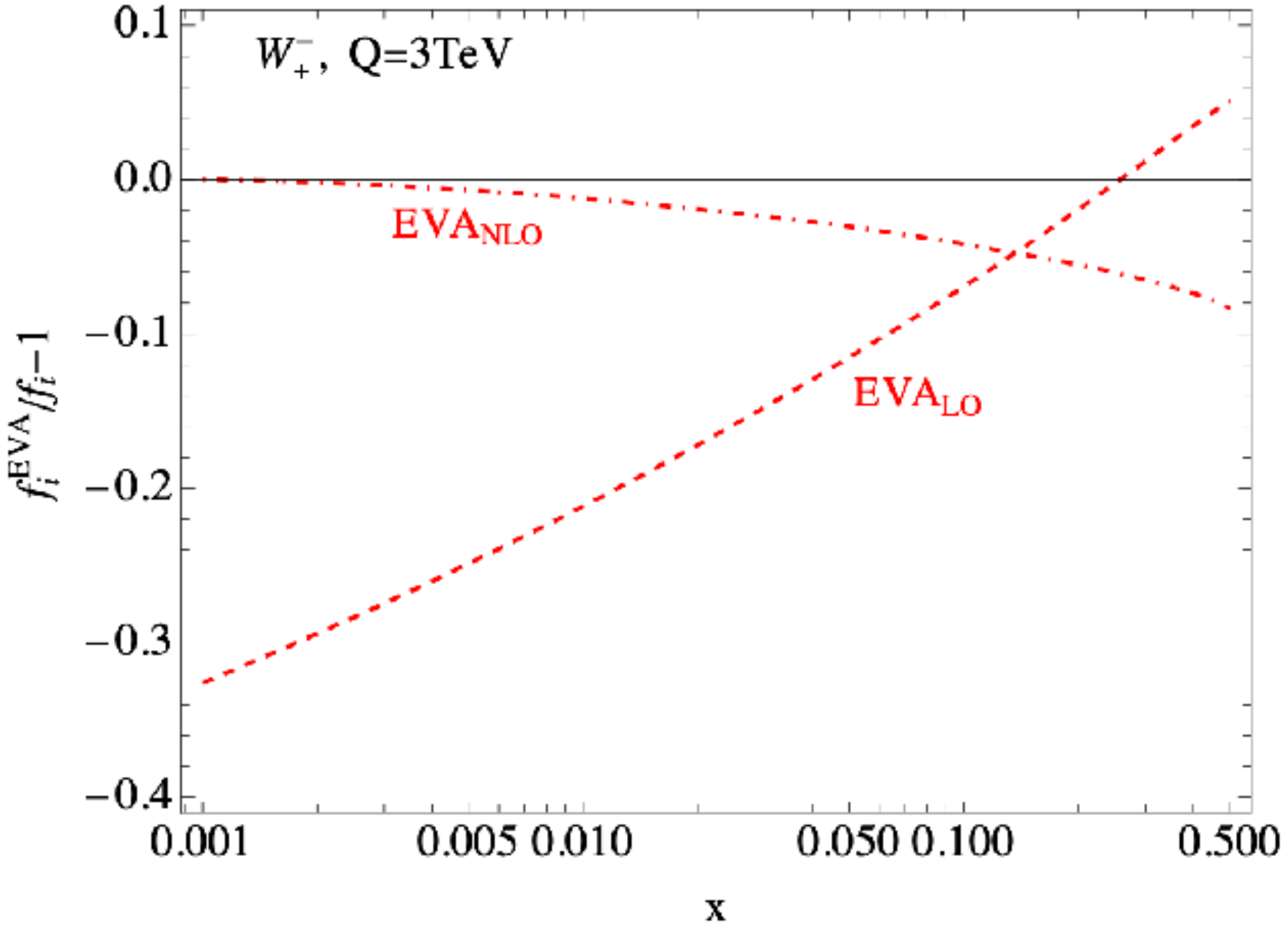
The **effect of finite EW masses is sizeable** even at TeV scales.

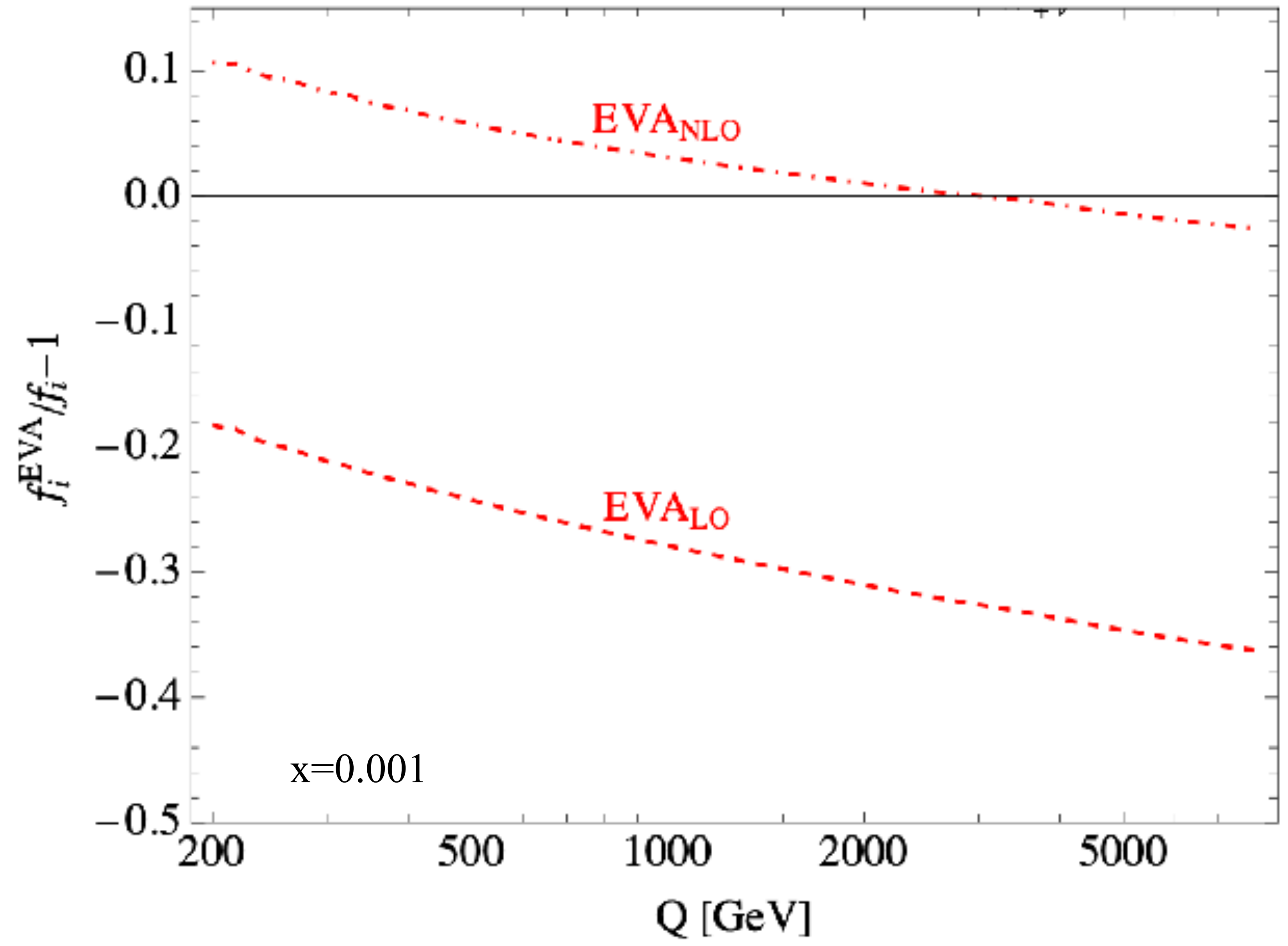
The kinematical effect of the mass of particle C is instead negligible in the collinear limit

$$E_C = (z\text{-}x)\, E > m_C \qquad z \geq x + \frac{m_C}{E}$$

For $E \gg p_T, m$, **we can neglect this effect**.

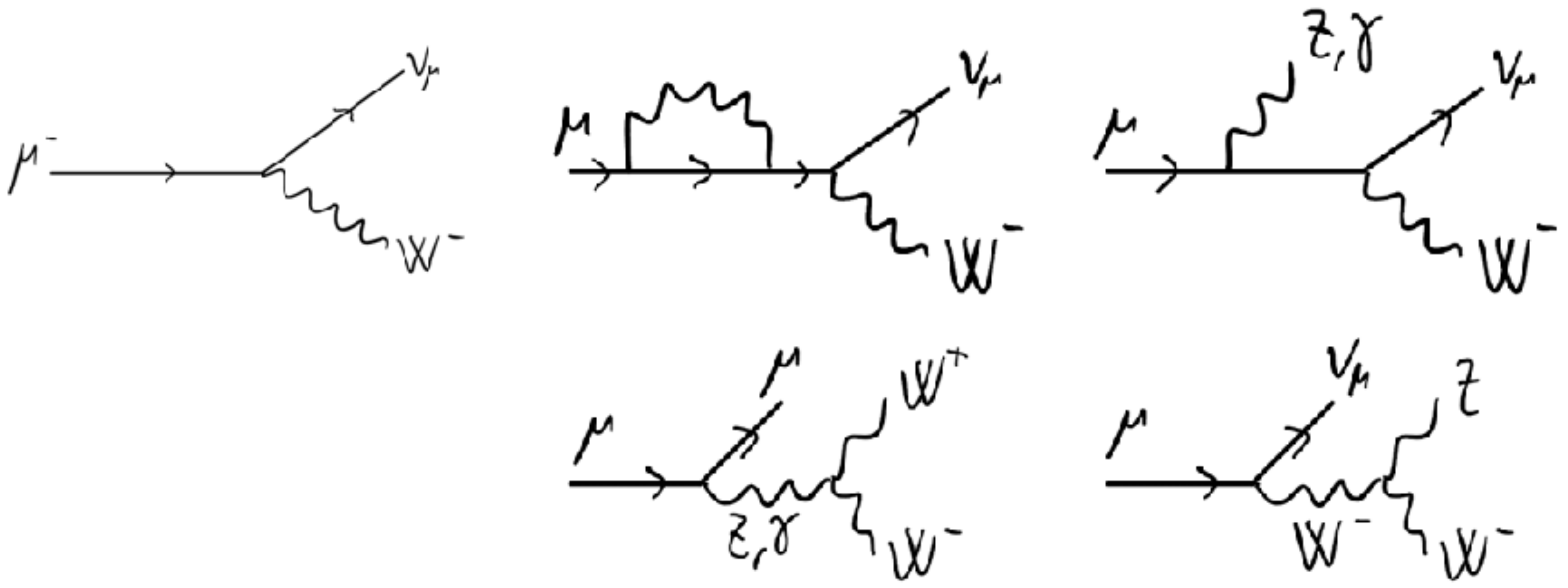
LePDF vs. EVA





The **deviation becomes larger at small x and at large scales** (Sudakov double logs are absent in EVA).

We improve EVA by computing iteratively the $\boldsymbol{W^-_+}$ **PDF at** $\mathbf{O(\alpha^2)}$. *



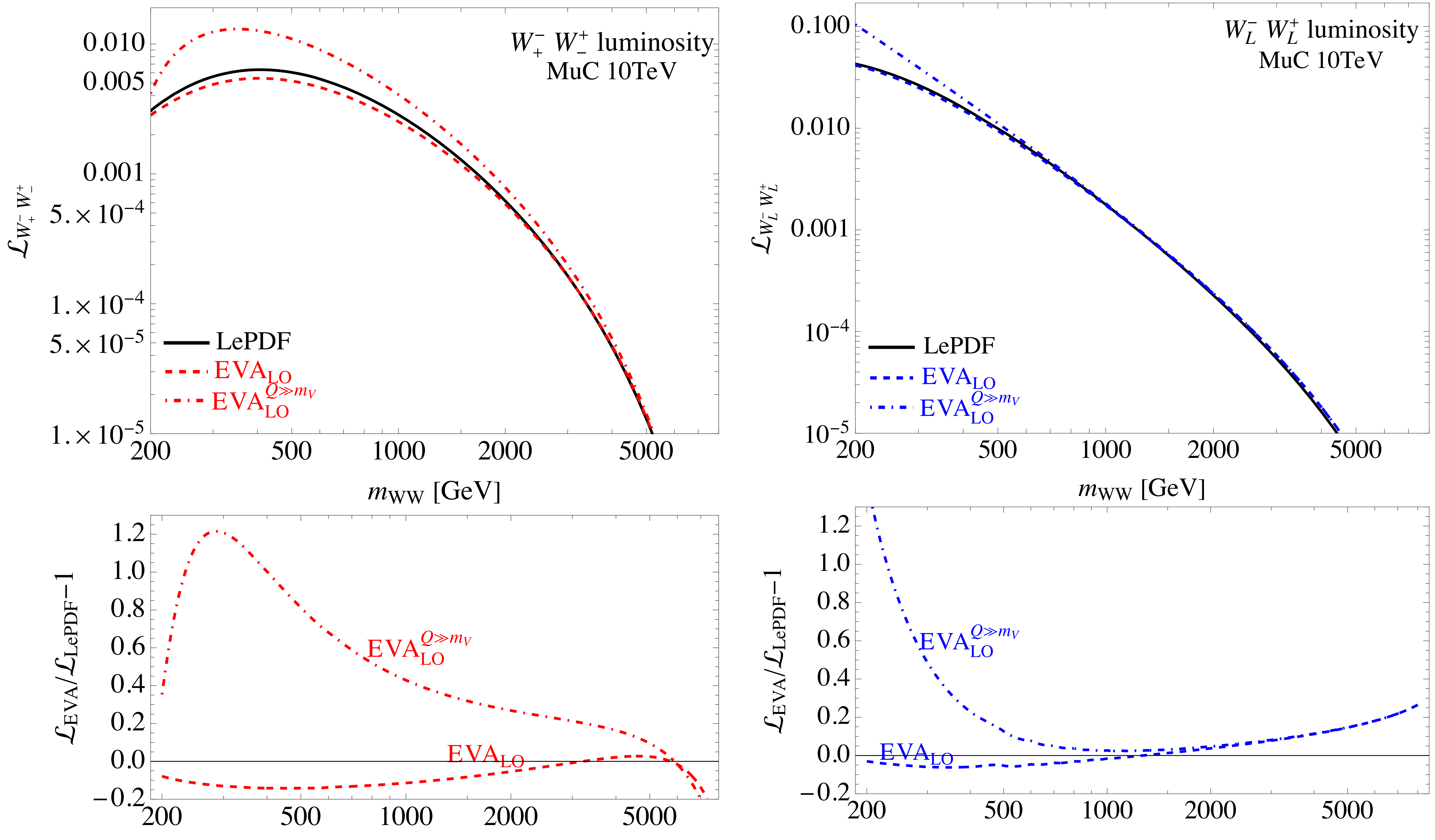
$$f^{(\alpha)}_{\mu_L}(x,t) \simeq \int_{t_{m_W}}^{t} dt' \left(\frac{1}{2} P^v_{\mu_L}(t')\delta(1-x) + \frac{\alpha_\gamma}{4\pi} P^V_{ff}(x) + \frac{\alpha_2}{4\pi c_W^2}(Q^Z_{\mu_L})^2 P^V_{ff}(x) \right) ,$$

$$f^{(\alpha^2)}_{W^-_+}(x,t) \simeq \int_{t_{m_W}}^{t} dt' \left(P^v_{W^-_+} f^{(\alpha)}_{W^-_+} + \frac{\alpha_2}{4\pi} P^f_{V_+ f_L} \otimes f^{(\alpha)}_{\mu_L} + \frac{\alpha_2}{2\pi} c_W^2 P_{V_+ V_s} \otimes (f^{(\alpha)}_{W^-_s} + f^{(\alpha)}_{Z_s}) + \right.$$
$$\left. + \frac{\alpha_\gamma}{2\pi} P_{V_+ V_s} \otimes (f^{(\alpha)}_{W^-_s} + f^{(\alpha)}_{\gamma_s}) + \frac{\sqrt{\alpha_\gamma \alpha_2}}{2\pi} c_W P_{V_+ V_s} \otimes f^{(\alpha)}_{Z/\gamma_s} \right) .$$

* for simplicity, in the NLO part we take the Q ≫ m_W and x ≪ 1 limit in the LO EVA expression.

Several **double logs appear at this order**,
we find a **much improved agreement** with the LePDF resummation.

LePDF vs. EVA: WW Luminosity



At the level of **parton luminosity**:

- for $\mathbf{W_T W_T}$: $\mathrm{EVA_{LO}}$ is accurate to **~15%**
- for $\mathbf{W_L W_L}$: $\mathrm{EVA_{LO}}$ is accurate to **~5%**
- The $\mathbf{Q \gg m_V}$ approximation does not reproduce well the complete result, with **O(1) differences** up to large scales (particularly for transverse modes).

$\mathrm{EVA_{LO}}$

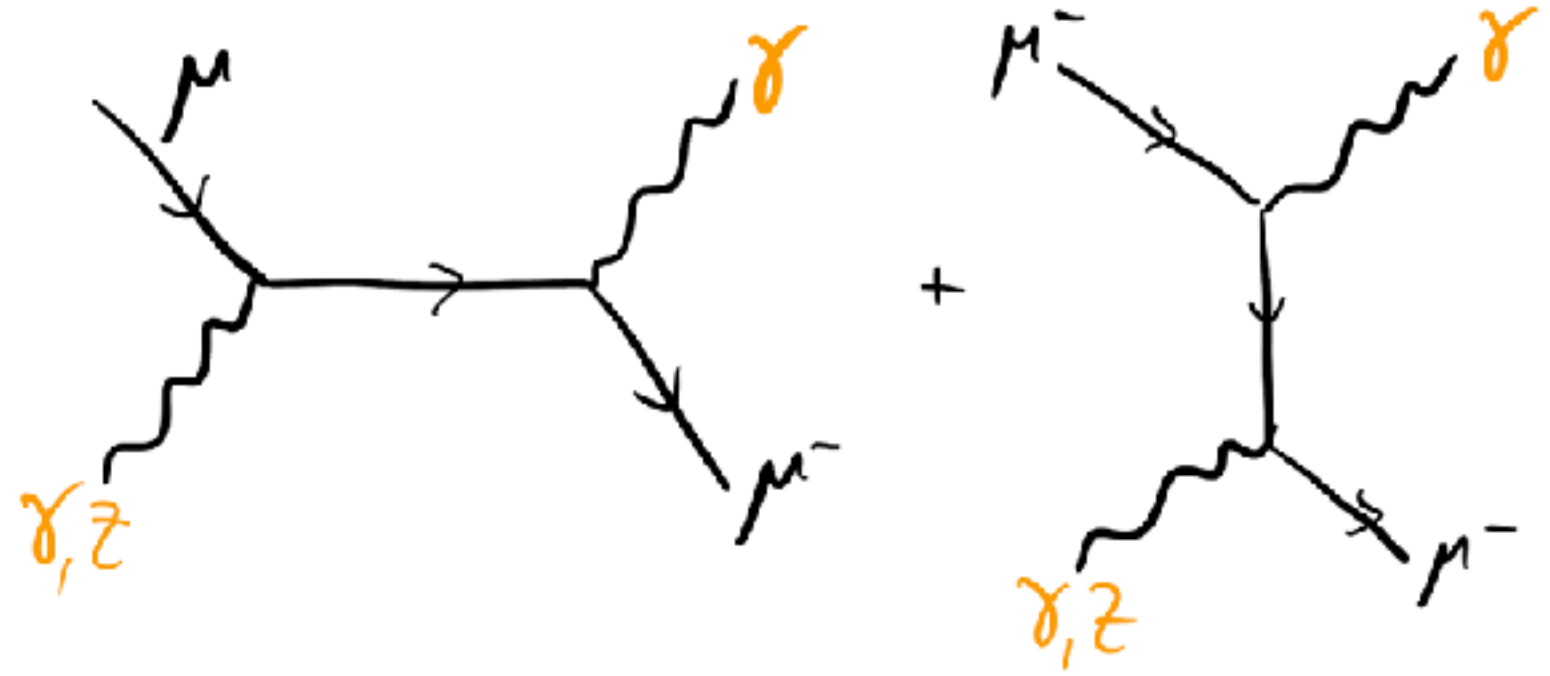
$$f^{(\alpha)}_{W^-_\pm}(x,Q^2) = \frac{\alpha_2}{8\pi} P^f_{V_\pm f_L}(x)\left(\log\frac{Q^2+(1-x)m_W^2}{m_\mu^2+(1-x)m_W^2} - \frac{Q^2}{Q^2+(1-x)m_W^2}\right)$$

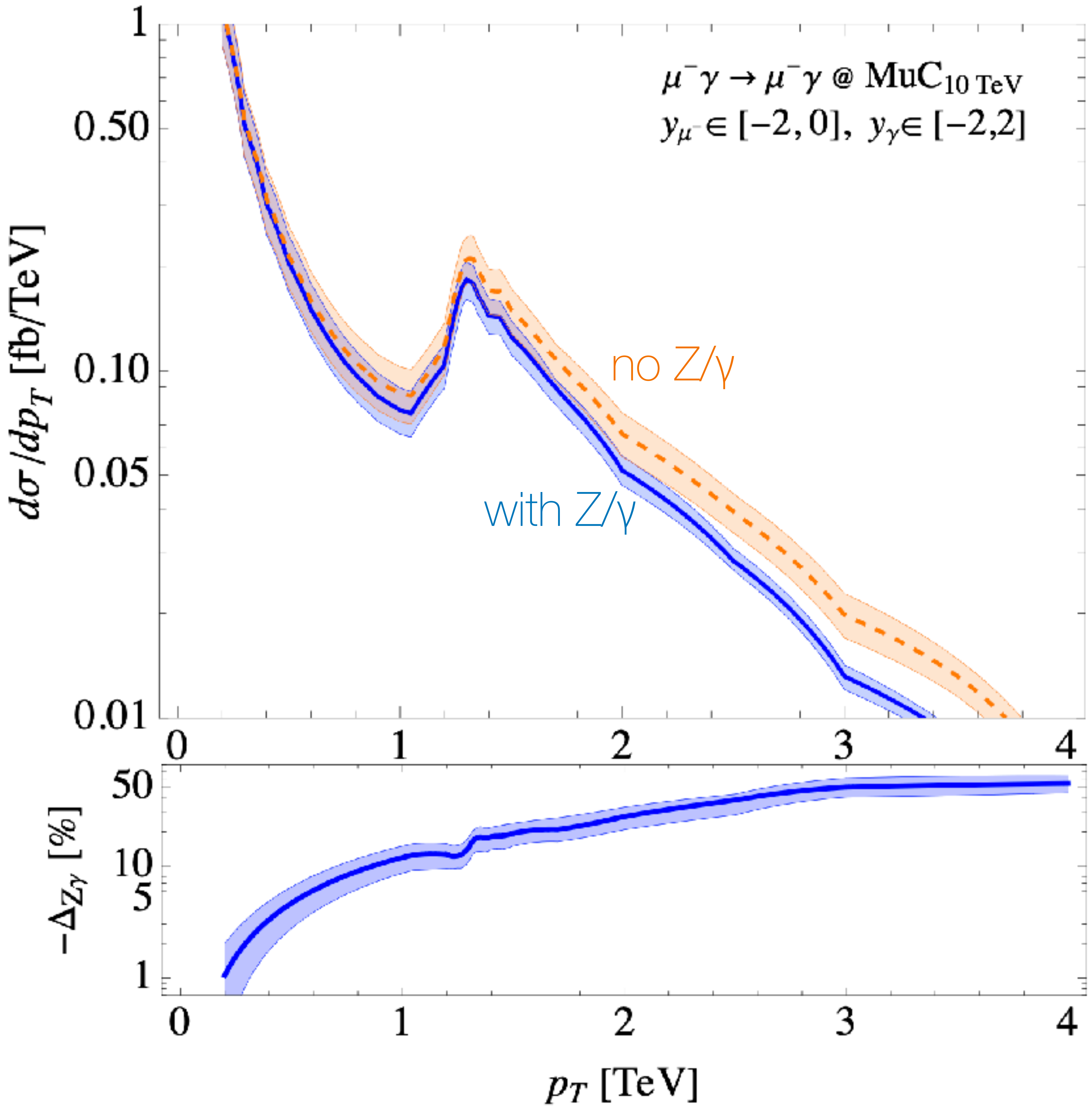
$\mathrm{EVA_{LO}}^{mV\to 0}$

$$f^{(\alpha)}_{W^-_\pm}(x,Q^2) \approx \frac{\alpha_2}{8\pi} P^f_{V_\pm f_L}(x)\log\frac{Q^2}{m_W^2}$$

Implemented in `MadGraph5_aMC@NLO`
Ruiz, Costantini, Maltoni, Mattelaer [2111.02442]

Compton Scattering @ MuC

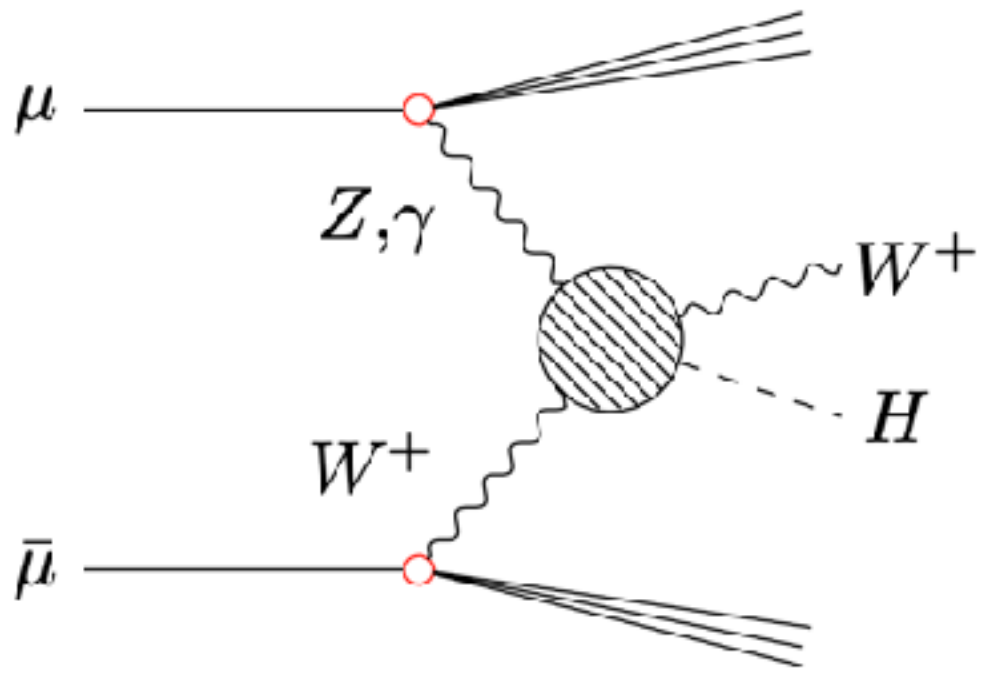




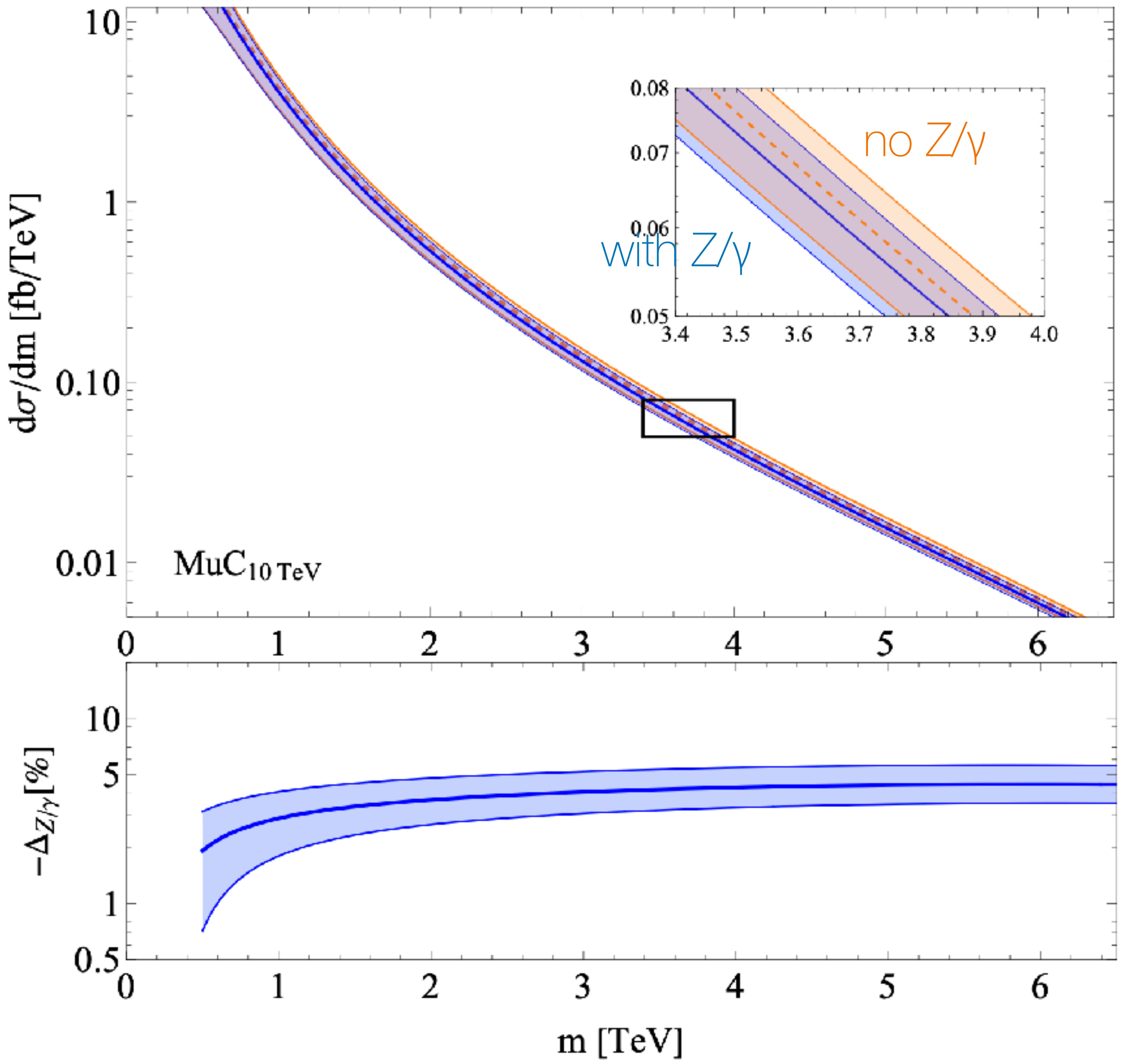
The peak at around $p_T \sim 1350$ GeV is due to the fact that, for those values of p_T the kinematical configuration with $x_1 = 1$ (x_1 being the Bjorken variable for the incoming muon) enters the range of rapidities included in the integration.
For $x_1 \approx 1$ the μ^- PDF gets the large enhancement due to it being the valence parton, remnant of the Dirac delta that describes the zeroth order PDF of the muon.

WH production @ MuC

Consider **associated W H production at a MuC**



While at present the effect is washed out by the scale uncertainties, these are expected to be reduced in the future, since one of the main goals of muon colliders is to perform measurements of EW processes at high energy with O(1%) precision.

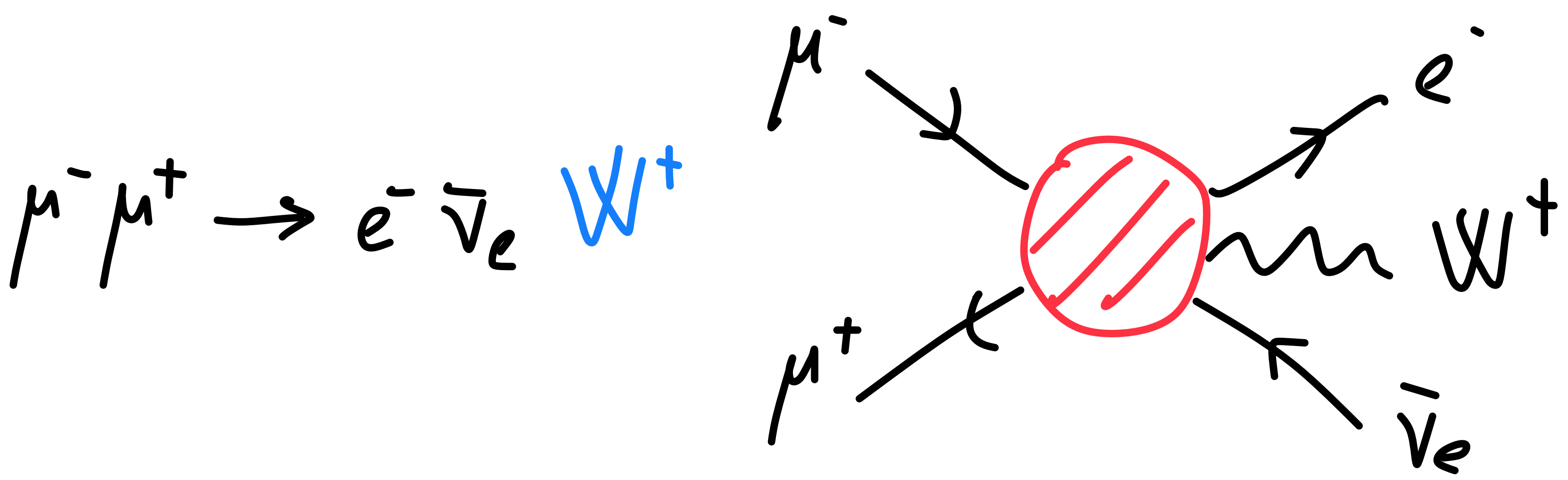


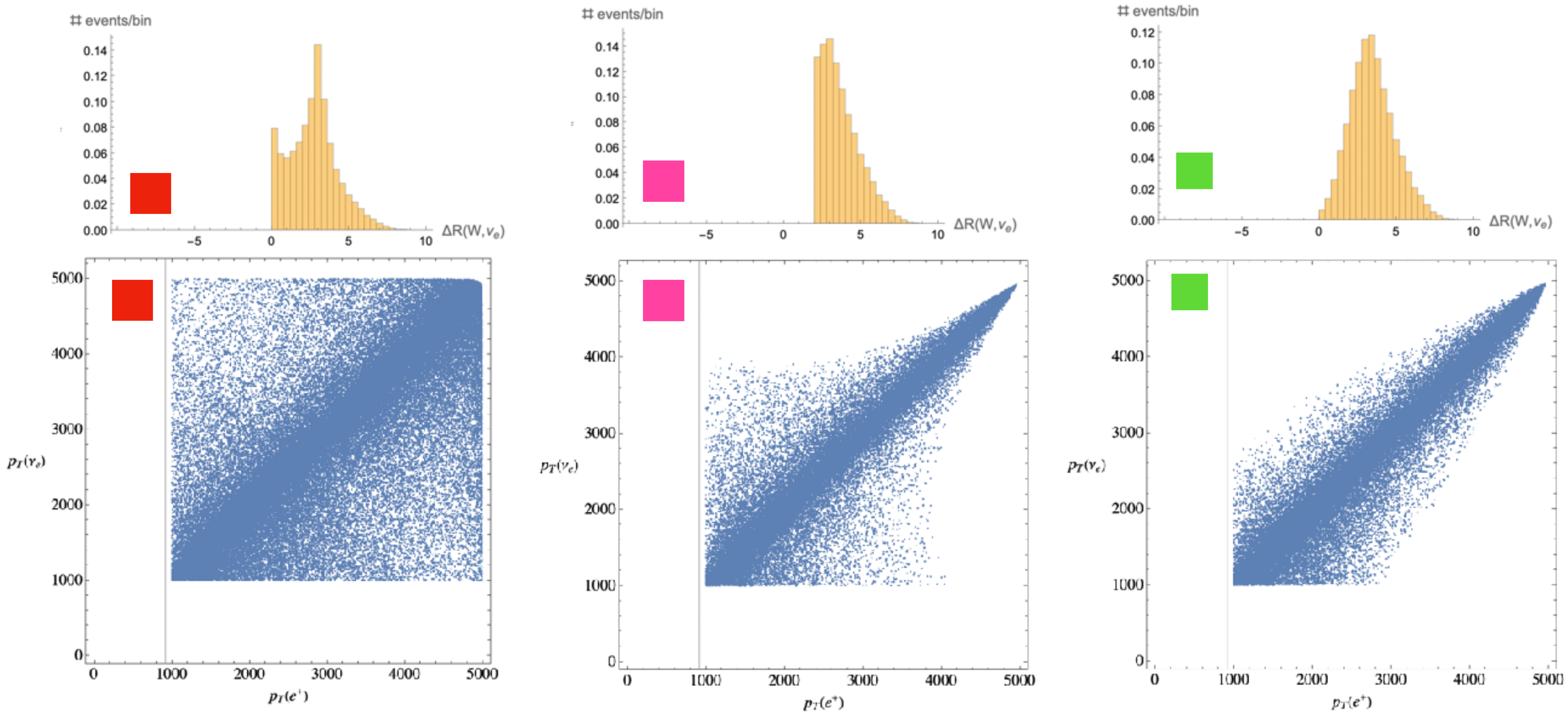
e+v: comparison with MadGraph

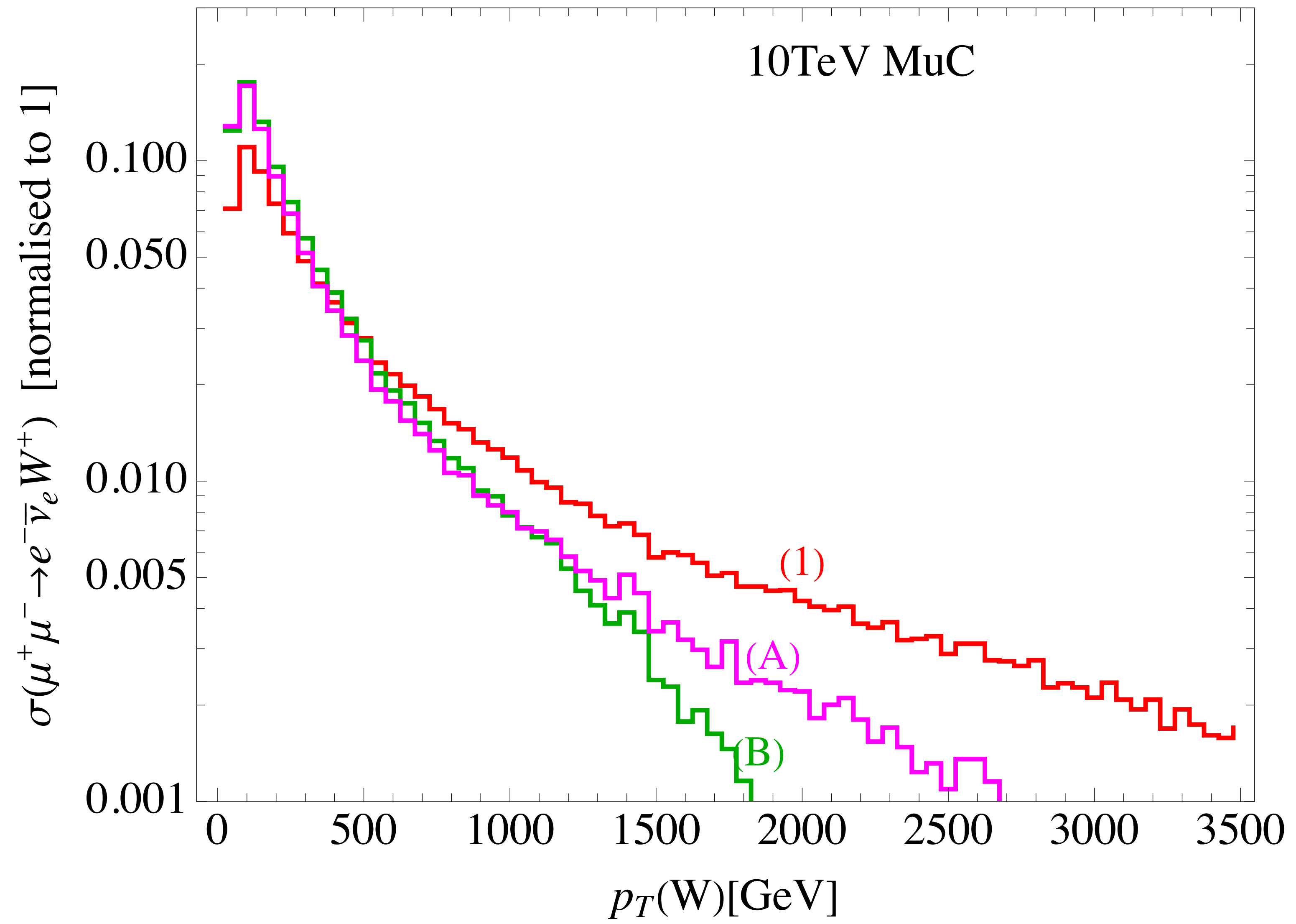
$|y_e| < 2\ ,\quad p_T^e > 1\,\mathrm{TeV}\ ,\quad p_T^\nu > 1\,\mathrm{TeV}\ ,\quad M(e,\nu_e) > 500\,\mathrm{GeV}$

A $\Delta R(i,j) > 2$ or 1.5

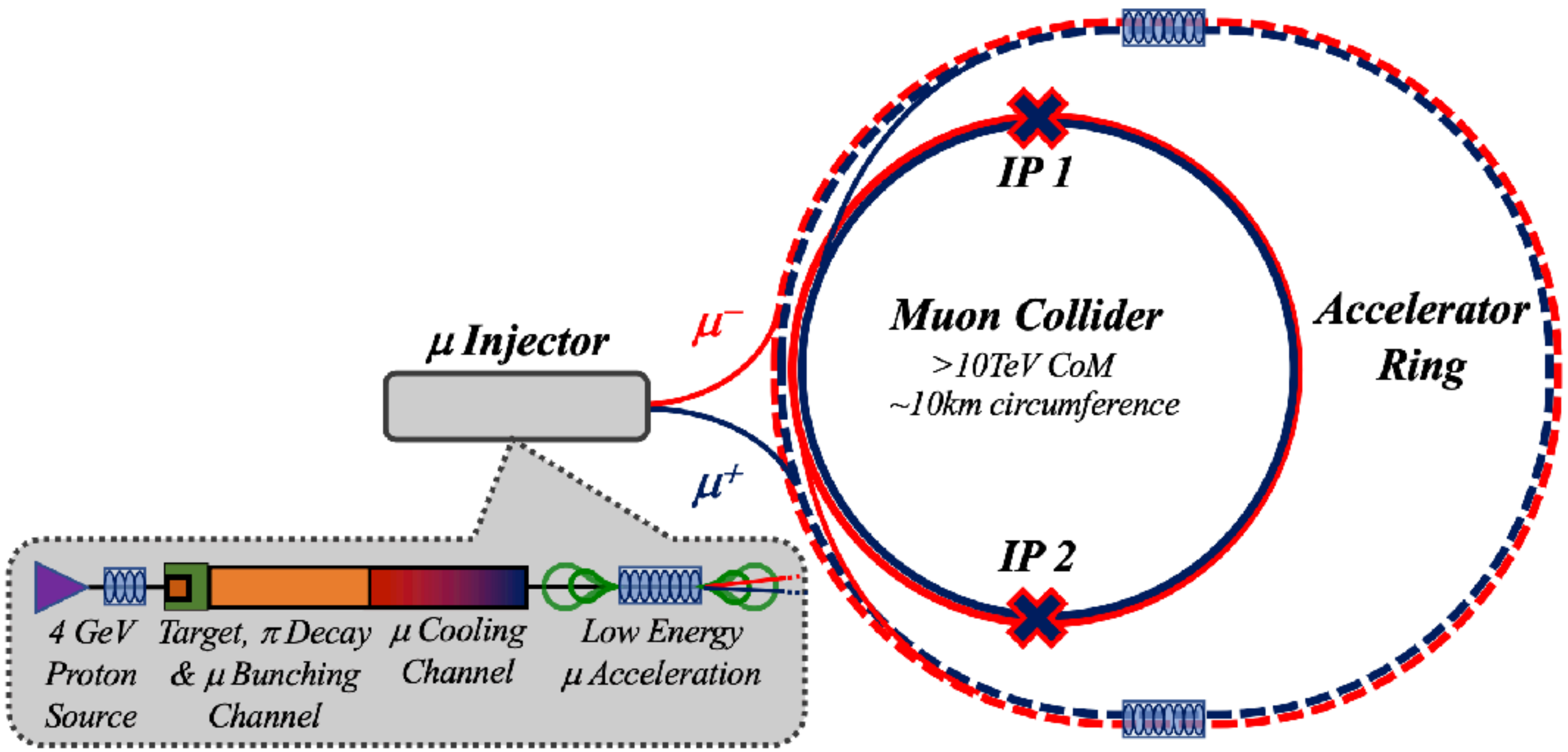
B $|\eta_W| > 1.5$ or 1.0







High Energy Muon Collider



There could be a **staged development**, with a 3 TeV phase first and a 10 TeV later.

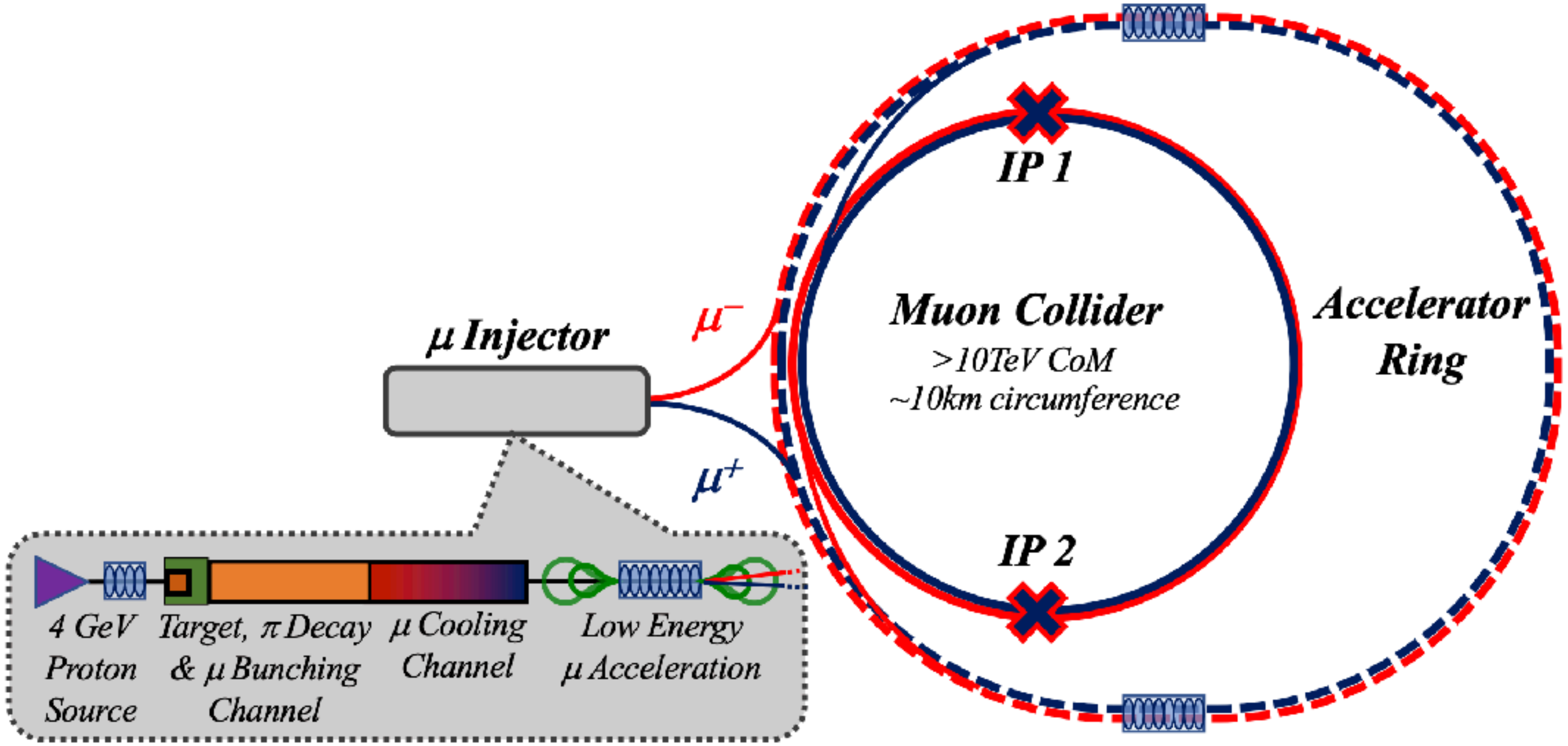
Several components could be re-used.

3TeV ~ 4.5 Km circumference

10TeV ~ 10 Km circumference

30TeV ?

High Energy Muon Collider



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3TeV ~ 4.5 Km circumference

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30TeV ?

A Muon Collider collaboration has been created at CERN.

EU Design Study for a MuC has been approved.

International
UON Collider
Collaboration

For more info:

See Snowmass reports 2203.08033, 2203.07224, 2203.07256, 2203.07261 and Refs. therein
Here a recent GGI Tea Break Focus Meeting on Muon Colliders: https://youtu.be/17JoTculs6k