NLO mixed QCD-EW corrections to Higgs gluon fusion

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in collaboration with

K. Melnikov (TTP, KIT) & L. Tancredi (CERN)

Topics

Looking for new physics

- Existence of physics beyond the Standard Model
- Lack of direct evidences of new physics (at the LHC)

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New physics from investigating known processes at higher precision

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Goal

Provide theoretical predictions for Higgs physics
with uncertainty lower than $O(1\%)$

Higgs at the LHC [Anastasiou. . . ,2009][LHC H CSWG,2017][Mistlberger,2018]

Gluon fusion

Dominant channel for H production at the LHC: $\sim$ 90% of σ_{tot} at 13 TeV

- $\sim$ 95% pure QCD
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Theoretical uncertainties

δ_{scale}	δ_{trunc}	$\delta_{\text{PDF-TH}}$	$\delta_{\text{QCD-EW}}$	$\delta_{t,\,b,\,c}$	δ_{1/m_t}
$\sim$ 2%	—	1.16%	1%	0.83%	1%

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$$\sigma^{\text{QCD-EW}} = \underbrace{\sigma_{\text{LO}}^{\text{QCD-EW}}}_{\text{Exact}} + \underbrace{\sigma_{\text{NLO}}^{\text{QCD-EW}}}_{m_{W,Z} \gg m_H} + \dots$$

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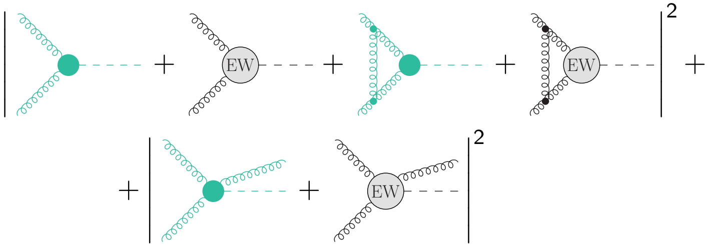
Compute $\sigma_{\text{NLO}}^{\text{QCD-EW}}$ for physical values of $m_{W,Z}$ and m_H

NLO building blocks

[Aglietti. . . ,2006][Degrassi. . . ,2004]

$$\sigma_{\text{NLO}}^{\text{QCD-EW}} = \int_0^1 \int_0^1 f(x_1) f(x_2) \overline{\sigma}_{\text{NLO}} \, \mathrm{d}x_2 \mathrm{d}x_1$$

$\overline{\sigma}_{\text{NLO}} \propto$

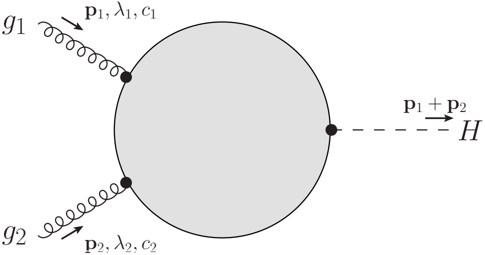


Pure QCD known up to N^3LO (here: $m_t \to +\infty$)

QCD-EW known in a physical region at LO

$gg \to H$ NLO QCD-EW amplitude

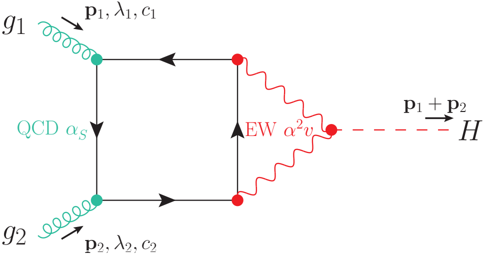
$gg \to H$: general structure



$$\mathcal{M}^{\,c_1 c_2}_{\,\lambda_1 \lambda_2} = \delta^{c_1 c_2} \epsilon_{\lambda_1}(\mathbf{p}_1) \cdot \epsilon_{\lambda_2}(\mathbf{p}_2) \mathcal{F}$$

$gg \to H$ NLO QCD-EW amplitude

$gg \to H$: Leading Order

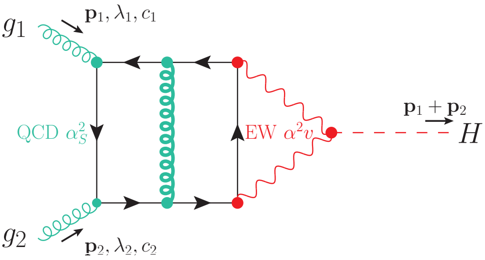


$$\mathcal{M}_2{}^{c_1 c_2}_{\lambda_1 \lambda_2} = \delta^{c_1 c_2} \epsilon_{\lambda_1}(\mathbf{p}_1) \cdot \epsilon_{\lambda_2}(\mathbf{p}_2) \mathcal{F}_2(s, m_W, m_Z)$$

- H couples to EW vector bosons: t suppressed (large mass)
- Light quarks taken to be massless (W: $\{u, d, s, c\}$; Z: $\{u, d, s, c, b\}$)
- W and Z never in the same diagram: two scales s & m^2

$gg \to H$ NLO QCD-EW amplitude

$gg \to H$: virtual NLO



$$\mathcal{M}3^{c_1 c_2}_{\lambda_1 \lambda_2} = \delta^{c_1 c_2} \epsilon_{\lambda_1}(\mathbf{p}_1) \cdot \epsilon_{\lambda_2}(\mathbf{p}_2) \mathcal{F}_3(s, m_W, m_Z)$$

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Feynman Integrals

- ${\mathcal{M}_3}^{c_1 c_2}_{\lambda_1 \lambda_2}$: 47 3-loop Feynman Diagrams for both W and Z EW bosons

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Tensorial Decomposition

$$\mathcal{F}_3(s, m_W, m_Z) = \mathcal{C}_V\, A^V_{3\text{L}} = \frac{\epsilon^{*\lambda_1}(\mathbf{p}_1) \cdot \epsilon^{*\lambda_2}(\mathbf{p}_2)}{2(1-\epsilon)} \frac{\delta_{c_1 c_2}}{N_C^2 - 1} \mathcal{M}3^{c_1 c_2}_{\lambda_1 \lambda_2}$$

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- $A^V_{3\text{L}}(s, m)$: sum of $\sim 10\,000$ 3-loop Feynman Integrals

Feynman Integral

$$\mathcal{I}(s, m, \epsilon) = \int \frac{\mathrm{d}^D k_1 \mathrm{d}^D k_2 \mathrm{d}^D k_3}{[\mathrm{i}\pi^{2-\epsilon}\Gamma(1+\epsilon)]^3} \prod_{j=1}^{J} \frac{1}{\mathcal{D}_j^{a_j}}$$

- $\mathcal{D}_j = (\alpha_{j1} p_1 + \alpha_{j2} p_2 + \beta_{j1} k_1 + \beta_{j2} k_2 + \beta_{j3} k_3)^2 - \gamma_j m^2$
- $a_j \in \mathbb{Z}$ (scalar products expressed via $a_j < 0$)

Master Integrals

[Chetyrkin. . . ,1981][Gehrmann. . . ,1999]

Not all FIs are independent!

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Feynman Integrals Relations

- Integration-by-Parts Identities

$$\int \frac{\partial}{\partial k^\mu} \left(q^\mu \prod_{j=1}^{J} \frac{1}{\mathcal{D}_j^{a_j}} \right) [\mathrm{d}^D k] = 0, \qquad q^\mu = k^\mu, p^\mu$$

- Lorentz Invariance
- Symmetries
- Dim. Reg.

System of linear relations among FIs

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Basis of functions for $A_{3\mathrm{L}}$: all FIs are linear combinations of MIs

- $A_{3\mathrm{L}}(s, m)$: linear combination of 95 Master Integrals

Evaluation of Master Integrals

$$\mathcal{I}(s, m, \epsilon)$$

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1. Change of variables to only one dimensionful parameter

$$y := \frac{\sqrt{1-4m^2/s}-1}{\sqrt{1-4m^2/s}+1} \qquad\qquad s$$

By dimensional analysis the s-dependence is determined

$$\mathcal{I}(s, y, \epsilon) = (-s)^{a-3\epsilon}\mathcal{J}(y, \epsilon)$$

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Solving Differential Equations w.r.t. dynamical variables of the MIs

Differential Equations [Kotikov,1991][Remiddi,1997][Gehrmann. . . ,1999]

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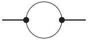
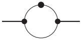
1. Differentiate the MIs w.r.t. masses or scalar kinematic invariants

$$\frac{\partial}{\partial\left(m^2\right)}$$  $$= -2$$ 

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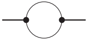
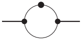
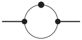
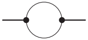
$\frac{\partial}{\partial\left(m^2\right)}$  $= -2$ 

2. Apply IBPs to recover the MIs (MI for subtopologies may arise)

 $= -\frac{1-2\epsilon}{4m^2+s}$  $- \frac{1-\epsilon}{m^2\left(4m^2+s\right)}$ 

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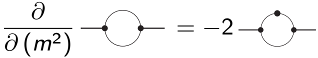

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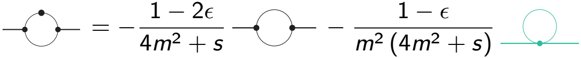
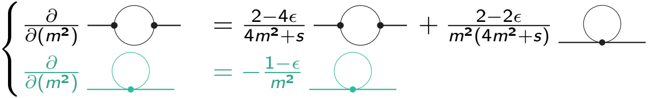


3. Include in cascade DEs for all the MIs in the subtopologies



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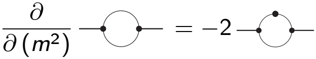

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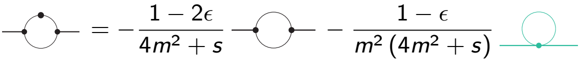

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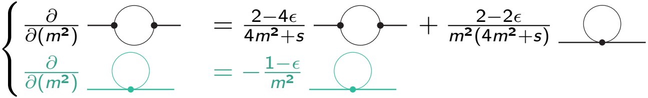
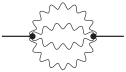


System of linear Partial Differential Equations

$$\frac{\partial \mathbf{J}(y,\epsilon)}{\partial y} = A(y,\epsilon)\mathbf{J}(y,\epsilon)$$

A simple form for FIs

[Henn,2013][Argeri. . . ,2014]



$$= \epsilon^{-1} \; \frac{s^2}{36} + \epsilon^0 \; s^2 \frac{71 - 18\log(-s)}{216} +$$

$$+ \epsilon^1 \quad s^2 \left[\frac{\log^2(-s)}{8} - \frac{71\log(-s) + \pi^2}{72} + \frac{3115}{1296} \right] +$$

$$+ \epsilon^2 \quad s^2 \left[-\frac{\log^3(-s)}{8} + \frac{71\log^2(-s)}{48} + \frac{\pi^2\log(-s)}{24} + \right.$$

$$\left. - \frac{3115\log(-s)}{432} - \frac{7\zeta(3)}{9} - \frac{71\pi^2}{432} + \frac{109403}{7776} \right] +$$

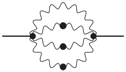
$$+ \quad O\left(\epsilon^3\right)$$

Generic FI: collection of rational and transcendental functions of the kinematics, ϵ-poles, and constants with no simple pattern at fixed ϵ-order

A simple form for FIs

[Henn,2013][Argeri. . . ,2014]

$\epsilon^3 (-s)$



$$= \epsilon^0 \quad 1 + $$
$$+ \epsilon^1 \quad [-3 \log(-s)] + $$
$$+ \epsilon^2 \quad \frac{9 \log^2(-s) - \pi^2}{2} + $$
$$+ \epsilon^3 \quad \left[\frac{-9 \log^3(-s) - 3\pi^2 \log(-s)}{2} - 28\zeta(3) \right] + O\left(\epsilon^4\right)$$

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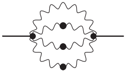
$\epsilon^3(-s)$ 

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- Constants from logs: $\pi \rightsquigarrow \log(-1)$, $\zeta(2k) \rightsquigarrow \pi^{2k} \rightsquigarrow \log^{2k}(-1)$

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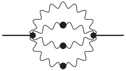
$\epsilon^3 (-s)$  $= \epsilon^0 \quad 1 +$

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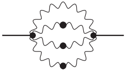
$\epsilon^3 (-s)$  $= \epsilon^0 \quad 1 +$

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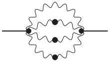
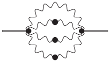
- $\frac{\partial}{\partial s}\Big[\epsilon^3(-s)$  $\Big] = \frac{-3\epsilon}{s}\Big[\epsilon^3(-s)$  $\Big]$

Uniformly Transcendental functions [Henn,2013][Di Vita...,2014]

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Weight W

Number of nested integrations over $\mathrm{d}\log R(\xi)$, $R(\xi)$ rational functions

$$F_n(y) = \int_0^y \dots \int_0^{\xi_n} \mathrm{d}\log R_n(\xi) \dots \mathrm{d}\log R_1(\xi) \quad \Rightarrow \quad W(F_n) := n$$

- Weight w functions in rational points give weight w constants
  $$W(\mathbb{Q}) = 0, \quad W(\pi) = W(\gamma_E) = 1, \quad W(\zeta(n)) = n$$
- $W(F_a\, F_b) = W(F_a) + W(F_b)$

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UT function

Function having a finite ϵ-expansion with weight n coefficients at order ϵ^n

The UT Cauchy problem

[Remiddi. . . ,1999][Henn,2013][Lee,2014]

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Differential Equations

$$\frac{\mathrm{d}}{\mathrm{d}y}\mathbf{F}(y,\epsilon) = \epsilon \sum_{a=1}^{A} B_a \frac{\mathrm{d}\log R_a(y)}{\mathrm{d}y}\mathbf{F}(y,\epsilon)$$

The UT Cauchy problem

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Canonical form — homogeneous dependence over ϵ

Fuchsian system — only simple poles in y (partial fractioning on d logs)

The UT Cauchy problem

[Remiddi. . . ,1999][Henn,2013][Lee,2014]

Differential Equations

$$\frac{\mathrm{d}}{\mathrm{d}y}\mathbf{F}(y,\epsilon) = \epsilon \sum_{a=1}^{A} B_a \frac{\mathrm{d}\log R_a(y)}{\mathrm{d}y}\mathbf{F}(y,\epsilon)$$

Canonical form — homogeneous dependence over ϵ

Fuchsian system — only simple poles in y (partial fractioning on $\mathrm{d}\log$s)

Boundary Conditions

To fix the integration constants the solution of the DEs is compared in $y \to y_0$ to a boundary function

$$\lim_{y\to y_0}\left[\mathbf{F}(y,\epsilon) - \mathbf{L}(y,\epsilon)\right] = 0$$

If y_0 is a rational point, $\mathbf{L}(y \to y_0, \epsilon)$ is also a UT expression

Canonical fuchsian solution

[Goncharov,1994][Remiddi...,1999][Henn,2013][Argeri...2014][Lee,2014]

Canonical form solution as a Dyson series in ϵ

$$\mathbf{F}(y,\epsilon) = \mathcal{P}_y e^{\epsilon \int A(\xi)\,\mathrm{d}\xi}\,\mathbf{F}_0(\epsilon) =$$

$$= \mathbf{F}_0^{(0)} + \epsilon\left[\int_y A(\xi_1)\mathbf{F}_0^{(0)}\,\mathrm{d}\xi_1 + \mathbf{F}_0^{(1)}\right] +$$

$$+\,\epsilon^2\left[\int_y A(\xi_1)\int_{\xi_1} A(\xi_2)\mathbf{F}_0^{(0)}\,\mathrm{d}\xi_2\mathrm{d}\xi_1 + \int_y A(\xi_1)\mathbf{F}_0^{(1)}\,\mathrm{d}\xi_1 + \mathbf{F}_0^{(2)}\right] + O\left(\epsilon^3\right)$$

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Fuchsian system iterated integrations as Goncharov Polylogarithms

$$G(\mathbf{m}_w; y) := \begin{cases} \frac{1}{w!}\log^w y & \text{if } \mathbf{m} = (0,\ldots,0) \\ \int_0^y \frac{1}{\xi - m_w} G(\mathbf{m}_{w-1};\xi)\,\mathrm{d}\xi & \text{if } \mathbf{m} \neq (0,\ldots,0) \end{cases}$$

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Integration constant — $\mathbf{J}_0^{(n)}$ rational combination of weight n constants

Evaluation of the Form Factor

[Vermaseren,1989][Nogueira,1993]
[Laporta,2001][Vollinga. . . ,2004][von Manteuffel. . . ,2012]

UT Solution

GPLs	known functions, arbitrary numerical precision available
UT BCs	small class of constants, efficient PSLQ matching

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From Feynman Diagrams to Master Integrals

QGRAF	Feynman diagrams generation
FORM	color, Lorentz and trace structure evaluation
Reduze	reduction of FIs to a basis of MIs (via Laporta's algorithm)

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From Master Integrals to UT Functions

A UT basis of MIs is not assured to exist for every process, but in many cases of interest can be found

STILL NOT STRAIGHTFORWARD

UT Solution

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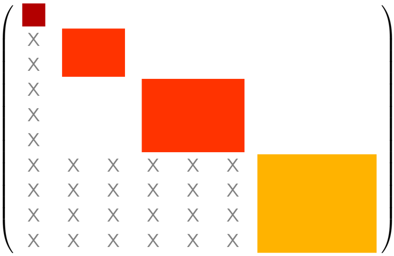
The plan for 3-loop NLO QCD-EW

DEs have a natural block-triangular form

- Off-diagonal terms related to subtopologies
- MIs with fewer denominators have simpler equations
- Blocks correspond to topologies requiring more than 1 MIs (up to 4)
- Different topologies with same number of denominators are not related

$$\frac{\mathrm{d}\mathbf{J}(y,\epsilon)}{\mathrm{d}y} =$$



$$\mathbf{J}(y,\epsilon)$$

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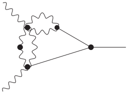
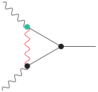
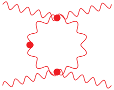
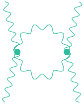
Bottom-up approach

Put into a UT form MIs having few denominators and off-diagonal terms, then modify one by one higher topologies

A two-steps approach

[Argeri. . . ,2014][Gehrmann. . . ,2014] [Lee,2014][Primo. . . ,2016][Gituliar. . . ,2017][Frellesvig. . . ,2017][Meyer,2017]

1. Study of $\mathbf{J}(y, \epsilon)$ $\quad A(y,\epsilon) \rightsquigarrow A_0(y) + \epsilon A_1(y)\,[+\dots]$
   - Building blocks

$\epsilon^4\,(1-2\epsilon)$  $\Leftarrow\ \epsilon^2$  $\otimes\ \epsilon$  $\otimes\ \epsilon\,(1-2\epsilon)$ 

   - Maximal cut
     - All possible propagators are put on-shell
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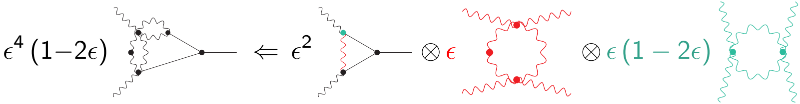

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2. Study of $A(y,\epsilon)$ $\qquad A_0(y) + \epsilon A_1(y)\,[+\dots] \rightsquigarrow \epsilon\, B_a\, \mathrm{d}_y \log R_a(y)$
   - Integrating away of $A_0(y)$

     Fuchsian structure can be spoiled: logs in $A_1(y)$
   - Algebraic techniques: `Fuchsia` & `CANONICA`

     BC can become non-UT: rescaling of lower UT MIs by ϵ-polynomials

Canonical Fuchsian DEs at 3 loops

$$\mathrm{d}\mathbf{F}(y,\epsilon) = \epsilon \; \Big[\; B_+ \, \mathrm{d}\log(1-y) \; + B_r \, \mathrm{d}\log(y^2 - y + 1) \; + \\ + B_- \, \mathrm{d}\log(y+1) \; + B_0 \, \mathrm{d}\log y \; \Big] \mathbf{F}(y,\epsilon)$$

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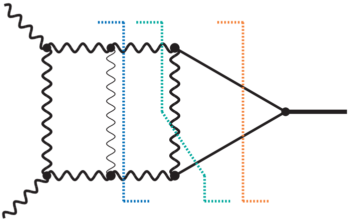

s	0	m^2	$4m^2$	$[\infty]$
y	$+1$	$\mathrm{e}^{\mathrm{i}\pi/3}$	-1	$[0]$
Kernel	$\frac{1}{\xi-1}$	$\frac{2\xi-1}{\xi^2-\xi+1}$	$\frac{1}{\xi+1}$	$\left[\frac{1}{\xi}\right]$

Boundary Conditions

[Smirnov,2002]

$\mathrm{d}\log(y^2 - y + 1)$: efficient numerical matching using PSLQ algorithm at

$$y := \frac{\sqrt{1 - 4m^2/s} - 1}{\sqrt{1 - 4m^2/s} + 1} \to 1$$

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  - Large momentum cannot be created, destroyed or provided by external legs: it must form at least one closed flow along the internal lines.

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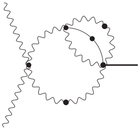
- Taylor expansion in $\sim \sqrt{s}$ to generate the large-mass integrals

A 3-loop example



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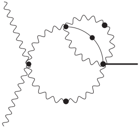
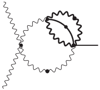
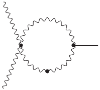
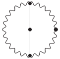
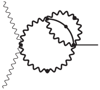
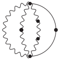
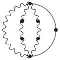


Two non-vanishing configurations

-  $\rightarrow$  $\times$ 
-  $\rightarrow$  $+ \; s\frac{2(1+\epsilon)}{2-\epsilon}$  $+ \; \mathcal{O}\left(\frac{(-s)^2}{(m^2)^4}\right)$

Since loops with large momentum detach from the main graph
all boundary terms consist of tadpoles and massless bubbles or triangles

Solutions at 3 loops

$\mathcal{F}(s,m)$ may contain FIs with ϵ^{-6} poles: solutions required up to ϵ^{6} terms (and weight $w=6$)

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Constant terms

$\mathbf{F}_0^{(w)}$ are all (simple) $\mathbb{Q}$-linear combinations of the following constants

w	0	1	2	3	4	5	6
Values	1		π^2	$\zeta(3)$	π^4	$\pi^2\zeta(3)$ $\zeta(5)$	π^6 $\zeta^2(3)$

Final expression for the Form Factor

$$\mathcal{F}(s, m_W, m_Z) = -\mathrm{i}\frac{\alpha^2\alpha_S(\mu)v}{64\pi\sin^4\theta_W}\sum_{V=W,Z} C_V A(m_V^2/s, \mu^2/s)$$

- $C_W = 4$
- $C_Z = \dfrac{2}{\cos^4\theta_W}\left(\dfrac{5}{4} - \dfrac{7}{3}\sin^2\theta_W + \dfrac{22}{9}\sin^4\theta_W\right)$

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- α_S renormalization: elimination of UV pole
- Real corrections: subtraction on IR pole MISSING

Finite values for the amplitude

[Catani,1998]

It is possible to extract a meaningful finite value from IR divergent A_{3L} thanks to the universal structure of QCD IR behavior at NLO

Catani's subtraction

$$A_{3L} = \mathbf{I}_g^{(1)} A_{2L} + A_{3L}^{\text{fin}}$$

$$\mathbf{I}_g^{(1)} = \left(-\frac{s}{\mu^2}\right)^{-\epsilon} \frac{e^{\epsilon\gamma_E}}{\Gamma(1-\epsilon)} \left[-\frac{C_A}{\epsilon^2} - \frac{\beta_0}{\epsilon}\right]$$

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$s = \mu = m_H = 125.09\,\text{GeV},\ m_W = 80.385\,\text{GeV},\ m_Z = 91.1876\,\text{GeV},\ N_C = 3,\ N_f = 5$

$$\begin{aligned}
A_{2L}(m_Z^2/m_H^2, 1) &= -6.880846 \quad -i\,0.5784119 \\
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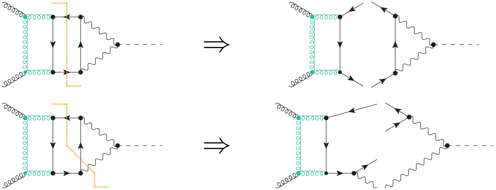
Analysis of the imaginary part

Diagram level Imaginary parts: on-shell intermediate particles



$\Rightarrow \quad s > 0$

$\Rightarrow \quad s > m^2$

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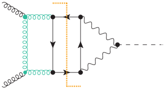
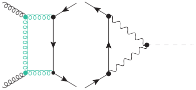
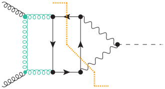
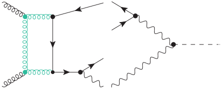
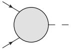
 $\Rightarrow$  $\Rightarrow$ $s > 0$

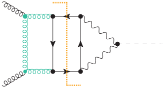
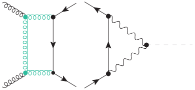
 $\Rightarrow$  $\Rightarrow$ $s > m^2$

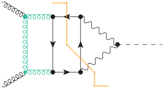
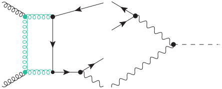
Amplitude level Not all intermediate configurations are allowed

H cannot couple to massless fermions  $= 0$

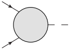
Analysis of the imaginary part

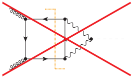
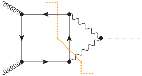
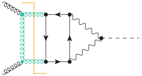
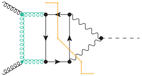
Diagram level Imaginary parts: on-shell intermediate particles

 $\Rightarrow$  $\Rightarrow$ $s > 0$

 $\Rightarrow$  $\Rightarrow$ $s > m^2$

Amplitude level Not all intermediate configurations are allowed

H cannot couple to massless fermions  $= 0$

	$s = 0$	$s = m^2$		
2 loops			$\Rightarrow$	$-\mathrm{i}\,2.302953$
3 loops			$\Rightarrow$	$-\mathrm{i}\,54.02989$

Taking into account the real emissions

Real emissions

- Challenging problem: 2-loop, with more than one scale (s, m^2, E_g)
  - Larger number of MIs
  - Few techniques for DEs with more than one scale
  - GPLs may be not enough to express the solution of the system

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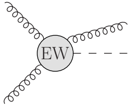
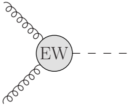
Soft limit





$$\left| \text{[diagram]} \right|^2 \underbrace{=}_{E_g \to 0} \frac{\alpha_S}{4\pi} C_A \frac{2\, p_1 \cdot p_2}{p_1 \cdot p_4\; p_2 \cdot p_4} \left| \text{[diagram]} \right|^2 + O\left(p_4^{-1}\right)$$

σ_{NLOr} factorizes into eikonal factor (given only by external legs) and σ_{LO}, further contributions are suppressed by E_g

Cross-Section at NLO

[de Florian. . . ,2012][Forte. . . ,2013]

Hadronic cross-section: the soft-gluon approximation is employed

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Soft-gluon approximation

$$\sigma = \int_0^1 \int_0^1 f(x_1, \mu)\, f(x_2, \mu)\, \sigma_{\mathrm{LO}}\, z\, G(z, \mu, \alpha_S)\, \mathrm{d}x_2 \mathrm{d}x_1$$

- $z := m_H^2/(S_{\mathrm{h}} x_1 x_2)$, $\quad gg \to H$ energy
- $G = \delta(1-z) + \frac{\alpha_S}{2\pi}\left[8 C_A \mathcal{D}_1 + \left(\frac{2\pi^2}{3} C_A + \frac{\sigma_{\mathrm{NLO}}^{\mathrm{fin}}}{\sigma_{\mathrm{LO}}}\right)\delta(1-z)\right]$
- $\mathcal{D}_1 = \left[\frac{\log(1-z)}{1-z}\right]_+ + (2 - 3z + 2z^2)\frac{\log[(1-z)/\sqrt{z}]}{1-z} - \frac{\log(1-z)}{1-z}$
- $\sigma_{\mathrm{NLO}}^{\mathrm{fin}}$ is the NLO finite reminder from Catani's formula

Numerical values for the cross-section

[LHC H CSWG,2017]

QCD vs. QCD-EW

$$\sigma_{\text{LO}}^{\text{QCD}} = 20.6\,\text{pb} \qquad \sigma_{\text{LO}}^{\text{QCD-EW}} = 21.7\,\text{pb} \quad \Rightarrow \quad +5.3\% \text{ at LO}$$

$$\sigma_{\text{NLO}}^{\text{QCD}} = 37.0\,\text{pb} \qquad \sigma_{\text{NLO}}^{\text{QCD-EW}} = 39.0\,\text{pb} \quad \Rightarrow \quad +5.5\% \text{ at NLO}$$

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Robust result

- Internal consistency check: standard and improved $\mathcal{D}_1$ give the same increase between σ_{LO} and σ_{NLO}
- PDFs suppress large energy for the extra gluon

Outlook

- NLO virtual corrections to QCD-EW $gg \to H$ have been evaluated
- In soft-gluon limit

$$\sigma_{\text{NLO}}^{\text{QCD-EW}} = \sigma_{\text{NLO}}^{\text{QCD}} + 2.0\,\text{pb}$$

Theoretical uncertainties now

δ_{scale}	δ_{trunc}	$\delta_{\text{PDF-TH}}$	$\delta_{\text{QCD-EW}}$	$\delta_{t,\,b,\,c}$	δ_{1/m_t}
$\sim 2\%$	—	1.16%	$\ll 1\%$	0.83%	1%

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- Next step: determination of the real corrections

$$gg \to Hg \qquad qg \to Hq \qquad \bar{q}g \to H\bar{q} \qquad q\bar{q} \to Hg$$

  - Necessary ingredient for further improvements (no soft-gluon approximation)
  - Interesting problem both for physics and mathematics

Thank you for your attention

Karlsruher Institut für Technologie

BACKUP SLIDES

Lower set: analysis of the MIs [Argeri...,2014][Gehrmann...,2014]

Non-UT parts are removed modifying the MIs or by integration

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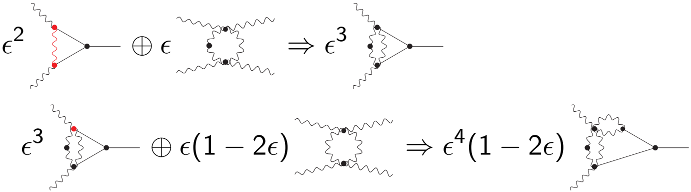
Non-UT parts are removed modifying the MIs or by integration

1. Adjusting powers of denominators in the MIs

Guiding principles

Building blocks composition of canonical functions



No UV divergencies the total mass dimension must be negative

Lower set: analysis of the MIs [Argeri...,2014][Gehrmann...,2014]

Non-UT parts are removed modifying the MIs or by integration

1. Adjusting powers of denominators in the MIs
2. Rescaling by $\epsilon^{a_1}(c_1 + \epsilon c_2)^{a_2}$ to obtain an ϵ-linear system

$$\frac{\mathrm{d}\tilde{\mathbf{J}}(y,\epsilon)}{\mathrm{d}y} = [A_0(y) + \epsilon A_1(y)]\tilde{\mathbf{J}}(y,\epsilon)$$

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1. Adjusting powers of denominators in the MIs
2. Rescaling by $\epsilon^{a_1}(c_1 + \epsilon c_2)^{a_2}$ to obtain an ϵ-linear system
3. Integrating out the ϵ-independent part

$$\frac{\mathrm{d}\mathbf{F}(y,\epsilon)}{\mathrm{d}y} = \epsilon \hat{S}^{-1}_{A_0}(y) A_1(y) \hat{S}_{A_0}(y) \mathbf{F}(y,\epsilon)$$

$$\mathbf{F}(y,\epsilon) = \hat{S}^{-1}_{A_0}(y) \tilde{\mathbf{J}}(y,\epsilon)$$

$$\hat{S}_{A_0}(y) = \sum_{k=0}^{+\infty} \int_{y_0}^{y} A_0(\xi_1) \ldots \int_{y_0}^{\xi_{k-1}} A_0(\xi_k)\, \mathrm{d}\xi_k \ldots \mathrm{d}\xi_1$$

- $\hat{S}_{A_0}(y)$ obtained by means of Magnus Series or direct integration
- Fuchsianity not always conserved: logarithms may arise from integration

Higher set: analysis of the DEs

[Lee,2014][Primo...,2016][Gituliar...,2017][Frellesvig...,2017][Meyer,2017]

Algebraic operations on the matrix of coefficients: `Fuchsia` & `CANONICA`

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By operating only on the matrix of coefficients, boundary conditions can be spoiled, becoming non-UT

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  MIs too complicated for direct inspection

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Maximal Cut sufficient to find good candidates for Fuchsia or CANONICA