Global $SU(2)_L \otimes$ BRST Symmetry and it's WTIs

Ozenc GUNGOR, CWRU

OG, BWL, GDS, ARXIV:1711.07349

RELATED: BWL, GDS, PRD.96.065003

OUTLINE

- Commutation of BRST and Global transformations
- ‘t Hooft Gauge condition and on-shell current conservation
- Off-shell, 1-soft-π Green’s function identities
- On-shell T-Matrix element identities
- Conclusions/Outlook

COMMUTATION OF BRST AND GLOBAL TRANSFORMATIONS

$$L_{R_\xi} + L_{FP} = s\left[\vec{\bar{\eta}} \cdot \left(\partial_\mu \vec{W}^\mu + \xi M_W \vec{\pi} + \frac{1}{2}\xi \vec{b}\right)\right]$$

The Rest is the usual SU(2) Lagrangian without fermions.

$$s\vec{W}_\mu = \partial_\mu \vec{\omega} + g_2 \vec{W}_\mu \times \vec{\omega} \qquad \delta_L \vec{W}_\mu = g_2 \vec{W}_\mu \times \vec{\Omega}$$

$$sH = -\frac{1}{2} g_2 \vec{\pi} \cdot \vec{\omega} \qquad \delta_L H = -\frac{1}{2} g_2 \vec{\pi} \cdot \vec{\Omega}$$

$$s\vec{\pi} = \frac{1}{2} g_2 (H\vec{\omega} + \vec{\pi} \times \vec{\omega}) \qquad \delta_L \vec{\pi} = \frac{1}{2} g_2 (H\vec{\Omega} + \vec{\pi} \times \vec{\Omega})$$

$$s\vec{\omega} = -\frac{1}{2} g_2 \vec{\omega} \times \vec{\omega} \qquad \delta_L \vec{\omega} = g_2 \vec{\omega} \times \vec{\Omega}$$

$$s\vec{\bar{\eta}} = \vec{b} \qquad \delta_L \vec{\bar{\eta}} = 0$$

$$s\vec{b} = 0 \qquad \delta_L \vec{b} = 0$$

The anti-ghost field is a SU(2) singlet.

$$\delta_L \left(L_{R_\xi} + L_{FP}\right) \neq 0$$

$$[\delta_L, s]\left\{\vec{W}_\mu, H, \vec{\pi}, , \vec{\omega}, \vec{\bar{\eta}}, \vec{b}\right\} = 0$$

$$[\delta_L, s]\left(L_{R_\xi} + L_{FP}\right) = 0$$

‘T HOOFT GAUGE CONDITION AND ON-SHELL CURRENT CONSERVATION

- Landau gauge is implemented by

$$\langle 0|T\left[\left(\partial_\mu \vec{W}^\mu(z)\right) h(x_1)\ldots h(x_N)\vec{\pi}(y_1)\ldots\vec{\pi}(y_M)\right]|0\rangle_C = 0$$

- Divergence of the global SU(2) current is

$$\partial_\mu J_L^\mu = \frac{1}{2}M_W\left(\vec{\pi}\times\partial_\mu\vec{W}^\mu + H\partial_\mu\vec{W}^\mu\right)$$

- The global SU(2) current is conserved on connected amplitudes

$$\langle 0|T\left[\left(\partial_\mu \vec{J}_L^\mu(z)\right) h(x_1)\ldots h(x_N)\vec{\pi}(y_1)\ldots\vec{\pi}(y_M)\right]|0\rangle_C = 0$$

OFF-SHELL, 1-SOFT-Π GREEN'S FUNCTION IDENTITIES

- Using $\langle 0|T\left[\left(\partial_\mu \vec{J}^\mu_L(z)\right) h(x_1)\ldots h(x_N)\vec{\pi}(y_1)\ldots\vec{\pi}(y_M)\right]|0\rangle_C = 0$, we can derive

$$\begin{aligned}
&\langle H\rangle \Gamma^{t_1\ldots t_M t}_{N,M+1}(p_1,\ldots,p_N;0,q_1,\ldots,q_M)\\
&\quad = \sum_{m=1}^{M} \delta^{t t_m} \Gamma^{t_1\ldots \hat{t_m}\ldots t_M}_{N+1,M-1}(p_1,\ldots,p_N,q_M;q_1,\ldots,\hat{q_m},\ldots,q_M)\\
&\quad - \sum_{n=1}^{N} \Gamma^{t_1\ldots t_M t}_{N-1,M+1}(p_1,\ldots,\hat{p_n},\ldots,p_N;q_1,\ldots,q_M,p_n)
\end{aligned}$$

- $\Gamma_{N,M}$ are $1-\phi-I$ connected, amputated Green's functions with M π legs $(q_1,\ldots,q_M)$, N h legs $(p_1,\ldots,p_N)$
- Free parameters are defined by

$$\Gamma_{0,4}(;0,0,0,0) \equiv -2\lambda_\phi^2$$
$$\Gamma_{0,2}(;0,0) \equiv -2m_\pi^2$$

OFF-SHELL, 1-SOFT-Π GREEN'S FUNCTION IDENTITIES

THE SCALAR - SECTOR EFFECTIVE LAGRANGIAN

- Up to operators with dim ≤ 4

$$L_\phi^{eff} = \Gamma_{1,0}(0;)h + \frac{1}{2!}\Gamma_{2,0}(p,-p;)h^2 + \frac{1}{2!}\Gamma_{0,2}^{t_1 t_2}(;q,-q)\pi^{t_1}\pi^{t_2} + \frac{1}{3!}\Gamma_{3,0}(0,0,0;)h^3 + \frac{1}{2!}\Gamma_{1,2}^{t_1 t_2} h\pi^{t_1}\pi^{t_2} + \frac{1}{4!}\Gamma_{4,0}(0,0,0,0;)h^4 + \frac{1}{2!\,2!}\Gamma_{2,2}^{t_1 t_2} 0,0;0,0 h^2\pi^{t_1}\pi^{t_2} + \frac{1}{4!}\Gamma^{t_1 \ldots t_4}(;0,0,0,0)\pi^{t_1}\pi^{t_2}\pi^{t_3}\pi^{t_4}$$

- Using WTI with N + M ≤ 4

$$L_\phi^{eff,kinetic} = \frac{1}{2}\left(\Gamma_{0,2}(;p,-p) - \Gamma_{0,2}(;0,0)\right)h^2 + \frac{1}{2}\left(\Gamma_{0,2}(;q,-q) - \Gamma_{0,2}(;0,0)\right)\vec{\pi}^2$$

$$V_\phi^{eff} = m_\pi^2\left(\frac{h^2+\vec{\pi}^2}{2} + \langle H\rangle h\right) + \lambda_\pi^2\left(\frac{h^2+\vec{\pi}^2}{2} + \langle H\rangle h\right)^2$$

ON-SHELL T-MATRIX ELEMENT IDENTITIES

- Using on-shell current conservation we can also derive

$$\langle H\rangle T^{tt_1\ldots t_M}_{N,M+1}(p_1,\ldots,p_N;0,q_1,\ldots,q_M)(2\pi)^4\delta^4\left(\sum_{n=1}^{N}p_n+\sum_{m=1}^{M}q_m\right)\Bigg|^{p_1^2=\ldots=p_N^2=m_h^2}_{q_1^2=\ldots=q_M^2=0} = 0$$

- **N=0, M=1 case** $\langle H\rangle T^{t_1t_2}_{0,2}(;0,0)=0$

- $\langle H\rangle \Gamma_{0,2}(;0,0)=\langle H\rangle m_\pi^2=0$

ON-SHELL T-MATRIX ELEMENT IDENTITIES

THE SCALAR - SECTOR EFFECTIVE LAGRANGIAN

- $V_\phi^{eff} = \lambda_\phi^2 \left(\frac{h^2 + \vec{\pi}^2}{2} + \langle H \rangle h \right)^2$ after using $\langle H \rangle \Gamma_{0,2}(;0,0) = \langle H \rangle m_\pi^2 = 0$

- $\Gamma_{1,0}(0;) = \langle H \rangle \Gamma_{0,2}(;0,0) = 0$, tadpoles automatically vanish.

- Holds for CP conserving SM with Dirac neutrino masses.
- Holds for a general gauge semi-simple gauge group.

CONCLUSIONS/OUTLOOK

- The off-shell WTIs and on-shell T-Matrix elements:
  - Determine the ϕ - sector effective Lagrangian fully
  - Automatically makes tadpoles vanish

- Generalize result to CP-breaking (QCD angle) cases
- Generalize to $\xi \neq 0$