Charmed Baryon to Strange Baryon Decay using QCD Sum Rules

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Bigger Picture

- **What I am interested in:**
  - non-perturbative QCD & color-confinement

- **Why?**
  - confinement mechanism:
    - What is the detailed structure? Can we derive it from first principles?
    - theory of non-standard matter
  - physics of nuclei:
    - internucleon distances $\sim$ nucleon size, then why don't all quarks inside form a 'commune'?
    - Why mass of nucleus $\approx AM_N$?
  - controversial resonances & exotic hadrons:
    - tetraquarks, glueballs
    - $d^*(2380)$, $X(3872)$, $X(3915)$, $Z(3930)$, $Y(3930)$, $Z_1^+(4050)$ *etc.*

Bigger Picture

How I approach these questions?

- lattice formulations
  - Scattering in LQCD for multiple partial waves and decay channels: *Nuclear Physics B 924 (2017) 477*

- collider physics
  - B Production In pp and AA Collisions: *Int. J. Theo. Phys. 56 (2017) 3649*

- QCD sum rules
  - Lambda baryon masses from QCD sum rules: *arXiv:1711.02164*

This Talk

estimate the rate of Cabibbo-favored weak decay: $\Lambda_c^+ \to \Lambda_s^0 \pi^+$

Quantum Chromodynamics (QCD) Basics

QCD: Theory of Strong Interactions

- a non-Abelian gauge theory with a local $SU(3)$ symmetry group
- invariance under local rotations of *color* indices
- at each space-time point, choose complex 3×3 matrix $\Omega(x)$ as element of $SU(3)$
- RG transformation makes coupling scale-dependent, α_s becomes large at $Q^2 \sim 1\ \mathrm{GeV} \Rightarrow$ infrared slavery

QCD sum rules

Main Idea

- assuming only color-confinement, a large amount of predictions can be made
- attack the problem from short distance side and gradually move to large distance
- asymptotic freedom starts to break down and power correction emerges

QCD sum rules

LHS: Wilson's Operator Product Expansion

- hadrons are represented by their interpolating quark current *e.g.*

$$\eta^{\text{nucleon}} = \epsilon_{abc}\left[u^a(x)C\gamma^\mu u^b(x)\right]\gamma_5\gamma_\mu d^c(x)$$

- write some correlation functions of these colorless currents

$$T_{\mu\nu\ldots}\Pi_2^j(q^2) = i\int d^4x e^{iq.x}\langle 0|T\left\{\eta_\Gamma(x)\bar{\eta}_\Gamma(0)\right\}|0\rangle$$

where $j_\Gamma = \bar{q}_i\Gamma q_j$ is a meson current, Γ encodes a tensor structure

- expand on basis of local, gauge-invariant operators (condensates):

$$i\int d^4x e^{iq.x}\langle 0|T\left\{\eta_\Gamma(x)\bar{\eta}_\Gamma(0)\right\}|0\rangle = C_I^\Gamma(q;\mu)I + \sum_n C_n^\Gamma(q;\mu)\langle\hat{O}_n(\mu)\rangle\ ,$$

where there coefficients will be evaluated using perturbation theory

QCD sum rules

RHS: Phenomenological Model

- correlation functions are related to the *spectral density* of the theory.
- for a two-point function, the spectral density $\rho(q^2) \sim Im\Pi(q^2)$.
- in narrow resonance approximation, we consider the model

$$Im\Pi(q^2) = \frac{\pi}{e_q^2} \sum_R \frac{m_R^2}{g_R^2}\, \delta(q^2 - m_R^2) + \frac{1}{4\pi}\left(1 + \frac{\alpha_s}{\pi}\right) \theta(q^2 - s_0)$$

where R denotes resonance.

- dispersion relation

$$\Pi(Q^2) = \frac{1}{\pi} \int_0^\infty ds \frac{Im\Pi(s)}{s + Q^2 - i\epsilon}, \qquad Q^2 = -q^2$$

QCD sum rules

Borel Improvement

- insertion of more bubbles leads to softer momenta $\Rightarrow$ coupling gets increasingly large and assumes leading logs $\Rightarrow$ factorial divergence in $\Pi\left[\alpha_s(q^2)\right]$

$$\alpha_s(q^2) \approx \frac{\alpha_s(\mu)}{1 - \frac{\beta_0}{4\pi}\alpha_s(\mu)\ln\left(\frac{\mu^2}{q^2}\right)} \to \sum_0^\infty \alpha_s(\mu)^{n+1}\left(\frac{\beta_0}{4\pi}\ln\frac{\mu^2}{q^2}\right)^n$$

- convergence achieved through Borel transformation

$$\tilde{\Pi}(M_B^2) = \mathcal{B}_{M_B^2}\left[\Pi(Q^2)\right] = \lim_{Q^2, n\to\infty; Q^2/n = M_B^2} \frac{(Q^2)^{n+1}}{n!}\left(-\frac{d}{dQ^2}\right)^n \Pi(Q^2)$$

Applying QCD sum rules

Estimation of lambda baryon masses

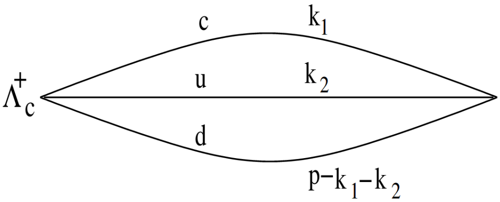
- lambda baryon current:

$$\eta_h(x) = \epsilon^{abc}\left[u^{aT}(x)C\gamma_\mu d^b(x)\right]\gamma^5\gamma^\mu h^c(x),$$

where $h(x)$ denotes the heavy quark; a, b, c are color indices; C is charge conjugation.

- Two point correlator

$$\Pi_2(p) \quad = \quad i\int d^4x e^{ip.x}\langle 0|T\left[\eta_h(x)\bar{\eta}_h(0)\right]|0\rangle$$



Applying QCD sum rules

Estimation of lambda baryon masses

- the perturbative diagram gives the dominant contribution
- peel off any term that leads to power divergence because they vanish with Borel transform anyway
- the hadronic mass of the baryon is found from the minima of the Borel-transformed two-point function
- no free parameter used other than current quark masses

Applying QCD sum rules

Estimation of lambda baryon masses

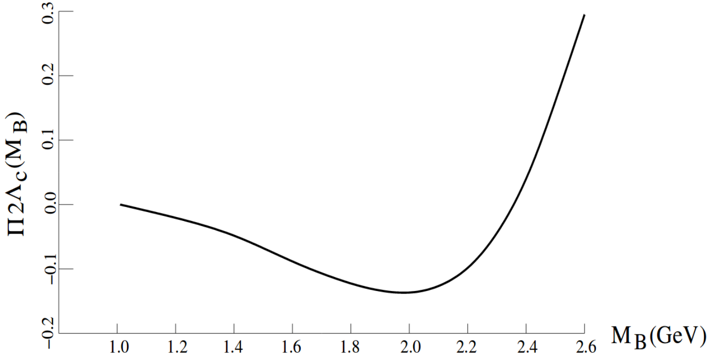
$$\Pi_2(M_B) = \frac{8M_B^2}{(4\pi)^6}\int_0^1 d\alpha d\kappa \frac{g(\alpha,\kappa)M_5(\alpha) + 3m^3 M_2(\alpha,\kappa)}{g(\alpha,\kappa)^3}\ e^{-b(\alpha,\kappa)^2/M_B^2}$$

where

$$\begin{aligned}
g(\alpha,\kappa) &= \kappa + \alpha(1-\alpha)(1-\kappa), \\
b(\alpha,\kappa)^2 &= g(\alpha,\kappa)\left[\frac{M_c^2}{\alpha(1-\alpha)(1-\kappa)} + \frac{m^2}{\kappa\alpha(1-\alpha)}\right], \\
M_2(\alpha,\kappa) &= \left(1 - \frac{1}{4\alpha(1-\alpha)}\right)\left[b(\alpha,\kappa)^2 - M_B^2\right], \\
M_5(\alpha) &= 2m^4 M_c - 3m^3 M_c^2 + \frac{3M_c m^4}{2\alpha(1-\alpha)} + \frac{3m^3 M_c^2}{4\alpha(1-\alpha)} - \frac{M_c m^4}{4\alpha^2(1-\alpha)^2}.
\end{aligned}$$

Applying QCD sum rules: arXiv:1711.02164v1

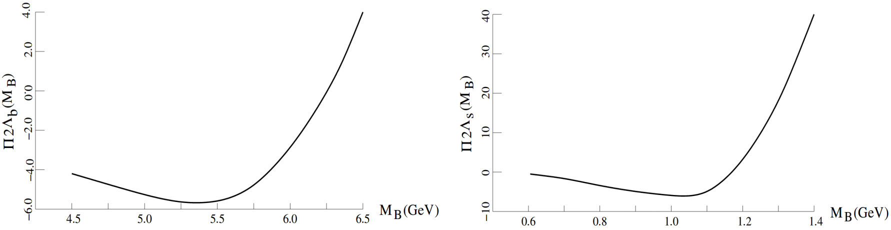
Estimation of Λ_c^+ baryon mass



Two-point correlators for Λ_c as a function of Borel mass

Applying QCD sum rules: arXiv:1711.02164v1

Estimation of Λ_b^0, Λ_s^0 baryon masses



Two-point correlators for Λ_b and Λ_s as a function of Borel mass

- $M_{\Lambda_c^+} = (2.01 \pm 0.30)$ GeV; $M_{\Lambda_c^+}^{\text{Exp}} = (2.28646 \pm 0.00014)$ GeV
- $M_{\Lambda_b^0} = (5.34 \pm 0.80)$ GeV; $M_{\Lambda_b^0}^{\text{Exp}} = (5.61958 \pm 0.00017)$ GeV
- $M_{\Lambda_s^0} = (1.05 \pm 0.16)$ GeV; $M_{\Lambda_s^0}^{\text{Exp}} = (1.115683 \pm 0.000006)$ GeV

A Brief Digression

Charge-radius of hadron from finite-volume residual

- Estimated hadron mass using only the perturbative diagram is consistently smaller than physical mass: confinement
- The difference provides us information about charge radius, form factor etc.
- Vacuum is sqeezed in confined volume $\Rightarrow$ constituent quarks are propagating in sqeezed vacuum $\Rightarrow$ pole of the hadrons get shifted to the heavier side
- work in progress

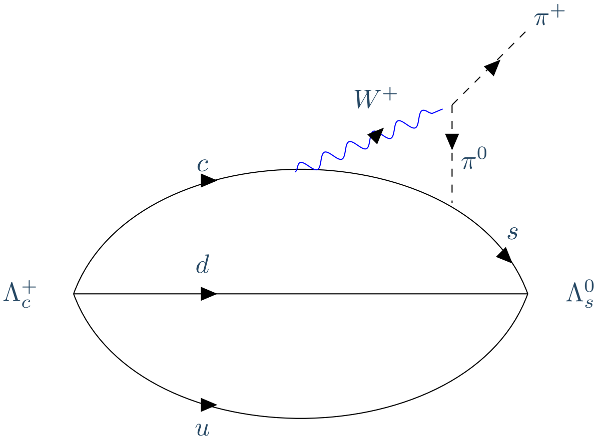
Applying QCD sum rules II

Cabibbo-favored weak decay of charmed lambda: $\Lambda_c^+ \to \Lambda_s^0\, \pi^+$

- Λ_c^+ (mass $2286.46\ \mathrm{MeV/c^2}$, $I(J^P) = 0(\frac{1}{2}^+)$) was the first charmed baryon ever to be discovered
- yet to map out all decay modes, mainly because it has many available modes
- we estimate the weak decay $\Lambda_c^+ \to \Lambda_s^0 \pi^+$ using QCD sum rules
- dynamic properties of Λ_c^+ can be extracted too using QCD sum rules

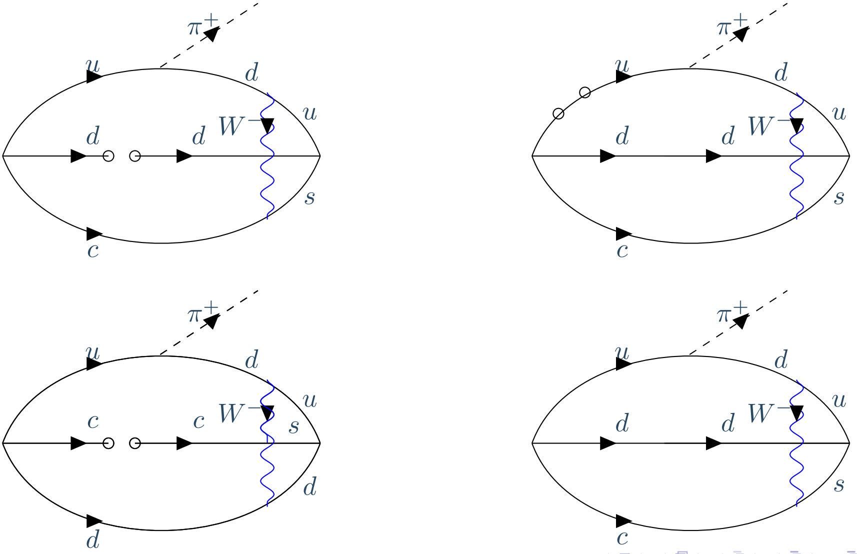
Applying QCD sum rules II

Leading Order Feynman Diagram corresponding $\Lambda_c^+ \to \Lambda_s^0\ \pi^+$



Applying QCD sum rules II

A few higher order diagrams



Applying QCD sum rules II

Cabibbo-favored weak decay of charmed lambda: $\Lambda_c^+ \to \Lambda_s^0\, \pi^+$

- need to include some new phenomenological parameters in OPE in presence of an external field
- construct a three-point function using the Λ_c^+ and Λ_s^0 current and the weak Hamiltonian

$$H_W = \frac{G_F}{\sqrt{2}} J^\mu J_\mu^\dagger$$

- J^μ is the flavor-changing current

$$J^\mu = \bar{s}\gamma^\mu(1-\gamma_5)c\; V_{cs} + \bar{d}\gamma^\mu(1-\gamma_5)u\; V_{ud}$$

where V_{ud} and V_{cs} are Cabibbo-Kobayashi-Maskawa matrix elements.

Applying QCD sum rules II

The Three-point function

- The three-point function in presence of an external field is given by

$$\Pi_3(p,q) \;=\; i\int d^4x d^4y e^{ip.x+iq.y}\,\Pi_3(x,y)$$

- p and q are four-momentum of Λ_c^+ and π^+. Also

$$\Pi_3(x,y) = \langle 0|T\left[\eta_{\Lambda_c^+}(x)H_W(y)\bar{\eta}_{\Lambda_s^0}(0)\right]|0\rangle_{\pi^+}$$

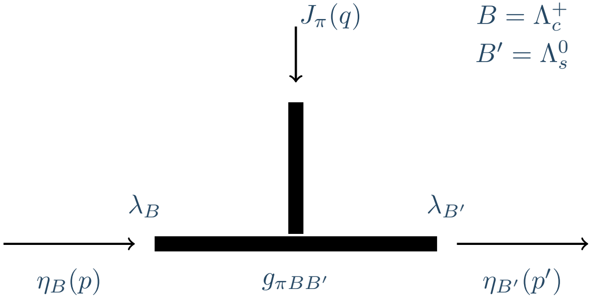
Applying QCD sum rules II

Phenomenological model for the three-point function

- double-pole at charmed and strange lambda masses in the narrow resonance approximation

$$\Pi_3(p,p') = -4\lambda_B \lambda_{B'} M_B M_{B'} \frac{1}{p^2 - M_B^2}\, g_{\pi BB'}(p^2)\, \frac{1}{p'^2 - M_{B'}^2} + \text{continuum}$$

$$B = \Lambda_c^+$$
$$B' = \Lambda_s^0$$



Conclusion

- To extract resonance information in non-perturbative QCD, we use lattice QCD, QCD sum rules *etc.*

- Lambda baryon masses were succesfully reproduced using two-point correlator constructed with colorless lambda baryon currents

  - $M_{\Lambda_c^+} = (2.01 \pm 0.30)\ \text{GeV};\qquad M^{\text{Exp}}_{\Lambda_c^+} = (2.28646 \pm 0.00014)\ \text{GeV}$
  - $M_{\Lambda_b^0} = (5.34 \pm 0.80)\ \text{GeV};\qquad M^{\text{Exp}}_{\Lambda_b^0} = (5.61958 \pm 0.00017)\ \text{GeV}$
  - $M_{\Lambda_s^0} = (1.05 \pm 0.16)\ \text{GeV};\qquad M^{\text{Exp}}_{\Lambda_s^0} = (1.115683 \pm 0.000006)\ \text{GeV}$

- The weak decay modes of charmed lambda, $\Lambda_c^+ \to \Lambda_s^0 \pi^+$, can be formalized in terms of a three-point correlation function of the Λ_c^+ and Λ_s^0 current operators and weak Hamiltonian H_W.

References

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