CONSTRAINING LEPTONIC FLAVOUR MODEL PARAMETERS

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NUTHEORIES

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Fermilab

Overview

- Basic principles underlying leptonic flavour models
- Model, its parameter space and its experimental constraints
- Tool chain and how to calculate exclusion regions
- Is there any complementary between these experiments?

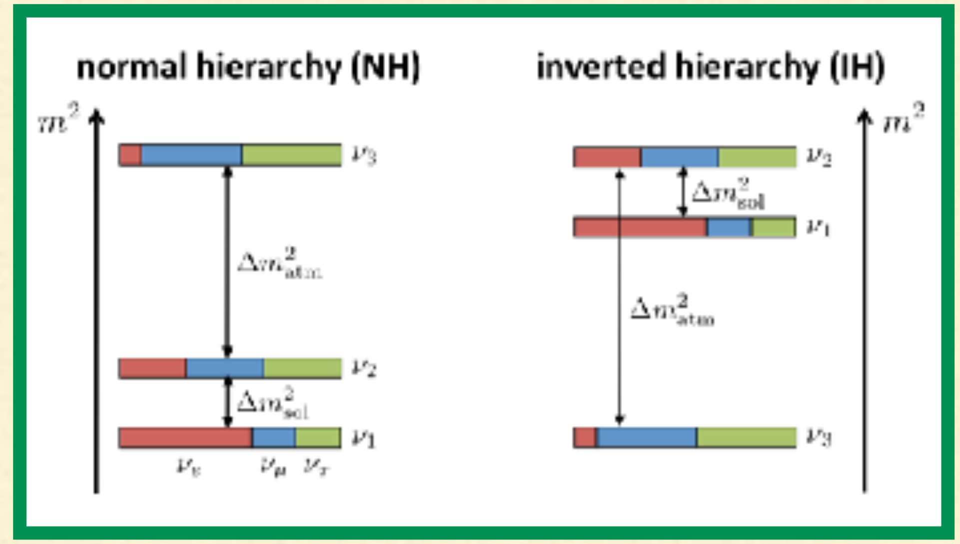
Neutrino Masses and Mixing

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \begin{pmatrix}U_{e1} & U_{e2} & U_{e3}\\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3}\\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3}\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}$$

flavour states

PMNS matrix

mass states



$$U m_\nu U^\dagger = m_{\nu \mathrm{diag}} \qquad P(\nu_\alpha \to \nu_\beta) = \left| \sum_i U_{\alpha i} U^*_{\tau i} e^{-i\frac{m_i^2}{2E}t} \right|^2$$

atmospheric | reactor | solar

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \begin{pmatrix}1 & 0 & 0\\ 0 & c_{23} & s_{23}\\ 0 & -s_{23} & c_{23}\end{pmatrix}\begin{pmatrix}c_{13} & 0 & -s_{13}e^{-i\delta}\\ 0 & 1 & 0\\ s_{13}e^{i\delta} & 0 & c_{23}\end{pmatrix}\begin{pmatrix}c_{12} & s_{12} & 0\\ -s_{12} & c_{12} & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}$$

T2K, MINOS, K2K

$40.3^\circ \leq \theta_{23} \leq 51.5^\circ$

Daya Bay, RENO, Double Chooz

$8.09^\circ \leq \theta_{13} \leq 8.98^\circ$

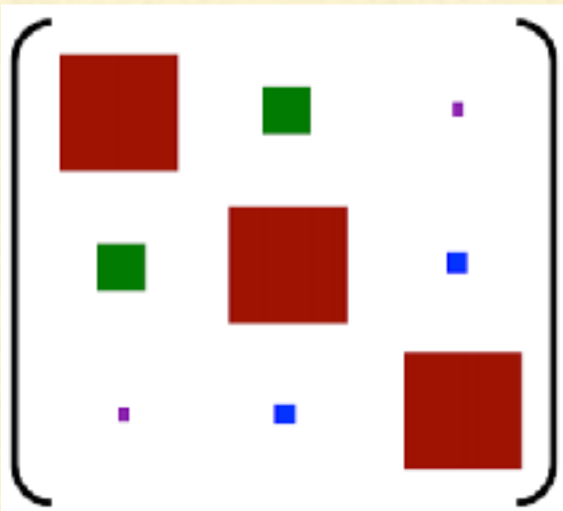
SuperK, KamLAND

$31.42^\circ \leq \theta_{12} \leq 36.05^\circ$

nu-fit Jan 2018

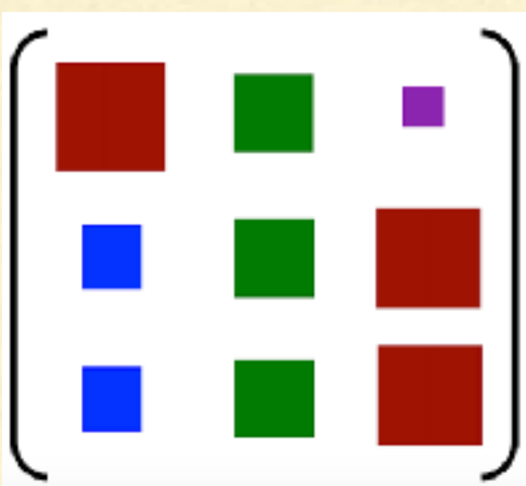
Flavour Model Motivation

quark mixing



Perturbed Identity Matrix
Small Mixing
small CPV

leptonic mixing



entries resemble CG coefficient of discrete groups

Anarchy

PMNS matrix described as the result of a random draw from unbiased 3 x 3 unitary matrix

Does not work for CKM

Hall, de Gouvea, Murayama

Symmetry

PMNS matrix results from the breaking of a non-Abelian symmetry at high energy scales

Difficult to apply to quark sector

Altarelli, Everett, Feruglio, King, Ding, Hagedorn, Petrov, M. C Chen, Harrison, Perkins, Scott, Luhn…….

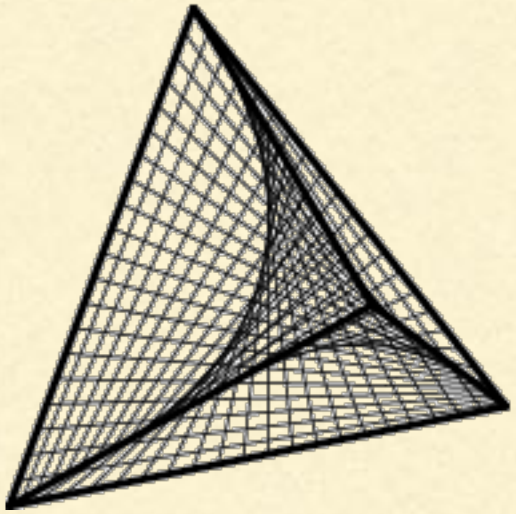
Flavour Model Paradigm

Energy

leptonic SU(2)$_L$ doublets in triplet of flavour group

Low energy effective theory is SM and Majorana mass term

G_F

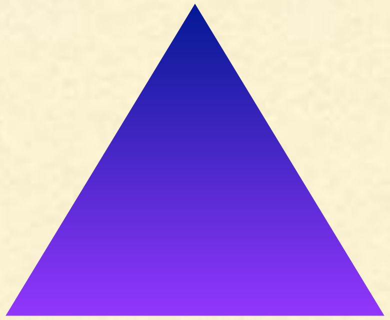


Non-Abelian Flavour Symmetry

vev of flavon breaks flavour symmetry

Abelian Residual Symmetry

G_ℓ



G_ν

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}$$

$$\ell_L \to T \ell_L$$

$$\nu_L \to S \nu_L$$

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$$

$$\omega = e^{\frac{2\pi i}{3}}$$

$$T^\dagger m_\ell m_\ell^\dagger T = m_\ell m_\ell^\dagger$$

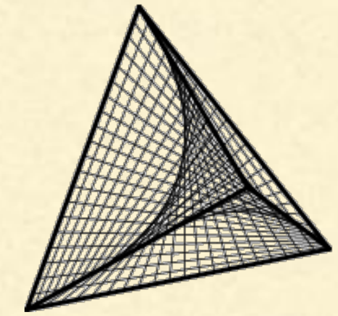
$$S^T m_\nu S = m_\nu$$

The model we constrain

1604.00925 and 1607.05599:
Pascoli and Zhou

$$\varphi = (\varphi_1, \varphi_2, \varphi_3)^T \sim 3,\ \chi = (\chi_1, \chi_2, \chi_3)^T \sim 3$$

$$\ell_L = (\ell_{eL}, \ell_{\mu L}, \ell_{\tau L})^T \sim 3,\ e_R \sim 1,\ \mu_R \sim 1',\ \tau_R \sim 1''$$



flavon pseudo-real triplets

$$-\mathcal{L}_l = \frac{y_e}{\Lambda}(\overline{\ell_L}\varphi)_{\mathbf{1}} e_R H + \frac{y_\mu}{\Lambda}(\overline{\ell_L}\varphi)_{\mathbf{1}''} \mu_R H + \frac{y_\tau}{\Lambda}(\overline{\ell_L}\varphi)_{\mathbf{1}'} \tau_R H + \text{h.c.}\,,$$

$$-\mathcal{L}_\nu = \frac{y_1}{2\Lambda\Lambda_{\mathrm{W}}}\left((\overline{\ell_L}\tilde{H}\tilde{H}^T\ell_L^c)_{\mathbf{3}_S}\chi\right)_{\mathbf{1}} + \frac{y_2}{2\Lambda_{\mathrm{W}}}(\overline{\ell_L}\tilde{H}\tilde{H}^T\ell_L^c)_{\mathbf{1}} + \text{h.c.}$$

Assume neutrino Majorana

$$\langle\varphi\rangle = (1,0,0)^T \frac{v_\varphi}{\sqrt{n}}$$

$$\langle\chi\rangle = (1,1,1)^T \frac{v_\chi}{\sqrt{3n}}$$

$$M_l = \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix} \frac{v v_\varphi}{\sqrt{2n}\Lambda}$$

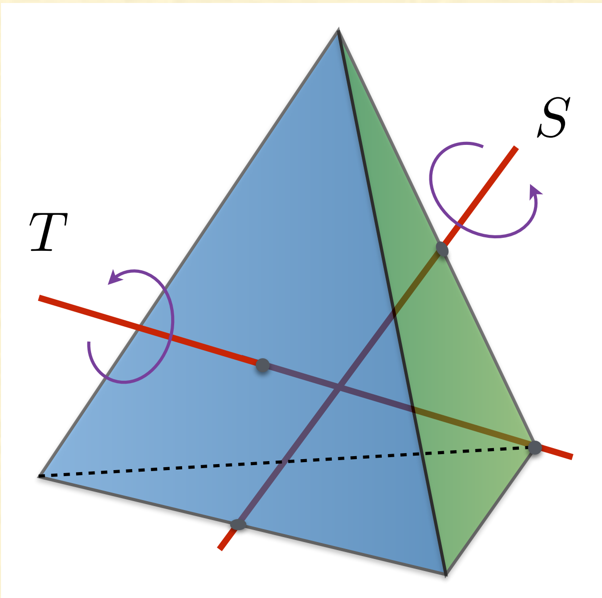
$$M_\nu = \begin{pmatrix} 2a+b & -a & -a \\ -a & 2a & -a+b \\ -a & -a+b & 2a \end{pmatrix}$$

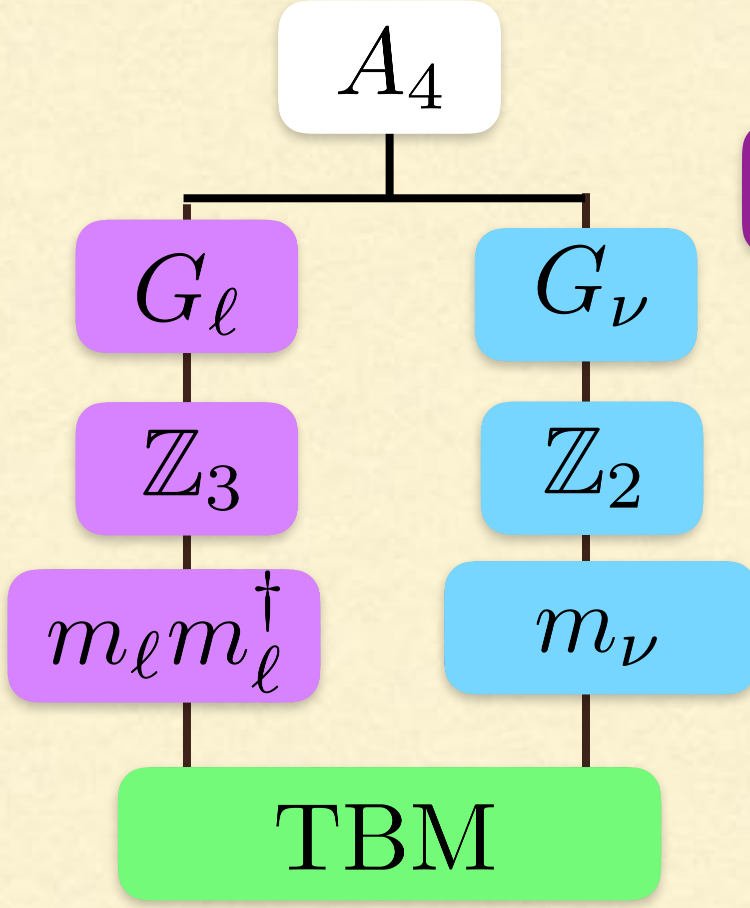
$$T\langle\varphi\rangle = \langle\varphi\rangle \qquad S\langle\chi\rangle = \langle\chi\rangle$$

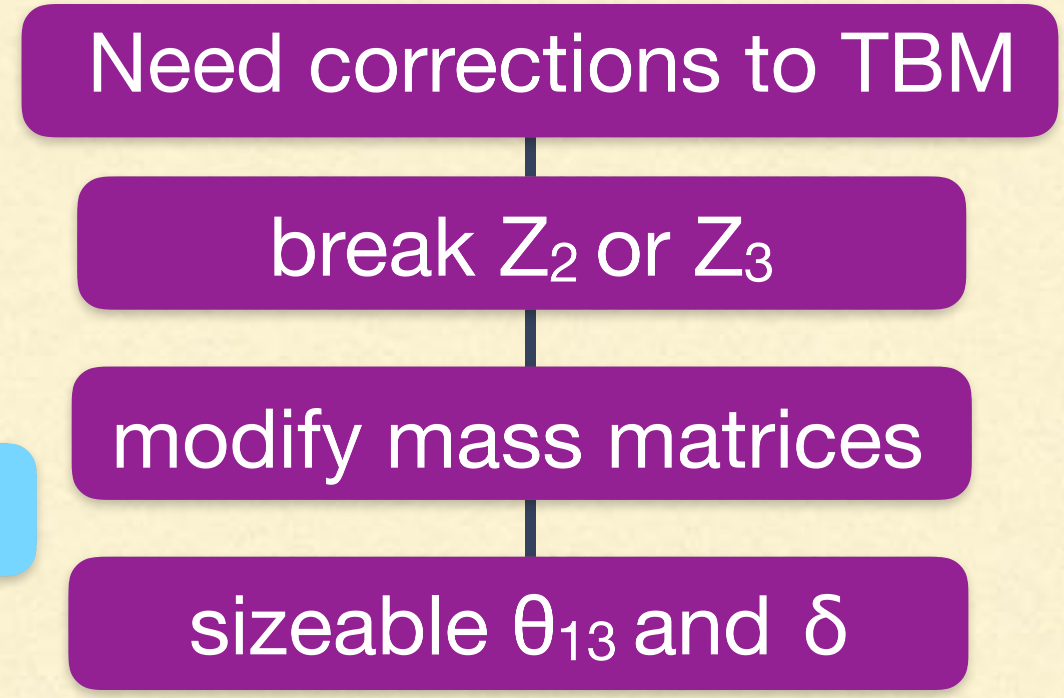
Results in TBM mixing of PMNS matrix

$a \equiv y_1 v_\chi v^2/(4\sqrt{3n}\Lambda\Lambda_{\mathrm{W}})$

$b \equiv y_2 v^2/2\Lambda_{\mathrm{W}}$







$$U_{TBM} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$V_{\not{\mathbb{Z}}_3}(\varphi) = \frac{1}{2} A(\varphi_2^2 + 2\varphi_1\varphi_2^*) + \text{h.c.},$$

function of the
6 model parameters

How does this flavour sector communicate with us?

Scalar Sector

$$V_{\text{cross}}(H, \varphi) = \frac{1}{2}\epsilon H^\dagger H (\varphi\varphi)_{\mathbf{1}}$$

Cross coupling between Higgs and flavons

$$\varphi_1 = v_\varphi + \tilde{\varphi}_1\,, \quad \varphi_2 = \epsilon_\varphi v_\varphi + \tilde{\varphi}_2\,.$$

$$(\varphi\varphi)_{\mathbf{1}} = \left(\varphi_1^2 + 2\varphi_2\varphi_2^*\right)$$

$$V(\varphi) = \frac{1}{2}\mu_\varphi^2 I_{1\varphi} + \frac{g_1}{4} I_{1\varphi}^2 + \frac{g_2}{4} I_{2\varphi},$$

$A_4 \times Z_2$ invariant potential

$$I_{1\varphi} = \varphi_1^2 + 2|\varphi_2|^2\,, \quad I_{2\varphi} = \frac{1}{3}\varphi_1^4 - \frac{2}{3}\varphi_1(\varphi_2^3 + \varphi_2^{*3}) + |\varphi_2|^4.$$

Scalar Sector

$$V_{\rm cross}(H,\varphi) = \frac{1}{2}\epsilon H^\dagger H (\varphi\varphi)_{\mathbf{1}}$$

Cross coupling between Higgs and flavons

$$\varphi_1 = v_\varphi + \tilde{\varphi}_1\,, \quad \varphi_2 = \epsilon_\varphi v_\varphi + \tilde{\varphi}_2\,.$$

$$(\varphi\varphi)_{\mathbf{1}} = \left(\varphi_1^2 + 2\varphi_2\varphi_2^*\right)$$

complex, 2 parameters

$$V(\varphi) = \frac{1}{2}\mu_\varphi^2 I_{1\varphi} + \frac{g_1}{4} I_{1\varphi}^2 + \frac{g_2}{4} I_{2\varphi},$$

A_4 x Z_2 invariant potential

$$I_{1\varphi} = \varphi_1^2 + 2|\varphi_2|^2\,, \quad I_{2\varphi} = \frac{1}{3}\varphi_1^4 - \frac{2}{3}\varphi_1(\varphi_2^3 + \varphi_2^{*3}) + |\varphi_2|^4.$$

Yukawa Sector

charged lepton flavour conserving

$$-\mathcal{L}^{\tilde{h},\tilde{\varphi}_1}_{\rm clfc} = \sum_{l=e,\mu,\tau} \frac{m_l}{v_H}\bar{l}l\tilde{h} + \frac{m_l}{v_\varphi}\bar{l}l\tilde{\varphi}_1 + \frac{m_l}{v_H v_\varphi}\bar{l}l\tilde{\varphi}_1\tilde{h}\,,$$

$$\begin{aligned}-\mathcal{L}^{\tilde{\varphi}_2}_{\rm clfv} = &\frac{m_e}{v_\varphi}\left(\overline{\mu_L}e_R\tilde{\varphi}_2 + \overline{\tau_L}e_R\tilde{\varphi}_2^*\right) + \frac{m_e}{v_H v_\varphi}\left(\overline{\mu_L}e_R\tilde{\varphi}_2 + \overline{\tau_L}e_R\tilde{\varphi}_2^*\right)\tilde{h}\\
&+\frac{m_\mu}{v_\varphi}\left(\overline{\tau_L}\mu_R\tilde{\varphi}_2 + \overline{e_L}\mu_R\tilde{\varphi}_2^*\right) + \frac{m_\mu}{v_H v_\varphi}\left(\overline{\tau_L}\mu_R\tilde{\varphi}_2 + \overline{e_L}\mu_R\tilde{\varphi}_2^*\right)\tilde{h}\\
&+\frac{m_\tau}{v_\varphi}\left(\overline{e_L}\tau_R\tilde{\varphi}_2 + \overline{\mu_L}\tau_R\tilde{\varphi}_2^*\right) + \frac{m_\tau}{v_H v_\varphi}\left(\overline{e_L}\tau_R\tilde{\varphi}_2 + \overline{\mu_L}\tau_R\tilde{\varphi}_2^*\right)\tilde{h} + \text{h.c.},\end{aligned}$$

Final state tau dominated

charged lepton flavour violating

Model Parameter Space

Parameter p	min(p)	max(p)
$\log_{10}(v_\varphi)$	1	3
$\log_{10}(\varepsilon)$	-3	0.5
$\log_{10}(g_1)$	-4	0
-$\log_{10}(g_2)$	-4	0
$\log_{10}(\lvert\epsilon_\varphi\rvert)$	-3	0.5
θ_φ	0	2π

Table 1: Parameter sampling boundaries.

1. Any flavon mass is too light, i.e. $m(s_i) < 10$ GeV, $i = 1 \cdots 3$.
2. All flavon masses are > 1 TeV.
3. Any flavon mass is too close to the Higgs — $|m(s_i) - m_H| < 5$ GeV for $i = 1, 2, 3$.
4. Any flavon mass which is not the Higgs is close to degenerate – $|m(s_i) - m(s_j)| <$ 100 MeV for $i, j = 1, 2, 3$.
5. $\lambda g < \frac{\varepsilon}{4}$
6. $g_1 + \frac{g_2}{3} < 0$

Conditions for Physicality

- **Model:** Z_3-breaking gives realistic flavour structure with minimal number of parameters

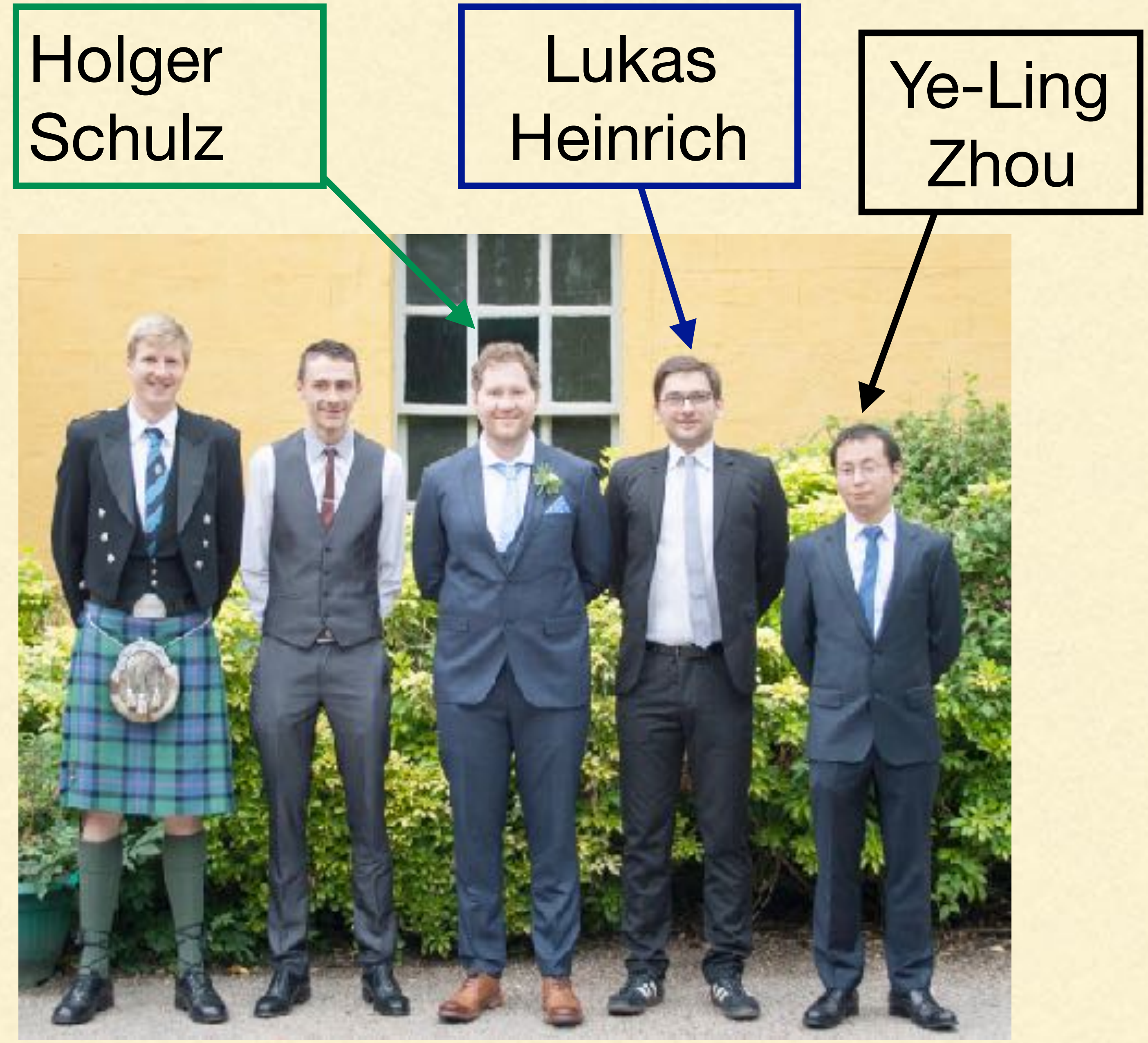
- **Identify:** collider signatures

- **Constraints:** Higgs-width, Higgs-scalar mixing, g-2, CLFV BRs.

- **Analysis:** recast 8 TeV ATLAS multi-lepton search

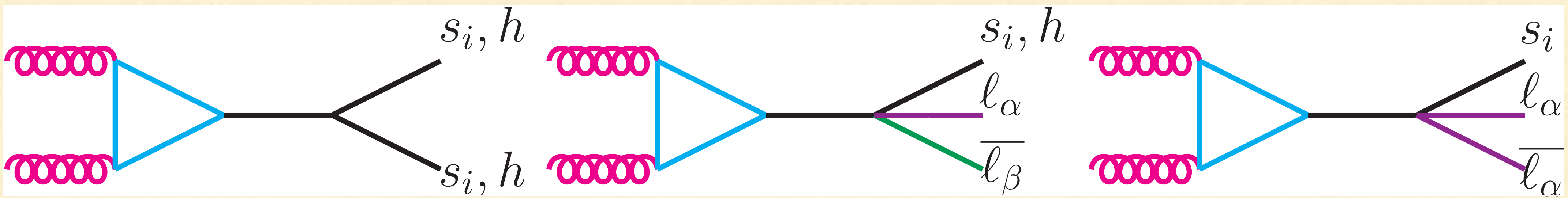
- **Tools:** MC event generation and CL_S method.

- **Results:** **1810.05648**



Collider Constraints

1. Flavons mix with the Higgs and decay via CLFV and CLFC processes.



2. Measured Higgs width ~ 22 MeV versus 4 MeV SM calculation

1405.3455

3. Make sure the Higgs is mostly comprised of the Higgs mass eigenstate.

Robens, Stefaniak, Pruna, Godunov, Roznanov, Vysotsky, Zhemchugov

1303.1150, 1501.02234, 1503.01618, 1502.01361

g-2 and MEG Experimental Constraints

0602035, 1311.2198

E821 (BNL) measures muon anomalous magnetic moment

$$\Delta a_\mu = a_\mu^{\rm exp} - a_\mu^{\rm SM} = (2.87 \pm 0.8) \times 10^{-9} \;\; (3.6\sigma)$$

MEG experiment measures μ→eɣ

$${\rm Br}\,(\mu \to e\gamma) \leq 4.2 \times 10^{-13}$$ at 90% C.L

1605.05081

g-2 and MEG Experimental Constraints

muon g-2 experiment based at Fermilab measures muon anomalous magnetic moment

$$\Delta a_\mu = a_\mu^{\rm exp} - a_\mu^{\rm SM} = (2.87 \pm 0.8) \times 10^{-9} \ (3.6\sigma)$$

MEG experiment measures μ→eɣ

$$\mathrm{Br}\,(\mu \to e\gamma) \leq 4.2 \times 10^{-13}$$ at 90% C.L

The above analytic constraints are functions of the 6 model parameters, see back up slides for formulas

The Collider Analysis in a nutshell

ATLAS Analysis: 8 TeV

1411.2921

Search for new phenomena in events with three or more charged leptons in pp collisions at $\sqrt{s}$ = 8 TeV with the ATLAS detector

20.3^{-1} fb

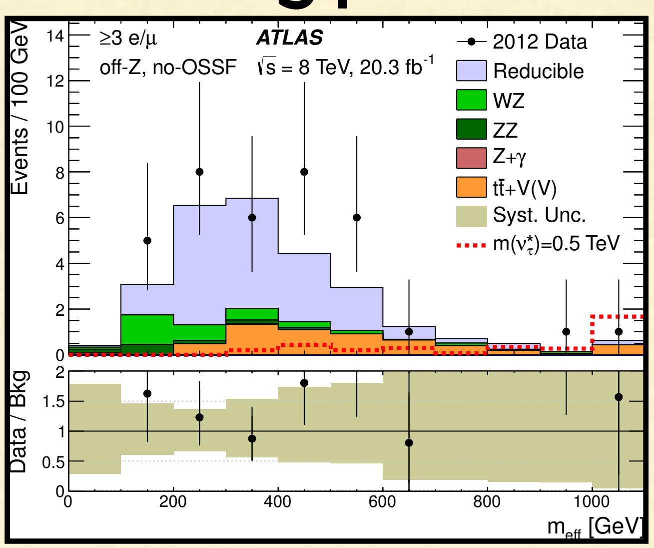
no-OSSF

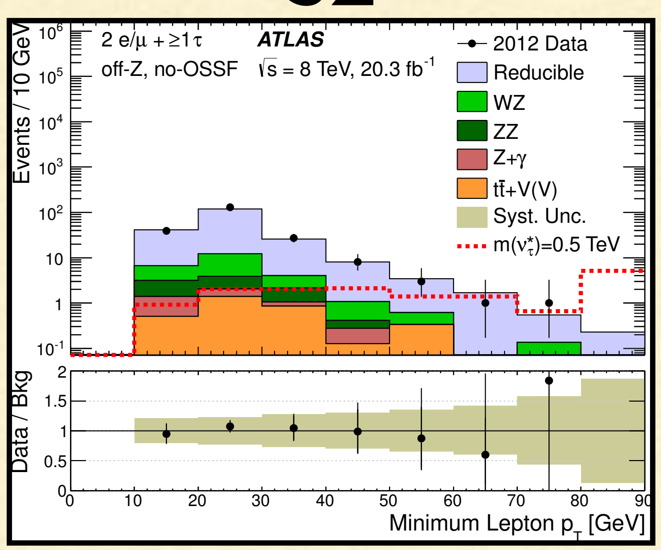
- no-OSSF ≥3e/μ — **S1**
- no-OSSF 2e/μ≥1τ_{had} — **S2**

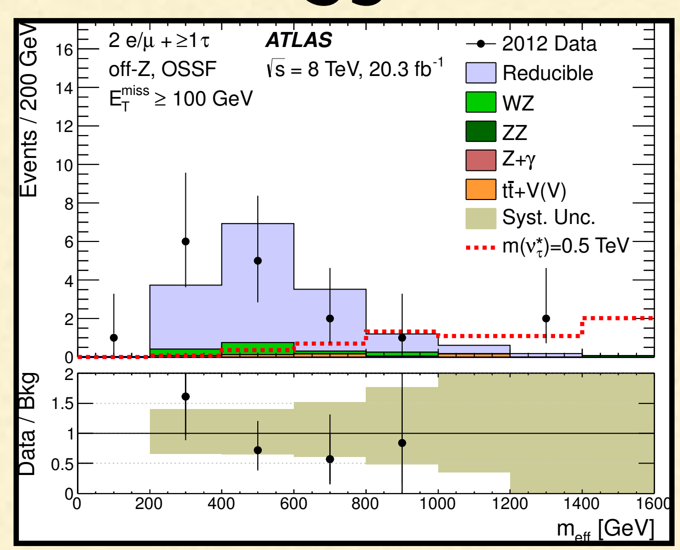
OSSF

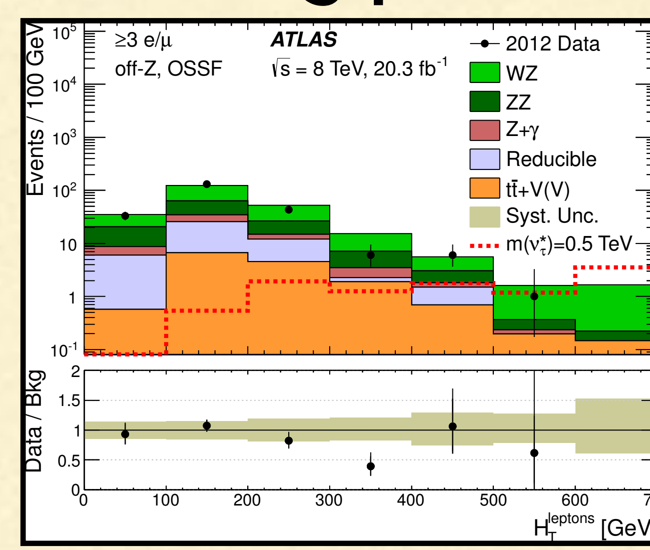
- OSSF ≥3e/μ — **S3**
- OSSF 2e/μ≥1τ_{had} — **S4**

distributions have not been corrected for detector effect i.e. not unfolded









m_{eff}: effective mass of event combining sum of jets, missing energy and lepton p_T

H_T^{lepton}: scalar sum of lepton p_T used to characterise event

ATLAS Analysis: 8 TeV

1411.2921

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OSSF

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S1

no-OSSF 2e/μ≥1τ$_{had}$

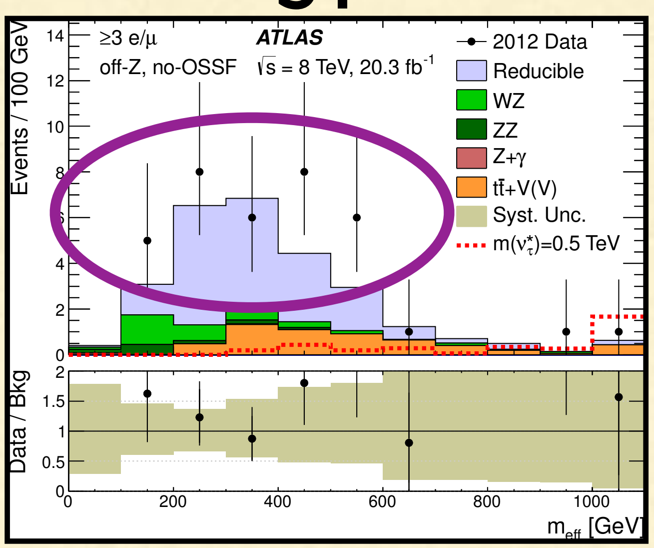
S2

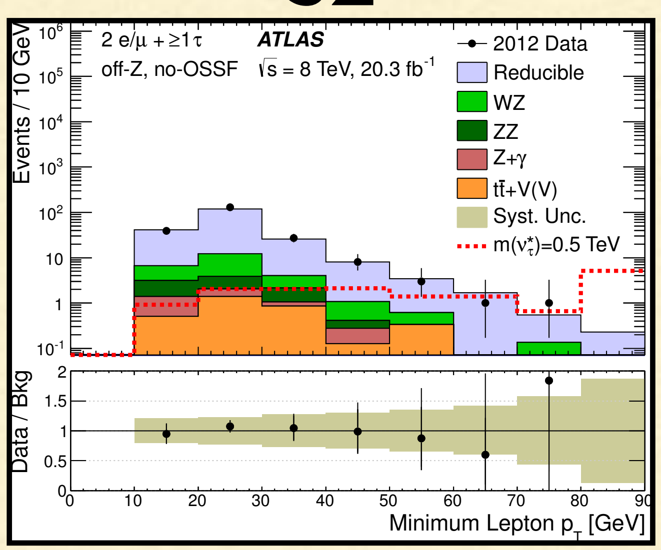
OSSF ≥3e/μ

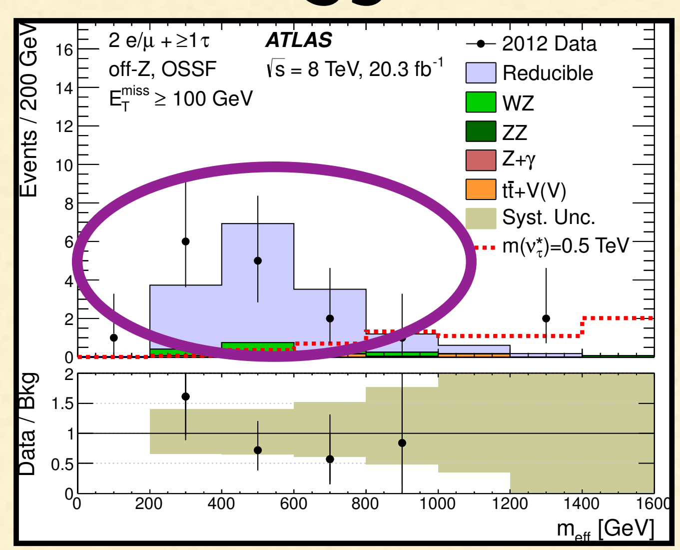
S3

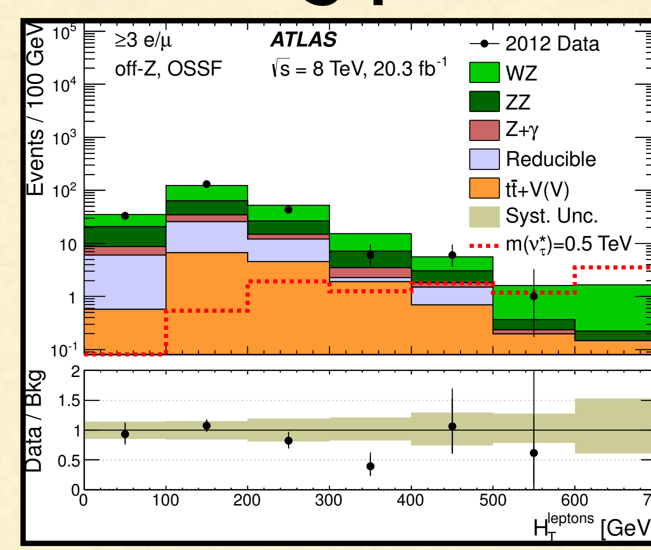
OSSF 2e/μ≥1τ$_{had}$

S4









m_{eff}: effective mass of event combining solar sum of jets, missing energy and lepton pT

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Tool Chain

1604.00925 and 1607.05599

Lagrangian

FeynRules
1406.3030

Sherpa
0811.4622

NNPDF3.0

https://github.com/diana-hep/pyhf

Calculate CLs

validated ATLAS analysis
1411.2921

ATLAS EXPERIMENT

Rivet
1003.0694

https://rivet.hepforge.org/

Constraining our 6D model PS

Analytic Constraints

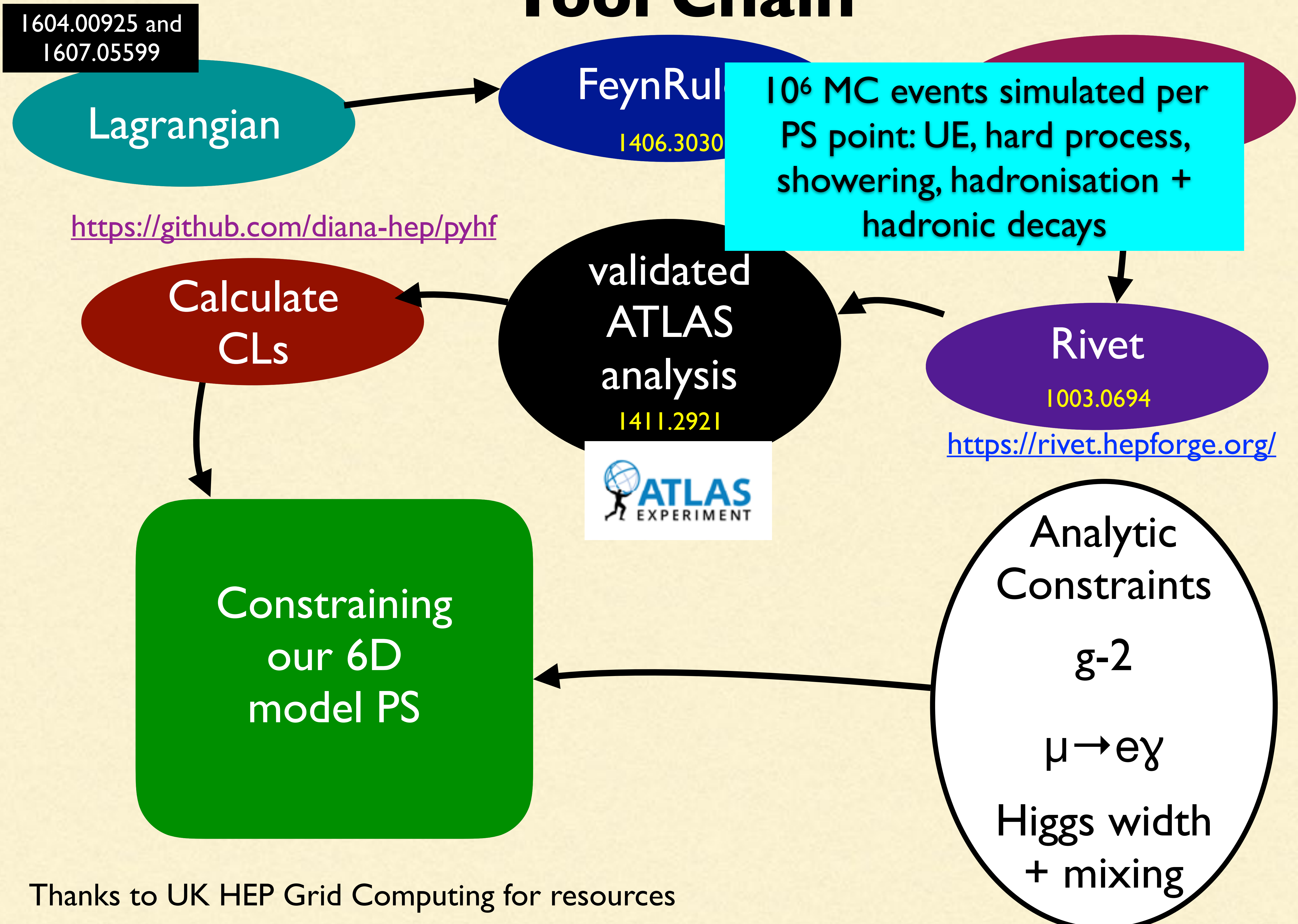
g-2

μ→eɣ

Higgs width + mixing

Thanks to UK HEP Grid Computing for resources

Tool Chain



Thanks to UK HEP Grid Computing for resources

CL_s Method for Recast

Calculated using PyHF:

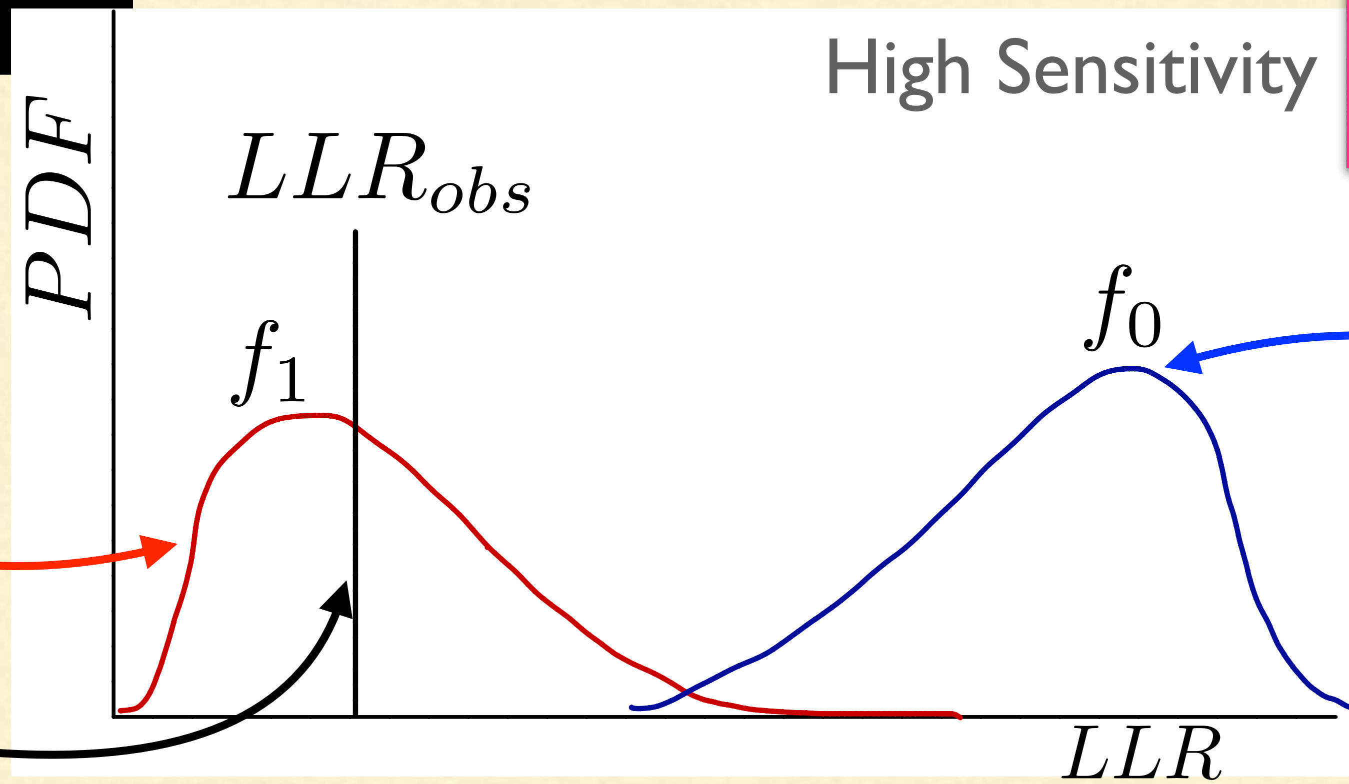
https://github.com/diana-hep/pyhf

PDF generated through possible fluctuations (Asimov data set) 1007.1727

signal+BG changes for each PS point

observed LLR (measurement)

BG only hypothesis (constant)



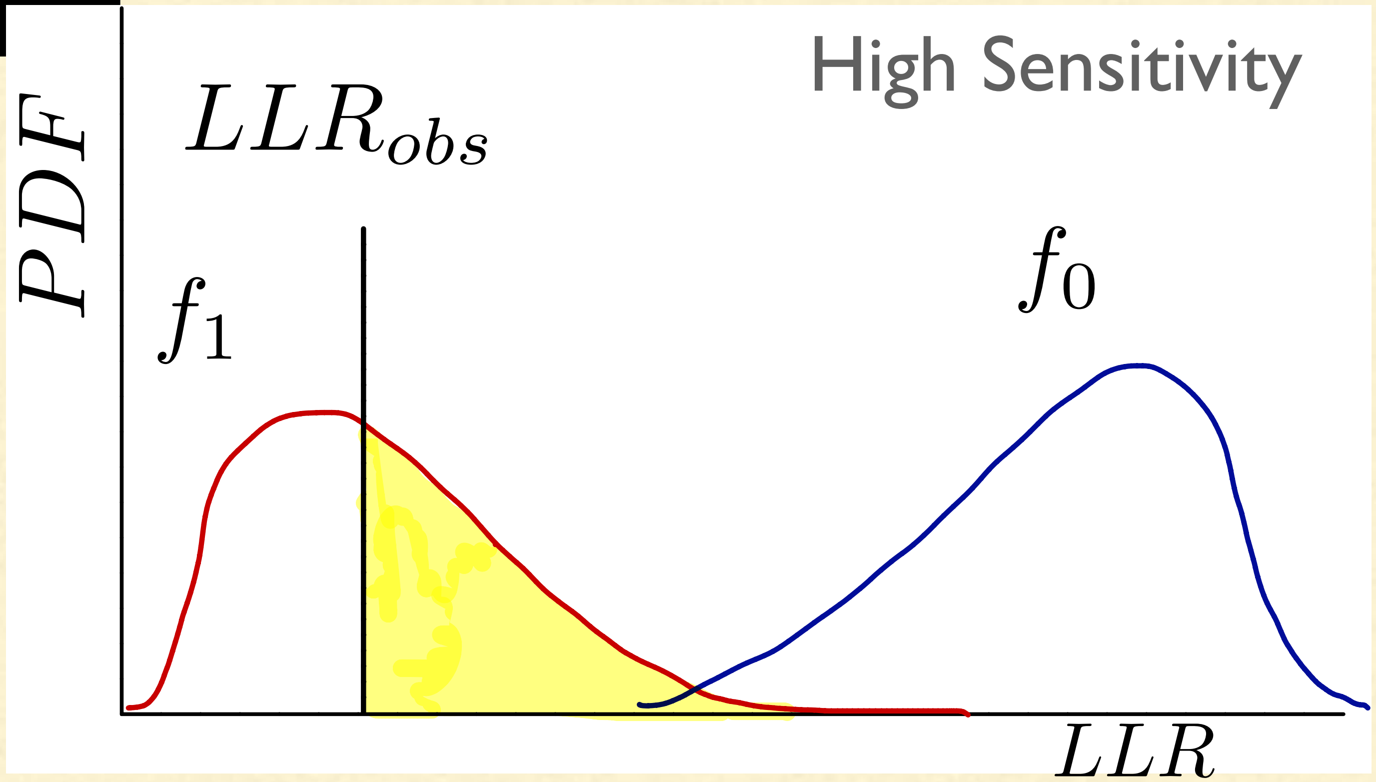
$$1 - CL_b \equiv \int_{-\infty}^{LLR_{obs}} f_0(LLR)dLLR \qquad CL_{s+b} \equiv \int_{LLR_{obs}}^{\infty} f_1(LLR)dLLR$$

$$CL_s = \frac{CL_{s+b}}{CL_b}$$

Frequentist is CL_{s+b} only

CL_s Method for Recast

PDF generated through possible fluctuations (Asimov data set) 1007.1727



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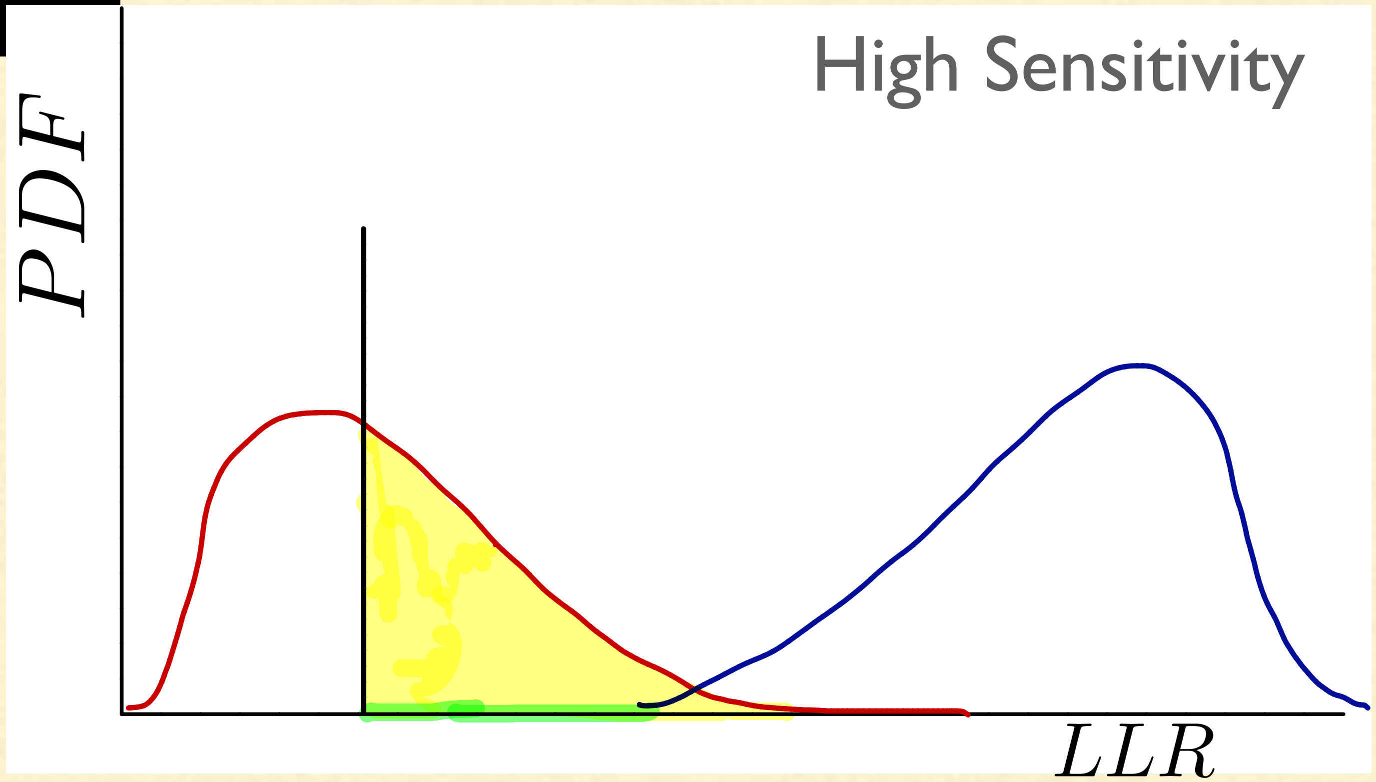


$$1 - CL_b \equiv \int_{-\infty}^{LLR_{obs}} f_0(LLR) dLLR$$

$$CL_{s+b} \equiv \int_{LLR_{obs}}^{\infty} f_1(LLR) dLLR$$

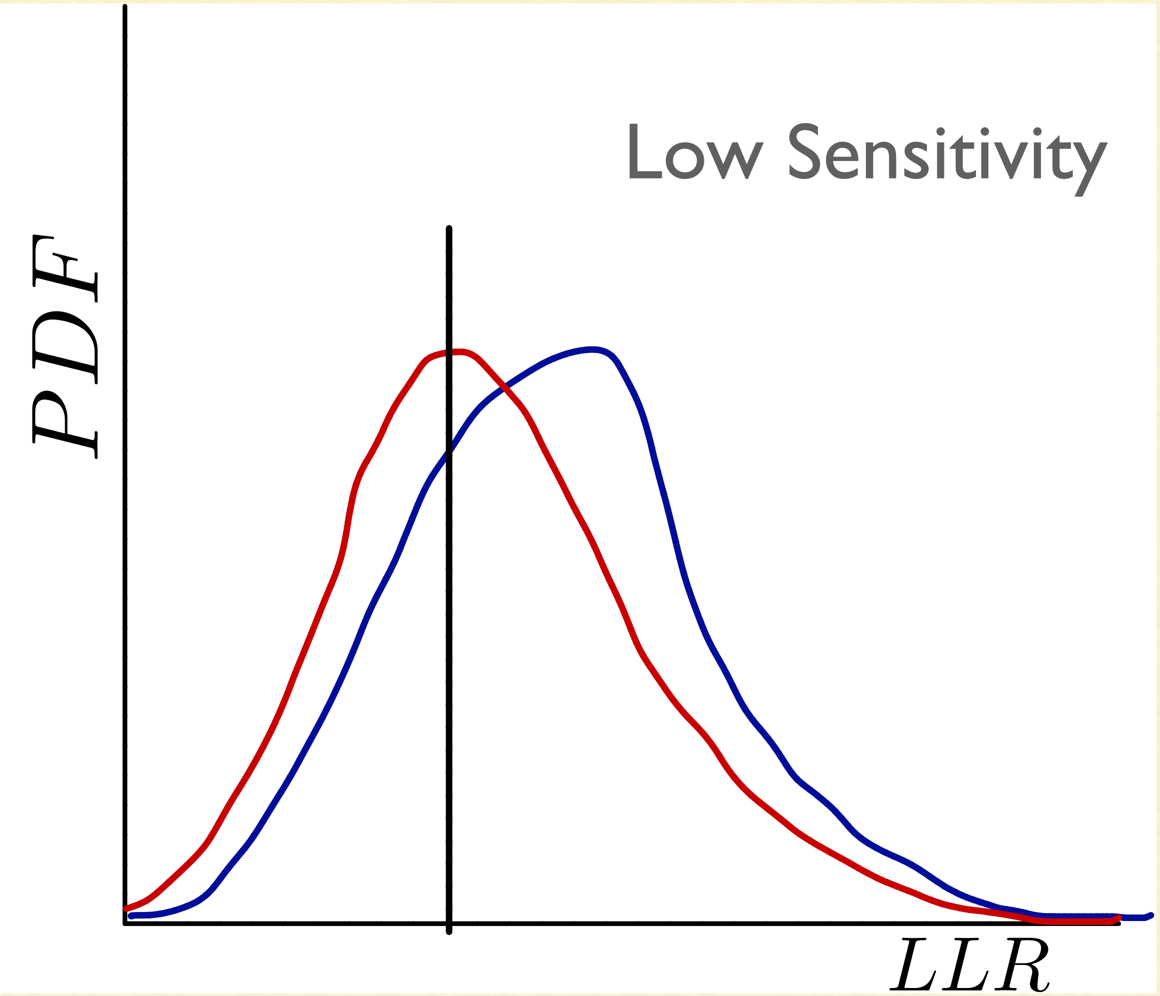
$$CL_s = \frac{CL_{s+b}}{CL_b}$$

Frequentist is CL_{s+b} only

$CL_S > 0.05$, H_1 cannot be excluded

CL_S Method for Recast

CL_S is conservative against overestimating exclusionary power in case of low signal sensitivity



CL_b becomes small therefore CL_s becomes large and H_1 cannot be excluded

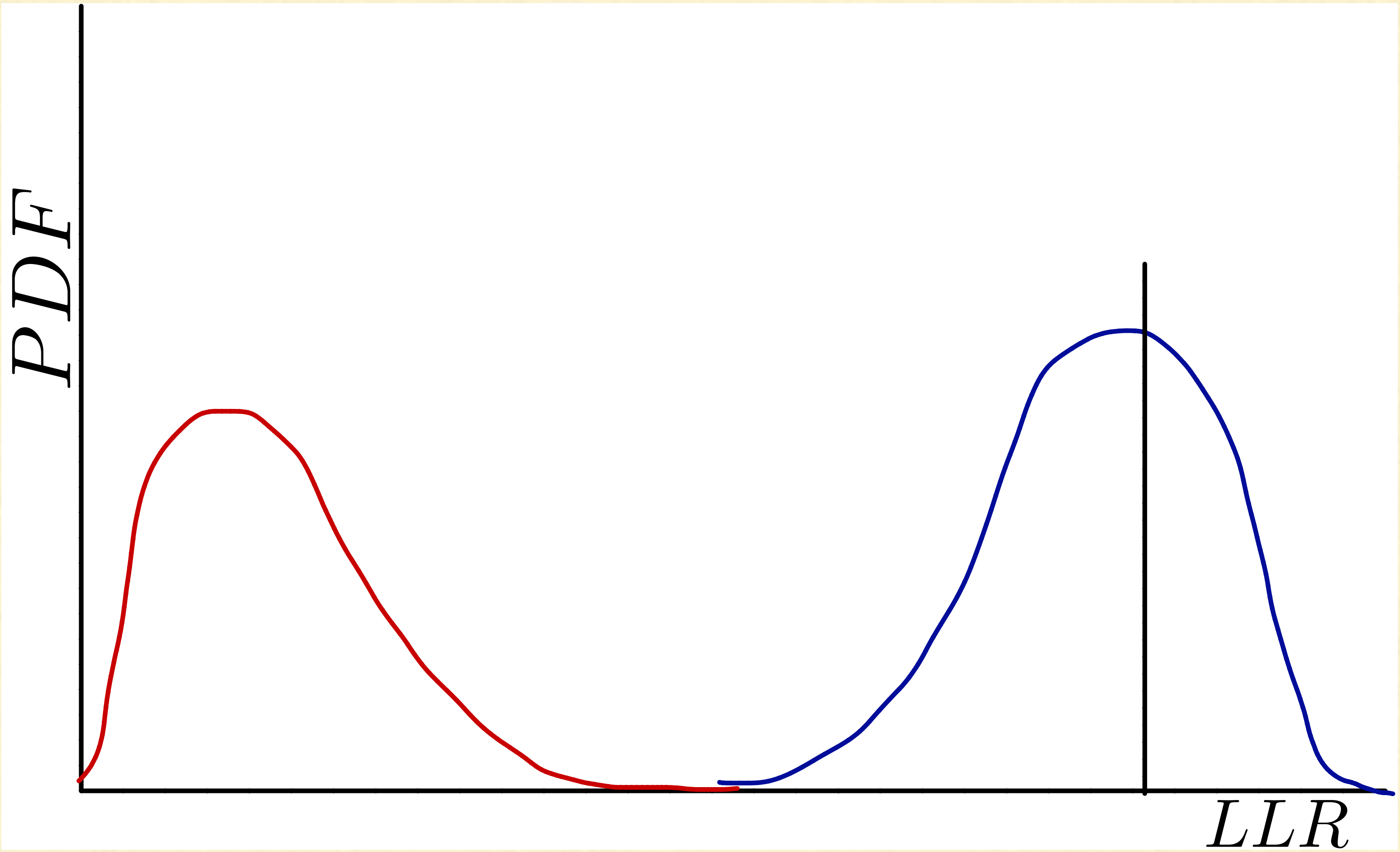
$$1 - CL_b \equiv \int_{-\infty}^{LLR_{obs}} f_0(LLR) dLLR \qquad CL_{s+b} \equiv \int_{LLR_{obs}}^{\infty} f_1(LLR) dLLR$$

$$CL_s = \frac{CL_{s+b}}{CL_b}$$

Frequentist is CL_{s+b} only

$CL_S > 0.05$, H_1 cannot be excluded

Many of our PS points



$$1 - CL_b \equiv \int_{-\infty}^{LLR_{obs}} f_0(LLR)dLLR \qquad CL_{s+b} \equiv \int_{LLR_{obs}}^{\infty} f_1(LLR)dLLR$$

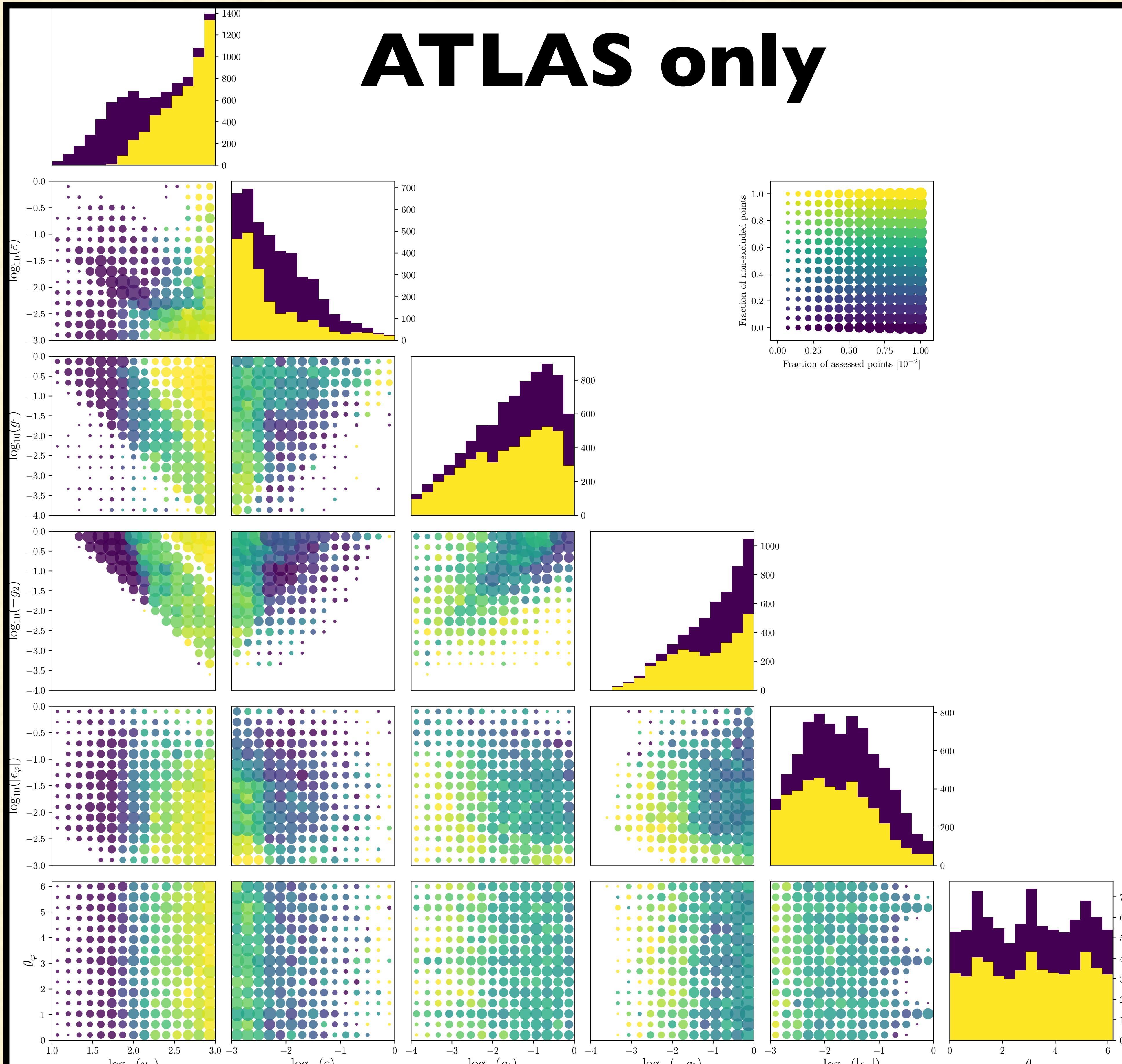
$$CL_s = \frac{CL_{s+b}}{CL_b}$$

Frequentist is CL_{s+b} only

$CL_S < 0.05$, H_1 can be excluded at 95% CL

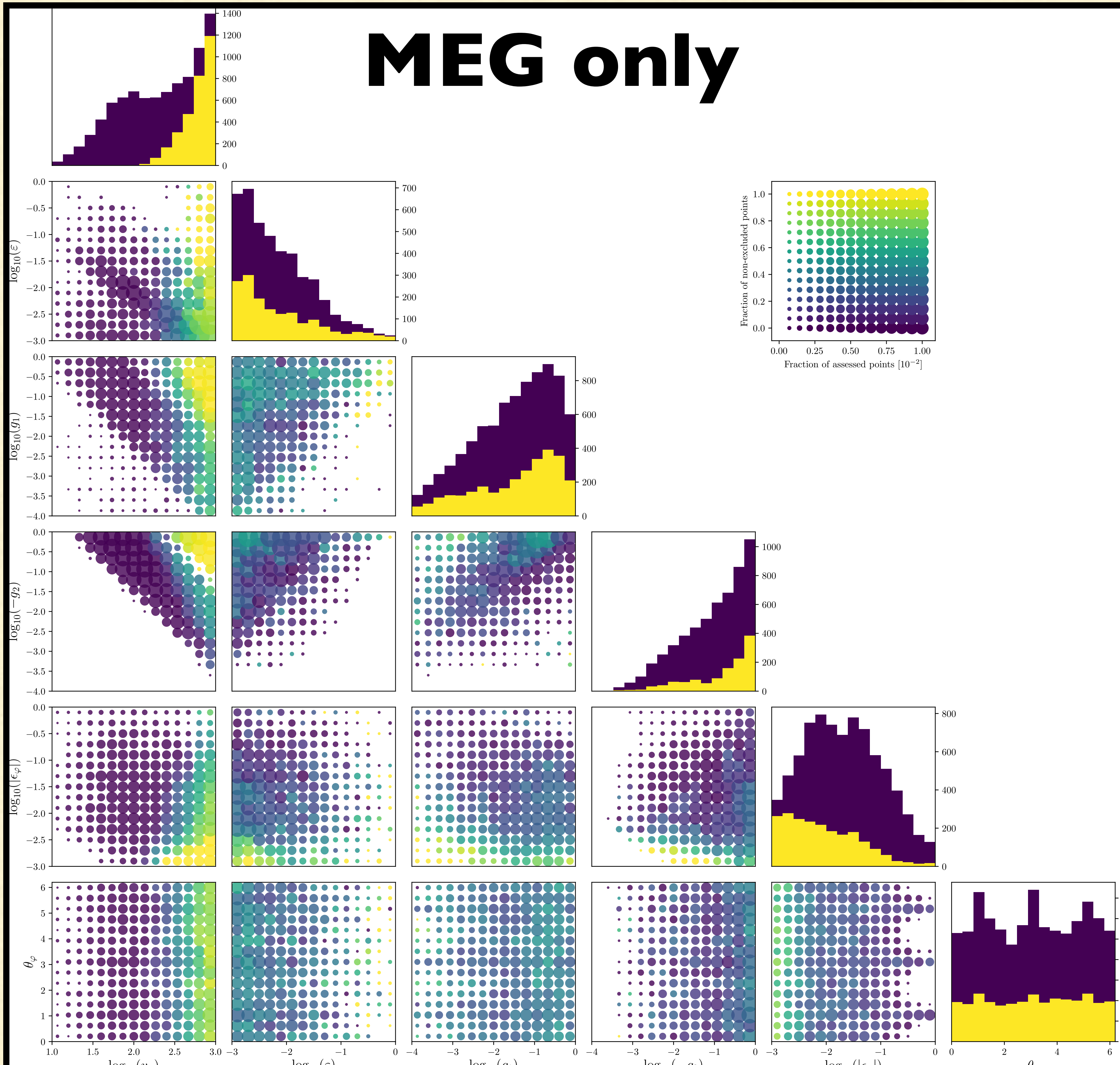
Results

ATLAS only



Fraction of non-excluded points
Fraction of assessed points $[10^{-2}]$
$\log_{10}(v_\varphi)$
$\log_{10}(\varepsilon)$
$\log_{10}(g_1)$
$\log_{10}(-g_2)$
$\log_{10}(|\epsilon_\varphi|)$
θ_φ

MEG only



$\log_{10}(v_\varphi)$
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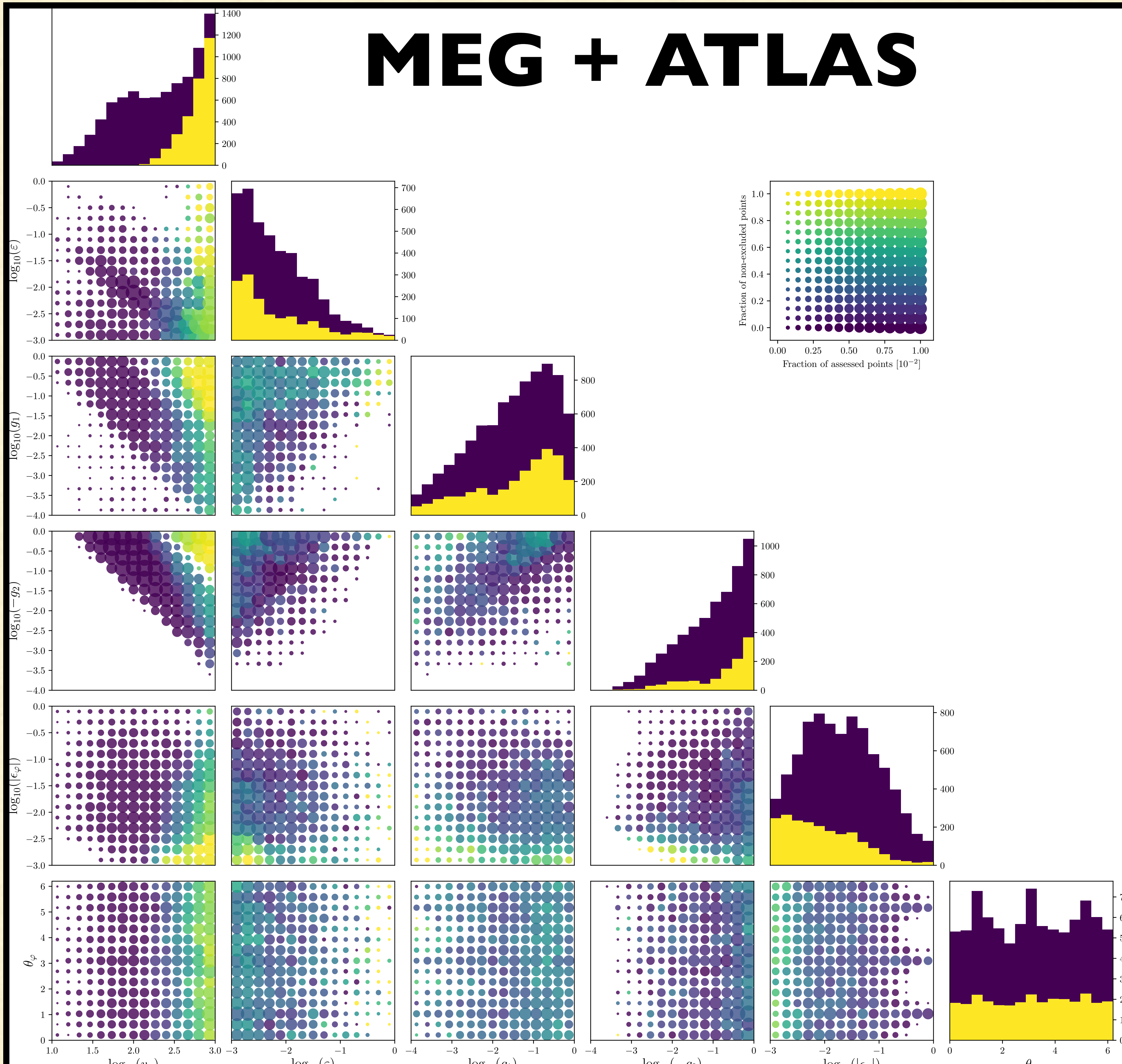
θ_φ
Fraction of non-excluded points
Fraction of assessed points $[10^{-2}]$

MEG + ATLAS



Fraction of non-excluded points
Fraction of assessed points [10^{-2}]
$\log_{10}(v_\varphi)$
$\log_{10}(\varepsilon)$
$\log_{10}(g_1)$
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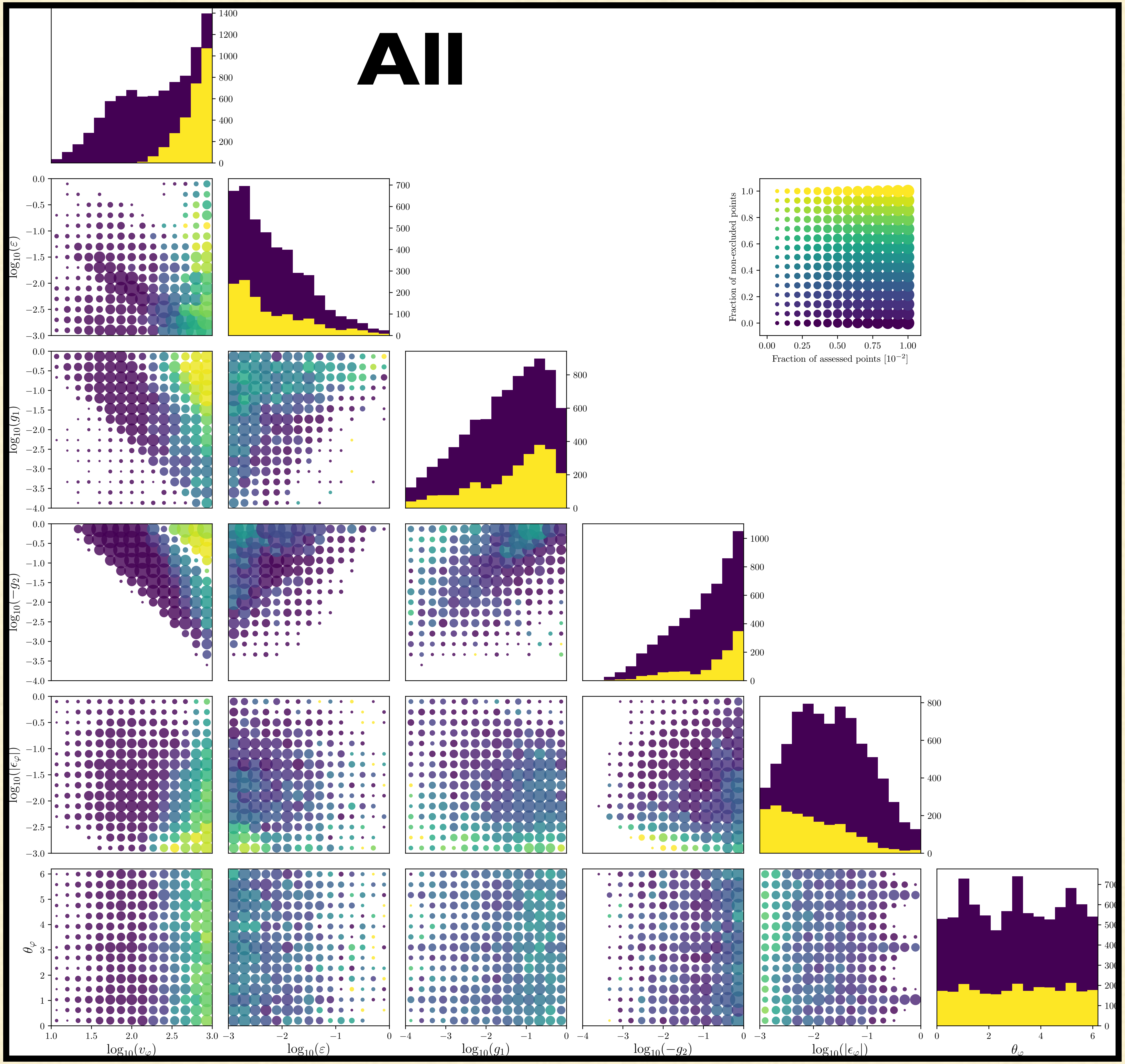
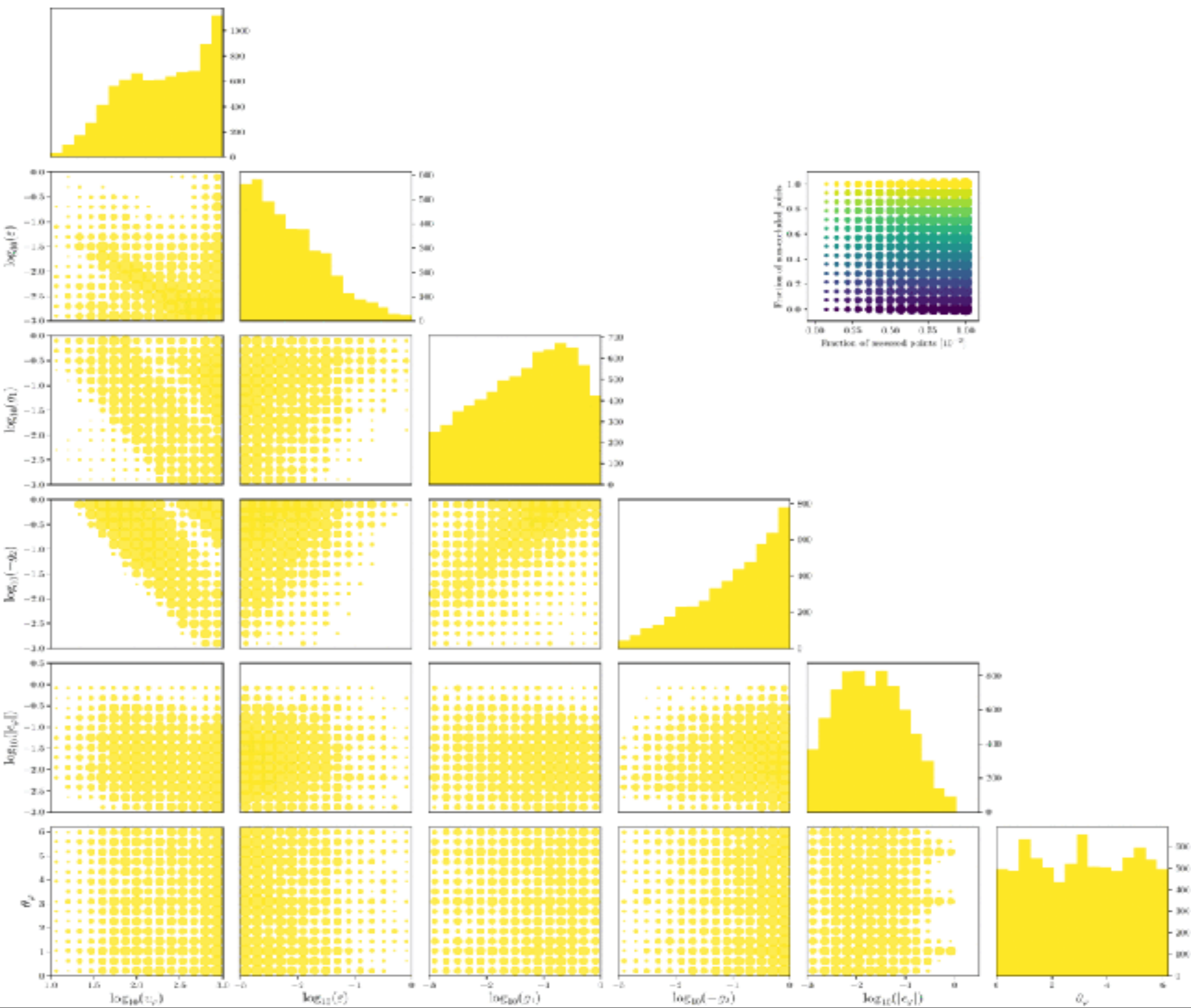
$\log_{10}(|\epsilon_\varphi|)$
θ_φ

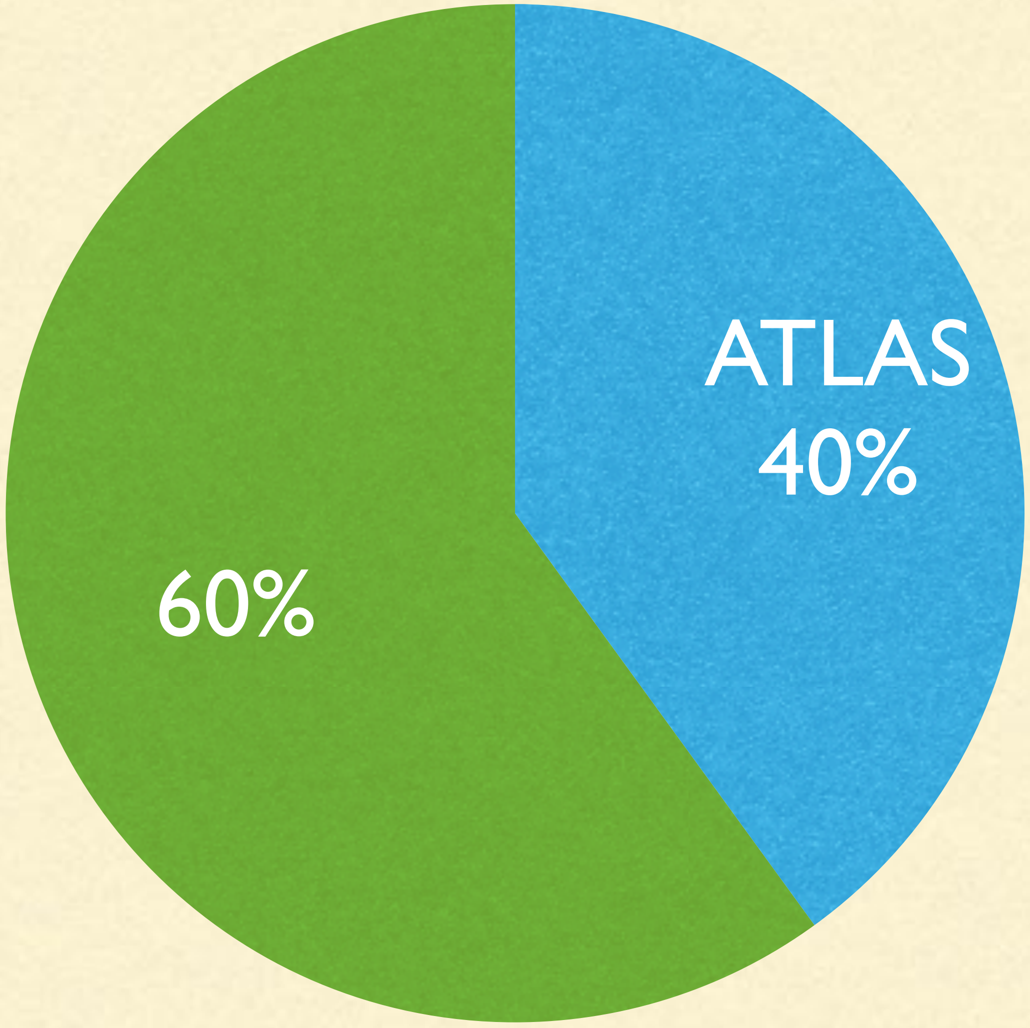
All

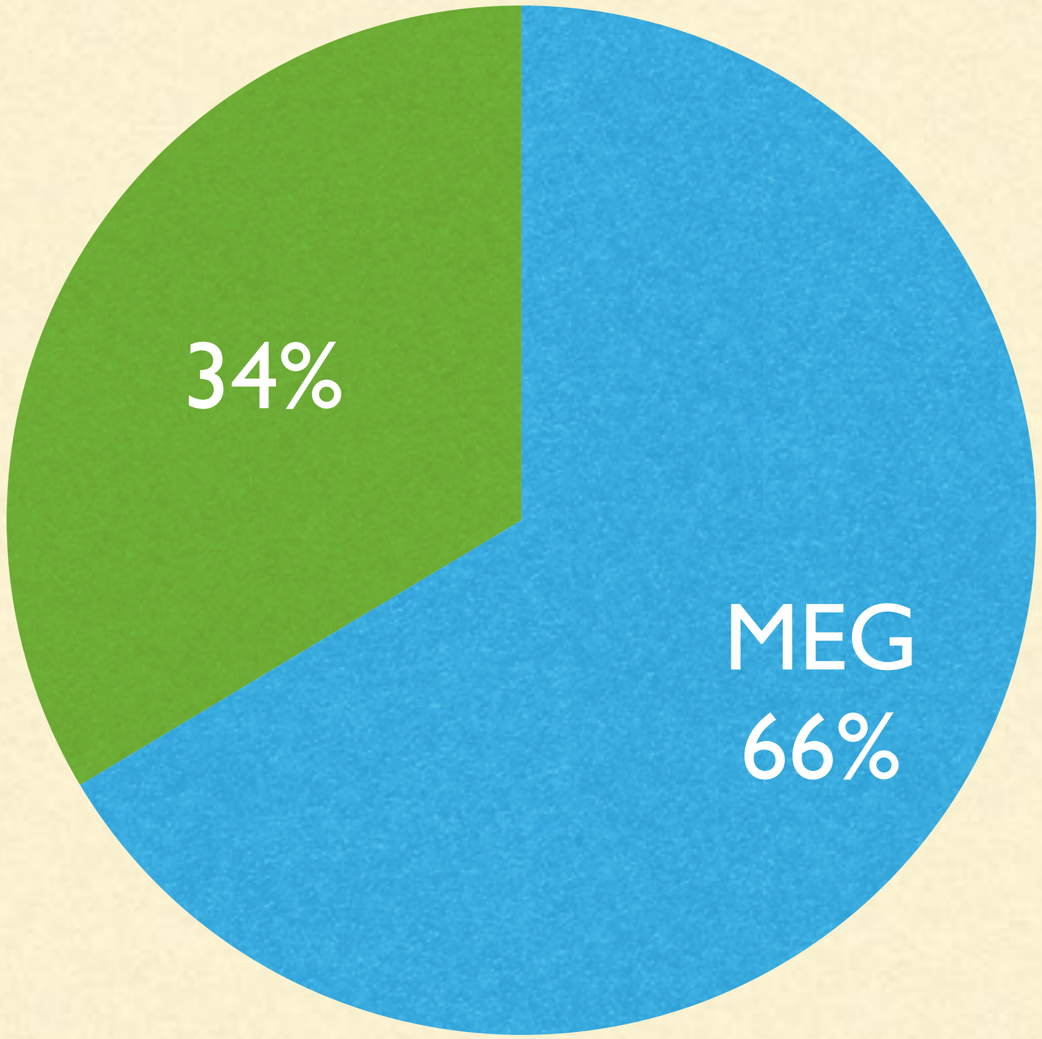

Fraction of non-excluded points
Fraction of assessed points $[10^{-2}]$
$\log_{10}(\varepsilon)$
$\log_{10}(g_1)$
$\log_{10}(-g_2)$
$\log_{10}(|\epsilon_\varphi|)$
θ_φ
$\log_{10}(v_\varphi)$



Exclusionary Power

Experimental data	Exclusion power [%]
MEG	65.6
ATLAS	40.0
Higgs-width	6.0
Higgs-mixing	1.7
$g-2$	0.7





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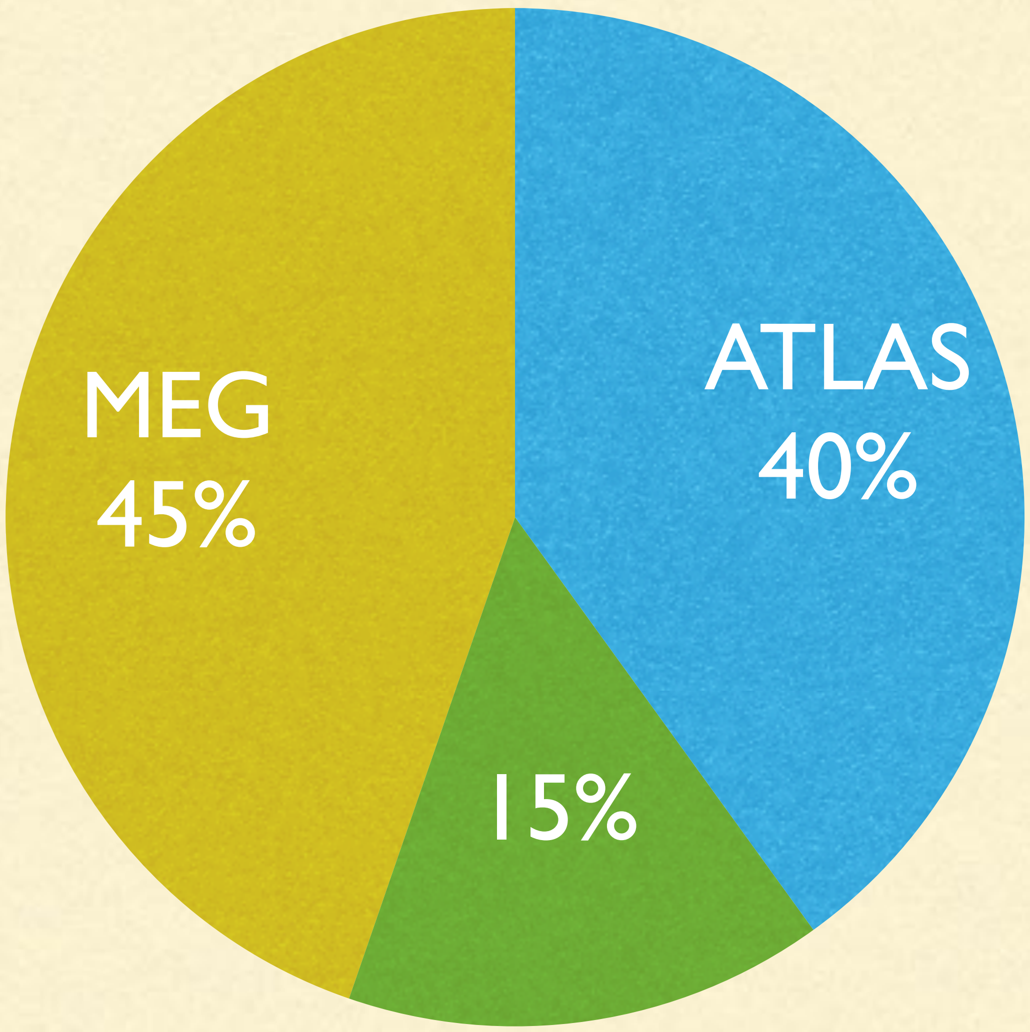
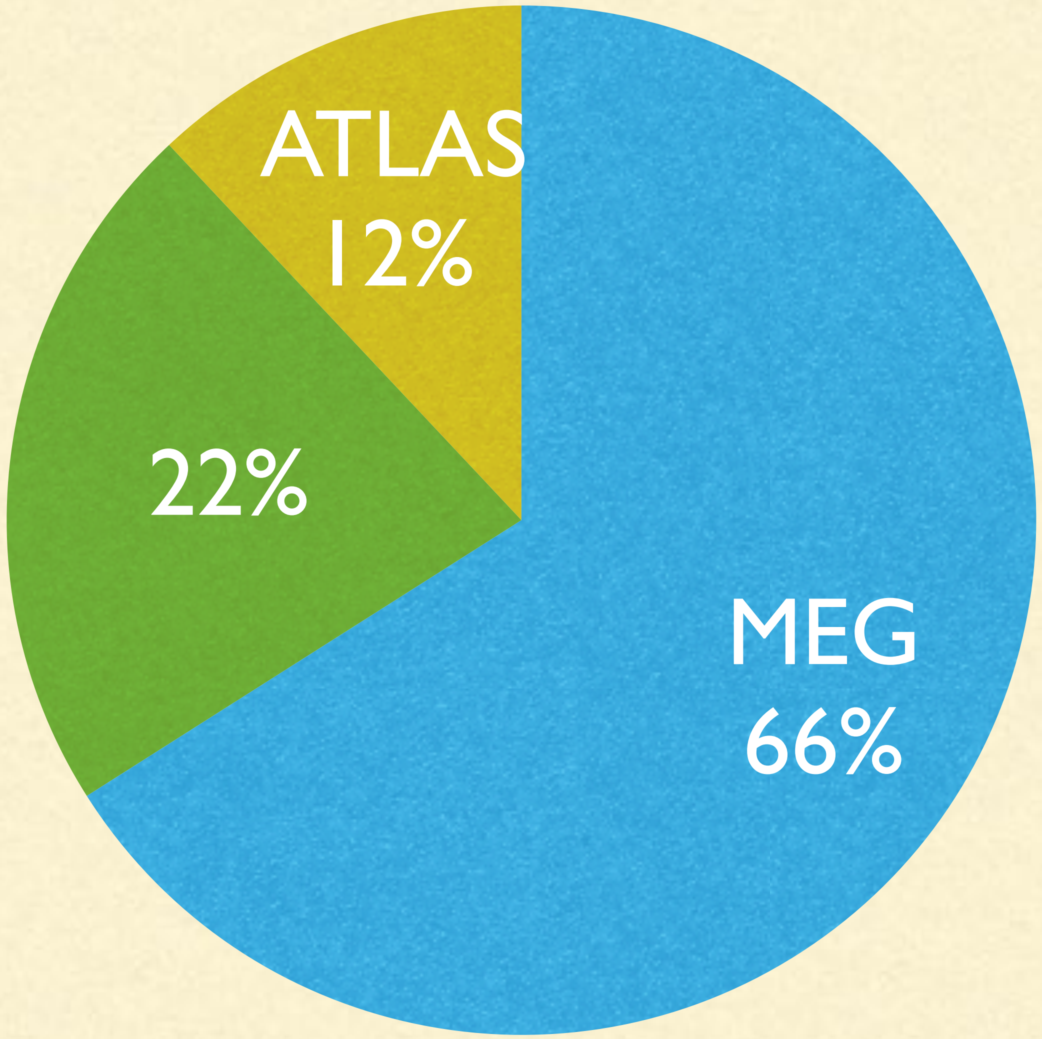




Conclusions

- A priori it is not clear the flavour breaking scale is to the GUT scale. Can we exclude a lower value of this scale?
- Experiments such as MEG place highly competitive constraints on flavour model P.S (we were skeptical the collider would be able to compete!)
- We demonstrated **collider searches** for high multiplicity leptonic final states **can compete and complement** MEG and g-2 experimental constraints.
- Why? The collider has sensitivity to flavon coupling to Higgs, MEG and g-2 are not.
- The chosen model P.S is largely excluded through synergy of these experiments.

Thank you!

Back-up Slides

Minimise the flavon and Higgs potential

$$\mu_H^2 + \lambda v_H^2 + \frac{1}{2}\epsilon v_\varphi^2 (1 + 2|\epsilon_\varphi|^2) = 0\,,$$

$$\mu_\varphi^2 + g_1 v_\varphi^2 (1 + 2|\epsilon_\varphi|^2) + \frac{1}{3} g_2 v_\varphi^2 [1 - \mathrm{Re}(\epsilon_\varphi^3)] + \frac{1}{2}\epsilon v_H^2 + A\epsilon_\varphi^* + A^*\epsilon_\varphi = 0\,,$$

$$\mu_\varphi^2 \epsilon_\varphi + g_1 v_\varphi^2 (1 + 2|\epsilon_\varphi|^2)\epsilon_\varphi + \frac{1}{2} g_2 v_\varphi^2 [-\epsilon_\varphi^{*2} + |\epsilon_\varphi|^2 \epsilon_\varphi] + \frac{1}{2}\epsilon\epsilon_\varphi v_H^2 + A + A^*\epsilon_\varphi^* = 0\,.$$

$$A\epsilon_\varphi^* + A^*\epsilon_\varphi^{*2} + 2\mathrm{Re}(A^*\epsilon_\varphi)|\epsilon_\varphi|^2 = \underbrace{-\frac{1}{2} g_2 v_\varphi^2 \epsilon_\varphi^{*3} + \frac{1}{3} g_2 v_\varphi^2 |\epsilon_\varphi|^2 \left[1 - \mathrm{Re}(\epsilon_\varphi^3) - \frac{3}{2}|\epsilon_\varphi|^2\right]}_{x}$$

$$A = \frac{\left(\epsilon_\varphi^*\right)^2 x^* - \epsilon_\varphi \left(x + 2i\,|\epsilon_\varphi|^2\,\Im\left[x\right]\right)}{|\epsilon_\varphi|^2 \left(-\,|\epsilon_\varphi|^2 + {\epsilon_\varphi^*}^3 + \epsilon_\varphi^3 - 1\right)}.$$

Back-up Slides

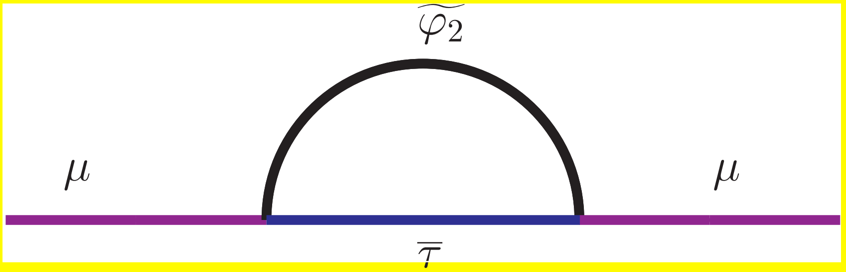
$$
\begin{aligned}
(M^2_{\tilde{\Phi}})_{11} &= 2\lambda v_H^2 \,, \\
(M^2_{\tilde{\Phi}})_{22} &= 2g v_\varphi^2 + \frac{1}{3} g_2 v_\varphi^2 \mathrm{Re}(\epsilon_\varphi^3) - 2\mathrm{Re}(A\epsilon_\varphi^*) \,, \\
(M^2_{\tilde{\Phi}})_{33} &= -\frac{1}{3} g_2 v_\varphi^2 [1 - \mathrm{Re}(\epsilon_\varphi^3)] + \frac{1}{2} g_2 v_\varphi^2 |\epsilon_\varphi|^2 - 2\mathrm{Re}(A\epsilon_\varphi^*) + \mathrm{Re}\left(-g_2 v_\varphi^2 (\epsilon_\varphi^* - \frac{1}{2}\epsilon_\varphi^2) + 2g_1 v_\varphi^2 \epsilon_\varphi^2 + A^* \right) \,, \\
(M^2_{\tilde{\Phi}})_{44} &= -\frac{1}{3} g_2 v_\varphi^2 [1 - \mathrm{Re}(\epsilon_\varphi^3)] + \frac{1}{2} g_2 v_\varphi^2 |\epsilon_\varphi|^2 - 2\mathrm{Re}(A\epsilon_\varphi^*) - \mathrm{Re}\left(-g_2 v_\varphi^2 (\epsilon_\varphi^* - \frac{1}{2}\epsilon_\varphi^2) + 2g_1 v_\varphi^2 \epsilon_\varphi^2 + A^* \right) \,, \\
(M^2_{\tilde{\Phi}})_{12} &= v_H v_\varphi \epsilon \,, \\
(M^2_{\tilde{\Phi}})_{13} &= \sqrt{2} v_H v_\varphi \epsilon \mathrm{Re}(\epsilon_\varphi) \,, \\
(M^2_{\tilde{\Phi}})_{14} &= \sqrt{2} v_H v_\varphi \epsilon \mathrm{Im}(\epsilon_\varphi) \,, \\
(M^2_{\tilde{\Phi}})_{23} &= \sqrt{2}\mathrm{Re}\left(2g_1 v_\varphi^2 \epsilon_\varphi - \frac{1}{2} g_2 v_\varphi^2 \epsilon_\varphi^{*2} + A \right) \,, \\
(M^2_{\tilde{\Phi}})_{24} &= \sqrt{2}\mathrm{Im}\left(2g_1 v_\varphi^2 \epsilon_\varphi - \frac{1}{2} g_2 v_\varphi^2 \epsilon_\varphi^{*2} + A \right) \,, \\
(M^2_{\tilde{\Phi}})_{34} &= \mathrm{Im}\left(-g_2 v_\varphi^2 (\epsilon_\varphi^* - \frac{1}{2}\epsilon_\varphi^2) + 2g_1 v_\varphi^2 \epsilon_\varphi^2 + A^* \right) \,, \qquad (2.19)
\end{aligned}
$$

Diagonalise mass matrix and ensure (1,1) entry is the Higgs mass

Relating gauge to mass basis

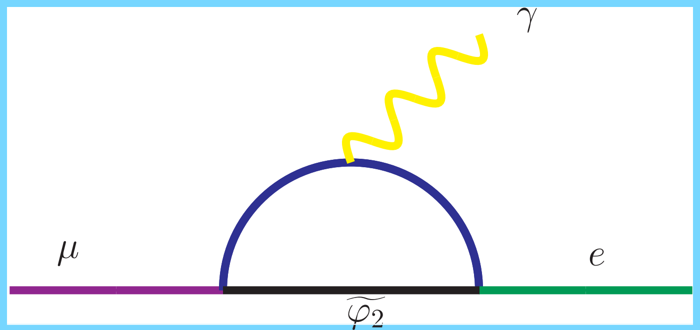
$$\begin{pmatrix} \tilde{h} \\ \tilde{\varphi_1} \\ \sqrt{2}\mathrm{Re}\left(\varphi_2\right) \\ \sqrt{2}\mathrm{Im}\left(\varphi_2\right) \end{pmatrix} = \begin{pmatrix} W_{00} & W_{01} & W_{02} & W_{03} \\ W_{10} & W_{11} & W_{12} & W_{13} \\ W_{20} & W_{21} & W_{22} & W_{23} \\ W_{30} & W_{31} & W_{32} & W_{33} \end{pmatrix} \begin{pmatrix} h \\ s_1 \\ s_2 \\ s_3 \end{pmatrix}$$

g-2 Constraint



$$\Delta a_\mu = \frac{m_\mu^2 m_\tau^2}{24\pi^2 v_\varphi^2}\Big[\frac{\left(|W_{13}|^2 - |W_{14}|^2\right)}{m_h^2} + \frac{\left(|W_{23}|^2 - |W_{24}|^2\right)}{m_{s_1}^2} + \frac{\left(|W_{33}|^2 - |W_{34}|^2\right)}{m_{s_2}^2} + \frac{\left(|W_{43}|^2 - |W_{44}|^2\right)}{m_{s_3}^2}\Big].$$

μ→eɣ Constraint



$$A\,(h) = \frac{1}{128\pi^2}\frac{1}{m_h^2 v_\varphi^2}\left[m_\mu m_\tau^2 G_2\left(\frac{m_\tau^2}{m_H^2}\right)(W_{13}+iW_{14})^2 - m_\mu m_\tau^2 \epsilon_\varphi^* G_2\left(\frac{m_\tau^2}{m_H^2}\right)\left(|W_{13}|^2+|W_{14}|^2\right)\right],$$

$$A\,(s_1) = \frac{1}{128\pi^2}\frac{1}{m_{s_1}^2 v_\varphi^2}\left[m_\mu m_\tau^2 G_2\left(\frac{m_\tau^2}{m_1^2}\right)(W_{23}+iW_{24})^2 - m_\mu m_\tau^2 \epsilon_\varphi^* G_2\left(\frac{m_\tau^2}{m_1^2}\right)\left(|W_{23}|^2+|W_{24}|^2\right)\right],$$

$$A\,(s_2) = \frac{1}{128\pi^2}\frac{1}{m_{s_2}^2 v_\varphi^2}\left[m_\mu m_\tau^2 G_2\left(\frac{m_\tau^2}{m_2^2}\right)(W_{33}+iW_{34})^2 - m_\mu m_\tau^2 \epsilon_\varphi^* G_2\left(\frac{m_\tau^2}{m_2^2}\right)\left(|W_{33}|^2+|W_{34}|^2\right)\right],$$

$$A\,(s_3) = \frac{1}{128\pi^2}\frac{1}{m_{s_3}^2 v_\varphi^2}\left[m_\mu m_\tau^2 G_2\left(\frac{m_\tau^2}{m_3^2}\right)(W_{43}+iW_{44})^2 - m_\mu m_\tau^2 \epsilon_\varphi^* G_2\left(\frac{m_\tau^2}{m_3^2}\right)\left(|W_{43}|^2+|W_{44}|^2\right)\right].$$

$$G_2(x) = -\log x - \frac{11}{6}$$

$$\Gamma\left(\mu\to e\gamma\right) = \frac{m_\mu^3|A|^2}{16\pi}, \qquad \Gamma\left(\mu\to e\overline{\nu}_e\nu_\mu\gamma\right) = \frac{G_F^2 m_\mu^5}{192\pi^3},$$