Lepton Number Violation and modified Higgs production in low-scale seesaw models

arXiv:1712.07611 – EPJC78(2018)795

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NuTheories workshop
09 November 2018

University of Pittsburgh

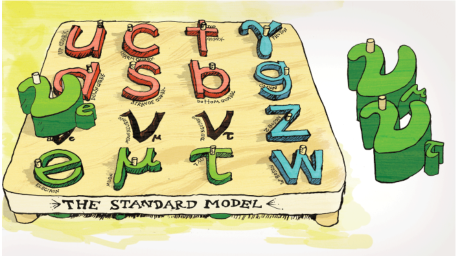
Massive neutrinos and New Physics

- Observation of ν oscillations
  $\Rightarrow$ at least 2 ν are massive
- Standard Model $L = \binom{\nu_L}{\ell_L}, \tilde{\phi} = \binom{H^{0*}}{H^-}$
  - No right-handed neutrino
    $\nu_R \rightarrow$ No Dirac mass term

$$\mathcal{L}_{\text{mass}} = -Y_\nu \bar{L} \tilde{\phi} \nu_R + \text{h.c.}$$

  - No Higgs triplet T
    $\rightarrow$ No Majorana mass term

$$\mathcal{L}_{\text{mass}} = -\frac{1}{2} f \bar{L} T L^c + \text{h.c.}$$



- Necessary to go beyond the Standard Model for ν mass
  - Radiative models
  - Extra-dimensions
  - R-parity violation in supersymmetry
  - Seesaw mechanisms $\rightarrow$ ν mass at tree-level a renormalizable way
    + BAU through leptogenesis

Dirac neutrinos ?

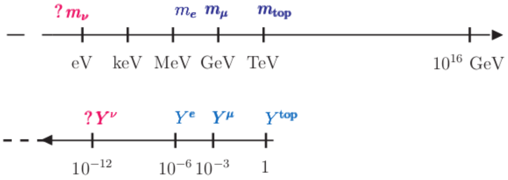
- Add gauge singlet (sterile), right-handed neutrinos ν_R $\Rightarrow \nu = \nu_L + \nu_R$

  $\mathcal{L}_{\text{mass}}^{\text{leptons}} = -Y_\ell \bar{L}\phi\ell_R - Y_\nu \bar{L}\tilde{\phi}\nu_R + \text{h.c.}$

  $\Rightarrow$ After electroweak symmetry breaking $\langle\phi\rangle = \binom{0}{\text{v}}$

  $\mathcal{L}_{\text{mass}}^{\text{leptons}} = -m_\ell \bar{\ell}_L \ell_R - m_D \bar{\nu}_L \nu_R + \text{h.c.}$

  $\Rightarrow$ 3 light active neutrinos: $m_\nu \lesssim 0.1\text{eV} \Rightarrow Y^\nu \lesssim 10^{-12}$



Majorana neutrinos ?

- Add gauge singlet (sterile), right-handed neutrinos ν_R
  $\mathcal{L}^{\text{leptons}}_{\text{mass}} = -Y_\ell \bar{L}\phi\ell_R - Y_\nu \bar{L}\tilde{\phi}\nu_R - \frac{1}{2}M_R\overline{\nu_R}\nu_R^c + \text{h.c.}$

  $\Rightarrow$ After electroweak symmetry breaking $\langle\phi\rangle = \binom{0}{\text{v}}$
  $\mathcal{L}^{\text{leptons}}_{\text{mass}} = -m_\ell \ell_L \ell_R - m_D \bar{\nu}_L \nu_R - \frac{1}{2}M_R\overline{\nu_R}\nu_R^c + \text{h.c.}$

  $3\,\nu_R \Rightarrow$ 6 mass eigenstates: $\nu = \nu^c$

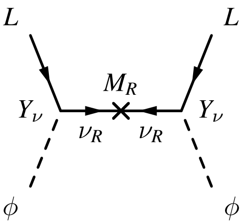
- $M_R\overline{\nu_R}\nu_R^c$ violates lepton number conservation $\Delta L = 2$

- ν_R gauge singlets
  $\Rightarrow M_R$ not related to SM dynamics, not protected by symmetries
  $\Rightarrow M_R$ between 0 and M_P

- $M_R \gg m_D \Rightarrow$ Type I seesaw mechanism

[Minkowski, 1977, Gell-Mann et al., 1979, Yanagida, 1979, Mohapatra and Senjanovic, 1980, Schechter and Valle, 1980]

Towards testable Type I variants



- Taking $M_R \gg m_D$ gives the "vanilla" type 1 seesaw
$$\mathrm{m}_\nu = -m_D M_R^{-1} m_D^T$$
$$\mathrm{m}_\nu \sim 0.1\,\mathrm{eV} \Rightarrow \left| \begin{array}{l} Y_\nu \sim 1 \quad \text{and} \quad M_R \sim 10^{14}\,\mathrm{GeV} \\ Y_\nu \sim 10^{-6} \text{ and } M_R \sim 10^2\ \mathrm{GeV} \end{array} \right.$$
- m_ν suppressed by small active-sterile mixing $m_D M_R^{-1}$
$\Rightarrow$ Suppressed heavy ν phenomenology
- Cancellation in matrix product to get large $m_D M_R^{-1}$
  - Lepton number, e.g. low-scale type I [Ilakovac and Pilaftsis, 1995] and others
    inverse seesaw [Mohapatra and Valle, 1986, Bernabéu et al., 1987]
    linear seesaw [Akhmedov et al., 1996, Barr, 2004, Malinsky et al., 2005]
  - Flavour symmetry, e.g. $A_4 \times \mathbb{Z}_2$ [Chao et al., 2010]
    A_4 or $\Sigma(81)$ [Chattopadhyay and Patel, 2017]
    $\mathbb{Z}(3)$ [Gu et al., 2009]
  - Gauge symmetry, e.g. $\mathrm{U}(1)_{B-L}$ [Pati and Salam, 1974] and others

$m_\nu = 0$ equivalent to conserved L for models with 3 ν_R
or less of equal mass [Kersten and Smirnov, 2007]

Extending the Kersten-Smirnov theorem

- Can the result of Kersten and Smirnov be generalized ?
- Are lepton number violating processes suppressed in all low-scale seesaw models ?

Theorem

If: - no cancellation between different orders of the seesaw expansion[a]
- no cancellations between different radiative orders[b]

Then $m_\nu = 0$ equivalent to having the neutrino mass matrix, in the basis $(\nu_L^C, \{\nu_{R,1}^{(1)}...\nu_{R,n}^{(1)}\}, \{\nu_{R,1}^{(2)}...\nu_{R,n}^{(2)}\}, \{\nu_{R,1}^{(3)}...\nu_{R,m}^{(3)}\})$

$$\tilde{M} = \begin{pmatrix} 0 & \alpha & \pm i\alpha & 0 \\ \alpha^T & M_1 & 0 & 0 \\ \pm i\alpha^T & 0 & M_1 & 0 \\ 0 & 0 & 0 & M_2 \end{pmatrix}, \quad (1)$$

for an arbitrary number of ν_R and to all radiative orders, with M_1 and M_2 diagonal matrices with positive entries and α a generic complex matrix.

[a] This is a necessary requirement to satisfy phenomenological constraints

[b] These are highly fine-tuned solution that cannot be achieved solely by specific textures of the neutrino mass matrix

Corollary on lepton number violation

Using a unitary matrix D, let us construct

$$Q = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \pm\frac{i}{\sqrt{2}}D & \frac{1}{\sqrt{2}}D & 0 \\ 0 & \frac{1}{\sqrt{2}}D & \pm\frac{i}{\sqrt{2}}D & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

then through a change of basis

$$Q^T \tilde{M} Q = \begin{pmatrix} 0 & \pm i\sqrt{2}(D^T\alpha^T)^T & 0 & 0 \\ \pm i\sqrt{2}D^T\alpha^T & 0 & \pm iD^T M_1 D & 0 \\ 0 & \pm iD^T M_1 D & 0 & 0 \\ 0 & 0 & 0 & M_2 \end{pmatrix} \sim \begin{pmatrix} 0 & M_D^T & 0 & 0 \\ M_D & 0 & M_R & 0 \\ 0 & M_R^T & 0 & 0 \\ 0 & 0 & 0 & M_2 \end{pmatrix}$$

- Similar to the L conserving limit of inverse and/or linear seesaw
- Explicitly L conserving taking the L assignment $(+1, -1, +1, 0)$

Corollary

The most general gauge-singlet neutrino extensions of the SM with no cancellation between different orders of the seesaw expansion, no fine-tuned cancellations between different radiative orders and which lead to three massless neutrinos are L conserving.

Eq. (1) as a sufficient condition

- Directly obtained from the corollary[1]

[1] In the seesaw limit, light neutrinos are Majorana fermions whose mass violate L conservation. Eq. (1) being equivalent to L conservation implies that the light neutrinos are massless.

Necessary condition: tree level

- At tree-level and for the first order of the seesaw expansion

$$\mathrm{m}_\nu \approx -m_D M_R^{-1} m_D^T$$

- If $m_D M_R^{-1} m_D^T = 0$ and using $Z = M_R^{-1} m_D^T$, then the exact block-diagonalisation of the full neutrino mass matrix gives
  [Korner et al., 1993, Grimus and Lavoura, 2000]

$$\begin{aligned}\mathrm{m}_\nu = & - \left(1 + Z^* Z^T\right)^{-\frac{1}{2}} Z^T m_D^T \left(1 + Z^\dagger Z\right)^{-\frac{1}{2}} \\ & - \left(1 + Z^T Z^*\right)^{-\frac{1}{2}} m_D Z \left(1 + Z Z^\dagger\right)^{-\frac{1}{2}} \\ & + \left(1 + Z^* Z^T\right)^{-\frac{1}{2}} Z^T M_R Z \left(1 + Z Z^\dagger\right)^{-\frac{1}{2}}\end{aligned}$$

- All terms contain $m_D M_R^{-1} m_D^T$ thus

$$\mathrm{m}_\nu = 0 \Rightarrow m_D M_R^{-1} m_D^T = 0$$

  to all orders of the seesaw expansion

An aside on the Kersten-Smirnov theorem

- Using tree-level contributions ($\mathrm{m}_\nu = 0 \Leftrightarrow m_D M_R^{-1} m_D^T = 0$), they get the general result if $\#\nu_R \leqslant 3$

$$m_D = m \begin{pmatrix} y_1 & y_2 & y_3 \\ a y_1 & a y_2 & a y_3 \\ b y_1 & b y_2 & b y_3 \end{pmatrix}, \quad \text{and} \quad \frac{y_1^2}{M_{R,1}} = \frac{y_2^2}{M_{R,2}} = \frac{y_3^2}{M_{R,3}}$$

- For $\#\nu_R > 3$, the system of linear equations in their proof is under-constrained
- In general, no symmetry is present. Necessary to assume degenerate heavy neutrinos to make a statement.
- Justify this by requiring radiative stability but approach based on running of the Weinberg operator
  $\rightarrow$ Works only if Higgs boson lighter than all heavy neutrinos

Necessary condition: one-loop level

- When $m_\nu = 0$ at tree-level, the one-loop induced masses are

$$\delta m_{ij} = \Re\left[\frac{\alpha_W}{16\pi^2 m_W^2} C_{ik} C_{jk} f\left(m_k\right)\right]$$

with C the mixing matrix in the neutral current and Higgs couplings and f the loop function

- In the basis where M_R is diagonal, the full neutrino mass matrix M is

$$M = \begin{pmatrix} 0 & m_{D1} & \dots & m_{Dn} \\ m_{D1}^T & \mu_1 & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ m_{Dn}^T & 0 & \dots & \mu_n \end{pmatrix}$$

and at the first order in the seesaw expansion

$$\delta m = 0 \Rightarrow \sum_{i=1}^{n} \mu_i^{-2} m_{Di} m_{Di}^T f\left(\mu_i\right) = 0$$

Necessary condition: one-loop level

- Cancellation could still come from summation of non-zero terms ☹
- But a rescaling $M \to \Lambda M$ does not affect the condition $m_\nu = \delta m = 0$
- $f(x)$ being monotonically increasing and strictly convex,

$$\sum_{i=1}^{n} \mu_i^{-2} m_{Di} m_{Di}^T f(\mu_i) = 0 \to \Lambda^{-2} \sum_{i=1}^{n} \mu_i^{-2} m_{Di} m_{Di}^T f(\Lambda \mu_i) = 0$$

generate linearly independent equations from which

$$m_\nu = 0 \Rightarrow m_{Di} m_{Di}^T = 0$$

since $\mu_i > 0, f(\mu_i) > 0$

From the necessary one-loop condition to the theorem

- We write $m_{Di}^T = (u^i, v^i, w^i)$, then

$$m_{Di} m_{Di}^T = \begin{pmatrix} u^{iT} u^i & u^{iT} v^i & u^{iT} w^i \\ v^{iT} u^i & v^{iT} v^i & v^{iT} w^i \\ w^{iT} u^i & w^{iT} v^i & w^{iT} w^i \end{pmatrix} = 0$$

- We construct $Y^i = u^{i*} u^{iT} + u^i u^{i\dagger}$. Imposing $u^{iT} u^i = 0$ and excluding the trivial solution $u^i = 0$, $\mathrm{rank}(Y^i) = 2$
- Y^i symmetric and real: we can build a basis of real orthogonal eigenvectors $b^i_{1 \ldots n_i}$. For the zero $n_i - 2$ eigenvalues,

$$Y^i b^i_k = 0 \Rightarrow ||u^i||^2 (u^{iT} b^i_k) = 0 \Rightarrow u^{iT} b^i_k = 0$$

- Then

$$u^{i'} = R^i_u u^i = \begin{pmatrix} b_1^{iT} u^i \\ b_2^{iT} u^i \\ b_3^{iT} u^i \\ \vdots \\ b_{n_i}^{iT} u^i \end{pmatrix} = \begin{pmatrix} u_1^{i'} \\ u_2^{i'} \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

From the necessary one-loop condition to the theorem

- Finally $u^{iT} u^i = 0 \Rightarrow u_2^{i'} = \pm i u_1^{i'}$
- Rinse and repeat for the other vectors, leaving M_R unaffected in the process, to get

$$m_{Di} = \begin{pmatrix} u_1^{i'} & \pm i u_1^{i'} & 0 & 0 & 0 & 0 & 0 & \dots & 0 \\ v_1^{i'} & \pm i v_1^{i'} & v_3^{i''} & \pm i v_3^{i''} & 0 & 0 & 0 & \dots & 0 \\ w_1^{i'} & \pm i w_1^{i'} & w_3^{i''} & \pm i w_3^{i''} & w_5^{i'''} & \pm i w_5^{i'''} & 0 & \dots & 0 \end{pmatrix}$$

- By rearranging the columns and rows, flavour-basis mass matrix becomes

$$M = \begin{pmatrix} 0 & \alpha & \pm i\alpha & 0 \\ \alpha^T & M_1 & 0 & 0 \\ \pm i\alpha^T & 0 & M_1 & 0 \\ 0 & 0 & 0 & M_2 \end{pmatrix} = \tilde{M} \quad \square$$

Consequences for phenomenology and model building

- Any symmetry that leads to massless light ν contains L as a subgroup or an accidental symmetry
- Prove the requirement of a nearly conserved L in low-scale seesaw models, baring fine-tuned solutions involving different radiative orders
- Smallness of the light neutrino mass related to the smallness of the L breaking parameter, or equivalently to the degeneracy of the heavy neutrinos in pseudo-Dirac pairs in low-scale seesaw models
- Expect L violating signatures to be suppressed
  $\rightarrow$ Suppressed LNV signatures at LHC [Kersten and Smirnov, 2007]
  $\rightarrow$ Assesment in progress for LNV meson decays
- Seems to be applicable to type III seesaw variants as well
  $\rightarrow$ Addendum to original paper in preparation

Searching for multi-TeV neutrinos

- How to search for heavy neutrinos with $m_N > \mathcal{O}(1\,\mathrm{TeV})$?

Use the Higgs sector to look for the effects of multi-TeV neutrinos

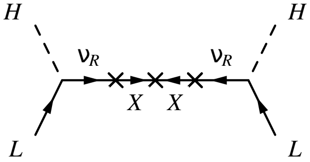
- $H\bar{\ell}_i\ell_j$:
  - Contribution negligible in the SM → evidence of new physics if observed
  - Complementary to other LFV searches
  - Large branching ratios are possible:
    $\mathrm{Br}(H \to \tau\mu) \sim 10^{-5}$ in ISS [Arganda, Herrero, Marcano, CW, 2015]
    $\mathrm{Br}(H \to \tau\mu) \sim 1\%$ in SUSY-ISS [Arganda, Herrero, Marcano, CW, 2016]
  - Sensitive to off-diagonal Yukawa couplings
- HHH:
  - Useful to validate the Higgs mechanism as the origin of EWSB
  - Deviations as large as $\sim +30\%$, within ILC and FCC-hh reach [Baglio, CW, 2017]
  - Sensitive to diagonal Yukawa couplings
- WWH production
  - Overlooked channel for BSM searches
  - t-channel process: different dependence on the heavy neutrino mass
  - Sensitive to diagonal Yukawa couplings

An example of low-scale seesaw models

- Type I seesaw variant with approximately conserved total lepton number
- Add right-handed neutrinos ν_R $(L = +1)$ and X $(L = -1)$
  [Mohapatra and Valle, 1986, Bernabéu et al., 1987]

$$\mathcal{L}_{inverse} = -Y_\nu \overline{L}\tilde{\phi}\nu_R - M_R \overline{\nu_R^c} X - \frac{1}{2}\mu_X \overline{X^c} X + \text{h.c.}$$

with $m_D = Y_\nu \mathrm{v}\,, M^\nu = \begin{pmatrix} 0 & m_D & 0 \\ m_D^T & 0 & M_R \\ 0 & M_R^T & \mu_X \end{pmatrix}$



$$m_\nu \approx \frac{m_D^2}{M_R^2}\mu_X$$

$$m_{N_1,N_2} \approx \mp M_R + \frac{\mu_X}{2}$$

2 scales: μ_X and M_R

- Decouple neutrino mass generation from active-sterile mixing
- Inverse seesaw: $Y_\nu \sim \mathcal{O}(1)$ and $M_R \sim 1\,\mathrm{TeV}$
  $\Rightarrow$ Expect large impact on the Higgs sector

Most relevant constraints for the ISS

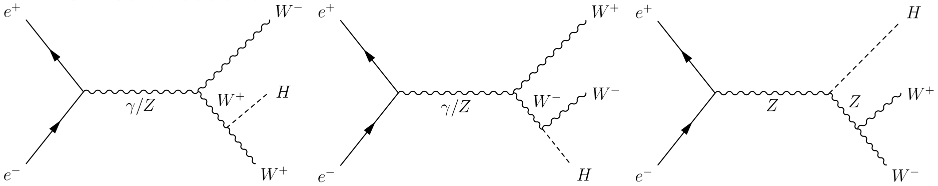
- Accommodate neutrino oscillation data using μ_X-parametrization
  [Arganda, Herrero, Marcano, **CW**, 2015; Baglio and **CW**, 2017]

$$\mu_X = M_R^T \, Y_\nu^{-1} \, U_{\text{PMNS}}^* m_\nu U_{\text{PMNS}}^\dagger \, Y_\nu^{T^{-1}} M_R \text{v}^2 \qquad \text{and beyond}$$

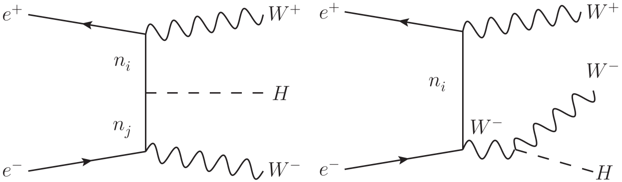
- Charged lepton flavour violation
  → For example: $\text{Br}(\mu \to e\gamma) < 4.2 \times 10^{-13}$ [MEG, 2016]
  ⇒ Avoid by using diagonal Y_ν and M_R
- Global fit to EWPO and low-energy data [Fernandez-Martinez et al., 2016]
- Electric dipole moment: 0 with real PMNS and mass matrices
- Invisible Higgs decays: $M_R > m_H$, does not apply
- Yukawa perturbativity: $|\frac{Y_\nu^2}{4\pi}| < 1.5$

WWH production

- Idea: Probe Y_ν at tree-level with off-shell N $\Rightarrow$ t-channel $e^+e^- \to W^+W^-H$
- Good detection prospects in SM [Baillargeon et al., 1994]
- SM contributions:

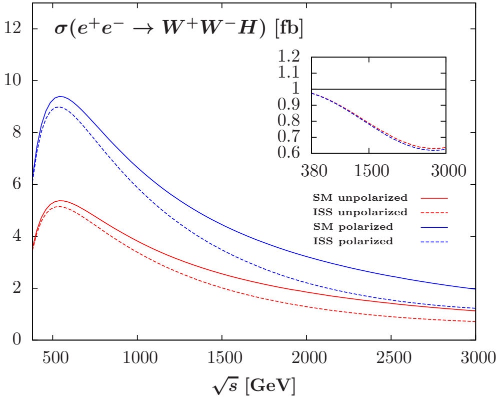


- SM+ISS contributions:



- SM electroweak corrections negligible for $\sqrt{s} > 600\,\mathrm{GeV}$ [Mao et al., 2009]
  $\Rightarrow$ neglected in our analysis

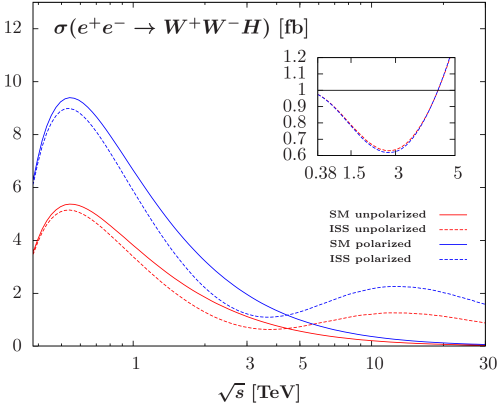
CoM energy dependence



- LO calculation, neglecting m_e
- Calculation done with `FeynArts`, `FormCalc`, `BASES`
- Deviation from the SM in the insert
- Polarized: $P_{e^-} = -80\%$, $P_{e^+} = 0$
- $\sigma(e^+e^- \to W^+W^-H)_{\text{pol}} \sim 2\sigma(e^+e^- \to W^+W^-H)_{\text{unpol}}$
- $Y_\nu = \mathbb{1}$, $M_{R_1} = 3.6\,\text{TeV}$, $M_{R_2} = 8.6\,\text{TeV}$, $M_{R_3} = 2.4\,\text{TeV}$

- Destructive interference between SM and heavy neutrino contributions
- Maximal deviation of -38% close to $3\,\text{TeV}$

What about novel accelerators and muon colliders ?



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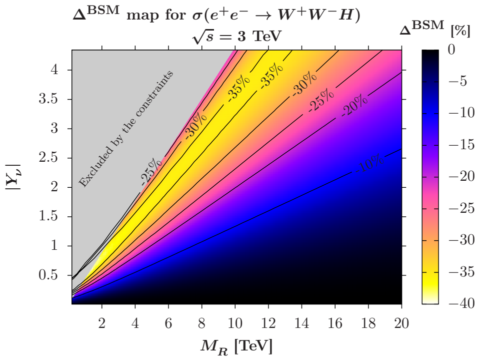
- $\sigma(e^+e^- \to W^+W^-H)_{\text{pol}} \sim 2\sigma(e^+e^- \to W^+W^-H)_{\text{unpol}}$
- $Y_\nu = \mathbb{1}, M_{R_1} = 3.6\,\text{TeV}, M_{R_2} = 8.6\,\text{TeV}, M_{R_3} = 2.4\,\text{TeV}$
- Enhancement from the t-channel heavy neutrinos going on-shell $\Rightarrow$ t-channel diagrams with heavy neutrino dominate at high $\sqrt{s}$

- ALIC $\to$ Accelerating gradient 1 GeV/m, linear e^+e^- collider at 10-50 TeV
  LEMMA $\to$ muon collider in LHC tunnel could reach 28 TeV

Results in the ISS



- $\Delta^{\rm BSM} = (\sigma^{\rm ISS} - \sigma^{\rm SM})/\sigma^{\rm SM}$
- Polarization $P_{e^-} = -80\%$

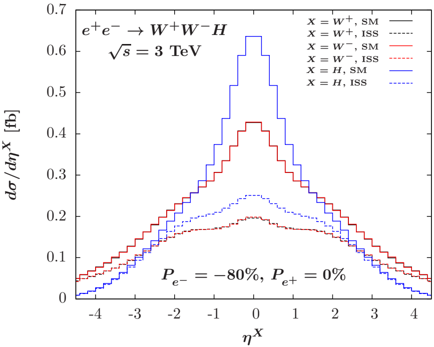
$$\mathcal{A}^{\rm ISS}_{\rm approx} = \frac{(1\text{ TeV})^2}{M_R^2}\mathrm{Tr}(Y_\nu Y_\nu^\dagger) \times \left(17.07 - \frac{19.79\text{ TeV}^2}{M_R^2}\right)$$

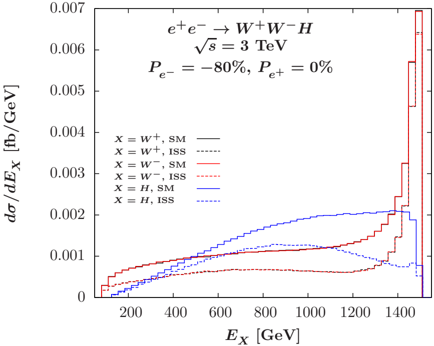
$$\Delta^{\rm BSM}_{\rm approx} = (\mathcal{A}^{\rm ISS}_{\rm approx})^2 - 11.94\,\mathcal{A}^{\rm ISS}_{\rm approx}$$

- Fit agrees within 1% for $M_R > 3\,\mathrm{TeV}$

- Maximal deviation of -38%, $\sigma^{\rm ISS}_{\rm pol} = 1.23$ fb
  $\rightarrow$ ISS induces sizeable deviations in large part of the parameter space
- Provide a new probe of the $\mathcal{O}(10)$ TeV region
  $\Rightarrow$ Complementary to existing observables

Enhancing the deviations



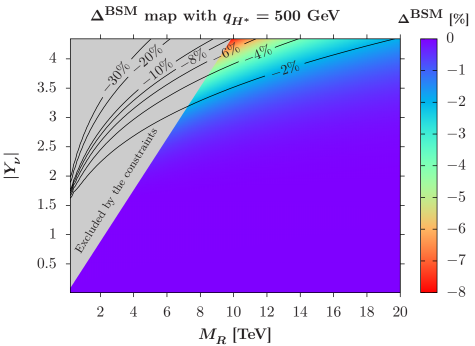


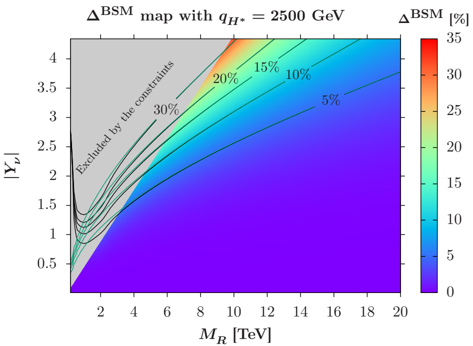
- Stronger destructive interference from ISS for: – central production
  – larger Higgs energy
- Cuts: $|\eta_H| < 1$, $|\eta_{W^\pm}| < 1$ and $E_H > 1$ TeV

	Before cuts	After cuts
σ_{SM} (fb)	1.96	0.42
σ_{ISS} (fb)	1.23	0.14
Δ^{BSM}	-38%	-66%

Comparison: λ_{HHH} in the ISS

[JHEP04(2017)038]





- $\Delta^{\mathrm{BSM}} = \frac{1}{\lambda_{HHH}^{1r,\mathrm{SM}}} \left(\lambda_{HHH}^{1r,\mathrm{full}} - \lambda_{HHH}^{1r,\mathrm{SM}} \right)$
- Diagonal Y_ν: full calculation in black, approximate formula in green
- Heavy ν effects at the limit of the ILC (10%) sensitivity [Fujii et al., 2015]
- Heavy ν effects clearly visible at FCC-hh (5%) [He et al., 2016]
- Sizeable deviation in a smaller part of the parameter space

Conclusions

- ν oscillations $\rightarrow$ New physics is needed to generate masses and mixing
- One of the simplest ideas: Add right-handed, sterile neutrinos
- Nearly conserved L is a cornerstone of low-scale type I seesaw variants
- Higgs physics provide new opportunities to test seesaw models
- Corrections to W^+W^-H production as large as -66% after cuts
- Maximal for diagonal Y_ν and provide new probes of the $\mathcal{O}(10)$ TeV region
- Next Step: Assess impact on LNV processes
  Sensitivity studies for W^+W^-H production

Backup slides

Cancellation between different seesaw orders

- To second order in the expansion

$$m_\nu^{(2)} = -m_\nu^{(1)} + \frac{1}{2}\left(m_\nu^{(1)}\theta + \theta^T m_\nu^{(1)}\right)$$

with $m_\nu^{(1)}$ the first order expression and θ is $Z^\dagger Z$ up to a unitary transformation

- Then

$$(m_\nu^{(2)})_{ii} = 0 \Leftrightarrow -\hat{m}_{\nu ii}^{(1)} + \hat{m}_{\nu ii}^{(1)}\theta_{ii} = 0$$

and $\theta_{ii} = 1$

- This contradicts [Fernandez-Martinez et al., 2016] which gives $||\theta|| \leqslant 0.0075$

Details of one-loop proof I

- The loop function is

$$f\left(m_k\right) = m_k\left(3m_Z^2 g_{kZ} + m_H^2 g_{kH}\right)$$

where

$$g_{ab} = \frac{m_a^2}{m_a^2 - m_b^2}\log\frac{m_a^2}{m_b^2}$$

which gives

$$U_l^T\left(1 + Z^T Z^*\right)^{-1} Z^T U_h^* f_h U_h^\dagger Z\left(1 + Z^\dagger Z\right)^{-1} U_l = 0$$

$$Z^T U_h^* f_h U_h^\dagger Z = 0$$

to the first order in the seesaw expansion

$$U_h \approx 1$$

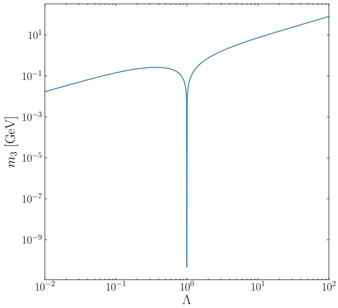
$$Z^T F_h Z = 0$$

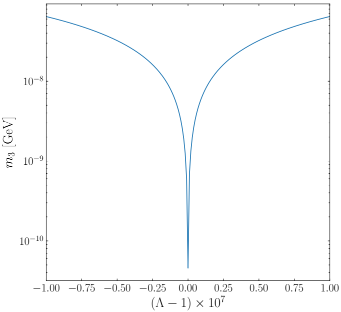
Details of one-loop proof II

- Once we have
$$u^{i'} = \left(u_1^{i'}, \pm i u_1^{i'}, 0, \ldots, 0\right)^T$$
Under this transformation, we have
$$u^{iT} v^i = 0 \rightarrow u'^{iT} v'^i = 0$$
leading us to conclude that
$$v^{i'} = \left(v_1^{i'}, \pm i v_1^{i'}, v_3^{i'}, v_4^{i'}, \ldots, v_{n_i}^{i'}\right)^T$$
- Similarly, we construct a second matrix R_v acting on $\left(v_3^{i'}, v_4^{i'}, \ldots, v_{n_i}^{i'}\right)^T$ such that $v^{i'}$ is reduced to
$$v^{i''} = \left(v_1^{i'}, \pm i v_1^{i'}, v_3^{i''}, \pm i v_3^{i''}, 0, \ldots, 0\right)^T$$
- Rinse and repeat for w

Fine-tuning

We adopt here the idea of [Lopez-Pavon et al., 2015], where the tree-level and one-loop contributions cancel.





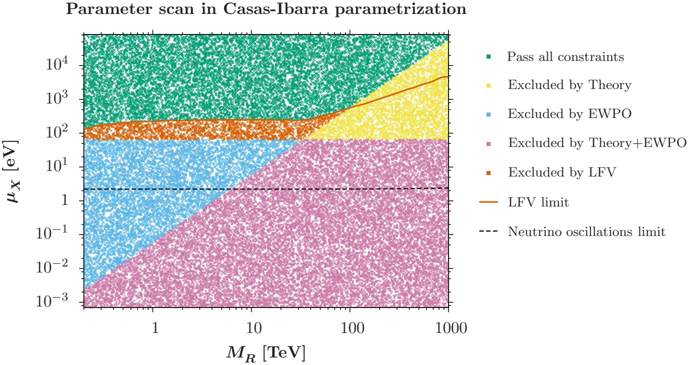
Evolution of m_3 as a function of the rescaling parameter Λ. Input masses and couplings where chosen to give $m_\nu = m_{\text{tree}} + m_{\text{1-loop}} = 0.046\,\text{eV}$ at $\Lambda = 1$.
A deviation of less then 10^{-7} here, is enough to spoil the cancellation and contradict experimental limits.

Next-order terms in the μ_X-parametrization

- Weaker constraints on diagonal couplings
  $\to$ Large active-sterile mixing $m_D M_R^{-1}$ for diagonal terms
- Previous parametrizations built on the 1st term in the $m_D M_R^{-1}$ expansion
  $\to$ Parametrizations breaks down
- Solution: Build a parametrization including the next order terms
- The next-order μ_X-parametrization is then

$$\mu_X \simeq \left(\mathbf{1} - \frac{1}{2} M_R^{*-1} m_D^\dagger m_D M_R^{T-1}\right)^{-1} M_R^T m_D^{-1} U_{\text{PMNS}}^* m_\nu U_{\text{PMNS}}^\dagger m_D^{T-1} M_R \times \left(\mathbf{1} - \frac{1}{2} M_R^{-1} m_D^T m_D^* M_R^{\dagger -1}\right)^{-1}$$

Impact of constraints



- Y_ν increases when M_R increases and/or μ_X decreases
- Strongest constraints:
  - Lepton flavour violation, mainly $\mu \to e\gamma$
  - Yukawa perturbativity (and neutrino width)
- Larger Y_ν (and effects) necessarily excluded by LFV constraints ?
  $\to$ Switch to μ_X-parametrization and use diagonal Y_ν