A new theorem for lepton number conservation in seesaw models

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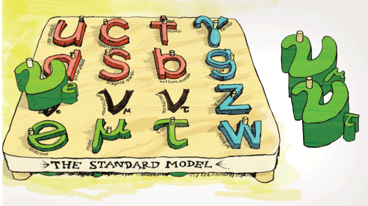
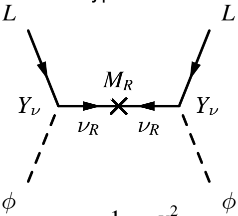
Massive neutrinos and New Physics

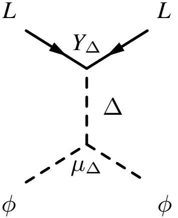
- Observation of ν oscillations
  $\Rightarrow$ at least 2 ν are massive
- BSM necessary for ν mass
  - Radiative models
  - Extra-dimensions
  - R-parity violation in supersymmetry
  - Seesaw mechanisms



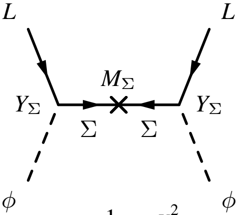
- 3 minimal tree-level seesaw models $\Rightarrow$ 3 types of heavy fields
  - type I: right-handed neutrinos, SM gauge singlets
  - type II: scalar triplets
  - type III: fermionic triplets



$$m_\nu = -\frac{1}{2} Y_\nu \frac{v^2}{M_R} Y_\nu^T$$

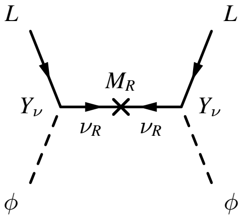


$$m_\nu = -2 Y_\Delta v^2 \frac{\mu_\Delta}{M_\Delta^2}$$



$$m_\nu = -\frac{1}{2} Y_\Sigma \frac{v^2}{M_\Sigma} Y_\Sigma^T$$

Towards testable Type I variants



- Taking $M_R \gg m_D$ gives the "vanilla" type 1 seesaw

$$\mathrm{m}_\nu = -m_D M_R^{-1} m_D^T$$

$$\mathrm{m}_\nu \sim 0.1\,\mathrm{eV} \Rightarrow \left| \begin{array}{l} Y_\nu \sim 1 \quad \text{and} \quad M_R \sim 10^{14}\,\mathrm{GeV} \\ Y_\nu \sim 10^{-6}\,\text{and} \quad M_R \sim 10^{2}\ \mathrm{GeV} \end{array} \right.$$

- m_ν suppressed by small active-sterile mixing m_D/M_R

- Cancellation in matrix product to get large m_D/M_R
  - Lepton number, e.g. low-scale type I [Ilakovac and Pilaftsis, 1995] and others
    inverse seesaw [Mohapatra and Valle, 1986, Bernabéu et al., 1987]
    linear seesaw [Akhmedov et al., 1996, Barr, 2004, Malinsky et al., 2005]
  - Flavour symmetry, e.g. $A_4 \times \mathbb{Z}_2$ [Chao et al., 2010]
    A_4 or $\Sigma(81)$ [Chattopadhyay and Patel, 2017]
    $\mathbb{Z}(3)$ [Gu et al., 2009]
  - Gauge symmetry, e.g. $\mathrm{U}(1)_{B-L}$ [Pati and Salam, 1974] and others

$m_\nu = 0$ equivalent to conserved L for models with 3 ν_R
or less of equal mass [Kersten and Smirnov, 2007]

Extending the Kersten-Smirnov theorem

- Can the result of Kersten and Smirnov be generalized ?
- Are lepton number violating processes suppressed in all low-scale seesaw models ?

Theorem

If: - no cancellation between different orders of the seesaw expansion[a]

- no cancellations between different radiative orders[b]

Then $m_\nu = 0$ equivalent to having the neutrino mass matrix, in the basis $(\nu_L^C, \{\nu_{R,1}^{(1)} \ldots \nu_{R,n}^{(1)}\}, \{\nu_{R,1}^{(2)} \ldots \nu_{R,n}^{(2)}\}, \{\nu_{R,1}^{(3)} \ldots \nu_{R,m}^{(3)}\})$

$$\tilde{M} = \begin{pmatrix} 0 & \alpha & \pm i\alpha & 0 \\ \alpha^T & M_1 & 0 & 0 \\ \pm i\alpha^T & 0 & M_1 & 0 \\ 0 & 0 & 0 & M_2 \end{pmatrix}, \tag{1}$$

for an arbitrary number of ν_R and to all radiative orders, with M_1 and M_2 diagonal matrices with positive entries and α a generic complex matrix.

[a] This is a necessary requirement to satisfy phenomenological constraints

[b] These are highly fine-tuned solution that cannot be achieved solely by specific textures of the neutrino mass matrix

Corollary on lepton number violation

Using a unitary matrix D, let us construct

$$Q = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \pm\frac{i}{\sqrt{2}}D & \frac{1}{\sqrt{2}}D & 0 \\ 0 & \frac{1}{\sqrt{2}}D & \pm\frac{i}{\sqrt{2}}D & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

then through a change of basis

$$Q^T \tilde{M} Q = \begin{pmatrix} 0 & \pm i\sqrt{2}(D^T\alpha^T)^T & 0 & 0 \\ \pm i\sqrt{2}D^T\alpha^T & 0 & \pm iD^T M_1 D & 0 \\ 0 & \pm iD^T M_1 D & 0 & 0 \\ 0 & 0 & 0 & M_2 \end{pmatrix} \sim \begin{pmatrix} 0 & M_D^T & 0 & 0 \\ M_D & 0 & M_R & 0 \\ 0 & M_R^T & 0 & 0 \\ 0 & 0 & 0 & M_2 \end{pmatrix}$$

- Similar to the L conserving limit of inverse and/or linear seesaw
- Explicitly L conserving taking the L assignment $(+1, -1, +1, 0)$

Corollary

The most general gauge-singlet neutrino extensions of the SM with no cancellation between different orders of the seesaw expansion, no fine-tuned cancellations between different radiative orders and which lead to three massless neutrinos are L conserving.

Eq. (1) as a sufficient condition

- Directly obtained from the corollary[1]

[1] In the seesaw limit, light neutrinos are Majorana fermions whose mass violate L conservation. Eq. (1) being equivalent to L conservation implies that the light neutrinos are massless.

Necessary condition: tree level

- At tree-level and for the first order of the seesaw expansion

$$\mathrm{m}_\nu \approx -m_D M_R^{-1} m_D^T$$

- If $m_D M_R^{-1} m_D^T = 0$ and using $Z = M_R^{-1} m_D^T$, then the exact block-diagonalisation of the full neutrino mass matrix gives

  [Korner et al., 1993, Grimus and Lavoura, 2000]

$$\begin{aligned} \mathrm{m}_\nu = &- \left(1 + Z^* Z^T\right)^{-\frac{1}{2}} Z^T m_D^T \left(1 + Z^\dagger Z\right)^{-\frac{1}{2}} \\ &- \left(1 + Z^T Z^*\right)^{-\frac{1}{2}} m_D Z \left(1 + Z Z^\dagger\right)^{-\frac{1}{2}} \\ &+ \left(1 + Z^* Z^T\right)^{-\frac{1}{2}} Z^T M_R Z \left(1 + Z Z^\dagger\right)^{-\frac{1}{2}} \end{aligned}$$

- All terms contain $m_D M_R^{-1} m_D^T$ thus

$$\mathrm{m}_\nu = 0 \Rightarrow m_D M_R^{-1} m_D^T = 0$$

  to all orders of the seesaw expansion

Necessary condition: one-loop level

- When $m_\nu = 0$ at tree-level, the one-loop induced masses are

$$\delta m_{ij} = \Re\left[\frac{\alpha_W}{16\pi^2 m_W^2} C_{ik} C_{jk} f\left(m_k\right)\right]$$

with C the mixing matrix in the neutral current and Higgs couplings and f the loop function

- In the basis where M_R is diagonal, the full neutrino mass matrix M is

$$M = \begin{pmatrix} 0 & m_{D1} & \dots & m_{Dn} \\ m_{D1}^T & \mu_1 & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ m_{Dn}^T & 0 & \dots & \mu_n \end{pmatrix}$$

and at the first order in the seesaw expansion

$$\delta m = 0 \Rightarrow \sum_{i=1}^{n} \mu_i^{-2} m_{Di} m_{Di}^T f\left(\mu_i\right) = 0$$

Necessary condition: one-loop level

- Cancellation could still come from summation of non-zero terms ☹
- But a rescaling $M \to \Lambda M$ does not affect the condition $m_\nu = \delta m = 0$
- $f(x)$ being monotonically increasing and strictly convex,

$$\sum_{i=1}^{n} \mu_i^{-2} m_{Di} m_{Di}^T f(\mu_i) = 0 \to \Lambda^{-2} \sum_{i=1}^{n} \mu_i^{-2} m_{Di} m_{Di}^T f(\Lambda \mu_i) = 0$$

generate linearly independent equations from which

$$m_\nu = 0 \Rightarrow m_{Di} m_{Di}^T = 0$$

since $\mu_i > 0, f(\mu_i) > 0$

Necessary condition: one-loop level

- From a bit of algebra and by excluding trivial solutions,

$$m_{Di} m_{Di}^T = 0 \Rightarrow$$

$$m_{Di} = \begin{pmatrix} u_1^{i'} & \pm i u_1^{i'} & 0 & 0 & 0 & 0 & 0 & \dots & 0 \\ v_1^{i'} & \pm i v_1^{i'} & v_3^{i''} & \pm i v_3^{i''} & 0 & 0 & 0 & \dots & 0 \\ w_1^{i'} & \pm i w_1^{i'} & w_3^{i''} & \pm i w_3^{i''} & w_5^{i'''} & \pm i w_5^{i'''} & 0 & \dots & 0 \end{pmatrix}$$

- By rearranging the columns and rows, flavour-basis mass matrix becomes

$$M = \begin{pmatrix} 0 & \alpha & \pm i\alpha & 0 \\ \alpha^T & M_1 & 0 & 0 \\ \pm i\alpha^T & 0 & M_1 & 0 \\ 0 & 0 & 0 & M_2 \end{pmatrix} = \tilde{M} \quad \square$$

Conclusions

- Any symmetry that leads to massless light neutrinos contains L as a subgroup or an accidental symmetry
- Spectrum in the L conserving limit: 3 massless light neutrinos + heavy Dirac neutrinos + decoupled neutrinos
- Nearly conserved L is a cornerstone of low-scale type I seesaw variants
- Smallness of the light neutrino mass related to the smallness of the L breaking parameter, or equivalently to the degeneracy of the heavy neutrinos in pseudo-Dirac pairs in low-scale type I seesaw variants
- Expect L violating signatures to be suppressed
  $\rightarrow$ Needs to be quantitatively assessed
- Seems to be applicable to type III seesaw variants as well
  $\rightarrow$ Currently investigating it

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Backup slides

Cancellation between different seesaw orders

- To second order in the expansion

$$m_\nu^{(2)} = -m_\nu^{(1)} + \frac{1}{2}\left(m_n^{(1)} u\theta + \theta^T m_\nu^{(1)}\right)$$

with $m_\nu^{(1)}$ the first order expression and θ is $Z^\dagger Z$ up to a unitary transformation

- Then

$$(m_\nu^{(2)})_{ii} = 0 \Leftrightarrow -\hat{m}_{lii}^{(1)} + \hat{m}_{lii}^{(1)}\theta_{ii} = 0$$

and $\theta_{ii} = 1$

- This contradicts [Fernandez-Martinez et al., 2016] which gives $||\theta|| \leqslant 0.0075$

An aside on the Kersten-Smirnov theorem

- Using tree-level contributions ($\mathrm{m}_\nu = 0 \Leftrightarrow m_D M_R^{-1} m_D^T = 0$), they get the general result if $\#\nu_R \leqslant 3$

$$m_D = m \begin{pmatrix} y_1 & y_2 & y_3 \\ a y_1 & a y_2 & a y_3 \\ b y_1 & b y_2 & b y_3 \end{pmatrix}, \quad \text{and} \quad \frac{y_1^2}{M_{R,1}} = \frac{y_2^2}{M_{R,2}} = \frac{y_3^2}{M_{R,3}}$$

- For $\#\nu_R > 3$, the system of linear equations in their proof is under-constrained
- In general, no symmetry is present. Necessary to assume degenerate heavy neutrinos to make a statement.
- Justify this by requiring radiative stability but approach based on running of the Weinberg operator
  → Works only if Higgs boson lighter than all heavy neutrinos

Details of one-loop proof I

- The loop function is

$$f\left(m_k\right) = m_k\left(3m_Z^2 g_{kZ} + m_H^2 g_{kH}\right)$$

where

$$g_{ab} = \frac{m_a^2}{m_a^2 - m_b^2}\log\frac{m_a^2}{m_b^2}$$

which gives

$$U_l^T\left(1 + Z^T Z^*\right)^{-1} Z^T U_h^* f_h U_h^\dagger Z\left(1 + Z^\dagger Z\right)^{-1} U_l = 0$$

$$Z^T U_h^* f_h U_h^\dagger Z = 0$$

to the first order in the seesaw expansion

$$U_h \approx 1$$

$$Z^T F_h Z = 0$$

Details of one-loop proof II

- We write $m_{Di}^T = (u^i, v^i, w^i)$, then

$$m_{Di} m_{Di}^T = \begin{pmatrix} u^{iT} u^i & u^{iT} v^i & u^{iT} w^i \\ v^{iT} u^i & v^{iT} v^i & v^{iT} w^i \\ w^{iT} u^i & w^{iT} v^i & w^{iT} w^i \end{pmatrix} = 0$$

- We construct $Y^i = u^{i*} u^{iT} + u^i u^{i\dagger}$. Imposing $u^{iT} u^i = 0$ and excluding the trivial solution $u^i = 0$, $\mathrm{rank}(Y^i) = 2$
- Y^i symmetric and real: we can build a basis of real orthogonal eigenvectors $b^i_{1\ldots n_i}$. For the zero $n_i - 2$ eigenvalues,

$$Y^i b^i_k = 0 \Rightarrow ||u^i||^2 (u^{iT} b^i_k) = 0 \Rightarrow u^{iT} b^i_k = 0$$

- Then

$$u^{i'} = R^i_u u^i = \begin{pmatrix} b_1^{iT} u^i \\ b_2^{iT} u^i \\ b_3^{iT} u^i \\ \vdots \\ b_{n_i}^{iT} u^i \end{pmatrix} = \begin{pmatrix} u_1^{i'} \\ u_2^{i'} \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

Details of one-loop proof III

- Once we have

$$u^{i'} = \left(u_1^{i'}, \pm i u_1^{i'}, 0, \ldots, 0\right)^T$$

Under this transformation, we have

$$u^{iT} v^i = 0 \to u'^{iT} v'^i = 0$$

leading us to conclude that

$$v^{i'} = \left(v_1^{i'}, \pm i v_1^{i'}, v_3^{i'}, v_4^{i'}, \ldots, v_{n_i}^{i'}\right)^T$$

- Similarly, we construct a second matrix R_v acting on $\left(v_3^{i'}, v_4^{i'}, \ldots, v_{n_i}^{i'}\right)^T$ such that $v^{i'}$ is reduced to

$$v^{i''} = \left(v_1^{i'}, \pm i v_1^{i'}, v_3^{i''}, \pm i v_3^{i''}, 0, \ldots, 0\right)^T$$

- Rinse and repeat for w

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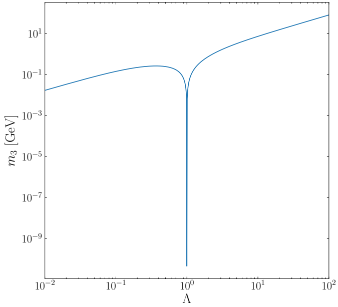
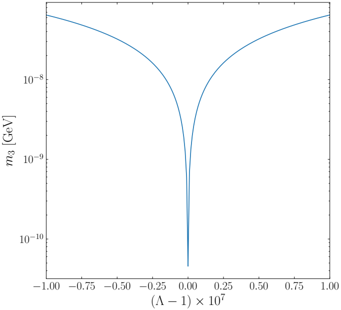
Fine-tuning

We adopt here the idea of [Lopez-Pavon et al., 2015], where the tree-level and one-loop contributions cancel.





Evolution of m_3 as a function of the rescaling parameter Λ. Input masses and couplings where chosen to give $m_\nu = m_{\text{tree}} + m_{\text{1-loop}} = 0.046\,\text{eV}$ at $\Lambda = 1$.
A deviation of less then 10^{-7} here, is enough to spoil the cancellation and contradict experimental limits.

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