Annihilation rates of wino dark matter from an effective field theory approach

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Talk outline

Wino dark matter

-Overview of conventional tools of calculation

-Bound state effects on annihilation

-Zero-range effective field theory, a three part story

- Part I Framework [arXiv: 1706.02253] *Pub. JHEP*
- Part II Coulomb Resumation [arXiv: 1708.07155] *Pub. JHEP*
- Part III Annihilation Effects [arXiv: 1712.07142] *Pub. JHEP pending* **This talk**

-Analytic results for inclusive and partial annihilation rates and Sommerfeld enhancements for wino dark matter

Wino dark matter

- Motivation: "WIMP" miracle: TeV-scale particle with weak-scale cross section naturally produces the observed dark matter density

- Fundamental theory can be either:
  - Minimal extension of the Standard Model to include one additional $SU(2)$ triplet
  - MSSM where the Lightest Supersymmetric Particle is a wino-like neutralino, all other SUSY particles at a higher scale
  - Either case: refer to the dark matter candidate as a 'wino'

$$\tilde{w} = \left(\tilde{w}^{+} \quad \underbrace{\tilde{w}^{0}}_{\text{Dark matter candidate}} \quad \tilde{w}^{-} \right)$$

- Wino masses:
  - Neutral wino mass $M\sim$ few TeV, charged winos $M + \delta$
  - Radiative corrections give $\delta = 170$ MeV, insensitive to M

(Pierce et al. NPB 1997)

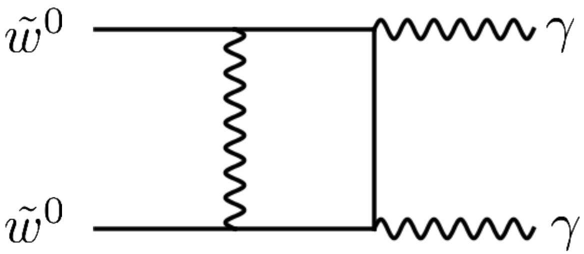
Wino interactions and nonperturbative effects

A pair of neutral winos can annihilate into a pair of electroweak gauge bosons

$$\left.\begin{aligned} \tilde{w}^0\tilde{w}^0 &\to Z^0 Z^0 \\ &\to W^+W^- \end{aligned}\right\} \text{Continuous } \gamma\text{-ray and positron signals}$$

$$\left.\begin{aligned} \tilde{w}^0\tilde{w}^0 &\to \gamma\gamma \\ &\to \gamma Z^0 \end{aligned}\right\} \text{Monochromatic } \gamma\text{-ray signals}$$

Leading-order (LO) annihilation cross-section for a pair of photons:



$$(v\sigma_{\text{ann}})_{\text{LO}} \sim \alpha^2\alpha_2^2/m_W^2$$

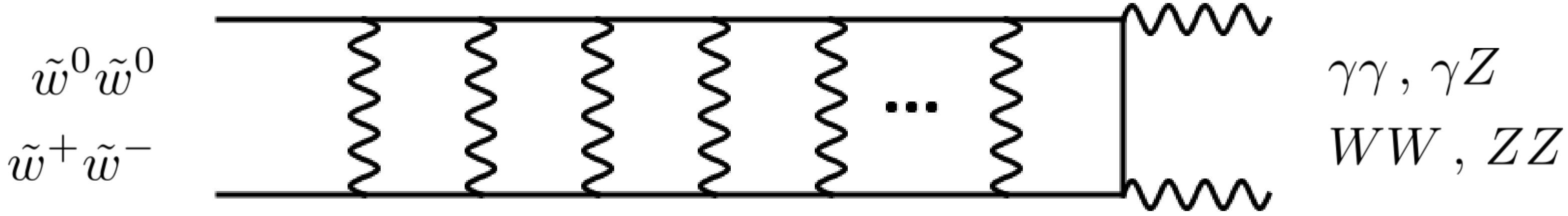
(Hisano et al. PRD 2005)

$(v\sigma_{\text{ann}})_{\text{LO}}$ **exceeds unitarity bound** $4\pi/vM^2$ for sufficiently large M!

Higher order diagrams must be included to calculate the annihilation rate

Wino interactions and nonperturbative effects

Higher order diagrams for direct pair annihilation involve exchanges of EW gauge bosons:



Ladder diagrams must be summed to all orders to compute annihilation rate

- Each 'rung' of the ladder gives a factor of $\alpha_2 M/m_W$
- For large enough M, $\alpha_2 M/m_W \sim 1$

(Hisano et al. PRD 2005)

- The annihilation cross sections receive enhancements: the "Sommerfeld enhancements"

Difficult to calculate in a fundamental quantum field theory (requires summing diagrams to all orders)

- Winos are non-relativistic, $v_{\rm rel}{\sim}10^{-3}$
- Employ a coupled-channel Schrödinger equation

Solving the Schrödinger equation

Numerically solve a coupled-channel Schrödinger equation:

$$\Bigg[\underbrace{\frac{-1}{M}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\left(\frac{d}{dr}\right)^2+\begin{pmatrix}0 & 0\\ 0 & 2\delta\end{pmatrix}+\boldsymbol{V}(r)}_{\text{Hermitian}}\underbrace{-\,i\frac{\delta(r)}{2\pi r^2}\boldsymbol{\Gamma}}_{\text{Anti-Hermitian}}\Bigg]r\begin{pmatrix}R_0(r)\\ R_1(r)\end{pmatrix}=E\,r\begin{pmatrix}R_0(r)\\ R_1(r)\end{pmatrix}$$

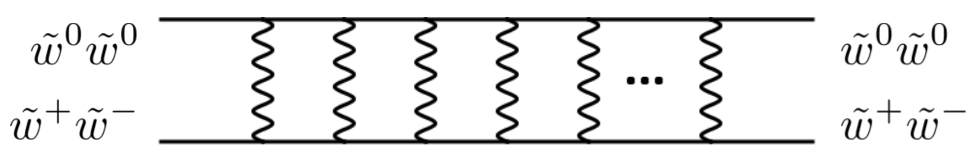
$$\boldsymbol{\Gamma}=\frac{\pi\alpha_2^2}{2M^2}\begin{pmatrix}2 & \sqrt{2}\\ \sqrt{2} & 3\end{pmatrix}$$

(Hisano et al. PRD 2005)

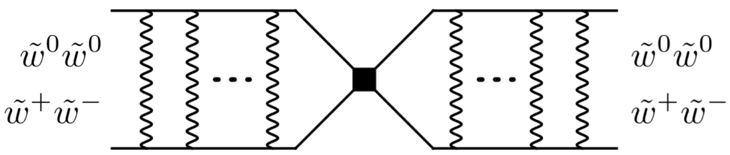
Real potential $\boldsymbol{V}(r)$ describes scattering between wino pairs through electroweak interactions:

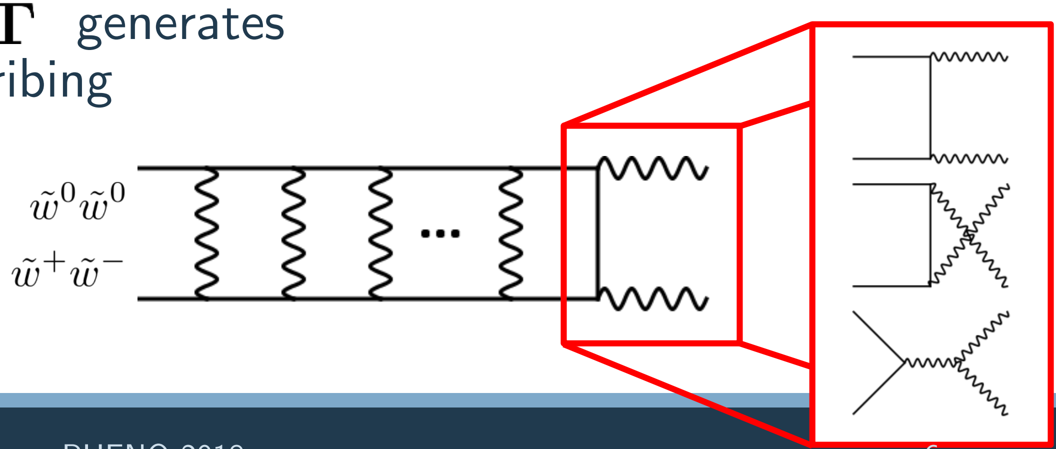
$$\boldsymbol{V}(r)=\begin{pmatrix}0 & -\sqrt{2}\,\alpha_2\,\frac{e^{-m_W r}}{r}\\ -\sqrt{2}\,\alpha_2\,\frac{e^{-m_W r}}{r} & -\frac{\alpha}{r}-\alpha_2\,c_W^2\,\frac{e^{-m_Z r}}{r}\end{pmatrix}$$

W , γ , Z exchange



Anti-Hermitian term with matrix $\boldsymbol{\Gamma}$ generates a ‘hard annihilation vertex,’ describing wino-pair annihilation:

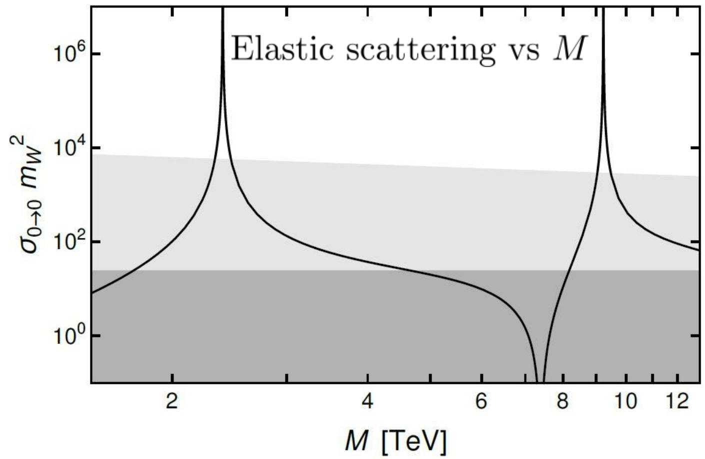




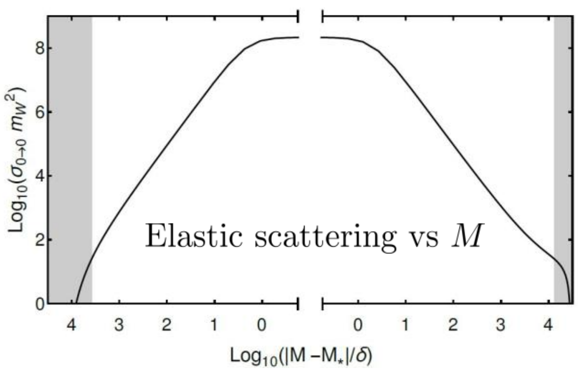
Solving the Schrödinger equation

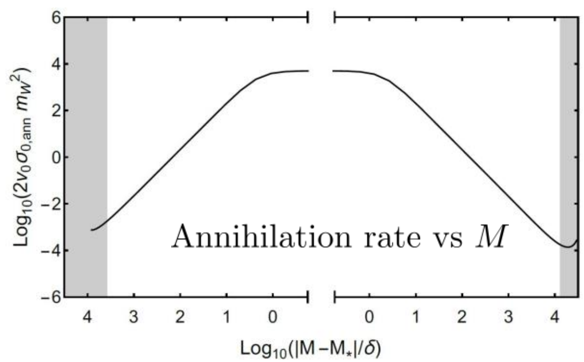
Short-range interactions produces a sequence of **critical masses** where a zero-energy resonance exists at the scattering threshold

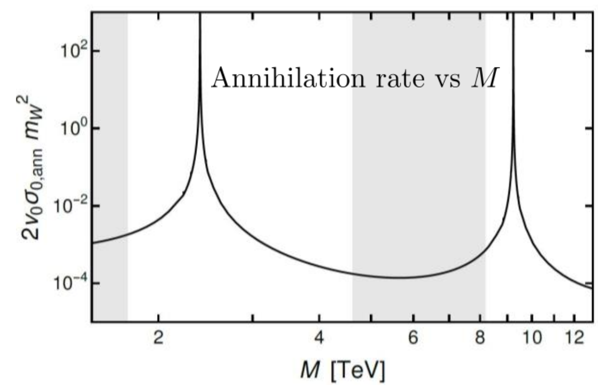
- Elastic cross section and annihilation rate is resonantly enhanced: **resonant Sommerfeld enhancement!**
- First critical mass at $M_* = 2.4$ TeV
- Resonant enhancement is regulated at low energy by the imaginary part of the potential



Log-Log plots approaching peaks above and below M_*







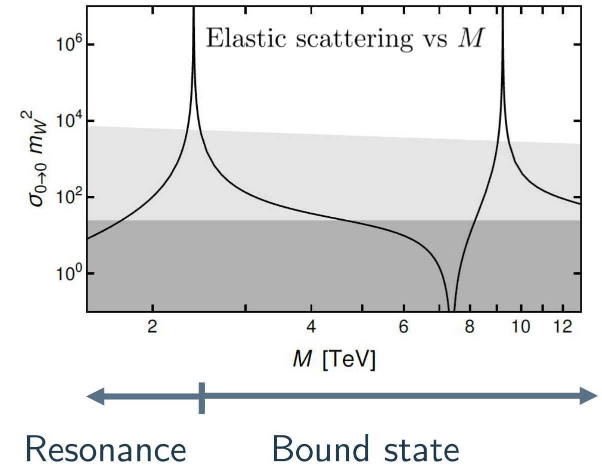
$$\sigma_{0\to0}(E=0)m_W^2\Big|_{M=M_*} = 2.2\times10^8$$

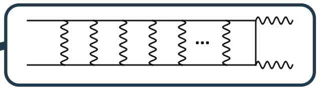
$$2v_0\,\sigma_{0,\mathrm{ann}}(E=0)m_W^2\Big|_{M=M_*} = 5.0\times10^3$$

Motivating an analytic approach: Bound state annihilation

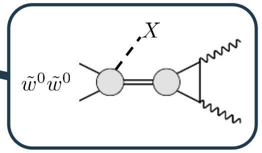
Dramatic enhancement of the elastic cross section and annihilation rates occurs at a critical mass M_* where there is a **zero-energy resonance** at the scattering threshold.

- For $M > M_*$, this resonance is a **bound state**
- Once a bound state forms, it can annihilate into electroweak gauge bosons
- Annihilation of dark matter through bound states **adds** to the overall annihilation rate, **increasing** theoretical predictions, thus **tightening** constraints!

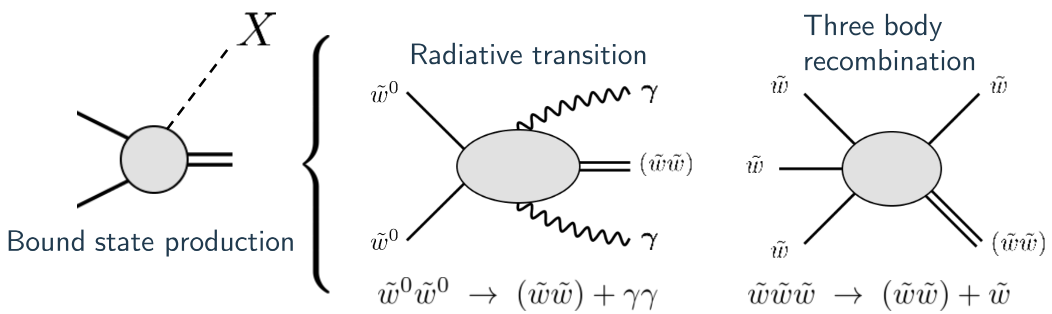




$$\sigma v[\tilde{w}^0\tilde{w}^0 \to \gamma\gamma]_{\text{total}} = \sigma v[\tilde{w}^0\tilde{w}^0 \to \gamma\gamma]_{\text{direct}} + \sigma v[\tilde{w}^0\tilde{w}^0 \to (\tilde{w}\tilde{w}) + X \to \gamma\gamma]$$



Calculations of the bound state rates can be simplified by using a new tool: **Zero-Range Effective Field Theory**



Brief overview of Zero-Range Effective Field Theory (ZREFT)

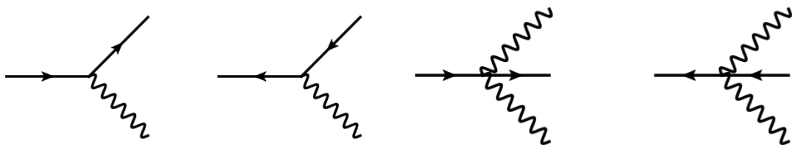
Lagrangian:

$$\mathcal{L} = \tilde{w}^{0\dagger}\left(i\partial_0 + \frac{\boldsymbol{\nabla}^2}{2M}\right)\tilde{w}^0 + \sum\nolimits_{\pm} \tilde{w}^{\pm\dagger}\left(iD_0 + \frac{\boldsymbol{D}^2}{2M} - \delta\right)\tilde{w}^{\pm} + \mathcal{L}_{\mathrm{EM}} + \mathcal{L}_{\text{zero-range}}$$

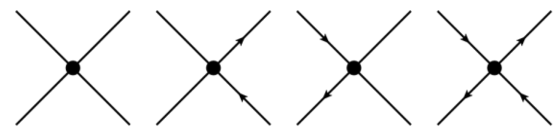
- Photon interactions arise from covariant derivatives for charged winos:

$$D_0\tilde{w}^{\pm} = (\partial_0 \pm ieA_0)\tilde{w}^{\pm} \quad \boldsymbol{D}\tilde{w}^{\pm} = (\boldsymbol{\nabla} \mp ie\boldsymbol{A})\tilde{w}^{\pm}$$

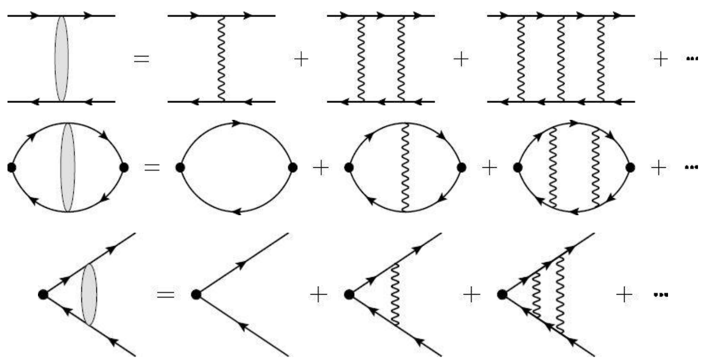
- Single and double photon vertices:

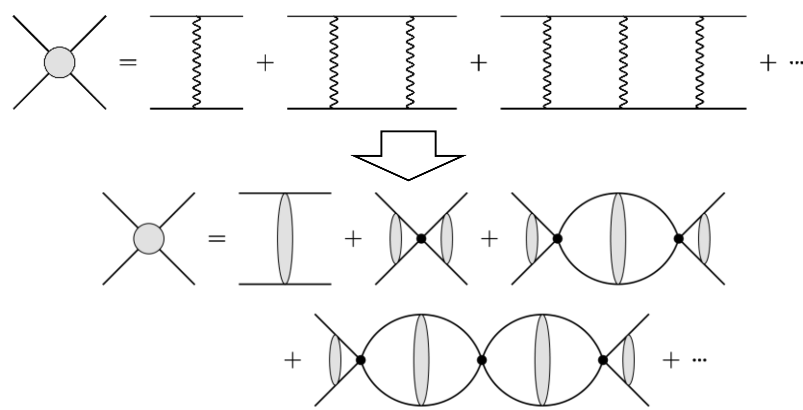


- Zero-range contact interactions for pairs of winos:



- Non-perturbative electroweak interactions reproduced by summing bubble diagrams to all orders
- Must resum over any number of photons exchanged between charged winos





Brief overview of Zero-Range Effective Field Theory (ZREFT)

Results and ideas from past work (Part I 1706.02253 and Part II 1708.07155):

- Power counting of the EFT is governed by its renormalization group fixed points
- Appropriate fixed point to expand around is where resonant scattering occurs in a linear combination of the neutral-wino channel $\tilde{w}^0\tilde{w}^0$ and charged-wino channel $\tilde{w}^+\tilde{w}^-$ with a mixing angle ϕ **(Lensky and Birse, EPJ 2011)**
- At leading order in the power counting, the mixing angle is the only free parameter and a scattering parameter γ_0 determined numerically from the Schrödinger equation
- Mixing angle is determined by matching low-energy behavior of the neutral-wino scattering

The result is a parametrization of the amplitudes for transitions between the neutral- and charged-wino channels:

$$\underbrace{\mathcal{T}_{00}(E)}_{\tilde{w}^0\tilde{w}^0\to\tilde{w}^0\tilde{w}^0} = \frac{8\pi/M}{L_0(E)} \quad , \quad \underbrace{\mathcal{T}_{01}(E) = \mathcal{T}_{10}(E)}_{\tilde{w}^0\tilde{w}^0\to\tilde{w}^+\tilde{w}^-\,,\,\tilde{w}^+\tilde{w}^-\to\tilde{w}^0\tilde{w}^0} = \frac{(4\sqrt{2}\,\pi/M)t_\phi W_1(E)}{L_0(E)} \quad , \quad \underbrace{\mathcal{T}_{11}(E)}_{\tilde{w}^+\tilde{w}^-\to\tilde{w}^-\tilde{w}^+} = \mathcal{A}_C(E) + \frac{(4\pi/M)t_\phi^2 W_1^2(E)}{L_0(E)}$$

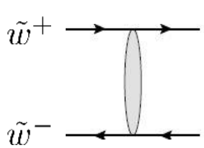
Labels:

$0 \, : \, \tilde{w}^0\tilde{w}^0$

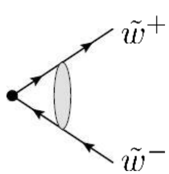
$1 \, : \, \tilde{w}^+\tilde{w}^-$

$$L_0(E) = -\gamma_0 + t_\phi^2\left[K_1(E) - K_1(0)\right] + \sqrt{-ME - i\varepsilon}$$

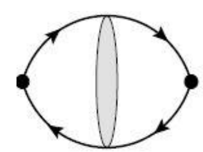
$\mathcal{A}_C(E)$ comes from



$W_1(E)$ comes from



$K_1(E)$ comes from

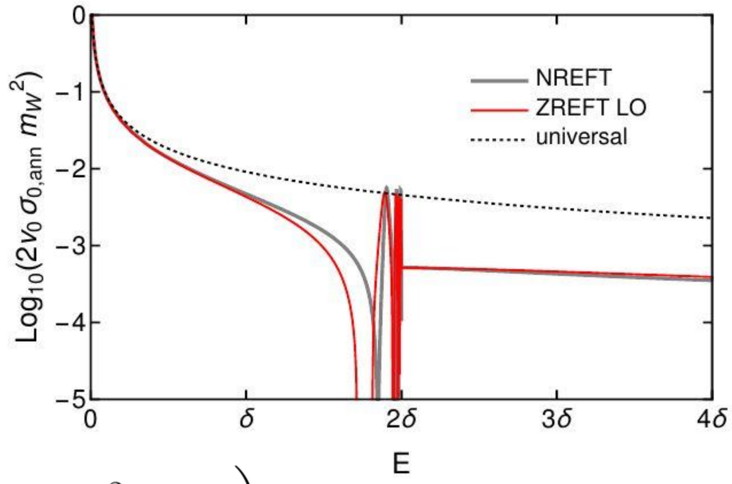


Annihilation rates in ZREFT

The Optical Theorem relates the total cross section to the forward scattering amplitude:

$$\sigma_{i,\mathrm{tot}}(E) = \frac{1}{v_i(E)} \mathrm{Im}\mathcal{T}_{ii}(E)$$

Labels:
$0 : \tilde{w}^0\tilde{w}^0$
$1 : \tilde{w}^+\tilde{w}^-$



Subtracting the contribution from wino-pair final states gives the **inclusive** annihilation cross sections:

$$\sigma_{i,\mathrm{ann}}(E) = \frac{1}{v_i(E)} \left(\mathrm{Im}\mathcal{T}_{ii}(E) - \frac{M^2}{8\pi} \left|\mathcal{T}_{i0}(E)\right|^2 v_0(E) - \frac{M^2}{4\pi} \left|\mathcal{T}_{i1}(E)\right|^2 v_1(E) \right)$$

Using the leading order expressions for the transition amplitudes $\mathcal{T}$, we get the results for the inclusive annihilation rate for neutral winos (which are the dark matter candidates):

$$2v_0\sigma_{0,\mathrm{ann}}(E) = \frac{16\pi/M}{|L_0(E)|^2} \mathrm{Im}\Big[\gamma_0 - \left(t_\phi^2 - |t_\phi^2|\right)\left[K_1(E) - K_1(0)\right]\Big]$$

From the annihilation rates, an expression for the inclusive Sommerfeld enhancement can be derived:

$$S(v) = \frac{8M}{\alpha_2^2\,|L_0(E)|^2} \Big[\mathrm{Im}[\gamma_0] - \mathrm{Im}[t_\phi^2]\left(\mathrm{Re}[K_1(E)] - K_1(0)\right)\Big]$$

Annihilation rates in ZREFT

The **partial** annihilation rates can be determined by first resolving the matrix into its contributions from particular annihilation products:

$$\mathbf{\Gamma} = \mathbf{\Gamma}^{(\gamma\gamma)} + \mathbf{\Gamma}^{(\gamma Z)} + \mathbf{\Gamma}^{(ZZ)} + \mathbf{\Gamma}^{(WW)}$$

(Hisano et al. PRD 2005)

For final states including monochromatic photons, we have

$$\mathbf{\Gamma}^{(\gamma X)} \equiv 2\mathbf{\Gamma}^{(\gamma\gamma)} + \mathbf{\Gamma}^{(\gamma Z)} = \frac{2\pi\alpha_2^2 s_w^2}{M^2} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

Solving the Schrödinger equation with this matrix and performing the matching for the mixing angle results in the Sommerfeld enhancement factor for neutral-wino annihilation producing a monochromatic photon signal:

$$\boxed{S(v) = \frac{2m_W^2}{\alpha_2^4 s_w^2 M\,|L_0(E)|^2}\left[\mathrm{Im}[\gamma_0]^{(\gamma X)} - \mathrm{Im}[t_\phi^2]^{(\gamma X)}\left(\mathrm{Re}[K_1(E)] - K_1(0)\right)\right]}$$

Conclusions

The addition of the effects of wino-pair annihilation completes the three part story of Zero Range Effective Field Theory for Resonant Wino Dark Matter

- Annihilation effects are introduced by analytically continuing real scattering parameters to complex values

ZREFT includes the important nonperturbative behavior the electroweak interaction has on wino reactiosn and produces **analytic results** for cross sections, annihilation rates, and Sommerfeld enhancements

Parameters of the effective theory are obtained by matching the low-energy scattering behavior

- Low-energy behavior is important for nonrelativistic dark matter in halos

ZREFT is generalizable and adaptable to other models of dark matter where there are resonant S-wave interactions

- Future work can be done to develop the ZREFT for higgsino dark matter among others

ZREFT can be used to more easily study effects of **bound state production** on indirect detection signals