Eikonal fit and comprehensive fit to high energy data for σ, ρ, and B: An update

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Outline

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Introduction

- In 2015, we did an eikonal fit to data on the total and elastic scattering cross sections σ_{tot} and σ_{elas}, the ratios ρ of the real to the imaginary parts of the forward elastic scattering amplitudes, and the logarithmic slopes B, for proton-proton and antiproton-proton scattering at the center-of-mass energies are from 6 GeV to 57 GeV.
- We also performed the comprehensive fits to data on proton-proton and antiproton-proton scattering - σ_{tot} ,σ_{elas}, σ_{inelas}, ρ and B .

M. Block, L. Durand, P. Ha, and F. Halzen,
PR **D 92**, 014030 (2015);
PR **D 92**, 114021 (2015).

Introduction

- At present, new data are available. For instance, the total proton-proton cross section at W=13 TeV (by TOTEM) and 95 TeV (by Telescope Array), and the latest measurements of the inelastic cross section at W= 8 TeV (by TOTEM and ATLAS) and 13 TeV (by TOTEM, CMS, and ATLAS), … .
- There is a need to update our fits to high energy data on for σ_{tot} , σ_{elas}, σ_{inelas}, ρ and B, including the new data.
- The stability of the fits is good and the obtained results agree well with the predictions of earlier fits.
- This work again confirms the evidence for the proton asymptotically becoming a black disk of gluons.

Eikonal Model

- Consider p-p and pbar-p scattering at high energies. Neglecting the small effects of the nucleons spins, we describe the scattering amplitude and cross sections in an impact parameter .
- The spin-independent eikonal scattering amplitude and differential elastic scattering amplitude are

$$f(s,t) = i\int_0^\infty dbb\,(1-e^{i\chi(b,s)})J_0(b\sqrt{-t}), \quad (1)$$

$$\frac{d\sigma}{dt}(s,t) = \pi\left|f(s,t)\right|^2. \quad (2)$$

s =W^2= 4(p^2 + m^2) - square of total energy in the C.M. system,
p – the C.M. momentum of either incident particle,
b = j/p , j is the partial wave angular momentum,
t = -2p^2 (1 – cos θ) – the invariant 4 - momentum transfer.

Eikonal Model

$\chi(b,s) = \chi_R + i\,\chi_I$ - the eikonal function

$$\sigma_{elas}(s) = 2\pi \int_0^\infty dbb \left|1 - e^{i\chi}\right|^2 \quad , \qquad (3)$$

$$\sigma_{tot}(s) = 4\pi\, \mathrm{Im}\, f(s,0) = 4\pi \int_0^\infty dbb\,(1 - \cos\chi_R e^{-\chi_I}), \qquad (4)$$

$$\sigma_{inelas}(s) = \sigma_{tot} - \sigma_{elas} = 2\pi \int_0^\infty dbb\,(1 - e^{-2\chi_I}), \qquad (5)$$

$$\rho = \mathrm{Re}\, f(s,0) / \mathrm{Im}\, f(s,0)$$

$$= -\int_0^\infty dbb\, e^{-\chi_I} \sin\chi_R \Big/ \int_0^\infty dbb\,(1 - \cos\chi_R e^{-\chi_I}), \qquad (6)$$

$$\mathrm{B} = \frac{d}{dt}\left[\ln\frac{d\sigma}{dt}(s,t)\right]$$

$$\approx \frac{1}{2}\int_0^\infty dbb^3\,(1 - e^{-\chi_I}) \Big/ \int_0^\infty dbb\,(1 - e^{-\chi_I}). \qquad (7)$$

Eikonal Fit: An update

$$\chi_{p\bar{p}}(b,W) = (\chi_E(b,W) + \chi_O(b,W))/2\ , \quad (8)$$

$$\chi_{pp}(b,W) = (\chi_E(b,W) - \chi_O(b,W))/2\ . \quad (9)$$

where

$$\chi_E(b,W) = i\left[\sigma_{qq}(\tilde{W})A(b,\mu_{qq}) + \sigma_{qg}(\tilde{W})A(b,\mu_{qg}) + \sigma_{gg}(\tilde{W})A(b,\mu_{gg})\right], \quad (10)$$

$$\chi_O(b,W) = -C_5\Sigma_{gg}\left(\frac{m_0}{\tilde{W}}\right)^{2-2\alpha_1} A(b,\mu_{odd})\ . \quad (11) \qquad \tilde{W} = We^{-i\pi/4}$$

$A(b, \mu)$ – overlap functions,
σ_{ij} – describing interactions between components i and j.

- We fit data on total cross sections for W $\geq$ 5.3 GeV and the elastic cross sections, ρ and B for energies W $\geq$ 10 GeV.
- Fix σ_{tot} at W = 4 GeV to match the results obtained from low-energy data.

Eikonal Fit: An update

- 9 parameter fit
- 189 datum points
- Seive algorithm eliminates 14 outliers
- dof = 166; χ^2 /dof = 0.985
- $\mathcal{R}\ \chi^2$ /dof =1.08
- Fixed parameters

$m_0 = 0.6$ GeV, $W_0 = 4$ GeV,

$\mu_{gg} = 0.705$ GeV, $\mu_{qq} = 0.89$ GeV,

$\mu_{odd} = 0.6$ GeV, $\alpha_S = 0.5$,

$\Sigma_{gg} = 19.635\ \text{GeV}^{-2}$.

- The fitted parameters

$C_0 = 6.790 \pm 0.07$

$C_1 = 26.80 \pm 0.02$

$C_2 = -0.187 \pm 0.0004$

$C_3 = -2.480 \pm 0.004$

$C_4 = 13.75 \pm 0.013$

$C_5 = -26.13 \pm 0.02$

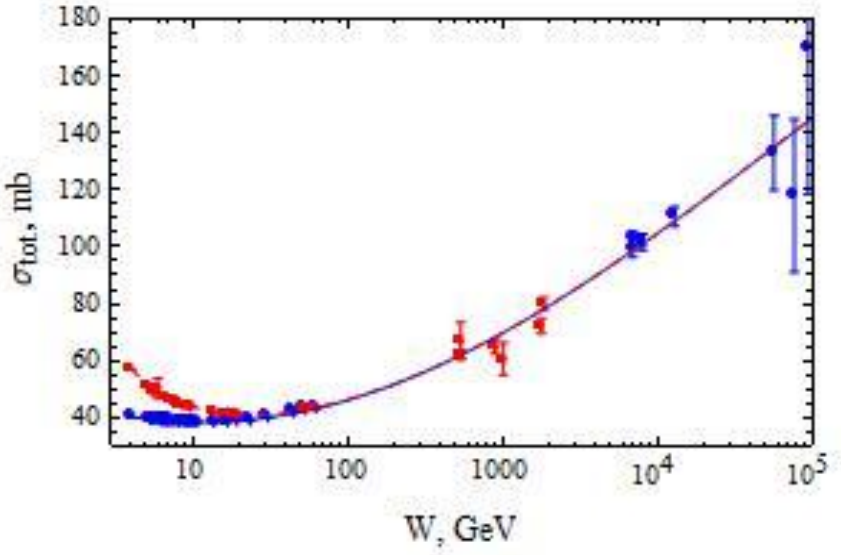
$\alpha_1 = 0.3188 \pm 0.0003$

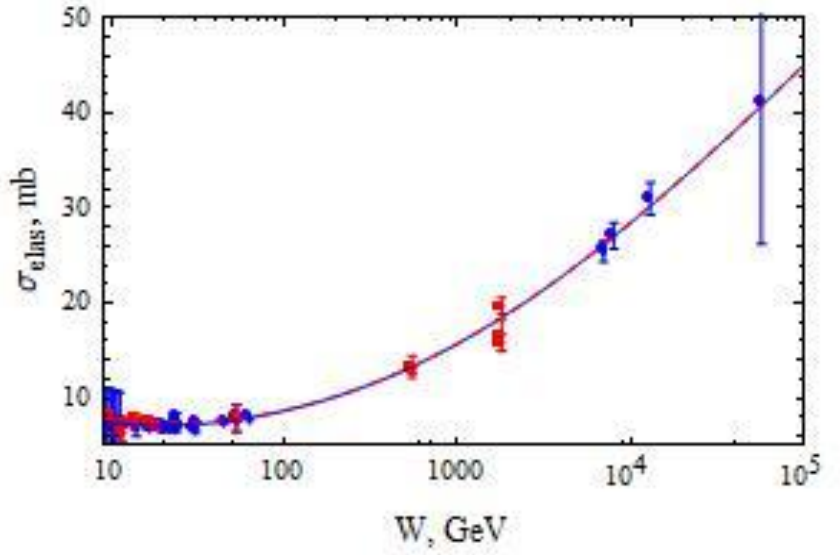
$\alpha_2 = 0.4866 \pm 0.0001$

$\beta_1 = 0.1474 \pm 0.0002$

Eikonal Fit: An update

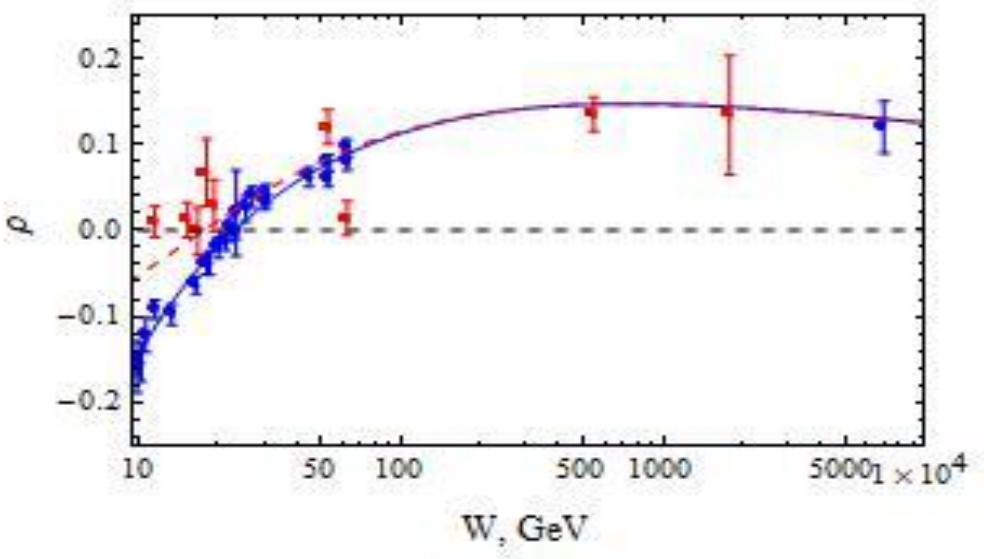
- Top: fits to σ_{tot} for p-p (blue dots and solid line) and pbar-p (red squares and dashed line) scattering.
- Bottom: fits to σ_{elas} for p-p (blue dots and solid line) and pbar-p(red squares and dashed line) scattering.

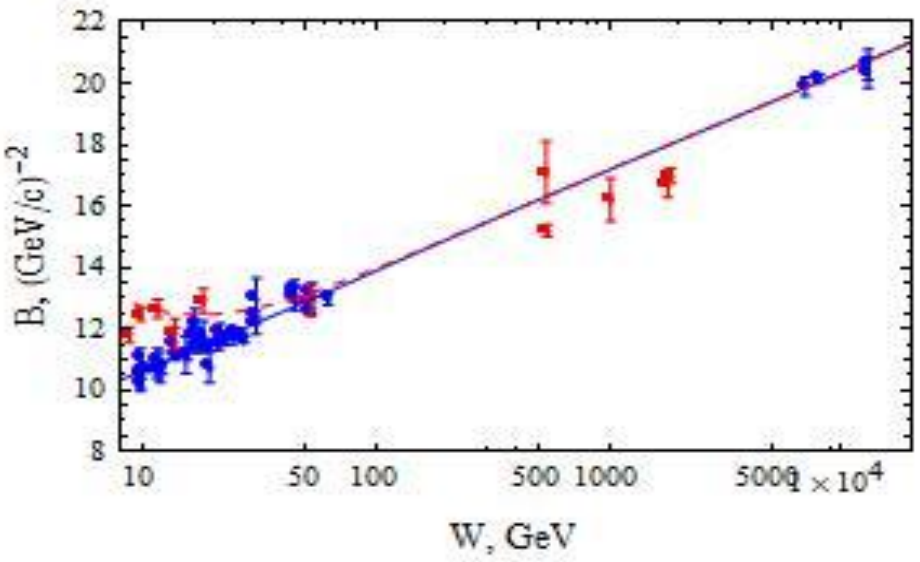




Eikonal Fit: An update

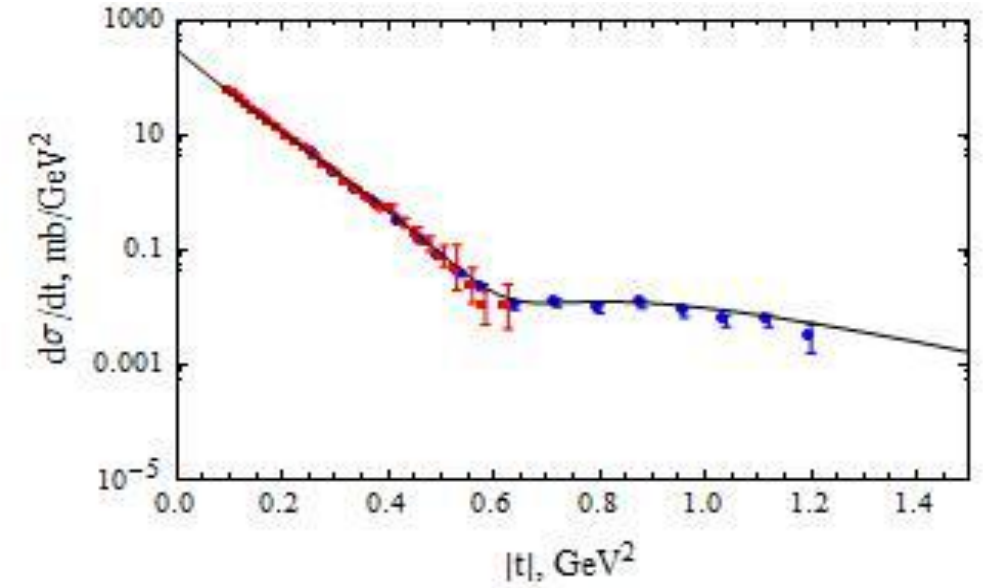
- Top: fits to the ratios ρ for p-p (blue dots and solid line) and pbar-p (red squares and dashed line) scattering. The horizontal dashed line is at ρ=0 .
- Bottom: fits to B for p-p (blue dots and solid line) and pbar-p(red squares and dashed line) scattering.

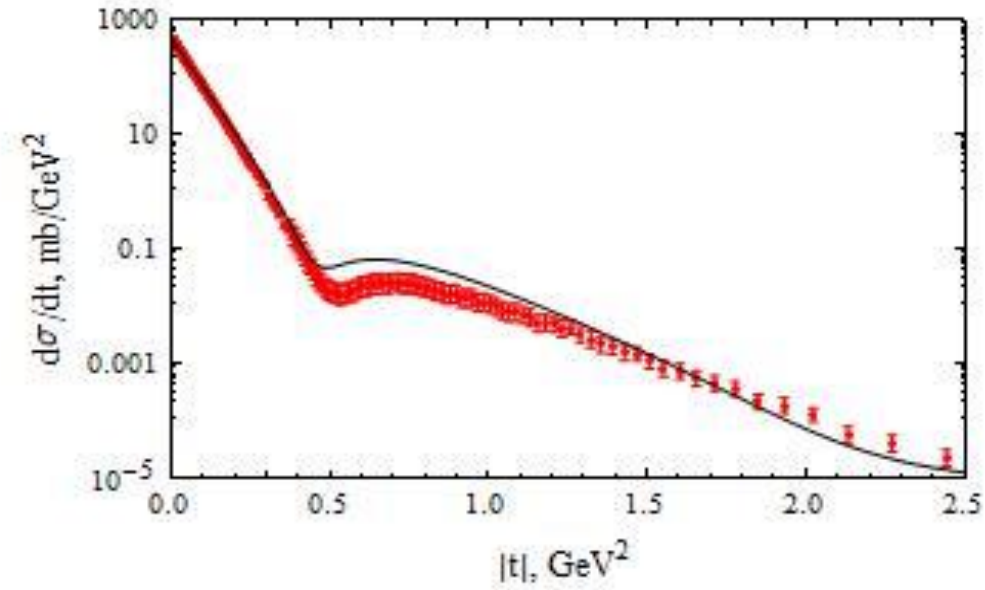




Eikonal Fit: An update

- Top: The differential cross section dσ/dt from the E710 experiment at W=1800 GeV.
- Bottom: dσ/dt from the TOTEM experiment W=7000 GeV





Eikonal Fit: An update

- The fit is excellent.
- Conclusions on the edge in the scattering amplitude and on the asymptotic black-disk limit are unchanged.

Comprehensive Fit: Parametrization

- We use Block-Cahn parametrization (see Rev. Mod. Phys. 57, 563 (1985)).
- The Block-Cahn analysis assumed a $\ln^2 s$ bound on the growth of the cross sections at high energy as implied by the Froissart bound.
- The analysis also use the Regge-like terms which are important at low energies.
- The phase of the scattering amplitude at high energies and the corresponding expression for ρ followed from the constraints imposed by analyticity and the crossing symmetry.
- We extend the parametrizations to the elastic and inelastic cross sections and the B parameter.

Comprehensive Fit: Parametrization

- The Block-Cahn parametrization

$$\sigma_{tot}^{\pm}(\nu) = c_0 + c_1 \ln(\tilde{\nu}) + c_2 \ln^2(\tilde{\nu}) + \beta(\tilde{\nu})^{\mu-1} \pm \delta(\tilde{\nu})^{\alpha-1} \quad (8)$$

$$\sigma_{elas}^{\pm}(\nu) = b_0 + b_1 \ln(\tilde{\nu}) + b_2 \ln^2(\tilde{\nu}) + \beta_e(\tilde{\nu})^{\mu-1} \pm \delta_e(\tilde{\nu})^{\alpha-1} \quad (9)$$

$$B^{\pm}(\nu) = a_0 + a_1 \ln(\tilde{\nu}) + a_2 \ln^2(\tilde{\nu}) + \beta_B(\tilde{\nu})^{\mu-1} \pm \delta_B(\tilde{\nu})^{\alpha-1} \quad (10)$$

$$\rho^{\pm}(\nu) = \frac{1}{\sigma_{tot}^{\pm}(\nu)}\left[\frac{\pi}{2}c_1 + \pi c_2 \ln(\tilde{\nu}) - \beta\cot\left(\frac{\pi\mu}{2}\right)(\tilde{\nu})^{\mu-1} + \frac{4\pi}{\nu}\mathrm{f}_+(0) \pm \delta\tan\left(\frac{\pi\alpha}{2}\right)(\tilde{\nu})^{\alpha-1}\right] \quad (11)$$

$\tilde{\nu} = (\nu/m) = \left(s - 2m^2\right)/2m^2$

18 parameters

\+ for pp ; - for $\bar{p}p$

Comprehensive Fit: Constraints

- Two low-energy constraints.

  1) The “analyticity constraint” of Block and Halzen requires that the fits reproduce the values of the total cross sections at a transition point far enough above the resonance region.

  2) The finite energy sum rule (FESR) of Igi and Ishida relates the low- and high energy regions.

- Two other rather subtle constraints hold for the coefficients of the Regge-like terms.
- Using the constraints, number of the fitting parameters will be reduced.

Comprehensive Fit: An update

- The data we use in our analysis consists of results on σ_{tot} for W $\geq$ 6.0 GeV, σ_{inel} for W $\geq$ 540 GeV, σ_{elas} for W $\geq$ 30 GeV , and ρ and B for W $\geq$ 10 GeV.
- We did a global fit using the seive algorithm to eliminate 9 outliers among 163 datum points.
- 14 fitting parameters; dof = 140
- χ^2 /dof = 0.941; $\mathcal{R}$ χ^2 /dof =1.04: excellent fit.

Comprehensive Fit: An update

- The fitting parameters a_2 , b_2, and c_2 are

$$a_2 = 0.0316 \pm 0.0077$$
$$b_2 = 0.1364 \pm 0.0123$$
$$c_2 = 0.2528 \pm 0.0279$$

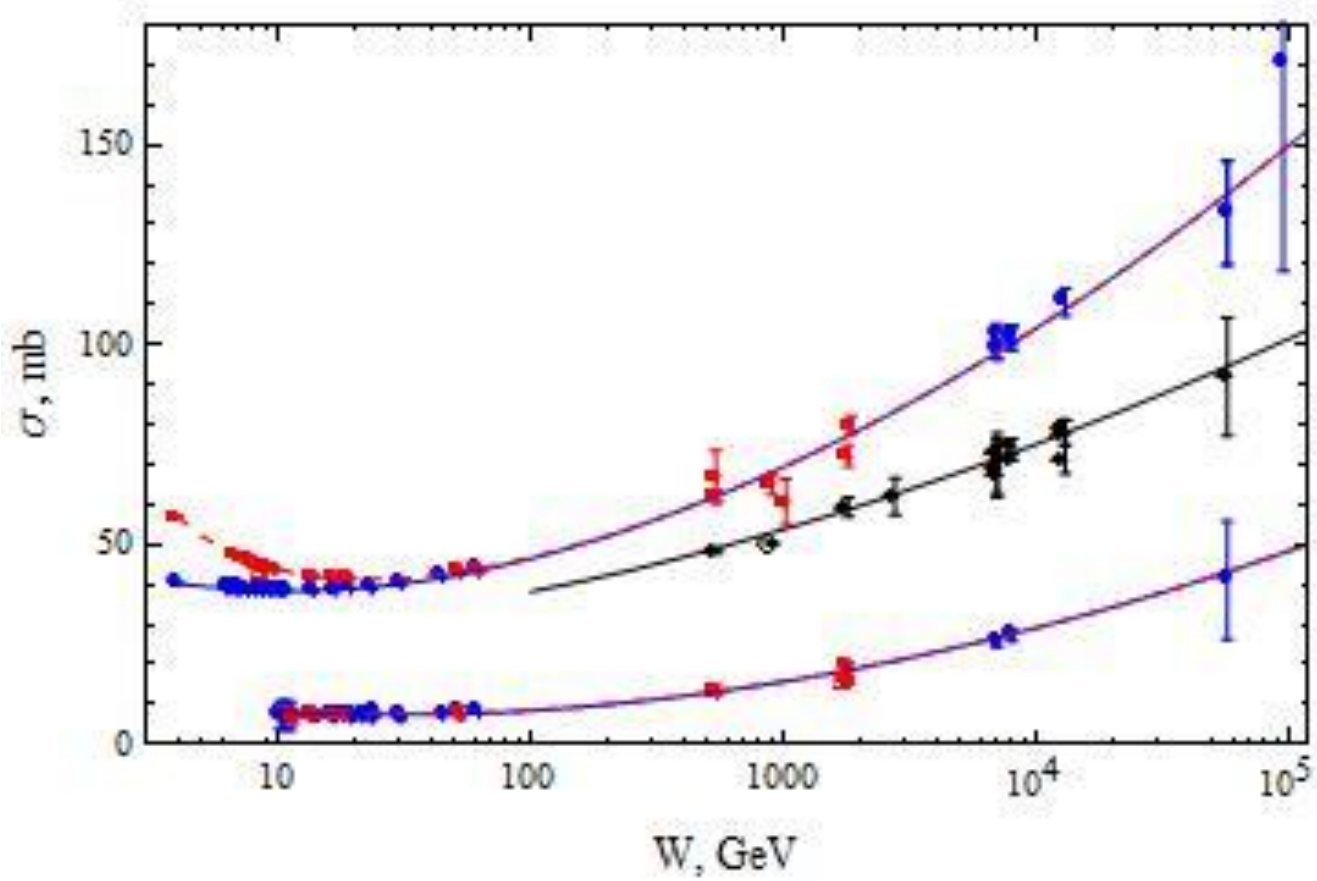
- Note that

$$\frac{\sigma_{elas}}{\sigma_{tot}} \to \frac{b_2}{c_2} = \frac{0.1364}{0.2528} = 0.540 \pm 0.077, \text{ as } s \to \infty \quad (12)$$

- The deviation of this from the expected value of 1/2 for black-disk scattering at infinity is well within the uncertainty of the fit.
- Also, the ratio of the fitted value of B to $\sigma_{tot}/8\pi$ agrees well with its expected value of 1 at high energy

$$(0.3894)8\pi\frac{a_2}{c_2} = 1.223 \pm 0.326, \text{ as } s \to \infty \quad (13)$$

Comprehensive Fit: An update



Fits, top to bottom, to the total, inelastic, and elastic scattering cross sections

Conclusions

- We have updated our eikonal and comprehensive fits. The fits are stable and their results are consistent with those from our earlier fits.
- This work again gives strong evidence that p-p and pbar-p scattering can be described asymptotically as black-disk scattering at very high energies.

Acknowledgements

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Some new data

At W=95 TeV, Telescope Array: $\sigma_{tot}^{p\bar{p}} = 170^{+48}_{-44}[\text{stat}]^{+19}_{-17}[\text{sys}]$ mb.

At W= 13 TeV, TOTEM: σ_{tot} = (110.6 ± 3.4) mb, σ_{el} = (31.0 ± 1.7) mb,

σ_{inel} = (79.5 ± 1.8) mb, B = (20.36 ± 0.5) GeV^{-2} .

At W= 8 TeV, TOTEM: σ_{tot} = (102.4 ± 2.1) mb.

At W= 7 TeV, TOTEM: σ_{tot} = (102.9 ± 2.3) mb.

- See also Table I in D. Fagundes *et al*., Phys. Rev. **D 96**, 054010 (2017).