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Minimal $SU(4)$ models & B-meson anomalies

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Lepton Flavor group

$$\mathcal{L}_{Y-\text{lept}}^{\text{SM}} = \bar{L} Y_l e_R \Phi$$

$$Y_l \stackrel{!}{=} 0$$

$$m_l \stackrel{!}{=} 0$$

~~SM~~

Sensu lato:

$$U(3)_{L_L} \times U(3)_{e_R}$$

$$L \rightarrow U \bullet L$$

$$e_R \rightarrow U' \bullet e_R$$

FLAVOR PHYSICS
refers here!

$$\cap \quad U' = U$$

Sensu mezzostrieto:

$$U(3)_{\text{LF}}$$

FLAVOR VIOLATION
refers there!

$$U = \text{diag}(e^{i\alpha}, e^{i\beta}, e^{i\gamma})$$

$$\cap \quad \det U = 1$$

Sensu stricto:

$$U(1)_e \times U(1)_\mu \times U(1)_\tau$$

✓ SM

$$\cap \quad U = e^{i\alpha} \mathbb{1}$$

~~ν oscillations~~

✓ all other experiments

Lepton Flavor Universality:

$$SU(3)_{\text{LFU}} \quad Y_l \stackrel{!}{=} \mathbb{1}_y$$

$$m_e \stackrel{!}{=} m_\mu \stackrel{!}{=} m_\tau$$

~~SM~~

Lepton
number:

$$U(1)_L$$

✓ SM

✓ SM ⊕ Dirac ν_R

✓ all experiments

✓ SM @ $E \gg \Delta m_l$

~~B-meson decays~~

LFUV in B -meson decays

(Cf. today morning talks for details)

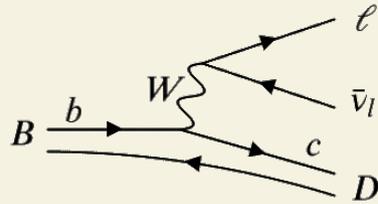
Charged current anomaly

$$R_{D^{(*)}} = \frac{\text{BR}(B \rightarrow D^{(*)} \tau \bar{\nu})}{\text{BR}(B \rightarrow D^{(*)} l \bar{\nu})}$$

$$R_{D^{(*)}}^{\text{exp}} > R_{D^{(*)}}^{\text{exp2019}} \gtrsim R_{D^{(*)}}^{\text{SM}}$$

$$R_D^{\text{SM}} = 0.299 \pm 0.003$$

$$R_{D^*}^{\text{SM}} = 0.258 \pm 0.005$$



Pre-Moriond2019 average:

$$R_D = 0.407 \pm 0.039 \pm 0.024$$

$$R_{D^*} = 0.306 \pm 0.013 \pm 0.007$$

LHCb: [1904.08794]

$$R_D = 0.307 \pm 0.037 \pm 0.016$$

$$R_{D^*} = 0.283 \pm 0.018 \pm 0.014$$

Gone?

Neutral current anomaly

$$R_{K^{(*)}} = \frac{\text{BR}(B \rightarrow K^{(*)} \mu^+ \mu^-)}{\text{BR}(B \rightarrow K^{(*)} e^+ e^-)}$$

$$R_{K^{(*)}}^{\text{exp}} < R_{K^{(*)}}^{\text{SM}} = 1$$

Pre-Moriond2019:

$$R_K^{\text{exp}} = 0.745_{-0.074}^{+0.090} \pm 0.036$$

$$R_{K^*}^{\text{exp}} = 0.685_{-0.069}^{+0.113} \pm 0.047$$

LHCb: [1903.09252]

$$R_K = 0.846_{-0.054}^{+0.060} \pm 0.016$$

Support from various b-s- μ - μ probes

- angular obs. in $B \rightarrow K^* \mu \mu$ (P_5')

- $B_s \rightarrow \mu \mu$

LF[U]V in LeptoQuark interactions

$$\mathcal{L}_{LQ-int} = (\bar{d} \quad \bar{s} \quad \bar{b}) \begin{pmatrix} y_{de} & y_{d\mu} & y_{d\tau} \\ y_{se} & y_{s\mu} & y_{s\tau} \\ y_{be} & y_{b\mu} & y_{b\tau} \end{pmatrix} \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix} LQ^{+2/3}$$

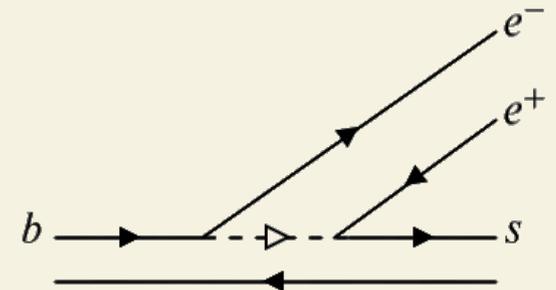
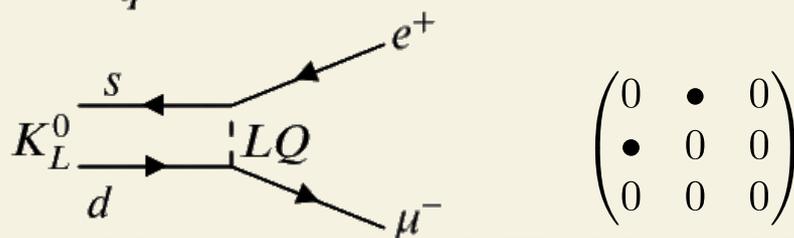
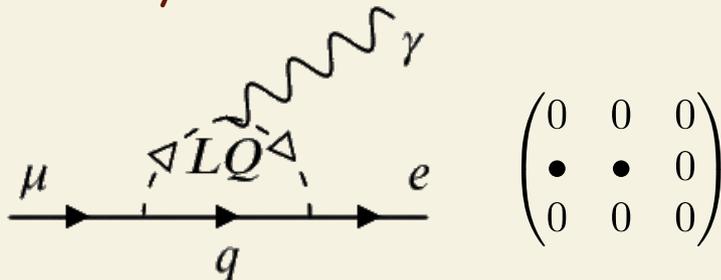
Lepton Flavor violation

Lepton Flavor Universality violation



(At least) two nonzero columns

Two (or more) columns differ



$$\begin{pmatrix} 0 & 0 & 0 \\ \bullet & 0 & 0 \\ \bullet & 0 & 0 \end{pmatrix}$$

Minimal $SU(4)$ model(s)

A. D. Smirnov (et al.)

- The Minimal quark - lepton symmetry model and the limit on Z' mass
Phys. Lett. B346 (1995) 297-302 arXiv: hep-ph/9503239
- Phys. Lett. B431 (1998) 119-126 arXiv: hep-ph/9805339
- Mod. Phys. Lett. A20 (2005) 3003-3012 arXiv: 0511149
- Mod. Phys. Lett. A23 (2008) 2907-2913 arXiv: 0807.4486
- Mod. Phys. Lett. A24 (2009) 1199-1207 arXiv: 0902.2931
- Mod. Phys. Lett. A31 (2016) 39, 1650224 arXiv: 1610.08409
- Vector leptoquark mass limits and branching ratios of $K_L^0, B^0, B_s \rightarrow l_i^+ l_j^-$ decays
with account of fermion mixing...
Mod. Phys. Lett. A33 (2018) 1850019 arXiv: 1801.02895

vanilla
version

P. Fileviez Pérez, M. B. Wise

- Low-scale quark-lepton unification
Phys. Rev. D88 (2013) 057703 arXiv: 1307.6213

inverse
seesaw

T. Faber, W. Porod, M. Hudec, M. Malinský, H. Kolečová, F. Staub

- A unified LQ model confronted with lepton non-universality in B-meson decays
Phys. Lett. B 787 (2018) 159-166 arXiv: 1808.05511
- Collider phenomenology of a unified LQ model arXiv: 1812.07592

Minimal $SU(4)$ model(s)

- $SU(4)_C \times SU(2)_L \times U(1)_R$

$$A_\mu^{SU(4)} = \begin{pmatrix} G_\mu + \frac{1}{\sqrt{12}} A_\mu^{15} & U_{1\mu} \\ U_{1\mu}^\dagger & -\frac{3}{\sqrt{12}} A_\mu^{15} \end{pmatrix}$$

Quark-Lepton unification

natural LeptoQuark environment

- No extra charged fermions

mandatory: ν_R

optional: extra singlet \rightarrow inverse seesaw

$$F_{(4,2,0)} = \begin{pmatrix} Q \\ L \end{pmatrix}$$

$$f_{u(4,1,+1/2)}^c = (u_R^c \quad \nu_R^c)$$

$$f_{d(4,1,+1/2)}^c = (d_R^c \quad e_R^c)$$

$$N_{(1,1,0)}$$

- Minimal scalar sector

Cascade SSB

$$\chi_{(4,1,+1/2)} = \begin{pmatrix} \bar{S}_1^\dagger(3,1,+2/3) \\ \chi^0(1,1,0) \end{pmatrix}$$

$$H_{(1,2,+1/2)}$$

Independent fermion masses

$$\Phi_{(15,2,+1/2)} = \begin{pmatrix} G_{(8,2,+1/2)} + \frac{H_2}{\sqrt{12}} & R_2(3,2,+7/6) \\ \tilde{R}_2^\dagger(\bar{3},2,-1/6) & -\frac{3}{\sqrt{12}} H_2(1,2,+1/2) \end{pmatrix}$$

LeptoQuarks in the model

- vector $U_1 \sim (3, 1, +2/3)$
 - Unitary interaction matrix \rightarrow LFV inevitable $\rightarrow m_{U_1} > 60$ TeV [1801.02895]
- scalar $\bar{S}_1 \sim (3, 1, +2/3)$
 - Goldstone boson eaten by U_1
- scalar $\widetilde{R}_2 \sim (3, 2, -1/6)$
 - Unable to accommodate $R(K^*)$ [1704.05438]
- scalar $R_2 \sim (3, 2, +7/6)$
 - able to accommodate $R(K)$ & $R(K^*)$ [1704.05438]
 - also used to explain $R(D)$ & $R(D^*)$ [1806.05689]

$$m_G^2 + 2m_{\hat{H}}^2 \sin^2 \beta = \frac{3}{2}(m_{R_2}^2 + m_{\widetilde{R}_2}^2)$$

The R_2 LeptoQuark & $R(K^{(*)})$ anomaly

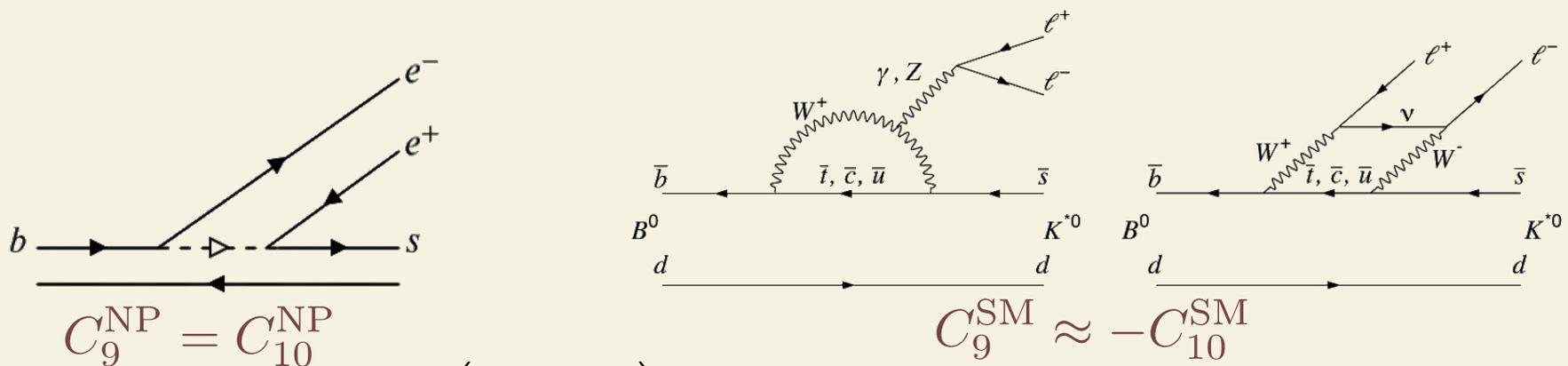
$$R_2 = \begin{bmatrix} R^{5/3} \\ R^{2/3} \end{bmatrix} \sim (3, 2, +\frac{7}{6}) \quad R_{K^{(*)}} = \frac{\text{BR}(B \rightarrow K^{(*)} \mu^+ \mu^-)}{\text{BR}(B \rightarrow K^{(*)} e^+ e^-)} \lesssim 1$$

$$\mathcal{L}_{R_2} = \left(R^{2/3} \overline{d}_L + R^{5/3} \overline{u}_L V_{\text{CKM}} \right) Y_4 e_R + \overline{u}_R Y_2 \left(\nu_L R^{2/3} + e_L R^{5/3} \right) + \text{h.c.}$$

Different chiralities than weak interactions

→ sums incoherently with SM contributions

→ can improve $R(K)$ iff coupling to electrons



Ideal case: $Y_4 = \begin{pmatrix} 0 & 0 & 0 \\ \bullet & 0 & 0 \\ \bullet & 0 & 0 \end{pmatrix} \longrightarrow R_K \approx \frac{|C^{\text{SM}}|^2}{|C^{\text{SM}}|^2 + |C_{ee}^{\text{NP}}|^2}$

Constraints on the R_2 LQ interaction from extended gauge symmetry

The relevant Yukawa matrix must satisfy

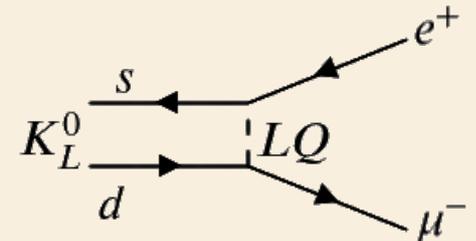
$$Y_4 = \frac{U \hat{M}_d - \hat{M}_l V}{\sqrt{2/3} v_{\text{ew}} \cos \beta} \propto \begin{matrix} & e_R & \mu_R & \tau_R \\ d_L & (u_{de} m_d - v_{de} m_e) & (u_{d\mu} m_d - v_{d\mu} m_\mu) & (u_{d\tau} m_d - v_{d\tau} m_\tau) \\ s_L & (u_{se} m_s - v_{se} m_e) & (u_{s\mu} m_s - v_{s\mu} m_\mu) & (u_{s\tau} m_s - v_{s\tau} m_\tau) \\ b_L & (u_{be} m_b - v_{be} m_e) & (u_{b\mu} m_b - v_{b\mu} m_\mu) & (u_{b\tau} m_b - v_{b\tau} m_\tau) \end{matrix}$$

Freedom: $U, V =$ arbitrary unitary matrices; $\cos \beta$; m_{LQ}

Special case: Low-energy $SO(10) \rightarrow U = V$

LFV inevitable

$$\begin{matrix} & e_R & \mu_R & \tau_R \\ d_L & (u_{de}(m_d - m_e)) & (u_{d\mu}(m_d - m_\mu)) & (u_{d\tau}(m_d - m_\tau)) \\ s_L & (u_{se}(m_s - m_e)) & (u_{s\mu}(m_s - m_\mu)) & (u_{s\tau}(m_s - m_\tau)) \\ b_L & (u_{be}(m_b - m_e)) & (u_{b\mu}(m_b - m_\mu)) & (u_{b\tau}(m_b - m_\tau)) \end{matrix}$$



model with $R_K = 0.8$ EXCLUDED

Only working scenario:

$$Y_4 = \sqrt{\frac{3}{2}} \frac{1}{v_{\text{ew}} \cos \beta} \begin{pmatrix} 0 & 0 & m_\tau \sin \phi \\ m_s/\sqrt{2} & 0 & m_\tau \cos \phi \\ m_b/\sqrt{2} & 0 & -m_b/\sqrt{2} \end{pmatrix}$$

NO muon flavor violation

Summary of Results

	Vanilla minimal model	Minimal model + inverse seesaw - SO(10)	Minimal model + inverse seesaw
R(D ^(*))	X	X	X
R(K ^(*))	X	X	<p style="text-align: center;">✓</p> <ul style="list-style-type: none"> • $1 \times 10^{-8} < \tau \rightarrow e \gamma < 3 \times 10^{-8}$ discovery@BelleII • $\tau \rightarrow eee, \tau \rightarrow e\mu\mu$ @BelleII • $\tau \rightarrow e\pi\pi$ possible @BelleII • 2/3 LQ decaying to $be^+ : b\tau^+ : j\tau^+ = 1 : 1 : 1.17$ @pp-colliders
b-s- μ - μ	X	X	X

Thank you

Matěj Hudec

Scalar potential

$$\begin{aligned}
 V = & \mu_H^2 |H|^2 + \mu_\chi^2 |\chi|^2 + \mu_\Phi^2 \text{Tr}(|\Phi|^2) + \lambda_1 |H|^2 |\chi|^2 + \lambda_2 |H|^2 \text{Tr}(|\Phi|^2) + \lambda_3 |\chi|^2 \text{Tr}(|\Phi|^2) \\
 & + (\lambda_4 H_i^\dagger \chi^\dagger \Phi^i \chi + \text{h.c.}) + \lambda_5 H_i^\dagger \text{Tr}(\Phi_j^\dagger \Phi^i) H^j + \lambda_6 \chi^\dagger \Phi^i \Phi_i^\dagger \chi + \lambda_7 |H|^4 + \lambda_8 |\chi|^4 + \lambda_9 \text{Tr}(|\Phi|^4) \\
 & + \lambda_{10} (\text{Tr}|\Phi|^2)^2 + (\lambda_{11} H_i^\dagger \text{Tr}(\Phi^i \Phi^j) H_j^\dagger + \lambda_{12} H_i^\dagger \text{Tr}(\Phi^i \Phi^j \Phi_j^\dagger) + \lambda_{13} H_i^\dagger \text{Tr}(\Phi^i \Phi_j^\dagger \Phi^j) + \text{h.c.}) \\
 & + \lambda_{14} \chi^\dagger |\Phi|^2 \chi + \lambda_{15} \text{Tr}(\Phi_i^\dagger \Phi^j \Phi_j^\dagger \Phi^i) + \lambda_{16} \text{Tr}(\Phi_i^\dagger \Phi^j) \text{Tr}(\Phi_j^\dagger \Phi^i) + \lambda_{17} \text{Tr}(\Phi_i^\dagger \Phi_j^\dagger) \text{Tr}(\Phi^i \Phi^j) \\
 & + \lambda_{18} \text{Tr}(\Phi_i^\dagger \Phi_j^\dagger \Phi^i \Phi^j) + \lambda_{19} \text{Tr}(\Phi_i^\dagger \Phi_j^\dagger \Phi^j \Phi^i)
 \end{aligned}$$



$$m_G^2 = \left(\frac{\sqrt{3}\lambda_4}{4} \tan \beta - \frac{3}{8} (\lambda_6 + \lambda_{14}) \right) v_\chi^2,$$

$$m_{R_2}^2 = \left(\frac{\sqrt{3}\lambda_4}{4} \tan \beta + \frac{\lambda_{14} - 3\lambda_6}{8} \right) v_\chi^2,$$

$$m_{\tilde{R}_2}^2 = \left(\frac{\sqrt{3}\lambda_4}{4} \tan \beta + \frac{\lambda_6 - 3\lambda_{14}}{8} \right) v_\chi^2,$$

$$m_{\hat{H}}^2 = \frac{\sqrt{3}\lambda_4}{2 \sin(2\beta)} v_\chi^2,$$



$$m_G^2 + 2m_{\hat{H}}^2 \sin^2 \beta = \frac{3}{2} (m_{R_2}^2 + m_{\tilde{R}_2}^2)$$

Some more eqns

Fermions	Scalars
$F_{(4,2,0)} = \begin{pmatrix} Q \\ L \end{pmatrix}$	$\chi_{(4,1,+1/2)} = \begin{pmatrix} \bar{S}_1^\dagger_{(3,1,+2/3)} \\ \chi^0_{(1,1,0)} \end{pmatrix}$
$f_u^c_{(\bar{4},1,-1/2)} = (u^c \quad \nu^c)$	$H_{(1,2,+1/2)}$
$f_d^c_{(\bar{4},1,+1/2)} = (d^c \quad e^c)$	$\Phi_{(15,2,+1/2)} = \begin{pmatrix} G_{(8,2,+1/2)} + \frac{1}{\sqrt{12}}H_2 & R_2_{(3,2,+7/6)} \\ \tilde{R}_2^\dagger_{(\bar{3},2,-1/6)} & \frac{-3}{\sqrt{12}}H_2_{(1,2,+1/2)} \end{pmatrix}$
$N_{(1,1,0)}$	

$$-\mathcal{L}_Y = f_u^c Y_1 H F + f_u^c Y_2 \Phi F + f_d^c Y_3 H^\dagger F + f_d^c Y_4 \Phi^\dagger F + f_u^c Y_5 \chi N + \frac{1}{2} N \mu N + \text{h.c.}$$

Scalar extensions of SM

$\Psi(Y)_L^{3B}$	$\bar{\nu}_R(0)_{-1}^0$	$\bar{e}_R(-1)_{-1}^0$	$L^j(-\frac{1}{2})_1^0$	$\bar{d}_{R\beta}(\frac{1}{3})_0^{-1}$	$\bar{u}_{R\beta}(-\frac{2}{3})_0^{-1}$	$Q^{j\beta}(\frac{1}{6})_0^1$
$Q^{i\alpha}(\frac{1}{6})_0^1$	$\tilde{R}_{2\alpha i}^\dagger(-\frac{1}{6})$	$R_{2\alpha i}^\dagger(-\frac{7}{6})$	$S_{1\alpha}\varepsilon_{ij}(\frac{1}{3})$ $S_{3\alpha ij}$	$H_i^\dagger\delta_\alpha^\beta(-\frac{1}{2})$ $G_{\alpha j}^\dagger$	$H^j\varepsilon_{ij}\delta_\alpha^\beta(\frac{1}{2})$ $X_\alpha^{j\beta}\varepsilon_{ij}$	$S_1^{\dagger\gamma}\varepsilon_{\alpha\beta\gamma}\varepsilon_{ij}$ $\chi_{\alpha\beta}\varepsilon_{ij}(-\frac{1}{3})$ $S_{3ij}^{\dagger\gamma}\varepsilon_{\alpha\beta\gamma}$ $X_{\alpha\beta ij}$
$\bar{u}_{R\alpha}(-\frac{2}{3})_0^{-1}$	$\bar{S}_1^{\dagger\alpha}(\frac{2}{3})$	$S_1^{\dagger\alpha}(-\frac{1}{3})$	$R_2^{\alpha i}\varepsilon_{ij}(\frac{7}{6})$	$S_{1\gamma}\varepsilon^{\alpha\beta\gamma}$ $\chi^{\dagger\alpha\beta}(\frac{1}{3})$	$\tilde{S}_{1\gamma}\varepsilon^{\alpha\beta\gamma}$ $\chi^{\dagger\alpha\beta}(\frac{4}{3})$	
$\bar{d}_{R\alpha}(\frac{1}{3})_0^{-1}$	$S_1^{\dagger\alpha}(-\frac{1}{3})$	$\tilde{S}_1^{\dagger}(-\frac{4}{3})$	$\tilde{R}_2^{\alpha i}\varepsilon_{ij}(\frac{1}{6})$	$\chi_\gamma^\dagger\varepsilon^{\alpha\beta\gamma}$ $\chi^{\dagger\alpha\beta}(-\frac{2}{3})$		
$L^i(-\frac{1}{2})_1^0$	$H^j\varepsilon_{ij}(\frac{1}{2})$	$H_i^\dagger(-\frac{1}{2})$	$\varphi\varepsilon_{ij}(1)$ Δ_{ij}			
$\bar{e}_R(-1)_{-1}^0$	$\varphi^\dagger(-1)$	$\varphi^\dagger(-2)$				
$\bar{\nu}_R(0)_{-1}^0$	$\varphi(0)$					

Baryon & lepton numbers

	G	G_{SM}	G_{vac}	$[B-L]$	F	M	B	L, L'
Fermions								
$F_L = \begin{pmatrix} Q \\ L \end{pmatrix}$	$(4, 2, 0)$	$Q \ (3, 2, 1/6)$ $L \ (1, 2, -1/2)$	$u \ (3, 2/3)$ $d \ (3, -1/3)$ $\nu \ (1, 0)$ $e \ (1, -1)$	$\begin{pmatrix} +1/3 \\ -1 \end{pmatrix}$	+1	+1	$\begin{pmatrix} +1/3 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ +1 \end{pmatrix}$
$f_u^c = (u^c \ \nu^c)$	$(\bar{4}, 1, -1/2)$	$u^c \ (\bar{3}, 1, -2/3)$ $\nu^c \ (1, 1, 0)$	$(\bar{3}, -2/3)$ $(1, 0)$	$(-1/3 \ 1)$	-1	-1	$(-1/3 \ 0)$	$(0 \ -1)$
$f_d^c = (d^c \ e^c)$	$(\bar{4}, 1, 1/2)$	$d^c \ (\bar{3}, 1, 1/3)$ $e^c \ (1, 1, 1)$	$(\bar{3}, 1/3)$ $(1, 1)$	$(-1/3 \ 1)$	-1	-1	$(-1/3 \ 0)$	$(0 \ -1)$
N	$(1, 1, 0)$	$(1, 1, 0)$	$(1, 0)$	0	+1	0	0	0, +1
Scalars								
$\chi = \begin{pmatrix} \bar{S}_1^\dagger \\ \chi^0 \end{pmatrix}$	$(4, 1, 1/2)$	$\bar{S}_1^\dagger \ (3, 1, 2/3)$ $\chi^0 \ (1, 1, 0)$	$(3, 2/3)$ $(1, 0)$	$\begin{pmatrix} +1/3 \\ -1 \end{pmatrix}$	0	+1	$\begin{pmatrix} +1/3 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ +1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \end{pmatrix}$
H	$(1, 2, 1/2)$	$(1, 2, 1/2)$	$H_1^+ \ (1, 1)$ $H_1^0 \ (1, 0)$	0	0	0	0	0
$\Phi = \begin{pmatrix} G & R_2 \\ \tilde{R}_2^\dagger & 0 \end{pmatrix}$ $+\sqrt{2}T^{15}H_2$	$(15, 2, 1/2)$	$R_2 \ (3, 2, 7/6)$ $\tilde{R}_2^\dagger \ (\bar{3}, 2, -1/6)$ $G \ (8, 2, 1/2)$ $H_2 \ (1, 2, 1/2)$	$R_2^{5/3} \ (3, 5/3)$ $R_2^{2/3} \ (3, 2/3)$ $\tilde{R}_2^{-1/3\dagger} \ (\bar{3}, 1/3)$ $\tilde{R}_2^{2/3\dagger} \ (\bar{3}, -2/3)$ $G^+ \ (8, 1)$ $G^0 \ (8, 0)$ $H_2^+ \ (1, 1)$ $H_2^0 \ (1, 0)$	$\begin{pmatrix} 0 & +4/3 \\ -4/3 & 0 \end{pmatrix}$	0	0	$\begin{pmatrix} 0 & +1/3 \\ -1/3 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & -1 \\ +1 & 0 \end{pmatrix}$
Gauge Bosons								
$A_\mu = \begin{pmatrix} G_\mu & X_\mu \\ X_\mu^* & 0 \end{pmatrix}$ $+T^{15}B'_\mu$	$(15, 1, 0)$	$G_\mu \ (8, 1, 0)$ $X_\mu \ (3, 1, 2/3)$ $B'_\mu \ (1, 1, 0)$	$(8, 0)$ $(3, 2/3)$ $(1, 0)$	$\begin{pmatrix} 0 & +4/3 \\ -4/3 & 0 \end{pmatrix}$	0	0	$\begin{pmatrix} 0 & +1/3 \\ -1/3 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & -1 \\ +1 & 0 \end{pmatrix}$
W_μ	$(1, 3, 0)$	$(1, 3, 0)$	$W_\mu^\pm \ (1, \pm 1)$ $W_\mu^3 \ (1, 0)$	0	0	0	0	0
B_μ	$(1, 1, 0)$	$(1, 1, 0)$	$(1, 0)$	0	0	0	0	0

$$M = (\# \text{ upper } SU(4) \text{ indices}) - (\# \text{ lower } SU(4) \text{ indices})$$