Ghost- and Tachyon-Free Regions of the Randall-Sundrum Model Parameter Space

[1802.05731]

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May 8th, 2018

Introduction to the RS Model

- Address the Gauge Hierarchy with a warped extra dimension:

$$ds^2 = e^{-2\sigma}\eta_{\mu\nu}dx^\mu dx^\nu - r_c^2 d\phi^2,$$

$$\sigma = kr_c|\phi|, \qquad kr_c \approx 10$$

- Two 4-D branes, the "Planck- (or UV)- brane" at $\phi = 0$, and the "TeV- (or IR)- brane" at $\phi = \pi$.
- By localizing the Higgs field near the TeV-brane, but allowing the graviton to propagate freely in the bulk, the electroweak scale is naturally suppressed by a factor of $\epsilon = e^{-kr_c\pi}$

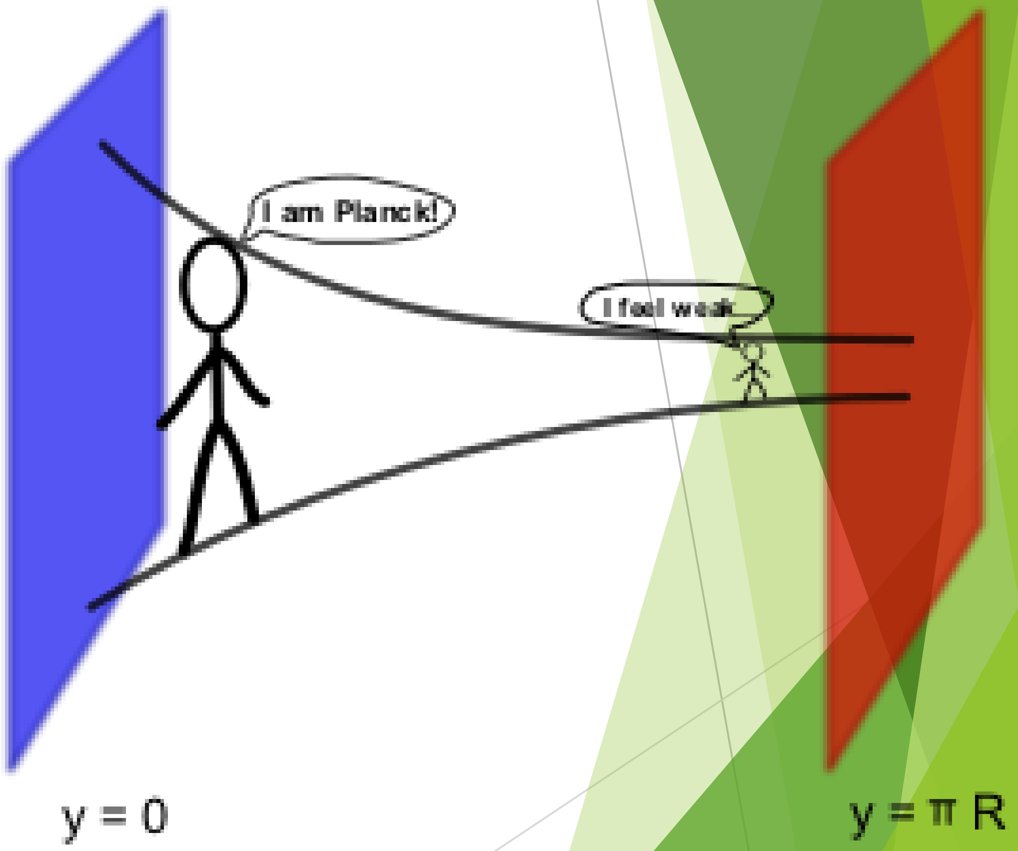


Image: http://resonaances.blogspot.com/2016/03/

SM Fields in the Bulk

- SM fields may also be placed in the bulk, allowing a larger parameter space and the resolution of other issues (e.g. fermion mass hierarchy)
- Example: A left-handed chiral fermion has the bulk action:

$$
\begin{aligned}
S_F = & \int d^4x \int r_c d\phi \sqrt{G} \left\{ V_N^M \left(\frac{i}{2}\bar{\Psi}\Gamma^N \partial_M \Psi + h.c.\right) + [2\tau_0/kr_c \;\; \delta(\phi) \right. \\
+ & \; 2\tau_\pi/kr_c \;\; \delta(|\phi| - \pi)] \, V_\nu^\mu (i\bar{\Psi}_L \gamma^\nu \partial_\mu \Psi_L + h.c.) \\
+ & \left. k \, sgn(\phi)(\eta + \frac{1}{2})\bar{\Psi}\Psi \right\} .
\end{aligned}
$$

- After integrating over the extra dimension, a single bulk field produces a “tower” of particles of different masses known as a Kaluza-Klein (KK) tower. Spectrum and normalizations of tower modes specified by orbifold (von Neumann) boundary conditions

Why Bust Ghosts and Chase Tachyons?

- Pheno restrictions (*e.g.*, precision electroweak) mean that all these extra parameters are often necessary!
- Allowed regions of parameter space limit some parameters' ability to address phenomenological bounds
- Care must be taken in any phenomenological analyses--we don't want to probe regions with tachyons!

Tachyons and Ghost States: Defining the Problem

- In the effective 4-D theory, BLKT's introduce the possibility of Kaluza-Klein tower modes with ghost-like normalizations or imaginary/complex mass.
- Masses given by roots of the equation ($m_n = x_n k e^{-kr_c\pi}$) :

$$\zeta_\eta(x_n) - \tau_\pi x_n \zeta_{1+\eta}(x_n) = 0, \qquad \zeta_q(z_n) \equiv \alpha_n J_q(z_n) + \beta_n Y_q(z_n)$$

- For convenience, define: $\epsilon = e^{-kr_c\pi}$
- Then the zero-mode (massless) state has a normalization constant proportional to:

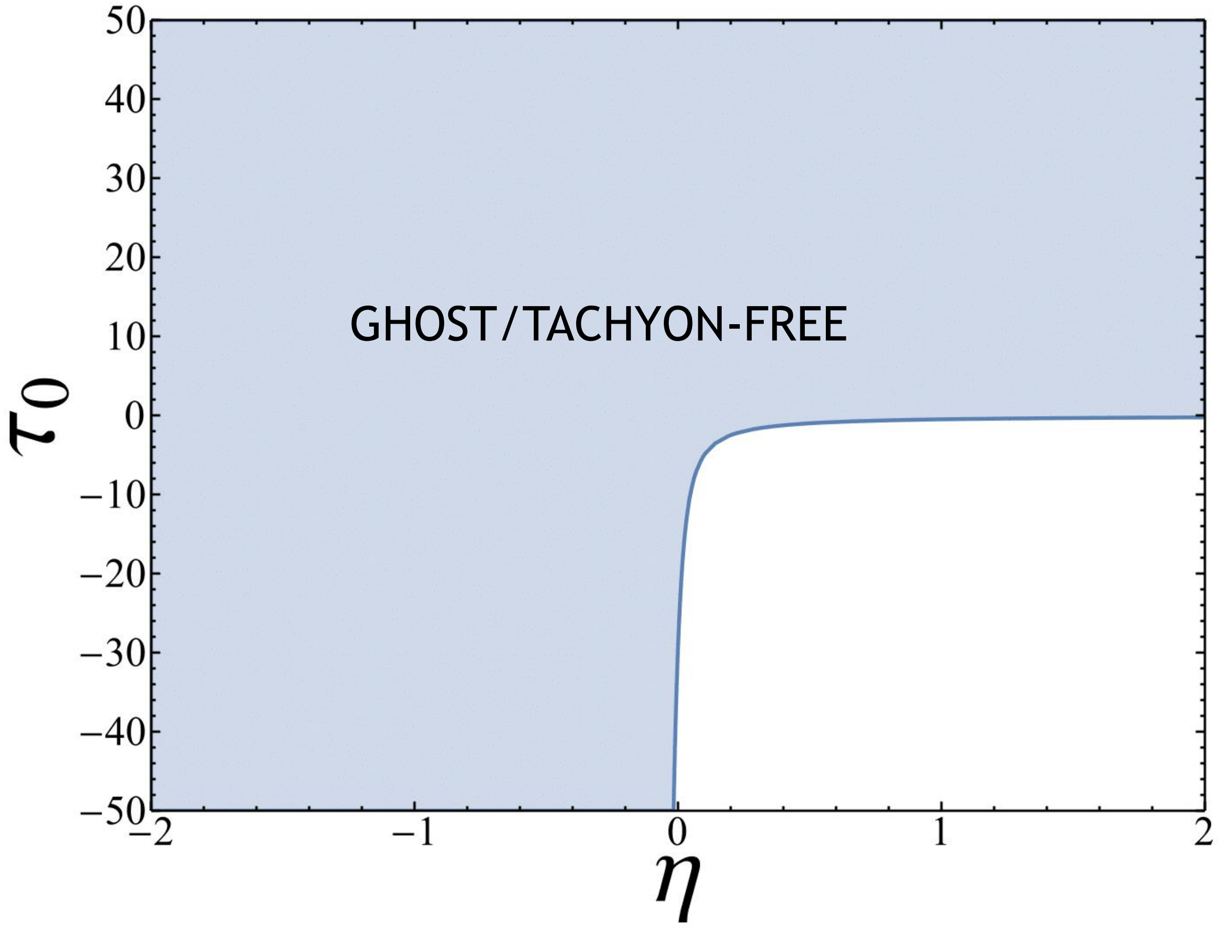
$$\sqrt{\frac{-2\eta}{(1-2\eta\tau_\pi)\epsilon^\eta - (1+2\eta\tau_0)\epsilon^{-\eta}}} \rightarrow \frac{-2\eta}{(1-2\eta\tau_\pi)\epsilon^\eta - (1+2\eta\tau_0)\epsilon^{-\eta}} > 0$$

Analysis Method

- In the absence of brane mass terms, it is provable that KK modes can have only purely real or purely imaginary mass. So to find tachyons, consider the case of purely imaginary mass.
- Then, assume that any physical tachyonic (imaginary) KK mass must be roughly on the scale $k\epsilon$, *i.e.*, $|m_{tachyon}| \ll k \sim M_{pl}$.
  - So, impose a cut-off *x* in the KK tower spectrum equation
- Expand the Bessel functions in the mass eigenvalue equations for a pure imaginary root, and impose no-ghost condition.
- Dropping terms suppressed by ϵ, we get that the tachyon- and ghost-free condition is:

$$\frac{1}{\pi\eta}\sum_{k=0}^{\infty}\left(\frac{x}{2}\right)^{2k}\frac{1}{k!}\left[\frac{\epsilon^{\eta}(1+2(k-\eta)\tau_{\pi})\Gamma(1-\eta)}{\Gamma(1+k-\eta)} - \frac{\epsilon^{-\eta}(1+2\eta\tau_0)(1+2k\tau_{\pi})\Gamma(1+\eta)}{\Gamma(1+k+\eta)}\right] < 0$$

Results: Fermions



- Separating into different regions of η, it is possible to derive:
- ALWAYS $\tau_\pi \geq 0$
- When $|\eta| < 1$:
  $$\tau_0 > \frac{1}{2\eta}\left[\left(\frac{\epsilon x_{max}}{2}\right)^{2\eta}\frac{\Gamma(1-\eta)}{\Gamma(1+\eta)} - 1\right]$$
- When $\eta \geq 1$: $\tau_0 > -\dfrac{1}{2\eta}$
- No condition when $\eta \leq -1$
- Pictured Left: Shaded region represents the ghost/tachyon-free region of parameter space

Gauge Bosons and Gravitons

- In the absence of bulk masses for gauge bosons and gravitons (forbidden on symmetry grounds), limits analogous to specific values of η
- For gauge bosons ($\eta = 0$):

$$\tau_0 > \gamma + \log(\epsilon) + \log(\frac{x_{max}}{2}), \tau_\pi \geq 0$$

- For gravitons ($\eta = 1$):

$$\tau_0 > -\frac{1}{2}, \qquad 0 \leq \tau_\pi \leq 1$$

- Upper limit on TeV-brane term comes from radion restrictions

Spontaneous Symmetry Breaking

- Including a TeV-brane-localized Higgs mechanism, the action for two bulk fermions, Q and q, coupled by a TeV-brane Yukawa coupling is:

$$S_{F,Yukawa} = S_F - \int d^4x \int r_c d\phi \sqrt{G} \left\{ \frac{2}{kr_c}\delta(|\phi| - \pi)e^{\sigma} \frac{v}{\sqrt{2}}[\overline{Q}_L Y q_R + \overline{q}_R Y^* Q_L] \right\}$$

- Extra terms modify boundary conditions, and might therefore influence the position/existence of tachyonic roots, or change the wave function normalizations (altering ghost states)
- Within the limit where $v \ll k\epsilon$, corrections to the wave function normalizations and locations of tachyonic roots are small, and cannot eliminate a ghost or tachyonic state without fine tuning.

Why do we care?

- RS parameter space is HUGE, we need to be aware of all constraints
- Comprehensive understanding of allowed parameter space permits a comprehensive probe
- Restrictions on BLKT's limit their model-building utility; provide more support for other tools to keep RS phenomenologically viable, *e.g.* bulk custodial symmetry for electroweak constraints