Progress on modeling atmospheric leptons at the surface and underground using MCEq; and CHROMO

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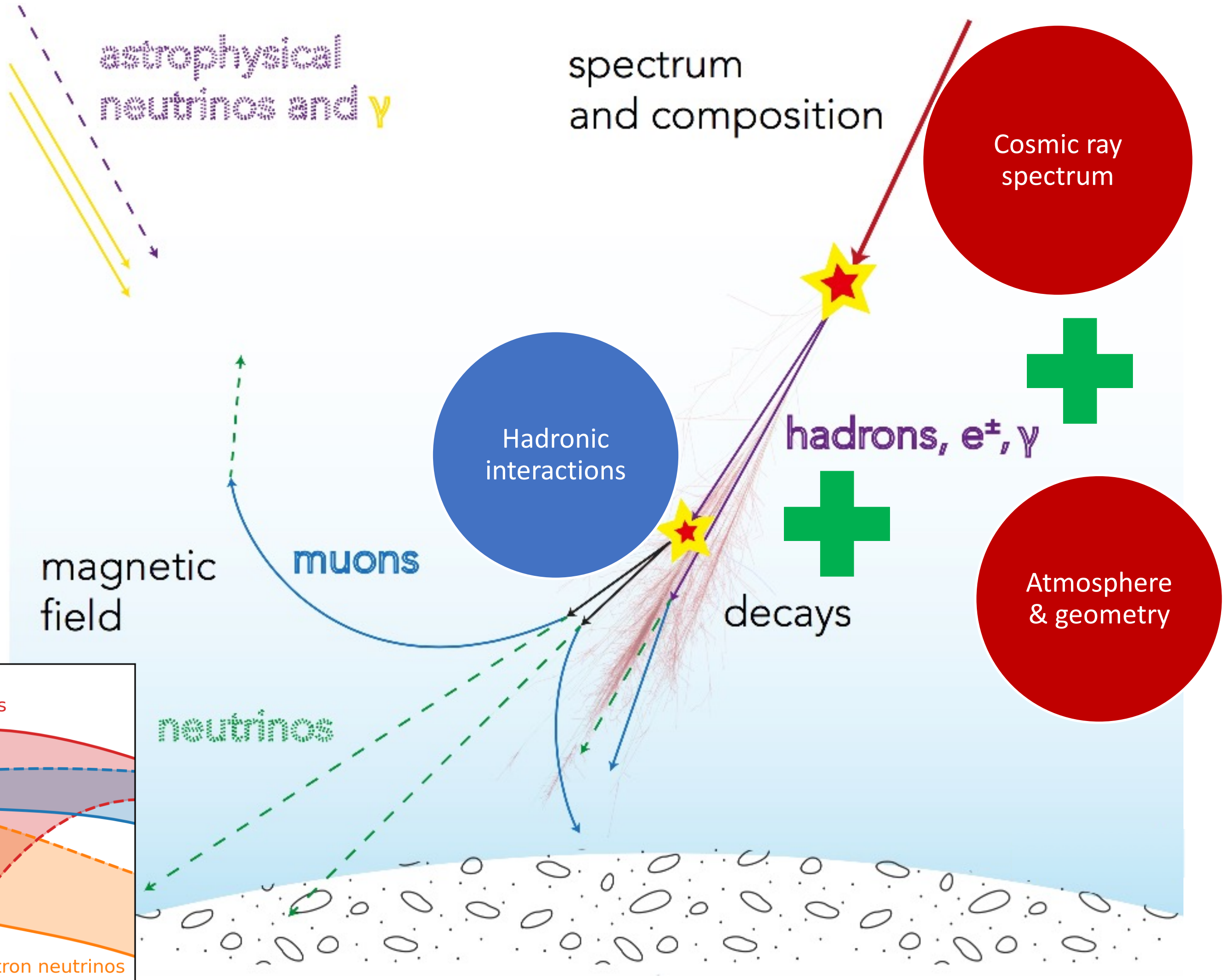
ISVHECRI, Puerto Valarta, MEXICO, 2024/07/12

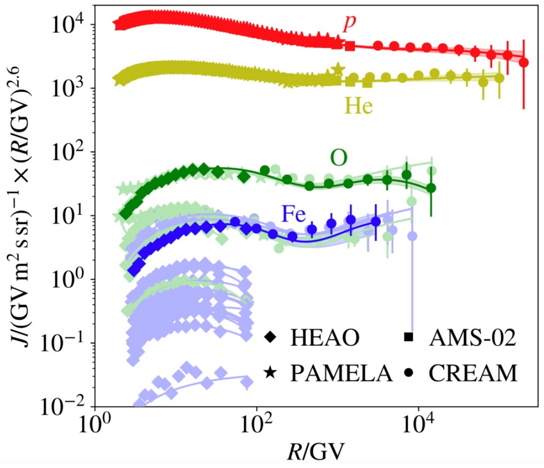
Overview

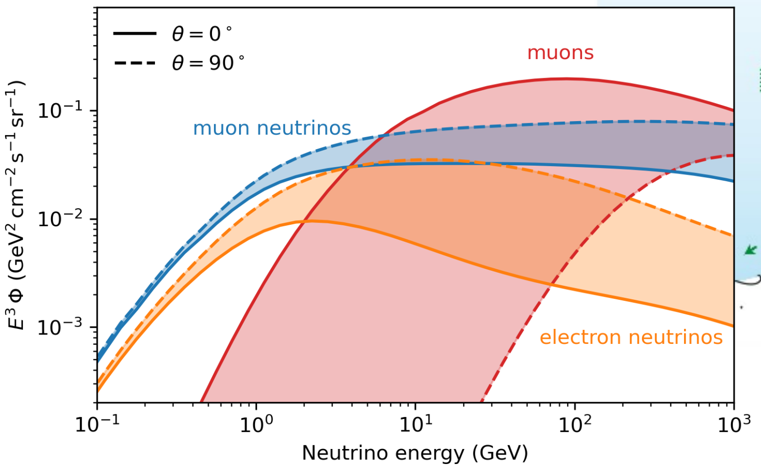
- New applications of the Matrix-Cascade Equations Code (MCEq)
  - **High-precision atmospheric lepton flux model DAEMONFLUX (w/ Juan Pablo Yanez Garza)**
  - **Deep underground muon using MUTE (w/ William Woodley and Marie-Cecile Piro)**
  - 2D MCEq (w/ Tetiana Kozynets and J. Koskinen) (see PRD 108 (2023) or 2306.15263)
- CHROMO: The Cosmic ray and HadROnic interactiOn MOnte-carlo frontend (w/ Anton Prosekin and Hans Dembinski)

Modeling inclusive leptons in the atmosphere

“inclusive” = integrated over CR energy and all other particles at the surface

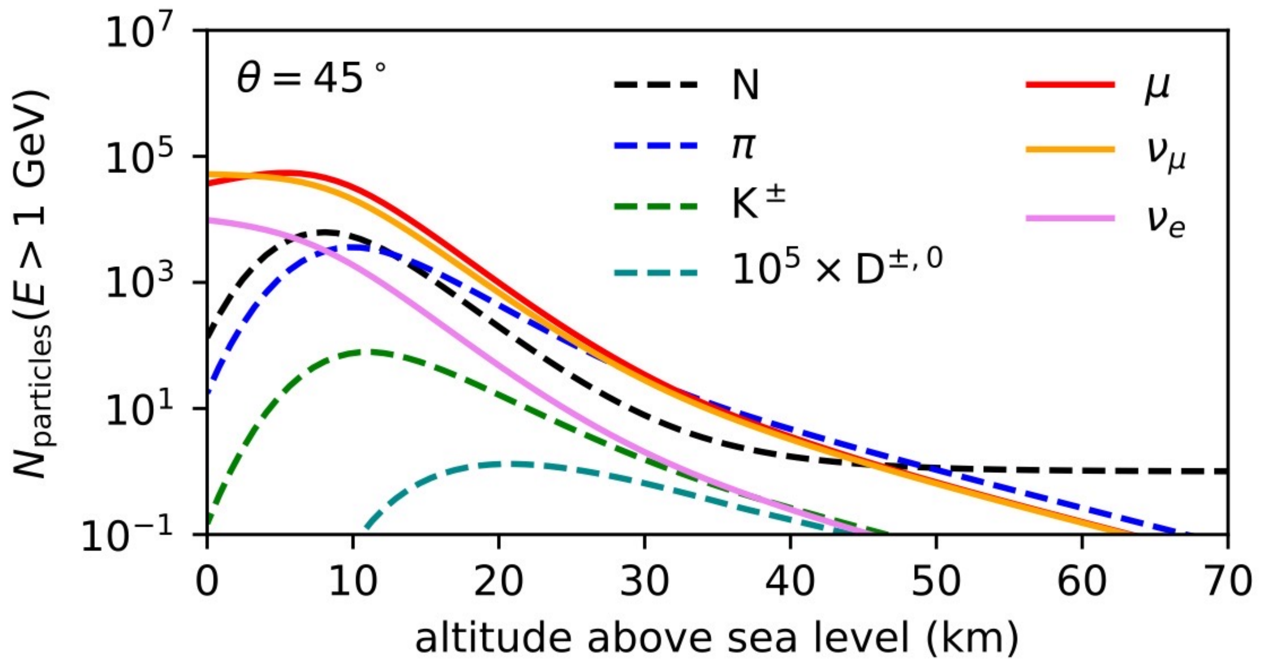






What is MCEq?

1. Open-source iterative cascade equation solver
2. Cascade equations = transport equations (solved by CORSIKA using a Monte Carlo method)
3. Mainly used in atmospheric lepton and neutrino telescope community
4. Potentially interesting for
   - Atmospheric leptons > 1 GeV
   - Underground muons
   - Cascade eqn. solver in CORSIKA8
   - Air shower & cosmic ray “theory”
   - Beyond standard model/Pheno
   - Astrophysics



Transport equations (hadronic cascade equations) in 1D

System of coupled non-linear PDE for each particle species *h* :

$$\frac{\mathrm{d}\Phi_h(E,X)}{\mathrm{d}X} = -\frac{\Phi_h(E,X)}{\lambda_{\mathrm{int},h}(E)} - \frac{\Phi_h(E,X)}{\lambda_{\mathrm{dec},h}(E,X)} - \frac{\partial}{\partial E}(\mu(E)\Phi_h(E,X)) + \sum_k \int_E^\infty \mathrm{d}E_k \frac{\mathrm{d}N_{k(E_k)\to h(E)}}{\mathrm{d}E}\frac{\Phi_k(E_k,X)}{\lambda_{\mathrm{int},k}(E_k)} + \sum_k \int_E^\infty \mathrm{d}E_k \frac{\mathrm{d}N^{\mathrm{dec}}_{k(E_k)\to h(E)}}{\mathrm{d}E}\frac{\Phi_k(E_k,X)}{\lambda_{\mathrm{dec},k}(E_k,X)}$$

cosmic ray physics

Interactions with air

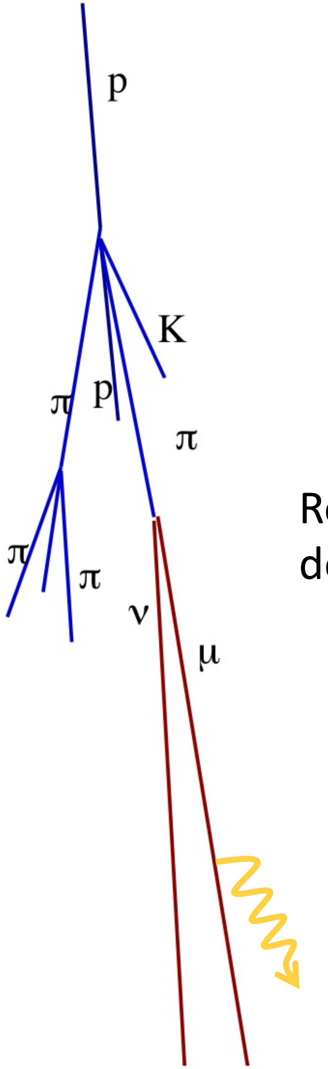
Decays

atmospheric physics

Continuous losses

particle physics

Re-injection from interactions

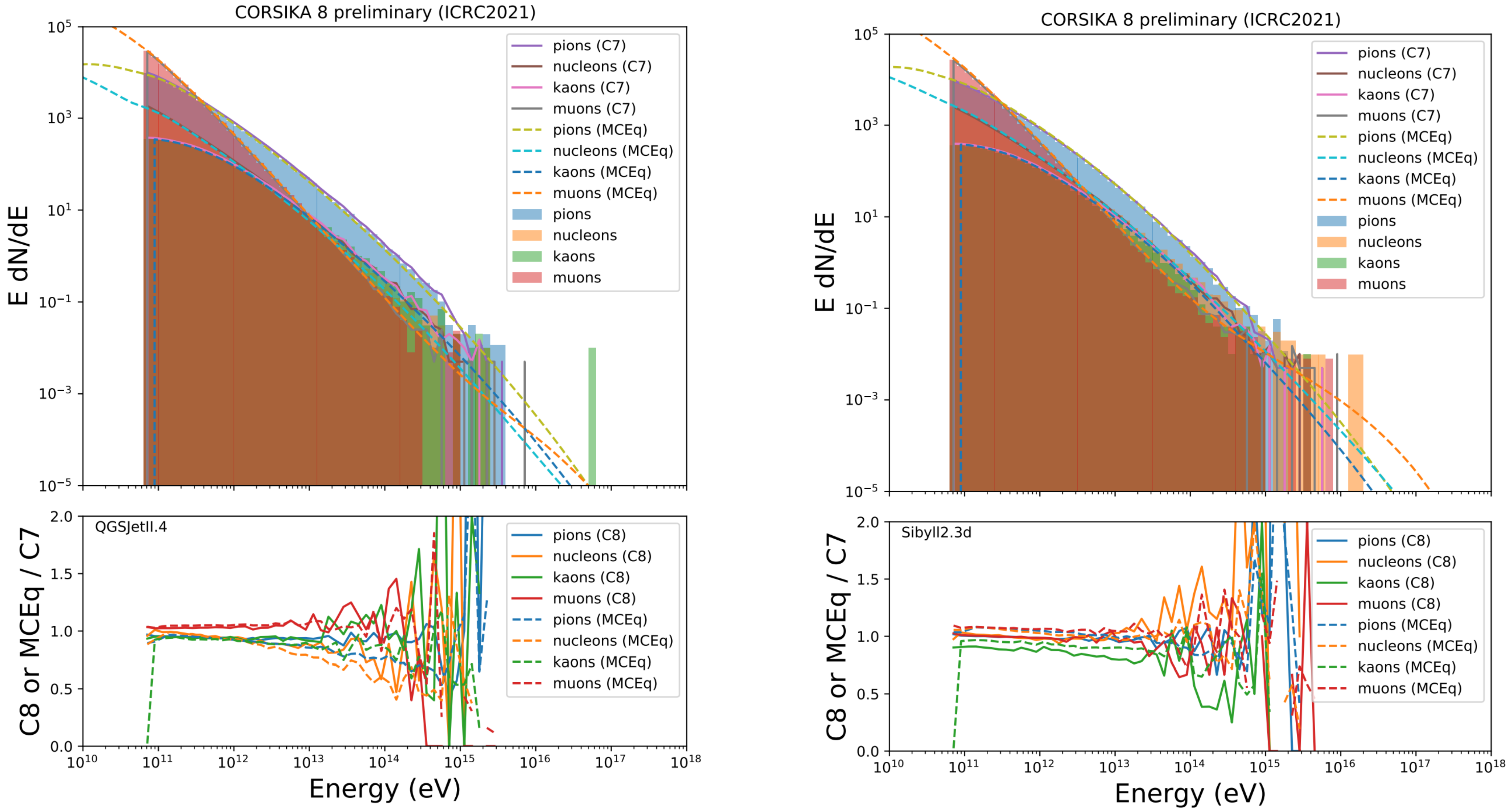


Re-injection from decays

$$X(h_0) = \int_0^{h_0} \mathrm{d}\ell\ \rho_{\mathrm{air}}(\ell)$$

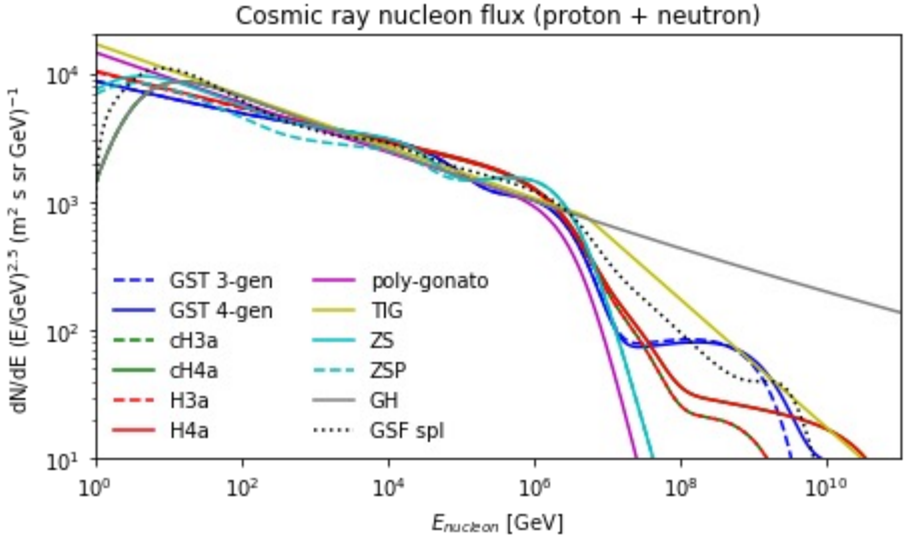
MCEq vs CORSIKA8 particle spectrum (for average air shower)

R. Ulrich et al. for C8 Coll.
PoS(ICRC 2021) 474



Available models

Cosmic ray flux models distributed in an independent crflux module.



Hadronic interaction models are:

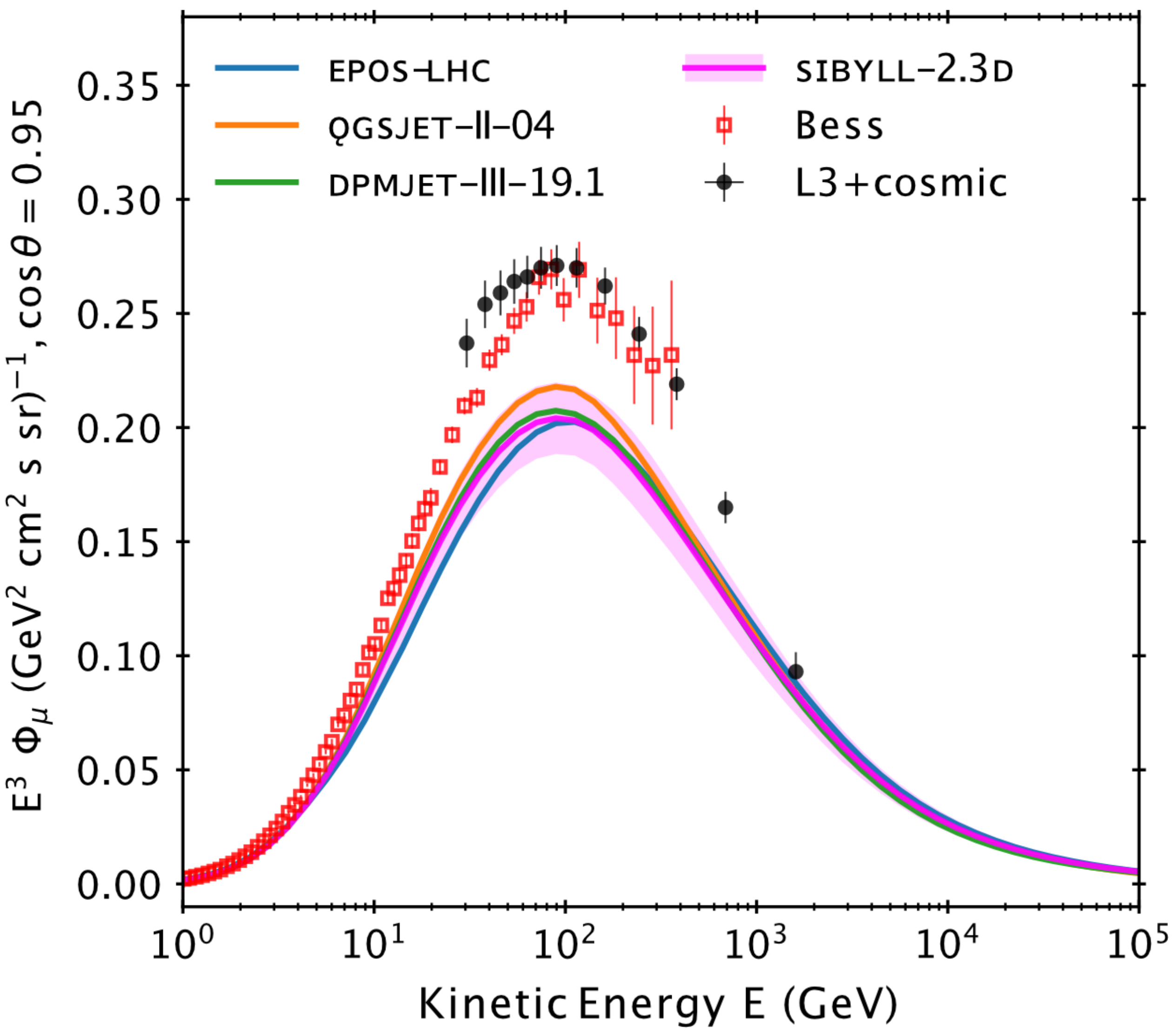
- SIBYLL*
- SIBYLL-2.3c/d + 2.1
- EPOS-LHC
- QGSJet-II-03/-04
- QGSJet-01c
- DPMJET-III-3.0.6
- DPMJET-III-19.1/-3
- FLUKA (work in progress)
- UrQMD (not public)
- Pythia 8 (not public)

Atmosphere models from

- CORSIKA7 (multiple locations)
- NRLMSISE-00 (global, “static”)
- Some special cases and interface to tabulated atm.

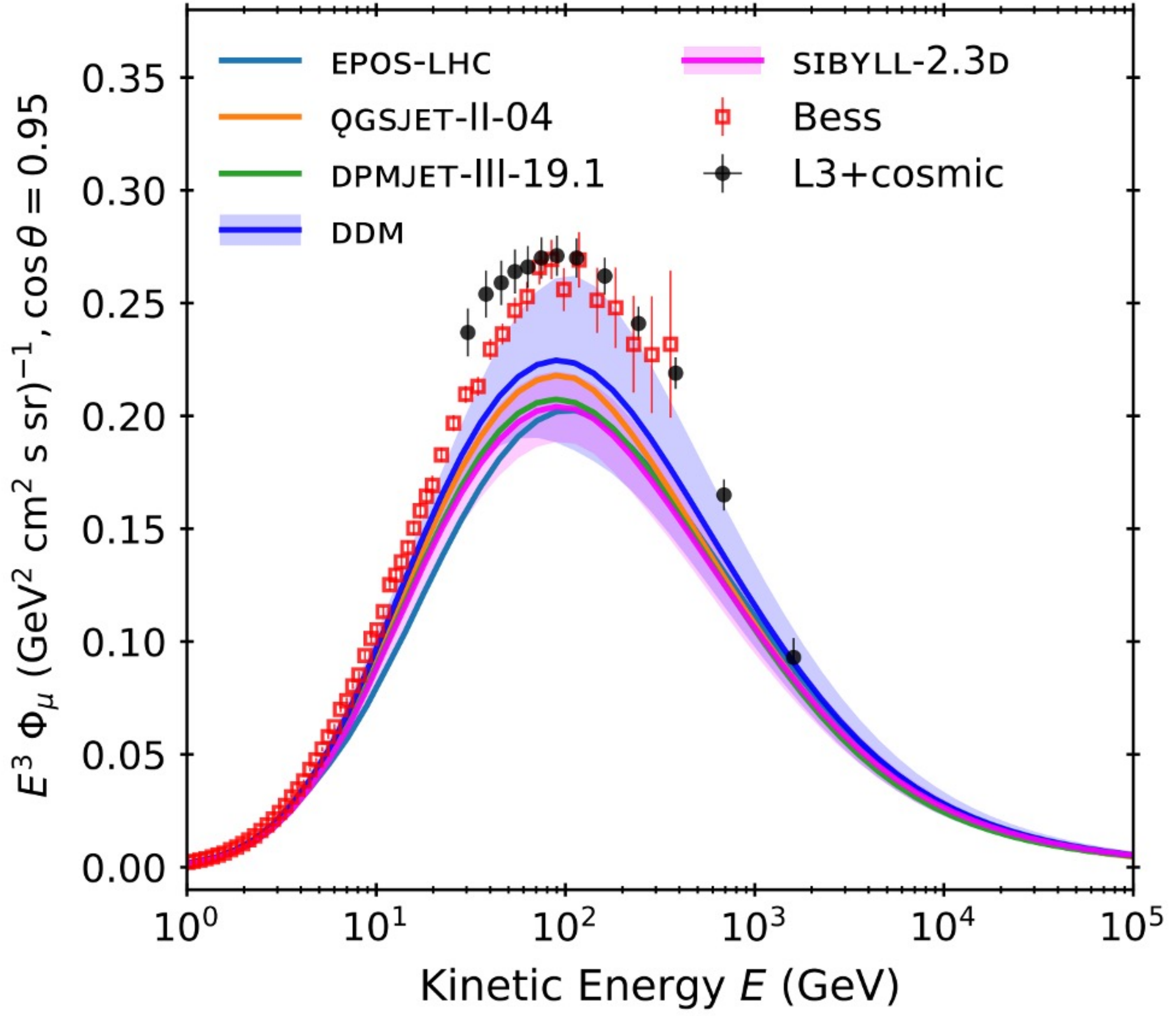
> BSD licensed @ https://github.com/mceq-project/MCEq

The comparisons with surface muon fluxes were known to undershoot data



The comparisons with surface muon fluxes were known to undershoot data

AF, Woodley, Piro, ApJ 2022 arXiv:2109.11559



…even when building a hadronic interaction model directly from NA49/61 data (DDM)

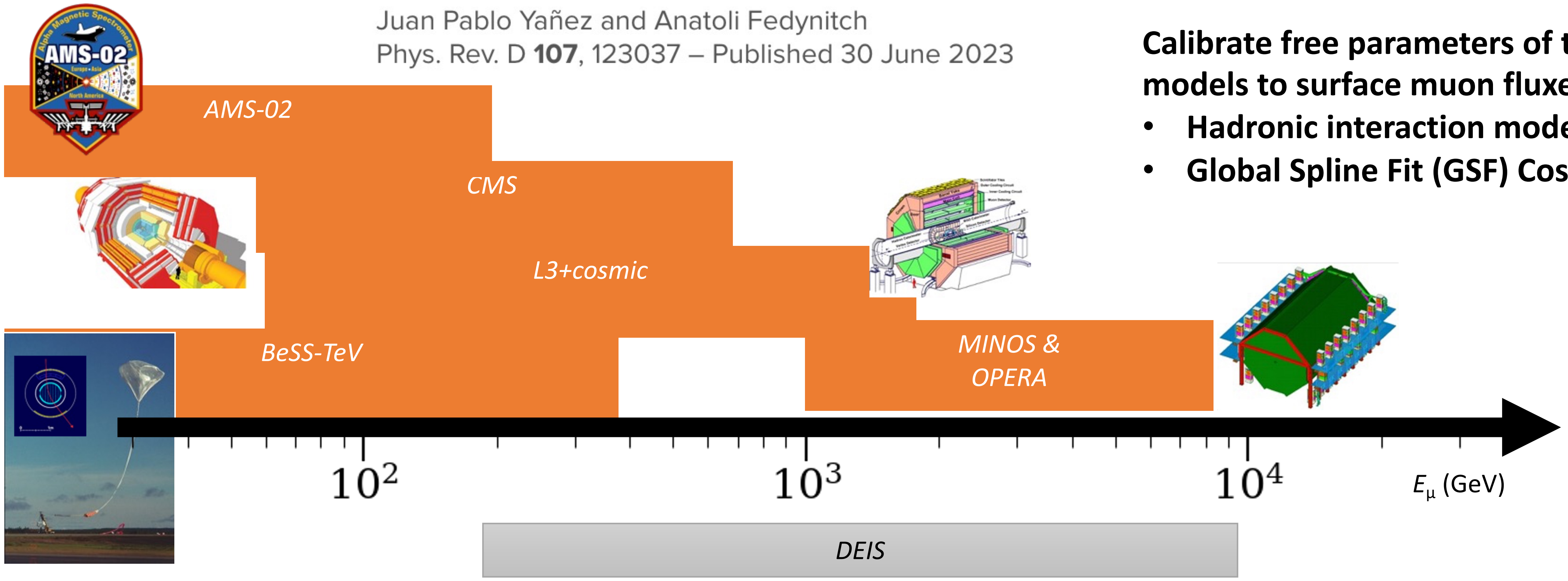
See AF & M. Huber, PRD, arXiv:2205.14766

`daemonflux`: Data-driven muon-calibrated neutrino flux

Juan Pablo Yañez and Anatoli Fedynitch
Phys. Rev. D **107**, 123037 – Published 30 June 2023

Calibrate free parameters of the data-driven models to surface muon fluxes:

- **Hadronic interaction model DDM**
- **Global Spline Fit (GSF) Cosmic Ray Flux**



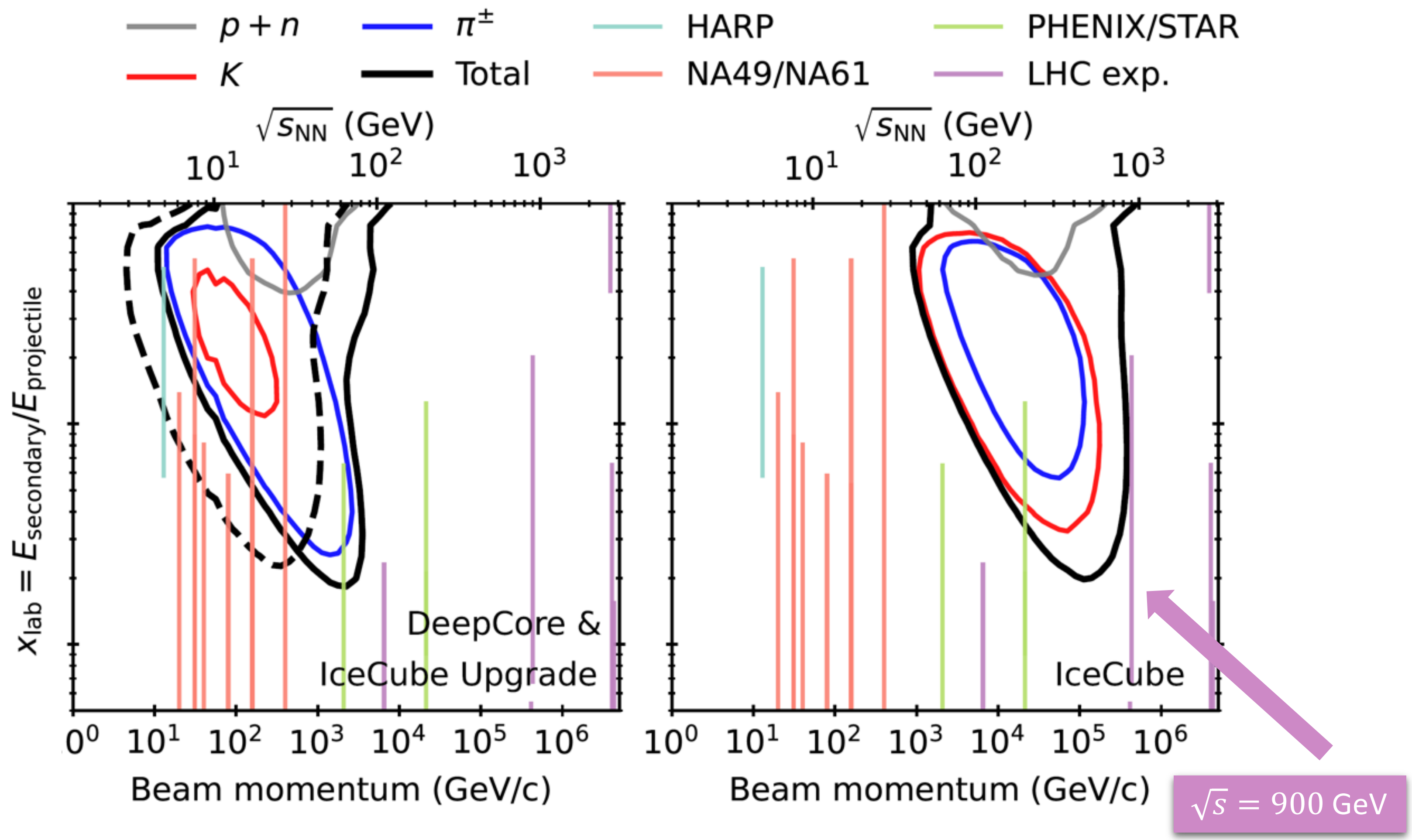
Experiment	Energy (GeV)	Measurements	Unit	Systematics	Location	Altitude	Zenith range
BESS-TeV [44]	0.6-400	Φ_μ	p_μ	C	36.2°N, 140.1°W	30 m	0-25.8°
CMS [45]	5-1000	R_{μ^+/μ^-}	p_μ	Q	46.31°N, 6.071°E	420 m	$p\cos\theta_z$
L3+C [46]	20-3000	Φ_μ,R_{μ^+/μ^-}	p_μ	C	46.25°N, 6.02°E	450 m	0-58°
DEIS [47]	5-10000	Φ_μ	p_μ	Q	32.11°N, 34.80°E	5 m	78.1-90°
MUTRON [48]	80-10000	R_{μ^+/μ^-}	p_μ	Q	35.67°N, 139.70°E	5 m	87-90°
MINOS [49]	1000-7000	R_{μ^+/μ^-}	E_μ	C	47.82°N, 92.24°W	5 m	unfolded
OPERA [50]	891-7079	R_{μ^+/μ^-}	E_μ	Q	42.42°N, 13.51°E	5 m	$E\cos\theta^*$

See also Albrecht et al., Muon Puzzle review, 2105.06148

Hadron production phase space seen by neutrino detectors

AF & M. Huber, arXiv:2205.14766

- Oscillation target energies covered by data from fixed target experiments
- IceCube energies not well covered by accelerator data
- LHC energies too high
- Shared production phase-space for parent mesons of muons and neutrinos
- Optimizal description of atm. Muon data → imporved atm. neutrinos



DeepCore :
tracks, E_{reco} < 60 GeV (osc.)

IceCube Northern Tracks
(muon neutrinos)

Contours = 90% of ν_μ events

See also Albrecht et al., Muon Puzzle review, 2105.06148

AF & M. Huber, arXiv:2205.14766

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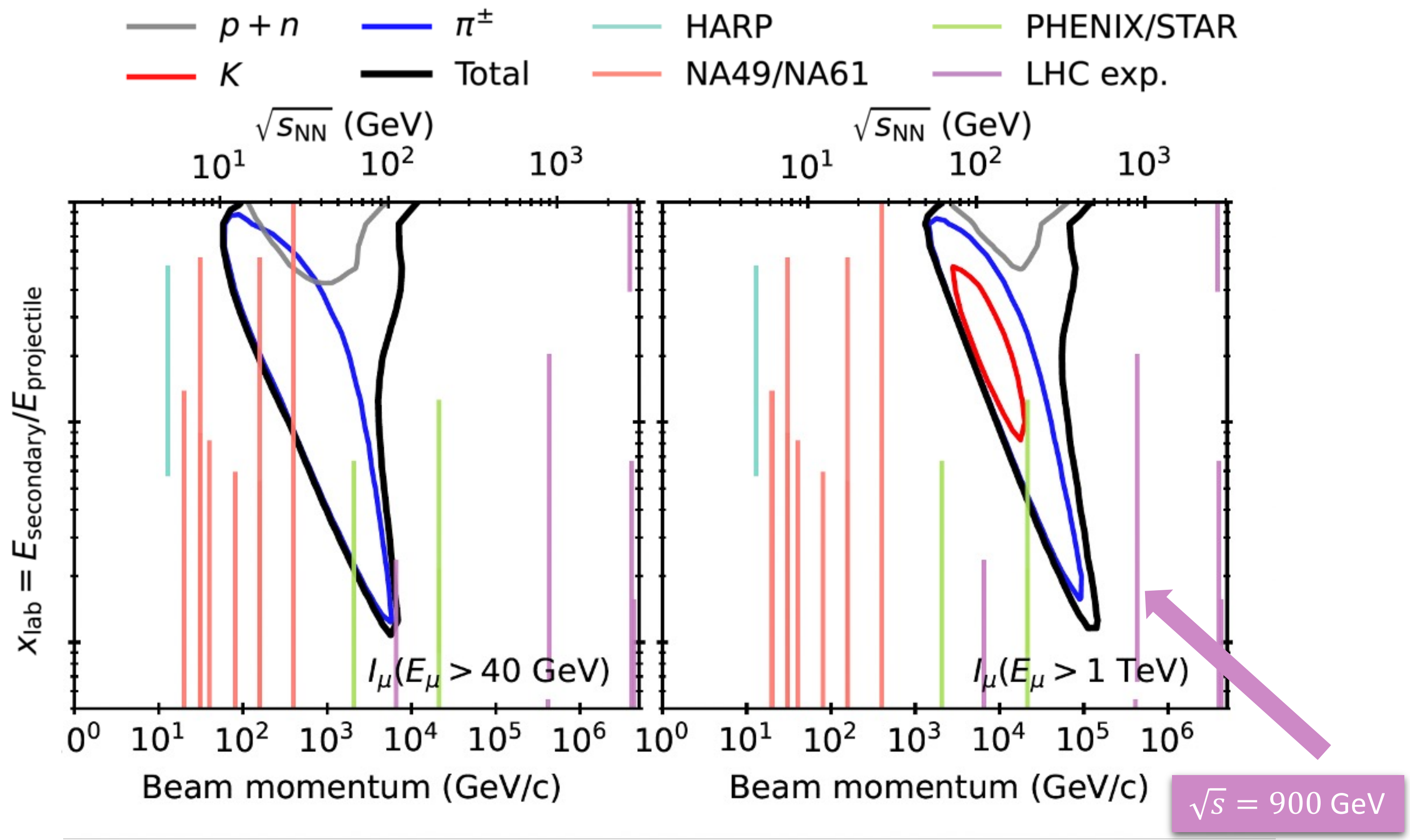
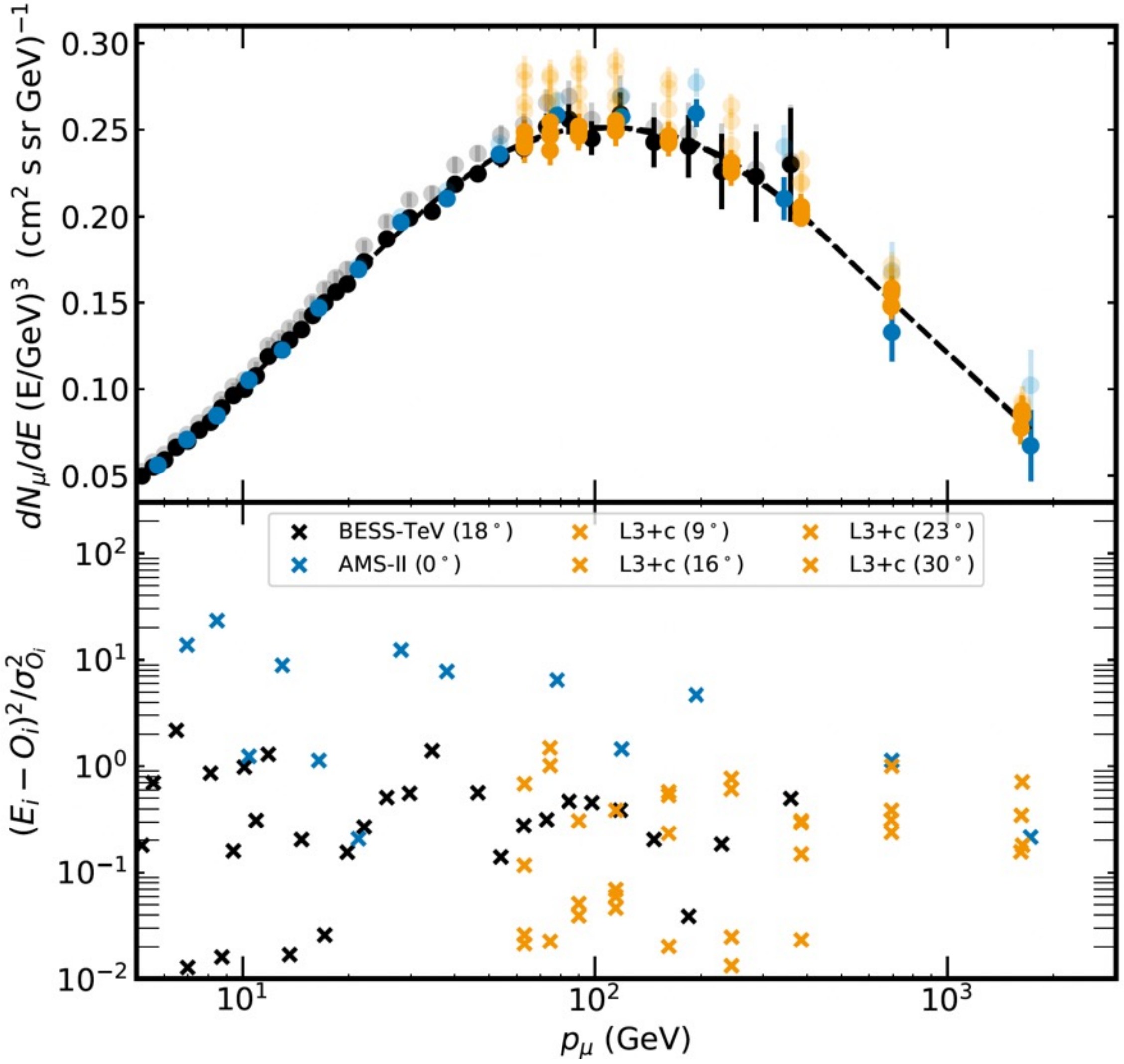


Integrated muon flux at the surface: E > 40 GeV

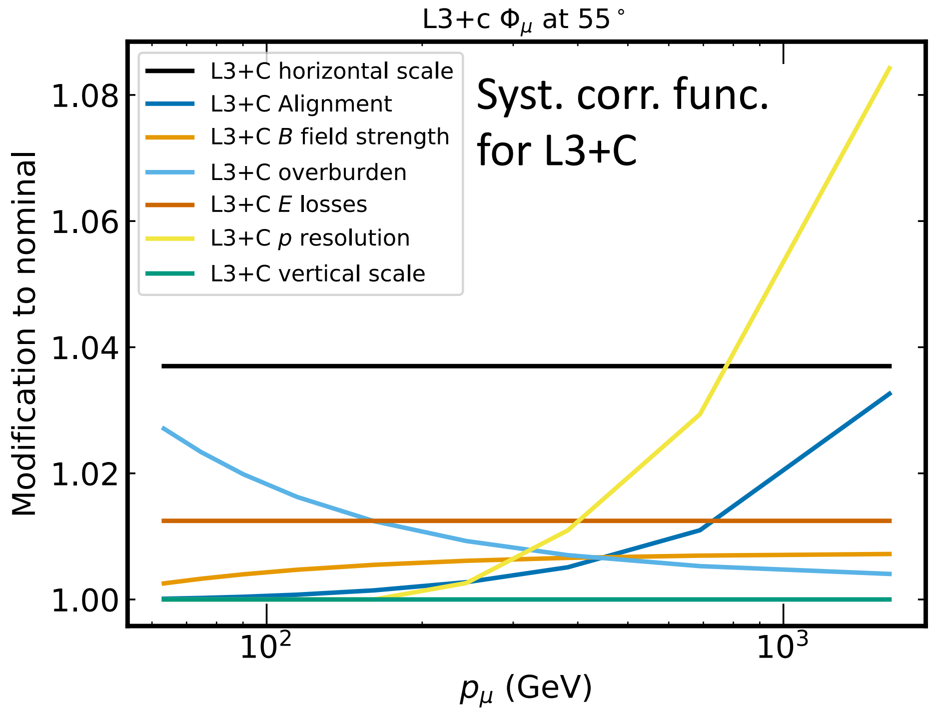
Integrated muon flux at the surface: E > 1 TeV

Contours = 90% of ν_μ events

Select data & test compatibility

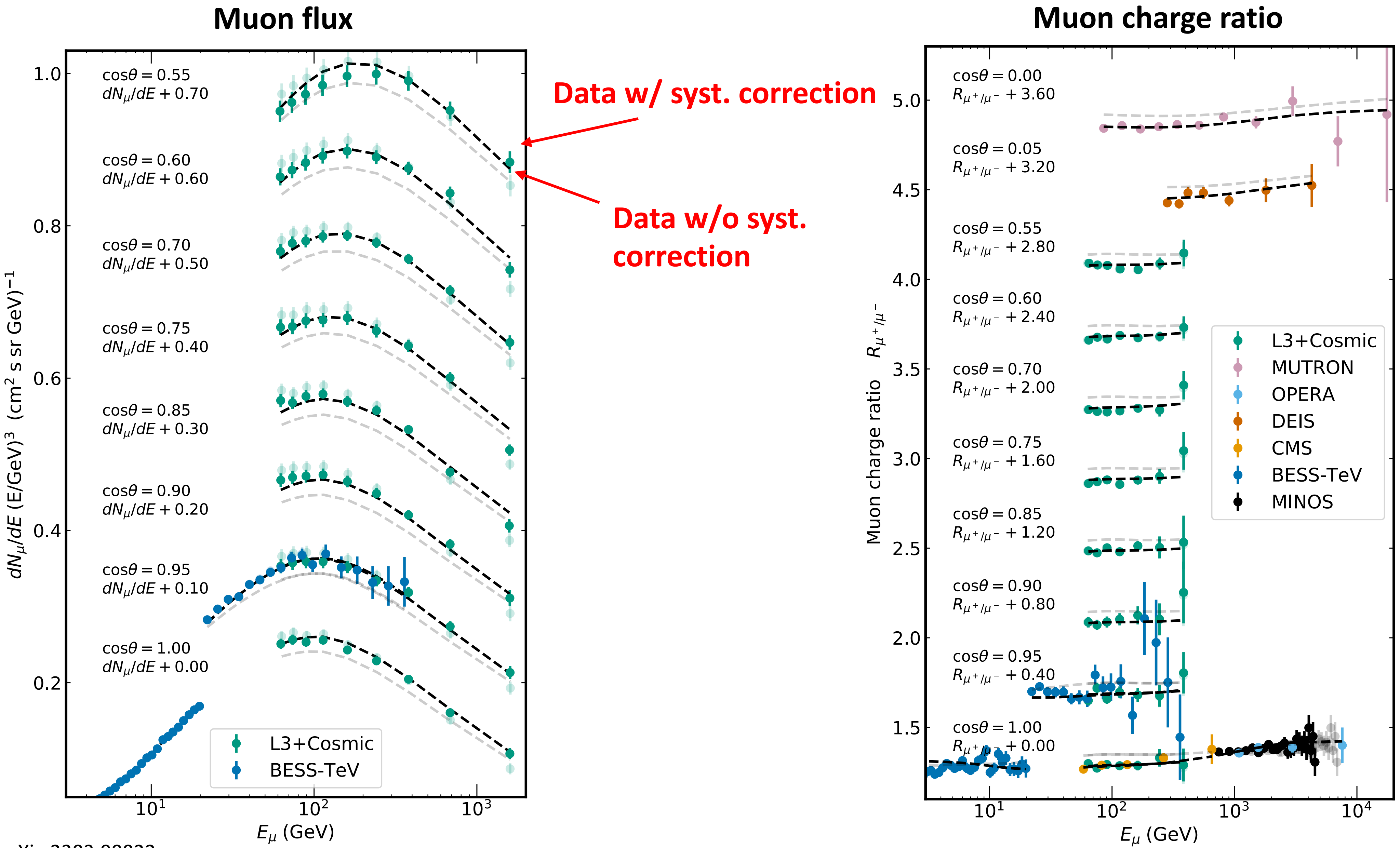


- Fit spline in common zenith band with the only requirement that flux has to be smooth. Fit systematic corrections.



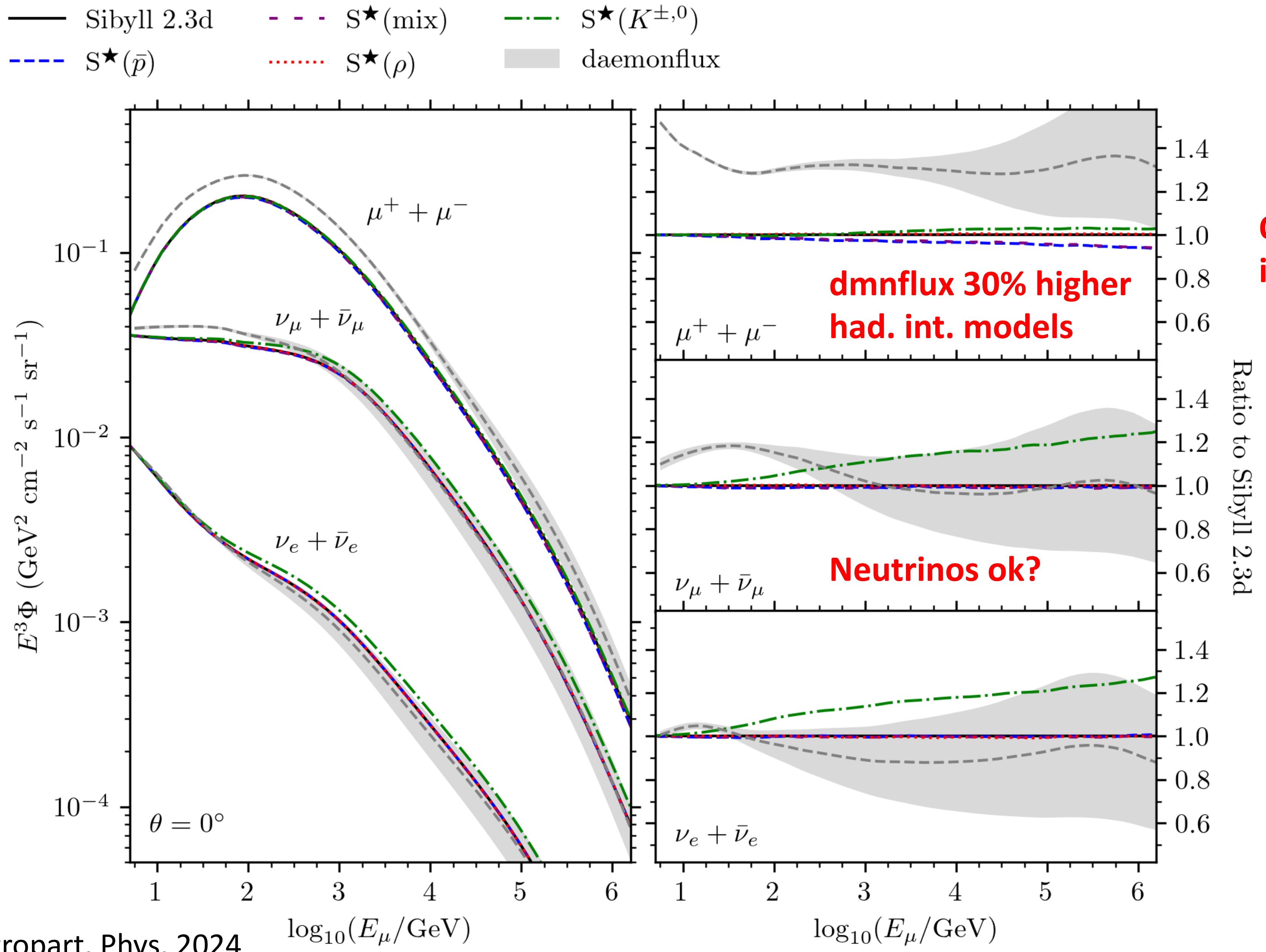
- **Exclude experiment**s, which either are
  - not mutually compatible, or
  - statistically not significant
- or
  - AMS (unpublished PhD thesis)
  - MARS (no competition to BESS)
  - MUTRON (unclear systematics)
  - DEIS (formally OK, but induces strange pulls)

Resulting muon fluxes and cross-calibrated data



J. P. Yanez & AF, arXiv:2303.00022

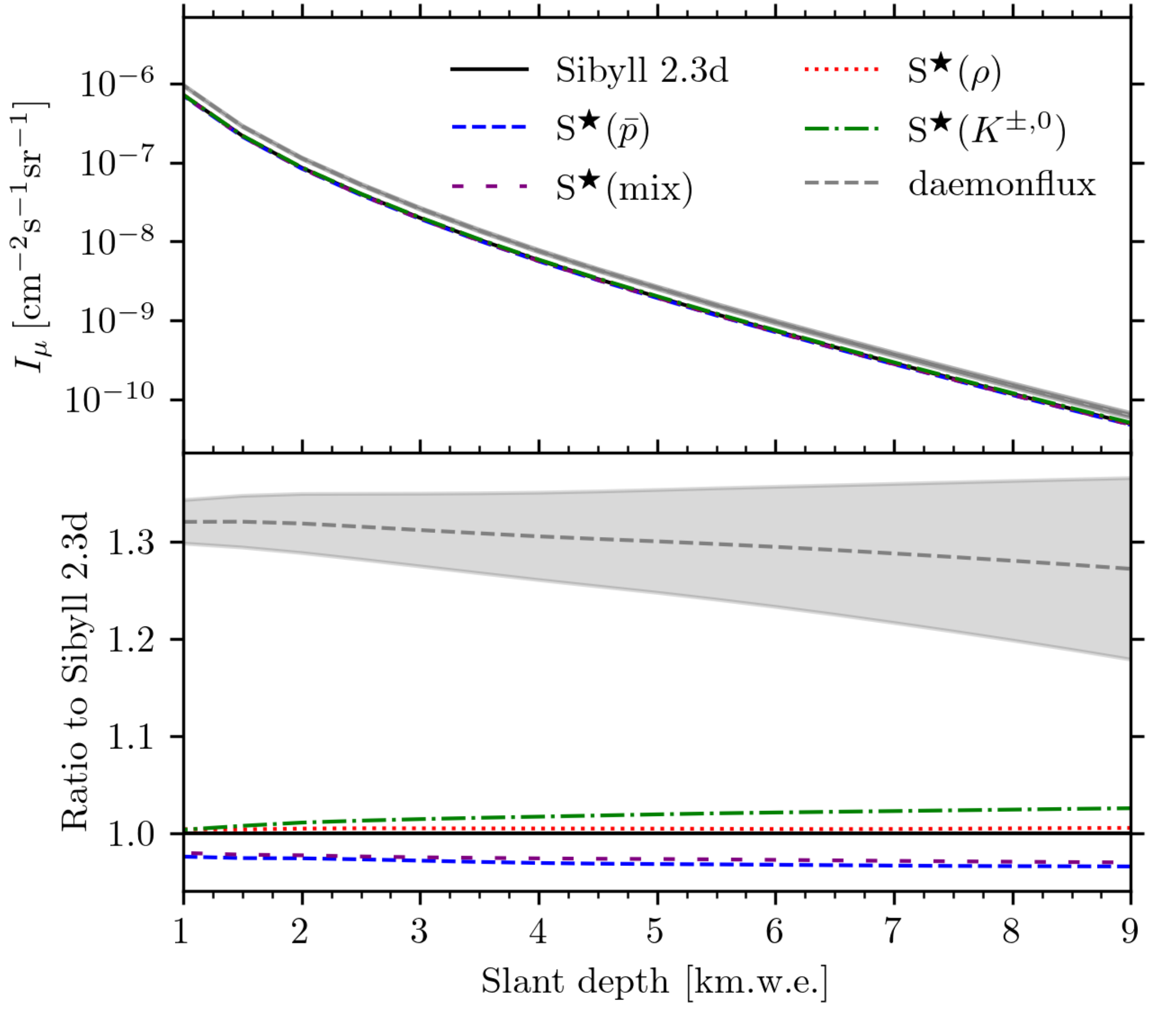
SIBYLL* vs data-driven muon-calibrated model (daemonflux)



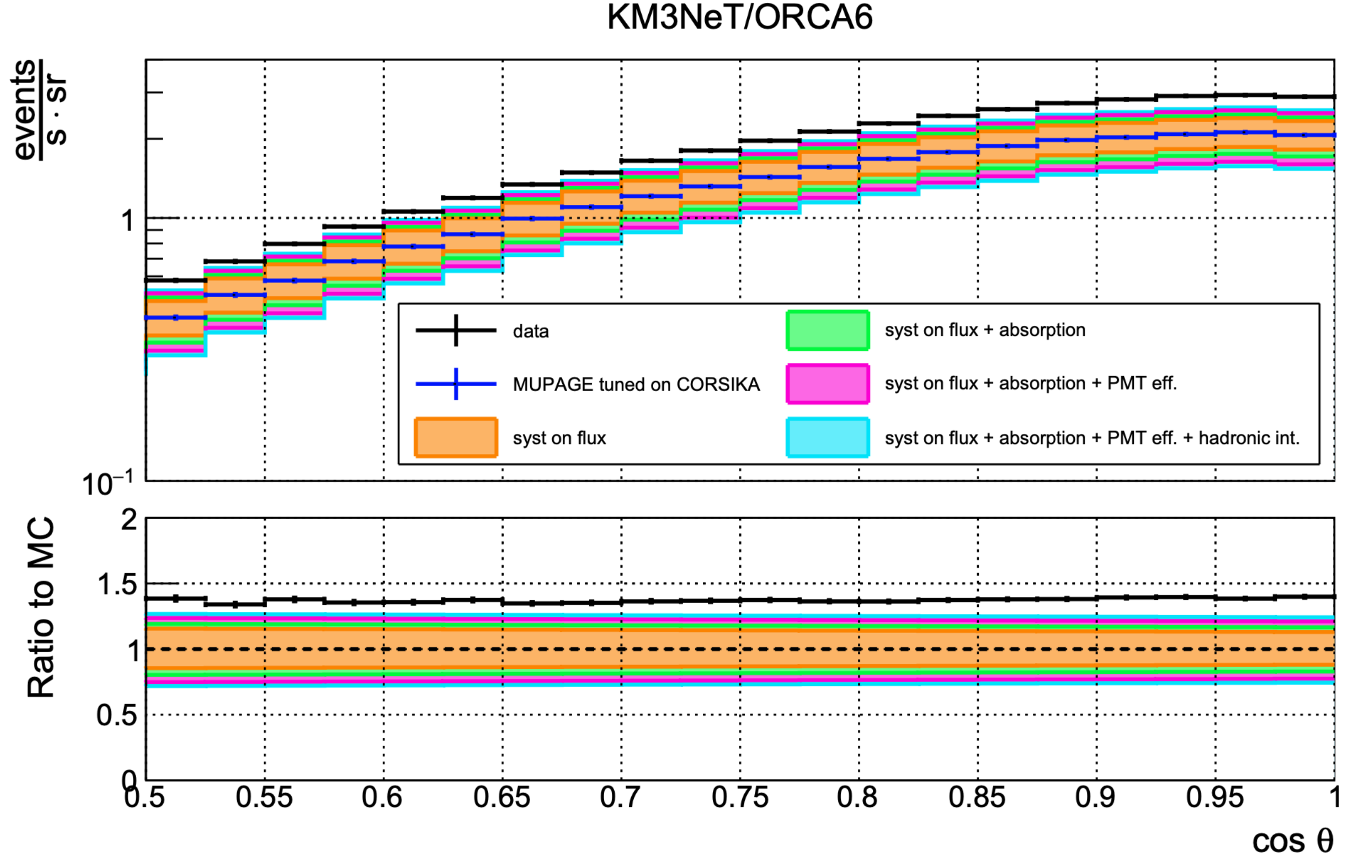
Other IA models in S* ballpark

F. Riehn, AF, R. Engel, Astropart. Phys. 2024

Daemonflux vs models underground and underwater data



A. Romanov et al. (KM3NeT), PoS(ICRC2023) 338 & 2403.11946



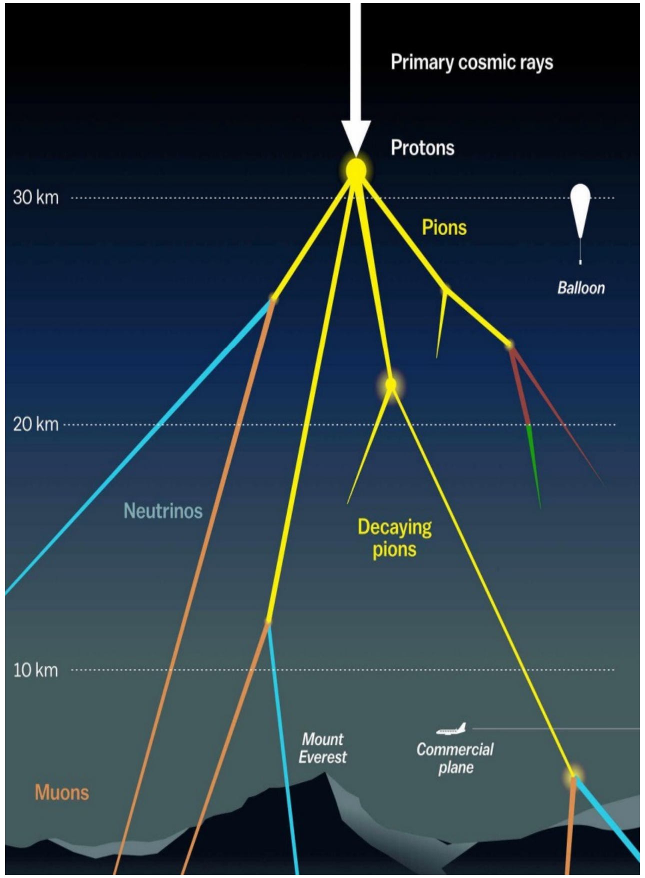
KM3NeT MC tuned to SIBYLL2.3c + GSF using CORSIKA

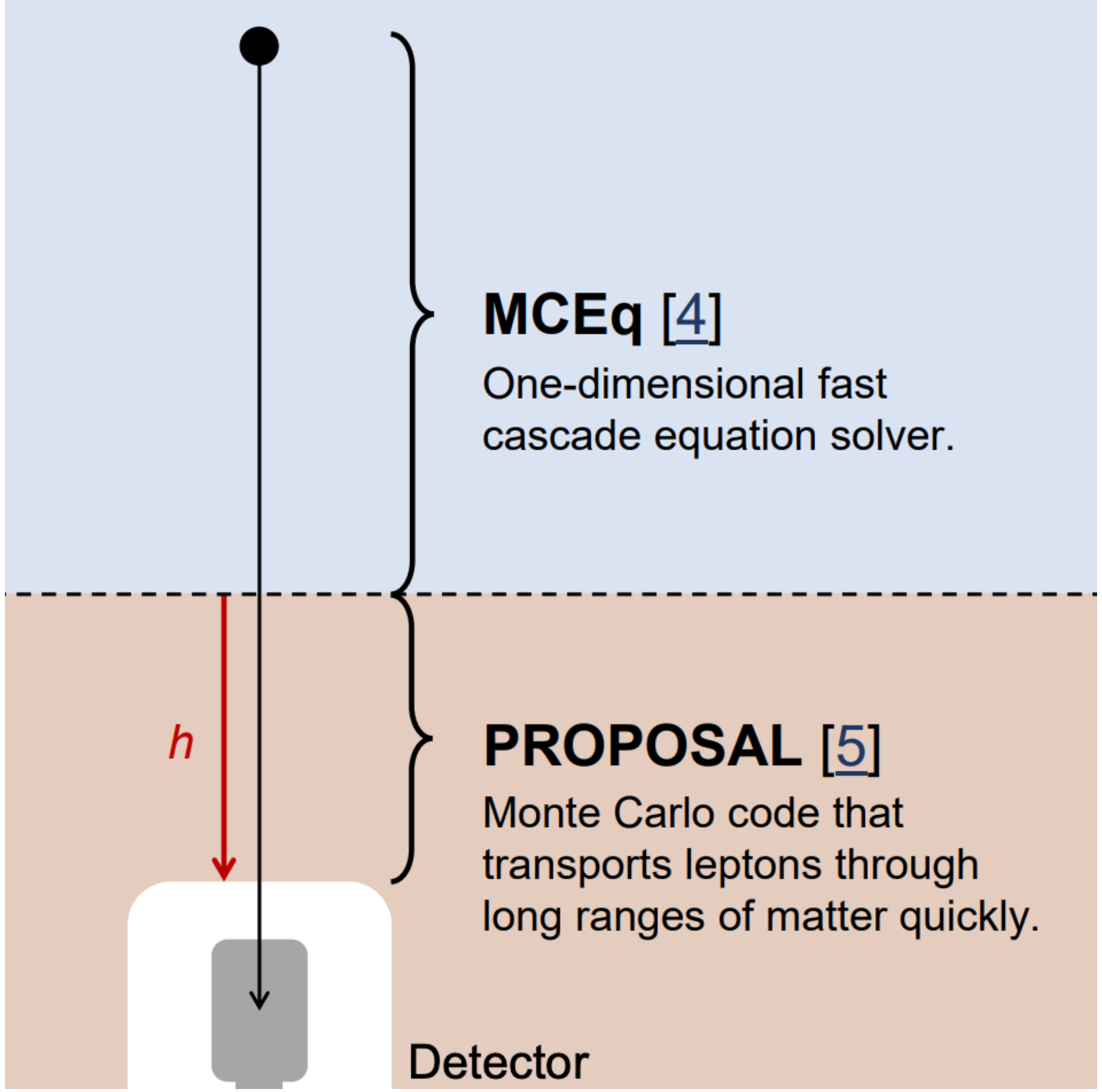
> 30% discrepancy!

F. Riehn, AF, R. Engel, Astropart. Phys. 2024

High energy constraints from underground μ?

W. Woodley (UofA), TeVPa 2022





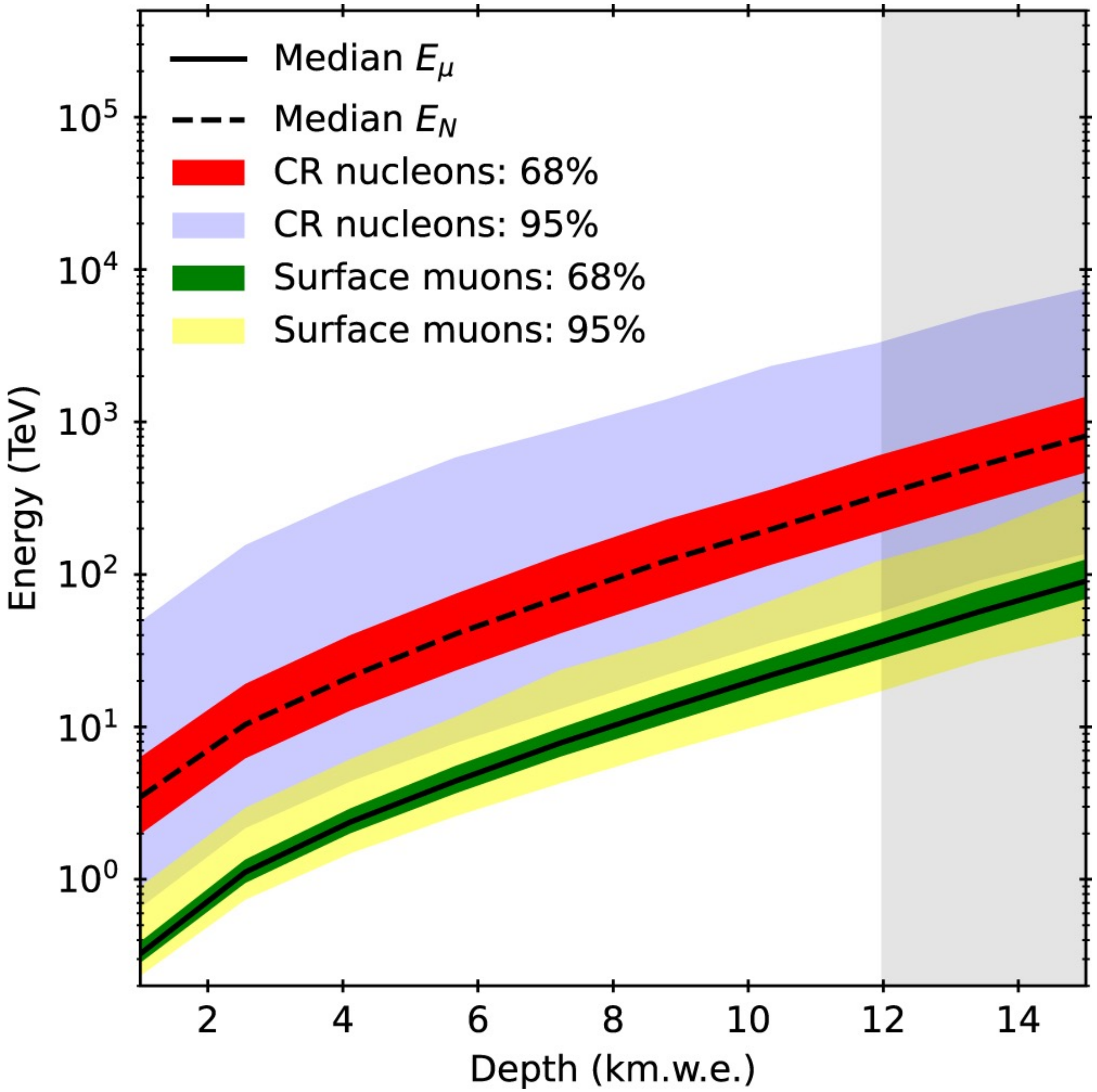
MUTE (Muon inTnsity codE)

https://github.com/wjwoodley/mute

AF, **W. Woodley**, M.-C. Piro, *ApJ* 928 27 (2022)

W. Woodley, AF, M.C. Piro, submitted to PRD, (2024) 2406.10339

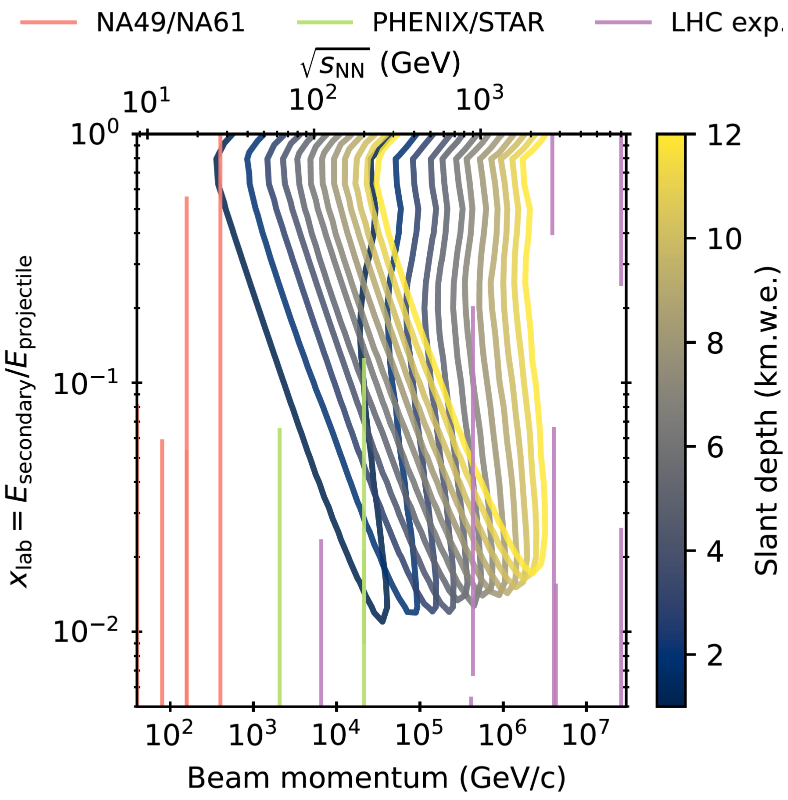
Relation of depth to surface and CR energy

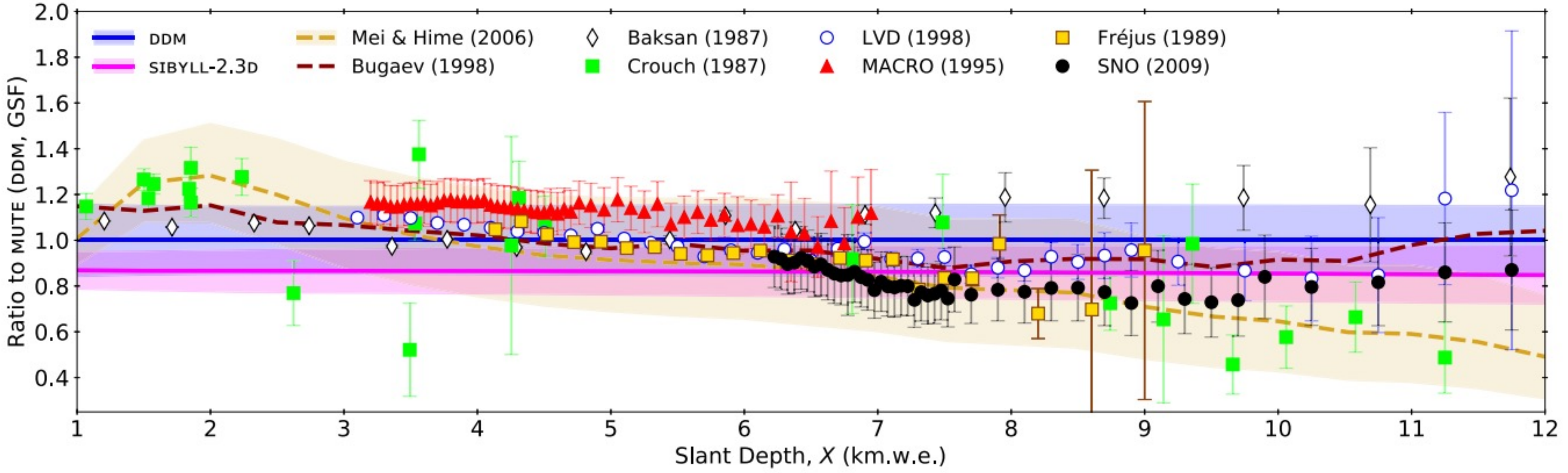


AF, W. Woodley, M.-C. Piro, *ApJ* **928** 27 (2022)

MUTE v1: Studying vertical-equivalent muon intensities

AF, **W. Woodley**, M.-C. Piro, *ApJ* **928** 27 (2022)

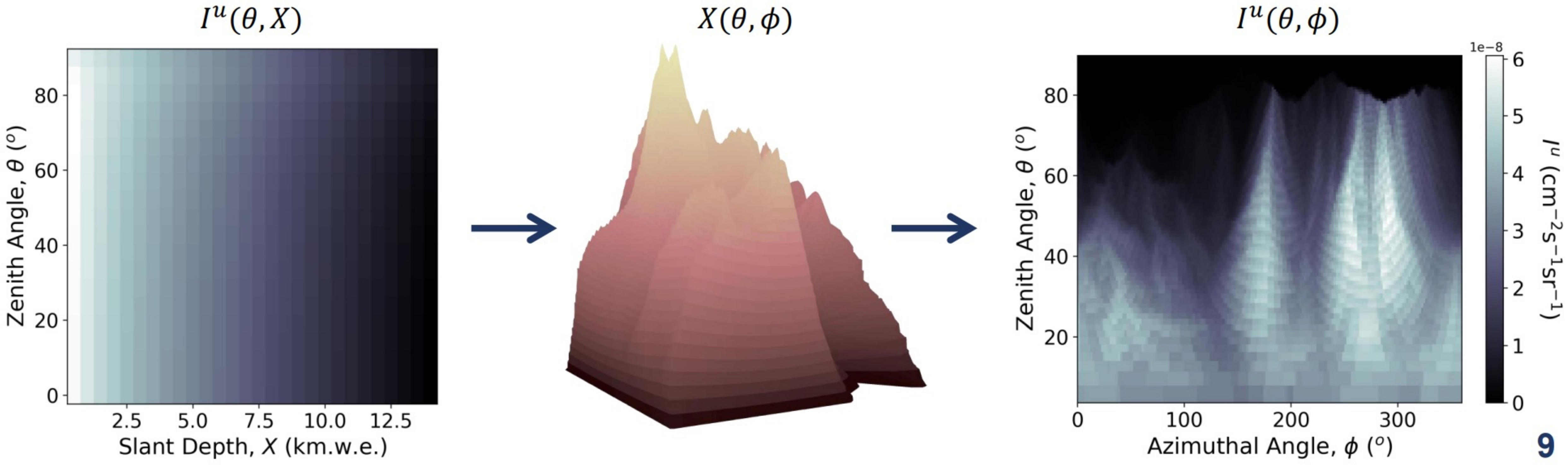




- Calculate vertical equivalent rate using MCEq + PROPOSAL using a convolution method
- Use vertical-equivalent data from underground labs
- Labs under mountains correct or “unfold” the data to equivalent depths
- → not very direct measurement

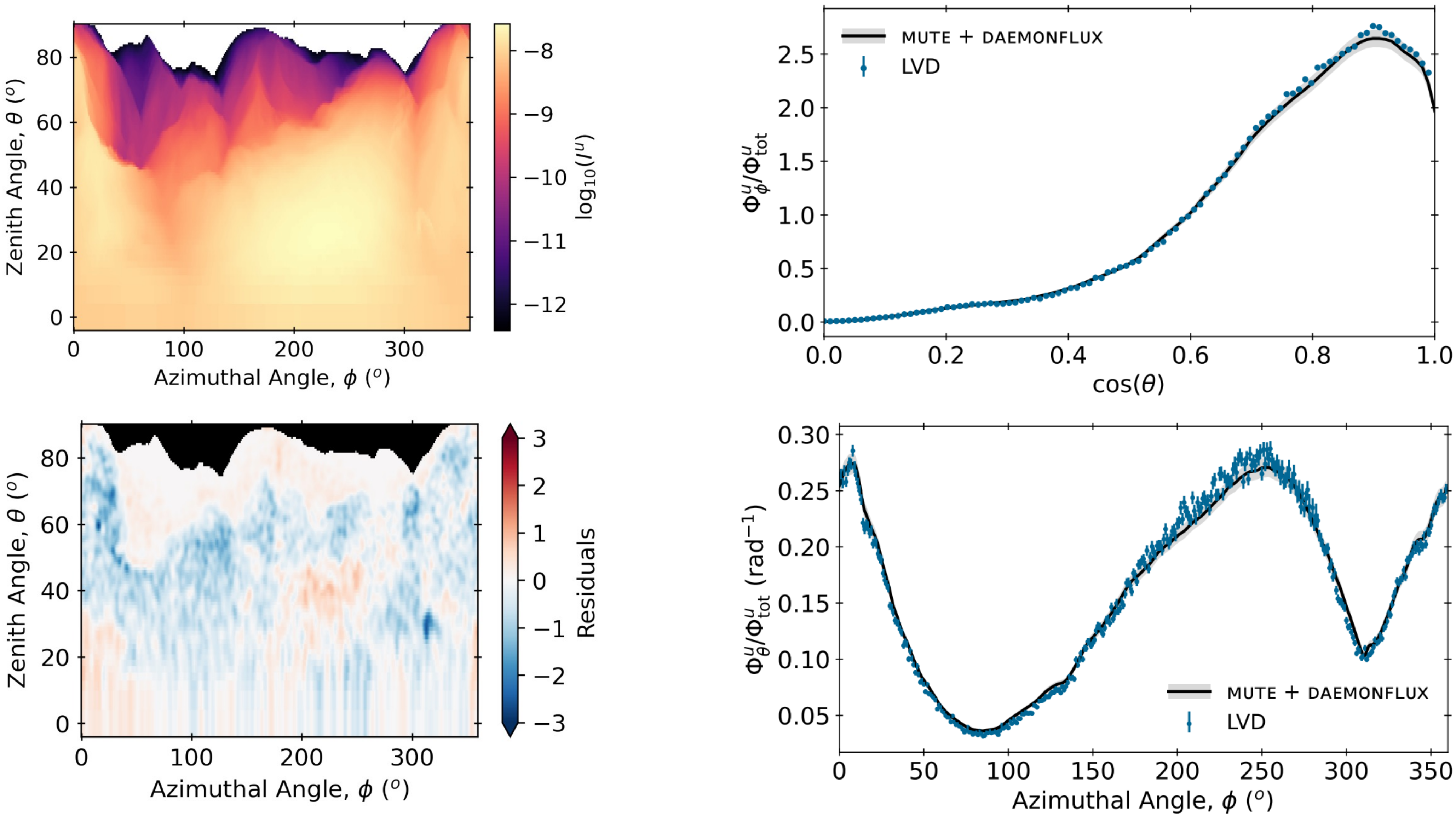
MUTE v2-3: Muon flux for labs under mountains

$$\Phi^u = \iint_\Omega I^u(X(\theta,\phi),\theta)\mathrm{d}\Omega.$$



W. Woodley, AF, M.C. Piro, submitted to PRD, (2024) 2406.10339

Angular dependence in LVD (Gran Sasso)

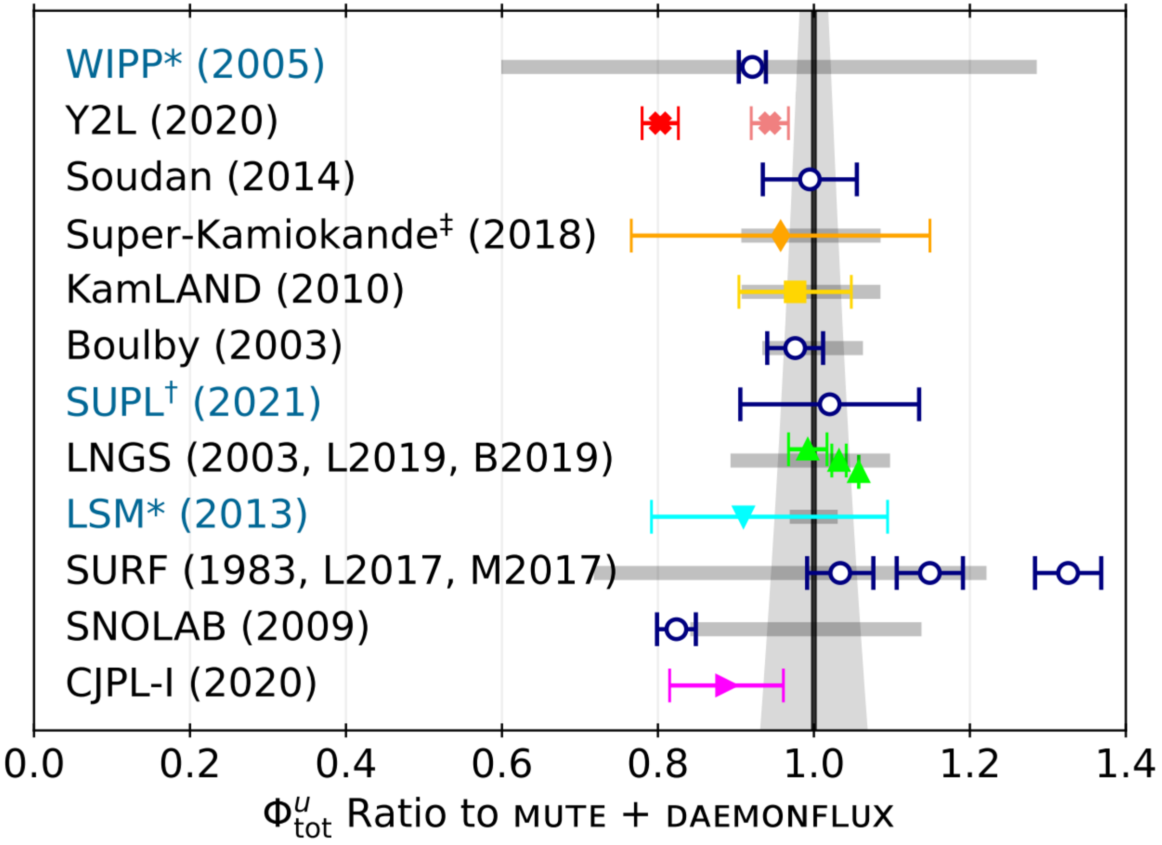


W. Woodley, AF, M.C. Piro, submitted to PRD, (2024) 2406.10339

Calculation of total muon fluxes

- Need to model chemical composition
- Data has to be corrected from multiple muon hits
- Overburden uncertainties complicated (often underestimated)
- Factoring in everything -> daemonflux is compatible < 6 km.w.e
- At large depths (~10-50 TeV @ surface) muon fluxes somewhat overestimated

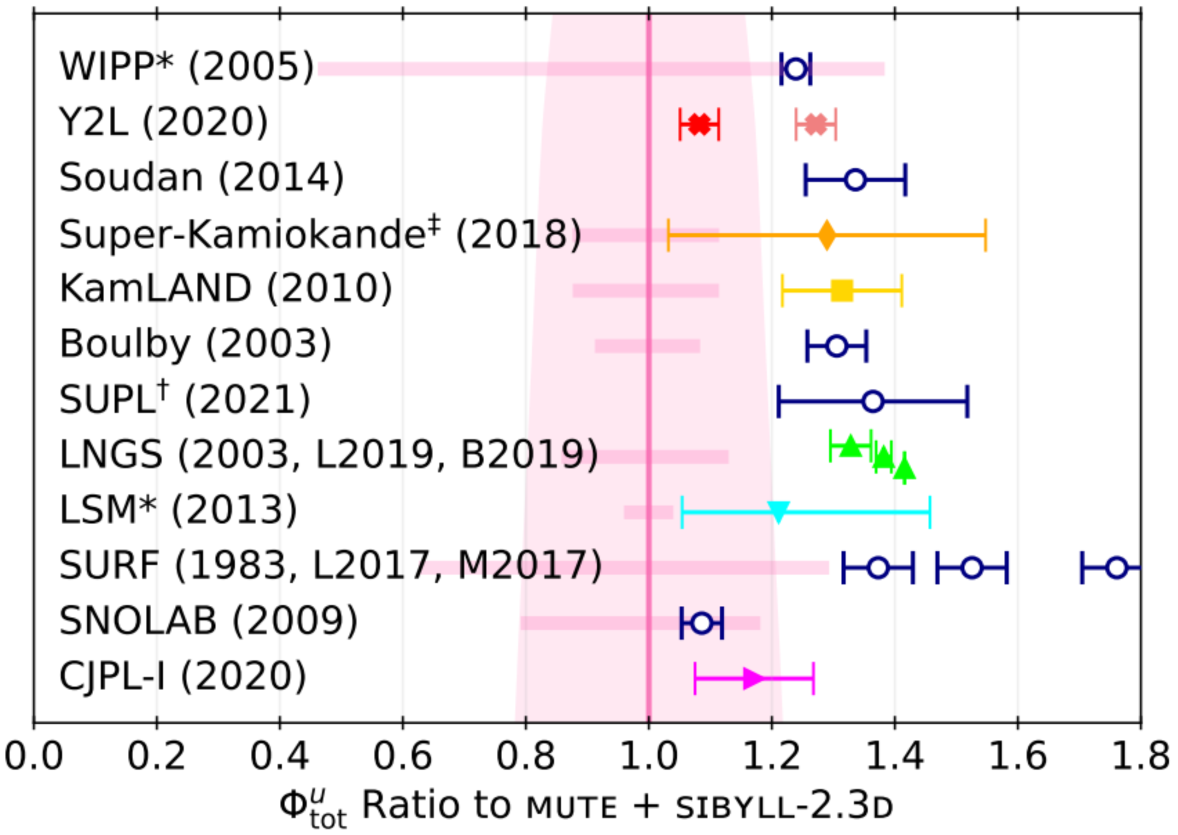
Daemonflux at the surface



W. Woodley, AF, M.C. Piro, submitted to PRD, (2024) 2406.10339

Calculation of total muon fluxes

SIBYLL 2.3d at the surface



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W. Woodley, AF, M.C. Piro, submitted to PRD, (2024) 2406.10339

Quick facts

Cosmic ray and HadROnic interactiOn MOnte-carlo frontend

Open-source code developed together with: **Hans Dembinski, Anton Prosekin** and others

- Python frontend to generators written in Fortran & C++
  - DPMJet-III*, PhoJet*, EPOS-LHC, Pythia-6.4, Pythia-8.3, QGSJet*, QGSJet- II*, SIBYLL*, SOPHIA, UrQMD 3.4 (* = several versions)
  - Use as Python library or command-line interface
- Open source development on Github
  - https://github.com/impy-project/chromo
  - BSD 3-clause license, contributions welcome
- Main authors
  - Anatoli Fedynitch (project lead), Hans Dembinski, Anton Prosekin
- Available on PyPI
  - Authors already use it for science projects
  - pip install chromo to install
  - For installation from source, see README.md

PoS(ICRC2023)189

See for more details A. Prosekin's talk at the "Workshop on the tuning of hadronic interaction models" in Wuppertal

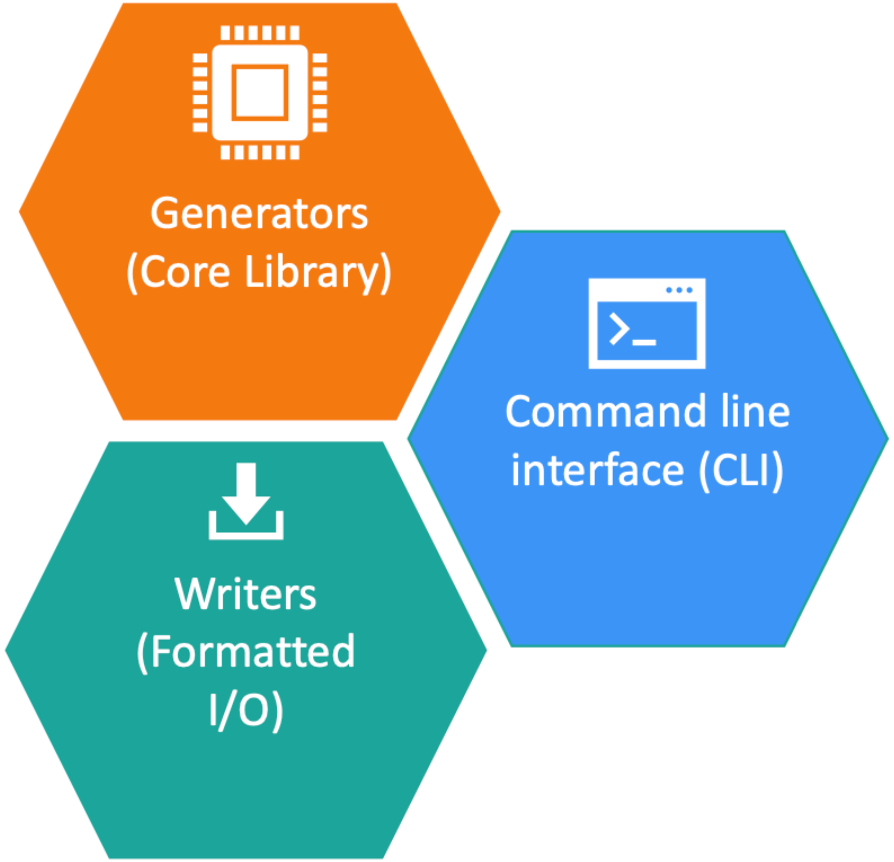
Supported models

DPMJET Models :	PYTHIA Models :	QGSJet Models :	SIBYLL Models :	Other Models:
• DPMJET-III 3.0.6 • PHOJET 1.12-35 • DPMJET-III 19.1 • PHOJET 19.1 • DPMJET-III 19.3 • PHOJET 19.3	• PYTHIA 6.4 • PYTHIA 8.3	• QGSJet-01 • QGSJet-II-03 • QGSJet-II-04	• SIBYLL-2.1 • SIBYLL-2.3 • SIBYLL-2.3c • SIBYLL-2.3d • SIBYLL*	• EPOS-LHC • SOPHIA 2.0 • UrQMD 3.4 • FLUKA (in progress)

MCEq matrices are calculated using this code.

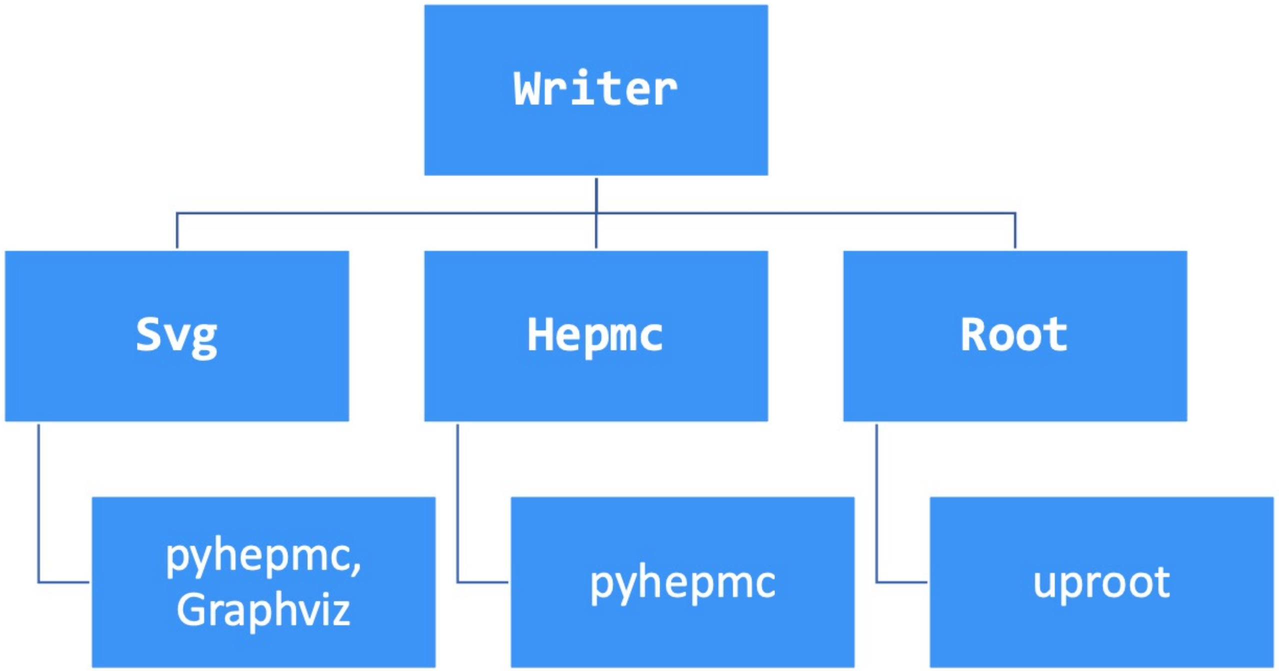
Main components

- Core library:
  - python scripts
  - jupiter notebooks
- Command line interface (CLI):
  - pipeline with other programs
  - drop-in substitution CRMC
- Writers (Formatted I/O representation of events)
  - SVG
  - Hepmc
  - Root



Events accessible directly in Python and in various common formats

- **`Writer`** is abstract class for wrapper classes over libraries that write to the corresponding formats



HepMC: Lingua franca for simulation software used at CERN

SIBYLL-2.1, pp, sqrt(s) = 20 GeV


```
HepMC::Version 3.02.05
HepMC::Asciiv3-START_EVENT_LISTING
T SIBYLL\|2.1\|
E 0 7 23
U GEV MM
P 1 0 2224 -8.9205041527748108e-02 1.3491769134998322e-01 2.0344371795654297e+00 2.3833706378936768e+00 1.2309999465942383e+00 2
P 2 0 111 8.9463070034980774e-02 -4.4863110780715942e-01 3.4179518222808838e+00 3.4519975185394287e+00 1.3496999442577362e-01 2
P 3 0 331 1.4990800619125366e-01 4.6003237366676331e-01 2.0358951091766357e+00 2.3069748878479004e+00 9.5749998092651367e-01 2
P 4 0 221 -1.6207817196846008e-01 -2.4423867464065552e-01 1.9545348882675171e+00 2.0511729717254639e+00 5.4879999160766602e-01 2
P 5 0 -211 -5.2947159856557846e-02 2.5346320867538452e-01 -5.1904711872339249e-02 2.9869863390922546e-01 1.3956999778747559e-01 1
P 6 0 111 -2.4904966354370117e-02 3.6575528979301453e-01 -1.8918764591217041e+00 1.9317893981933594e+00 1.3496999442577362e-01 2
P 7 0 2212 8.9764386415481567e-02 -5.2129906415939331e-01 -7.4990553855895996e+00 7.5760145187377930e+00 9.3826997280120850e-01 1
P 8 1 2212 -1.1027524620294571e-01 9.2852763831615448e-02 1.1708897352218628e+00 1.5073537826538086e+00 9.3826997280120850e-01 1
P 9 1 211 2.1069953218102455e-02 4.2065303772687912e-02 8.6355310678482056e-01 8.7602353096008301e-01 1.3956999778747559e-01 1
P 10 2 22 -1.5905208885669708e-02 -2.5956141948699951e-01 1.9417071342468262e+00 1.9595654010772705e+00 0.0000000000000000e+00 1
P 11 2 22 1.0539282113313675e-01 -1.8919278681278229e-01 1.4771823883056641e+00 1.4933792352676392e+00 0.0000000000000000e+00 1
P 12 3 211 1.3005101121962070e-02 9.5943860709667206e-02 2.2564361989498138e-01 2.8325849771499634e-01 1.3956999778747559e-01 1
P 13 3 -211 9.5603697001934052e-02 6.0167539864778519e-02 1.2169665843248367e-01 2.1799579262733459e-01 1.3956999778747559e-01 1
P 14 3 221 4.1300963610410690e-02 3.0392596125602722e-01 1.6885783672332764e+00 1.8057471513748169e+00 5.4879999160766602e-01 2
P 15 4 22 -1.3510279357433319e-01 4.4372059404850006e-02 1.5330873727798462e+00 1.5396685600280762e+00 0.0000000000000000e+00 1
P 16 4 22 -2.6978071779012680e-02 -2.8861477971076965e-01 4.2147985100746155e-01 5.1153844594955444e-01 0.0000000000000000e+00 1
P 17 6 22 -4.5049645006656647e-02 2.6162242889404297e-01 -1.5857362747192383e+00 1.6078042984008789e+00 0.0000000000000000e+00 1
P 18 6 22 2.0137846469879150e-02 1.0423322767019272e-01 -3.0665934085845947e-01 3.2451510429382324e-01 0.0000000000000000e+00 1
P 19 14 211 1.6239669173955917e-02 1.1180111020803452e-01 3.0710572004318237e-01 3.5670593380928040e-01 1.3956999778747559e-01 1
P 20 14 -211 -5.0198074430227280e-02 7.7754214406013489e-02 2.6761403679847717e-01 3.1666630506515503e-01 1.3956999778747559e-01 1
P 21 14 111 7.5260899960994720e-02 1.1438217759132385e-01 1.1139335632324219e+00 1.1324540376663208e+00 1.3496999442577362e-01 2
P 22 21 22 6.8888634443283081e-02 -3.4993162844330072e-03 5.4559254646301270e-01 5.5093902349472046e-01 0.0000000000000000e+00 1
P 23 21 22 6.3929148018360138e-03 1.1791287362575531e-01 5.6864660978317261e-01 5.8182567358016968e-01 0.0000000000000000e+00 1
HepMC::Asciiv3-END_EVENT_LISTING
```


Example

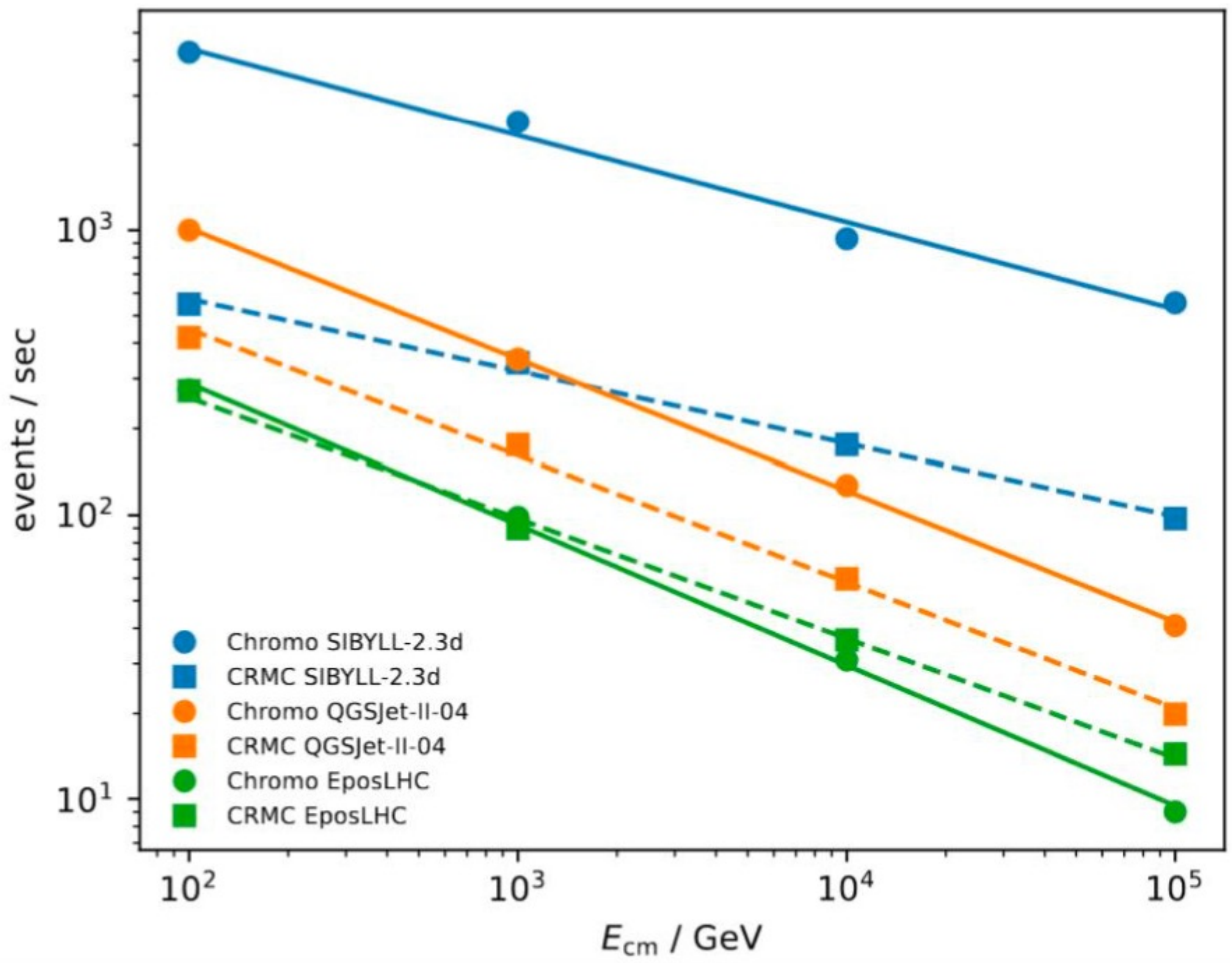
```python
with Hepmc("file.hepmc", model) as writer:
    writer.write(event)
```

Performance: CHROMO vs CRMC

- Python code "glue" fast compiled libraries written in Fortran/C++
- Runtime is limited below by the runtime of Fortran/C++ code performance of wrapped event generator
- NumPy array view (pointers) into hepevt common block if possible
- Avoid copy and hot Python loops
- Buffering of output
- Further optimization: put all heavy lifting of EventKinematics into C++ code

Cosmic Ray Monte Carlo Package, CRMC

Ulrich, Ralf[1]; Pierog, Tanguy[1]; Baus, Colin[1]



CHROMO is built for simplicity in application

Installation

```
(env_test) -bash-4.2$ pip install chromo
Collecting chromo
Downloading chromo-0.4.0-cp39-cp39-manylinux_2_17_x86_64.manylinux2014_x86_64.whl (23.7 MB)
   ━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━ 23.7/23.7 MB 4.7 MB/s eta 0:00:00
Installing collected packages: chromo
Successfully installed chromo-0.4.0
```

Typical workflow

```python
from chromo.kinematics import CenterOfMass
from chromo.models import EposLHC

kinematics = CenterOfMass(100, "p", "p")
event_generator = EposLHC(kinematics)

for event in event_generator(1000):
    # process the result of the collision
    # represented by ‘event‘ object
```

And that’s it: no compilation, no Fortran, and no specific knowledge required.

Event under the hood

```python
class EventData:
    """
    Data structure to keep filtered data.
    """
    generator: Tuple[str, str]
    kin: EventKinematics
    nevent: int
    impact_parameter: float
    n_wounded: Tuple[int, int]
    pid: np.ndarray
    status: np.ndarray
    charge: np.ndarray
    px: np.ndarray
    py: np.ndarray
    pz: np.ndarray
    en: np.ndarray
    m: np.ndarray
    vx: np.ndarray
    vy: np.ndarray
    vz: np.ndarray
    vt: np.ndarray
    mothers: Optional[np.ndarray]
    daughters: Optional[np.ndarray]
```

Event properties

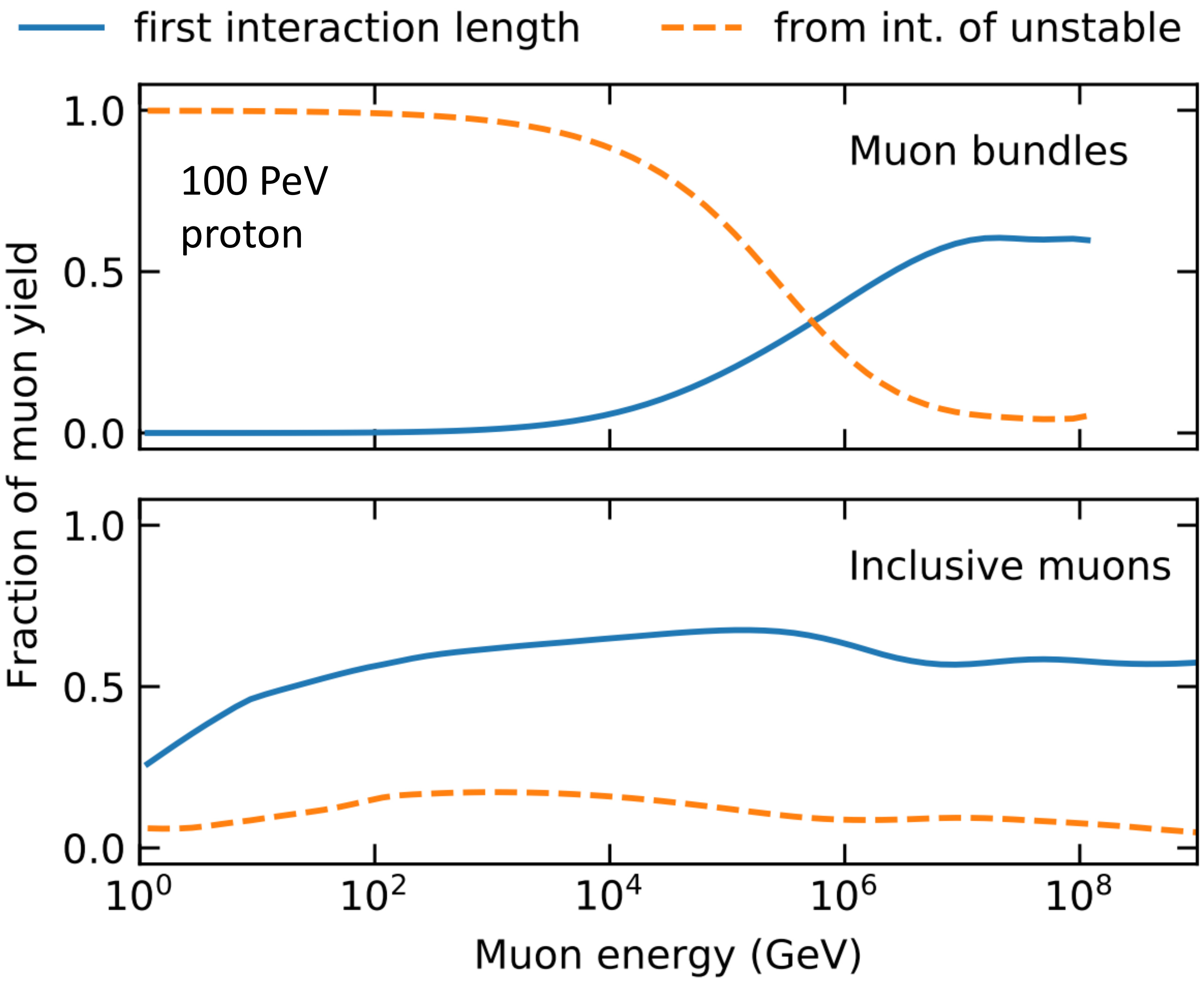
```python
class EventData:
    …
    @property
    def p_tot(self):
        """Return total momentum in
    GeV/c."""

    @property
    def eta(self):
        """Return pseudorapidity."""

    @property
    def y(self):
        """Return rapidity."""

    @property
    def xf(self):
        """Return Feynman x_F."""“

    …
```

Summary

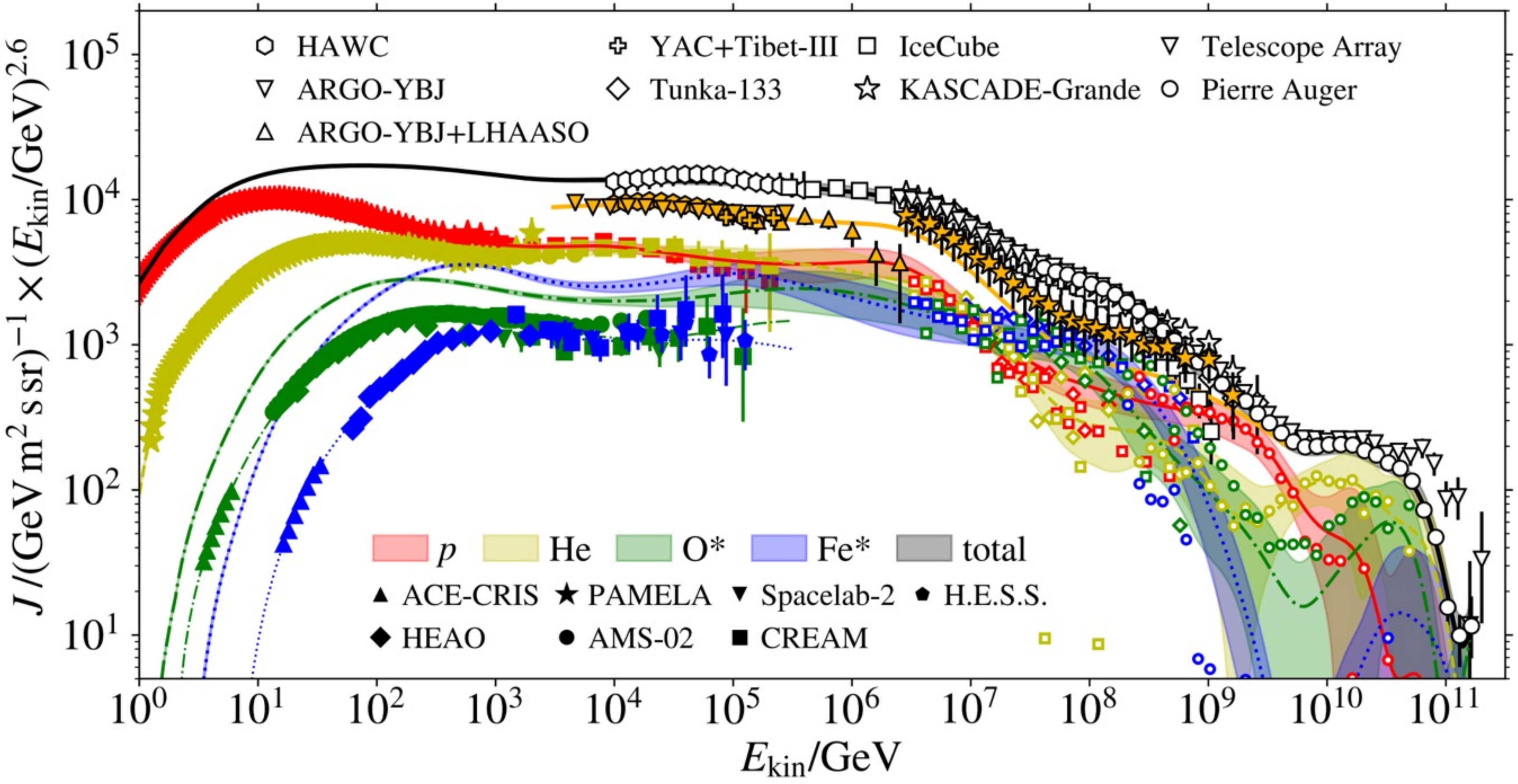
- MCEq is a generic tool, validated against data and other simulations
- Atm. Leptons are a different channel to study very forward hadronic interactions (mostly p-air)
- “Differences“ seen in comparisons with muon data at the surface and underground
- Validation/calibration via muon surface fluxes is challenging if performed rigorously (need to rely on old data and documentation; systematics often not discussed in detail)
- **Models 30-35% lower than muon data above a few GeV** (FLUKA is somewhat better at low energies but at higher energies also lower than data)
- Discrepancy in neutrinos (sensitive to kaon production) experimentally not established → needs more work
- Origin of discrepancies different from the muon excess in air showers (SIBYLL*)
- Underground/-water data confirms these findings with different detection principles

Inclusive atm. leptons air showers: different “astroparticle observable”

- Inclusive fluxes sensitive to ”first interaction”
- Air shower muons at the surface mostly from pion interactions
- Reason: competition between falling CR flux vs falling forward cross section
- Problems in incl. leptons distinct should be distinct from air showers



The Global Spline Fit (parameterization of CR fluxes)



Pros:

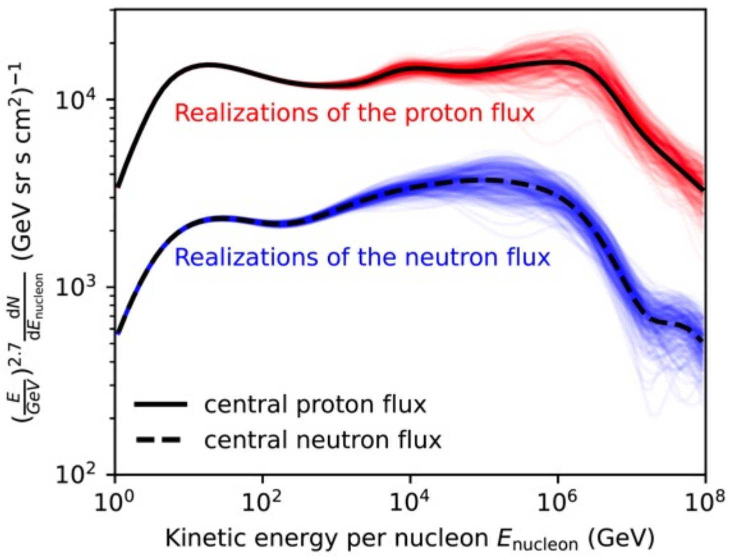
- Parameterizes data
- AND uncertainty
- AND covariance matrix
- Can be updated “easily”

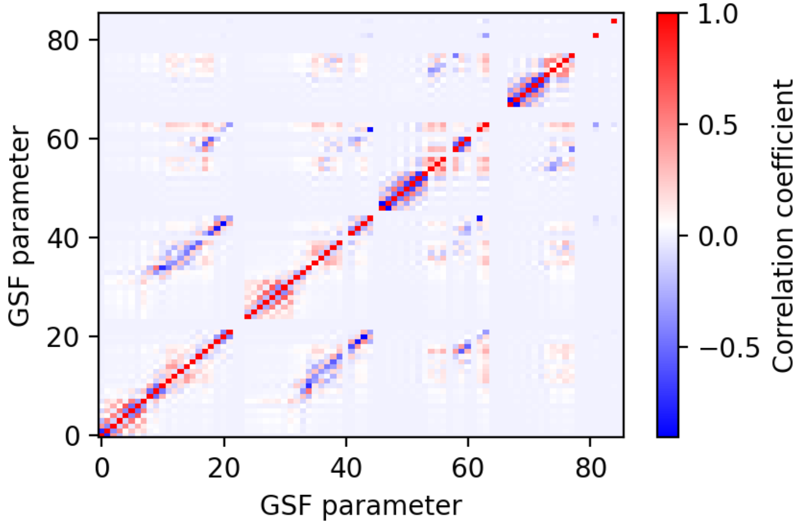
Cons:

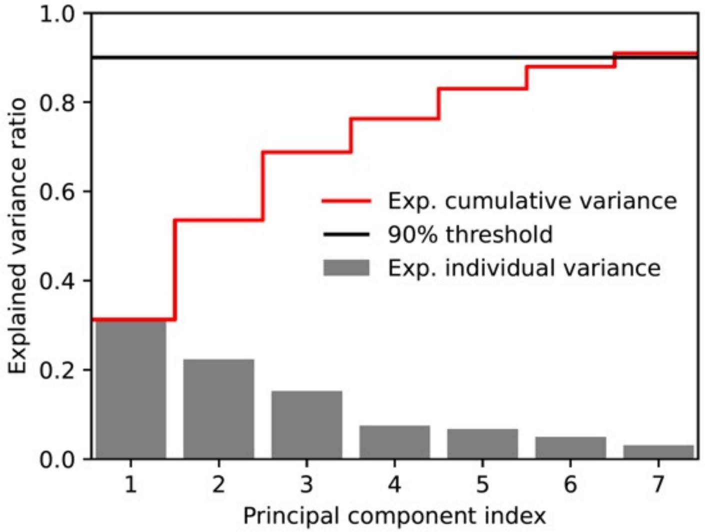
- Many parameters
- ~5 * 20 ☺
- Not all equally important for ν fluxes
- Splines somewhat sensitive choice

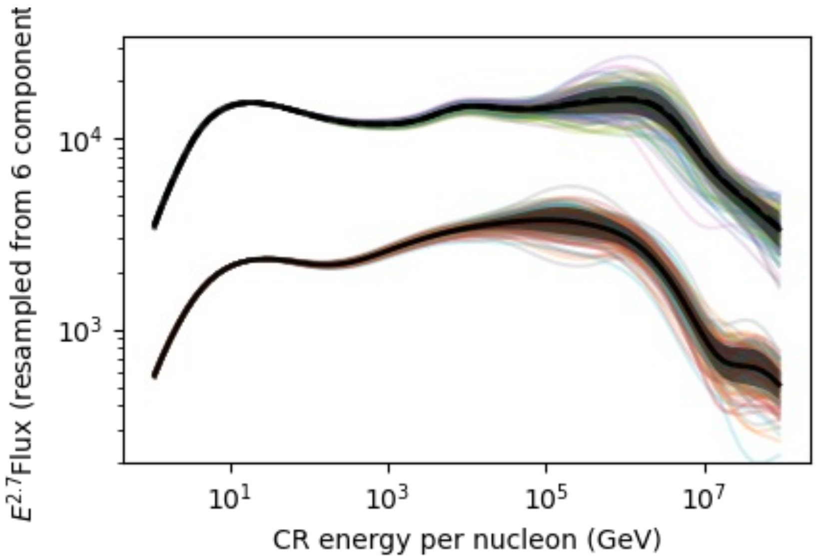
Dembinski, AF, Gaisser, ICRC 2017 & H. Dembinski 2019

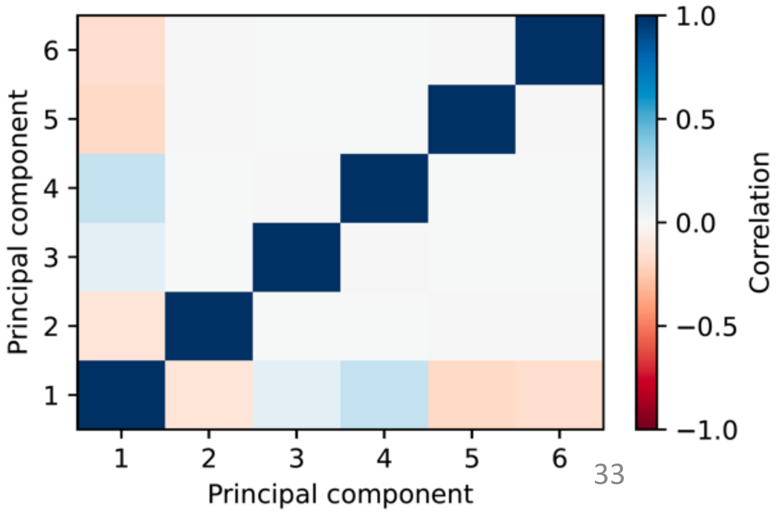
Dimensionality reduction of nucleon flux to 6 parameters





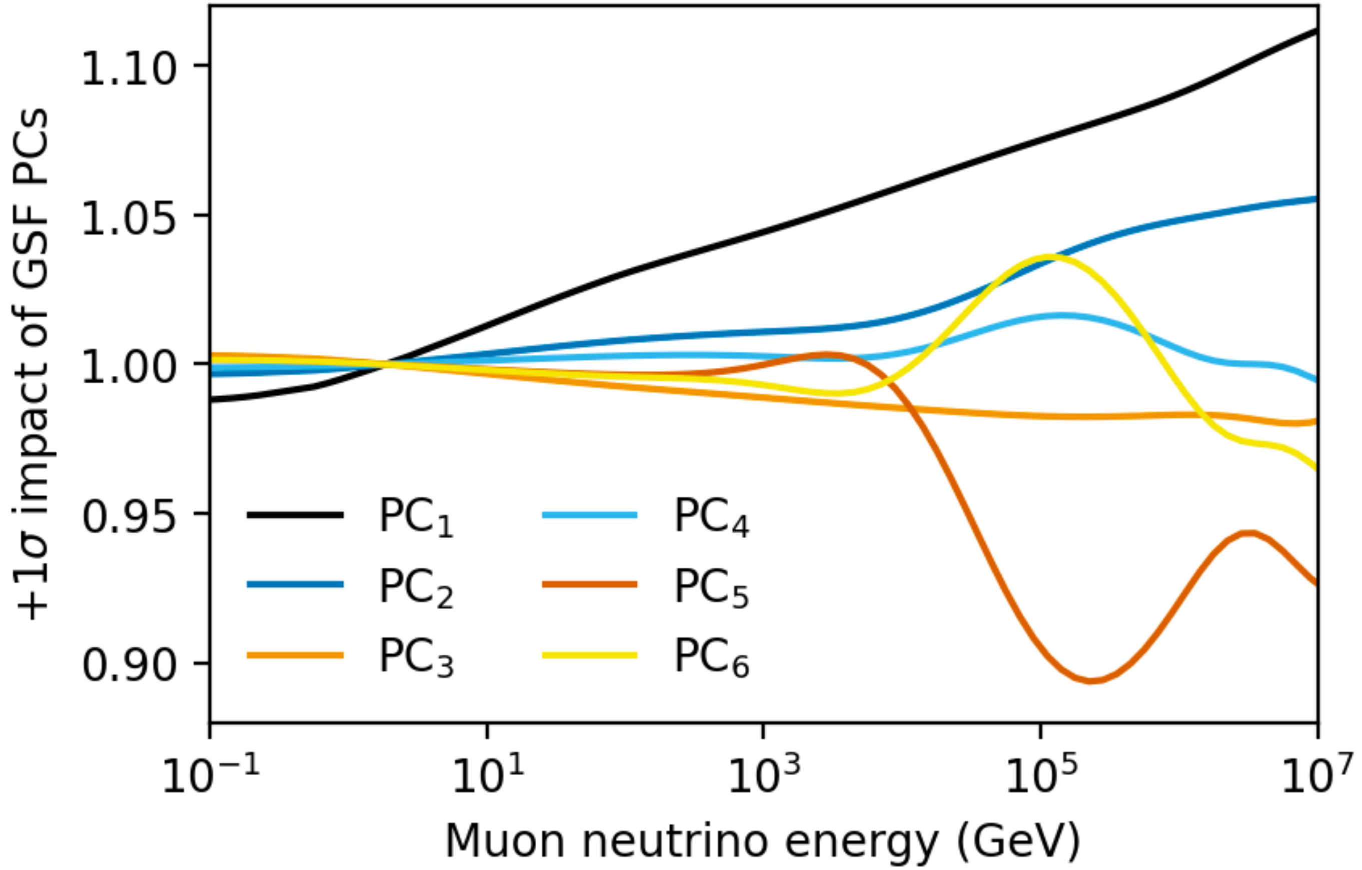






Principal components of CR nucleon fluxes

Zenith-averaged muon neutrinos



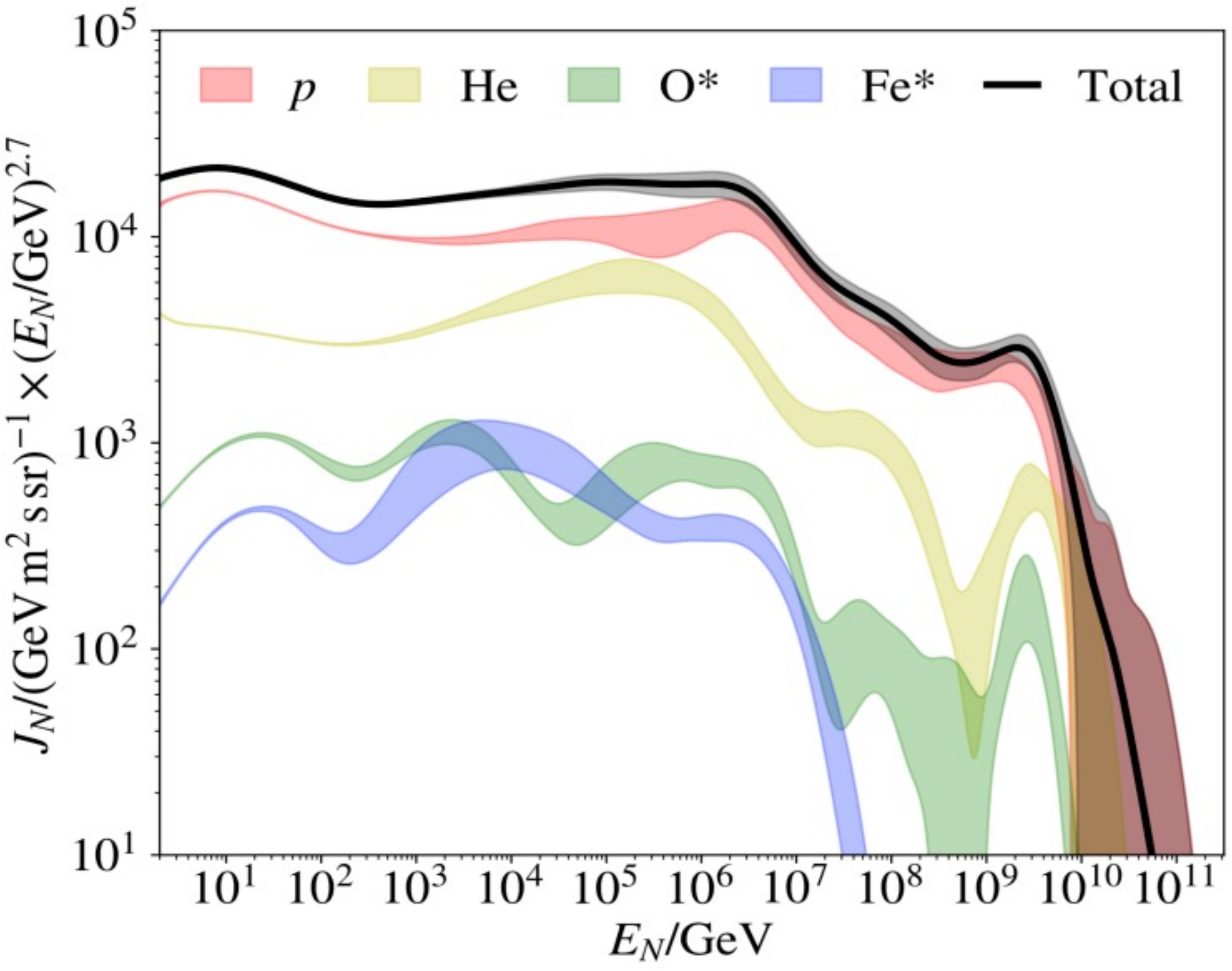
- Component 1 is a "global" spectral index correction
- Sum of components can reproduce 90% allowed shapes from the 1-sigma range of GSF

→ CR nucleon flux represented by weighted sum of 6 base vectors

→ GSF is meant to be updated once new data comes in

→ **Optimal CR nucleon flux model for neutrino flux calculations**

The Global Spline Fit – nucleon fluxes (MCEq input)



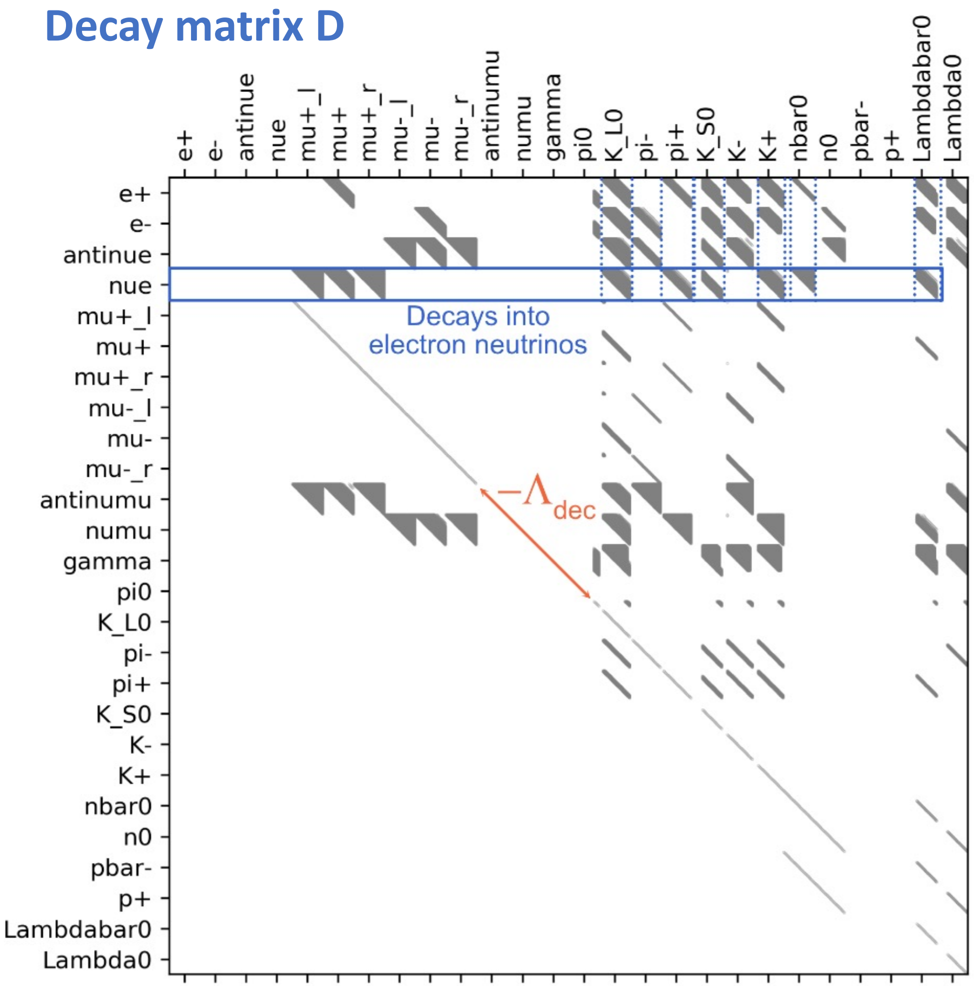
- Most contribution from proton and helium flux
- Correlations between H and He affect
  - CR neutron fraction
  - Muon charge ratio
  - Neutrino/Antineutrino ratio

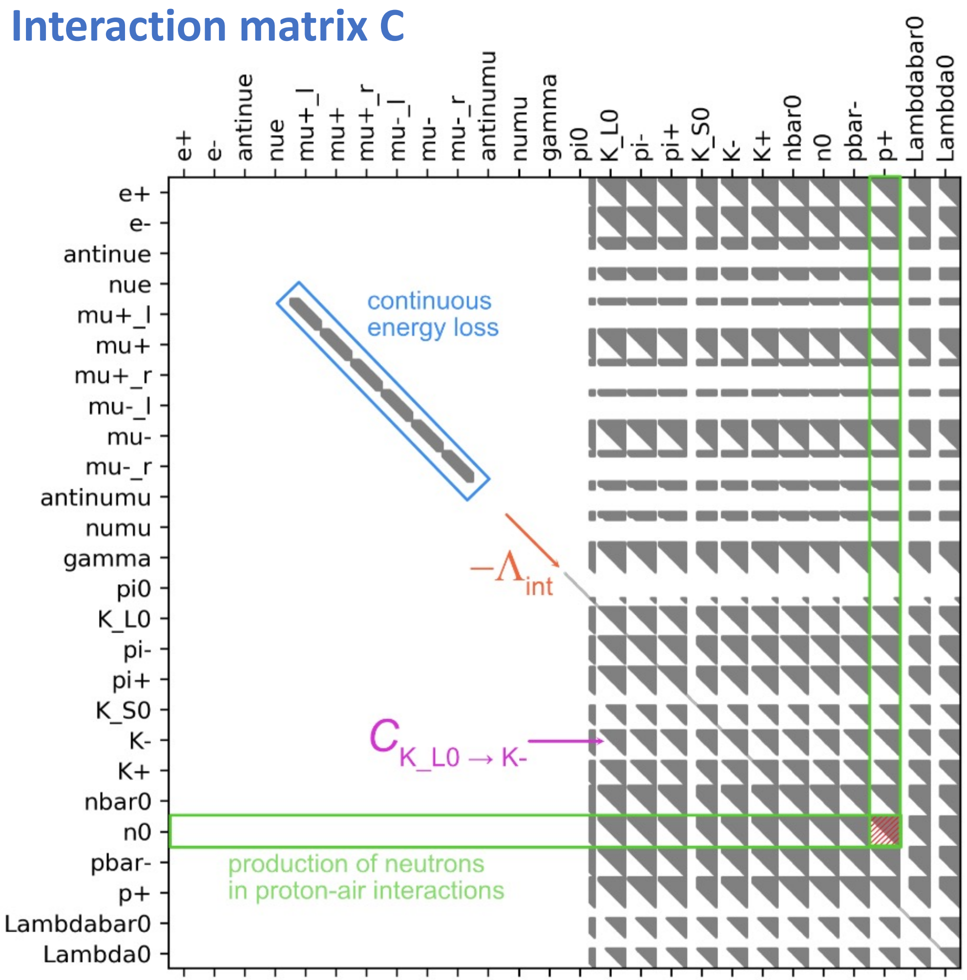
→ Need to model two correlated components

→ technically ~80 parameters

AF, Dembinski, Engel, Riehn, Gaisser, Stanev ICRC 2017

Sparse matrix structure





matrices are sparse → high performance

MCEq: Matrix Cascade Equations

A. Fedynitch, R. Engel, T. K. Gaisser, F. Riehn and S. Todor
PoS ICRC 2015, 1129 (2015), EPJ Web Conf. 99, 08001 (2015) and EPJ Web Conf. 116, 11010 (2016)

$$\frac{\mathrm{d}\Phi_h(E,X)}{\mathrm{d}X} = -\frac{\Phi_h(E,X)}{\lambda_{\mathrm{int},h}(E)} - \frac{\Phi_h(E,X)}{\lambda_{\mathrm{dec},h}(E,X)} - \frac{\partial}{\partial E}(\mu(E)\Phi_h(E,X)) + \sum_\ell \int_E^\infty \mathrm{d}E_\ell \frac{\mathrm{d}N_{\ell(E_\ell)\to h(E)}}{\mathrm{d}E} \frac{\Phi_\ell(E_\ell,X)}{\lambda_{\mathrm{int},l}(E_\ell)} + \sum_\ell \int_E^\infty \mathrm{d}E_\ell \frac{\mathrm{d}N^{\mathrm{dec}}_{\ell(E_\ell)\to h(E)}}{\mathrm{d}E} \frac{\Phi_\ell(E_\ell,X)}{\lambda_{\mathrm{dec},l}(E_\ell,X)}$$

$$\frac{\mathrm{d}\Phi^h_{E_i}}{\mathrm{d}X} = -\frac{\Phi^h_{E_i}}{\lambda^h_{\mathrm{int},E_i}} - \frac{\Phi^h_{E_i}}{\lambda^h_{\mathrm{dec},E_i}(X)} - \vec{\nabla}_i(\mu^h_{E_i}\Phi^h_{E_i}) + \sum_{E_k\geq E_i}^{E_N}\sum_\ell \frac{c_{\ell(E_k)\to h(E_i)}}{\lambda^\ell_{\mathrm{int},E_k}}\Phi^\ell_{E_k} + \sum_{E_k\geq E_i}^{E_N}\sum_\ell \frac{\mathrm{d}_{\ell(E_k)\to h(E_i)}}{\lambda^\ell_{\mathrm{dec},E_k}(X)}\Phi^\ell_{E_k}$$

State (or flux) vector

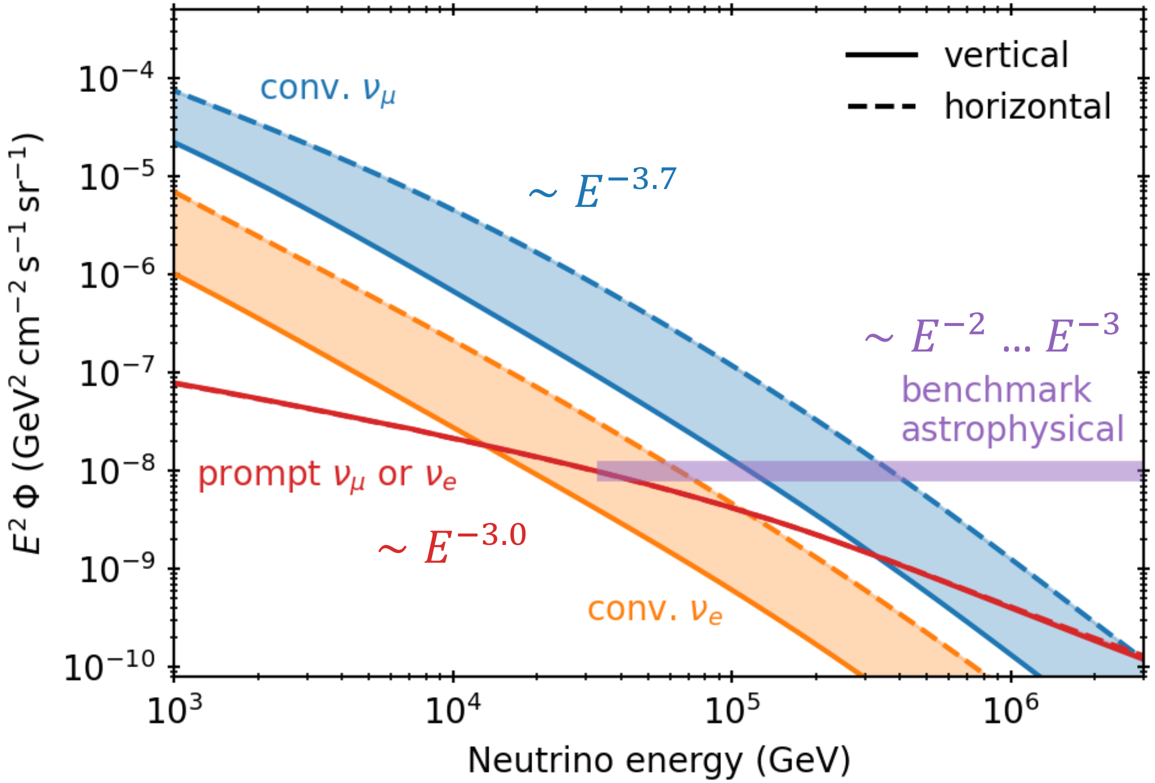
$$\vec{\Phi} = \left(\vec{\Phi}^{\mathrm{p}} \quad \vec{\Phi}^{\mathrm{n}} \quad \vec{\Phi}^{\pi^+} \quad \dots \quad \vec{\Phi}^{\bar{\nu}_\mu} \quad \dots\right)^T$$

$$\vec{\Phi}^{\mathrm{p}} = \left(\Phi^{\mathrm{p}}_{E_0} \quad \Phi^{\mathrm{p}}_{E_1} \quad \cdots \quad \Phi^{\mathrm{p}}_{E_N}\right)^T$$

"Matrix form"

$$\frac{\mathrm{d}}{\mathrm{d}X}\vec{\Phi} = -\vec{\nabla}_E(\mathrm{diag}(\vec{\mu})\vec{\Phi}) + (-\mathbf{1}+\mathbf{C})\mathbf{\Lambda}_{\mathrm{int}}\vec{\Phi} + \frac{1}{\rho(X)}(-\mathbf{1}+\mathbf{D})\mathbf{\Lambda}_{\mathrm{dec}}\vec{\Phi}$$

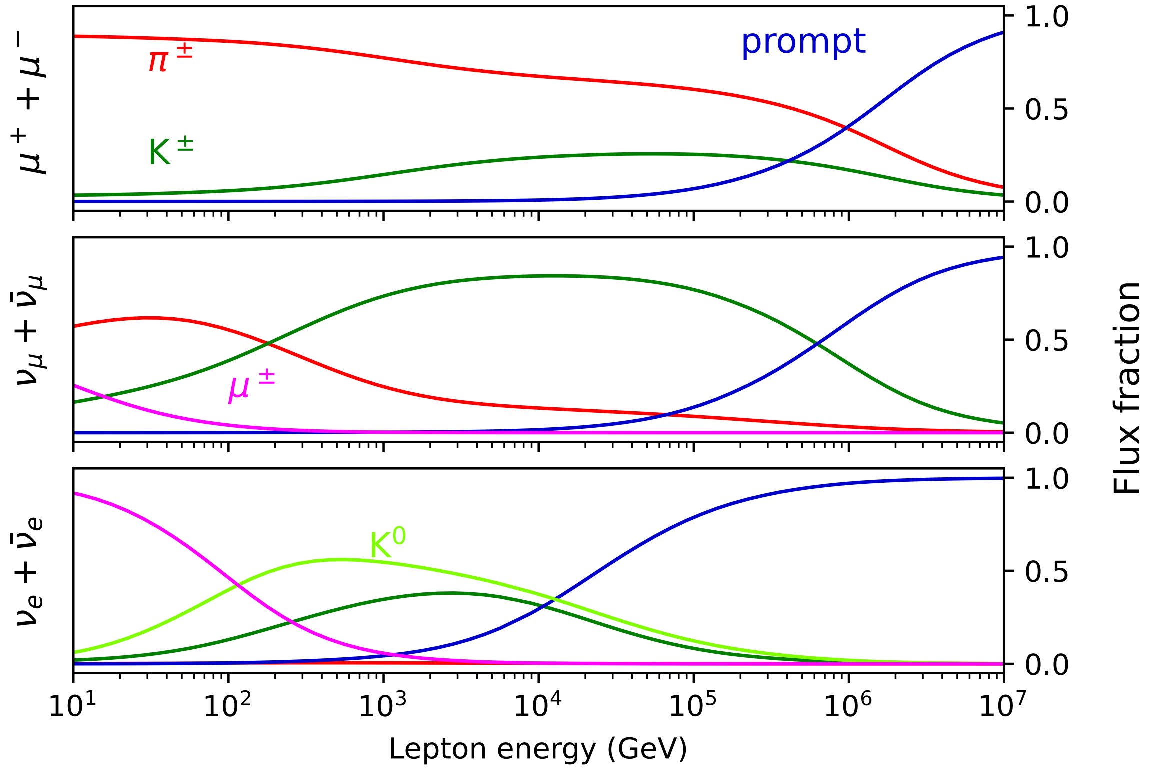
High energy lepton spectrum



Bands (zenith-enhancement):

- Lower boundary $\cos\theta = 1$, vertical
- Upper boundary $\cos\theta = 0$, horizontal

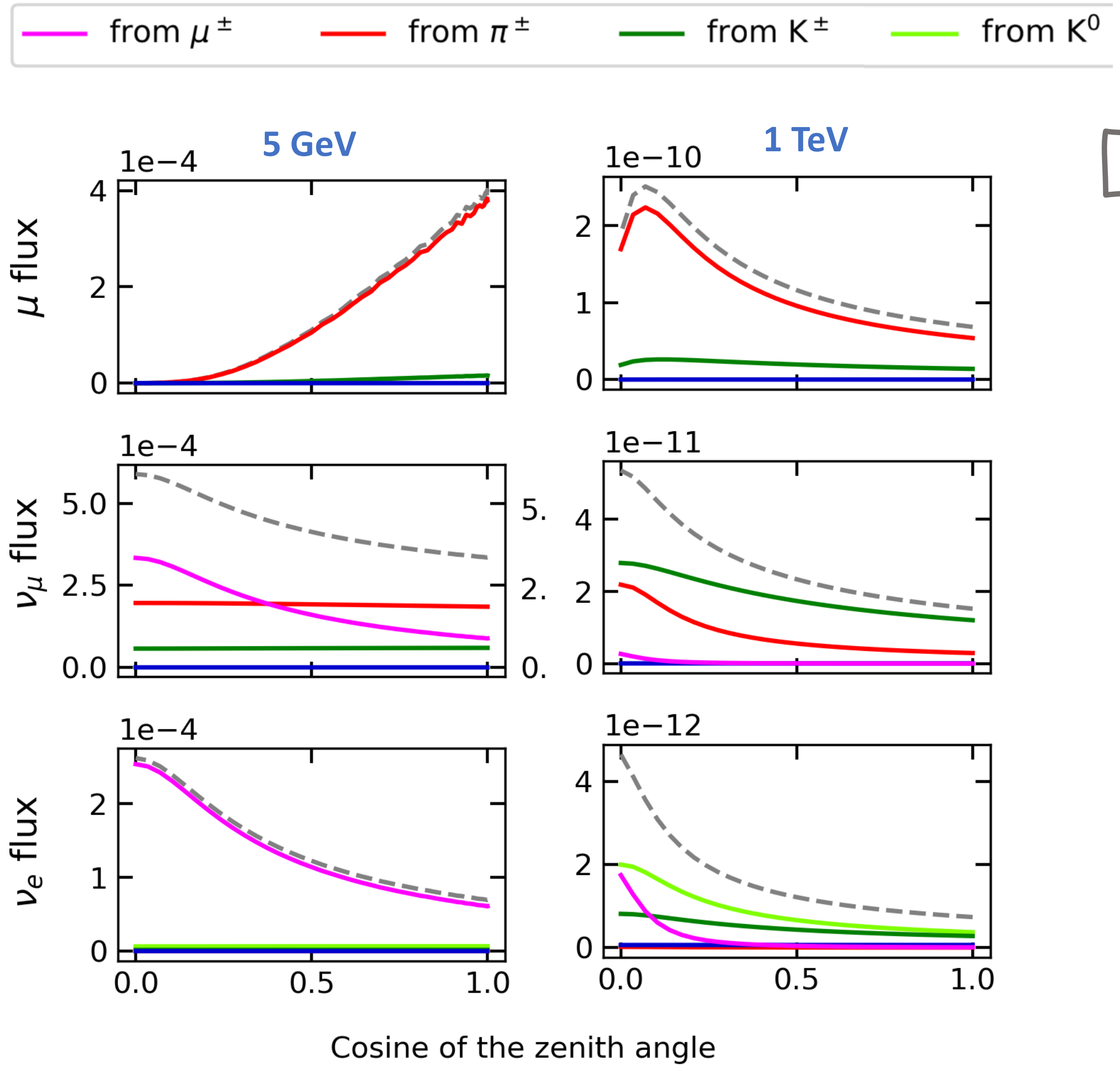
AF, F. Riehn, R. Engel, T.K. Gaisser, T. Stanev, PRD 100 2019

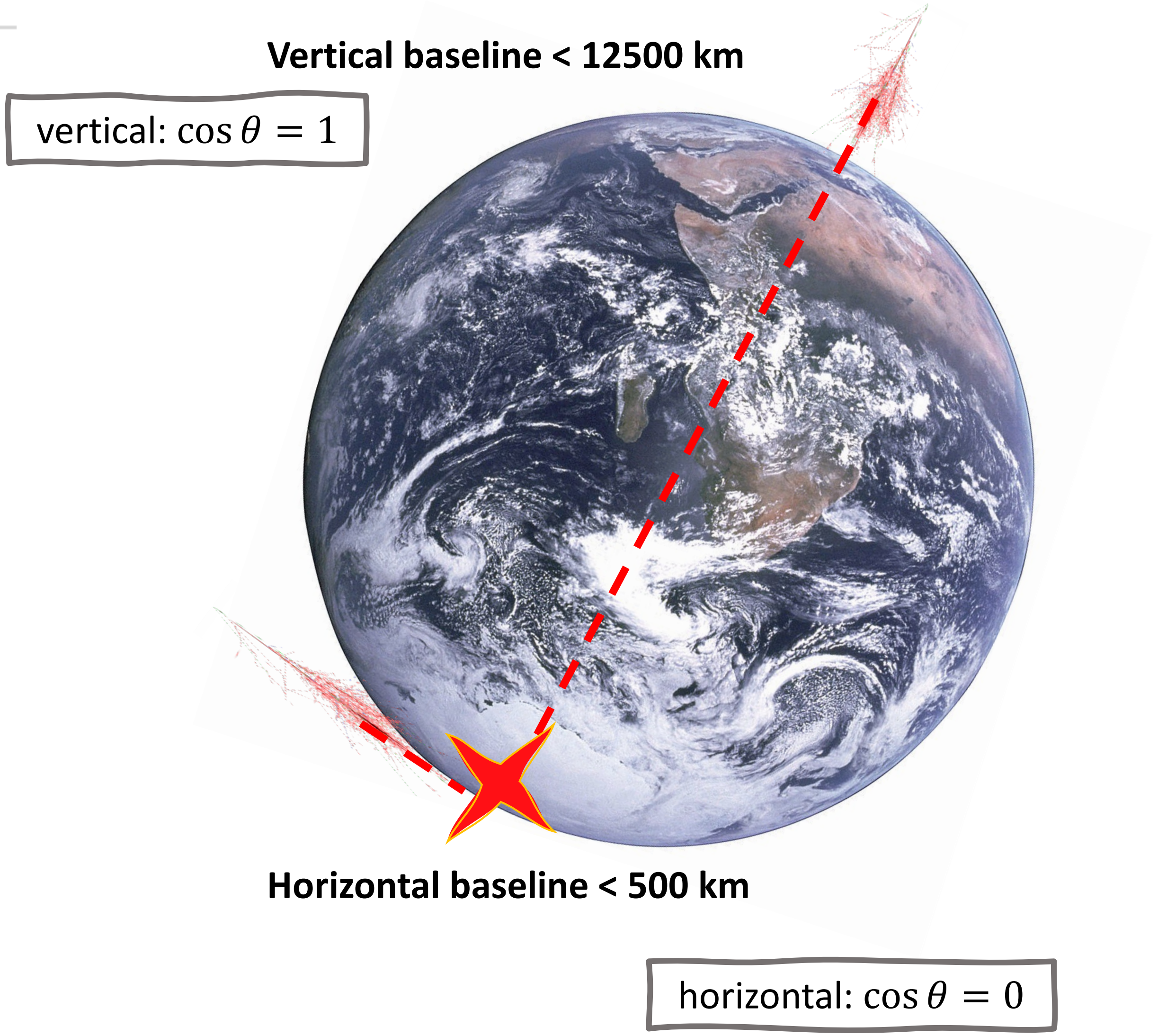


Different weight of hadrons in lepton production, due to:

- Hadron production cross sections
- Branching ratio & decay kinematics

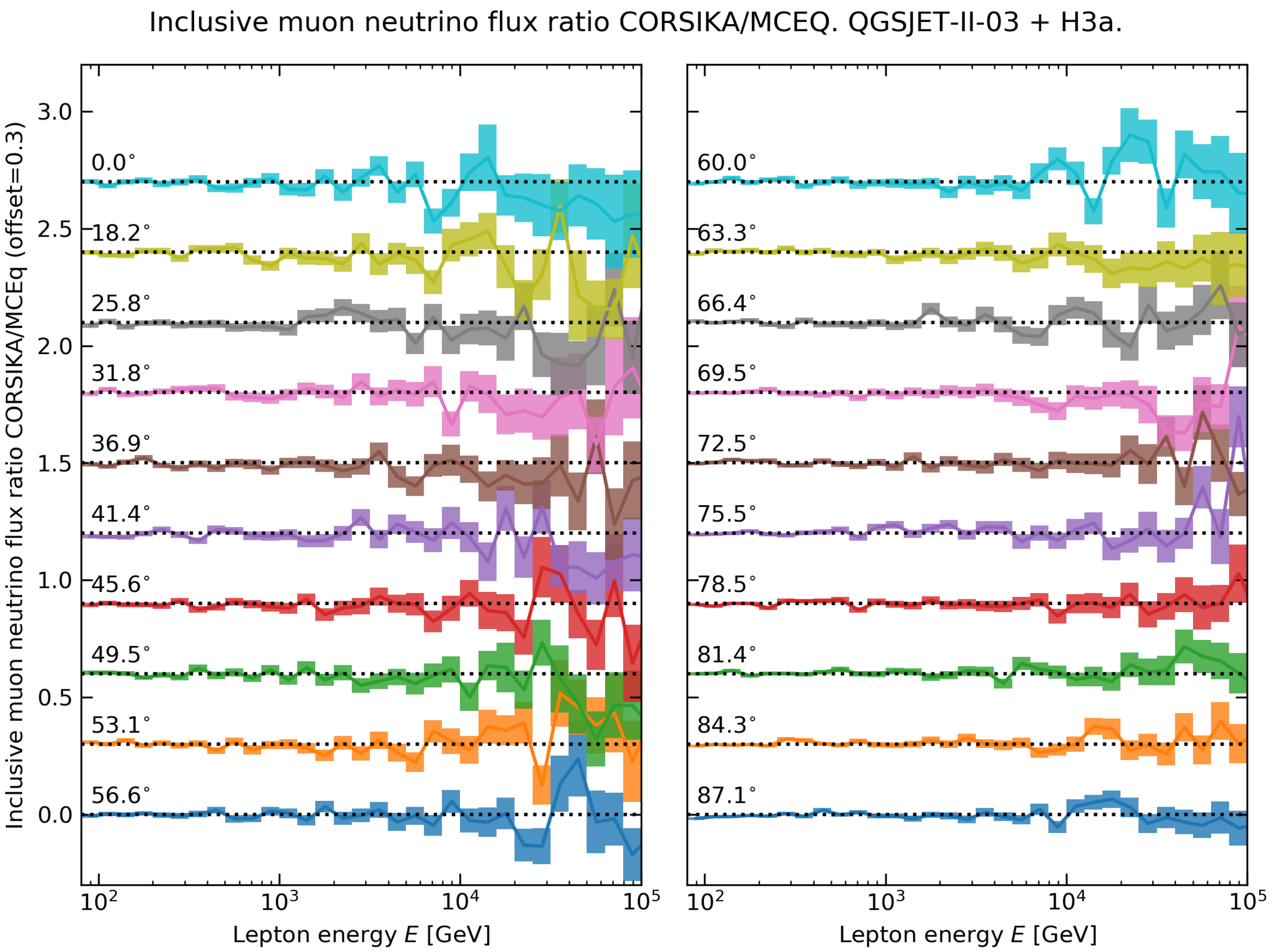
Zenith angle dependence at higher-E is sensitive to hadron production

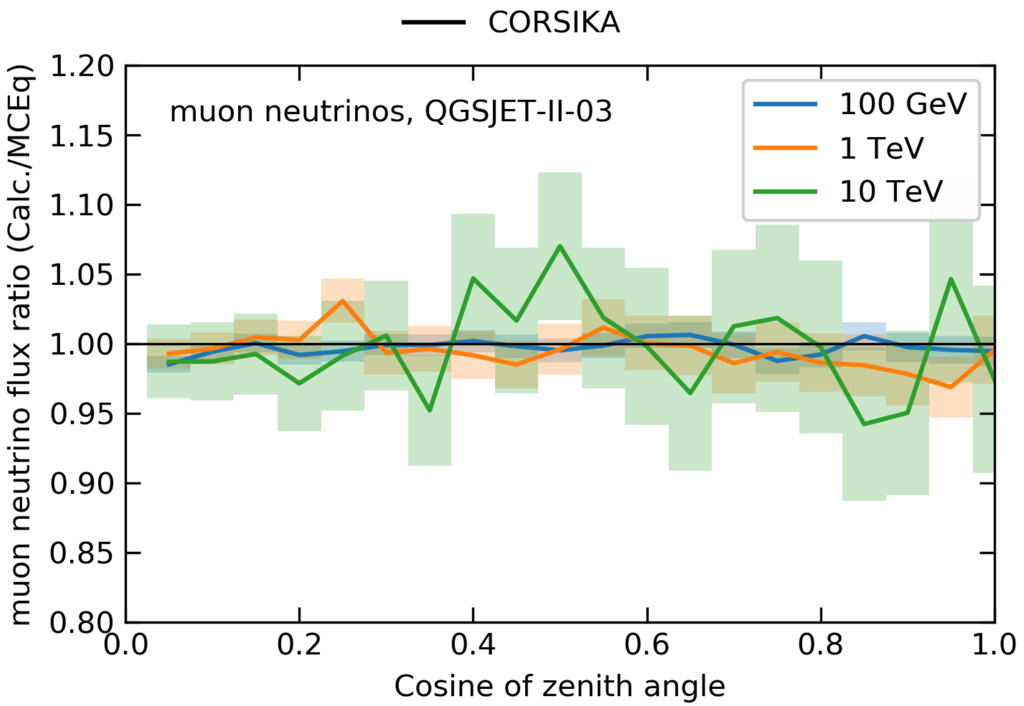




AF, F. Riehn, R. Engel, T.K. Gaisser, T. Stanev, PRD 100 2019

MCEq vs CORSIKA7 inclusive spectra

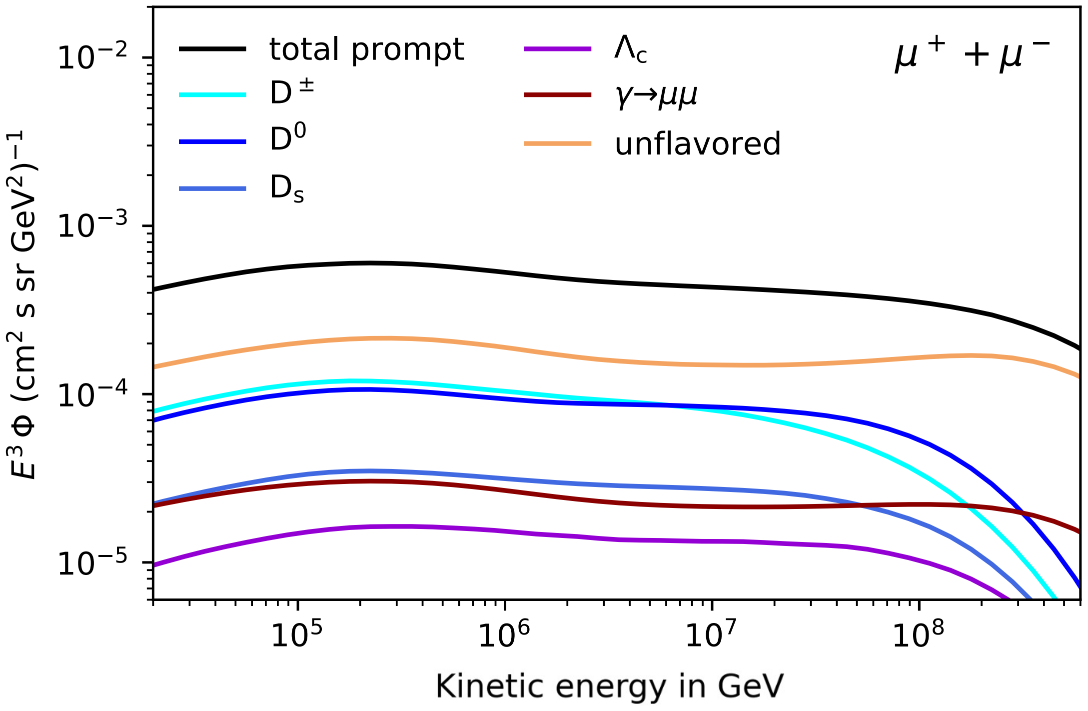




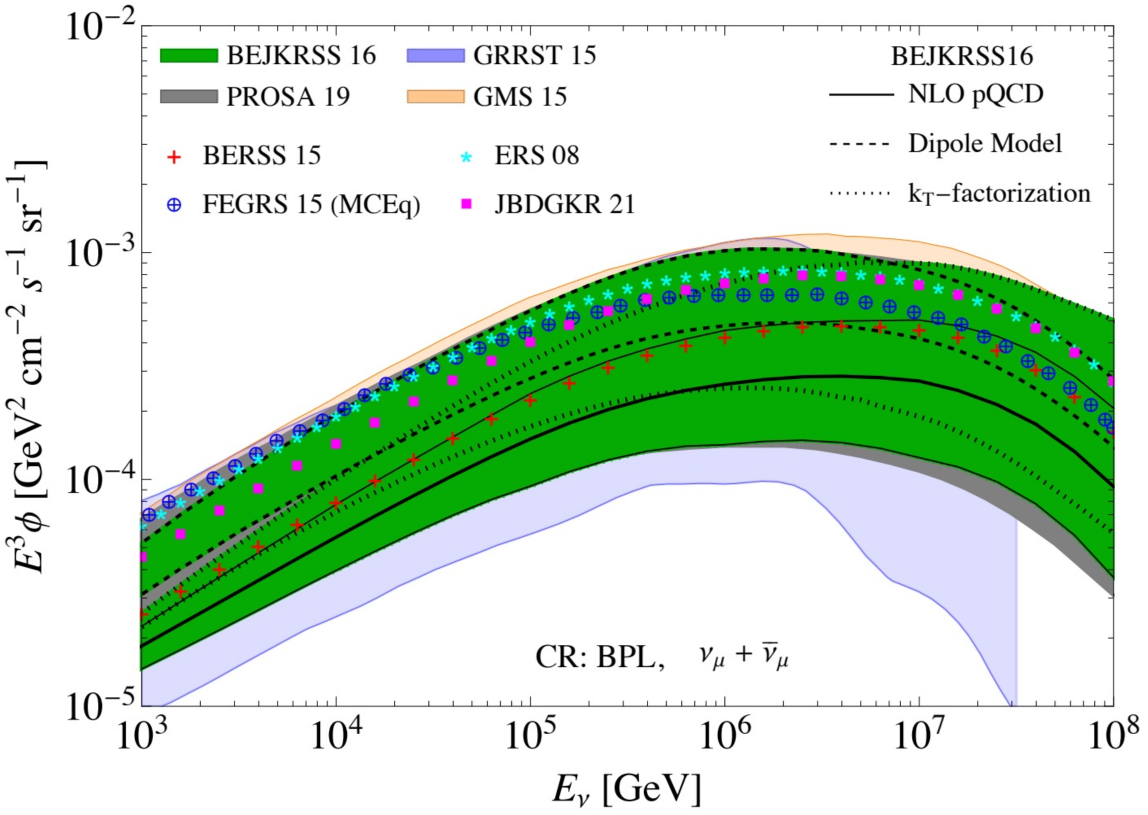
CORSIKA: A. Fedynitch, J. Becker Tjus and P. Desiati, PRD 2012

Above 100 TeV: territory of the (undiscovered) prompt muons and neutrinos

AF, F. Riehn, R. Engel, T.K. Gaisser, T. Stanev, PRD 100 2019



Forward Physics Facility Snowmass arXiv: 2203.05090

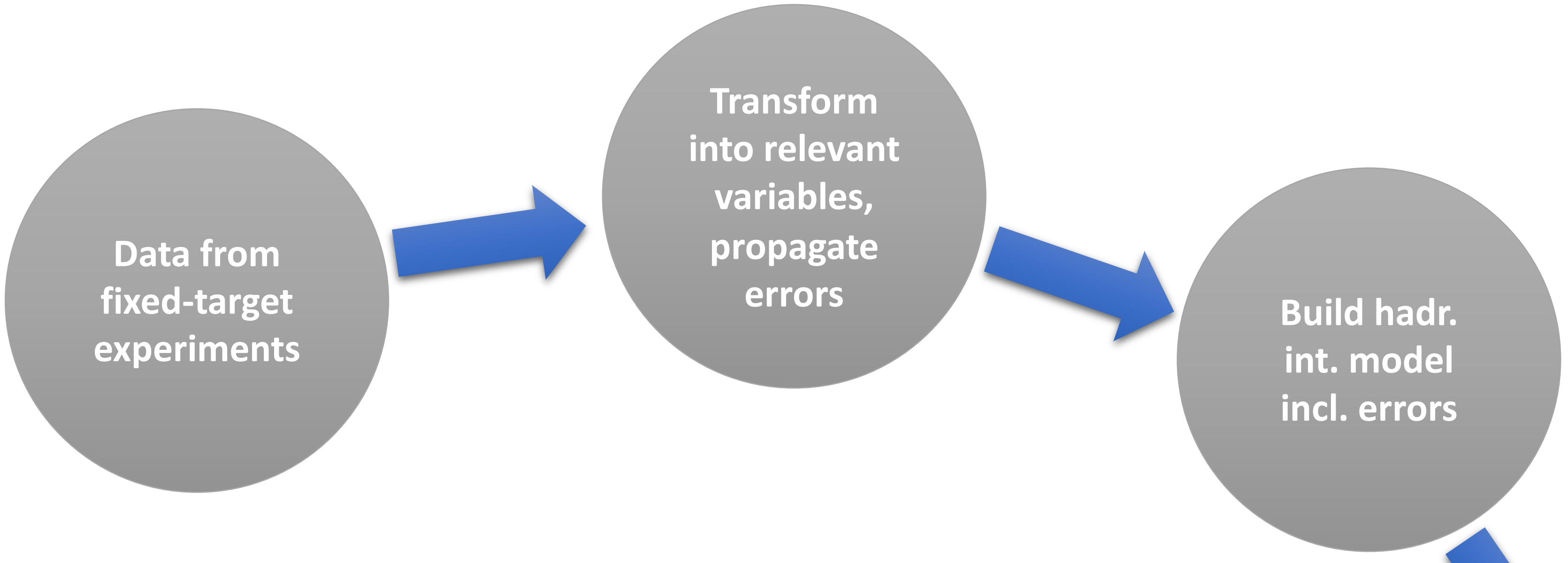


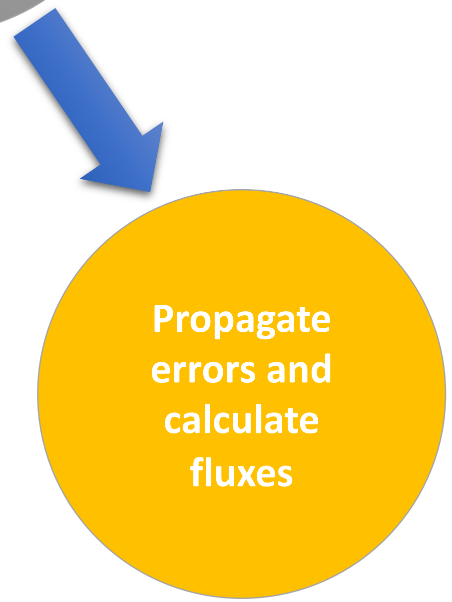
Prompt muons more production channels than prompt neutrinos:

- Rare decays of unflavored mesons *e.g.,* $\eta \rightarrow \mu^+\mu^-$
- EM pair production $\gamma \rightarrow \mu^+\mu^-$

- Large uncertainties from pQCD
- pQCD might be incomplete (intrinsic charm)
- The fragmentation (c $\rightarrow D$) function is a choice

Data-Driven Hadronic Interaction Model (DDM)



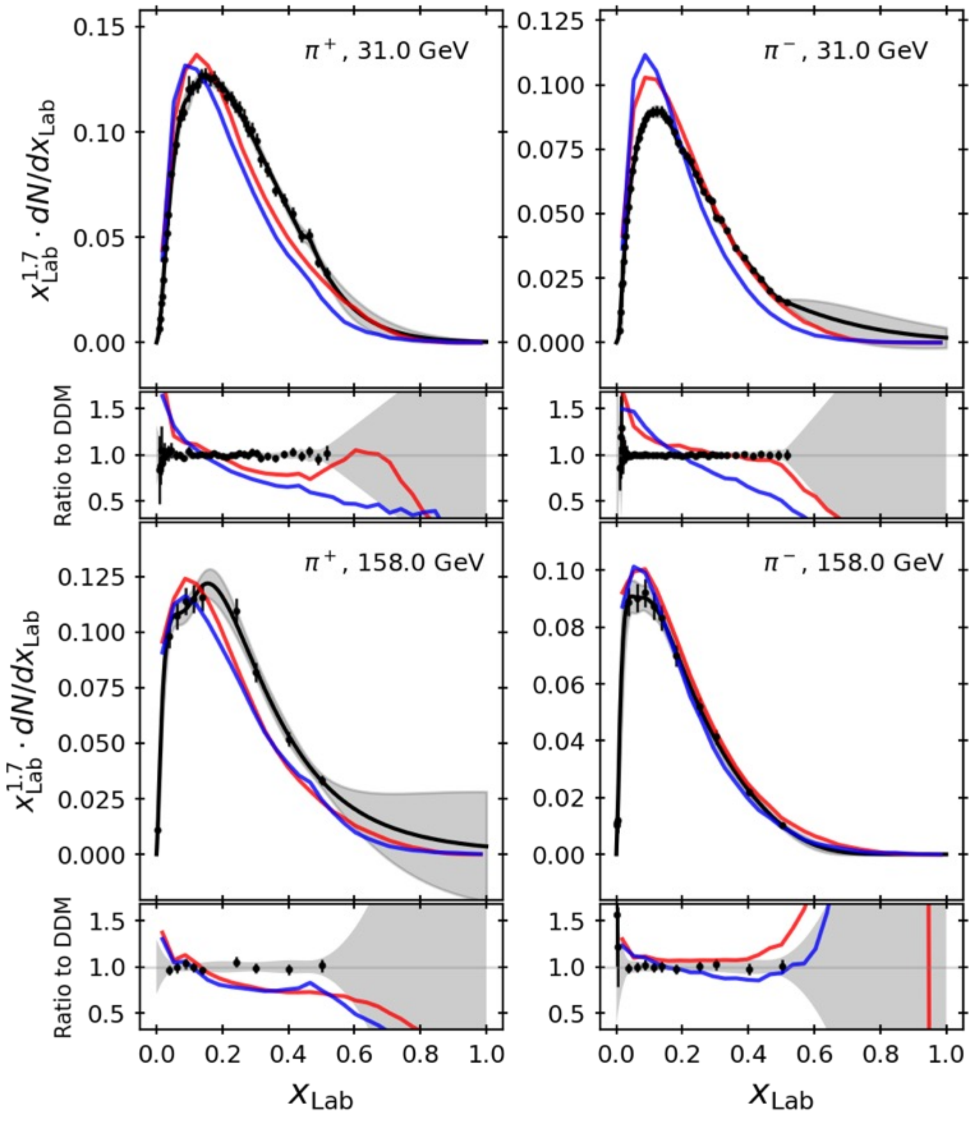


Based on modern proton-carbon data (and pp)

Experiment	beam	E_{beam}/GeV	Secondaries	Variables
NA49	pC	158	$\pi^{\pm}, \overset{(-)}{\mathrm{p}}, n$	$x_{\mathrm{F}}, p_{\perp}$
NA49	pp	158	$\mathrm{K}^{\pm}$	$x_{\mathrm{F}}, p_{\perp}$
NA61/SHINE	pC	31	$\pi^{\pm}, K^{\pm}, K_S^0, \Lambda$	p, θ
NA61/SHINE	π^{-}C	158, 350	$\pi^{\pm}, K^{\pm}, \overset{(-)}{\mathrm{p}}$	$p, p_{\perp}$

AF & M. Huber, PRD 106, 083018 (2022), arXiv:2205.14766

Data-driven Hadronic interaction model (DDM)



AF & M. Huber, PRD106 2022, arXiv:2205.14766

- Inclusive particle production cross sections from data at 31, 158 and 350 GeV (lab.)
- Interpolated linearly in log(E)
- Isospin symmetry for neutrons and π^-
- Feynman scaling at higher energy
- Errors from fiting splines to data
- Additional free parameters by “cloning” cross sections to higher energies

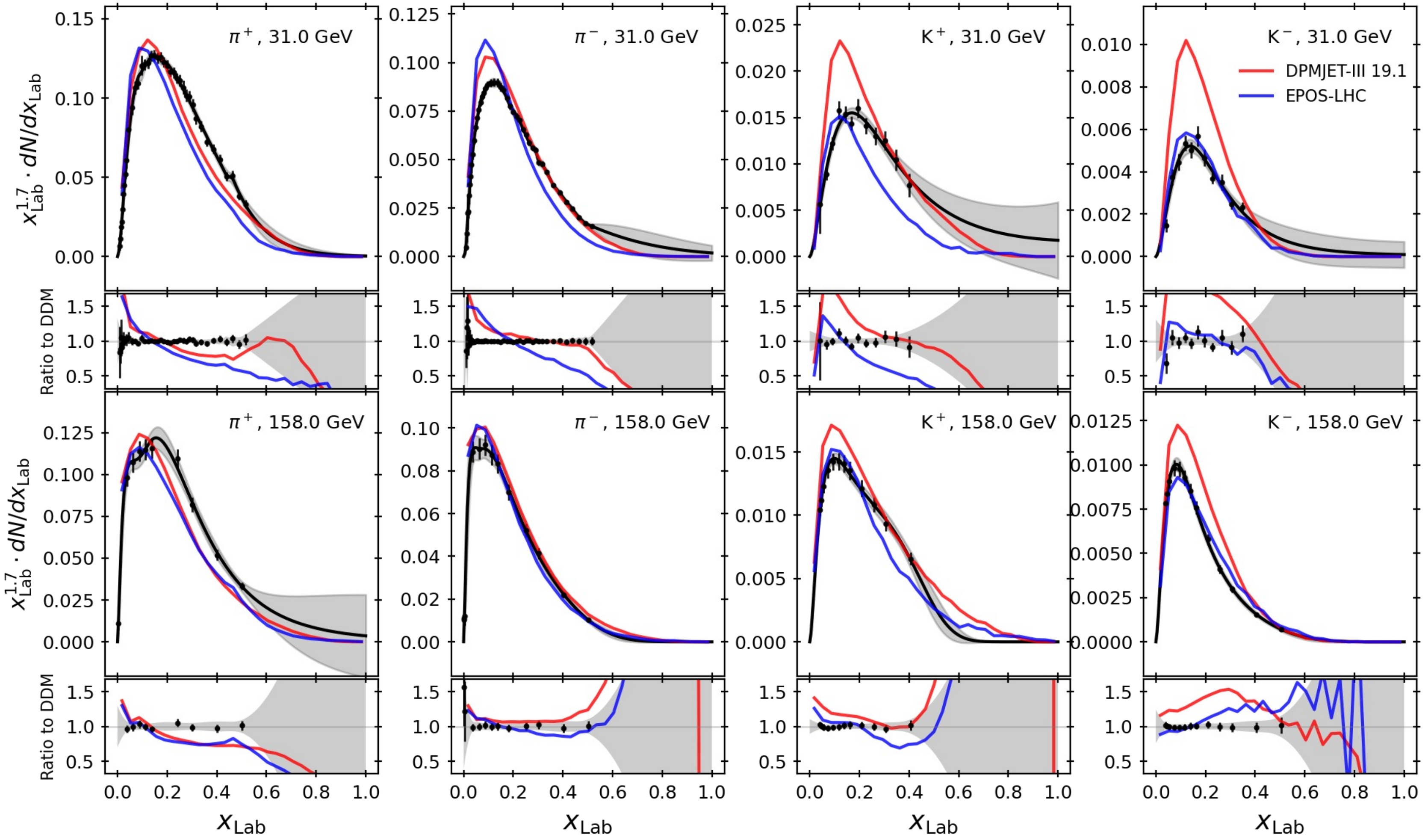
→ **Optimal hadronic model for calibrating neutrino flux calculations**

Experiment	beam	$E_{\mathrm{beam}}/\mathrm{GeV}$	Secondaries	Variables
NA49	pC	158	$\pi^{\pm}, \overset{(-)}{\mathrm{p}}, n$	$x_{\mathrm{F}}, p_{\perp}$
NA49	pp	158	$\mathrm{K}^{\pm}$	$x_{\mathrm{F}}, p_{\perp}$
NA61/SHINE	pC	31	$\pi^{\pm}, K^{\pm}, K_S^0, \Lambda$	p, θ
NA61/SHINE	π^-C	158, 350	$\pi^{\pm}, K^{\pm}, \overset{(-)}{\mathrm{p}}$	$p, p_{\perp}$

Data-driven model (DDM) built in incl. cross sections

NA49 & NA61 proton-carbon

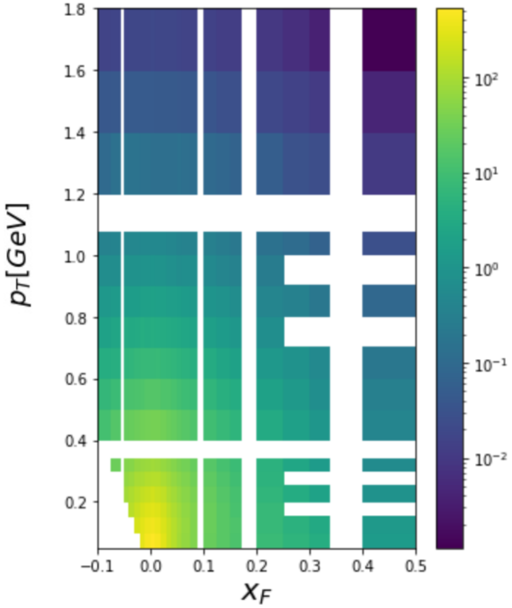
AF & M. Huber, PRD 106, 2022, arXiv:2205.14766



- **Uncertainties conservatively scale up** in absence of forward data
- K^{+-} data at 158 GeV extrapolated from pp→pC
  - → + 5-7% error from MC
- Carbon to air correction < 1%
- + proton and neutron secondaries , & π^- projectiles (not shown)
- Neutron (and π^+ projectiles) via isospin relations
- K^0 via isospin

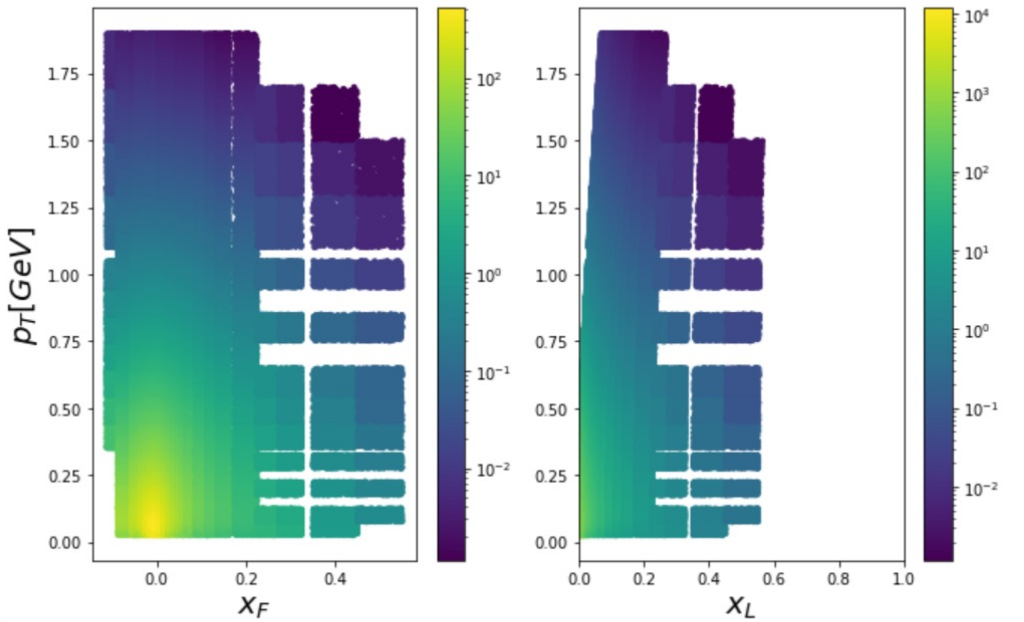
Building the DDM

NA49 proton-carbon @ 158 GeV

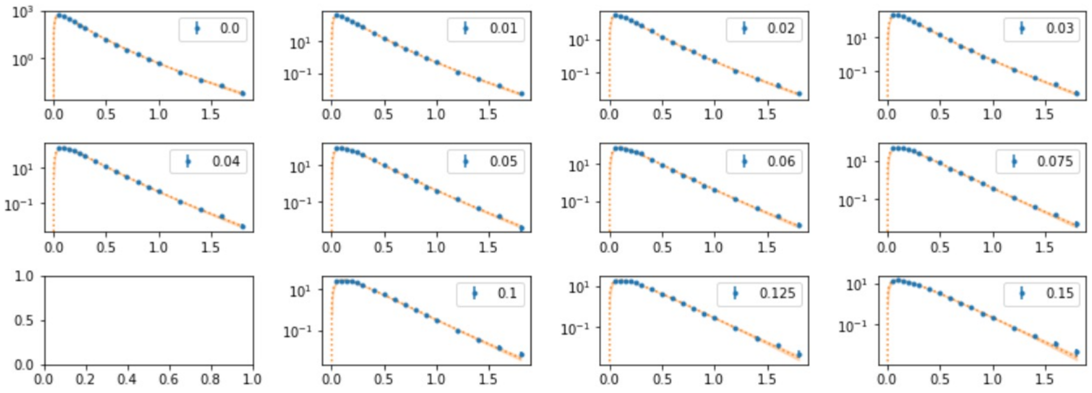


Sample from
$x_F = pz/sqrt(s)$ and convert into $x_L = E_{secondary}/E_{proj}$

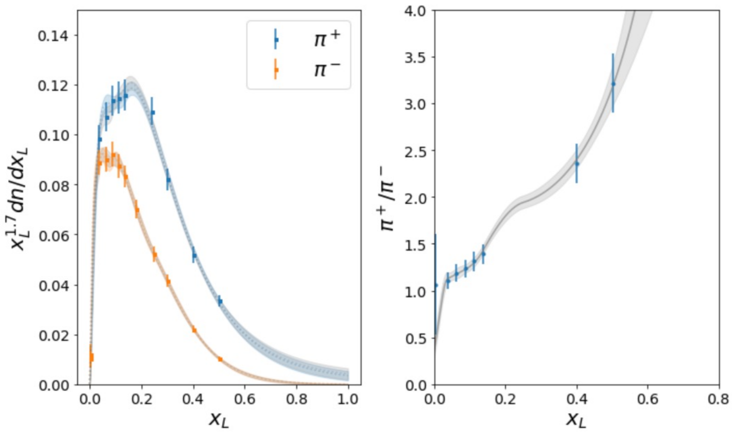
$$x_{Lab} = \frac{E_c}{E_a} = \frac{\gamma\sqrt{m_c^2 + \frac{1}{4}x_F^2 E_{c.m.}^2 + p_{c,T}^{*2}} + \frac{1}{2}\gamma\beta x_F^2 E_{c.m.}}{E_a}$$



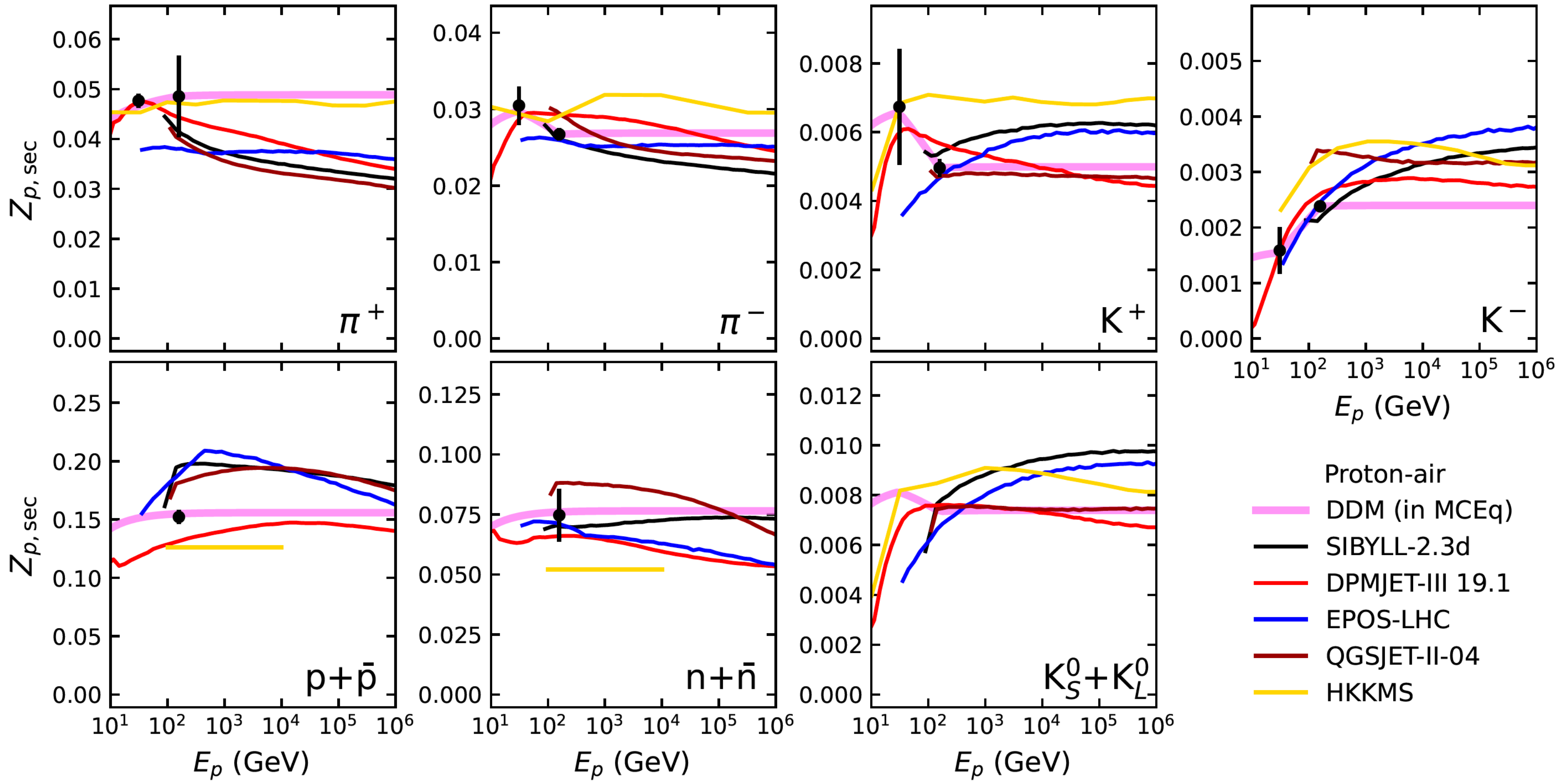
Fit dn/dx_L with splines, get covariance matrix



Fit pT in each x_F bin using $$\frac{dn}{dp_\perp} = a_0 p_\perp^{a_1}\ e^{a_2 p_\perp^{a_3}}$$



Energy inter- and extrapolation

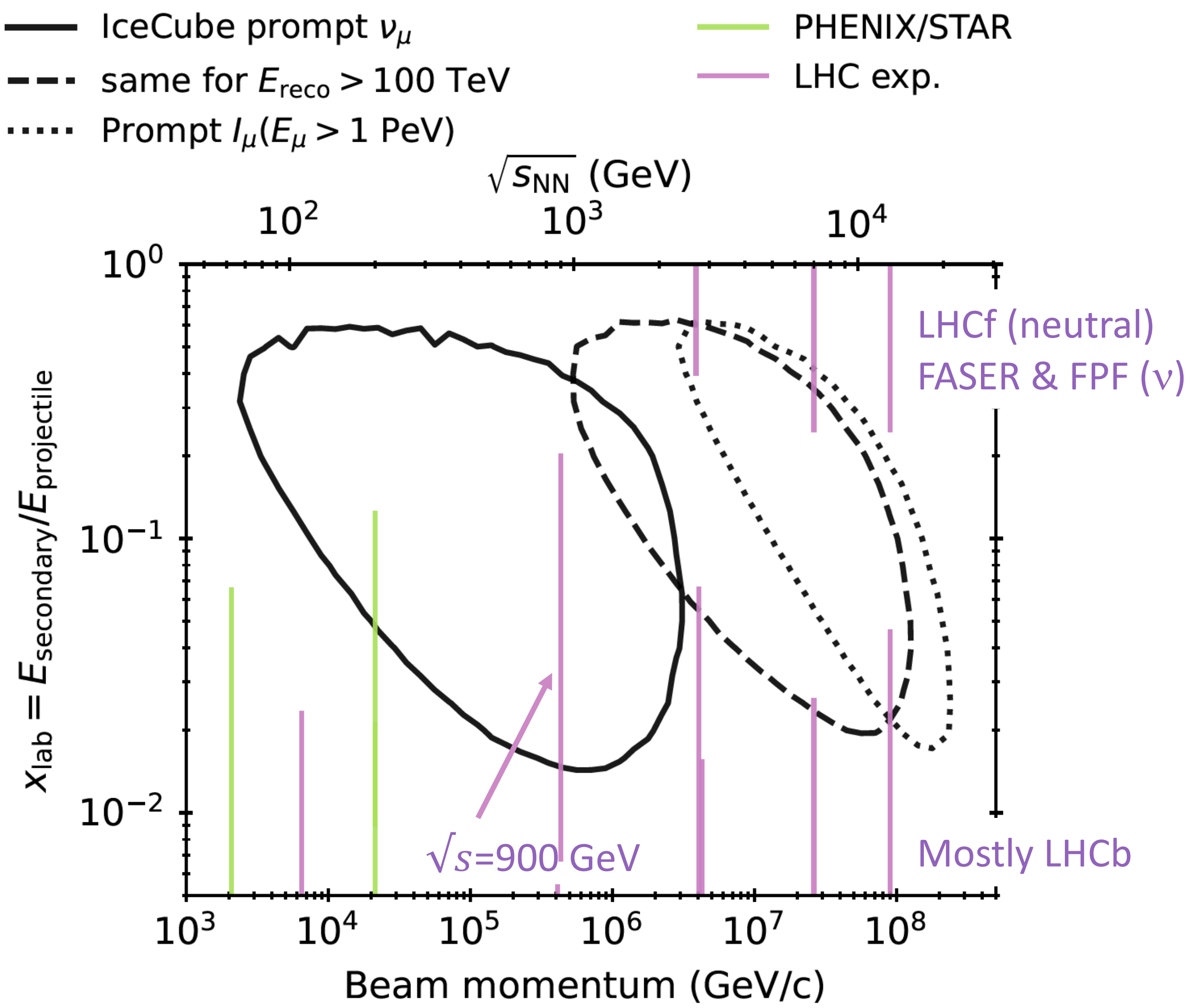


- 1 or 2 cross section "shapes" @ 31 & 158 GeV
- Interpolates linearly in log(E) between those
- Assumes Feynman scaling (shape of longitudinal spectrum constant)
- More points can be added to complicate energy dependence → **daemonflux**

Atm.-flux-relevant phase space
→ Spectrum-weighted moment:

$$Z_{Nh}(E_N) = \int_0^1 \mathrm{d}x_{\mathrm{Lab}}\; x_{\mathrm{Lab}}^{\gamma(E_N)-1} \frac{\mathrm{d}N_{N\to h}}{\mathrm{d}x_{\mathrm{Lab}}}(E_N)$$

Charm production cross section inaccessible to present-day colliders



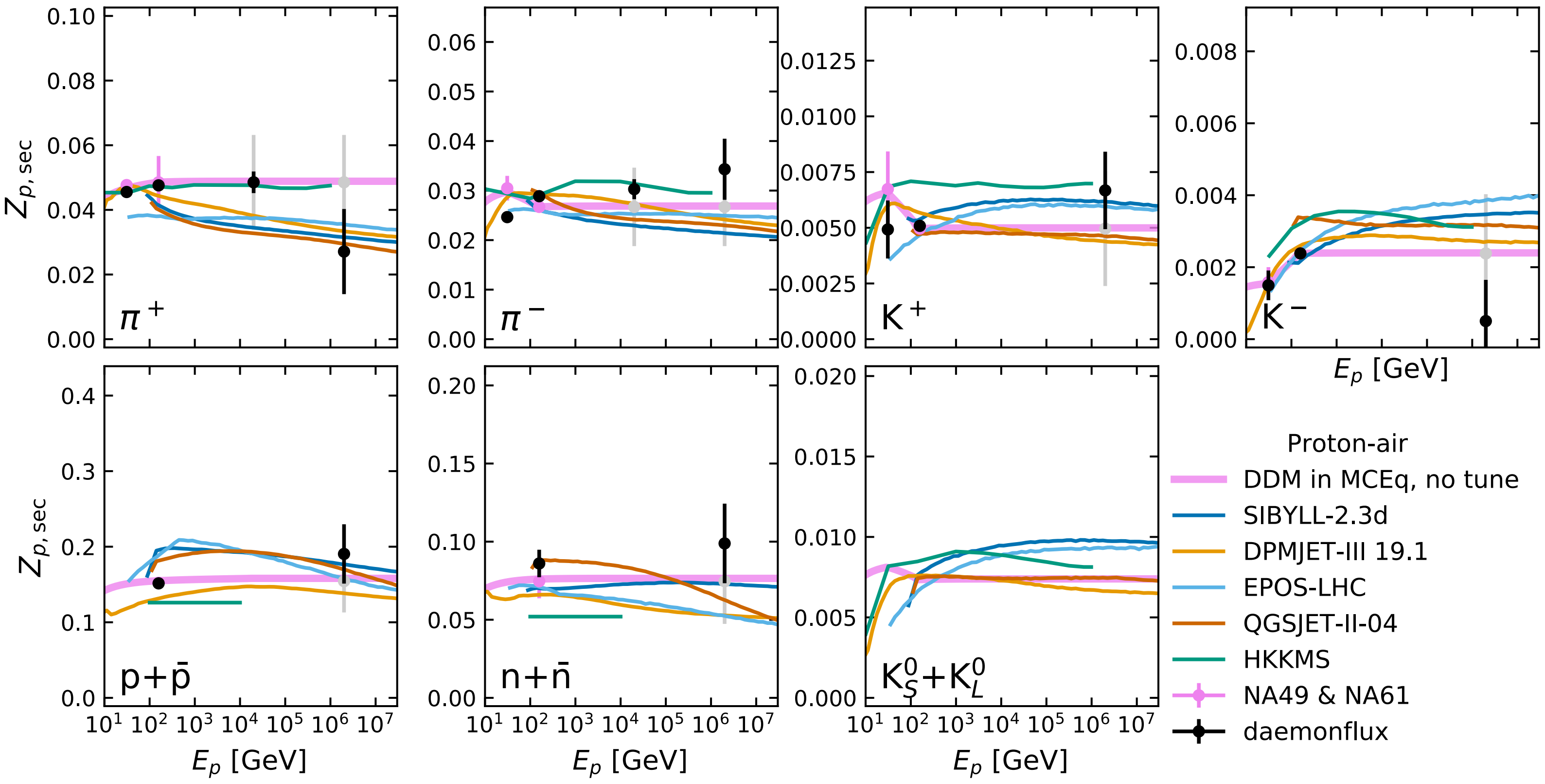
- Each line represents a collider running at fixed $\sqrt{s}$
- Gap in x between LHC coverage is due to the beam pipe
- Detectors need particle ID capability & sufficient luminosity
- Indirect constraints from new forward detectors like FASER and the proposed FPF (see 2203.05090)
- New insights expected from proton-oxygen collisions in Run3

AF & M. Huber, arXiv:2205.14766

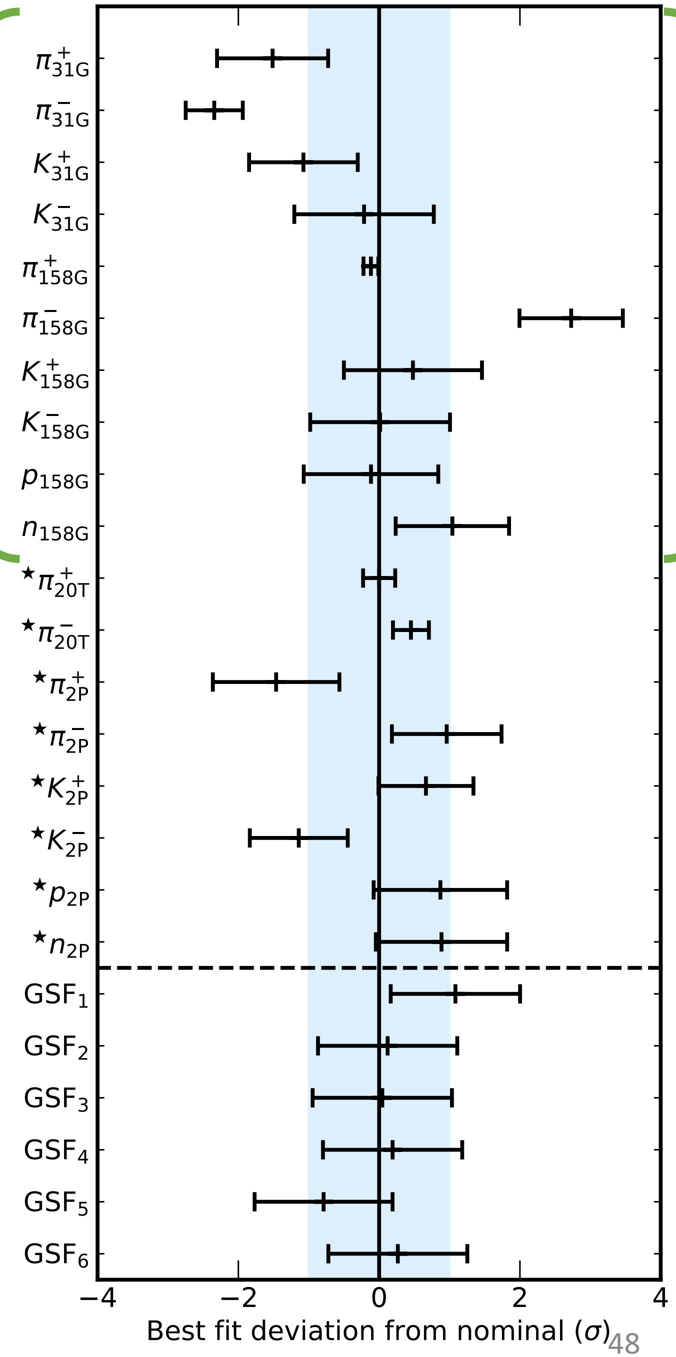
Neutrino contours: 90% events in IceCube

Muon contours: 90% of integral flux

Fitted parameter values



Accelerator constrained

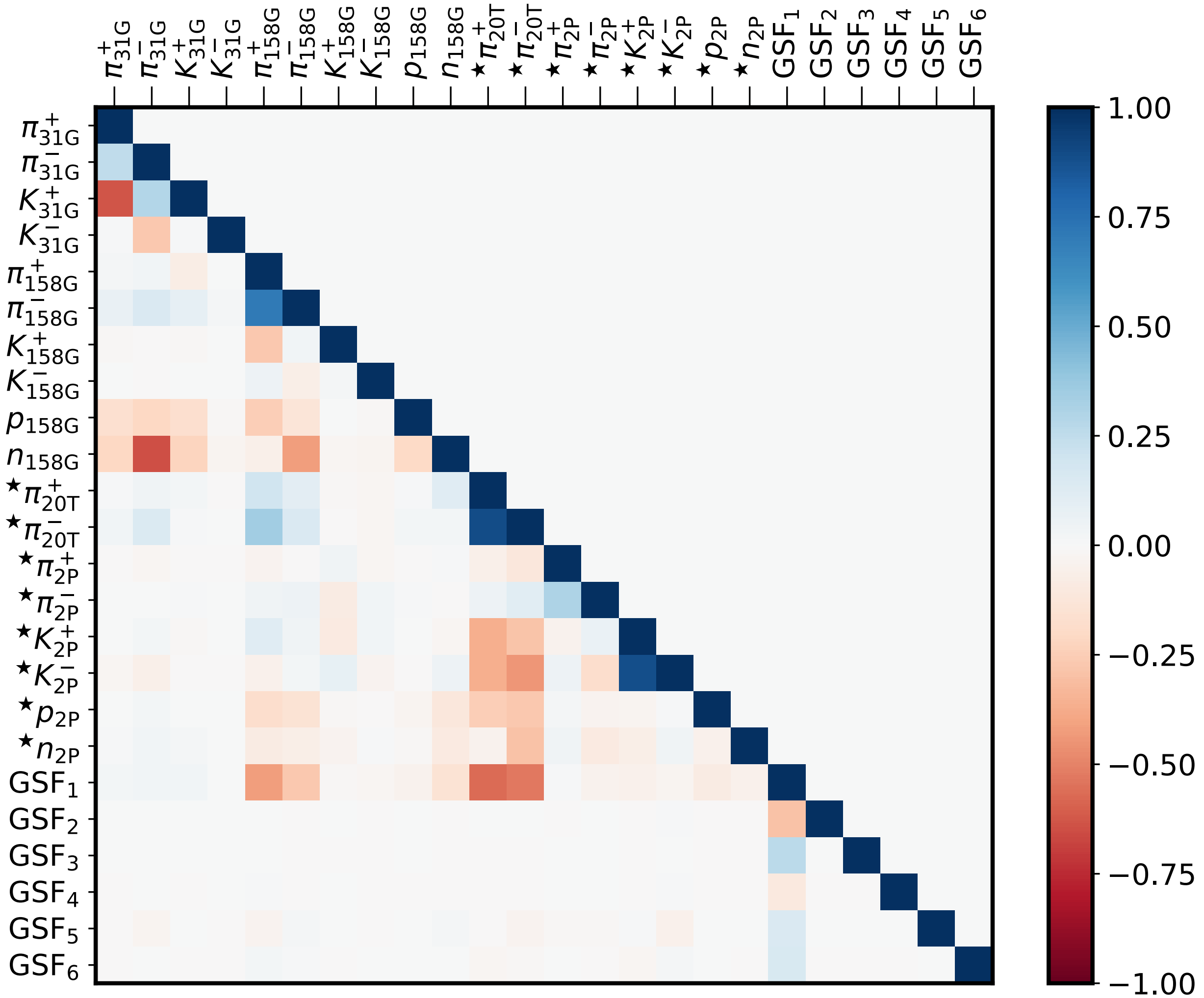


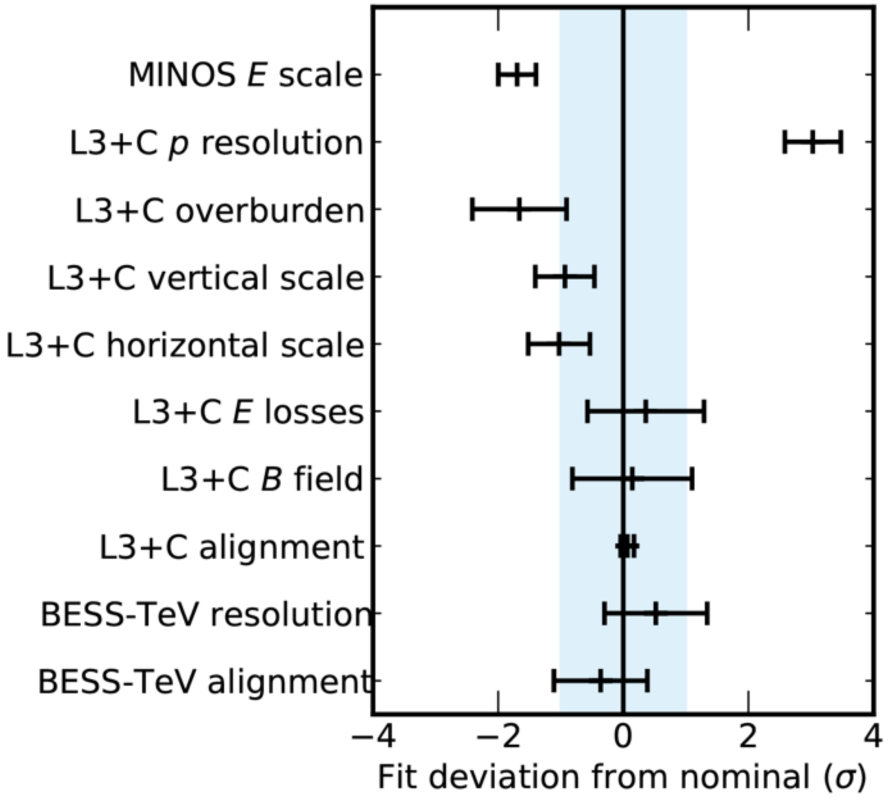
Atm.-flux-relevant phase space

→ Spectrum-weighted moment:

$$Z_{Nh}(E_N) = \int_0^1 \mathrm{d}x_{\mathrm{Lab}}\; x_{\mathrm{Lab}}^{\gamma(E_N)-1} \frac{\mathrm{d}N_{N\to h}}{\mathrm{d}x_{\mathrm{Lab}}}(E_N)$$

Systematic parameters and Fit quality





Physics parameter part of the correlation matrix: Total 34 parameters: 18 hadrons + 6 GSF + 10 experimental

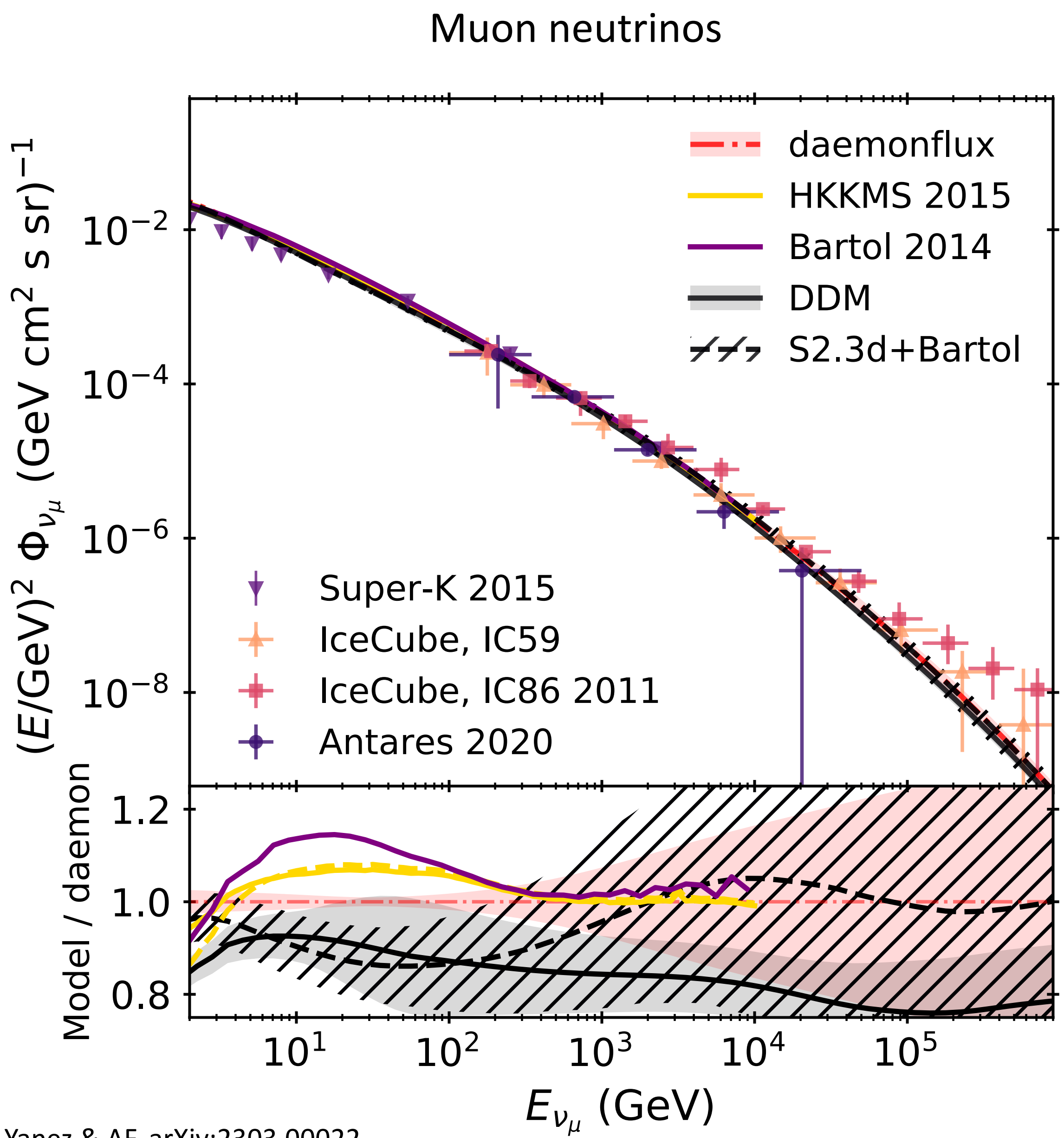
Chi^2 199/ 217 dof (approximate)
P-value = 81%

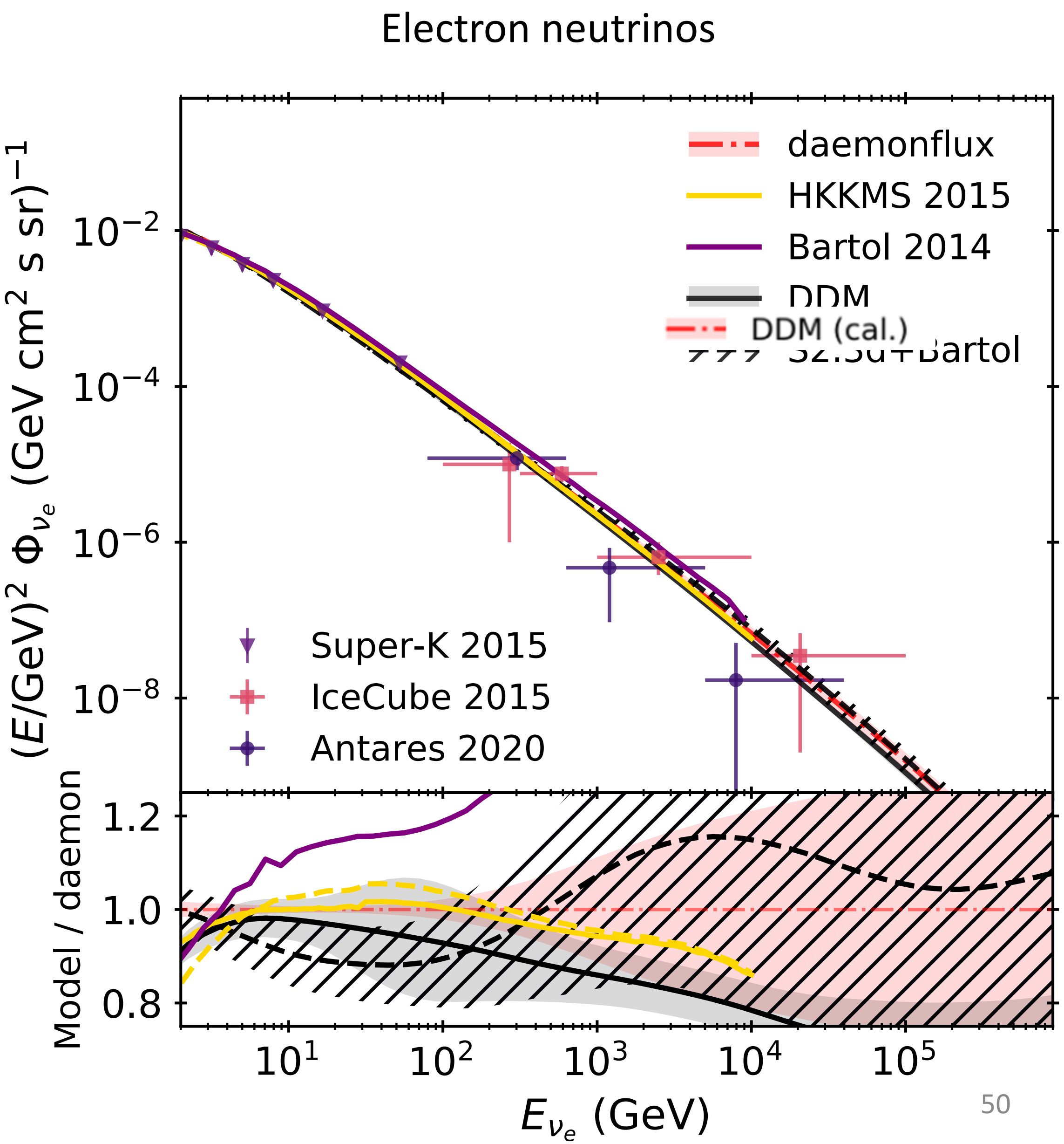
J. P. Yanez & AF, arXiv:2303.00022

Neutrino fluxes

hatched area: uncertainty from

Barr et al. PRD74, 094009 (2006) & AF, Huber PRD (2022)



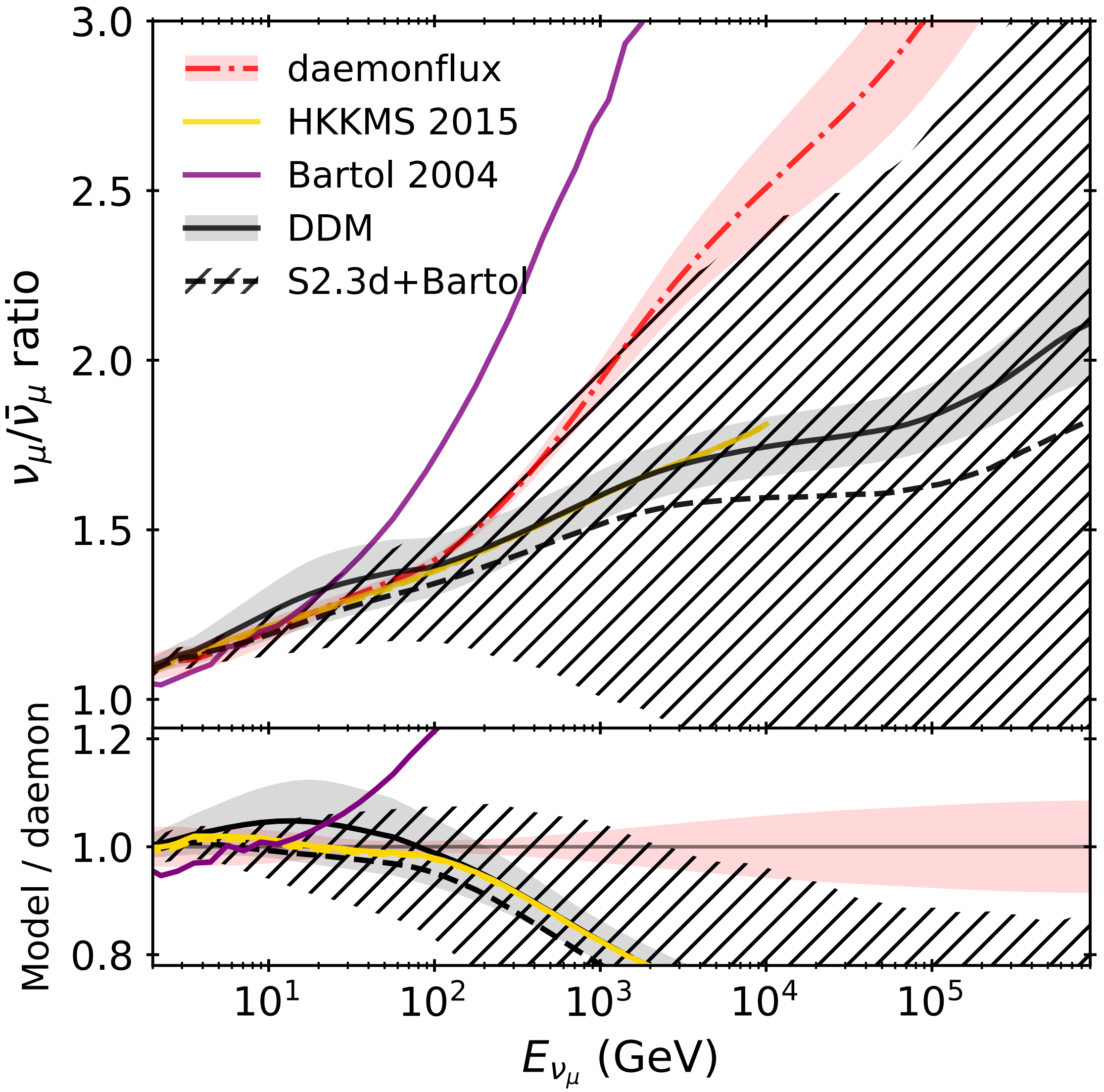


J. P. Yanez & AF, arXiv:2303.00022

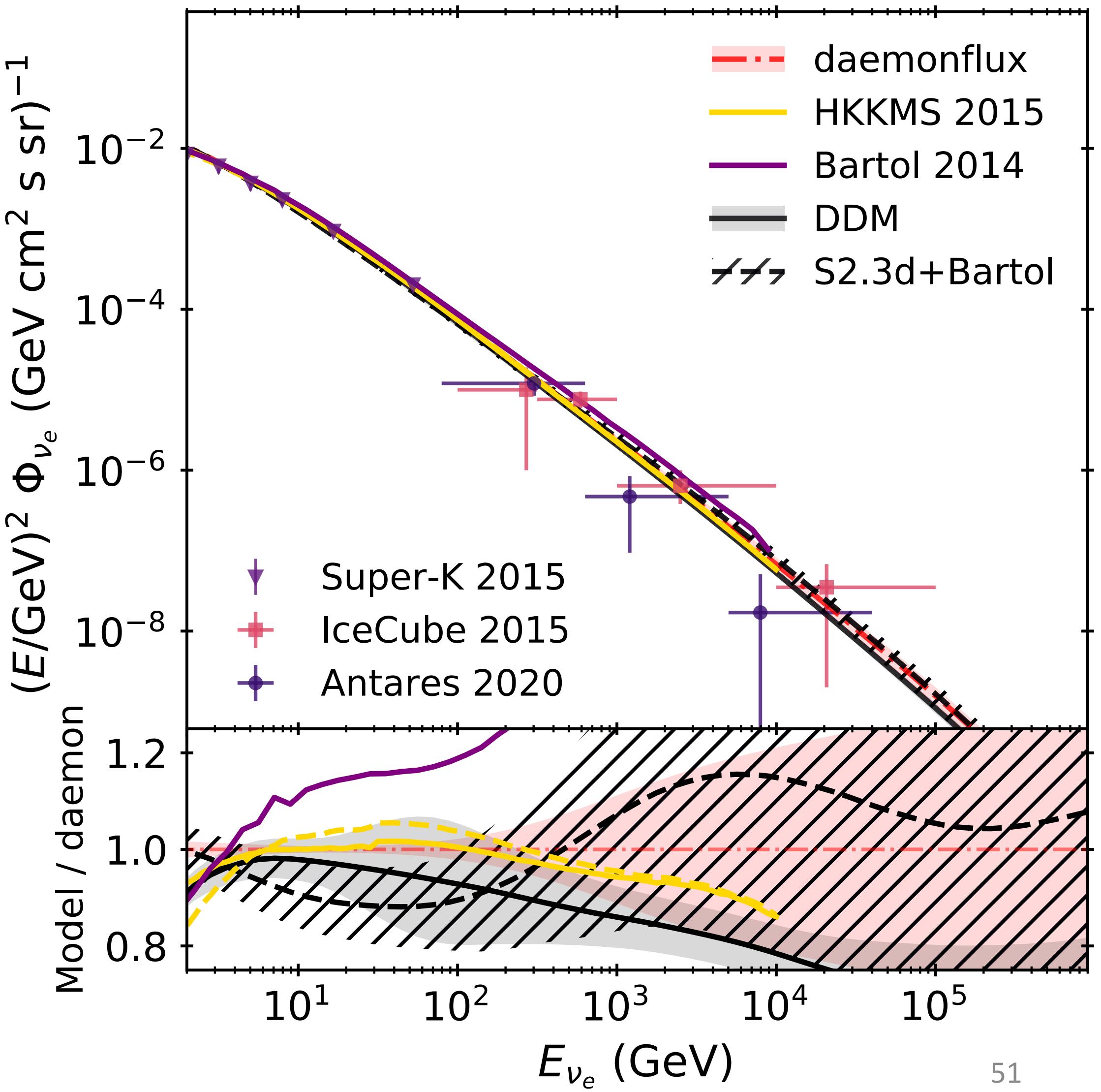
Neutrino ratios

hatched area: uncertainty from
Barr et al. PRD74, 094009 (2006) & AF, Huber PRD (2022)

Numu/numubar ratio

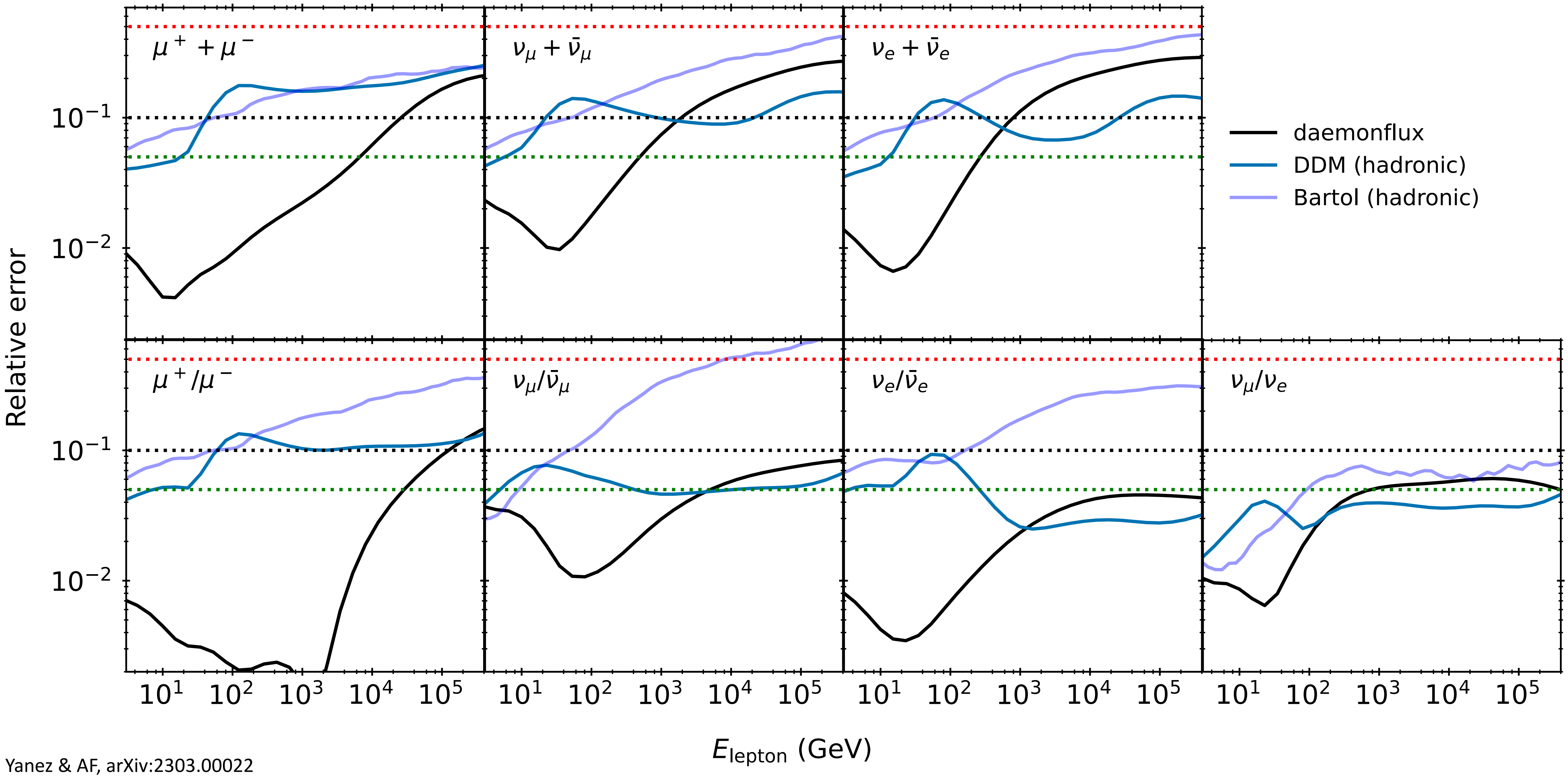


Flavor ratio



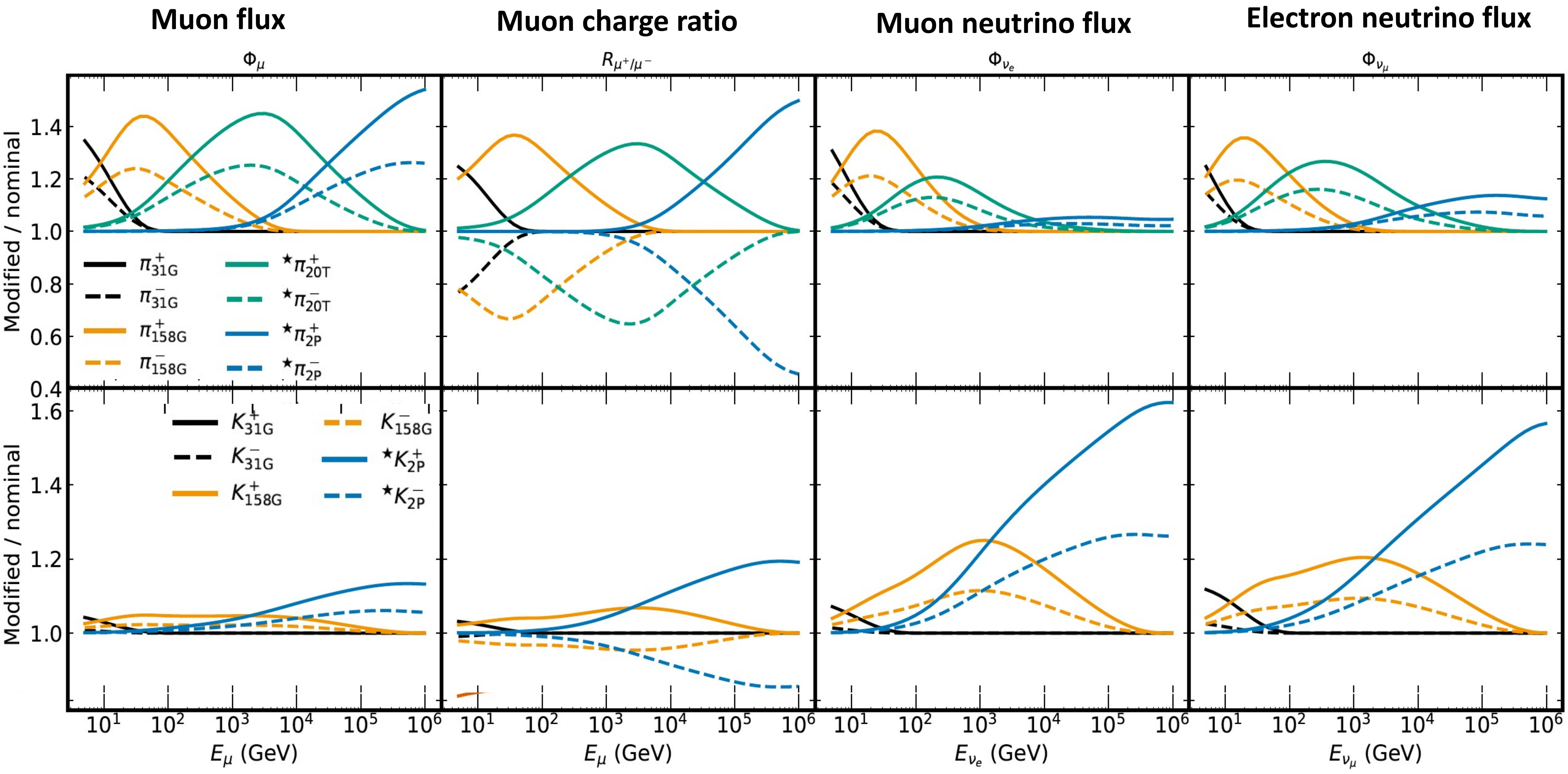
J. P. Yanez & AF, arXiv:2303.00022

Total uncertainty of daemonflux (DDM+GSF+Fit)



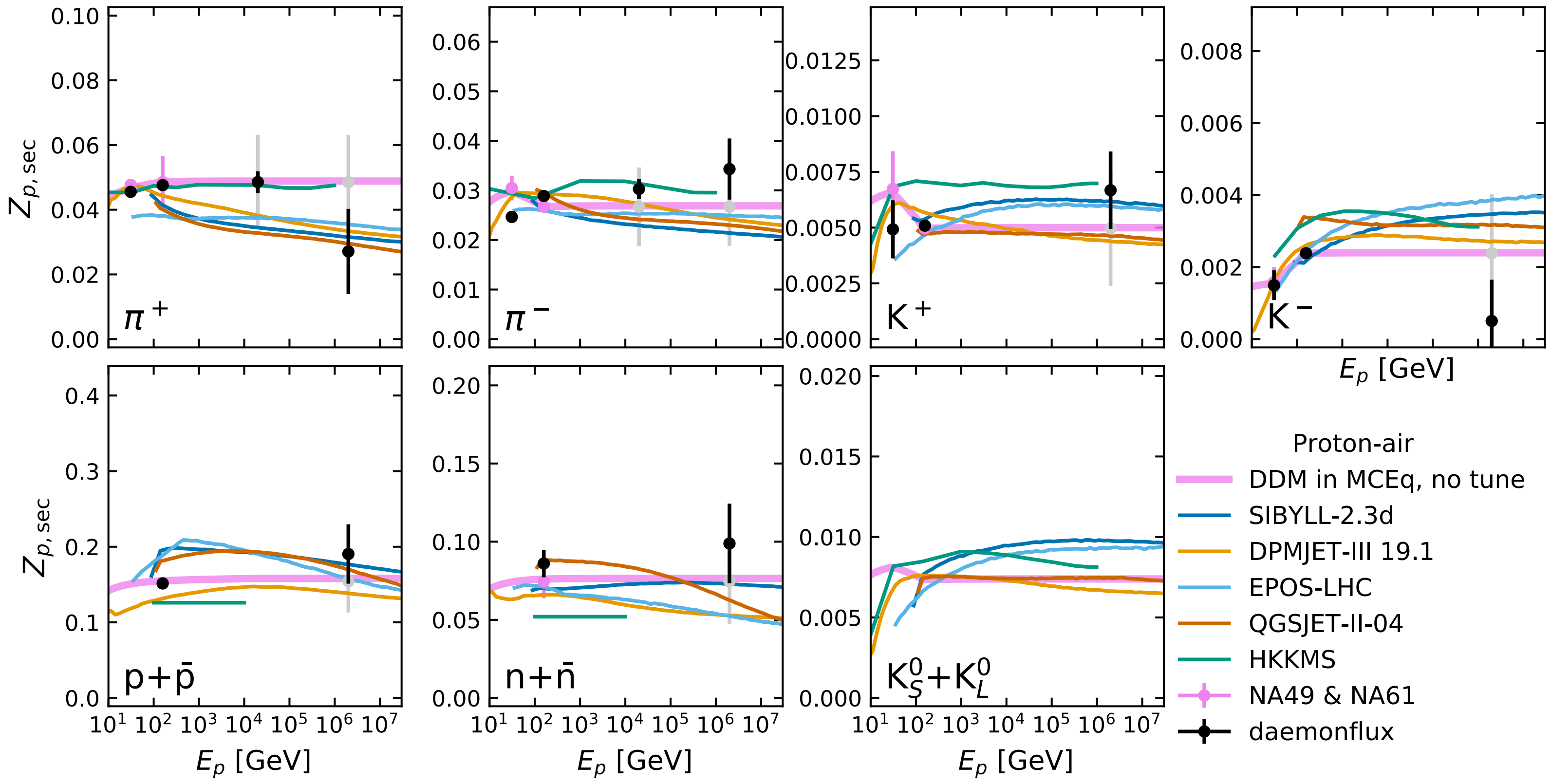
J. P. Yanez & AF, arXiv:2303.00022

Choice of extrapolation parameters above “DDM energies”

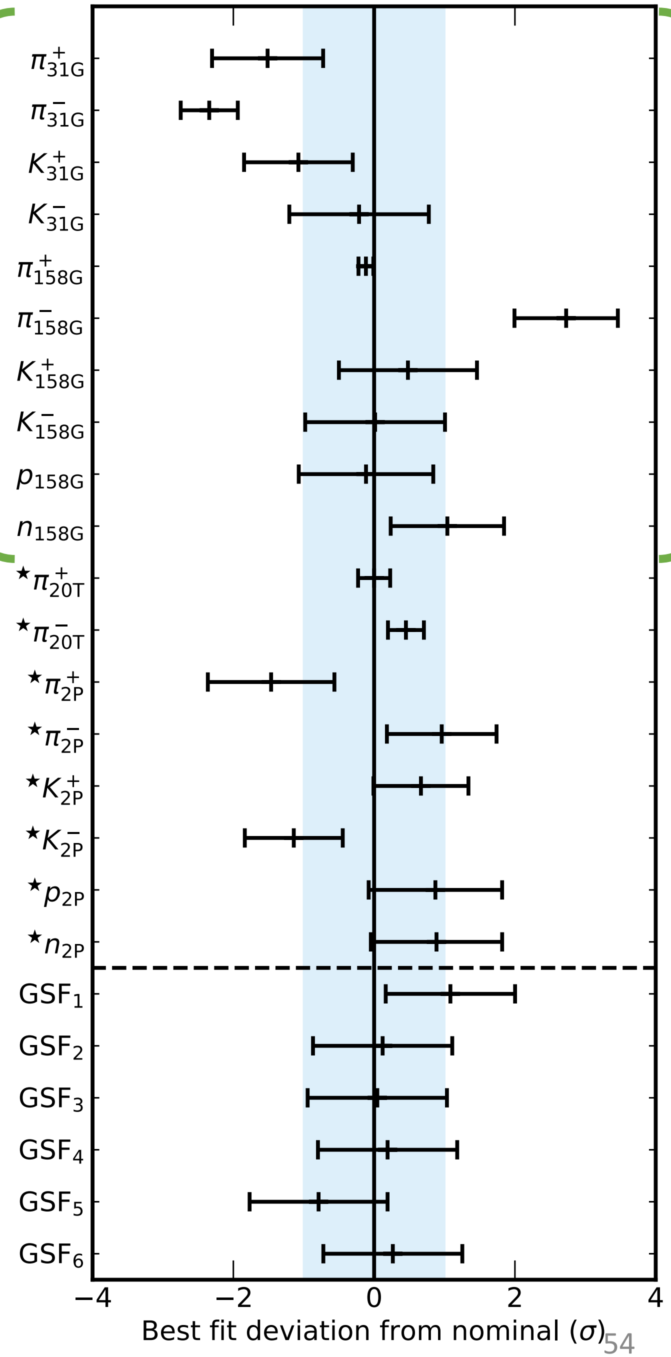


J. P. Yanez & AF, arXiv:2303.00022

Fitted parameter values



Accelerator constrained



J. P. Yanez & AF, arXiv:2303.00022

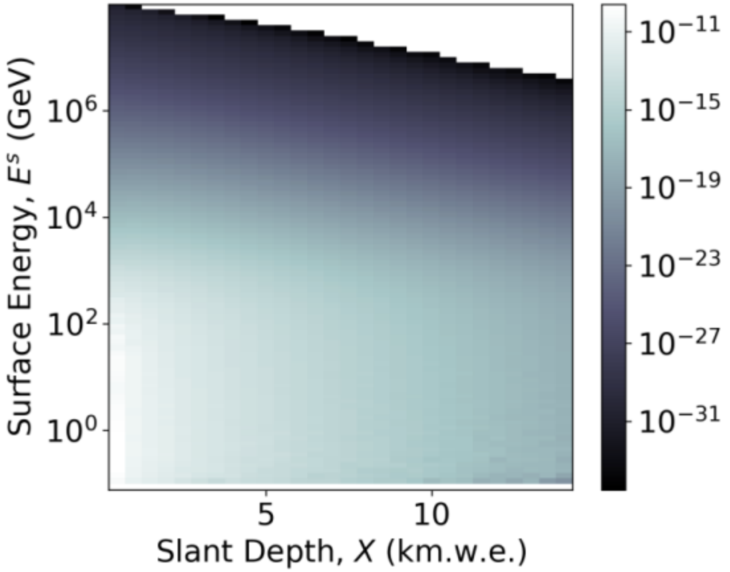
MUTE (Muon inTnsity codE): fast convolutions

AF, **W. Woodley**, M.-C. Piro, *ApJ* **928** 27 (2022)
W. Woodley, TeVPa 2022 and Woodley, AF, Piro in prep.

https://github.com/wjwoodley/mute

$$\Phi^u(E_j^u, X_k, \theta_k) = \sum_i \Phi^s(E_i^s, \theta_k) P(E_i^s, E_j^u, X_k) \left(\frac{\Delta E_i^s}{\Delta E_j^u}\right)$$

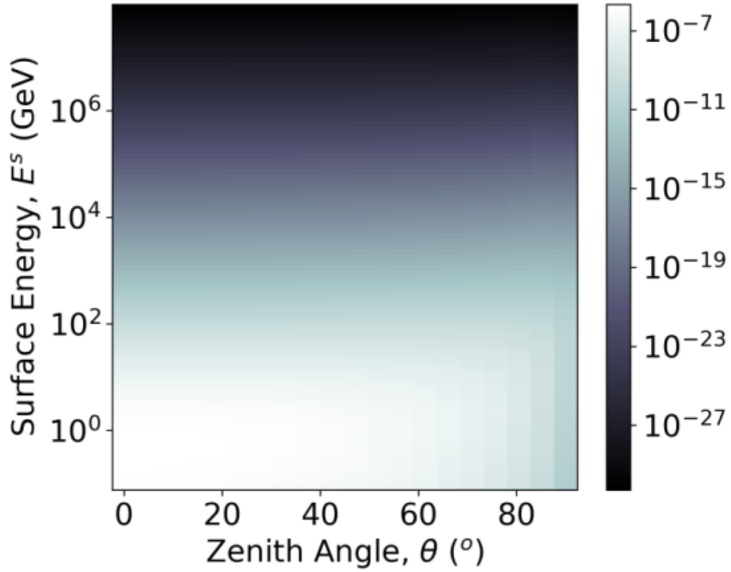
Φ^u



MUTE
Underground Fluxes

=

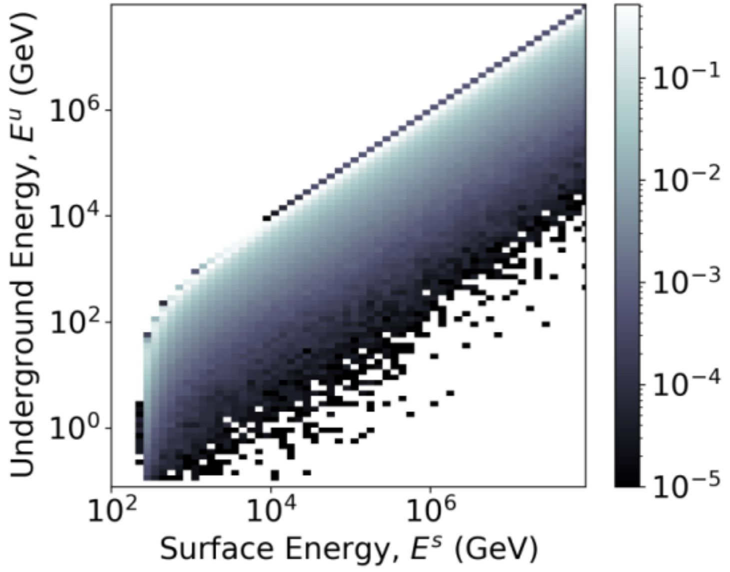
Φ^s



MCEq
Surface Fluxes

*

P



PROPOSAL
Transfer Tensor