The electroweak parton distribution functions

Part II: Necessity and application

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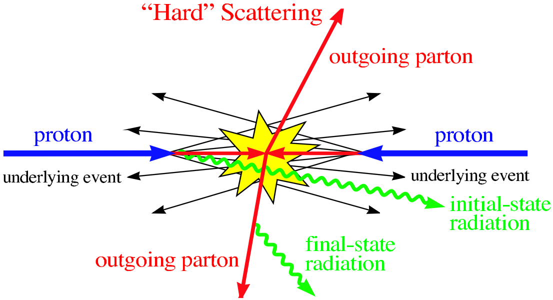
Ongoing work with Tao Han and Keping Xie.
Please also see Keping Xie's talk for Part I.

Background: what is parton distribution function (PDF) ?

According to QCD, protons are composite particles, made of "partons" of quark and gluons.



- Factorization formalism

$$\sigma(AB \to X) = \sum_{a,b} \int \mathrm{d}x_a \mathrm{d}x_b f_{a/A}(x_a, Q) f_{b/B}(x_b, Q) \hat{\sigma}(ab \to X)$$

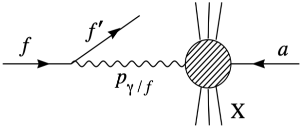
  - a, b are the "partons" from the beam particles A and B.
  - $f_{a/A}$, $f_{b/B}$ are the probabilities to find a parton a (b) from the beam particle A (B) with a momentum fraction x_a (x_b).

Weizäicker-Williams approximation

- "Equivalent photon approximation (EPA)"
  - Treat photon as a parton constituent in the electron

$$\sigma(e^- + a \to e^- + X) = \int \mathrm{d}x f_{\gamma/e}(x) \hat{\sigma}(\gamma a \to X)$$

$$f_{\gamma/e^-}(x) = \frac{\alpha}{\pi} P_{\gamma/e}(x) \ln\left(\frac{E_e}{m_e}\right)$$



  - $f_{\gamma/e}$ is the "photon PDF", $P_{\gamma/e}$ is the "splitting kernel"

- Splitting kernels of polarized electron and positron to photons

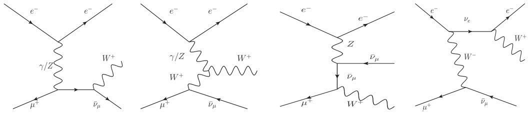
	e_L^-	e_R^-	$\langle e^- \rangle$	e_L^+	e_R^+	$\langle e^+ \rangle$
γ_-	$\frac{1}{x}$	$\frac{(1-x)^2}{x}$	$\frac{1+(1-x)^2}{2x}$	$\frac{(1-x)^2}{x}$	$\frac{1}{x}$	$\frac{1+(1-x)^2}{2x}$
γ_+	$\frac{(1-x)^2}{x}$	$\frac{1}{x}$	$\frac{1+(1-x)^2}{2x}$	$\frac{1}{x}$	$\frac{(1-x)^2}{x}$	$\frac{1+(1-x)^2}{2x}$
$\sum \gamma_\lambda$	$\frac{1+(1-x)^2}{x}$	$\frac{1+(1-x)^2}{x}$	$\frac{1+(1-x)^2}{x}$	$\frac{1+(1-x)^2}{x}$	$\frac{1+(1-x)^2}{x}$	$\frac{1+(1-x)^2}{x}$

Photon PDF is polarized if the electron beam is polarized (e.g. on ILC).

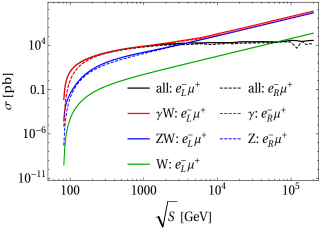
Test EPA on $e^-\mu^+ \to e^- W^+ \overline{\nu}_\mu$

The EPA photon PDF $f_{\gamma/e^-}(x)$ gives very good approximations at low energies.

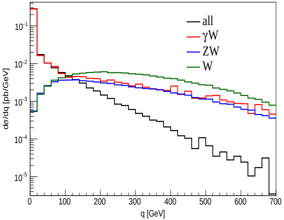
- Feynman diagrams

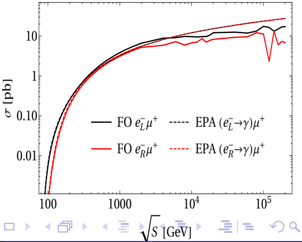


- Q: What if we continue to higher energies?
- A: Need to check
  - Need all 4 diagrams for gauge invariance
  - Unphysical results if only including $\gamma(W)$



- Q: Why EPA works?
- A: Forward γ dominates.





Factorization formalism

- The beam cross section

$$\sigma(e_h^- + X \to Y) = \sum_V \sum_\lambda \int \mathrm{d}x f_{V_\lambda/e_h^-}(x,Q)\hat{\sigma}(V_\lambda \to Y)$$

Note: In general, the relations $f_{V_-/e_L^-} \neq f_{V_+/e_R^-}, f_{V_+/e_L^-} \neq f_{V_-/e_R^-}$.

- Vector boson and coherence PDFs, $L_\gamma = \ln\left(Q^2/m_e^2\right)$, $L_Z = \ln\left(Q^2/m_Z^2\right)$,

$$f_V \sim \langle\Omega|V^{\mu\nu}V_{\mu\nu}|\Omega\rangle \sim \frac{g_V^2}{8\pi^2} P_{V/e} L_V,\ V = \gamma, Z$$

$$f_{\gamma Z/e} \sim \frac{1}{2}(\langle\Omega|A^{\mu\nu}Z_{\mu\nu}|\Omega\rangle + \langle\Omega|Z^{\mu\nu}A_{\mu\nu}|\Omega\rangle) \sim \frac{g_\gamma g_Z}{8\pi^2} P_{V/e} L_Z$$

Note: $f_{\gamma Z/e}$ can be either negative or positive, depending on g_γ and g_Z.

- Rotation between the (γ, Z) and (B, W^3) bases,

$$B = c_W A - s_W Z,\ W^3 = s_W A + c_W Z,$$

$$\begin{pmatrix} f_B \\ f_{W^3} \\ f_{BW^3} \end{pmatrix} = \begin{pmatrix} c_W^2 & s_W^2 & -2c_W s_W \\ s_W^2 & c_W^2 & 2c_W s_W \\ c_W s_W & -c_W s_W & c_W^2 - s_W^2 \end{pmatrix} \begin{pmatrix} f_\gamma(L_\gamma) \\ f_Z(L_Z) \\ f_{\gamma Z}(L_Z) \end{pmatrix}.$$

Partonic cross sections: $V + \mu^+ \to W^+ + \overline{\nu}_\mu$

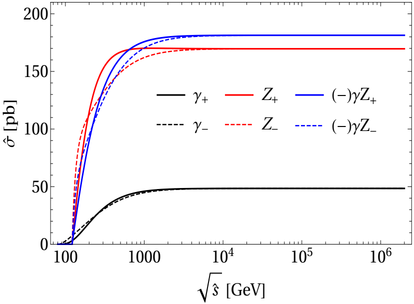
Relation between the partonic amplitude in (γ, Z) and (B, W^3) bases

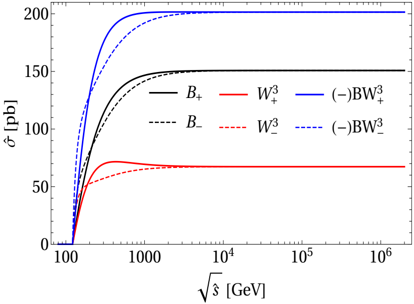
$$M_B = c_W M_\gamma - s_W M_Z, \; M_{W^3} = s_W M_\gamma + c_W M_Z$$

Partonic cross sections are defined as

$$\hat{\sigma}(V) \sim \int d\Phi M_V^\dagger M_V, \quad \hat{\sigma}(V_1 V_2) \sim \int d\Phi (M_{V_1}^\dagger M_{V_2} + M_{V_2}^\dagger M_{V_1}) \tag{1}$$

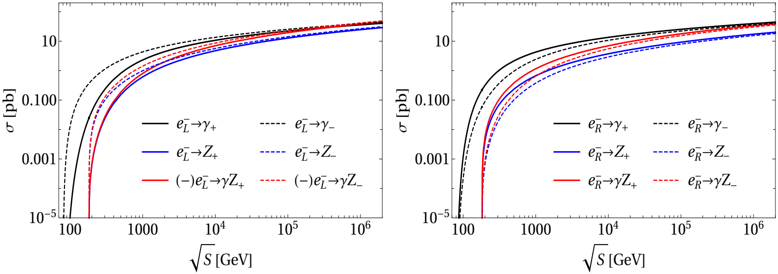
Note: $\hat{\sigma}(V_-) \neq \hat{\sigma}(V_+)$, so we need to take care on the PDFs part.





Beam cross sections in (γ, Z) basis

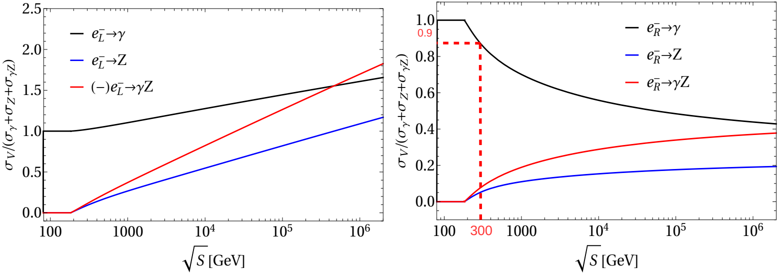
Cross sections of $e^-\mu^+ \to e^- W^+ \overline{\nu}_\mu$: $\sigma_{Tot} = \sigma_\gamma + \sigma_Z + \sigma_{\gamma Z}$



- When $Q^2 \geq m_Z^2$ ($\sqrt{S} \geq 2m_Z$), σ_Z and $\sigma_{\gamma Z}$ contribution appears.
- The EW PDFs have different effects on left-handed and right-handed electron beams, which agrees with the chiral nature of EW physics.
  - For both e_L^- and e_R^-, we see the polarization effect, i.e. $\sigma_{V_+} \neq \sigma_{V_-}$.
  - For e_L^-, the negative $\sigma_{\gamma Z}$ gives a destructive effect:
    - Below several TeV, $\sigma_Z + \sigma_{\gamma Z} \sim 0$, EPA work well, i.e. $\sigma_{Tot} \sim \sigma_\gamma$.
    - At higher energies, $\sigma_{\gamma Z} + \sigma_\gamma \sim 0$, so $\sigma_{Tot} \sim \sigma_Z$.
  - For e_R^-, constructive interference: $\sigma_{\gamma Z} > 0$ due to $f_{\gamma Z} < 0$ and $\hat{\sigma}_{\gamma Z} < 0$. $\sigma_{\gamma Z}$ and σ_Z are comparable to σ_γ. When $\sqrt{S} = 100$ TeV, $\sigma_{Tol} \sim 2\sigma_\gamma$.

Necessity of the full EW PDFs

Individual contributions of each component in (γ, Z) basis to the total cross section



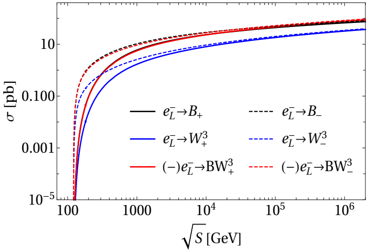
- $Q^2 < m_Z^2$, photon contribution dominates, EPA works good.
- $Q^2 \geq m_Z^2$, the full EW PDFs effects enter, need to consider σ_Z and $\sigma_{\gamma Z}$.
- For the e_L^- case, the negative $\sigma_{\gamma Z}$ cancels with σ_Z, making EPA a good approximation up to several TeVs.
- For the e_R^- case, the positive $\sigma_{\gamma Z}$ and the σ_Z increase quite fast. **The full EW PDFs effect is non-negligible even at intermediate energies**.
- At $\sqrt{S} = 300$ GeV, the photon contribution is less then 90%. This effect might be seen at colliders in the near future.

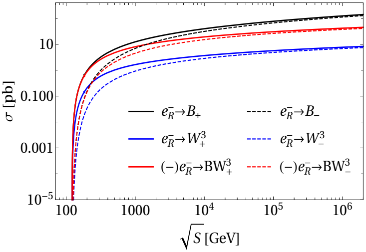
Beam cross sections in the gauge basis

Some new features in the (B, W^3) basis

- Though e_R^- does not couple to W^3, PDFs f_{W^3/e_R^-} and f_{BW^3/e_R^-} are non-zero due to the uncanceled residues of γ/Z in EW symmetry breaking

$$f_{W^3/e_R^-} \propto \ln \frac{m_Z^2}{m_e^2}, \quad f_{BW^3/e_R^-} \propto \ln \frac{m_Z^2}{m_e^2}$$





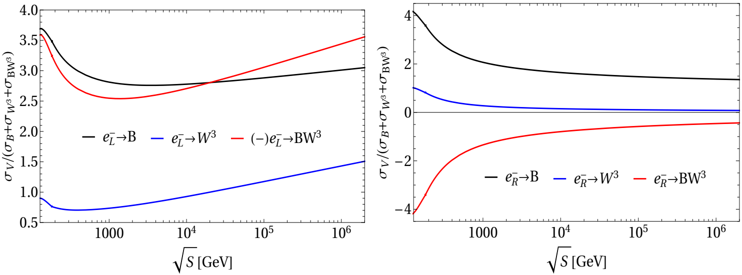
- Polarization effects
- All σ_{BW^3} are negative

- For e_L^-, σ_{BW^3} almost exactly cancels σ_B, so **roughly only σ_{W^3} remains**.
- For e_R^-, σ_{BW^3} and σ_B cancel in some degree. **σ_B dominates at high energies**.

Beam cross sections in (B, W^3) basis

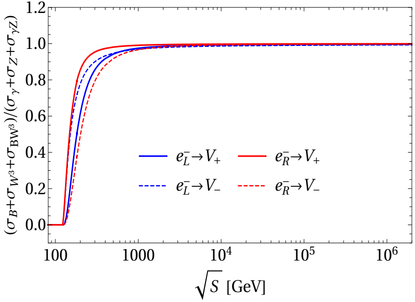
Individual contributions of each component in (B, W^3) basis to σ_{Tot}

- For the e_L^- case, there is cancellation between σ_B and σ_{BW^3}.
- For the e_R^- case
  - There are non-zero σ_{W^3} and σ_{BW^3} due to the non-zero f_{W^3/e_R^-} and f_{BW^3/e_R^-}.
  - Because of the $\ln(m_Z^2/m_e^2)$ behavior of f_{W^3/e_R^-} and f_{BW^3/e_R^-}, σ_B will dominate in very high energy range resulted from the $\ln Q^2$ enhancement.



Compare between (γ, Z) and (B, W^3) bases

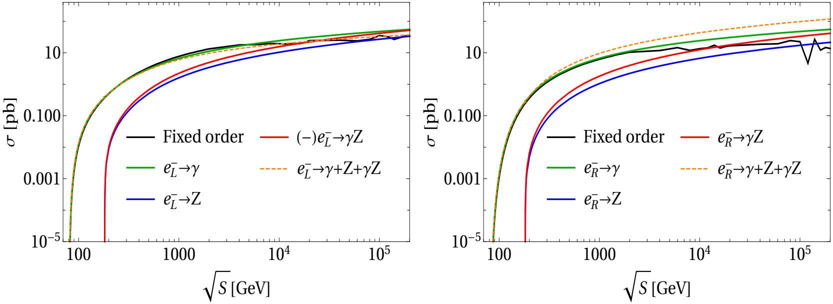
- In the low energy range, one should do calculations in the (γ, Z) basis.
- Starting from several TeV, the results in the two bases are consistent.
- In the high energy limit, (B, W) basis approaches to EW unbroken phase.



Factorization VS Fixed order

Compare the factorization results with the full fixed-order calculation.

- When $Q^2 < m_Z^2$, the equivalent photon works perfectly. We need to match the EW PDFs with the existing photon PDF and/or the full fixed-order calculation.
- At intermediate energies ($Q^2 \geq m_Z^2$), the full EW PDFs need to be considered.
- In the high energy limit ($Q^2 \gg m_Z^2$), the fixed-order calculations converge badly, which can be handled by the factorization method with the EW PDFs.



Summary and new features

Summary

- The traditional photon PDF works well in the low energy regime ($Q^2 < m_Z^2$).
- When $Q^2 \geq m_Z^2$, the Z and γZ mixing need to be taken into account.
- In the high energy limit, we can work in the gauge basis, which approaches to EW unbroken phase.
- The EW PDFs play an important role in studying future high-energy colliders.

New features

- Due to the parity violation, the EW PDFs should be polarized, e.g. one need to **separate** σ_{V_-} **and** σ_{V_+} in the factorization procedure.
- The γZ mixing contribution can be either negative (e_L^-) or positive (e_R^-).
- For the e_R^- case, the γZ is already around 10% to the total cross section for $\sqrt{S} = 300$ GeV.
- Because of the EW symmetry breaking, there are non-zero f_{W^3/e_R^-} and f_{BW^3/e_R^-} PDFs (proportional to $\ln(m_Z^2/m_e^2)$).

Polarization effect

A

$$\sigma = \int dx_1 dx_2 f_1(x_1, Q^2) f_2(x_2, Q^2) \langle\hat{\sigma}\rangle$$

$$f_i(x_i, Q^2) = f_i^+(x_i, Q^2) + f_i^-(x_i, Q^2)$$

$$\langle\hat{\sigma}\rangle = \frac{1}{4}(\hat{\sigma}^{+,+} + \hat{\sigma}^{+,-} + \hat{\sigma}^{-,+} + \hat{\sigma}^{-,-})$$

B

$$\sigma = \sum_{h_1,h_2} \int dx_1 dx_2 f_1^{h_1}(x_1, Q^2) f_2^{h_2}(x_2, Q^2) \hat{\sigma}^{h_1,h_2} \qquad h_1, h_2 = 0, \pm 1$$

Focus on pure transverse case

$$\sigma_T = \sum_{h_1,h_2} \int dx_1 dx_2 f_1^{h_1}(x_1, Q^2) f_2^{h_2}(x_2, Q^2) \hat{\sigma}^{h_1,h_2}$$

$$= \int dx_1 dx_2 f_1^+(x_1, Q^2) f_2^+(x_2, Q^2) \hat{\sigma}^{+,+} + (+ \to -, - \to +)$$

$$+ \int dx_1 dx_2 f_1^+(x_1, Q^2) f_2^-(x_2, Q^2) \hat{\sigma}^{+,-} + (+ \to -, - \to +)$$

$$= \int dx_1 dx_2 \left(f_1^T f_2^T \hat{\sigma}^{T,T} + f_1^T f_2^\Delta \hat{\sigma}^{T,\Delta} + f_1^\Delta f_2^T \hat{\sigma}^{\Delta,T} + f_1^\Delta f_2^\Delta \hat{\sigma}^{\Delta,\Delta} \right),$$

B1 B2 B3 B4

Introduce

$$f_i^T = f_i^+(x_i, Q^2) + f_i^-(x_i, Q^2), \quad f_i^\Delta = f_i^+(x_i, Q^2) - f_i^-(x_i, Q^2),$$

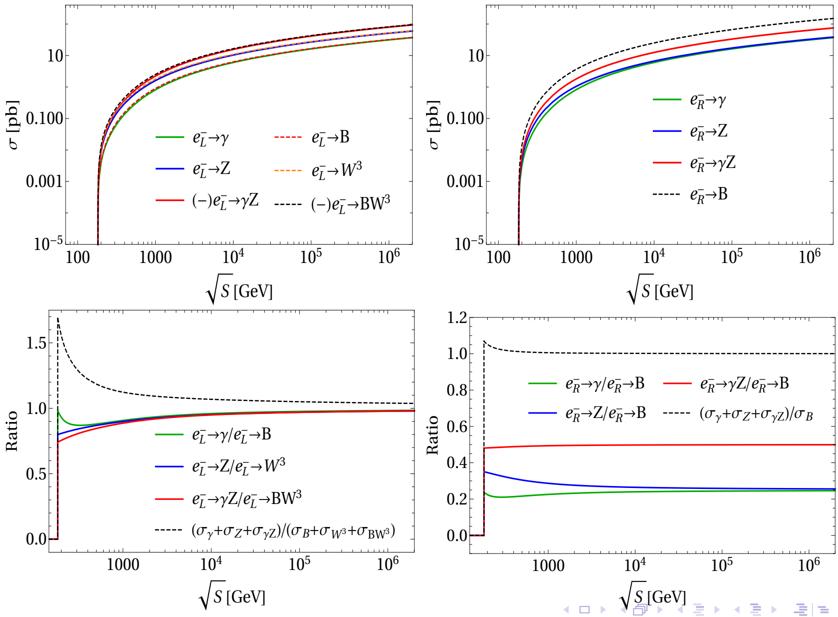
A=B1

B2, B3, B4 are missing, if we use A !

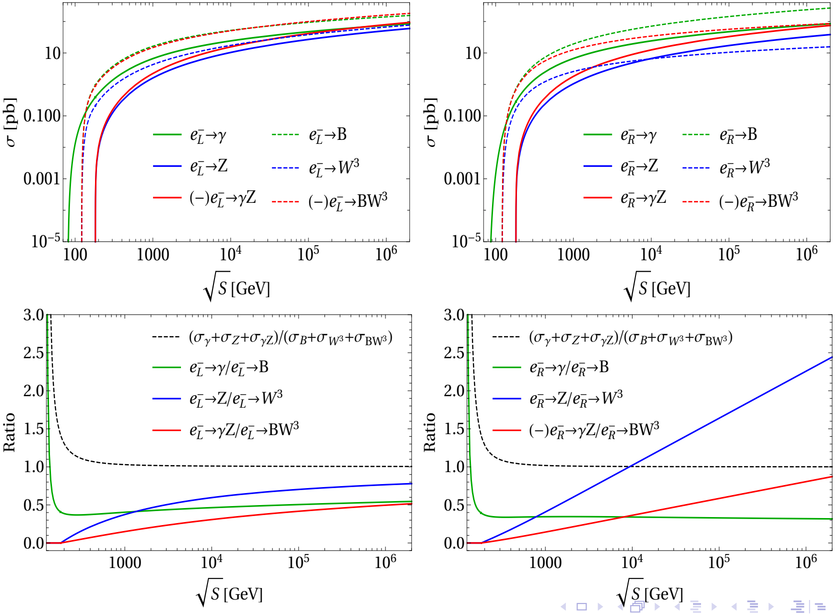
$$\hat{\sigma}^{T,T} = \frac{1}{4}\left(\hat{\sigma}^{+,+} + \hat{\sigma}^{+,-} + \hat{\sigma}^{-,+} + \hat{\sigma}^{-,-}\right), \quad \hat{\sigma}^{T,\Delta} = \frac{1}{4}\left(\hat{\sigma}^{+,+} - \hat{\sigma}^{+,-} + \hat{\sigma}^{-,+} - \hat{\sigma}^{-,-}\right)$$

$$\hat{\sigma}^{\Delta,T} = \frac{1}{4}\left(\hat{\sigma}^{+,+} + \hat{\sigma}^{+,-} - \hat{\sigma}^{-,+} - \hat{\sigma}^{-,-}\right), \quad \hat{\sigma}^{\Delta,\Delta} = \frac{1}{4}\left(\hat{\sigma}^{+,+} - \hat{\sigma}^{+,-} - \hat{\sigma}^{-,+} + \hat{\sigma}^{-,-}\right)$$

Beam cross sections for $L_1 = L_2$



Beam cross sections for $L_1 \neq L_2$



Necessity of full EW PDF, polarized