

# Quantum Critical Higgs Models

in the  $gg \rightarrow ZZ \rightarrow 4\ell$  channel

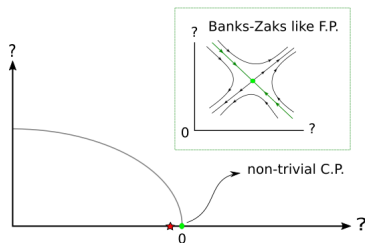
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May 6, 2018

# Why QCH?

- Hierarchy problem + LHC: we need to explore new directions.
- Georgi on unparticles: “it’s different; not just more particles”
- What is QCH?
  - Non-trivial critical point: Higgs sector is described by a (strongly coupled) CFT



- The strongly coupled theory produces a weakly coupled effective theory
- Scaling dimension is  $1 < \Delta < 2$  (In SM  $\Delta = 1 + \mathcal{O}(\alpha)$ )
- The CFT is softly broken at  $\mu > m_h$
- No new massless particles (dilaton)

# The 1PI Effective Action

$$S = S_{CFT} + S_{mix} + S_{elem.}$$

$$S \rightarrow S_{1PI} = \frac{1}{2\mathcal{Z}_h} \int \frac{d^4 p}{(2\pi)^4} h(p) \Sigma(p^2, \mu, \Delta) h(-p) + \dots$$

- $S_{1PI}$  is weakly coupled

Higgs observed pole:

- $S_{CFT} \rightarrow S_{1PI}$  : hierarchy solved.  
or
- $S_{CFT} + S_{mix} \rightarrow S_{1PI}$ : the case in our models.

# The CFT

- At high energies the CFT is restored:

$$\langle h(p)h(-p) \rangle = \frac{-i}{p^{2(2-\Delta)}}.$$

- consequently

$$\Sigma(p^2) = p^{2(2-\Delta)} \quad \text{as } p^2 \rightarrow \infty$$

- Unitarity requires:  $1 < \Delta$
- In QCH we have an action:  $\Delta < 2$  (unparticles)

# unHiggs

A 4D example: unparticles model with mass gap

- We break the CFT using the ansatz:

$$\langle h(p)h(-p) \rangle \propto \int_{\mu^2}^{\infty} (M^2 - \mu^2)^{\Delta-2} \frac{i}{p^2 - M^2 + i\epsilon} dM^2,$$

- The Kinetic term is

$$\Sigma(p^2) = -(\mu^2 - p^2)^{2-\Delta} + (\mu^2 - m_h^2)^{2-\Delta}$$

- Unitarizes  $WW \rightarrow WW$

# Higgs from AdS

- n-point functions can be calculated from:

$$\langle e^{-\int d^4x \phi_0 \mathcal{O}} \rangle_{CFT} = e^{-S_{gravity}[\phi]},$$

- The dual field has the scaling dimension

$$ds^2 = e^{-2A(z)} (\eta^{\mu\nu} dx_\mu dx_\nu - dz^2)$$

$$\Delta = 2 - \nu,$$

$$\nu = \sqrt{m^2 R^2 + 4}, \quad M^2(z) = m^2 + f(z)$$

- 

$$\Sigma(p^2) = \frac{R^3}{\epsilon^{3-2\Delta}} \partial_z \mathcal{K}(p^2, z = \epsilon) - m_{UV}^2$$

$$h_\epsilon = \epsilon^\Delta h_{QCH}, \quad p\epsilon \rightarrow 0$$

# Model I

“Holographic unHiggs”

hep-ph/0810.4940

- The background is assumed:

$$\begin{cases} A(z) = \log\left(\frac{z}{R}\right) + \frac{2}{3}\mu(z - R), \\ M^2(z) = e^{\frac{4}{3}\mu(z-R)}\left(m^2 - \frac{3z}{R^2}\right). \end{cases}$$

- After redefining the field and removing the regulating brane

$$h_\epsilon^2 = \frac{\epsilon^{4-2\nu} 2^{2\nu} \Gamma(\nu)}{R^3 \Gamma(1-\nu)} h^2$$

$$\Sigma(p^2) = -(\mu^2 - p^2)^\nu + (\mu^2 - m_h^2)^\nu$$

# Model II

“Dynamical Soft-Wall AdS/QCD”

hep-ph/0801.4383

- Constructed to study AdS/QCD
- Background is solved in the presence of a dilaton,  $\Phi$ , and a Tachyonic mode.

$$\begin{cases} A(z) = \log\left(\frac{z}{R}\right) \\ M^2(z) = m^2 \\ \Phi(z) = (2\mu z)^\alpha \end{cases} \leftarrow \begin{cases} \alpha = 1, & \text{continuum} \\ \alpha = 2, & \text{Regge} \end{cases}$$

$$S_5 = \int d^5x \sqrt{g} e^{-\Phi} \mathcal{L}$$

$$\Sigma(p^2) = (\mu^2 - p^2)^\nu \frac{\Gamma\left(\frac{1}{2} + \frac{3\mu}{2\sqrt{\mu^2 - p^2}} + \nu\right)}{\Gamma\left(\frac{1}{2} + \frac{3\mu}{2\sqrt{\mu^2 - p^2}} - \nu\right)} - (p^2 \rightarrow m_h^2)$$



# Model III

“ The AdS/CFT/Unparticle Correspondence”

hep-ph/0804.0424

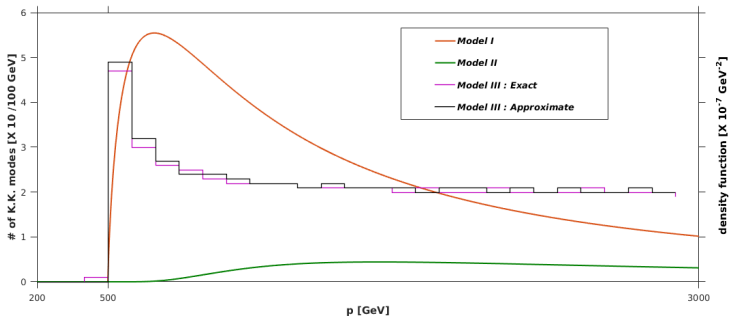
- An addition scalar field was added in the bulk
- The scalar acquires a z-dependent VEV

$$\begin{cases} A(z) = \log\left(\frac{z}{R}\right) \\ \phi(z) = \mu^2 R^{1/2} \left(\frac{z}{R}\right)^2 \\ M^2(z) = m^2 + R^{1/2} \phi(z) \end{cases}$$

- We further investigate the back-reaction of the scalar field on the geometry
- There is a singularity at  $z_s \approx (RM_5)^{3/4}/\mu$

$$\Sigma(p^2) = (\mu^2 - p^2)^\nu \left( 1 + \frac{2}{\Gamma(1-\nu)\Gamma(\nu)} \frac{K_\nu(\sqrt{\mu^2 - p^2} z_s)}{I_\nu(\sqrt{\mu^2 - p^2} z_s)} \right) - (p^2 \rightarrow m_h^2)$$

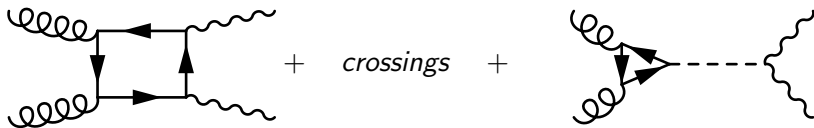
# The generic behavior of the spectral density functions



$$\langle h(p)h(-p) \rangle \propto \int_{\mu^2}^{\infty} \rho(M^2) \frac{i}{p^2 - M^2 + i\epsilon} dM^2,$$

# Interactions

- $gg \rightarrow ZZ$ :



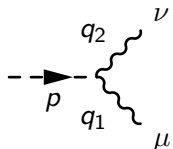
- We will focus on the  $hZZ$  coupling.

## Gauging the non-local action

- We can gauge the non-local action using Mandelstam's method (Wilson line)

$$S = \int d^4x \int d^4y \quad h(x) \Sigma(x-y) P \cdot e^{-ig \int_x^y A_\mu du^\mu} h(y)$$

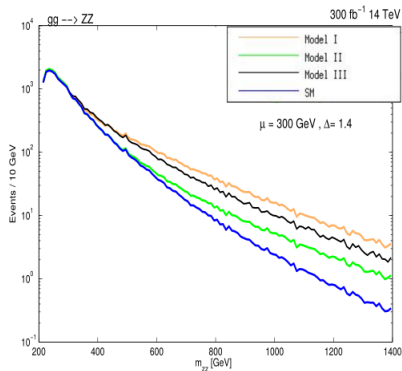
- $hZZ$  (Stancato's PhD thesis)

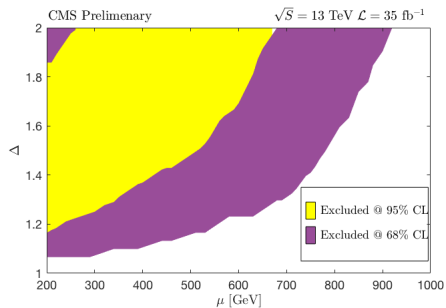
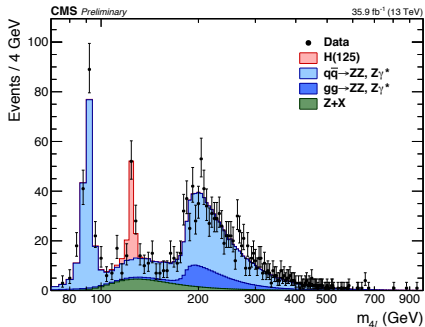


$$= -i \frac{gM_Z}{2c_w \mathcal{Z}_h} \left[ 2\eta^{\mu\nu} \frac{\Sigma(p) - \Sigma(0)}{p^2} + (2q_2 + q_1)^\mu q_2^\nu * \# + \dots \right]$$

# Results

- Plot has been generated using *ggZZ* program
- The Zs are on-shell
- No background



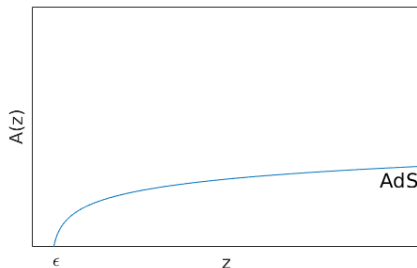


## AdS/Broken-CFT

$$ds^2 = e^{-2A(z)} (\eta^{\mu\nu} dx_\mu dx_\nu - dz^2)$$

AdS  $\iff$  CFT      energy scale  $\iff$   $1/z$

5D



4D dual

- UV cut-off =  $1/\epsilon$
- CFT not broken in IR
- e.g. RS II model

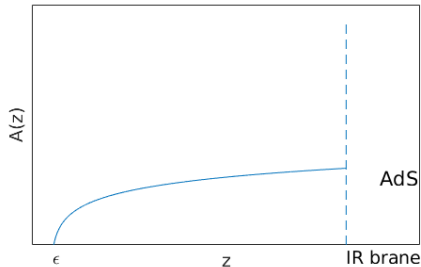
$$A(z)_{AdS} = \log(z/R) \quad , \quad R : \text{radius of AdS}$$

## AdS/Broken-CFT

$$ds^2 = e^{-2A(z)} (\eta^{\mu\nu} dx_\mu dx_\nu - dz^2)$$

AdS  $\iff$  CFT      energy scale  $\iff$   $1/z$

5D



4D dual

- UV cut-off =  $1/\epsilon$
- hard breaking of CFT
  - K.K. modes
- e.g. RS I model

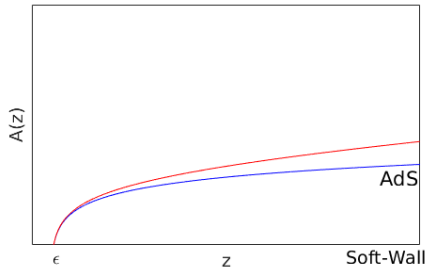


## AdS/Broken-CFT

$$ds^2 = e^{-2A(z)} (\eta^{\mu\nu} dx_\mu dx_\nu - dz^2)$$

AdS  $\iff$  CFT      energy scale  $\iff$   $1/z$

5D



4D dual

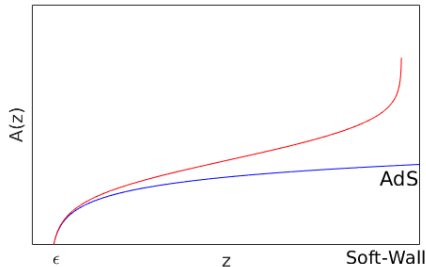
- UV cut-off =  $1/\epsilon$
- soft breaking of CFT
  - mass gap + no K.K. modes
- e.g. models I & II

## AdS/Broken-CFT

$$ds^2 = e^{-2A(z)} (\eta^{\mu\nu} dx_\mu dx_\nu - dz^2)$$

AdS  $\iff$  CFT      energy scale  $\iff$   $1/z$

5D



4D dual

- UV cut-off =  $1/\epsilon$
- soft + hard breaking of CFT  
- mass gap + K.K. modes
- e.g. model III

## Interactions from the bulk

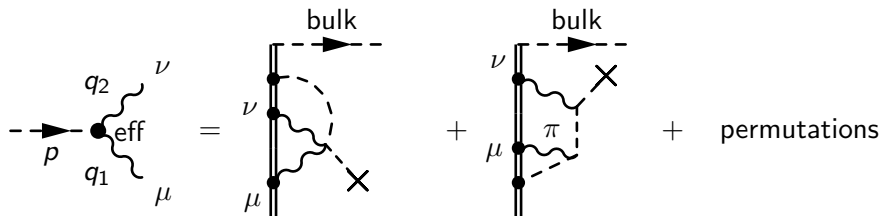
- We need to check for further contributions in AdS picture
- We remove the unwanted gauges by b.c.:  
 $SU(2)_L \times SU(2)_R \rightarrow SU(2)_L \times U(1)_Y$
- Boundary potential for the Higgs ignites SB in the bulk

$$\mathcal{L}_H = \frac{1}{2} \partial_M h \partial^M h - \frac{1}{2} M(z)^2 h^2 + \frac{1}{2} (V(z) + h)^2 \left( L_M^a - R_M^a \right)^2 + \dots$$

where

$$L_{UV\mu}^3 - R_{UV\mu}^3 = \frac{g}{c_w} Z_\mu$$

## Interactions from the bulk



- These will give what we already have found by imposing gauge invariance
- We can add gauge invariant coupling that further contribute to  $hZZ$ :

$$\mathcal{L}_{int} \propto H^2 F^2$$

- Our philosophy is to keep few free parameters and so will ignore these
- No further contribution at tree level

# Unitarity of $t\bar{t}WW$ scattering

hep-ph/1203.0312

- Non conformal case

$$\mathcal{L}_{Y,NC} = -\frac{m_t}{\mathcal{V}} ht\bar{t}$$

$$\mathcal{M}(t\bar{t} \rightarrow WW) = \sqrt{2}G_F m_t m_h^{4-2\Delta} (-1)^{\Delta-2} s^{\Delta-3/2}$$

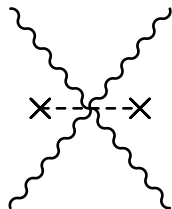
- The amplitude diverges for  $\Delta > 3/2$
- Propagating the top into the bulk we find

$$\mathcal{L}_{Y,C} = -\frac{m_t}{\mathcal{V}} F(p^2) ht\bar{t}$$

$$\lim_{p^2 \rightarrow \infty} F(p^2) \propto p^{1-\Delta}$$

# unitarity of WW scattering

- vertices with arbitrary number of gauge bosons



- The crosses indicate insertion of Higgs VEV

## SB in the Bulk

$$S = \int d^4x dz \sqrt{g} \left[ |D_M H|^2 - \frac{1}{4g_L^2} L_{MN}^a{}^2 - \frac{1}{4g_R^2} R_{MN}^a{}^2 - M(z)^2 + \mathcal{L}_{int}(H) \right] + \int d^4x \mathcal{L}_{boundary}$$

$$\mathcal{L}_{boundary} \supset -m_{UV}^2 H_0^2 + \lambda/4 H_0^4$$

$$\mathcal{H} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ h + v \end{pmatrix} + \Pi$$

# Peskin-Takeuchi S and T

$$T = 0$$

$$\alpha S = 4s_w^2 g^2 \partial_{p^2} \Pi_{WB}(p^2 = 0)$$

$$\Pi_{WB} = \frac{1}{g_L^2 + g_R^2} \left( \frac{v'(p^2, z = z_{UV})}{v(p^2, z = z_{UV})} - \frac{a'(p^2, z = z_{UV})}{a(p^2, z = z_{UV})} \right)$$

- $a$  and  $v$  are the solution to the axial and vector 5D fields

$$SU(2)_L \times SU(2)_R \rightarrow SU(2)_V$$



# "The Schrödinger potential"

- After a field re-difination

$$-\bar{h}'' + V\bar{h} = p^2\bar{h}$$

$$V = \frac{9}{4}A'^2 - \frac{3}{2}A'' + e^{-2A}M^2$$

- as  $z \rightarrow \infty$ :

$V \rightarrow \infty$       discretuum

$V \rightarrow 0$       continuum w/o mass gap

$V \rightarrow \mu$       continuum w/ mass gap