Innovating Slicing Methods for Systematic and Efficient Collider Phenomenology at N3LO

Gherardo Vita

CERN

QCD Seminar - Milano, 2 December 2024

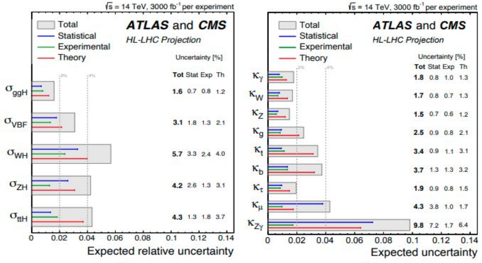
Based on:

"N3LO Power Corrections for 0-jettiness Subtractions With Fiducial Cuts" **GV** [2401.03017]

"Projection-to-Born-improved Subtractions at NNLO" Campbell, Neumann, **GV** [2408.05265]

Outline

- **Intro e Motivation**
  - Do we need N3LO cross sections?



	Q [GeV]	$\delta\sigma^{\text{NNLO}}$	$\delta\sigma^{\text{N}^3\text{LO}}$
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	100	-2.3%	-2.1%
CCDY(W^+)	30	-0.1%	-4.7%
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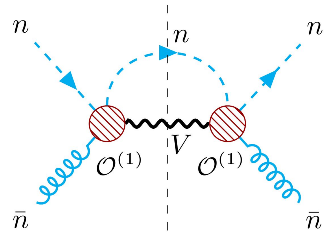
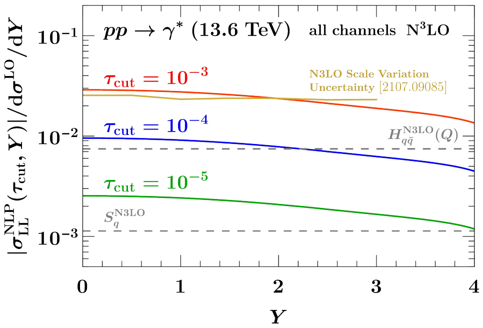
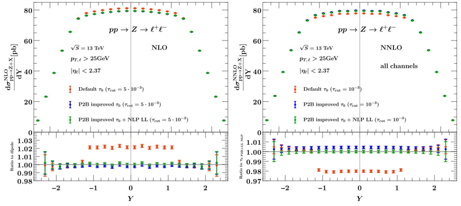
  - Slicing methods

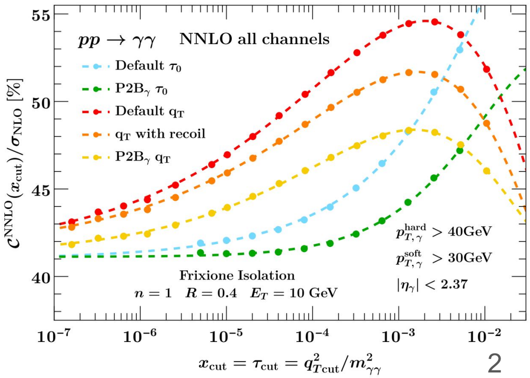
$$\sigma(X) = \underbrace{\int_0^{q_{T\text{cut}}} \mathrm{d}q_T \frac{\mathrm{d}\sigma^{\text{sing}}(X)}{\mathrm{d}q_T}}_{\text{Below the cut region}} + \underbrace{\int_{q_{T\text{cut}}} \mathrm{d}q_T \frac{\mathrm{d}\sigma(X)}{\mathrm{d}q_T}}_{\text{Above the cut region}} + \underbrace{\Delta\sigma(X, q_{T\text{cut}})}_{\text{Residual}}$$



- **Improving Slicing**
  - N3LO Power corrections for 0-jettiness subtraction
  - Projection-to-Born-improved Subtractions

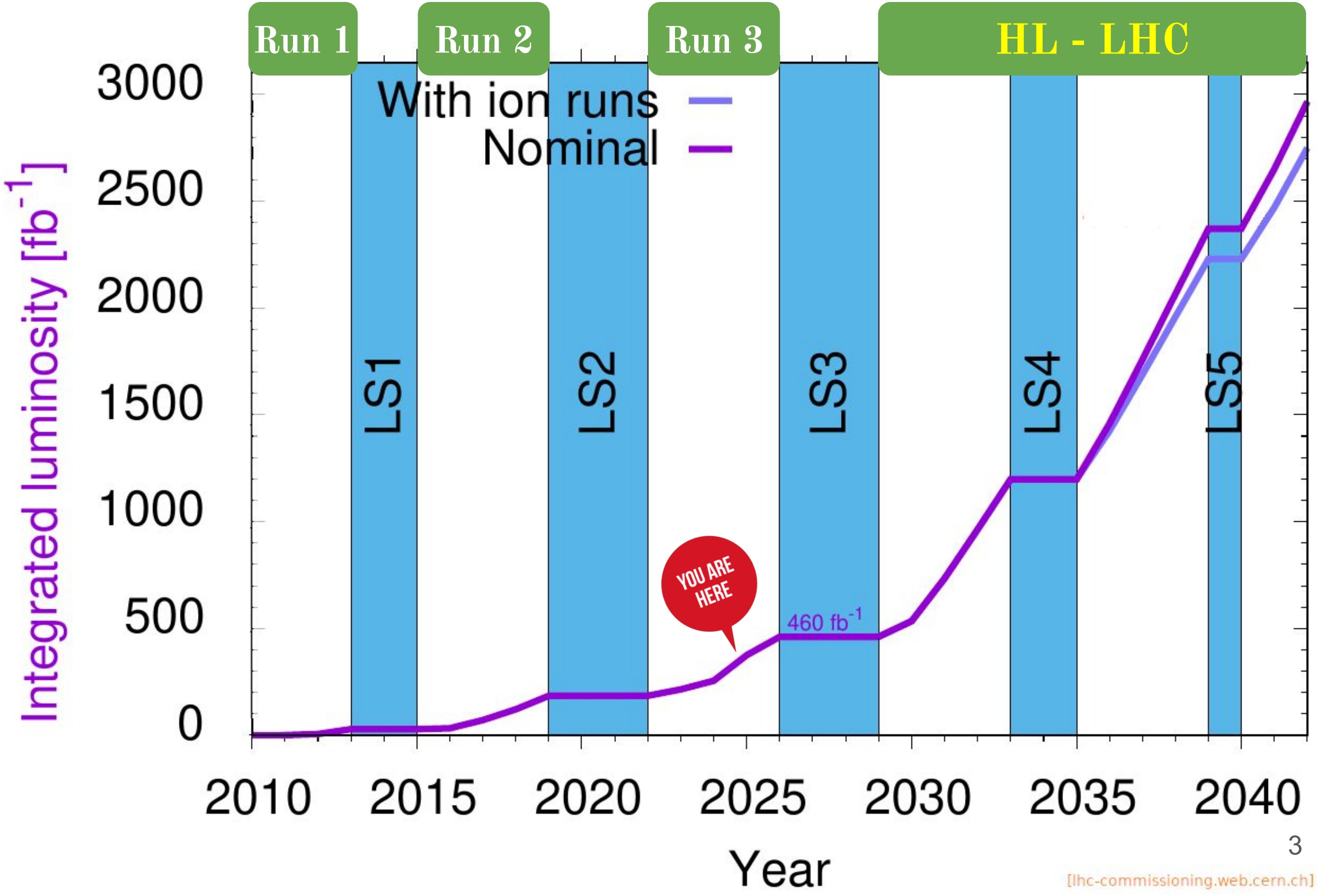






LHC Timeline

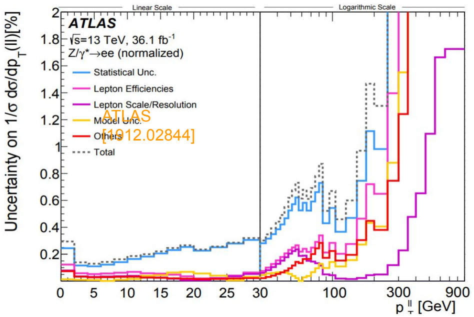
- **The LHC physics program is just at the beginning!**
- LHC analysis **today** are based on less than **150 fb^{-1}**
- **460 fb^{-1}** will be collected by the end of **2025**
- **3000 fb^{-1}** will be collected after **HL - LHC**

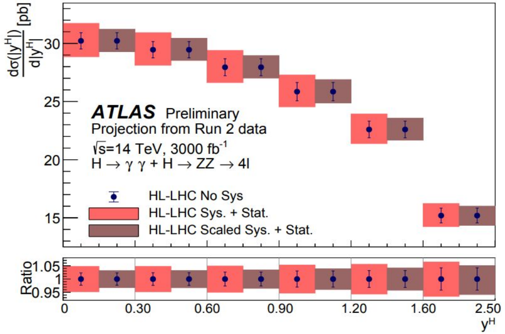


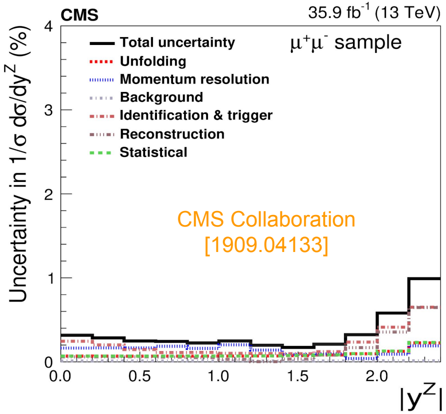
[lhc-commissioning.web.cern.ch]

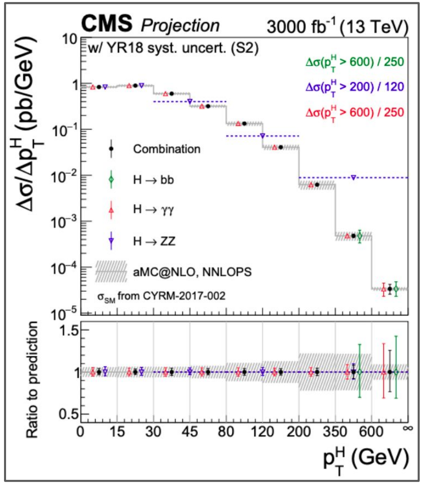
Testing the Standard Model at Colliders

Ability to test the SM at (sub)-percent accuracy!









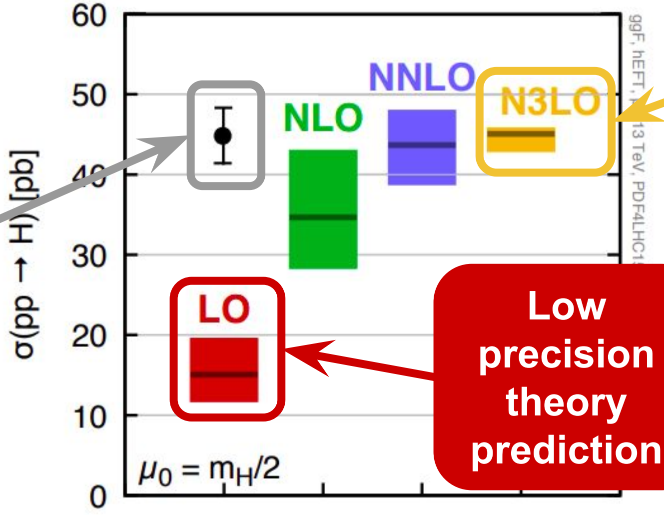
High experimental accuracy for processes sensitive to some of the most interesting aspects of contemporary particle physics:

- ➢ Probing Electroweak Symmetry Breaking
- ➢ Unveiling the Nature of Yukawa Interactions
- ➢ Testing the Boundaries of the Standard Model
- ➢ Measuring the Strength of the Strong Interaction
- ➢ Deepening our Understanding of the Proton Structure

Precision in Theoretical Predictions

To answer these fundamental questions we need comparable precision from the theory side!

Example: Higgs Production at the LHC



Experimental Measurement

Accurate theoretical prediction

Low precision theory prediction

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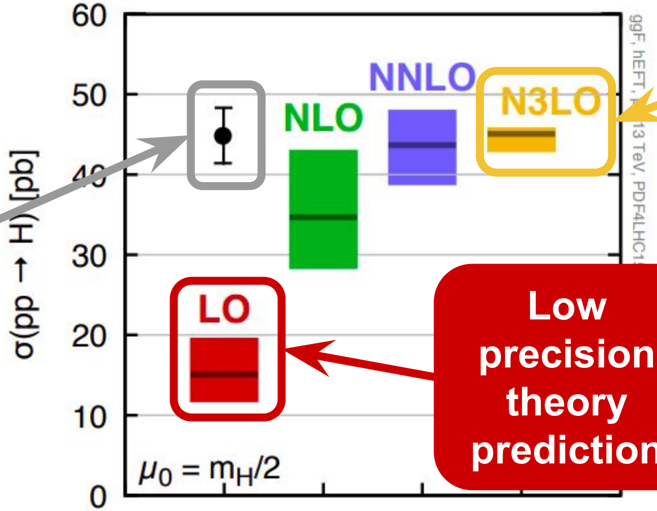

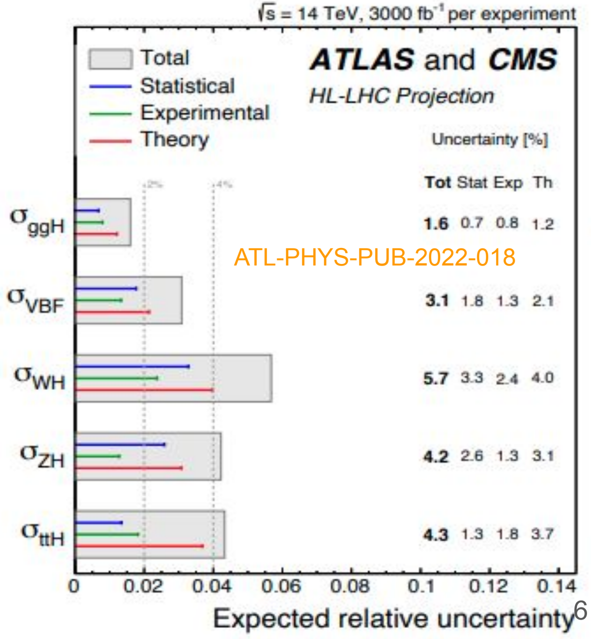

Recurring theme for HL - LHC Projections: Accuracy of Theory Predictions will be limiting factor in many analysis!

Standard Model Phenomenology at percent level

One way to achieve more accurate predictions is by advancing

$$\hat{\sigma}_{ab\to X} = \underbrace{\sigma_0}_{\text{LO}} + \underbrace{\alpha_s \sigma_1}_{\text{NLO}} + \underbrace{\alpha_s^2 \sigma_2}_{\text{NNLO}} + \underbrace{\alpha_s^3 \sigma_3}_{\text{N}^3\text{LO}} + \ldots$$

QCD Perturbation Theory

And many more things…

- **Corrections beyond massless QCD:** EWK and masses.
- **Determination of N3LO PDFs:** possibly with a good estimate of MHOU and systematic uncertainties from fitting procedure
- **Parton Showers:** Consistent combination of PS with fixed order calculations at N^3LO.
- **Resummation:** Complementing N^3LO computations and resummation techniques for infrared sensitive observables.
- **Uncertainties:** Deriving/defining reliable uncertainty estimates for theoretical computations at the percent level.
- **Factorisation Violation/Beyond Leading Power Factorisation:** Exploring the limitations of leading power perturbative descriptions of hadron collision cross sections.
- **Accessibility and User Friendliness:** Creating frameworks that make N^3LO (and NNLO) predictions easily accessible for comparison to experimental data.

Do we really need N3LO cross sections?

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Not so fast...

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- Let's look at some explicit example...

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Total Inclusive Cross sections

N3loxs
[Baglio, Duhr, Mistlberger, Szafron '22]

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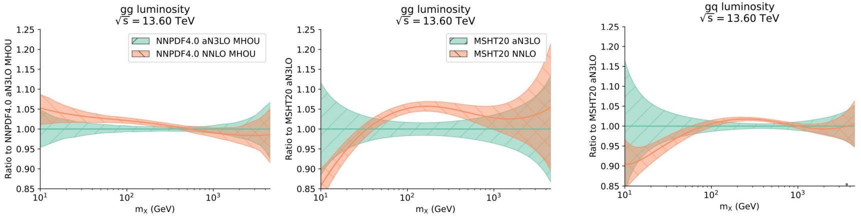
Parton Distribution Functions

[2411.05373] NNPDF & MSHT Collaborations

PDF set	$\sigma(gg \to h)$	$\sigma(h \text{ VBF})$
$\Delta^{\text{exact}}_{\text{NNLO}}$ (NNPDF4.0)	2.2%	1.3%
$\Delta^{\text{exact}}_{\text{NNLO}}$ (MSHT20)	5.3%	2.3%
$\Delta^{\text{exact}}_{\text{NNLO}}$ (combination)	3.3%	2.3%



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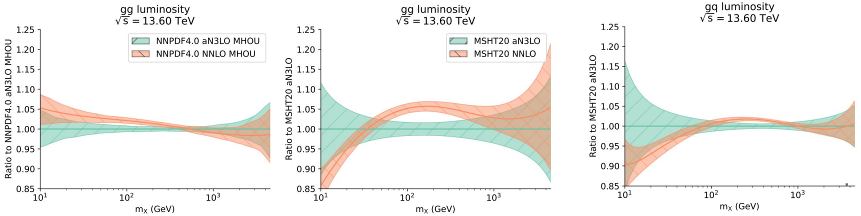
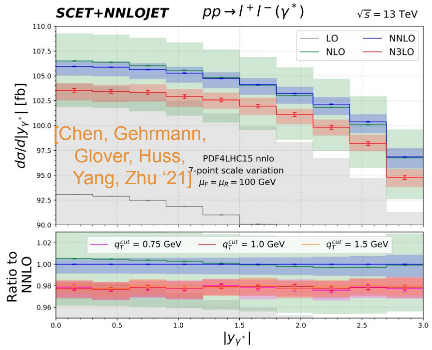
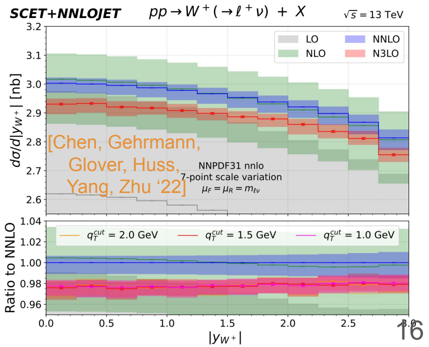




Differential / Fiducial Cross Sections

Order k	σ [pb] Symmetric cuts: N^{k}LO	σ [pb] Symmetric cuts: N^{k}LO+N^{k}LL	σ [pb] Product cuts: N^{k}LO	σ [pb] Product cuts: N^{k}LO+N^{k}LL
0	$721.16^{+12.2\%}_{-13.2\%}$	—	$721.16^{+12.2\%}_{-13.2\%}$	—
1	$742.80(1)^{+2.7\%}_{-3.9\%}$	$748.58(3)^{+3.1\%}_{-10.2\%}$	$832.22(1)^{+2.7\%}_{-4.5\%}$	$831.91(2)^{+2.7\%}_{-10.4\%}$
2	$741.59(8)^{+0.42\%}_{-0.71\%}$	$740.75(5)^{+1.15\%}_{-2.66\%}$	$831.32(3)^{+0.59\%}_{-0.96\%}$	$830.98(4)^{+0.74\%}_{-2.73\%}$
3	$722.9(1.1)^{+0.68\%}_{-1.09\%} \pm 0.9$	$726.2(1.1)^{+1.07\%}_{-0.77\%}$	$816.8(1.1)^{+0.45\%}_{-0.73\%} \pm 0.8$	$816.6(1.1)^{+0.87\%}_{-0.69\%}$

[Chen, Gehrmann, Glover, Huss, Monni, Re, Rottoli, Torrielli '22]



We really need N3LO cross sections for 1% accuracy

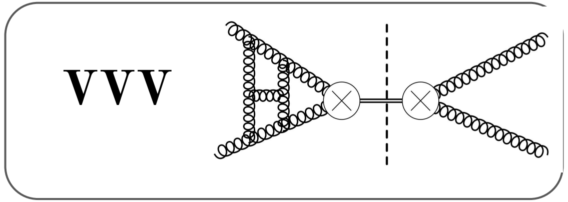
- Numerous cases now show that **N3LO corrections can easily exceed 1%**.

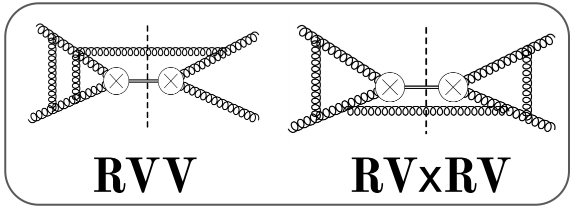
- This, at the very least, indicates that **to claim percent level accuracy** in QCD sensitive observables **N3LO corrections**, or a very good quantitative estimate of them, **must be included**.

- Therefore, a key aspect of the precision program at the HL-LHC will be the ability to **systematically incorporate N3LO contributions**.

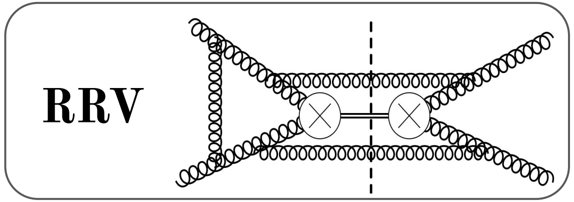
Cross Sections in Perturbative QCD

$$\sigma = f_1 \circ f_2 \int d\Phi \, |M|^2$$

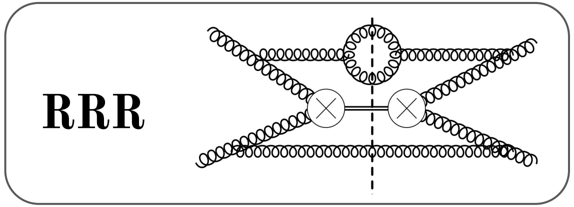




- Cross sections for LHC processes are obtained via **phase space integrals** over **amplitudes** (squared) convoluted with **Parton Distribution Functions (PDFs)**



- **IR divergences** at intermediate steps of the calculation **cancel only after summing** over all **real** and **virtual** contributions



- The complexity of cancellations grows dramatically with higher orders, making **systematization** of cross section calculations at NNLO very challenging and **at N3LO a monumental undertaking**

Slicing methods

- One way to deal with IR singularities for cross sections are **slicing methods**
- The idea behind is quite simple. Take the production of color singlet q at N3LO as example.
  - Find an observable x that isolates the Born configuration in a the region where the observable vanishes (think for example at the transverse momentum of q)
  - Reorganize the cross section separating out the region around the Born configurations

$$\sigma_q^{\mathrm{N^3LO}}(\mathcal{O}) = \int_0^{x_{\mathrm{cut}}} \mathrm{d}x \frac{\mathrm{d}\sigma_q^{\mathrm{N^3LO}}}{\mathrm{d}x}(\mathcal{O}) + \int_{x_{\mathrm{cut}}}^{x_{\mathrm{max}}} \mathrm{d}x \frac{\mathrm{d}\sigma_{q+\mathrm{jet}}^{\mathrm{NNLO}}}{\mathrm{d}x}(\mathcal{O})$$

Below the cut region:

Only region where **genuine N3LO cancellation of IR divergences** is necessary

Above the cut region:

- Resolved extra radiation => no events in Born configuration
- From a IR point of view this is an NNLO problem, so no N3LO subtraction needed to get this term

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Approximate the full distribution in x below the cut with its **Leading Power term** (obtained via resummation / factorization theorem in SCET)

$$\int_0^{x_{\rm cut}} dx \frac{d\sigma_q^{\rm N^3LO}}{dx}(\mathcal{O}) = \int_0^{x_{\rm cut}} dx \frac{d\sigma_{q,(\rm LP)}^{\rm N^3LO}}{dx}(\mathcal{O}) + \dots$$

$$\sigma_q^{\rm N^3LO}(\mathcal{O}) = \int_0^{x_{\rm cut}} dx \frac{d\sigma_q^{\rm N^3LO}}{dx}(\mathcal{O}) + \int_{x_{\rm cut}}^{x_{\rm max}} dx \frac{d\sigma_{q+\rm jet}^{\rm NNLO}}{dx}(\mathcal{O})$$

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Slicing methods

$$\sigma_q(\mathcal{O}) = \int_0^{x_{\rm cut}} \mathrm{d}x \frac{\mathrm{d}\sigma_q^{\rm sub}}{\mathrm{d}x}(\mathcal{O}) + \int_{x_{\rm cut}}^{x_{\rm max}} \mathrm{d}x \frac{\mathrm{d}\sigma_{q+\rm jet}}{\mathrm{d}x}(\mathcal{O}) + \int_0^{x_{\rm cut}} \mathrm{d}x \left[\frac{\mathrm{d}\sigma_q}{\mathrm{d}x}(\mathcal{O}) - \frac{\mathrm{d}\sigma_q^{\rm sub}}{\mathrm{d}x}(\mathcal{O})\right]$$

$$\equiv \sigma^{\rm sub}(x_{\rm cut}, \mathcal{O}) + \sigma^{\rm above}(x_{\rm cut}, \mathcal{O}) + \Delta\sigma(x_{\rm cut}, \mathcal{O})$$

Below the cut region:
- Singular distribution
- Contains most complicated cancellation of IR divergences
- Control it analytically via factorization theorems

Above the cut region:
- Resolved extra radiation
- Calculate with lower order subtraction schemes for process with jet (e.g. NNLOjet)

Slicing Residual/Error:
Non singular terms from below the cut that are **neglected** (aka *power corrections*).
Minimized by going to very small values of cut parameter

q_T Subtraction: [Catani, Grazzini '07]

N-Jettiness Subtraction: [Boughezal, Focke, Liu, Petriello '15] [Gaunt, Stahlhofen, Tackmann, Walsh '15]

- Extremely successful program for many color singlet (and top) processes at **NNLO** [MATRIX Collaboration] [DYTurbo]
- With ***N*-Jettiness** (or k_T-ness) ability to tackle also processes with **jets in the final state**

[Boughezal, Focke, Liu, Petriello + Campbell, Ellis, Giele '15, '16] [MCFM collaboration] [Geneva] [Buonocore, Grazzini, Haag, Rottoli, Savoini '22]

Slicing methods

$$\sigma_q(\mathcal{O}) = \int_0^{x_\mathrm{cut}} \mathrm{d}x \frac{\mathrm{d}\sigma_q^\mathrm{sub}}{\mathrm{d}x}(\mathcal{O}) + \int_{x_\mathrm{cut}}^{x_\mathrm{max}} \mathrm{d}x \frac{\mathrm{d}\sigma_{q+\mathrm{jet}}}{\mathrm{d}x}(\mathcal{O}) + \int_0^{x_\mathrm{cut}} \mathrm{d}x \left[\frac{\mathrm{d}\sigma_q}{\mathrm{d}x}(\mathcal{O}) - \frac{\mathrm{d}\sigma_q^\mathrm{sub}}{\mathrm{d}x}(\mathcal{O}) \right]$$

$$\equiv \underbrace{\sigma^\mathrm{sub}(x_\mathrm{cut}, \mathcal{O}) + \sigma^\mathrm{above}(x_\mathrm{cut}, \mathcal{O})}_{\sigma(x_\mathrm{cut}, \mathcal{O})} + \Delta\sigma(x_\mathrm{cut}, \mathcal{O})$$

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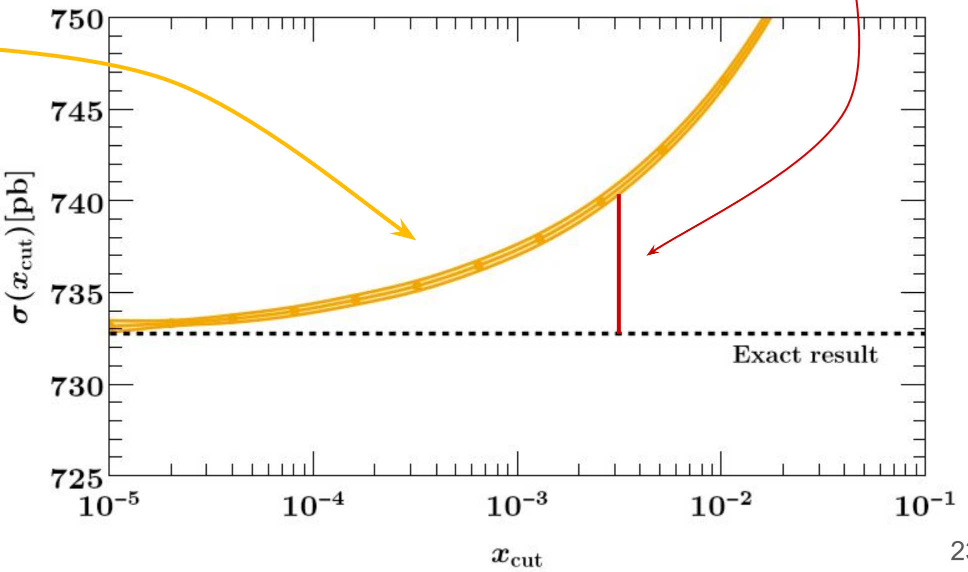
$$\sigma(x_{\rm cut}, \mathcal{O})$$



- Dependence on $x_{\rm cut}$ is artifact of slicing method
  => use it to estimate slicing error

Slicing methods

$$\sigma_q(\mathcal{O}) = \int_0^{x_{\rm cut}} dx \frac{d\sigma_q^{\rm sub}}{dx}(\mathcal{O}) + \int_{x_{\rm cut}}^{x_{\rm max}} dx \frac{d\sigma_{q+\rm jet}}{dx}(\mathcal{O}) + \int_0^{x_{\rm cut}} dx \left[\frac{d\sigma_q}{dx}(\mathcal{O}) - \frac{d\sigma_q^{\rm sub}}{dx}(\mathcal{O})\right]$$

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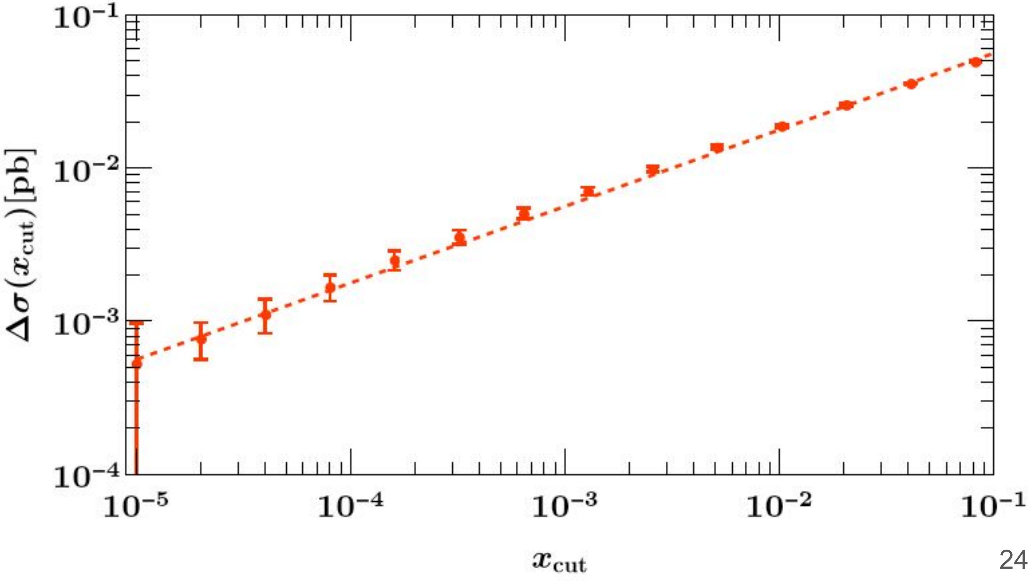
- Ideally, one should:
  - look directly at error in log-log plot
  - See the expected scaling
  - Stop at a cut value where the size of the estimated error is ok for the current study



Slicing, non-local subtractions, and local subtractions

- **In principle**, slicing methods are different from standard subtraction methods since the residual power corrections constitute an intrinsic error that is always present, which is not the case for a subtraction method.

- However, still in principle, with the exact same theoretical ingredients, one can very easily write a **(non-local) subtraction scheme** with **no residual power corrections**

$$\sigma_q(\mathcal{O}) = \int_0^{x_{\max}} \mathrm{d}x \Big[\underbrace{\frac{\mathrm{d}\sigma_q^{\mathrm{full}}}{\mathrm{d}x}(\mathcal{O})}_{\text{real contribution}} - \underbrace{\frac{\mathrm{d}\sigma_q^{\mathrm{sub}}}{\mathrm{d}x}(\mathcal{O})}_{\text{counterterm}} \Big] + \underbrace{\int_0^{x_{\mathrm{cut}}} \mathrm{d}x \frac{\mathrm{d}\sigma_q^{\mathrm{sub}}}{\mathrm{d}x}(\mathcal{O})}_{\text{Integrated counterterm}}$$

- In practice, **every implementation of a subtraction scheme** (local, non-local, slicing) has technical cut-offs that lead to the **neglect of subleading power terms**

- For standard NLO calculations, technical cutoffs of $\sim 10^{-6}$ in local subtractions are more than enough. For complicated NNLO final states and at N3LO these aspects are not as clear cut.

Extending Slicing to N3LO

- **Singular region** (i.e. below the cut) can be understood at all orders via *Leading power factorization theorems* in Soft and Collinear Effective Theory (**SCET**). For example $\mathbf{q_T}$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\mathrm{d}Y\,\mathrm{d}^2\vec{q}_T} = \sigma_0 \sum_{i,j} H_{ij}(Q^2,\mu) \int \mathrm{d}^2\vec{b}_T\, e^{\mathrm{i}\,\vec{q}_T\cdot\vec{b}_T}\, \tilde{B}_i\Big(x_1^B, b_T, \mu, \frac{\nu}{\omega_a}\Big) \tilde{B}_j\Big(x_2^B, b_T, \mu, \frac{\nu}{\omega_b}\Big) \tilde{S}(b_T,\mu,\nu)$$

Hard Function — **q_T Beam Functions** — **Soft Function**

N3LO ingredients q_T:

[Gehrmann, Glover, Huber, Ikizlerli, Studerus '10]

"TMDPDFs at N3LO" M.Ebert, B.Mistlberger, **GV** [2006.05329]

"Quark Transverse Parton Distribution at N3LO" [Luo, Yang, Zhu, Zhu '19]

[Li, Zhu '16]

N3LO ingredients 0-jettiness:

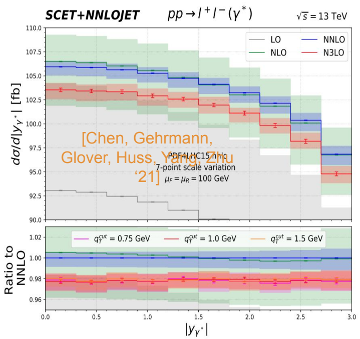
[Gehrmann, Glover, Huber, Ikizlerli, Studerus '10]

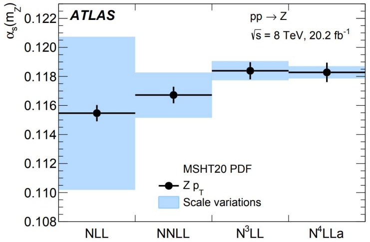
"N-Jettiness Beam Functions at N3LO" M.Ebert, B.Mistlberger, **GV** [2006.03056]

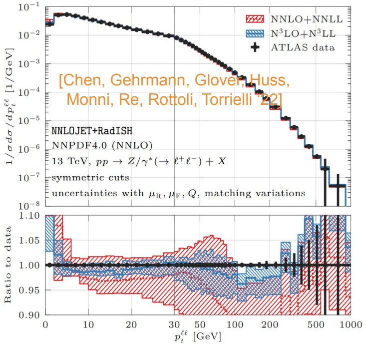
"Zero-jettiness soft function to third order in perturbative QCD" [Baranowski, Delto, Melnikov, Pikelner, Wang '24]

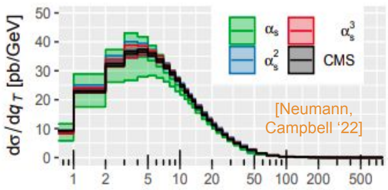
Precision Standard Model Phenomenology at N3LO

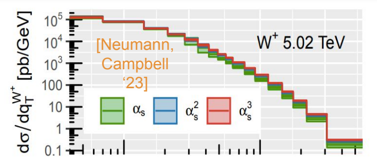
- **N3LO TMDPDF** were **last missing ingredient** for q_T slicing at N3LO
- Enabled N3LO predictions for differential and fiducial Drell-Yan and Higgs production
- Marked the advent of a new level of accuracy for the precision program at the LHC

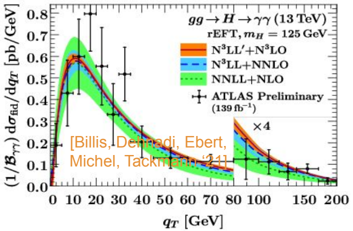


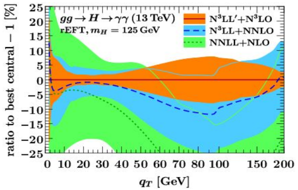












And many more:
[Ju, Schönherr '21]
[Camarda, Cieri, Ferrera '21]
[Re, Rottoli, Torrielli '21]
…

Precision Standard Model Phenomenology at N3LO

- **N3LO TMDPDF** were **last missing ingredient** for q_T slicing at N3LO
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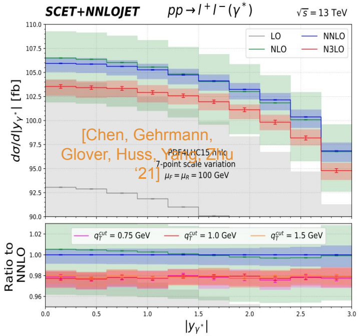

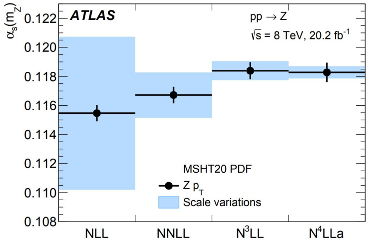

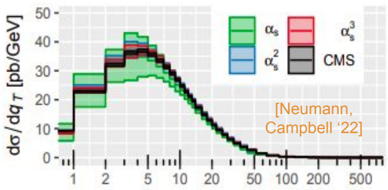

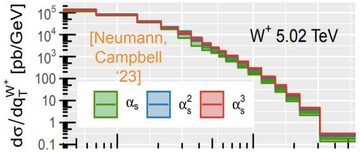

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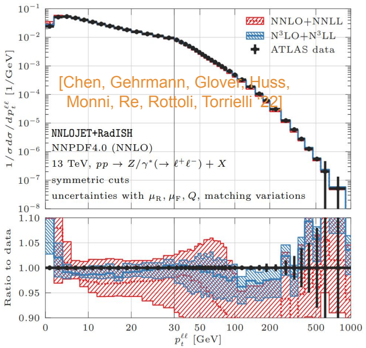
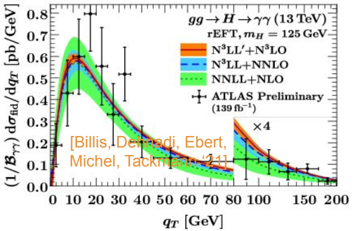
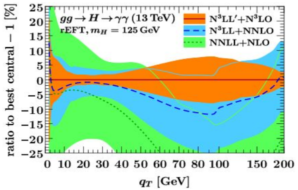






However…

- Numerical (slicing) error of these methods very difficult to control at this order
- Extreme push of NNLO+j predictions well into the IR needed (NNLOjet pushed to q_T = 0.5 GeV)
- Calculations take **O(10 million) CPU hours**
- Almost any change will require to run everything from scratch
- Other results use O(100k) CPU hours and stop at 5 GeV… this requires very delicate extrapolation to 0 to obtain finite results.
- Going forward, these facts pose issues for the practical usability of these predictions

In short, starting to think about how to move from

making fully differential N3LO predictions **possible**,

to

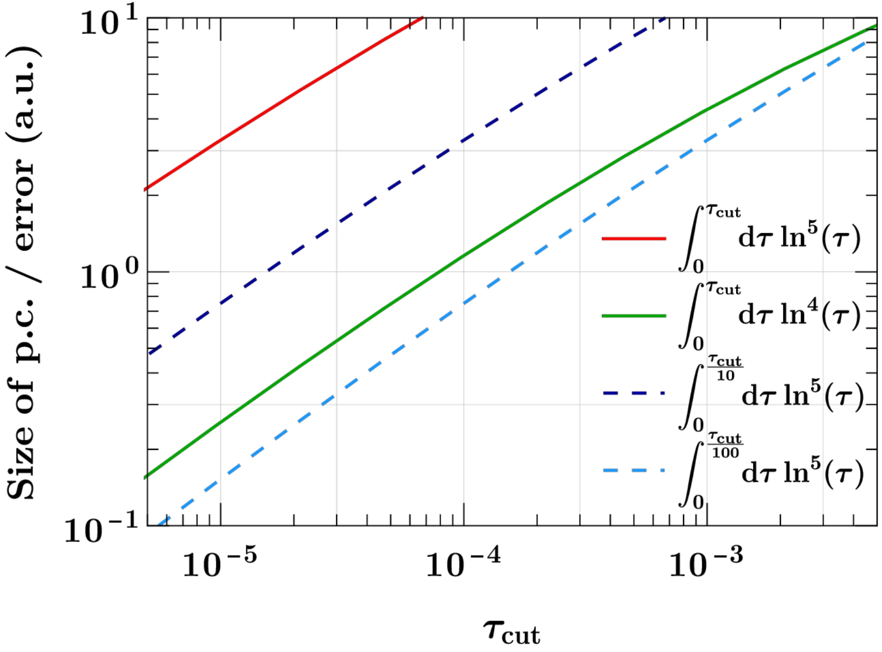
making N3LO predictions (more) **efficient**, **stable**, and **usable**

(at least for some color singlet processes…which may also turn out to be a necessary stepping stone to make other processes possible at N3LO)

Improving non-local subtraction methods: Power corrections

$$\Delta\sigma(\tau_{\mathrm{cut}}) = \int_0^{\tau_{\mathrm{cut}}} \mathrm{d}\tau \left[\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} - \frac{\mathrm{d}\sigma^{\mathrm{sub}}}{\mathrm{d}\tau}\right]$$

- At N3LO power corrections start with **5th power of log**
- Taking τ_{cut} small reduces single power, but increases size of log => very slow convergence
- Each order in the log equivalent to ~ a 10 fold reduction in τ_{cut}



$$\Delta\sigma^{N3LO}(\tau_{\mathrm{cut}}) \sim \left(\frac{\alpha_s}{4\pi}\right)^3 \int_0^{\tau_{\mathrm{cut}}} \mathrm{d}\tau \left(c_{3,5}^{\mathrm{NLP}} \ln^5 \tau + c_{3,4}^{\mathrm{NLP}} \ln^4 \tau + c_{3,3}^{\mathrm{NLP}} \ln^3 \tau + \ldots\right)$$

Improving non-local subtraction methods: Power corrections

$$\Delta\sigma(\tau_{\rm cut}) = \int_0^{\tau_{\rm cut}} d\tau \left[\frac{d\sigma}{d\tau} - \frac{d\sigma^{\rm sub}}{d\tau}\right]$$

- At ... h
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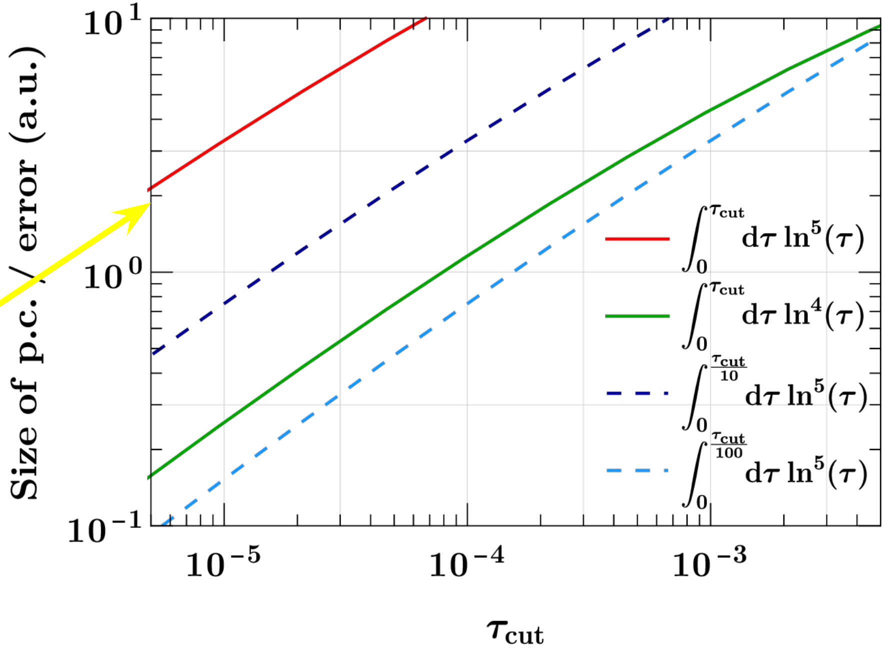
- Tak... t
  incr... ence
- Each ... to ∼ a 10 fold
  reduct...

Very straightforward way of improving slicing:

Obtain the leading logarithmic term at NLP analytically



$$\Delta\sigma^{N3LO}(\tau_{\rm cut}) \sim \left(\frac{\alpha_s}{4\pi}\right)^3 \int_0^{\tau_{\rm cut}} d\tau \left(c_{3,5}^{\rm NLP}\ln^5\tau + c_{3,4}^{\rm NLP}\ln^4\tau + c_{3,3}^{\rm NLP}\ln^3\tau + \dots\right)$$

0-Jettiness Power Corrections at N3LO [GV 2401.03017]

$$\Delta\sigma^{N3LO}(\tau_{\rm cut}) \sim \left(\frac{\alpha_s}{4\pi}\right)^3 \int_0^{\tau_{\rm cut}} \mathrm{d}\tau \left(c_{3,5}^{\rm NLP} \ln^5 \tau + c_{3,4}^{\rm NLP} \ln^4 \tau + c_{3,3}^{\rm NLP} \ln^3 \tau + \ldots \right)$$

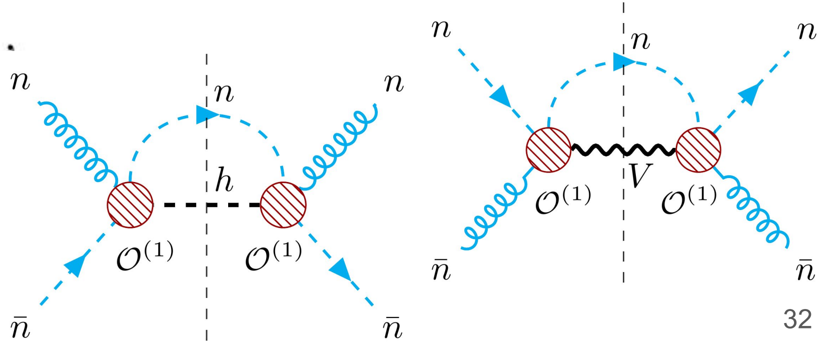
- For 0-jettiness, use consistency relations to relate full LL to RVV correction in collinear limit.
  [Moult, Rothen, Stewart, Tackmann, Zhu ‘16] [Moult, Stewart, **GV**, Zhu ‘19]

- Focus on Drell-Yan and Higgs production. Single collinear emission fully differential in rapidity:

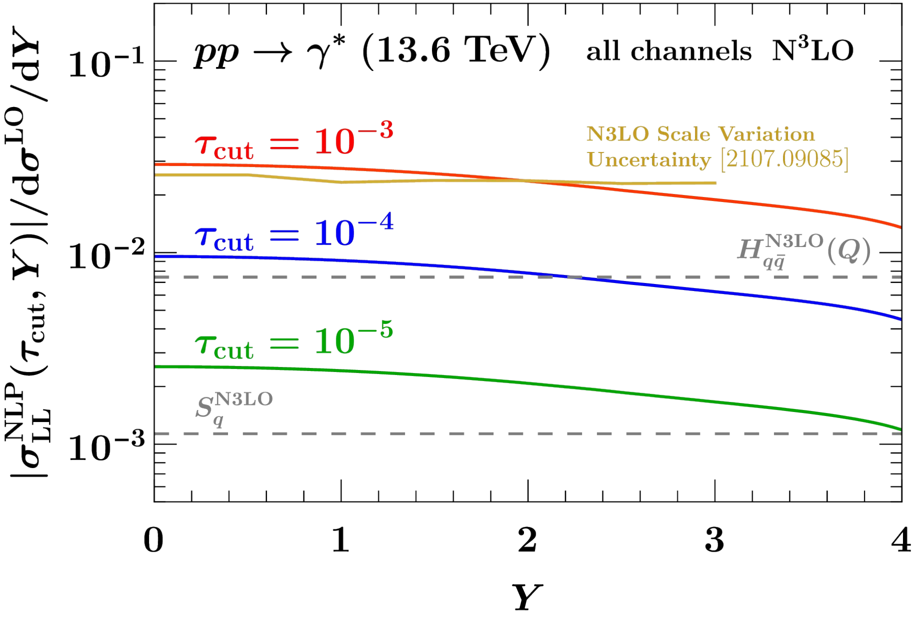
$$\frac{\mathrm{d}\sigma_n^{\rm NLP}}{\mathrm{d}Q^2\mathrm{d}Y\mathrm{d}\mathcal{T}} \sim \int_{x_a}^1 \frac{\mathrm{d}z_a}{z_a} \frac{(Q^2\tau)^{-\epsilon}}{(1-z_a)^{\epsilon}} \Bigg\{ \underbrace{\tau A^{(0)}(\tau, z_a, \epsilon)}_{\text{LP Matrix Element}} \underbrace{\left[-f_a\left(\frac{x_a}{z_a}\right) f_b(x_b) + f_a\left(\frac{x_a}{z_a}\right) x_b f_b'(x_b) \right]}_{\text{NLP Phase Space}} + \underbrace{f_a\left(\frac{x_a}{z_a}\right) f_b(x_b)}_{\text{LP Phase Space}} \underbrace{A^{(2)}(\tau, z_a, \epsilon)}_{\text{NLP Matrix Element}} \Bigg\} .$$

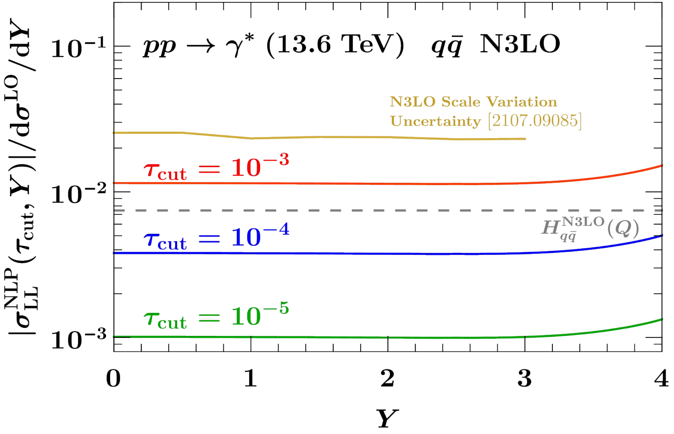
[Ebert, Moult, Stewart, Tackmann, **GV**, Zhu ‘18]

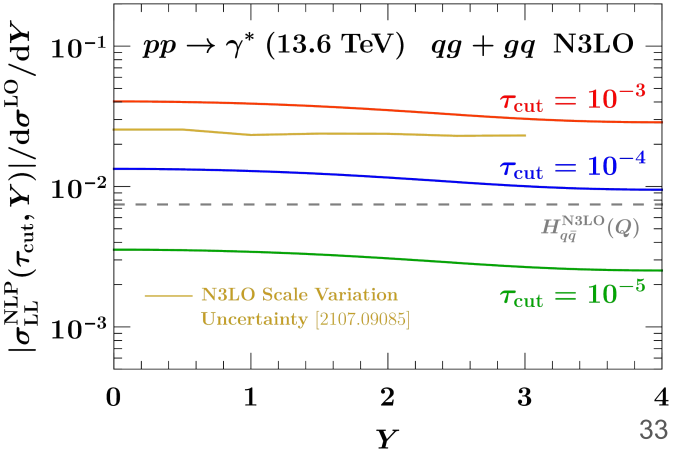
- LL contributions also from off-diagonal qg + gq channels via subleading power hard scattering operators and Lagrangian insertions



0-Jettiness Power Corrections at N3LO: Results for DY







- By the size of LL NLP: 0-jettiness with standard setup (only LP in subtraction term) would require $\tau_{cut} \sim 10^{-5}$ or even smaller.
- Off-diagonal channel has large power corrections (in line with empirical observation in q_T slicing at N3LO)

A word on linear vs quadratic power corrections

$$\text{0-jettiness}: \quad \Delta\sigma^{N3LO}(\tau_{\rm cut}) \sim \left(\frac{\alpha_s}{4\pi}\right)^3 \int_0^{\tau_{\rm cut}} d\tau \left(c_{3,5}^{\rm NLP} \ln^5\tau + c_{3,4}^{\rm NLP} \ln^4\tau + c_{3,3}^{\rm NLP} \ln^3\tau + \dots\right)$$

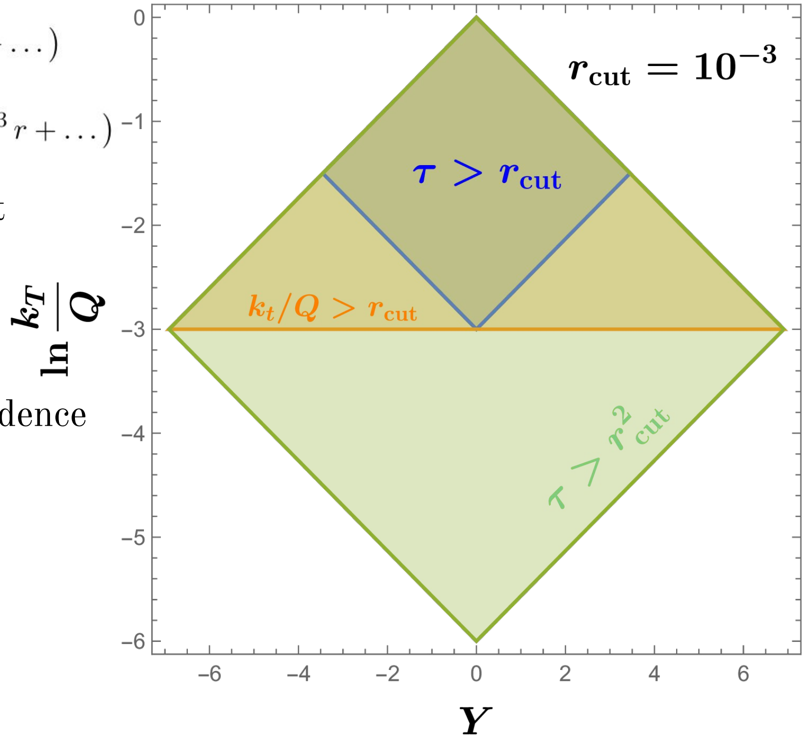
$$q_T: \quad \Delta\sigma^{N3LO}(q_{T\rm cut}) \sim \left(\frac{\alpha_s}{4\pi}\right)^3 \int_0^{q^2_{T\rm cut}/Q^2} dr \left(d_{3,5}^{\rm NLP} \ln^5 r + d_{3,4}^{\rm NLP} \ln^4 r + d_{3,3}^{\rm NLP} \ln^3 r + \dots\right)$$

- Scaling in q_T of the slicing param. may lead to the impression that q_T subtraction has *quadratic* power corrections, while jettiness has *linear* power corrections.

- But it all comes down to how one decides to treat the angle dependence

$$\tau = \frac{q_T}{Q} e^{-|Y|} \sim \begin{cases} \frac{q_T}{Q} & \text{soft emissions} \\ \frac{q_T^2}{Q^2} & \text{collinear emissions} \end{cases}$$

- In practice, key point is what is more challenging numerically for the above the cut code:
  - 0-jettiness: better suppression of collinear emissions
  - q_T: better suppression of wide angle soft emissions



Note: fiducial p.c. generating *linear* terms in q_T, go as $\sqrt{\tau_{\rm cut}}$ in the case of 0-jettiness

Ok, but what about fiducial power corrections?

Fiducial Power Corrections

- These are **purely kinematic effects**, but have very **large impact** on non-local subtractions due to non canonical scaling in the cut parameter.

- In short: **(Ebert, Tackmann) [1911.08486]**

$$\frac{\mathrm{d}\sigma^{(\mathrm{cuts})}(X)}{\mathrm{d}Q^2\mathrm{d}Y\,\mathrm{d}q_T^2} \sim \frac{1}{q_T^2}\frac{q_T}{Q}\,, \qquad \frac{\mathrm{d}\sigma^{(\mathrm{cuts})}(X)}{\mathrm{d}Q^2\mathrm{d}Y\,\mathrm{d}\mathcal{T}_0} \sim \frac{1}{\mathcal{T}_0}\sqrt{\frac{\mathcal{T}_0}{Q}}\,.$$

  - **Cuts on leptons** induce *linear* terms
    For q_T subtraction they can be captured analytically by a boost, but not for 0-jettiness.

  - **Photon Isolations** induce p.c. with wild and complicated scaling
    No simple boost trick to account for them.

$$\frac{\mathrm{d}\sigma^{(\mathrm{smooth})}(X)}{\mathrm{d}Q^2\mathrm{d}Y\,\mathrm{d}q_T^2} \sim \frac{R^2}{q_T^2}\left(\frac{q_T}{Q}\right)^{1/n}\left(\frac{Q}{E_T^{\mathrm{iso}}}\right)^{1/n}$$

$$\frac{\mathrm{d}\sigma^{(\mathrm{smooth})}(X)}{\mathrm{d}Q^2\mathrm{d}Y\,\mathrm{d}\mathcal{T}_0} \sim \begin{cases} \dfrac{R^2}{\mathcal{T}_0}\left(\dfrac{\mathcal{T}_0}{Q}\right)^{1+1/(2n)}\left(\dfrac{Q}{E_T^{\mathrm{iso}}}\right)^{1/n} \\[2ex] \dfrac{R^2}{\mathcal{T}_0}\left(\dfrac{\mathcal{T}_0}{Q}\right)^{1/n}\left(\dfrac{Q}{E_T^{\mathrm{iso}}}\right)^{1/n} \end{cases}$$

- So, although fiducial power corrections are more trivial conceptually, account for them comes first numerically compared to dynamical power corrections.

Projection to Born Improved Slicing

[Cacciari et al. '15]
[Ebert, Tackmann '19]
[**GV** '24]
[Campbell, Neumann, **GV** '24]

Cut-induced power corrections can be numerically accounted for by using
"Projection-to-Born Improved Slicing"

$$\sigma_{h,\,\mathrm{N^3LO}}(\mathcal{O}) = \sigma_{h,\,\mathrm{N^3LO}}(\tilde{\mathcal{O}}) + \sigma_{h+j,\,\mathrm{NNLO}}(\mathcal{O}-\tilde{\mathcal{O}})$$

P2B correction factor

Slicing calculation for Born projected observable

$$= \int_0^{\tau_{\mathrm{cut}}} \mathrm{d}\tau \frac{\mathrm{d}\sigma^{\mathrm{sub}}_{h,\,\mathrm{N^3LO}}}{\mathrm{d}\tau}(\tilde{\mathcal{O}}) + \int_{\tau>\tau_{\mathrm{cut}}} \mathrm{d}\sigma^{\mathrm{full}}_{h+j,\,\mathrm{NNLO}}(\tilde{\mathcal{O}})$$

Below the cut term

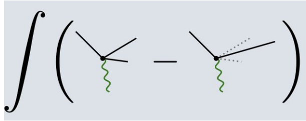
Above the cut term

$$+ \int_0^{\tau_{\mathrm{cut}}} \mathrm{d}\tau \left[\frac{\mathrm{d}\sigma^{\mathrm{full}}_{h,\,\mathrm{N^3LO}}}{\mathrm{d}\tau} - \frac{\mathrm{d}\sigma^{\mathrm{sub}}_{h,\,\mathrm{N^3LO}}}{\mathrm{d}\tau}\right](\tilde{\mathcal{O}})$$

Residual Error

$$+ \int \mathrm{d}\sigma^{\mathrm{full}}_{h+j,\,\mathrm{NNLO}}(\mathcal{O}-\tilde{\mathcal{O}})$$

P2B correction factor



Note:
Because of local cancellation using exact matrix elements, P2B is very efficient numerically.
Sometimes referred as the "*perfect*" subtraction scheme

Projection to Born Improved Slicing

[Cacciari et al. '15]
[Ebert, Tackmann '19]
[**GV** '24]
[Campbell, Neumann, **GV** '24]

Equivalently, perform standard slicing and correct with P2B only below the cut

$$\sigma_{h,\,\mathrm{N^nLO}}(\mathcal{O}) = \int_0^{x_\mathrm{cut}} \mathrm{d}x \frac{\mathrm{d}\sigma^\mathrm{sub}_{h,\,\mathrm{N^nLO}}}{\mathrm{d}x}(\mathcal{O}) + \int_{x>x_\mathrm{cut}} \mathrm{d}\sigma^\mathrm{full}_{h+j,\,\mathrm{N^{n-1}LO}}(\mathcal{O})$$

Below the cut term

Above the cut term

$$+ \int_0^{x_\mathrm{cut}} \mathrm{d}x \left[\frac{\mathrm{d}\sigma^\mathrm{full}_{h,\,\mathrm{N^nLO}}}{\mathrm{d}x} - \frac{\mathrm{d}\sigma^\mathrm{sub}_{h,\,\mathrm{N^nLO}}}{\mathrm{d}x} \right] (\tilde{\mathcal{O}})$$

Residual Error

$$+ \int_0^{x_\mathrm{cut}} \mathrm{d}\sigma^\mathrm{full}_{h+j,\,\mathrm{N^{n-1}LO}}(\mathcal{O} - \tilde{\mathcal{O}}) \,.$$

P2B correction factor below the cut

Projection to Born Improved Slicing

[Cacciari et al. '15]
[Ebert, Tackmann '19]
[GV '24]

C… using

This enable us to

- Focus on *analytic* calculation of *dynamical* power corrections
- *Numerically* treat *fiducial* power corrections efficiently with P2B method

…ction factor

Slicing calc…
Born pr…
observ…

…o$(\tilde{\mathcal{O}})$

…t term

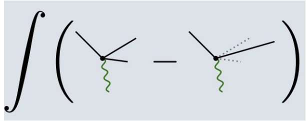
Note:
Because of local cancellation using exact matrix elements, P2B is very efficient numerically.
Sometimes referred as the "*perfect*" subtraction scheme

$$+\int_0^{\tau_{\rm cut}} \mathrm{d}\tau \left[\frac{\mathrm{d}\sigma^{\rm full}_{h,\,\mathrm{N^3LO}}}{\mathrm{d}\tau} - \frac{\mathrm{d}\sigma^{\rm sub}_{h,\,\mathrm{N^3LO}}}{\mathrm{d}\tau}\right](\tilde{\mathcal{O}})$$

Residual Error

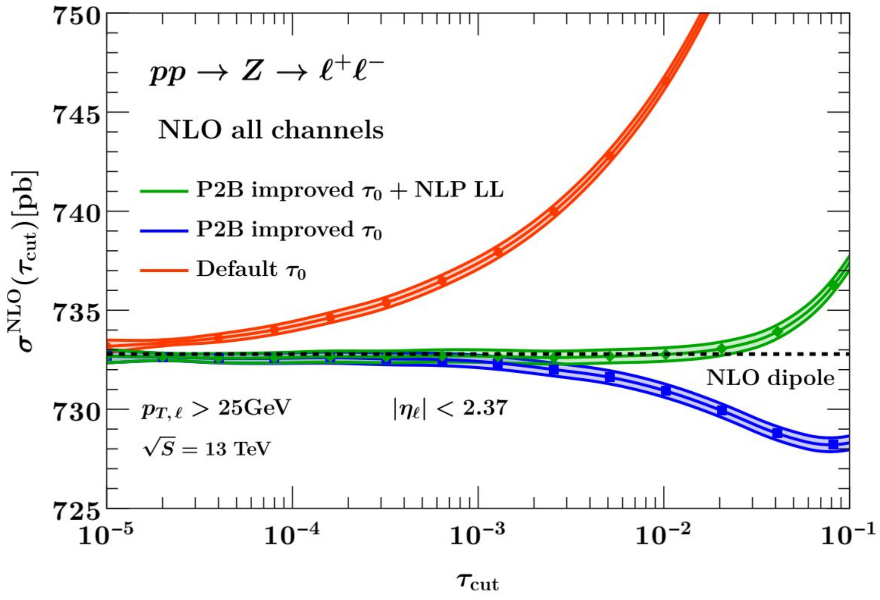
$$+\int \mathrm{d}\sigma^{\rm full}_{h+j,\,\mathrm{NNLO}}(\mathcal{O}-\tilde{\mathcal{O}})$$

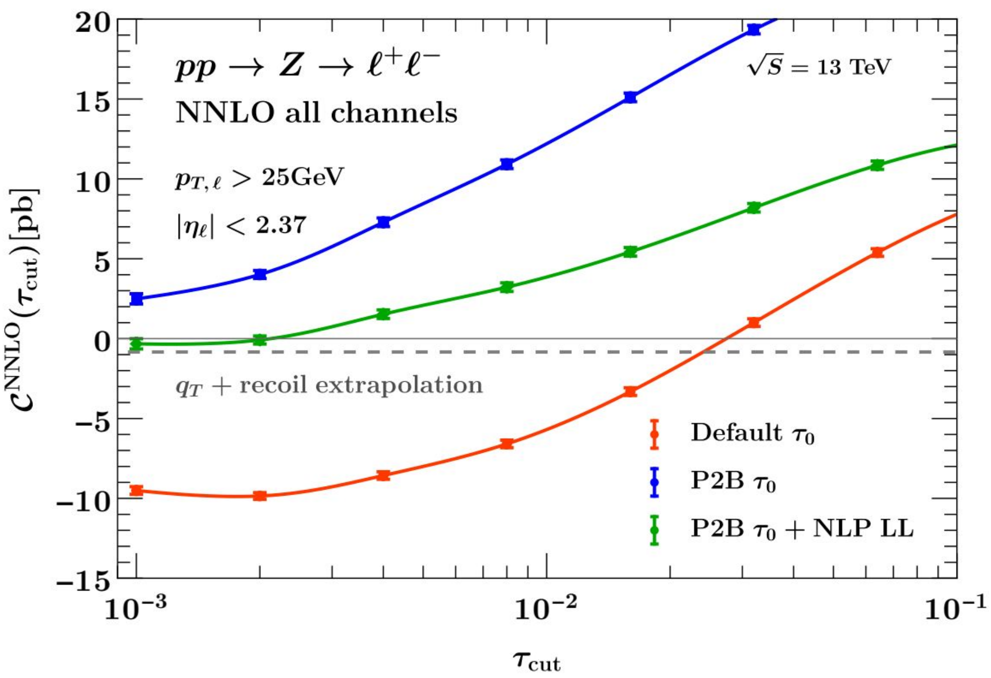
P2B correction factor



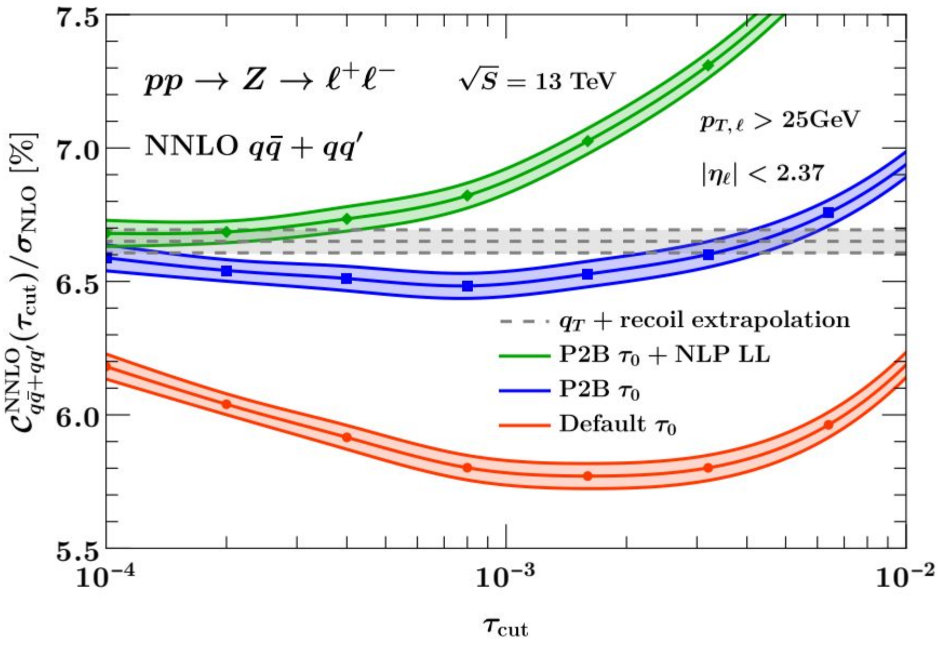
Projection-to-Born-improved Subtractions at NNLO

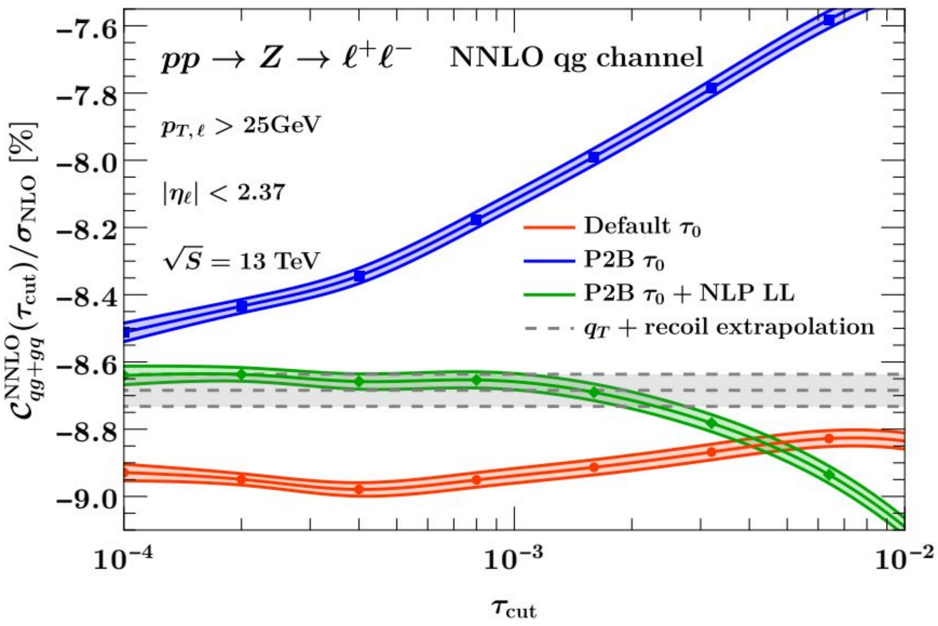
We studied this at NNLO in MCFM in 2408.05265





Projection-to-Born-improved Subtractions at NNLO

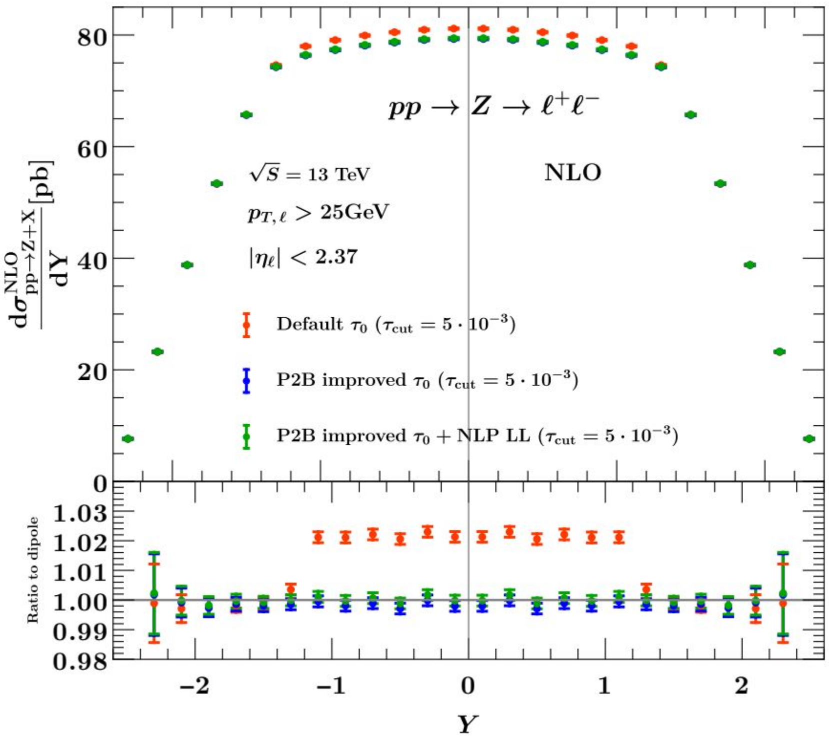


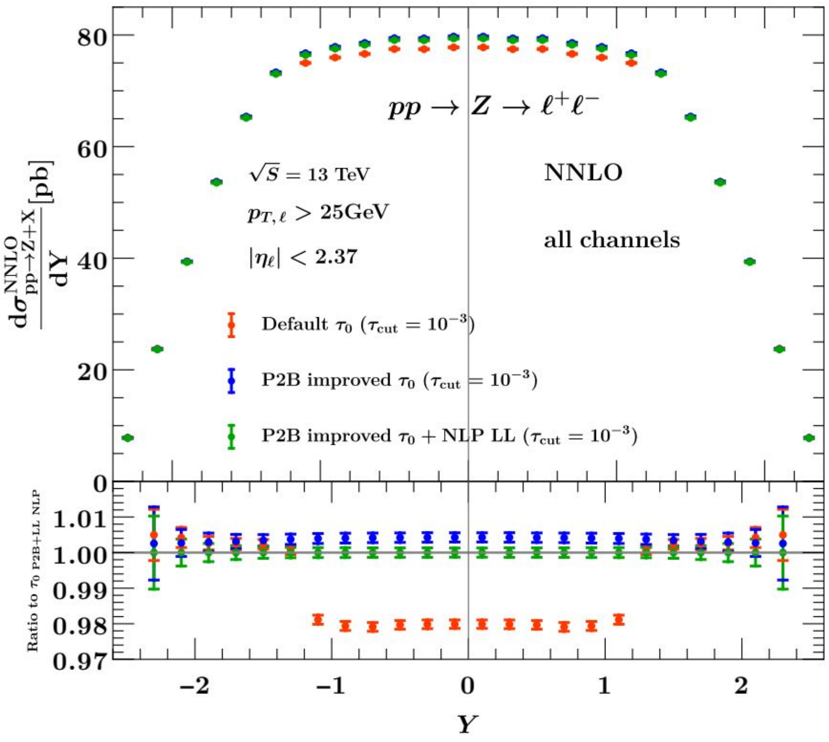


Extrapolation from q_T with recoil in MCFM taken as reference (excellent agreement with result from MATRIX based on same method)

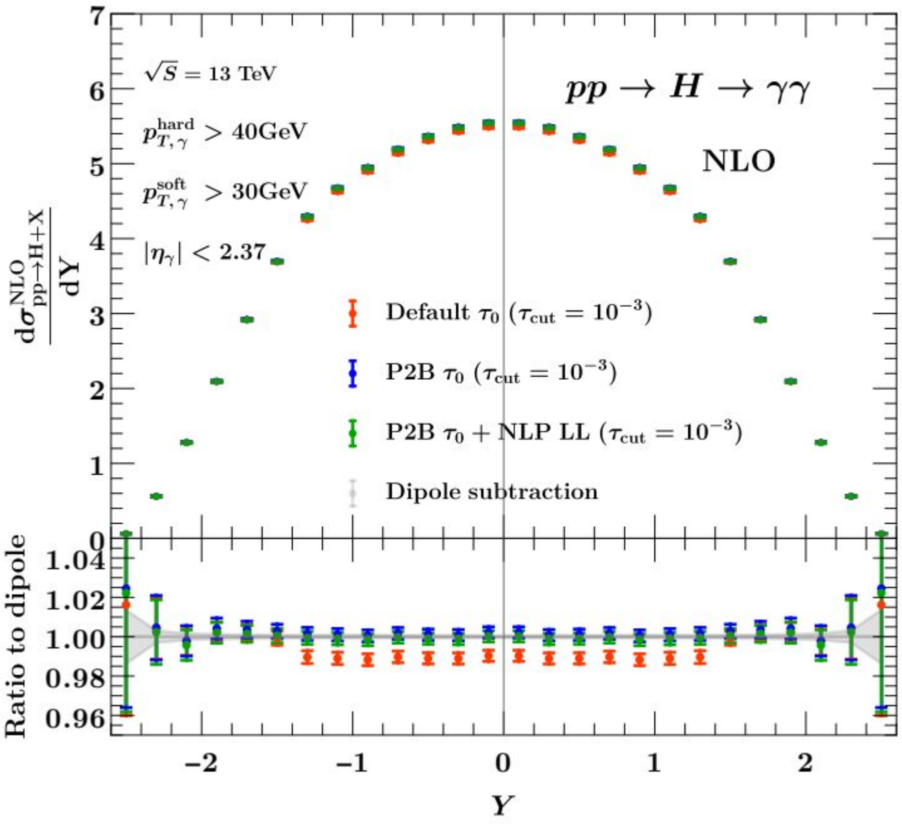
$pp \to Z \to \ell^+\ell^-$ NNLO coefficient	$q\bar{q} + qq'$	qg	gg
MCFM q_T + recoil	$48\,732 \pm 316$ fb	$-31\,819 \pm 175$ fb	$13\,870 \pm 25$ fb
MATRIX q_T + recoil	$48\,695 \pm 364$ fb	$-31\,798 \pm 131$ fb	$13\,786 \pm 205$ fb
Relative Difference	0.08 ± 0.99 %	0.07 ± 0.69 %	0.61 ± 1.53 %

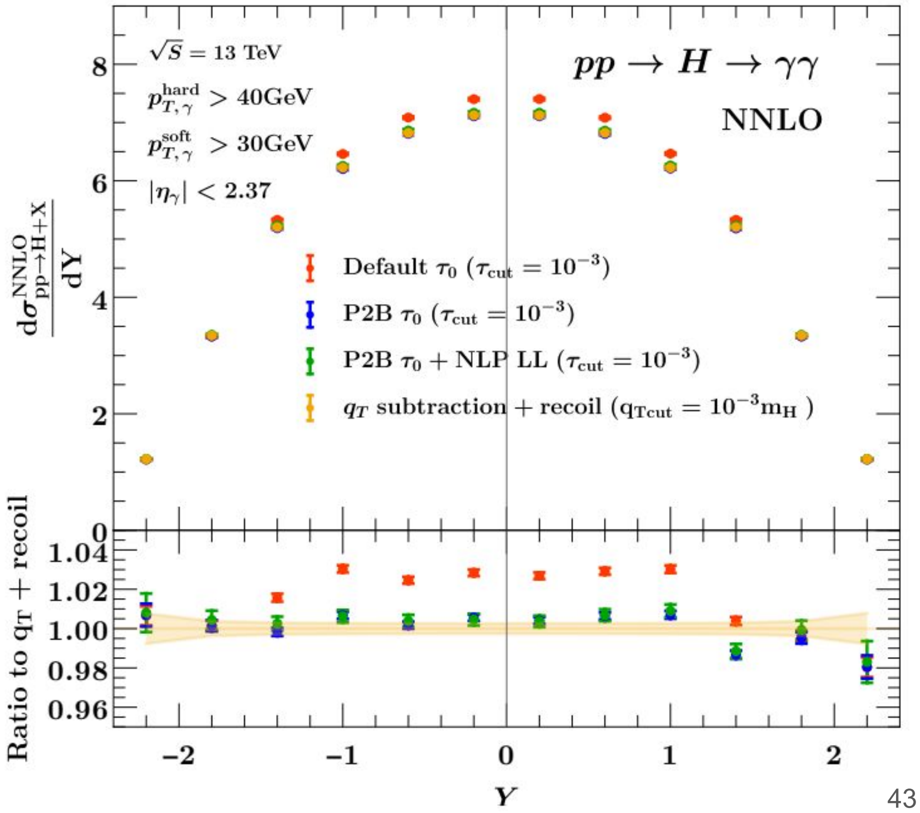
Projection-to-Born-improved Subtractions at NNLO





Projection-to-Born-improved Subtractions at NNLO



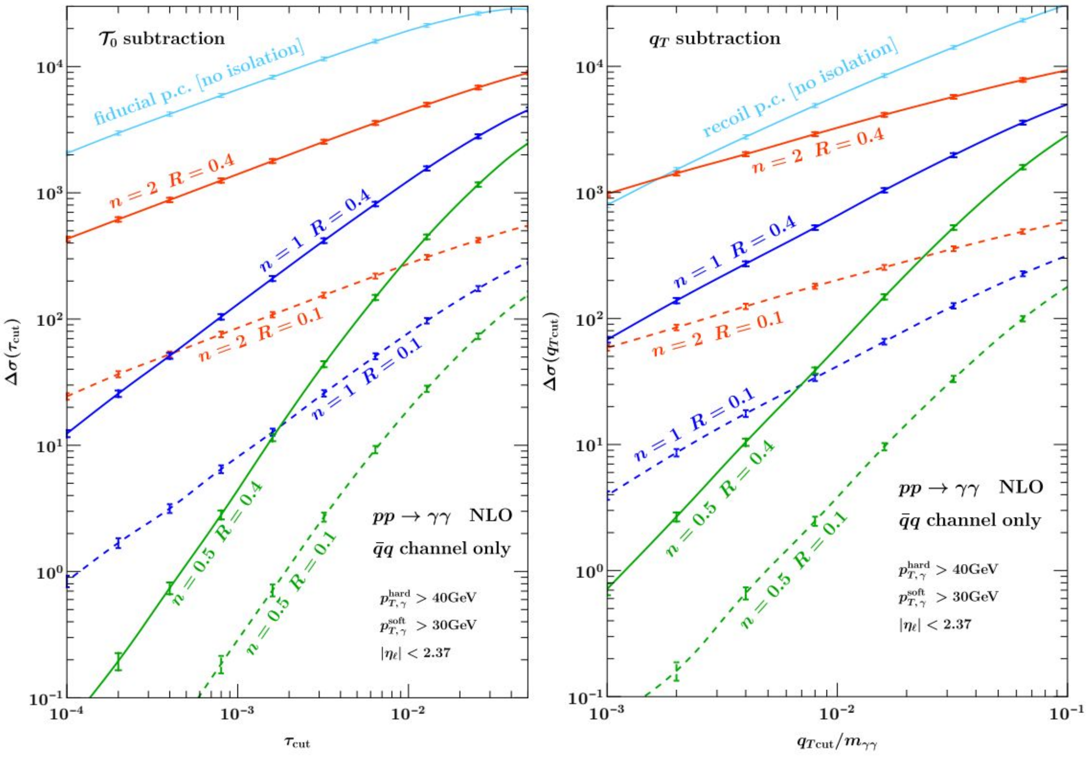


Di-photon

- Di-photon is notoriously challenging
- Isolation prescription required to avoid QED singularity when quarks are in the final state

$$\sum_{d(i,\gamma)\leq r} E_T^i \leq E_T^{\text{iso}} \left[\frac{1-\cos(r)}{1-\cos(R)}\right]^n , \qquad \forall r \leq R$$

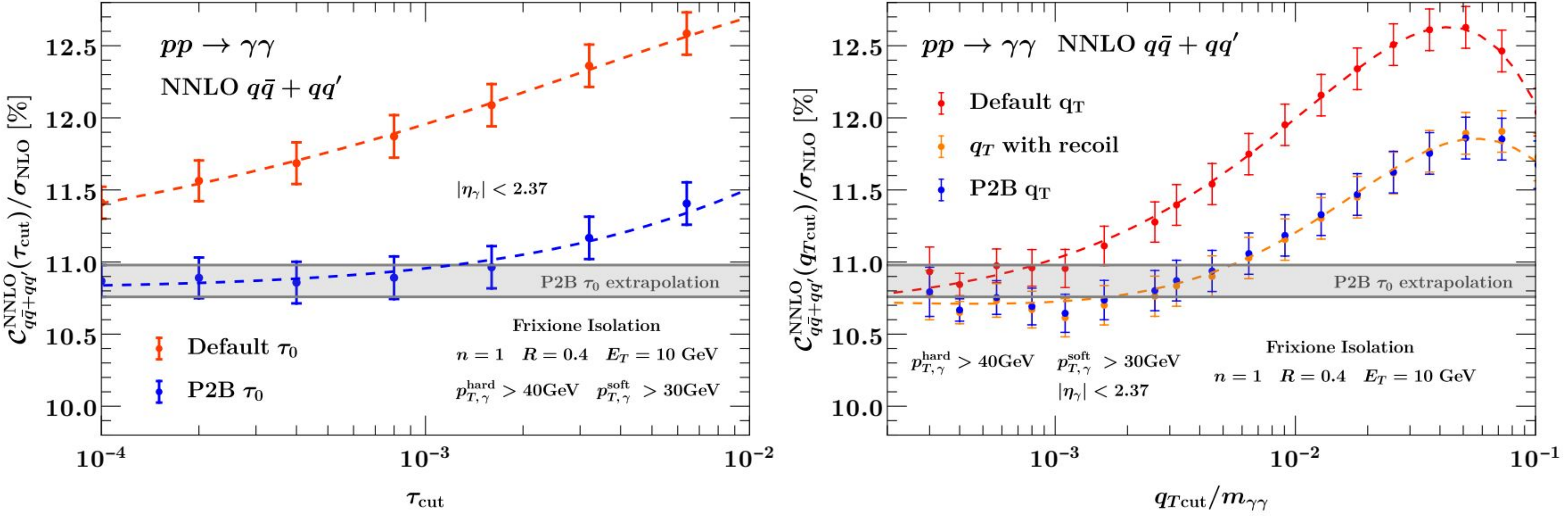
- Mix of isolation and kinematical cuts and large QCD corrections
- Large impact of the isolation parameters on the power corrections



$$\Delta\sigma^{\text{fiducial+hadronic}}_{p_T^\gamma+\text{iso},n=1} \sim \begin{cases} \sqrt{\frac{\mathcal{T}_{0\,\text{cut}}}{Q}} + \frac{\mathcal{T}_{0\,\text{cut}}}{Q} R^2 \frac{Q}{E_T^{\text{iso}}} + \frac{\mathcal{T}_{0\,\text{cut}}}{Q}\log(R) + \frac{\mathcal{T}_{0\,\text{cut}}}{Q}, & \text{0-jettiness subtraction} \\ \frac{q_{T\text{cut}}}{Q} + \frac{q_{T\text{cut}}}{Q} R^2 \frac{Q}{E_T^{\text{iso}}} + \frac{q_{T\text{cut}}}{Q}\log(R) + \frac{q_{T\text{cut}}^2}{Q^2}, & q_T\text{-subtraction} \end{cases}$$

Di-photon at NNLO: quark - anti quark channels

- For this channel, the P2B and recoil capture the fiducial and isolation power corrections



Di-photon at NNLO: fragmentation channel

- For fragmentation channel, the fiducial corrections due to the cuts on the of the photons are small, but the **isolation corrections are very large**

$$\sum_{d(i,\gamma)\le r} E_T^i \le E_T^{\mathrm{iso}} \left[\frac{1-\cos(r)}{1-\cos(R)}\right]^n , \qquad \forall r \le R$$

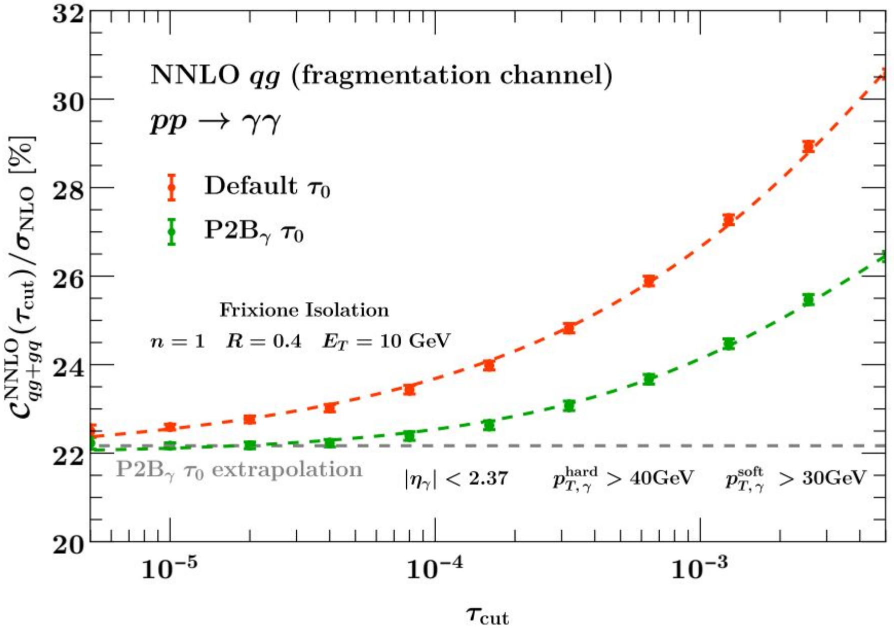
- To account for isolation p.c. we modified the P2B improved slicing by switching off the P2B counterterm. I'll refer to this prescription as $\mathbf{P2B}_{\gamma}$

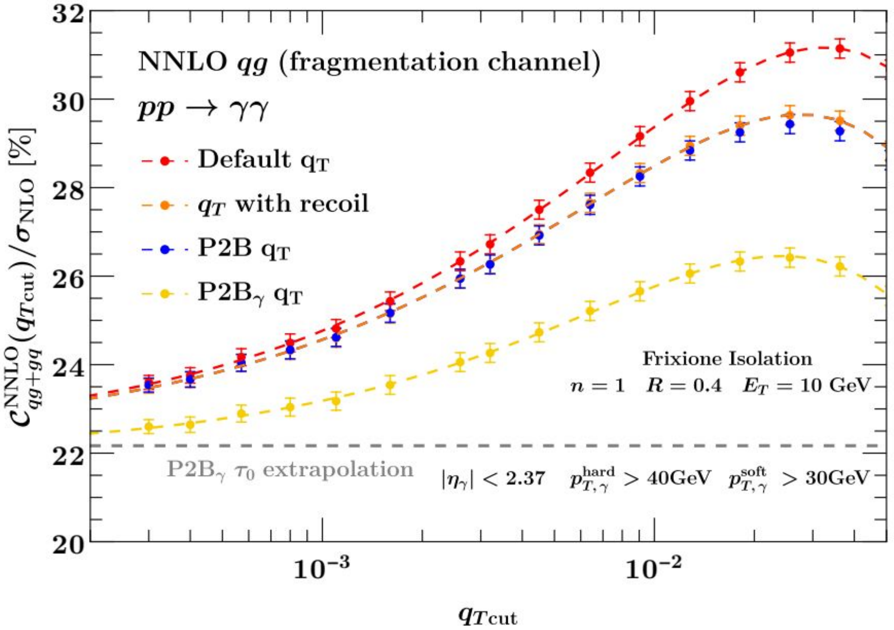
$$\int_0^{x_{\mathrm{cut}}} \mathrm{d}\sigma^{\mathrm{full}}_{h+j,\,\mathrm{N^{n-1}LO}}(\mathcal{O}-\tilde{\mathcal{O}}) \longrightarrow \int \mathrm{d}\sigma^{\mathrm{full}}_{h+j,\,\mathrm{N^{n-1}LO}}(\mathcal{O}-\tilde{\mathcal{O}}\theta(r>R))$$

- The point is that for realistic values of the isolation parameters the quark inside the cone of radius R must be soft so one can directly calculate this contribution numerically without a counterterm.

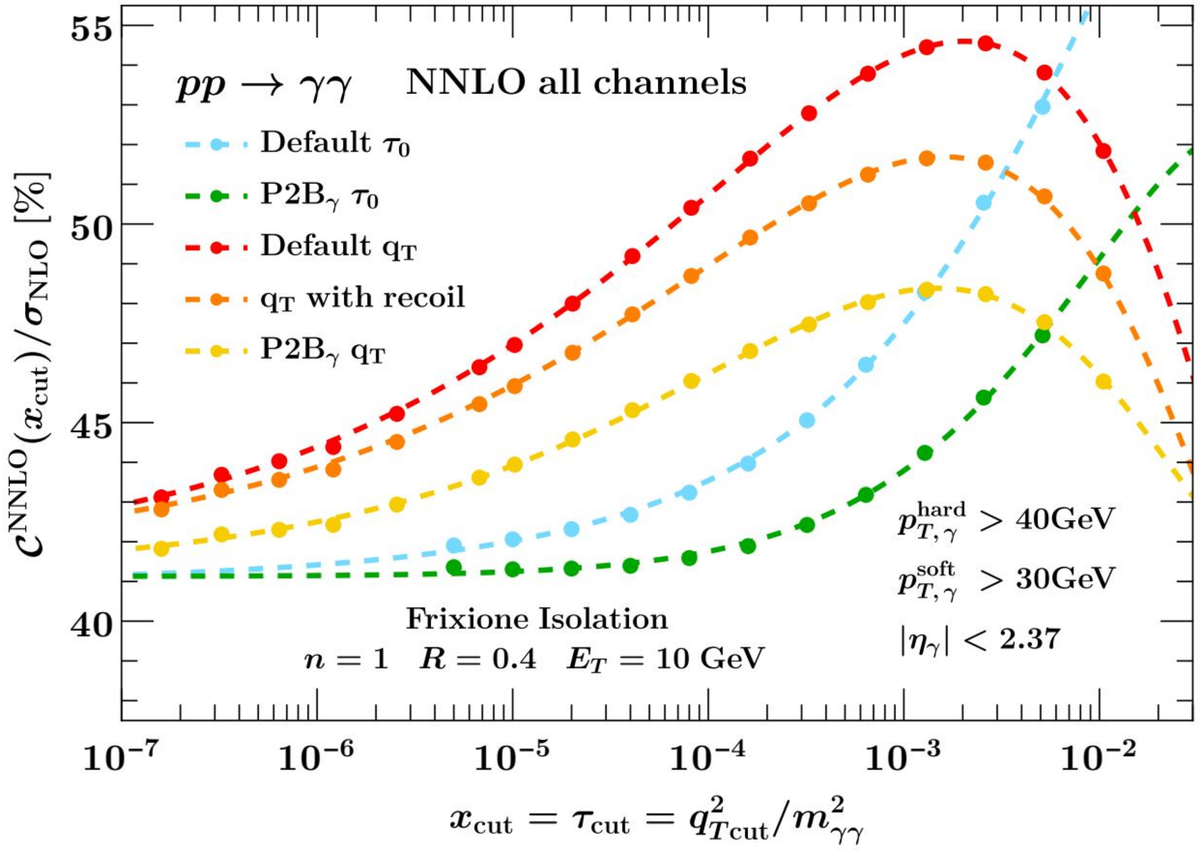
Di-photon at NNLO: fragmentation channel

- For fragmentation channel, the P2B and recoil corrections are small
- Numerically, it seems that **P2B$_\gamma$** is capturing an important amount of the effect both for 0-jettiness as well as for q_T subtraction.

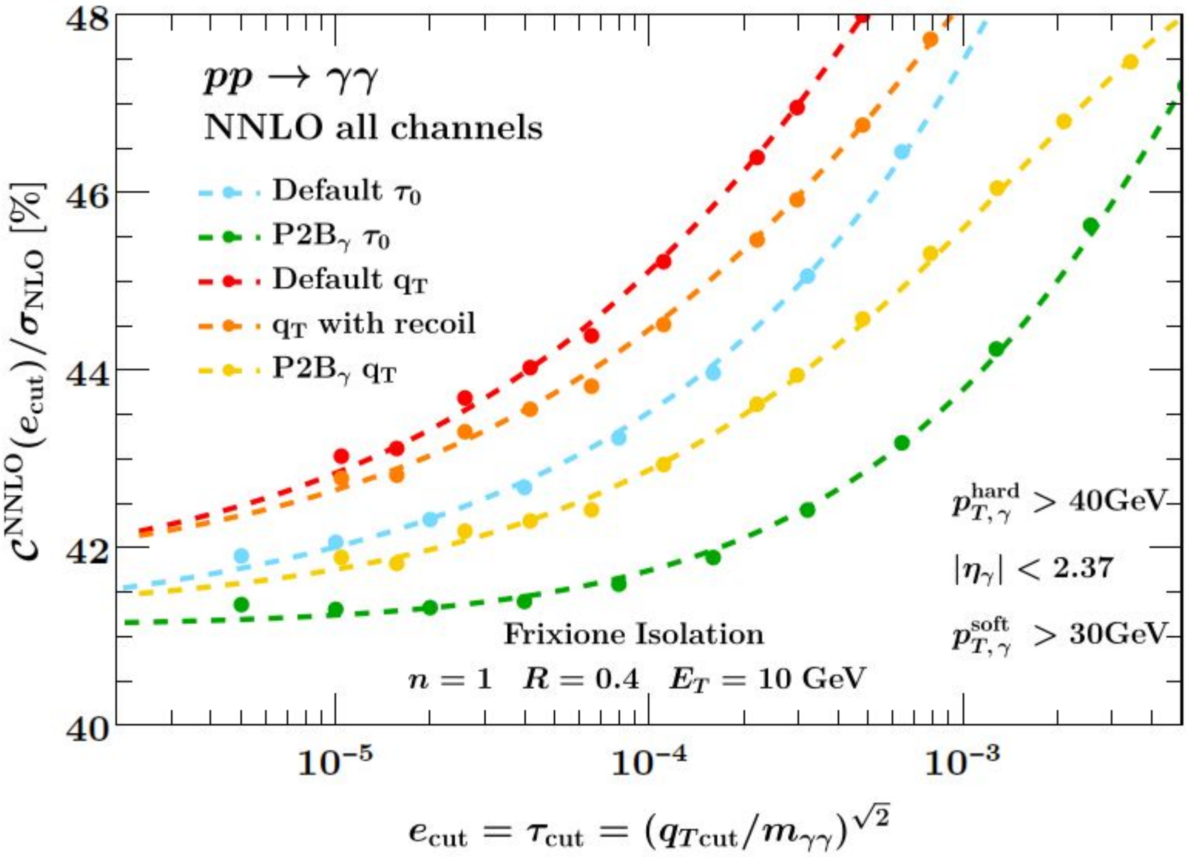




Di-photon at NNLO: all channels



Di-photon at NNLO: all channels

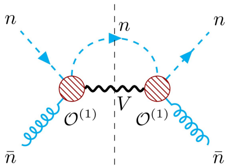


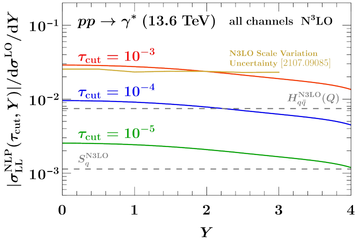
Conclusion

- ➢ Discussed challenges of N3LO calculations and slicing methods

$$\sigma(X) = \underbrace{\int_0^{q_{T\text{cut}}} \mathrm{d}q_T \frac{\mathrm{d}\sigma^{\text{sing}}(X)}{\mathrm{d}q_T}}_{\text{Below the cut region}} + \underbrace{\int_{q_{T\text{cut}}} \mathrm{d}q_T \frac{\mathrm{d}\sigma(X)}{\mathrm{d}q_T}}_{\text{Above the cut region}} + \underbrace{\Delta\sigma(X, q_{T\text{cut}})}_{\text{Residual}}$$

- ➢ Illustrated impact of the LL power corrections at N3LO for 0-jettiness for Drell-Yan and Higgs production





- ➢ Used P2B improved slicing to account for fiducial power corrections

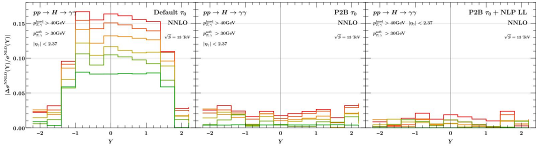
$$\sigma_{h,\text{N}^3\text{LO}}(\mathcal{O}) = \sigma_{h,\text{N}^3\text{LO}}(\tilde{\mathcal{O}}) + \sigma_{h+j,\text{NNLO}}(\mathcal{O} - \tilde{\mathcal{O}}) \quad \text{P2B correction factor}$$

Slicing calculation for Born projected observable

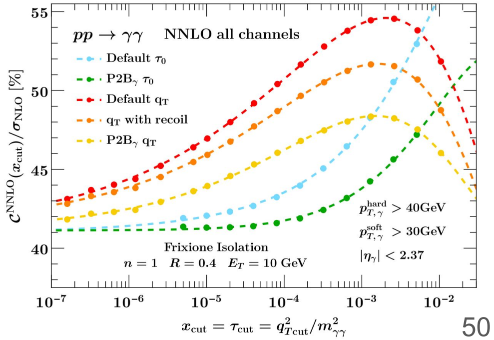
$$= \underbrace{\int_0^{\tau_{\text{cut}}} \mathrm{d}\tau \frac{\mathrm{d}\sigma^{\text{sub}}_{h,\text{N}^3\text{LO}}}{\mathrm{d}\tau}(\tilde{\mathcal{O}})}_{\text{Below the cut term}} + \underbrace{\int_{\tau>\tau_{\text{cut}}} \mathrm{d}\sigma^{\text{full}}_{h+j,\text{NNLO}}(\tilde{\mathcal{O}})}_{\text{Above the cut term}}$$

$$+ \int_0^{\tau_{\text{cut}}} \mathrm{d}\tau \left[\frac{\mathrm{d}\sigma^{\text{full}}_{h,\text{N}^3\text{LO}}}{\mathrm{d}\tau} - \frac{\mathrm{d}\sigma^{\text{sub}}_{h,\text{N}^3\text{LO}}}{\mathrm{d}\tau}\right](\tilde{\mathcal{O}}) \quad \text{Residual Error}$$

$$+ \int \mathrm{d}\sigma^{\text{full}}_{h+j,\text{NNLO}}(\mathcal{O} - \tilde{\mathcal{O}}) \quad \text{P2B correction factor}$$



- ➢ Presented prescription for isolation corrections in Diphoton production due to soft quark emissions



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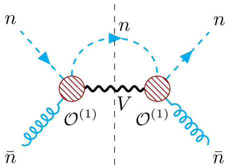

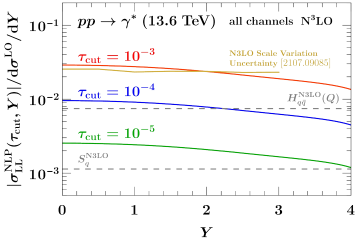

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Slicing calculation for Born projected observable; P2B correction factor

$$= \underbrace{\int_0^{\tau_{\text{cut}}} \mathrm{d}\tau \frac{\mathrm{d}\sigma^{\text{sub}}_{h,\,\text{N}^3\text{LO}}}{\mathrm{d}\tau}(\tilde{\mathcal{O}})}_{\text{Below the cut term}} + \underbrace{\int_{\tau > \tau_{\text{cut}}} \mathrm{d}\sigma^{\text{full}}_{h+j,\,\text{NNLO}}(\tilde{\mathcal{O}})}_{\text{Above the cut term}}$$

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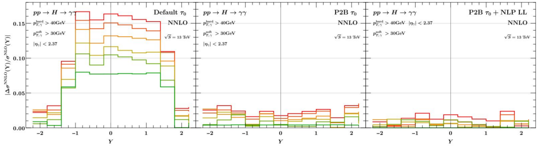

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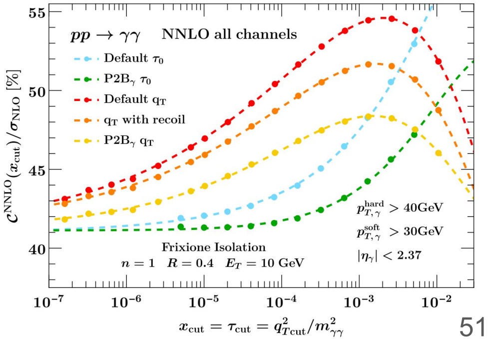


Thank you

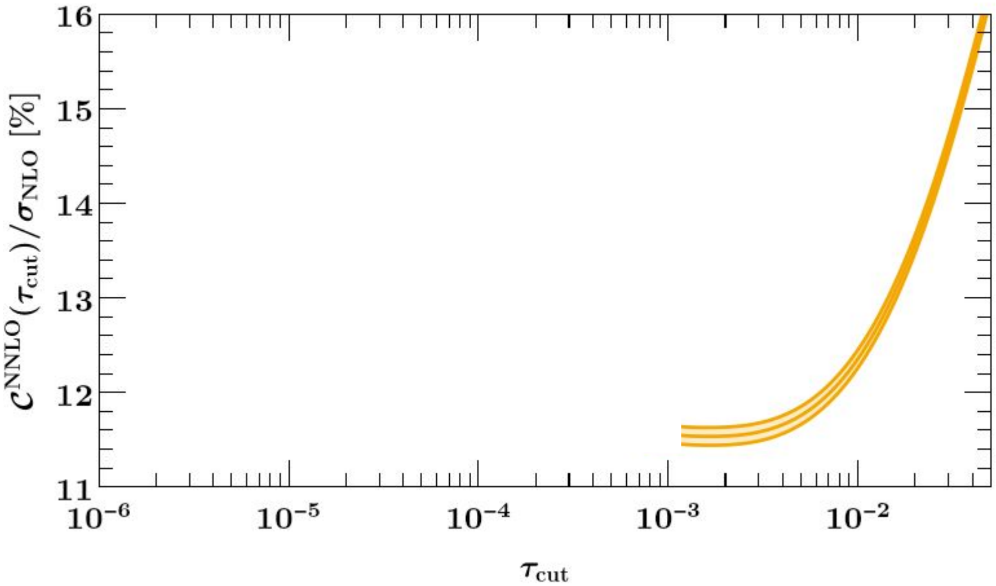
Backup

Slicing methods

$$\sigma_q(\mathcal{O}) = \int_0^{x_{\rm cut}} \mathrm{d}x \frac{\mathrm{d}\sigma_q^{\rm sub}}{\mathrm{d}x}(\mathcal{O}) + \int_{x_{\rm cut}}^{x_{\rm max}} \mathrm{d}x \frac{\mathrm{d}\sigma_{q+\rm jet}}{\mathrm{d}x}(\mathcal{O}) + \int_0^{x_{\rm cut}} \mathrm{d}x \left[\frac{\mathrm{d}\sigma_q}{\mathrm{d}x}(\mathcal{O}) - \frac{\mathrm{d}\sigma_q^{\rm sub}}{\mathrm{d}x}(\mathcal{O})\right]$$

$$\equiv \underbrace{\sigma^{\rm sub}(x_{\rm cut}, \mathcal{O}) + \sigma^{\rm above}(x_{\rm cut}, \mathcal{O})}_{\sigma(x_{\rm cut}, \mathcal{O})} + \Delta\sigma(x_{\rm cut}, \mathcal{O})$$

- Danger is fake plateau
- This is caused by a node in power corrections
- Resolved by lowering cut or calculating power corrections



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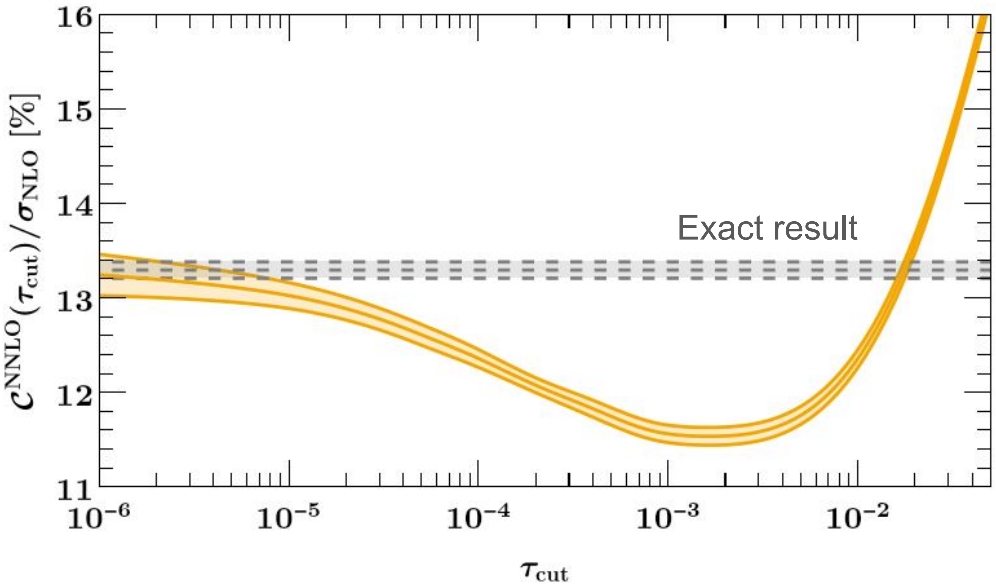
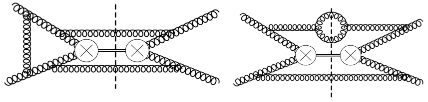
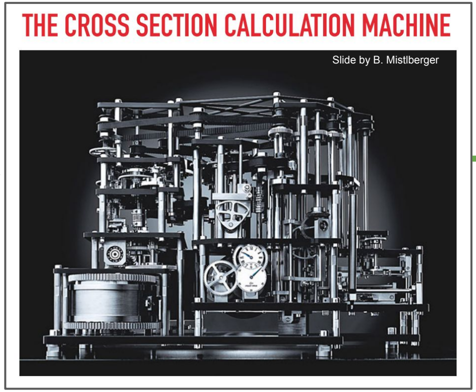


Beam Functions at N3LO

1 million 3-loop Feynman Diagrams





Collinear expansion of the partonic cross section for Drell Yan and Higgs at N3LO **differential in (Q_T, τ, z)**

project to $\boldsymbol{\tau}$

$$B_a(t_a, x_1^B, \mu)$$

project to $\mathbf{q_T}$

$$\tilde{B}_i\left(x_1^B, b_T, \mu, \frac{\nu}{\omega_a}\right)$$

"N-Jettiness Beam Functions at N3LO"

M.Ebert, B.Mistlberger, **GV** [2006.03056]

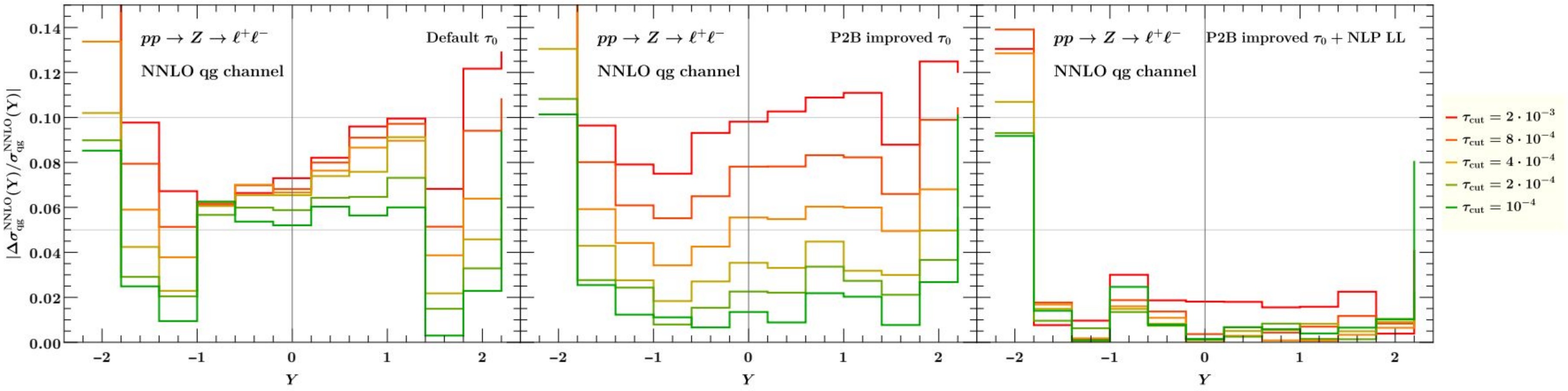
- Quark $\boldsymbol{\tau}$ **Beam Functions** (i.e. Quark N-Jettiness BF)
- Gluon $\boldsymbol{\tau}$ **Beam Functions** (i.e. Gluon N-Jettiness BF)

"Transverse Momentum Dependent PDFs at N3LO"

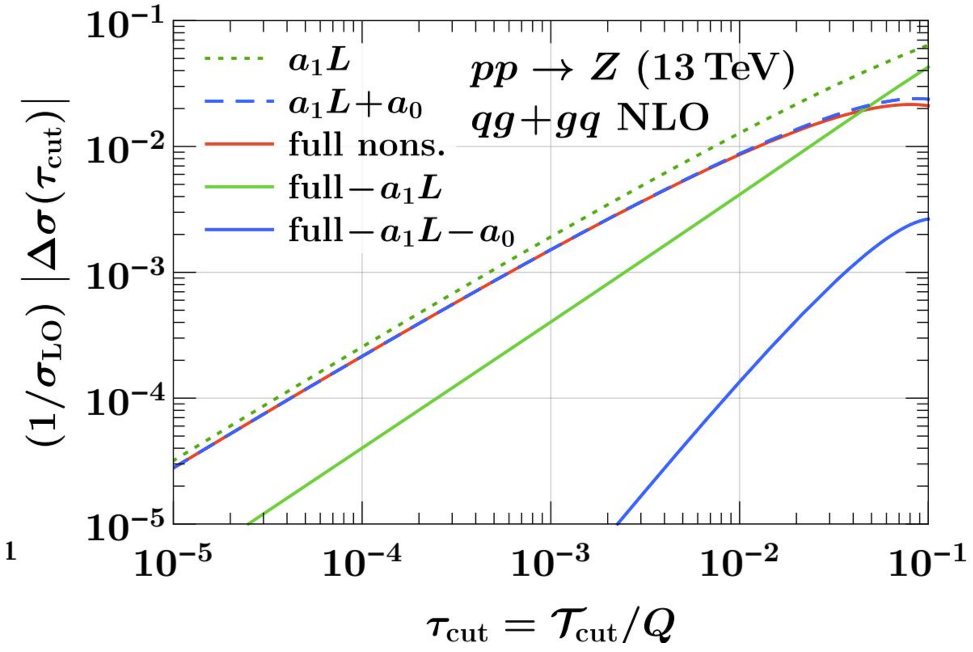
M.Ebert, B.Mistlberger, **GV** [2006.05329]

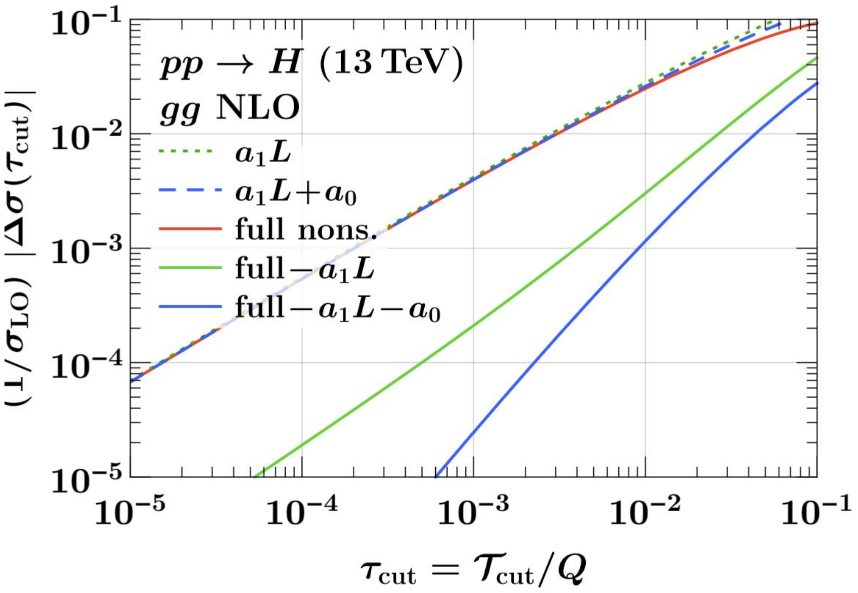
- Quark **TMDPDF** (Quark q_T Beam Function)
- Unpolarized **Gluon TMDPDF** (Gluon q_T Beam Function)

Projection-to-Born-improved Subtractions at NNLO



Log behaviour at NLP NLO

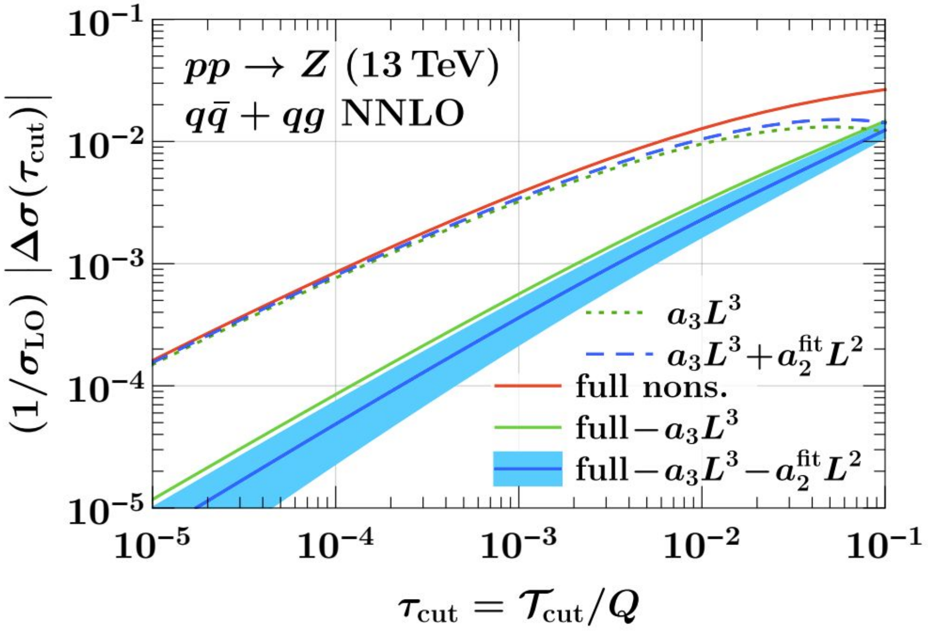




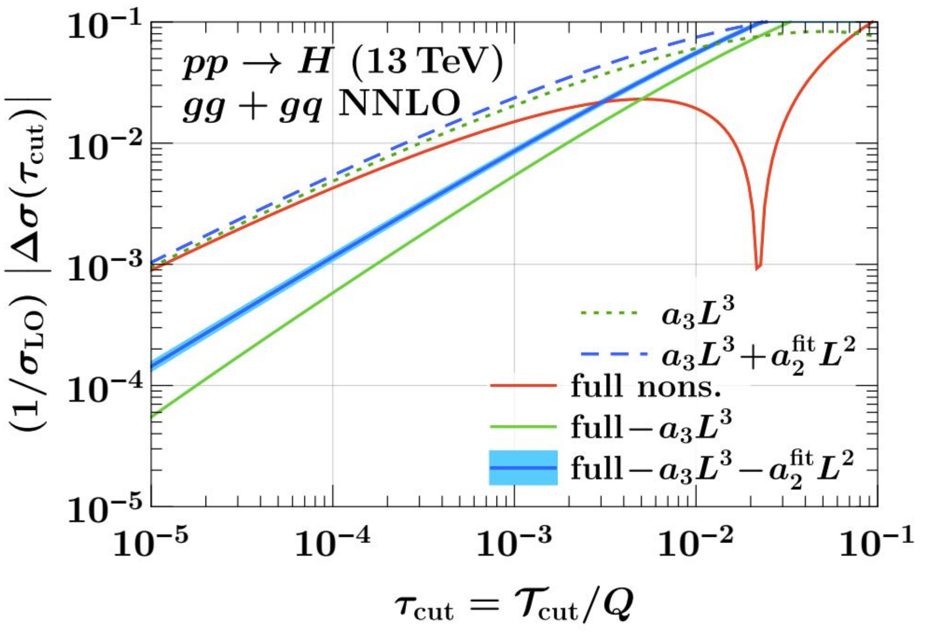
[1807.10764]

Log behaviour at NLP NNLO

[1612.00450]



[1710.03227]



Differential color singlet production at N3LO

- Two methods for differential N3LO predictions for color singlet:

Projection to Born

[Cacciari, Dreyer, Karlberg, Salam, Zanderighi '15]

$$\frac{d\sigma_F^{\mathrm{N}^k\mathrm{LO}}}{d\mathcal{O}} = \left(\frac{d\sigma_{F+\mathrm{jet}}^{\mathrm{N}^{(k-1)}\mathrm{LO}}}{d\mathcal{O}} - \frac{d\sigma_{F+\mathrm{jet}}^{\mathrm{N}^{(k-1)}\mathrm{LO}}}{d\tilde{\mathcal{O}}}\right) + \frac{d\sigma_F^{\mathrm{N}^k\mathrm{LO}}}{d\tilde{\mathcal{O}}}$$

Locally subtracted real emissions — Integrated counterterm

- **PRO:** Local counterterm is the full Matrix Element => Great numerical efficiency
- **Cons:** Integrated counterterm is very hard to obtain (analytic differential distribution at N3LO in full kinematics)

q_T or **0-jettiness** subtraction

q_T Subtraction: [Catani, Grazzini '07]

N-Jettiness: [Boughezal, Focke, Liu, Petriello '15] [Gaunt, Stahlhofen, Tackmann, Walsh '15]

$$\sigma(X) = \int_0^{q_{T\,\mathrm{cut}}} dq_T \frac{d\sigma^{\mathrm{sing}}(X)}{dq_T} + \int_{q_{T\,\mathrm{cut}}} dq_T \frac{d\sigma(X)}{dq_T} + \Delta\sigma(X, q_{T\,\mathrm{cut}})$$

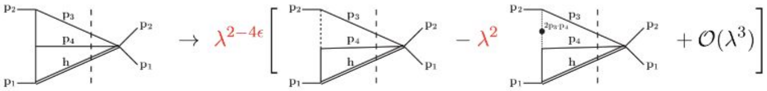
Below the cut region — Above the cut region — Residual

- **PRO:** Analytic control of IR divergences from EFT factorization thm. at Leading Power
- **Cons:** numerically challenging

Beam Functions calculation at N3LO

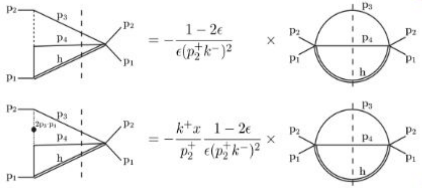
(Ebert, Mistlberger, **GV**) [2006.05329], [2006.03056]

- Calculation of the **collinear expansion of the partonic cross section** for DY and Higgs @N3LO **differential** in **(Q_T, τ, z)**
  - ~ 100k Feynman diagrams
  - Reverse unitarity for phase space integrals
  - Collinear Expansion at the XS level
    "Collinear expansion for color singlet cross sections" [Ebert, Mistlberger, **GV**]

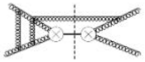


  - Reduction to basis of **Master Integrals** via Integration By Parts (IBPs) using Water

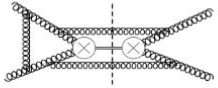
Expanded diagrams admit (simplified) **IBPs** identities



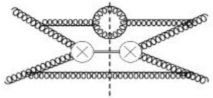
  - RVV: known in full kinematics
    [Duhr, Gehrmann] [Duhr, Gehrmann, Jaquier] [Dulat, Mistlberger]



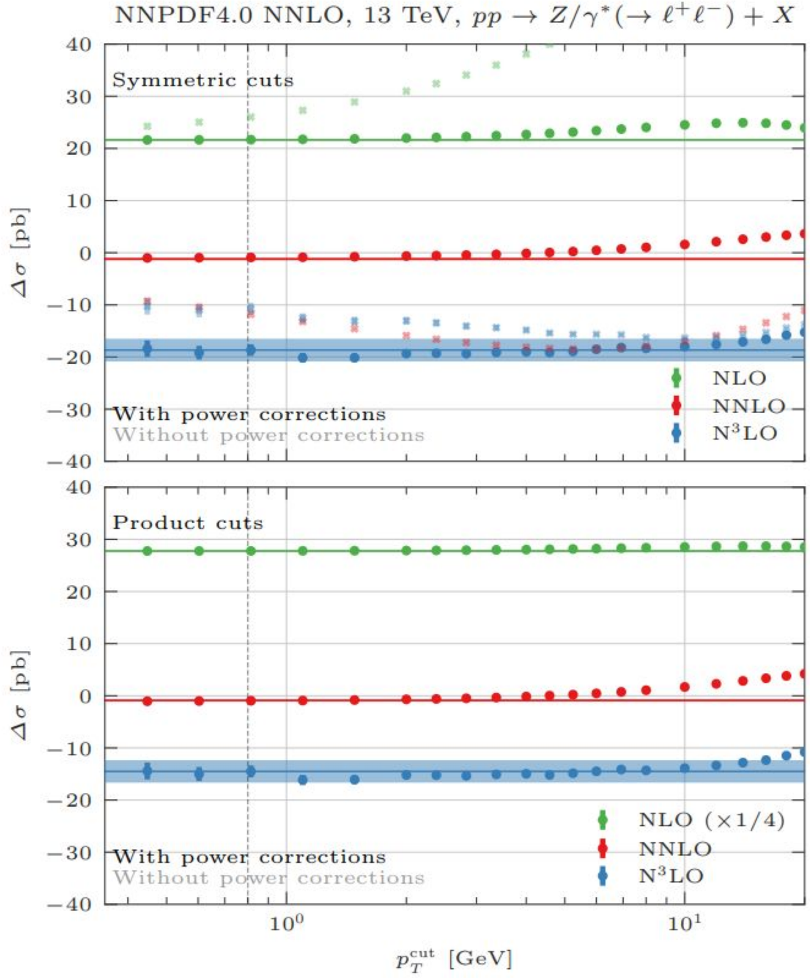
- **RRV**: 170 Collinear Master Integrals

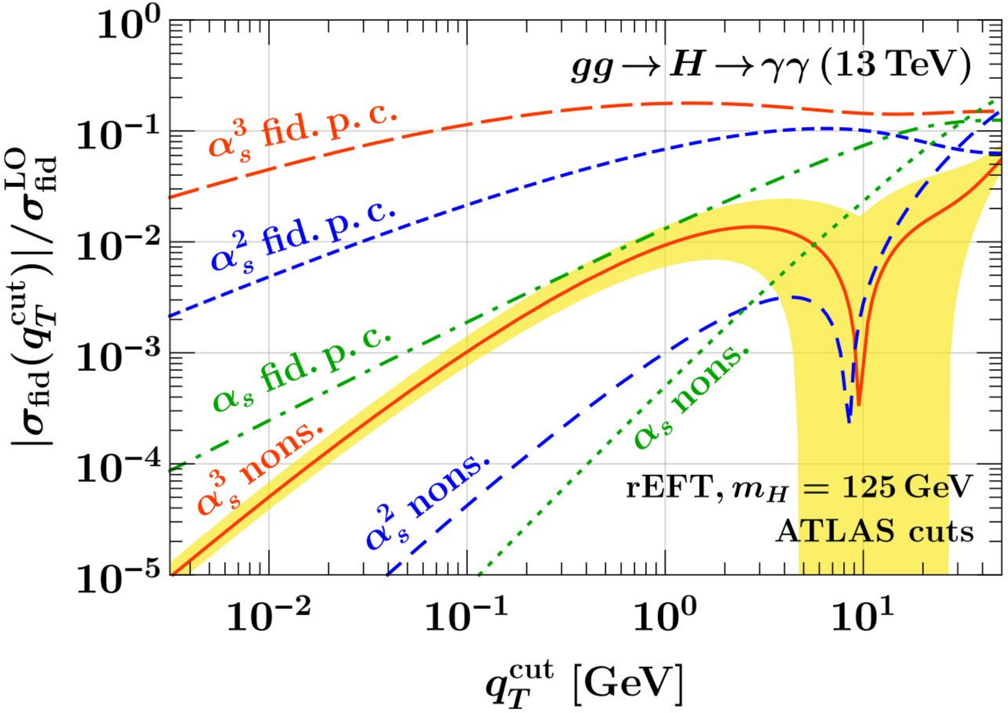


- **RRR**: 320 Collinear Master Integrals



- Derived system of Differential Equations for the Master Integrals
- System has 2 non trivial scales with algebraic dependence on the variables (not something solvable algorithmically)
- Algebraic sectors: constructed dlog integrand basis via calculation of **leading singularities** of candidate integrals on maximal cut surface
- Boundaries from soft integrals [Anastasiou, Duhr, Dulat, Mistlberger] and constraints on singular behavior


NNPDF4.0 NNLO, 13 TeV, $pp \to Z/\gamma^*(\to \ell^+\ell^-) + X$
Symmetric cuts
With power corrections
Without power corrections
NLO
NNLO
N³LO
Product cuts
NLO (×1/4)
$\Delta\sigma$ [pb]
p_T^{cut} [GeV]


$gg \to H \to \gamma\gamma$ (13 TeV)
α_s^3 fid. p. c.
α_s^2 fid. p. c.
α_s fid. p. c.
α_s^3 nons.
α_s^2 nons.
α_s nons.
rEFT, $m_H = 125$ GeV
ATLAS cuts
$|\sigma_{\text{fid}}(q_T^{\text{cut}})|/\sigma_{\text{fid}}^{\text{LO}}$
q_T^{cut} [GeV]