

Beyond the 3x3 neutrino paradigm with majorons

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Parametrization of our ignorance

“Known”: $M_\nu = U \text{diag}(m_1, m_2, m_3) U^T \simeq -m_D M_R^{-1} m_D^T$

 PMNS mixing matrix, 3x3 paradigm!

 seesaw mechanism

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seesaw mechanism

- Still 9 unknown parameters in type-I seesaw.
- Most common: couplings of N_R .

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PMNS mixing matrix, 3x3 paradigm!

seesaw mechanism

- Still 9 unknown parameters in type-I seesaw.

- Here: hermitian $m_D m_D^\dagger$.

[Davidson, Ibarra, hep-ph/0104076]

Beyond 3x3 paradigm!

- One-to-one: $\{m_D, M_R\} \leftrightarrow \{M_\nu, m_D m_D^\dagger\}$.

- (More general: $\{m_D, M_R\} \leftrightarrow \{M_\nu, m_D M_R^n m_D^\dagger\}$).

Majoronic seesaw

- 3 singlets N_R + new scalar $\sigma = (f + \sigma^0 + iJ)/\sqrt{2}$.

B-L breaking scale

Heavy scalar
(inflaton?)

Majoron

[Chikashige, Mohapatra, Peccei, '81; Schechter, Valle, '82]

(Even more beyond
3x3 paradigm!)

- Break $U(1)_{B-L}$ spontaneously:

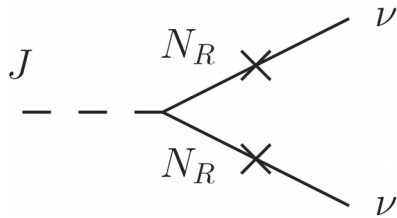
$$L = - \underbrace{\bar{L}yHN_R}_{m_D} - \frac{1}{2} \underbrace{\bar{N}_R^c \kappa \sigma N_R}_{M_R} + \text{h.c.}$$

$$m_D = y \langle H \rangle \quad M_R = \frac{\kappa f}{\sqrt{2}}$$

- Similar for *inverse* seesaw, *extended* seesaw,...
- Assume J is a *pseudo*-Goldstone: $m_J \neq 0$.

Majoron couplings

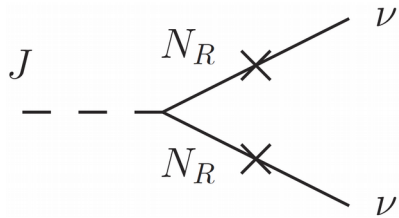
- Tree-level coupling only to neutrinos:



$$\frac{iJ}{2f} \bar{\nu}_\alpha^c \gamma_5 (m_D^* M_R^{-1} m_D^\dagger)_{\alpha\beta} \nu_\beta = -\frac{iJ}{2f} \sum_k \bar{\nu}_k \gamma_5 m_k \nu_k$$

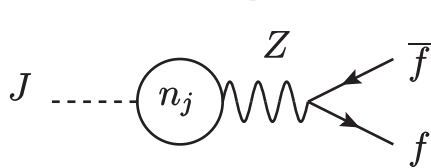
Majoron couplings

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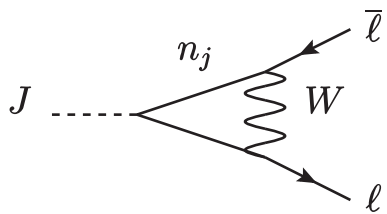
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- One loop:



$$\frac{iJ}{f} \bar{f} \gamma_5 f \frac{m_f T_3^f}{8\pi^2 v^2} \text{tr} (m_D m_D^\dagger)$$

Off-diagonal!

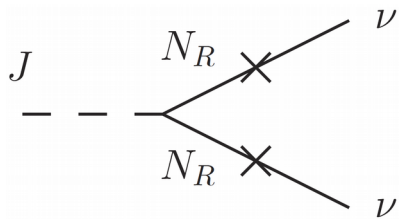


$$\frac{iJ}{f} \bar{l}_\alpha \left(\frac{m_\beta}{8\pi^2 v^2} P_R - \frac{m_\alpha}{8\pi^2 v^2} P_L \right) l_\beta (m_D m_D^\dagger)_{\alpha\beta}$$

[Heeck, Camilo Garcia-Cely, 1701.07209; see also Pilaftsis '94]

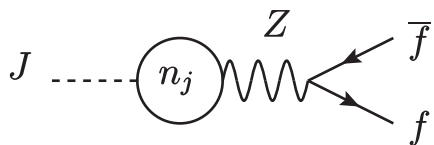
Majoron couplings

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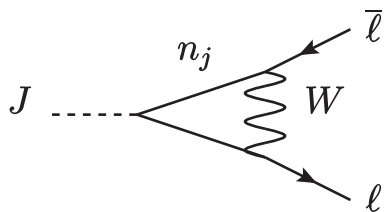
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- Two loop: $\Gamma(J \rightarrow \gamma\gamma) \simeq \frac{\alpha^2 \text{tr} (m_D m_D^\dagger)^2}{4096\pi^7} \frac{m_J^3}{v^4 f^2} \left| \sum_f N_c^f T_3^f Q_f^2 g \left(\frac{m_J^2}{4m_f^2} \right) \right|^2$

[Heeck, Camilo Garcia-Cely, 1701.07209; see also Pilaftsis '94]

Reconstruct seesaw?

- $\{m_D, M_R\} \leftrightarrow \{M_\nu, m_D m_D^\dagger\}$. [Davidson, Ibarra, hep-ph/0104076]
- $J_{\nu\nu}$ coupling to measure $U(1)_L$ scale f .
- Use $J\bar{f}f$ couplings to reconstruct

$$(m_D m_D^\dagger)_{\alpha\beta} = K_{\alpha\beta} v f = \begin{pmatrix} K_{ee} & |K_{e\mu}|e^{ia} & |K_{e\tau}|e^{ib} \\ |K_{e\mu}|e^{-ia} & K_{\mu\mu} & |K_{\mu\tau}|e^{ic} \\ |K_{e\tau}|e^{-ib} & |K_{\mu\tau}|e^{-ic} & K_{\tau\tau} \end{pmatrix} v f.$$

- Diagonal K entries from e.g. J_{ee} , $J_{\mu\mu}$, and $J_{\gamma\gamma}$ or J_{qq} .
- Light J: $|K_{\alpha\beta}| < 10^{-5}$ from stellar energy loss.

[Raffelt, e.g. 1205.1776]

$$f \sim 10^7 \text{ GeV}$$

Take from
axion/ALP
searches.

Lepton flavor violation

- Off-diagonal $|K_{\alpha\beta}|$ from LFV: $\ell_\alpha \rightarrow \ell_\beta J$.
- (Phase of off-diagonal $K_{\alpha\beta}$?)
- $J\bar{\ell}\ell'$ coupling can be *large* and of **arbitrary structure**.
- Similar couplings arise for familons or flavor Z' .
[Wilczek, '82; Reiss, '82; Grinstein, Preskill, Wise, 85; ...]
- Boson not necessarily massless: *pseudo*-Goldstone.
- Experimental signature depends on decay channel:

$$\ell \rightarrow \ell' J, \quad J \rightarrow \text{inv}, \ell'' \ell''', \gamma\gamma.$$

[Heeck, Rodejohann, 1710.02062]

$\ell \rightarrow \ell' J$ with $J \rightarrow$ invisible

- Standard LFV in seesaw:

$$\frac{\Gamma(\ell \rightarrow \ell' \gamma)}{\Gamma(\ell \rightarrow \ell' \nu_\ell \bar{\nu}_{\ell'})} \simeq \frac{3\alpha}{8\pi} |(m_D M_R^{-2} m_D^\dagger)_{\ell\ell'}|^2.$$

- Great signature, but requires light N_R .
- With majoron: look for **mono-energetic** lepton:

[Pilaftsis, '94; Feng, Moroi, Murayama, Schnapka, '98; Hirsch, Vicente, Meyer, Porod, '09]

$$\frac{\Gamma(\ell \rightarrow \ell' J)}{\Gamma(\ell \rightarrow \ell' \nu_\ell \bar{\nu}_{\ell'})} \simeq \frac{3}{16\pi^2} \frac{1}{m_\ell^2 f^2} |(m_D m_D^\dagger)_{\ell\ell'}|^2.$$

- If $M_R = \text{diag}(M)$: $\frac{\Gamma(\ell \rightarrow \ell' \gamma)}{\Gamma(\ell \rightarrow \ell' J)} \simeq 2\pi\alpha \frac{m_\ell^2}{M^2} \frac{f^2}{M^2} \begin{cases} \gg 1 & \text{for } M \ll f, \\ \ll 1 & \text{for } M \sim f \gg m_\ell. \end{cases}$

$\mu \rightarrow e J$ with $J \rightarrow$ invisible

- TWIST, '15: limits on different anisotropies.
- Chiral coupling $\bar{\mu} P_L e J$ suppresses sensitivity!

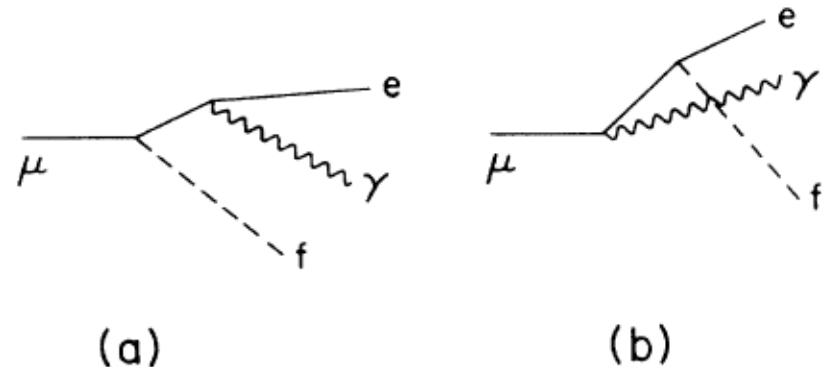
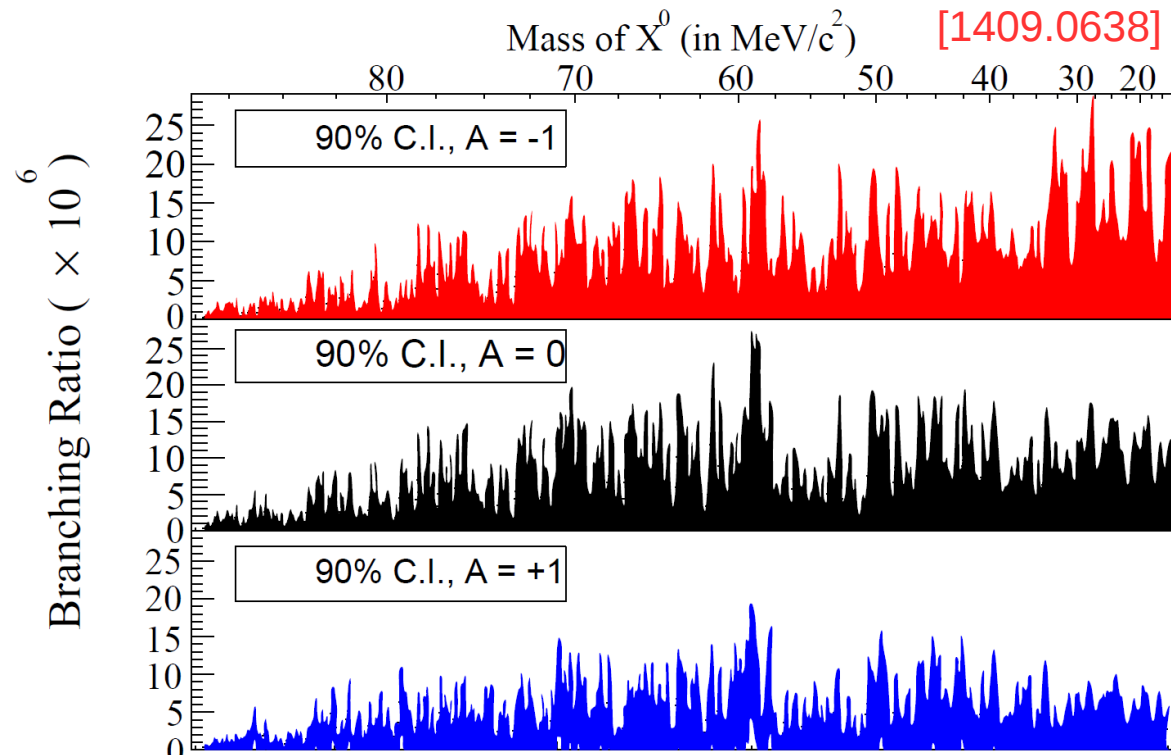
[Heeck, Garcia-Cely, 1701.07209]

- Bremsstrahlung is competitive: $\mu \rightarrow e J \gamma$.

[Goldman et al, '87]

- Approximate limit

$$\frac{|(m_D m_D^\dagger)_{\mu e}|}{v f} \lesssim 10^{-5}.$$



$\mu \rightarrow e J$ with $J \rightarrow$ invisible

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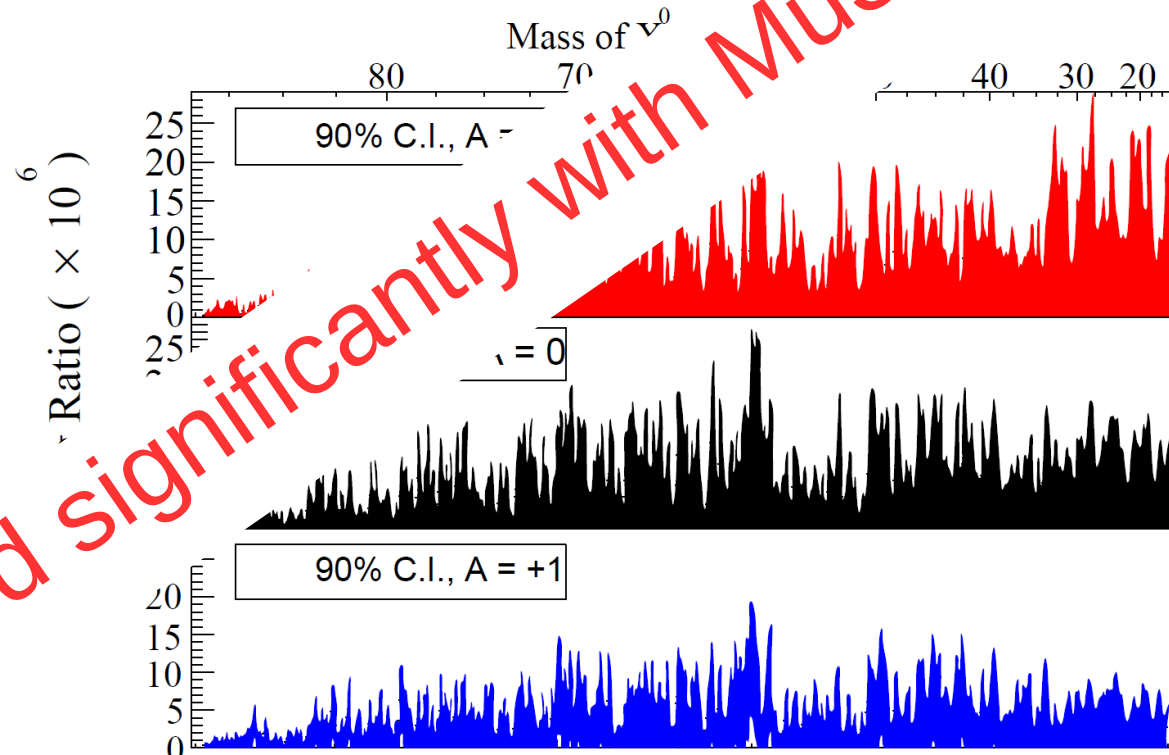
[Heeck, Garcia-Cely, 1701.07206]

- Bremsstrahlung competitive

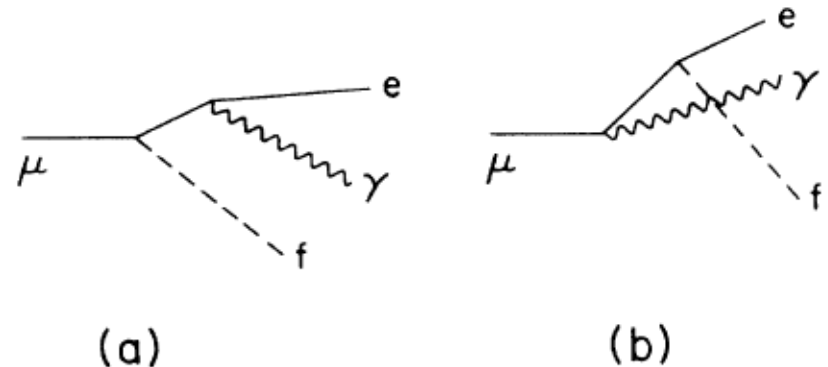
[Goldman et al.]

- Approximate limit

$$\left| \frac{\text{Im}(m_D^\dagger)_{\mu e}}{v f} \right| \lesssim 10^{-5}.$$



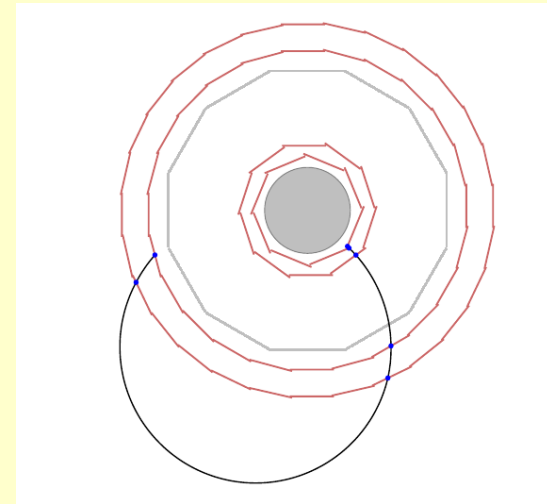
Can be improved significantly with Mu3e!



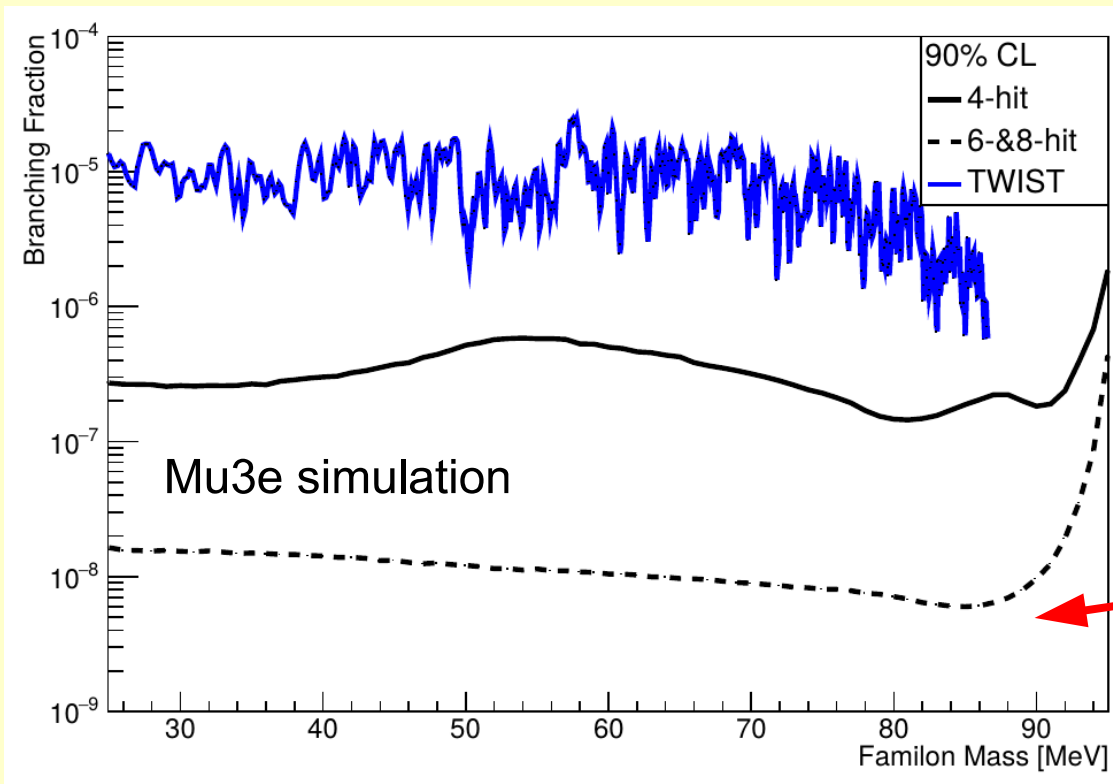


Searches for $\mu \rightarrow e X$ with Mu3e

- Full reconstruction of all Michel decays is a big challenge for data acquisition
- $B(\mu \rightarrow e X) \sim 10^{-8}$ at 90 % CL



recurling track in Mu3e



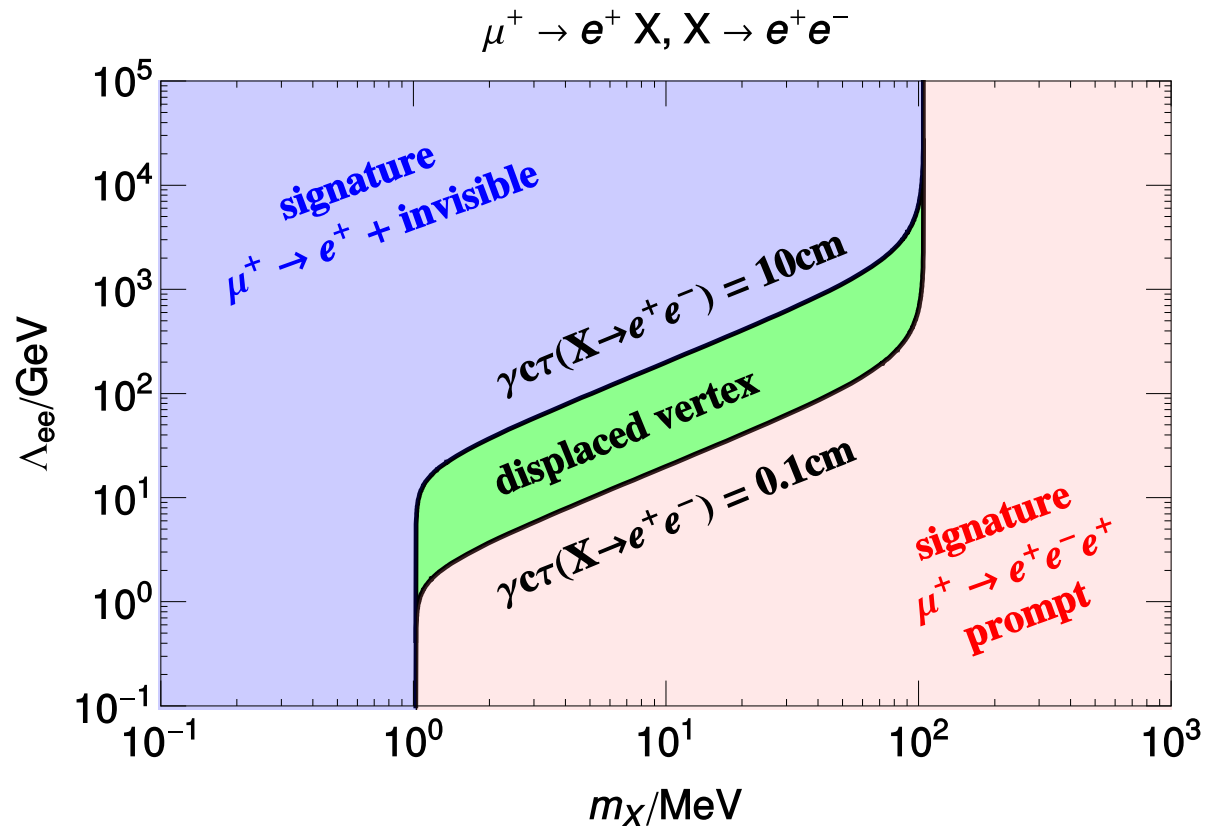
required full reconstruction of “recurlers”

$\mu \rightarrow e X$ with $X \rightarrow$ visible

- Take $X\bar{e}y_5e m_e/\Lambda_{ee}$.
- Decay length determines signature.
- Displaced vertex gives new observable.
[Heeck, Rodejohann, 1710.02062]

- Muon at rest:

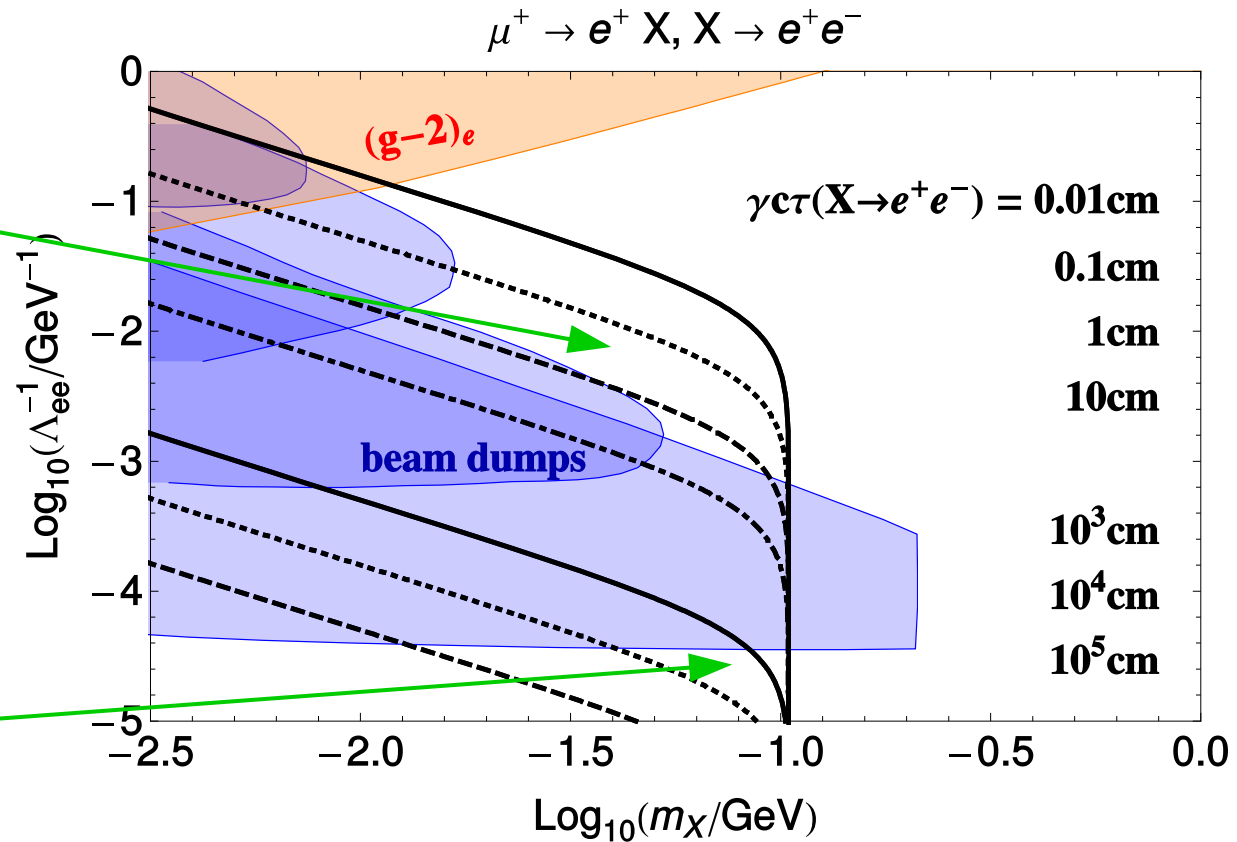
$$\gamma c\tau \simeq \frac{\pi m_\mu \Lambda_{ee}^2}{m_e^2 m_X^2} \simeq 2.5 \text{ cm} \left(\frac{\Lambda_{ee}}{100 \text{ GeV}} \right)^2 \left(\frac{10 \text{ MeV}}{m_X} \right)^2.$$



Sub-GeV X with ee coupling allowed?

$\mu \rightarrow e X$ with $X \rightarrow \bar{e}e$

- Decay length typically below cm. => looks prompt.
- Below beam dump: $\Lambda_{ee} > 30$ TeV; mostly invisible, but some DV!



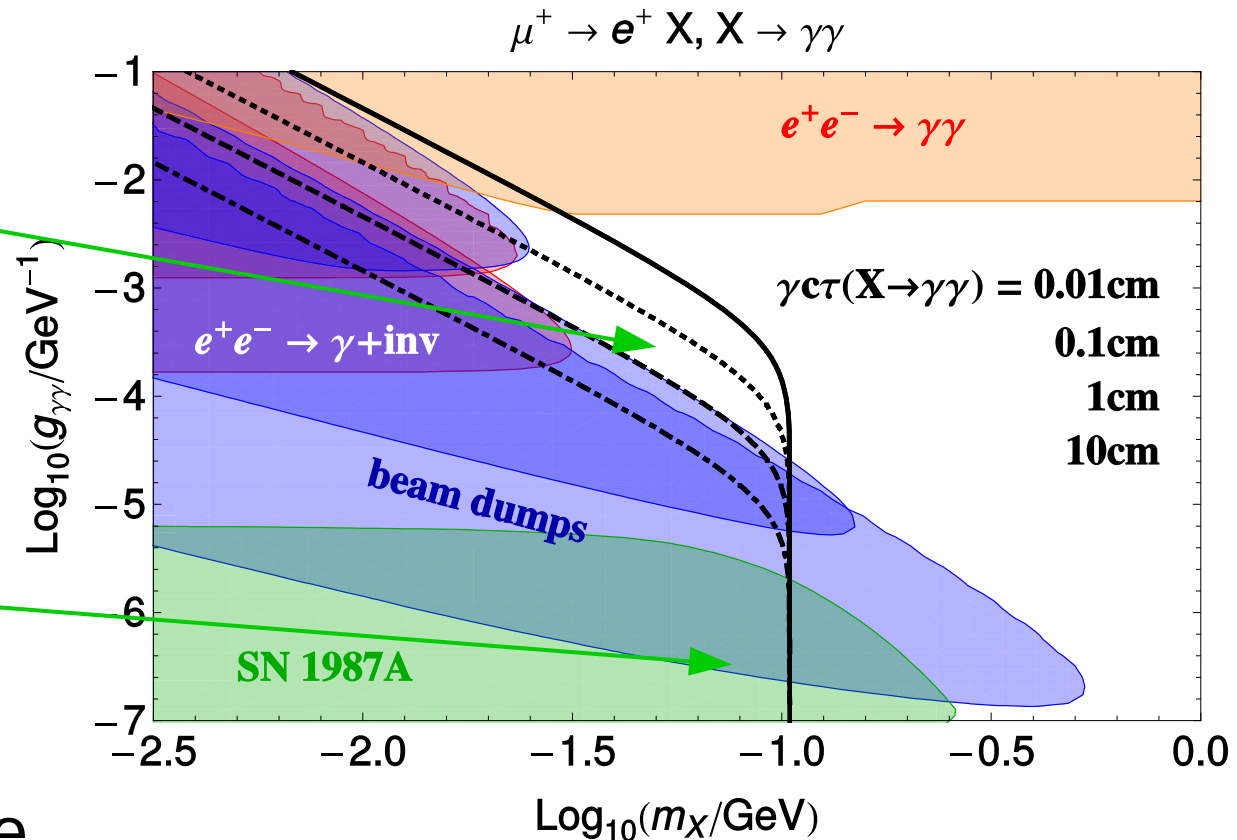
$$\text{BR}(\mu \rightarrow eX) \text{BR}(X \rightarrow ee) (1 - P(l_{\text{dec}}))$$

$$\simeq \text{BR}(\mu \rightarrow eX) \frac{l_{\text{dec}}}{\gamma_{CT}}$$

Possible in Mu3e!

$\mu \rightarrow e X$ with $X \rightarrow \gamma\gamma$

- Decay length always below cm. \Rightarrow looks prompt.
- Below beam dump: supernova constraints!
- Prompt channel still interesting, maybe MEG(II) or Mu3e extension?

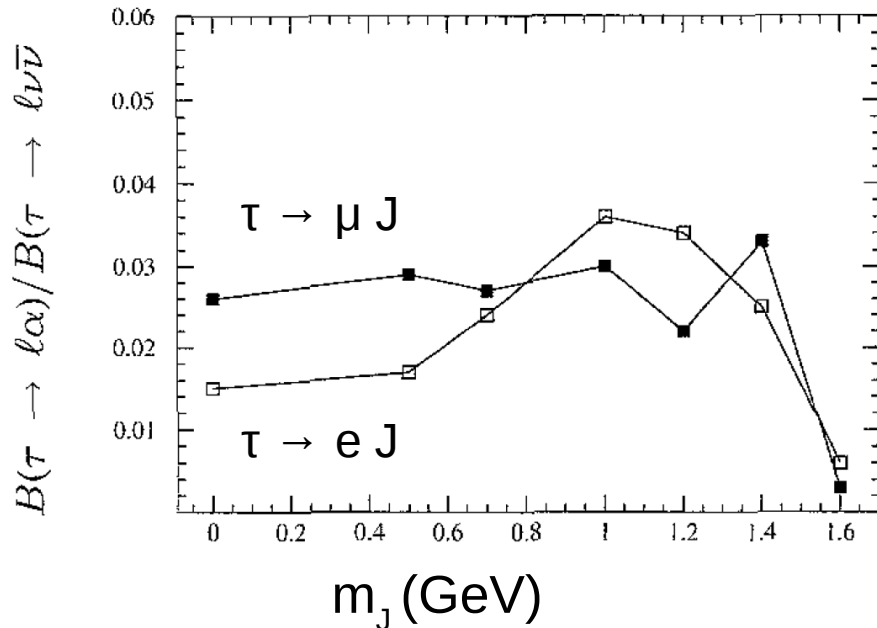


[Recent limits: Dolan et al, 1709.00009]

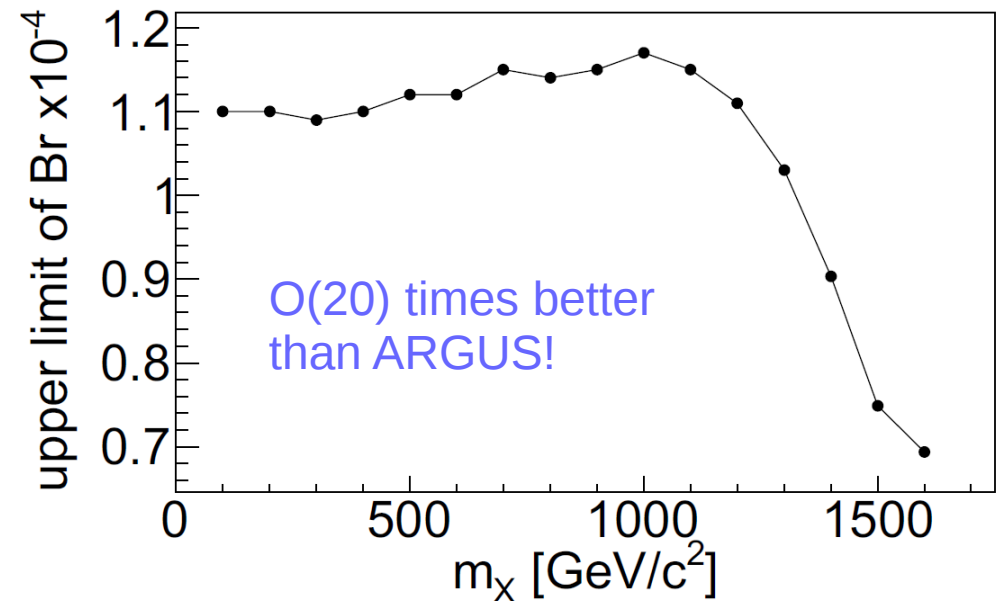
Muons difficult, taus easier.

$\tau \rightarrow \ell J$ with $J \rightarrow$ invisible

- ARGUS, '95; $5e5$ taus.



- Belle, '16 prelim.; $1e9$ taus.



- Also interesting for LFV Z'.

[Foot, He, Lew, Volkas, '94; Heeck, 1602.03810; Altmannshofer et al, 1607.06832]

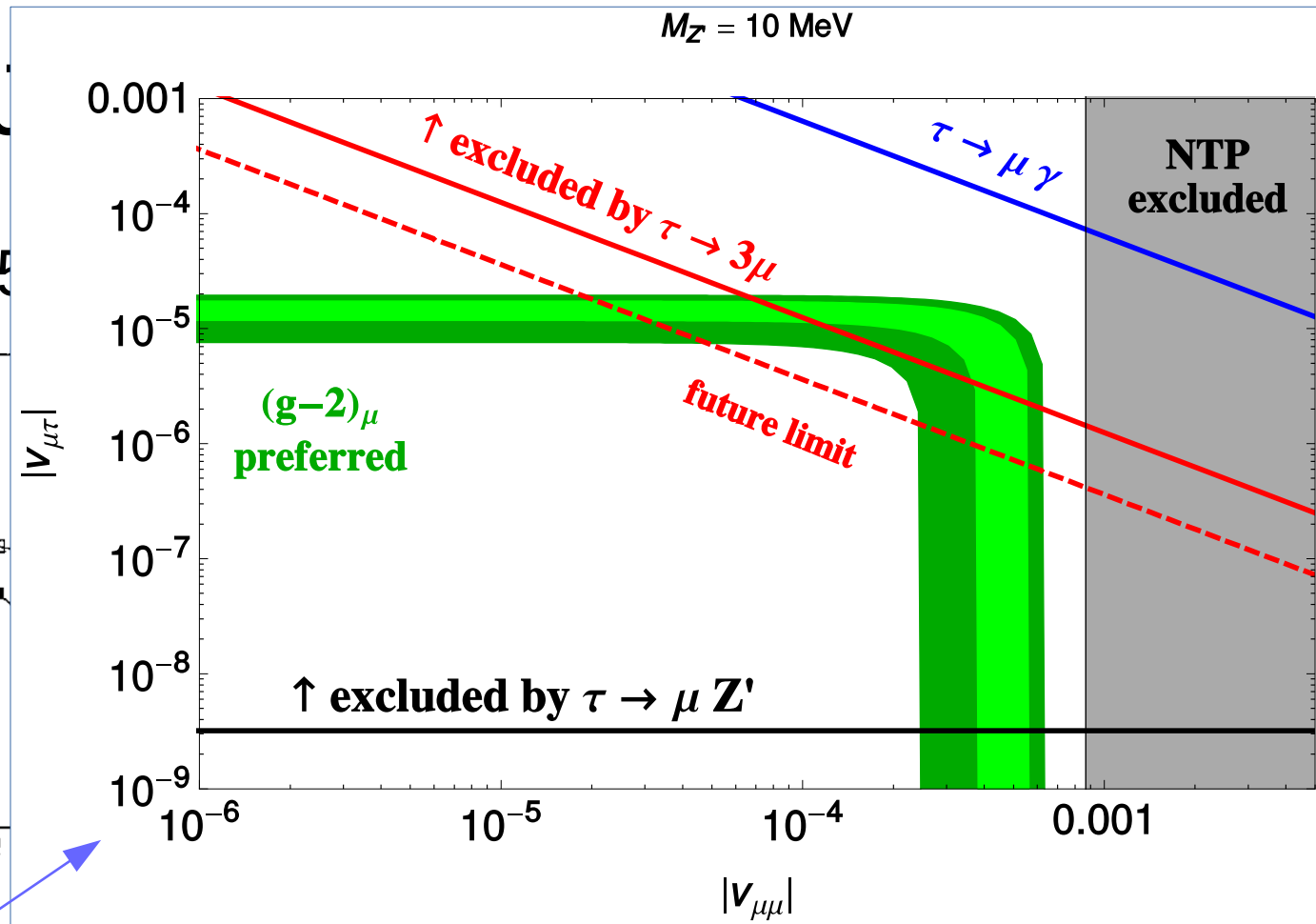
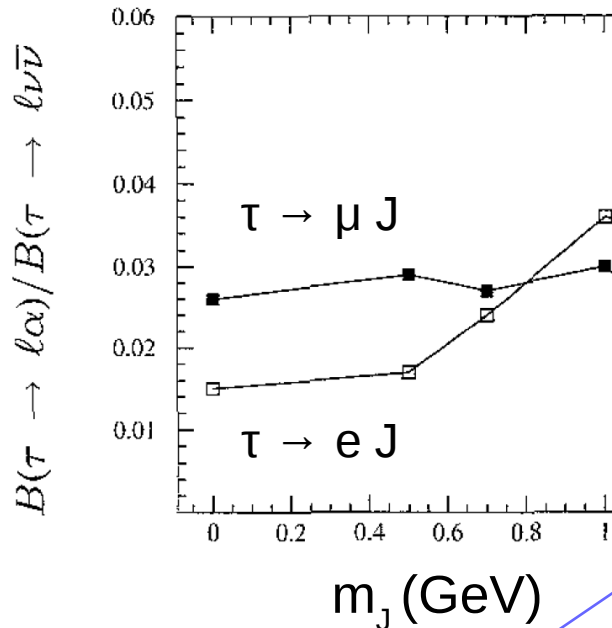
- Improvement with Belle-II.

$$\frac{|(m_D m_D^\dagger)_{\tau e}|}{vf} \lesssim 6 \times 10^{-3},$$

$$\frac{|(m_D m_D^\dagger)_{\tau \mu}|}{vf} \lesssim 9 \times 10^{-3}.$$

$\tau \rightarrow \ell J$ with J

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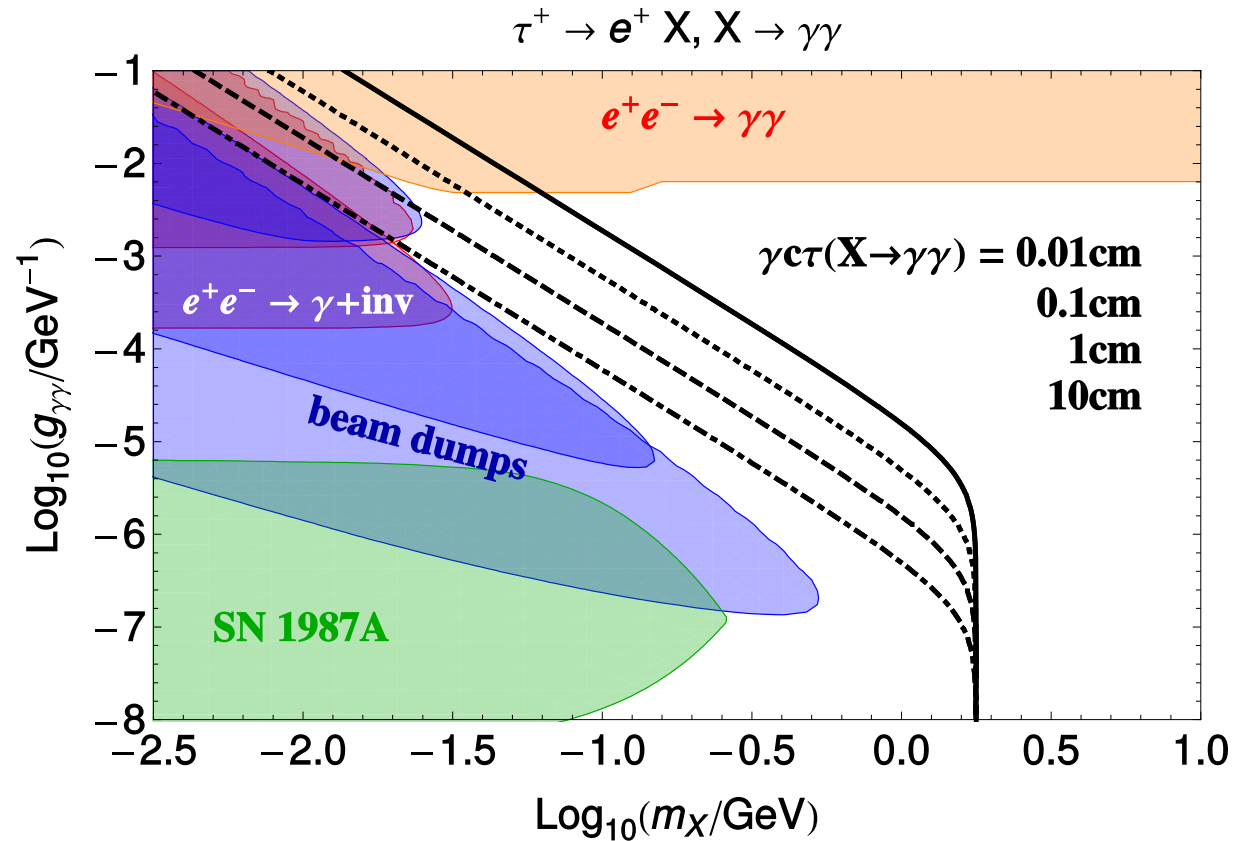
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$\tau \rightarrow e X$ with $X \rightarrow$ visible

- Tau at rest, higher X boost.
- Arbitrary decay lengths possible.
- Similar for $X \rightarrow ee, \mu\mu, \mu e$.
- Worthwhile in LHCb and Belle (II).



[Recent limits: Dolan et al, 1709.00009]

Muons difficult, taus easier...

Reconstruct seesaw

- $\{m_D, M_R\} \leftrightarrow \{M_\nu = -m_D M_R^{-1} m_D^T, m_D m_D^\dagger\}$. [Davidson, Ibarra, '01]
- $J_{\nu\nu}$ coupling to measure $U(1)_L$ scale f .
- J_{ff} couplings to reconstruct

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Difficult to probe $f > 10^7$ GeV. What happens there?

Reconstruct seesaw

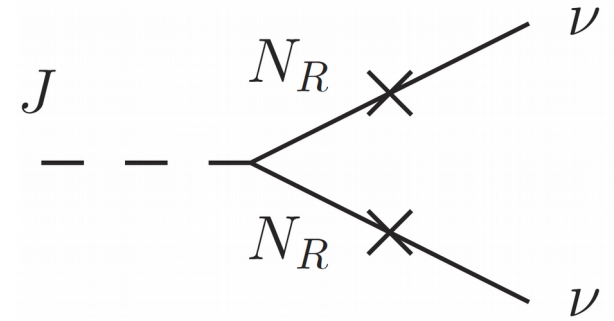
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Difficult to probe $f > 10^7$ GeV. What happens there?

Majoron dark matter!

Tree-level couplings



- $L = iJ \sum_k \left(\frac{m_k}{2f} \right) \bar{\nu}_k \gamma_5 \nu_k + \dots$

Tiny coupling: neutrino mass over B-L breaking scale!

- Long lifetime \rightarrow majoron dark matter!

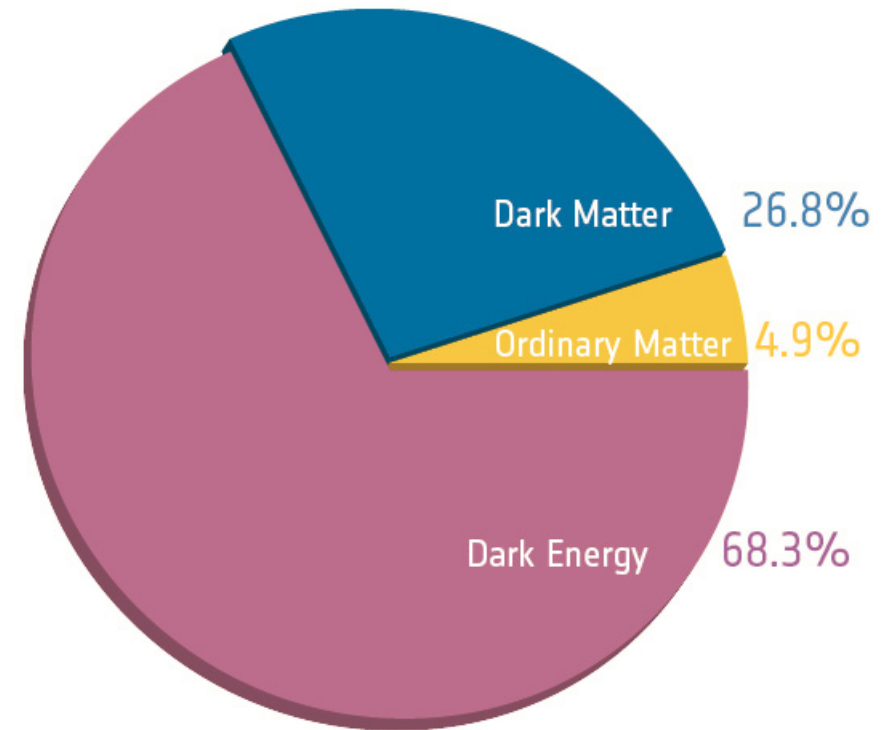
[Berezinsky, Valle '93; Lattanzi, Valle '07; Bazzocchi et al, '08; Queiroz, Sinha, '14]

$$\Gamma(J \rightarrow \nu\nu) \simeq \frac{1}{3 \times 10^{19} \text{s}} \left(\frac{m_J}{\text{MeV}} \right) \left(\frac{10^9 \text{GeV}}{f} \right)^2 \left(\frac{\sum_k m_k^2}{10^{-3} \text{eV}^2} \right)$$

- Signatures for $\text{MeV} < m_J$: $J \rightarrow \nu\nu, \gamma\gamma, \bar{f}f$.
- For $\text{keV} < m_J < \text{MeV}$: $J \rightarrow \gamma\gamma$. Maybe *warm* DM.

Dark matter abundance

- **Freeze out** via $\lambda J J \bar{H} H$:
 - $m_J \sim m_h/2$,
 - $m_J > 400$ GeV.



Dark matter abundance

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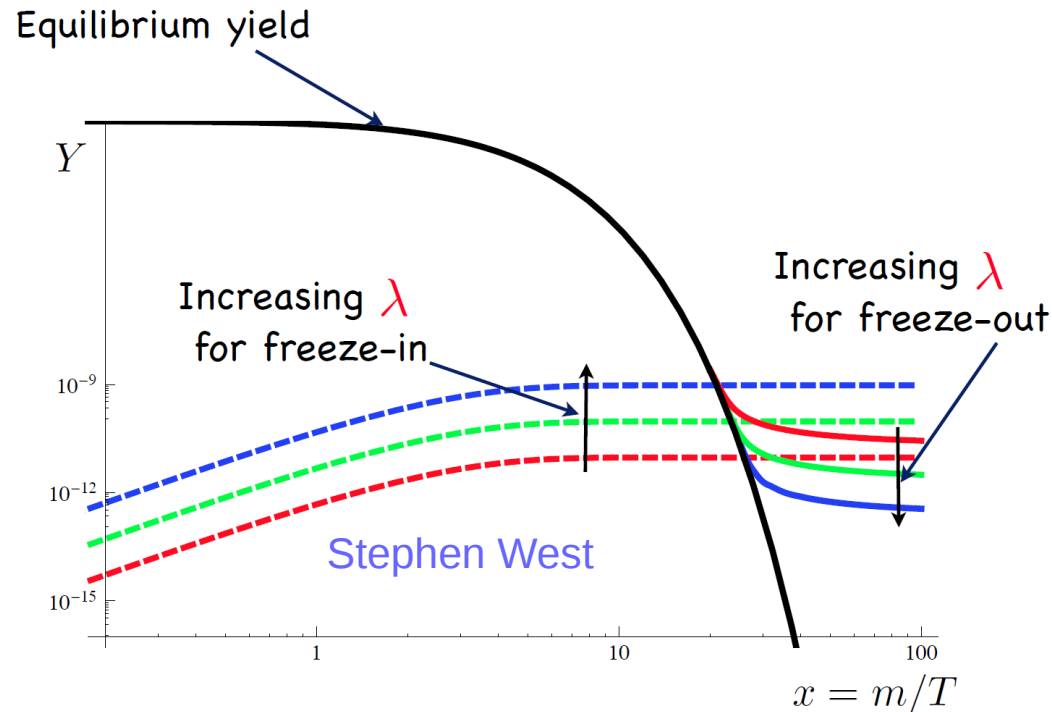
- $m_J \sim m_h/2$,
- $m_J > 400$ GeV.

- **Freeze in:**

$$\Omega_J \propto m_J \Gamma(h \rightarrow JJ)$$

$$\Rightarrow m_J \simeq \left(\frac{10^{-10}}{\lambda} \right)^2 \text{ MeV.}$$

[McDonald, '02; Hall, Jedamzik, March-Russell, West '10; Frigerio, Hambye, Masso, '11]



Lyman- α excludes $m_J < 12$ keV!
 Use different mechanism:
[JH, Teresi, 1706.09909, 1709.07283.](#)

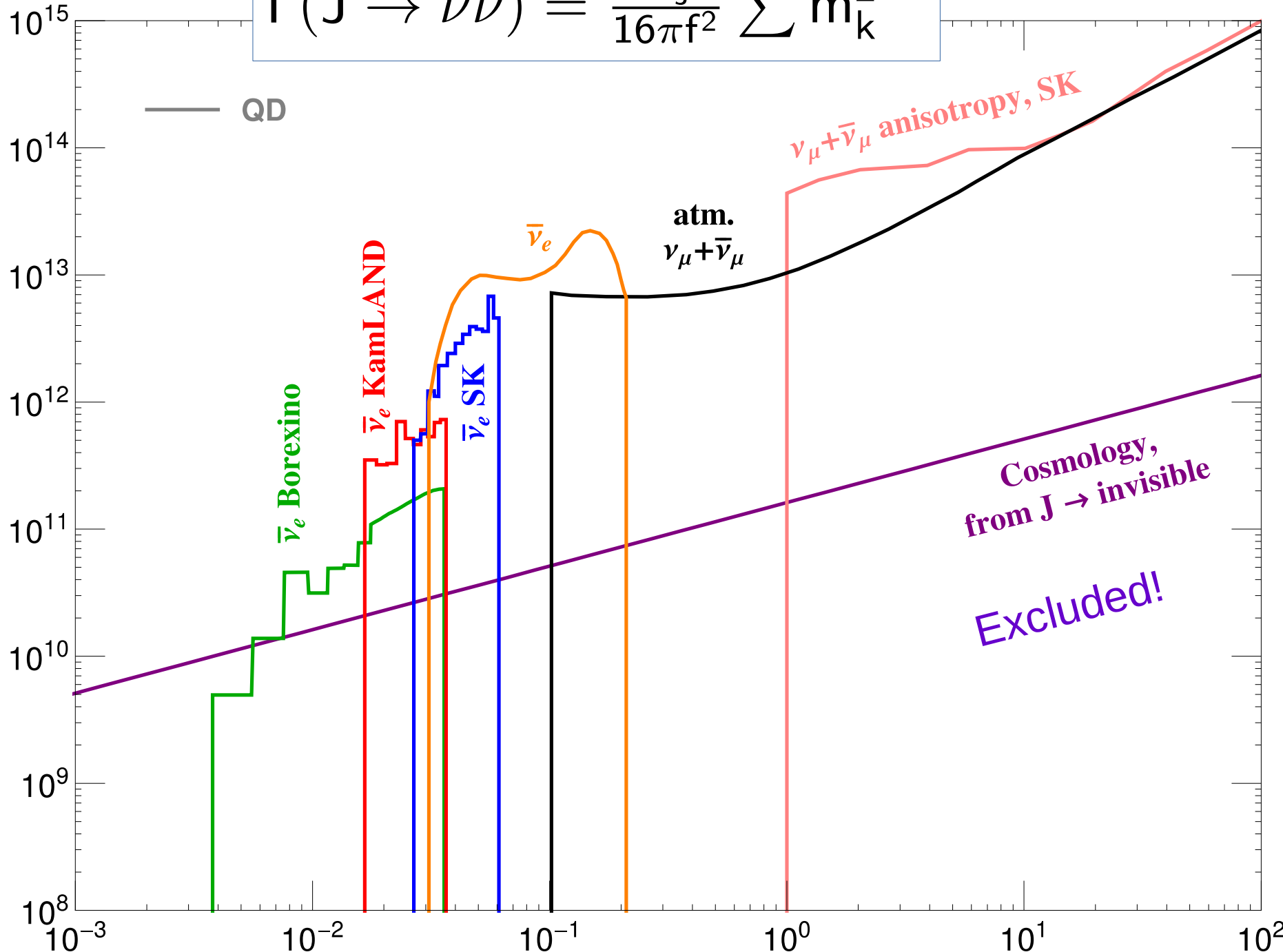
Indirect detection

$$\Gamma(J \rightarrow \nu\nu) \simeq \frac{1}{3 \times 10^{19} \text{s}} \left(\frac{m_J}{\text{MeV}} \right) \left(\frac{10^9 \text{GeV}}{f} \right)^2 \left(\frac{\sum_k m_k^2}{10^{-3} \text{eV}^2} \right)$$

- **General limit from DM \rightarrow invisible:** $\tau \gtrsim 10 \times \tau_{\text{Universe}}$.
[Audren, Lesgourgues, Mangano, Serpico, Tram, '14]
- **Can we observe the **neutrino lines**?**
 - $m_J > 10 \text{ TeV}$: **No**. Dominant decay is $J \rightarrow \nu\nu h$. \blacktriangleright no line!
[Dudas, Mambrini, Olive, '15]
 - Also want to avoid electroweak Bremsstrahlung.
[Kachelriess, Serpico, '07; Bell, Dent, Jacques, Weiler, '08; Queiroz, Yaguna, Weniger, '16]
 - For $\text{MeV} < m_J < 100 \text{ GeV}$: **Yes!**

$$\Gamma(J \rightarrow \nu\nu) = \frac{m_J}{16\pi f^2} \sum m_k^2$$

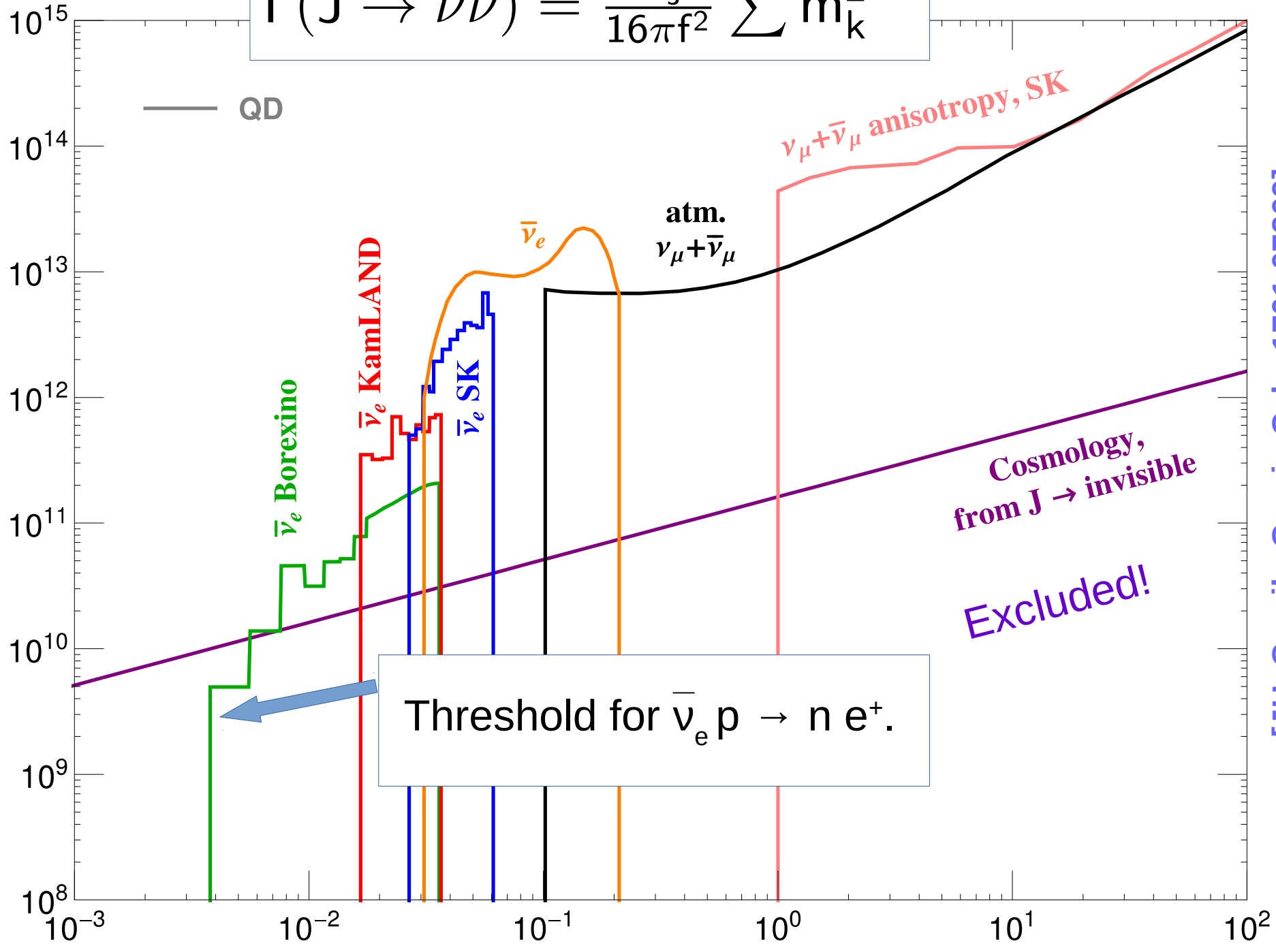
Lower limit on breaking scale f (GeV)



[JH, Camilo Garcia-Cely, 1701.07209]

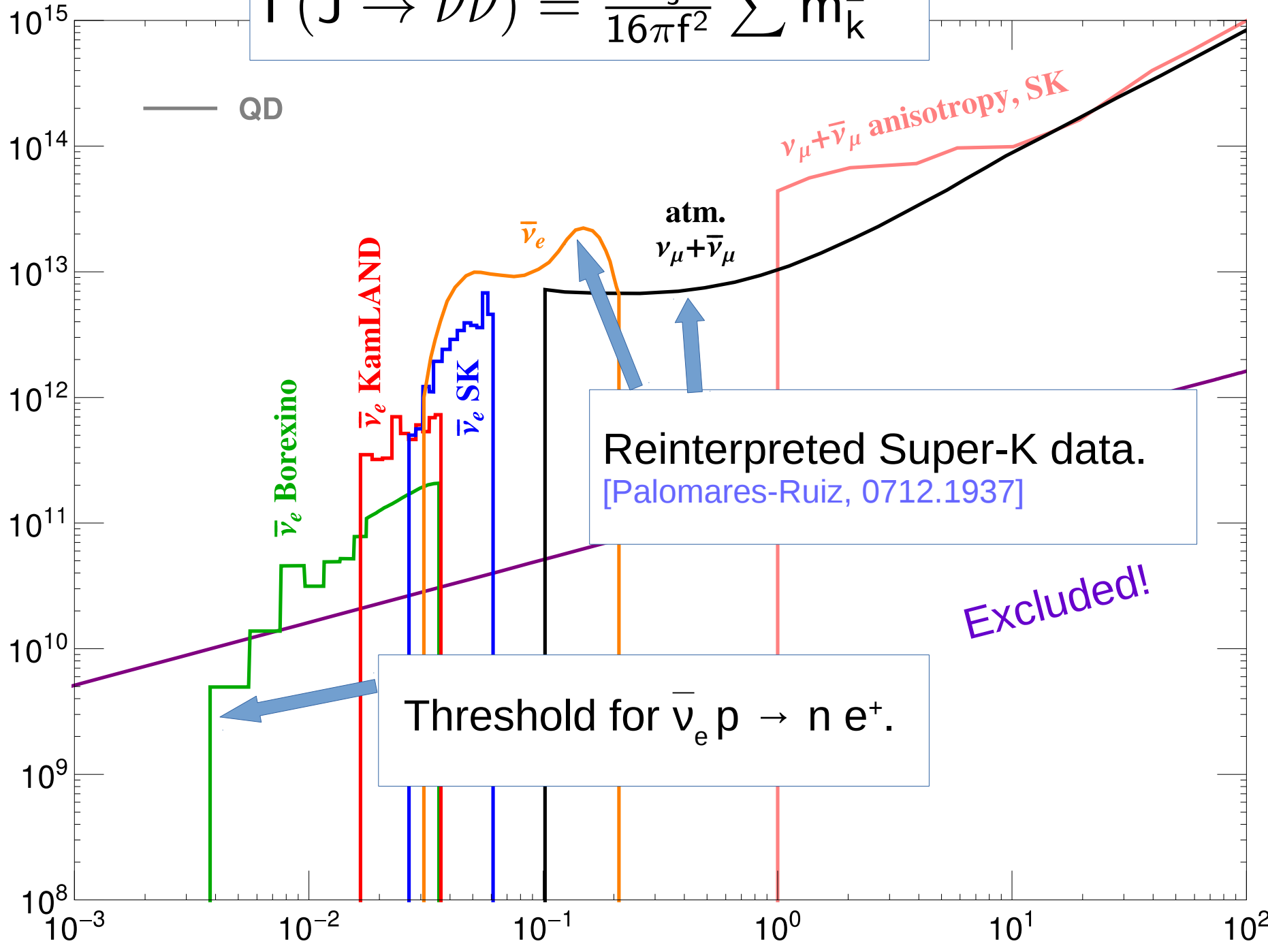
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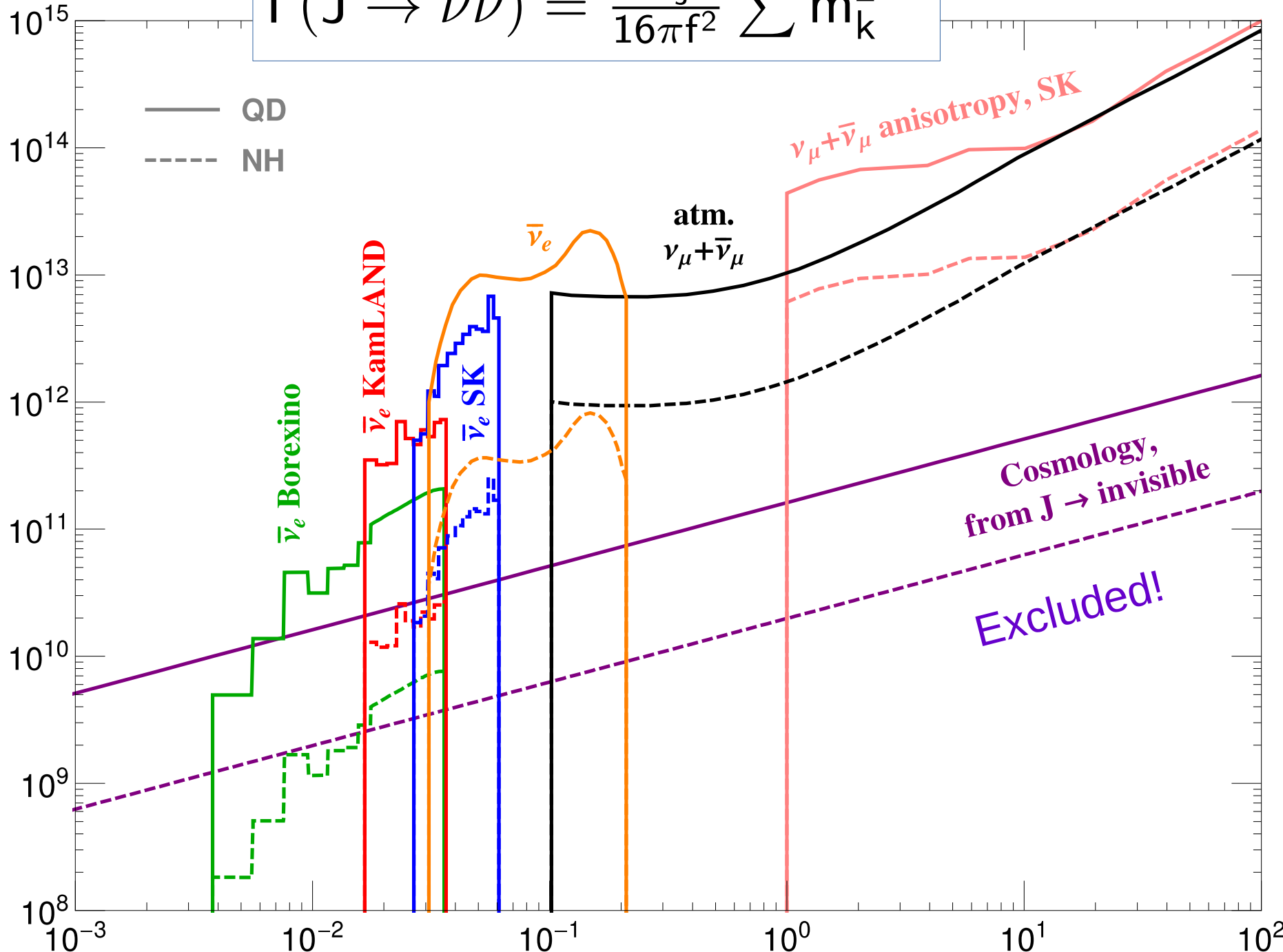
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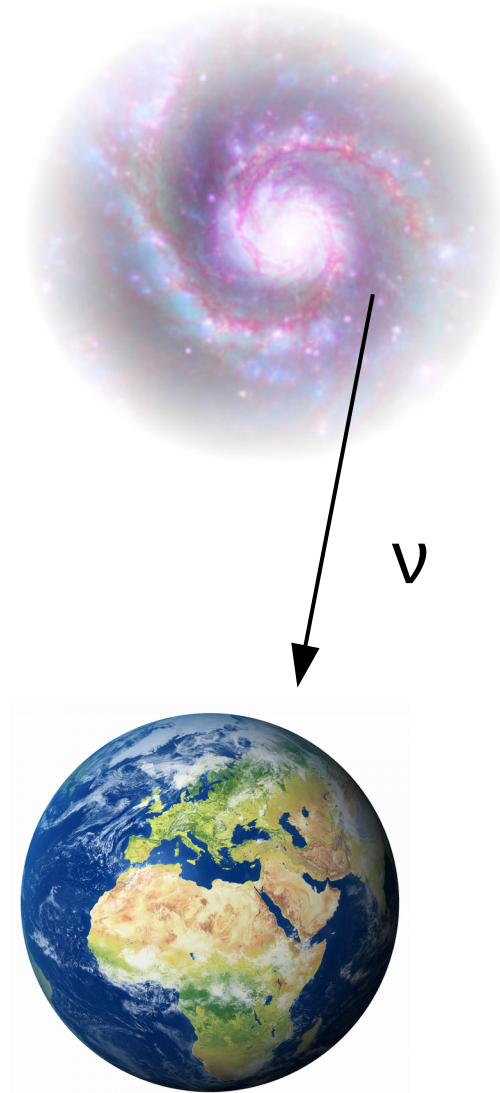


[JH, Camilo Garcia-Cely, 1701.07209]

Look for neutrinos from light DM!

- ν lines detectable down to **MeV**.
- For free in searches for diffuse supernova neutrino background.
- Borexino = indirect DM detector!
- Darwin, Hyper-K, JUNO, ... = indirect DM detectors.
- DM $\rightarrow \nu$ easily dominant channel, no SU(2) argument as for multi-TeV DM.

[El Aisati, Garcia-Cely, Hambye, Vanderheyden, 1706.06600]



Loop induced $J \rightarrow \gamma\gamma, \bar{q}q, \bar{\ell}\ell'$

- Tree-level J couplings $\propto M_\nu$ while loop level $\propto m_D m_D^\dagger$.
- E.g. diagonal J - f - f couplings with $K \equiv \frac{m_D m_D^\dagger}{v f}$

$$L = iJ\bar{q}\gamma_5 q \frac{m_q}{8\pi^2 v} (T_3^q \text{tr} K) + iJ\bar{\ell}\gamma_5 \ell \frac{m_\ell}{8\pi^2 v} (T_3^\ell \text{tr} K + K_{\ell\ell}).$$

- One-to-one mapping: $\{m_D, M_R\} \leftrightarrow \{M_\nu, m_D m_D^\dagger\}$.

[Davidson, Ibarra, hep-ph/0104076]

$J \rightarrow \gamma\gamma, \bar{q}q, \bar{\ell}\ell'$ are *complementary* to $\nu\nu$ channel!

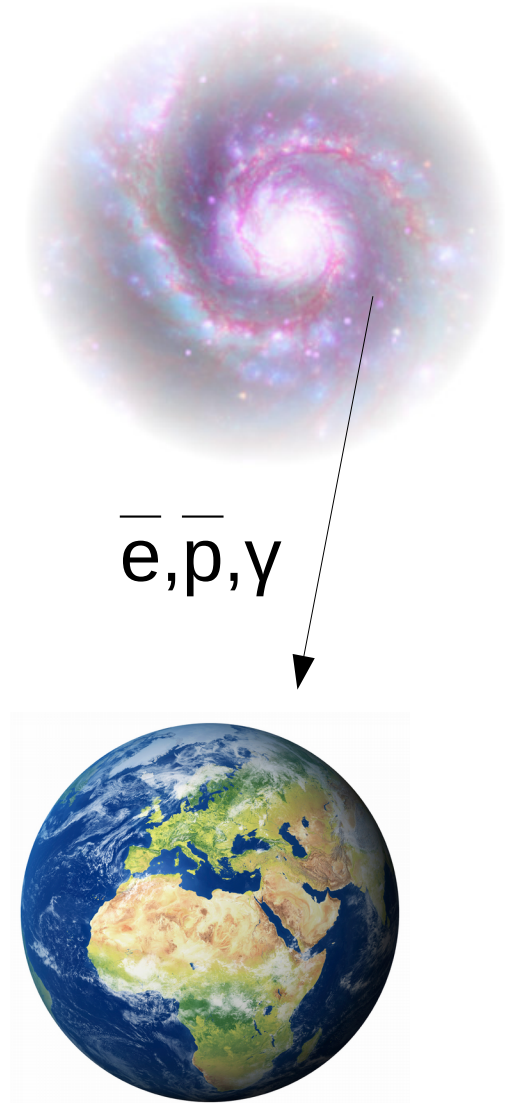
- One generation: $K \sim \frac{m_\nu M_R}{v f} \sim 10^{-13} M_R / f$.

[Chikashige, Mohapatra, Peccei, '81; Pilaftsis '94]

Indirect detection II

$$\Gamma(J \rightarrow \bar{f}f) \propto m_f^2 \mathcal{O}(K^2)$$

- DM \rightarrow $\tau\tau$, bb , tt , ... give
 - continuous γ spectrum:
Integral, Fermi-LAT.
 - anti-protons and positrons:
PAMELA, AMS-02.
- DM decay around $z \sim 1000$:
 - modification of CMB.
[\[Slatyer, Wu, 1610.06933\]](#)
 - independent of DM profile.
- DM $\rightarrow \gamma\gamma$ gives lines.

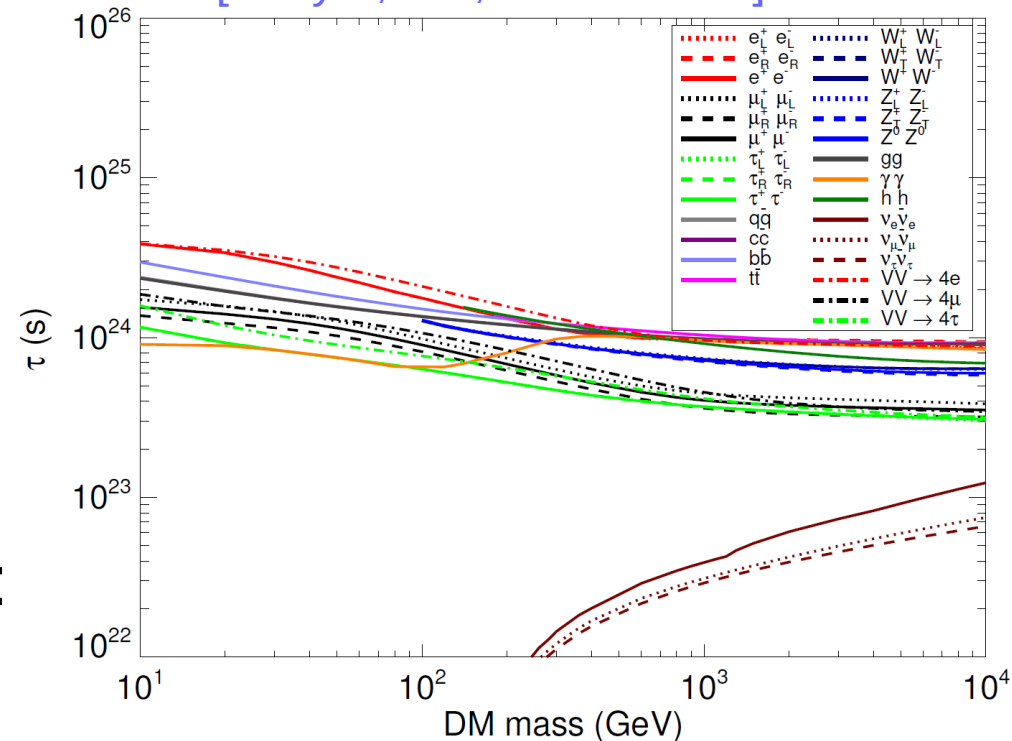


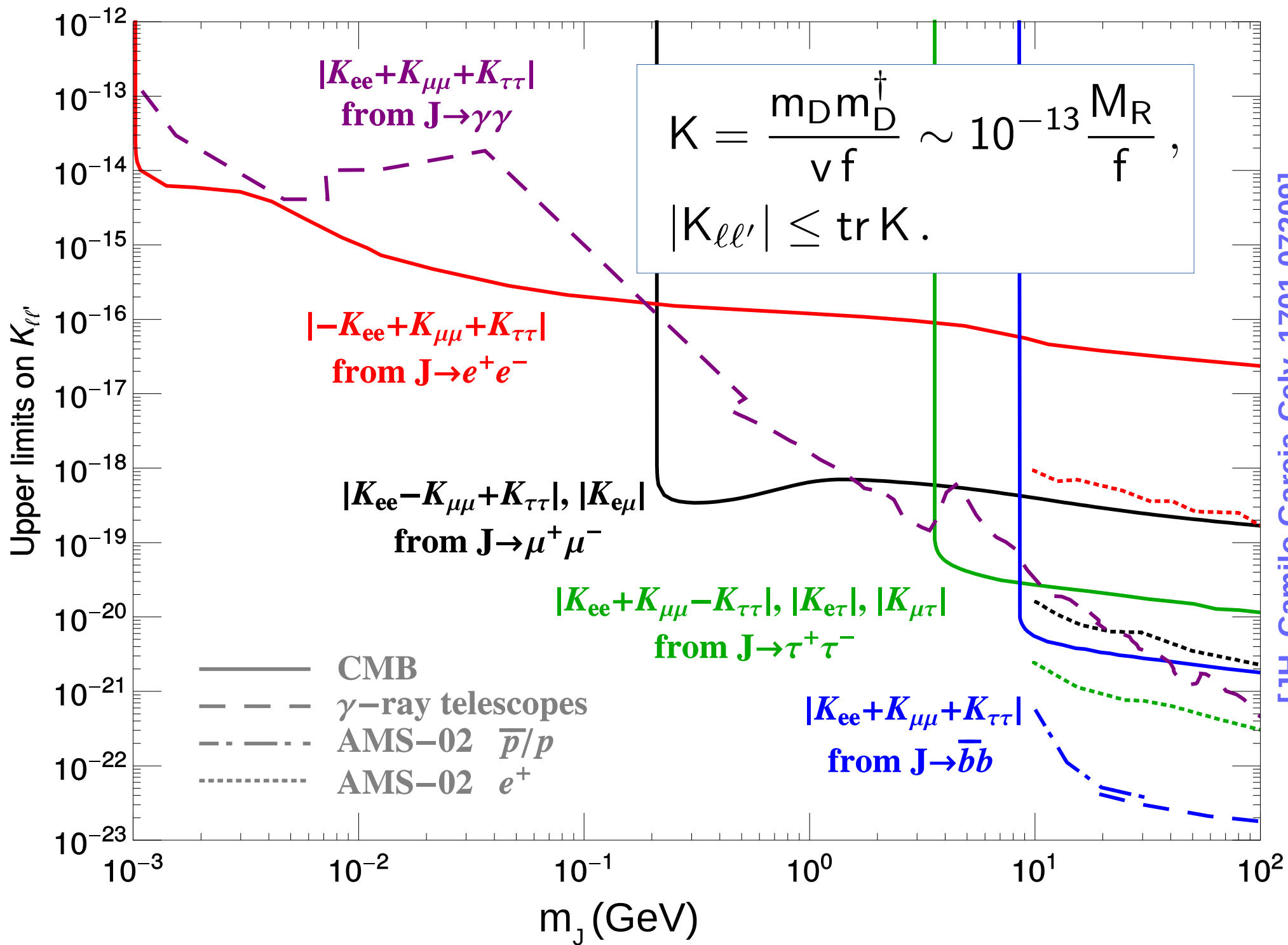
Indirect detection II

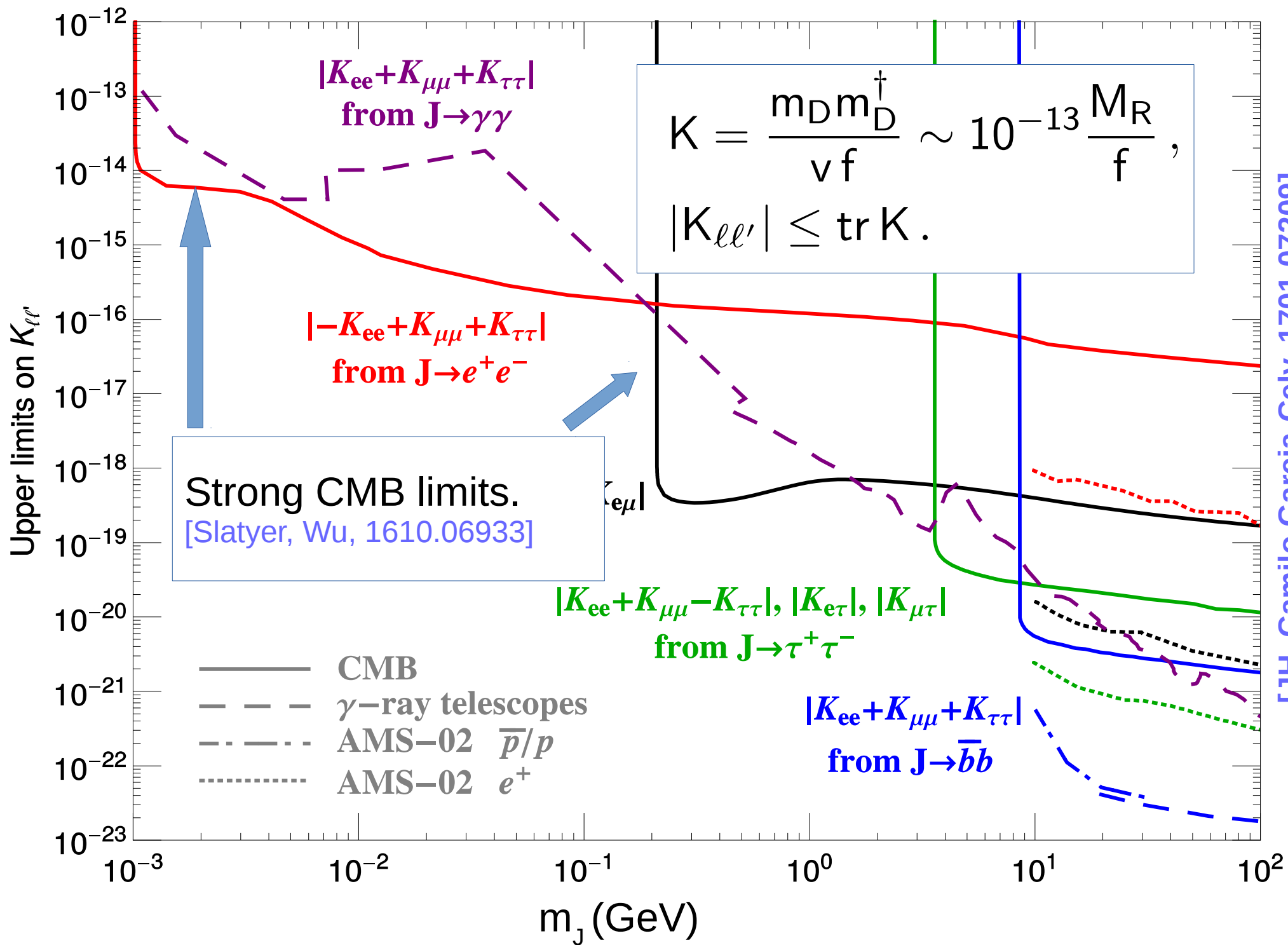
$$\Gamma(J \rightarrow \bar{f}f) \propto m_f^2 \mathcal{O}(K^2)$$

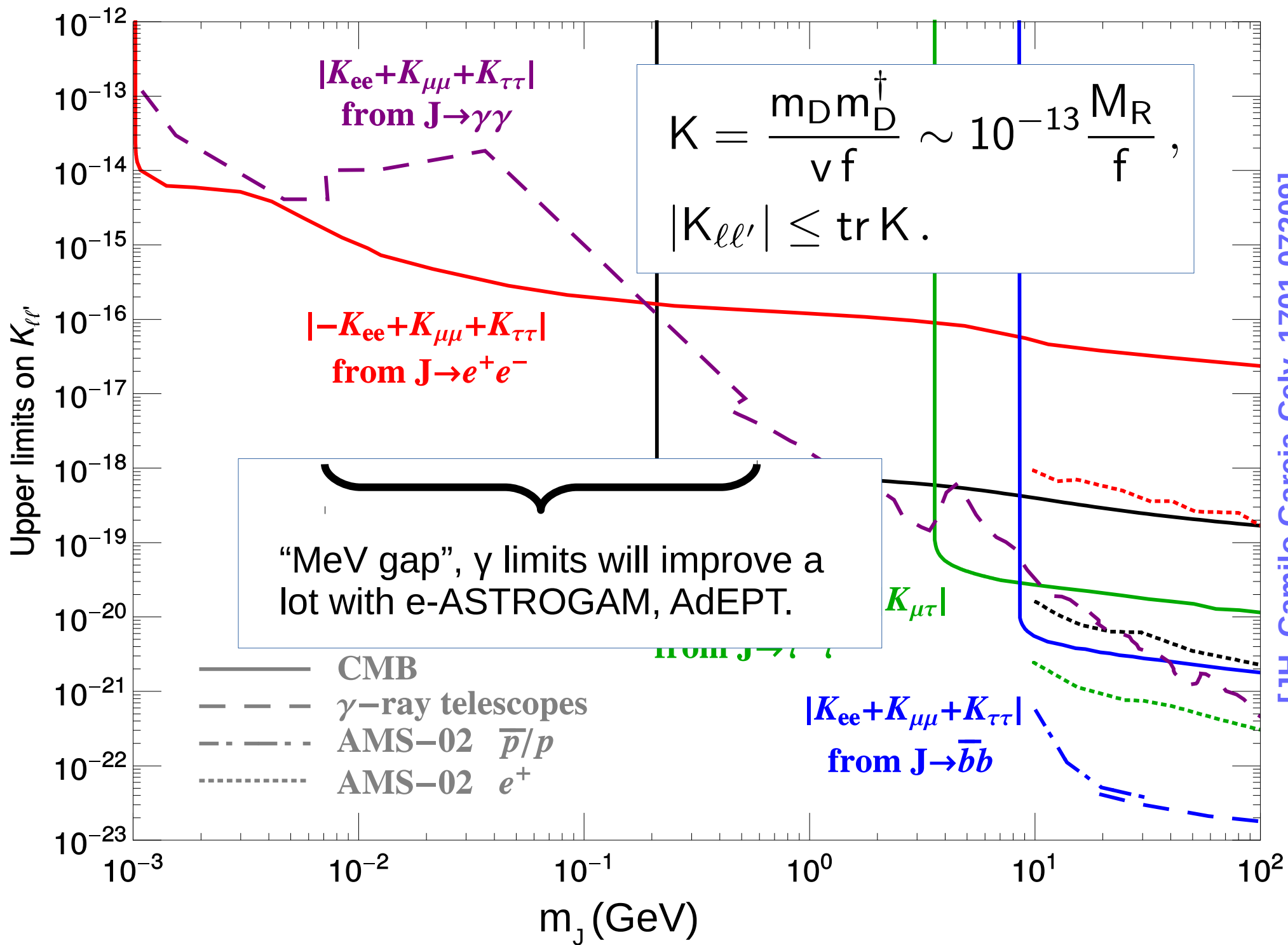
- DM $\rightarrow \tau\tau, bb, tt, \dots$ give
 - continuous γ spectrum: Integral, Fermi-LAT.
 - anti-protons and positrons: PAMELA, AMS-02.
- DM decay around $z \sim 1000$:
 - modification of CMB. [\[Slatyer, Wu, 1610.06933\]](#)
 - independent of DM profile.
- DM $\rightarrow \gamma\gamma$ gives lines.

[Slatyer, Wu, 1610.06933]



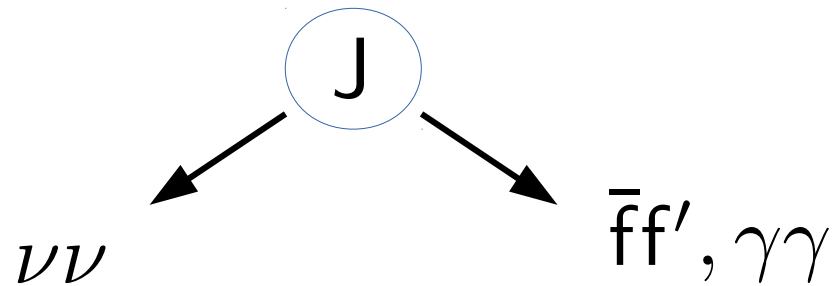






Is it possible to detect dark matter via **neutrinos** and **not** gamma-rays or anti-matter?

Yes!



depends on

$$M_\nu \simeq -m_D M_R^{-1} m_D^T.$$

depends on

$$m_D m_D^\dagger.$$

Independent / Complementary!

Summary

- Reconstruct seesaw to go beyond 3x3.
- Measure couplings of N_R or **majoron J!**
- Seesaw parameters encoded in $J\bar{f}f$ couplings.
- Axion-like particle pheno + LFV $\ell_\alpha \rightarrow \ell_\beta J$.
- J as long-lived dark matter: $J \rightarrow \nu\nu, \gamma\gamma, \bar{\ell}\ell', \bar{q}q$.
- Future: Mu3e, Belle-II, Hyper-K, DUNE,.....

Always look out for lines!

Backup

Majoron = DM

- Naturally light, long-lived DM candidate.
- Indirect detection possible:
 - $\text{MeV} < m_j$: $J \rightarrow \nu\nu, \gamma\gamma, \bar{f}f$.
 - $\text{keV} < m_j < \text{MeV}$: $J \rightarrow \gamma\gamma$. Maybe warm DM.
[\[JH, Daniele Teresi, 1706.09909, 1709.07283\]](#)

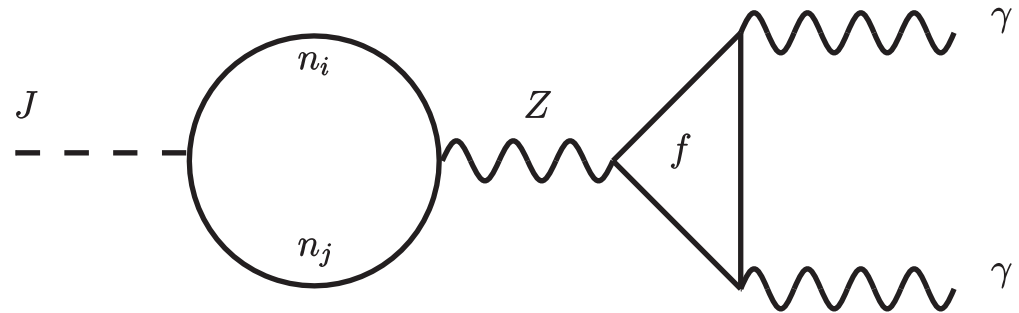
Majoron \neq DM

- Increase couplings to produce J in lab.
- Measure seesaw parameters.
[\[JH, work in progress\]](#)

Two-loop couplings

- Full calculation non-trivial, only do:

[JH, in progress]



- J-Z mixing formally similar to triplet majoron:

[Bazzocchi, Lattanzi, Riemer-Sørensen, Valle, 0805.2372]

$$\text{(two loop)} \quad \frac{\text{tr } K}{16\pi^2} \leftrightarrow \frac{2v_T^2}{v f} \cdot \quad \text{(one loop)}$$

- Gives the only DM signature for $m_J < \text{MeV}$.

[Lattanzi, Riemer-Sørensen, Tórtola, Valle, '13; Queiroz, Sinha, '14]

$$\Gamma(J \rightarrow \gamma\gamma) \simeq \frac{\alpha^2 (\text{tr } K)^2}{4096\pi^7} \frac{m_J^3}{v^2} \left| \sum_f N_c^f T_3^f Q_f^2 g \left(\frac{m_J^2}{4m_f^2} \right) \right|^2$$

Pseudo-Goldstone

- Spontaneous global U(1) breaking gives $m_J = 0$.
- Non-zero mass from:

- Breaking by gravity, e.g. wormholes,

$$m_J \sim M_{\text{Pl}} \exp \left[-\mathcal{O}(M_{\text{Pl}}/f) \right].$$

[Alonso, Urbano, 1706.07415]

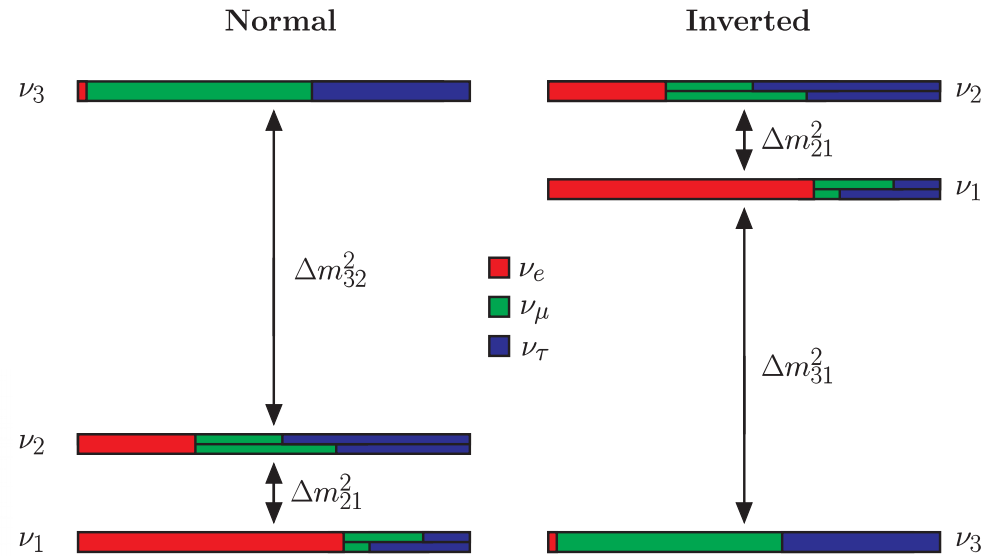
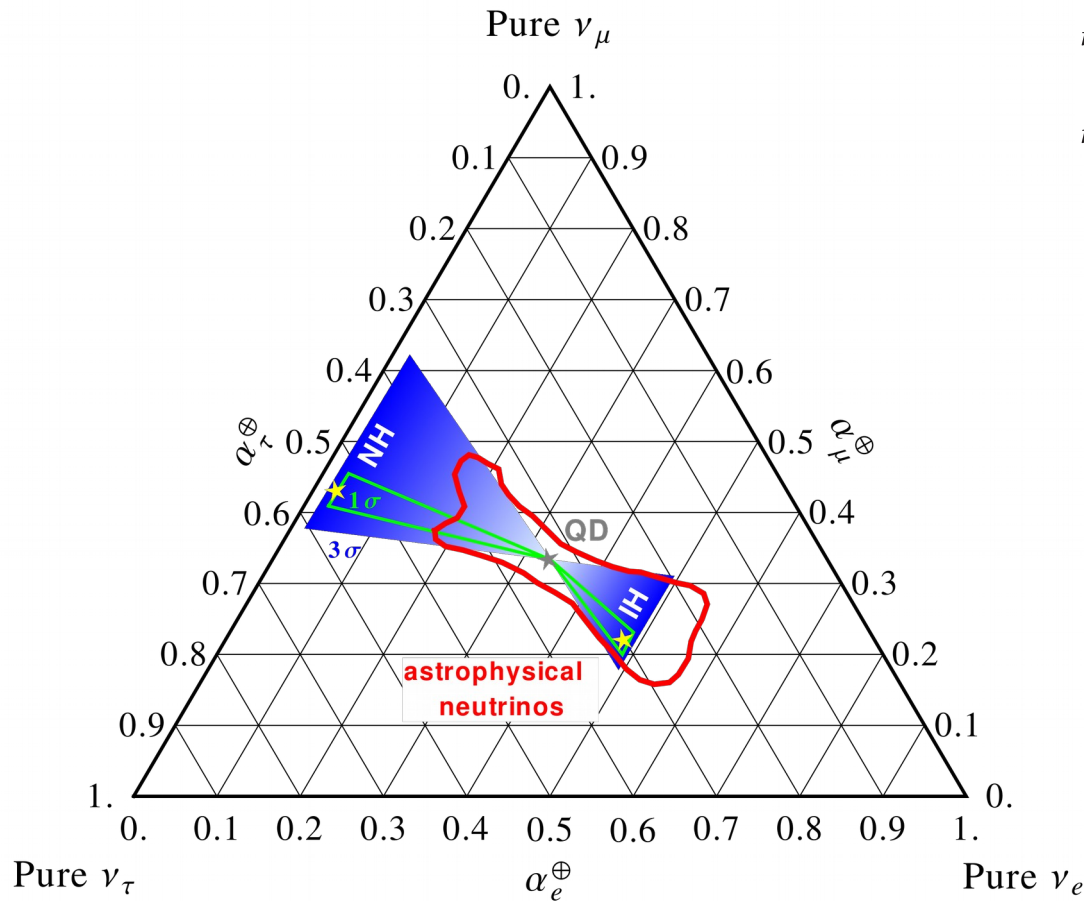
- Anomalies, e.g. if $U(1)_{\text{B-L}} = U(1)_{\text{PQ}}$.

[Mohapatra, Senjanovic '83; Langacker, Peccei, Yanagida '86; SMASH '16]

- Explicit breaking, e.g. $\Delta V = \frac{1}{2} m_J^2 J^2$.

Flavor of J $\rightarrow \nu_k \nu_k$

Mass eigenstates \rightarrow no oscillations!



Flavor ratios:

$$\alpha_e : \alpha_\mu : \alpha_\tau$$

$$\text{NH} : 0.03 : 0.43 : 0.54 ,$$

$$\text{IH} : 0.48 : 0.22 : 0.30 ,$$

$$\text{QD} : 0.33 : 0.33 : 0.33 .$$

[JH, Camilo Garcia-Cely, 1701.07209]