Localization, Revivals, and Ballistic spread in One-dimensional Electric Quantum Walks

Muhammad Sajid

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Outline

- Introduction to quantum walks (definitions, and properties
- Applications: A versatile tool for quantum simulation
- Quantum walks with artificial electric field

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Classical random walk

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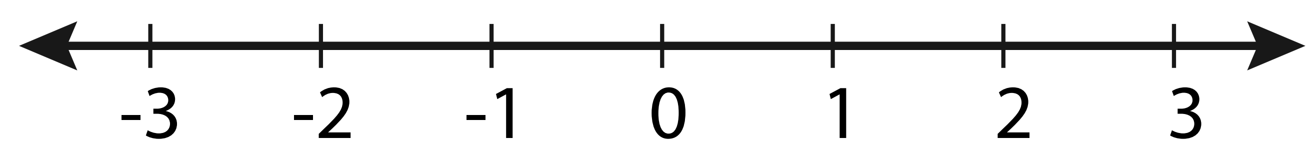
https://www.mit.edu/~kardar/teaching/projects/chemotaxis(AndreaSchmidt)/random.htm

Classical random walk

Random walk on an integer line:



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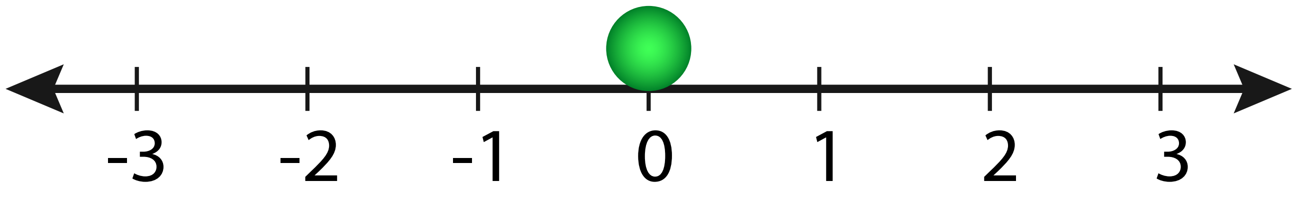

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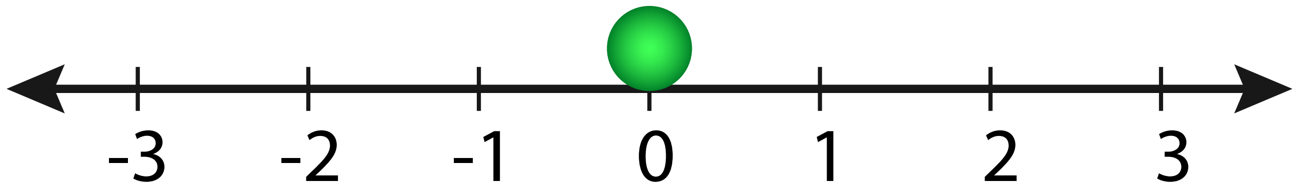

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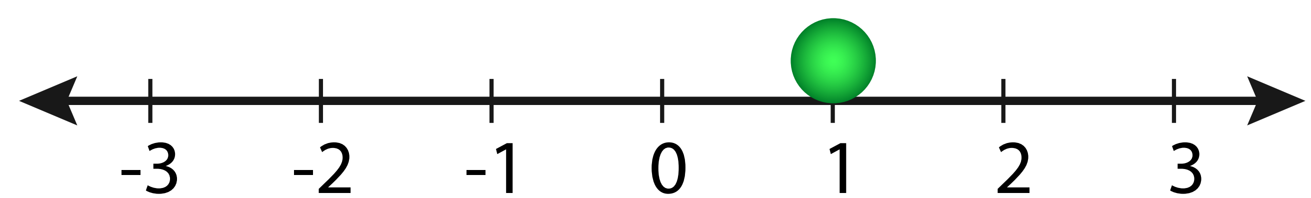

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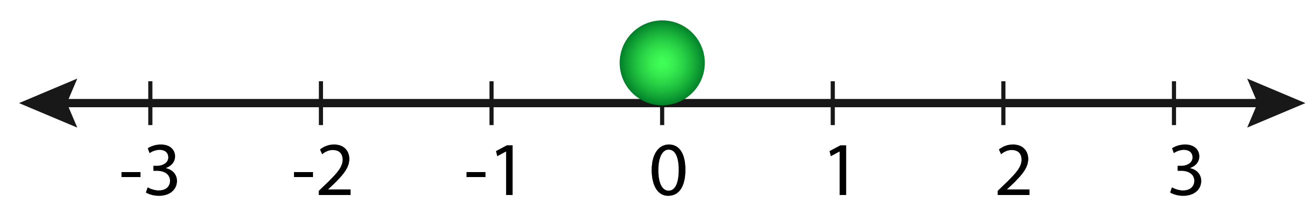

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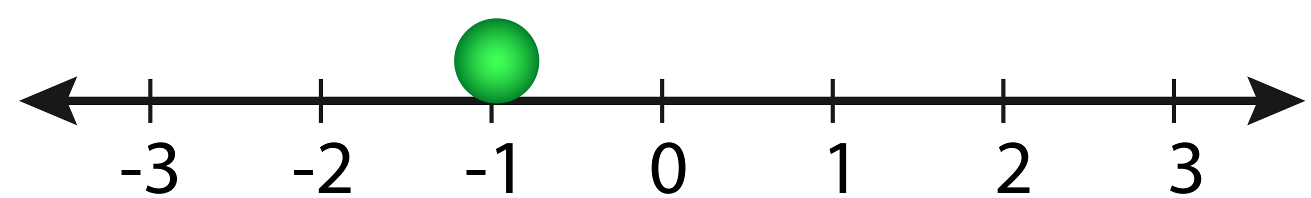
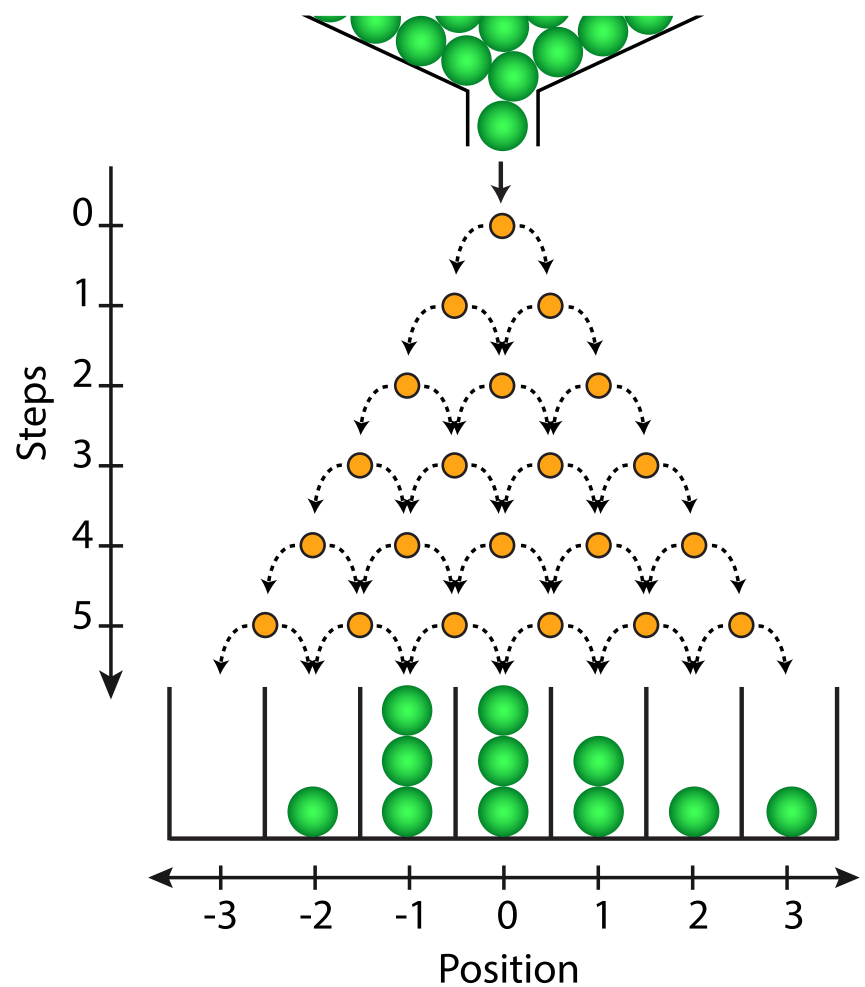
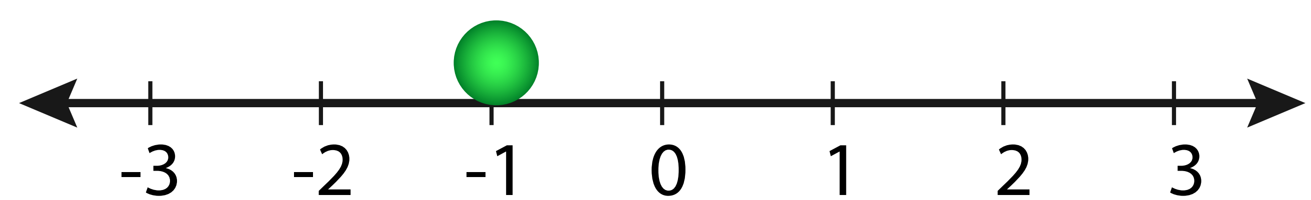


Classical random walk

Random walk on an integer line:

Galton board





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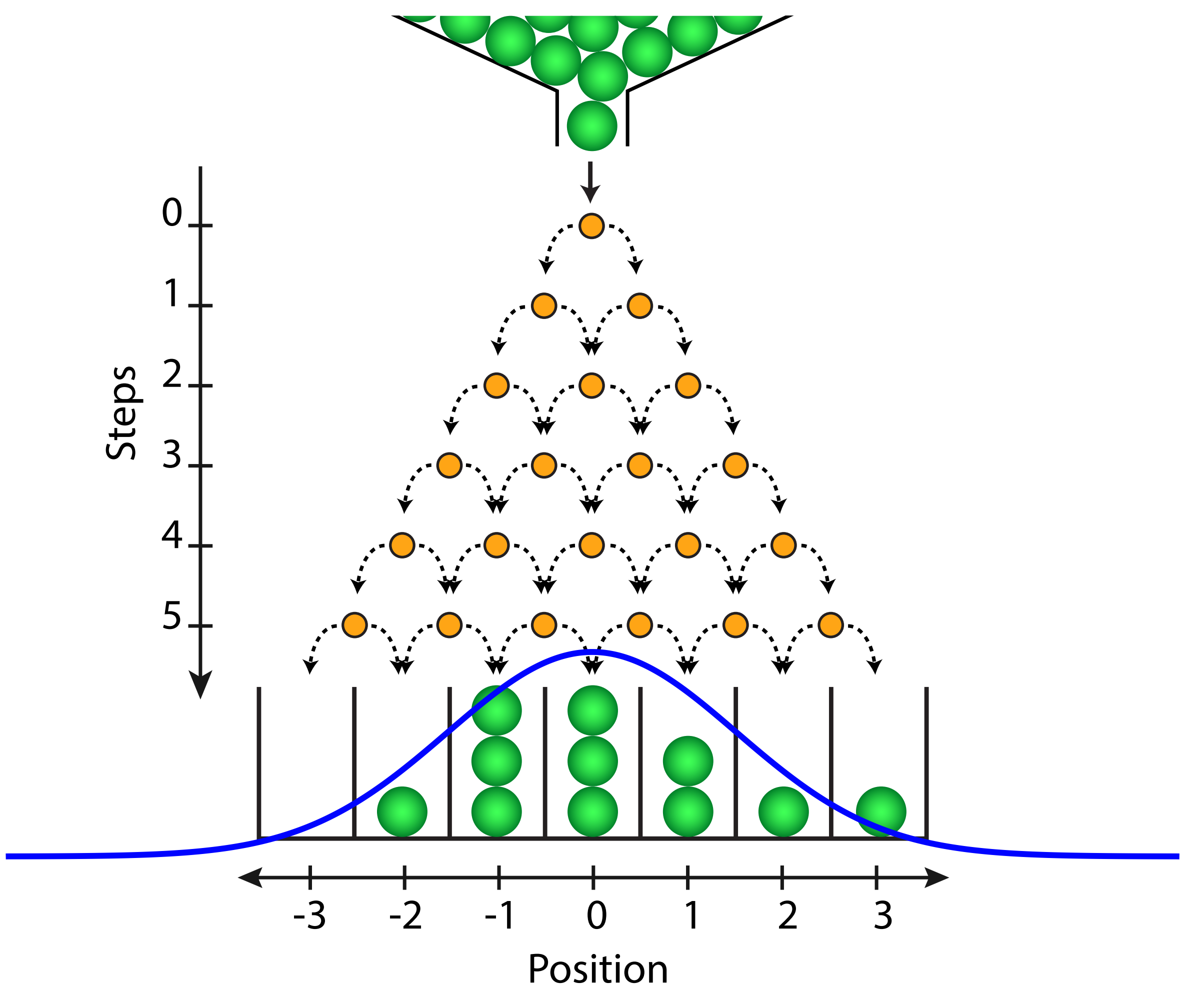
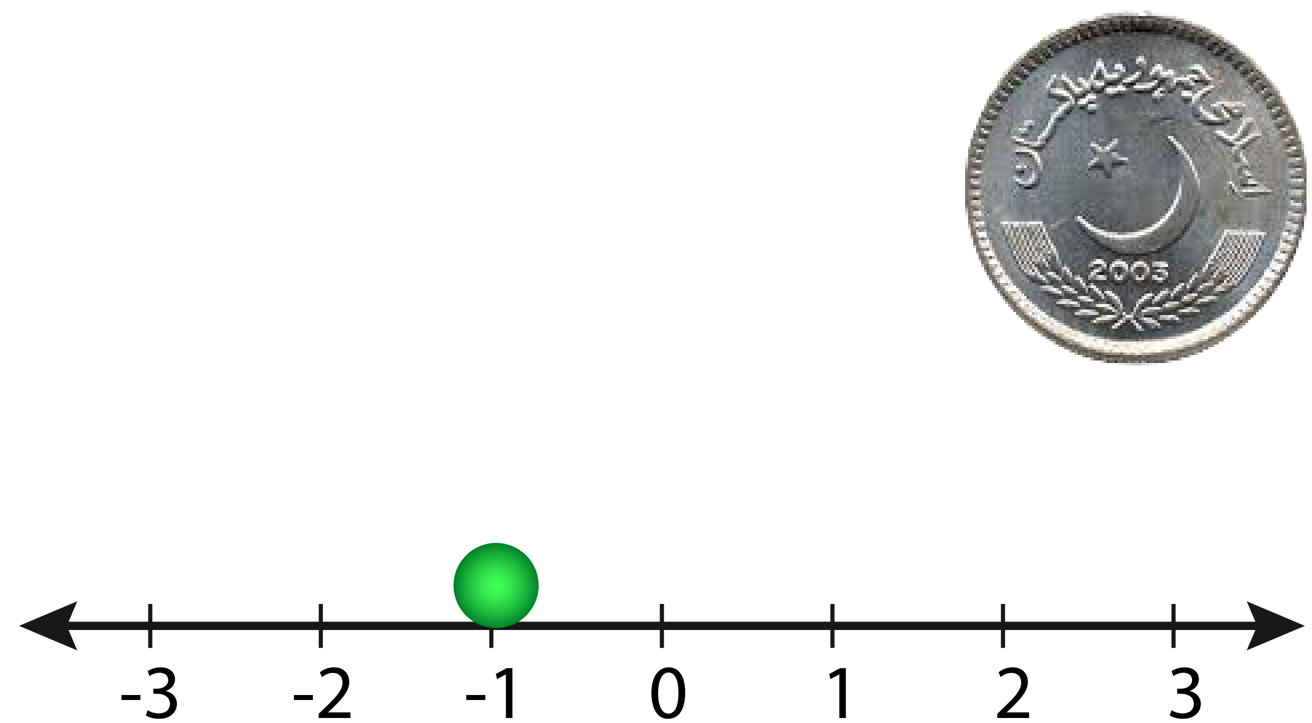


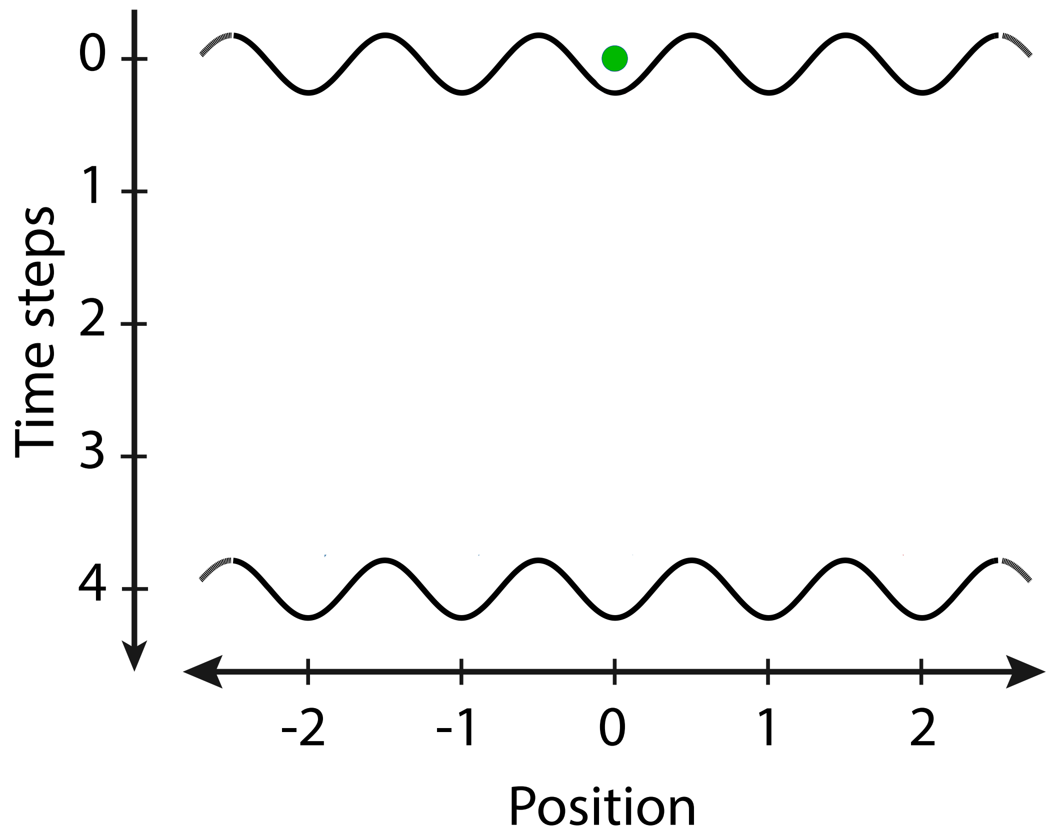


Classical random walk

Classical random walks have many applications in various fields of science.

Physics (Brownian motion) , economics (Price forecast in stock market), computer science (as basic framework for search algorithms).

Quantum walk in 1D



Karski et al., Science 325, 174 (2009)
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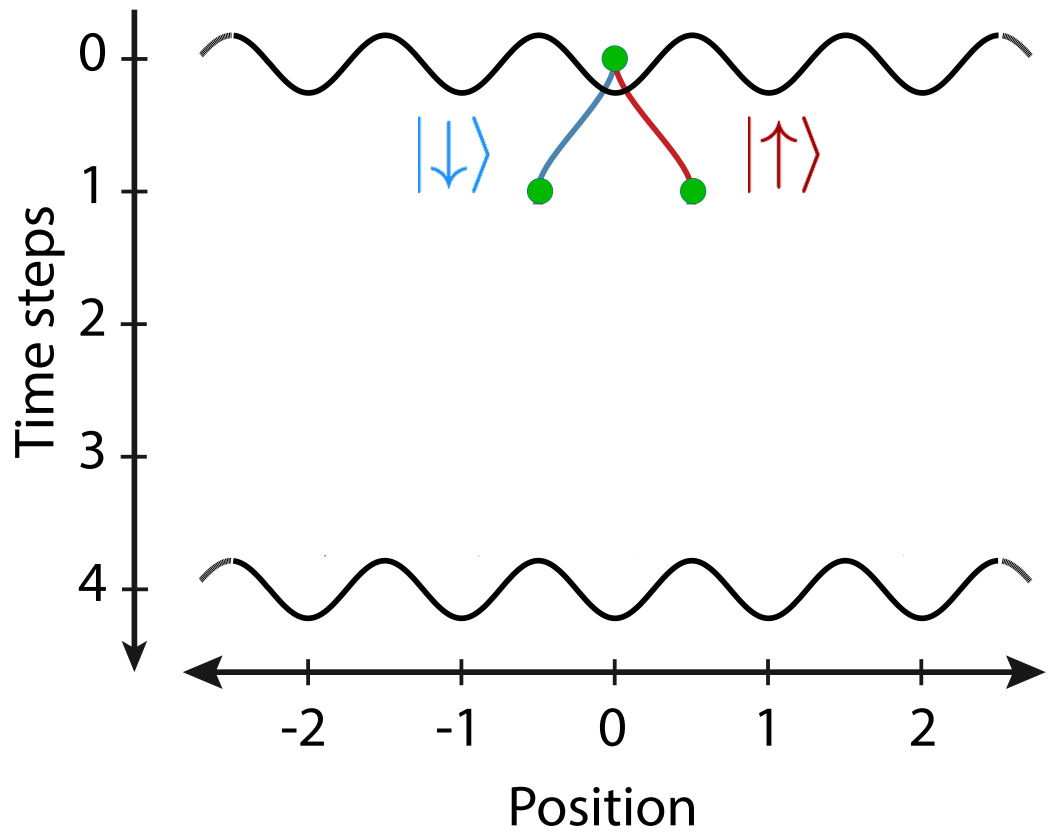

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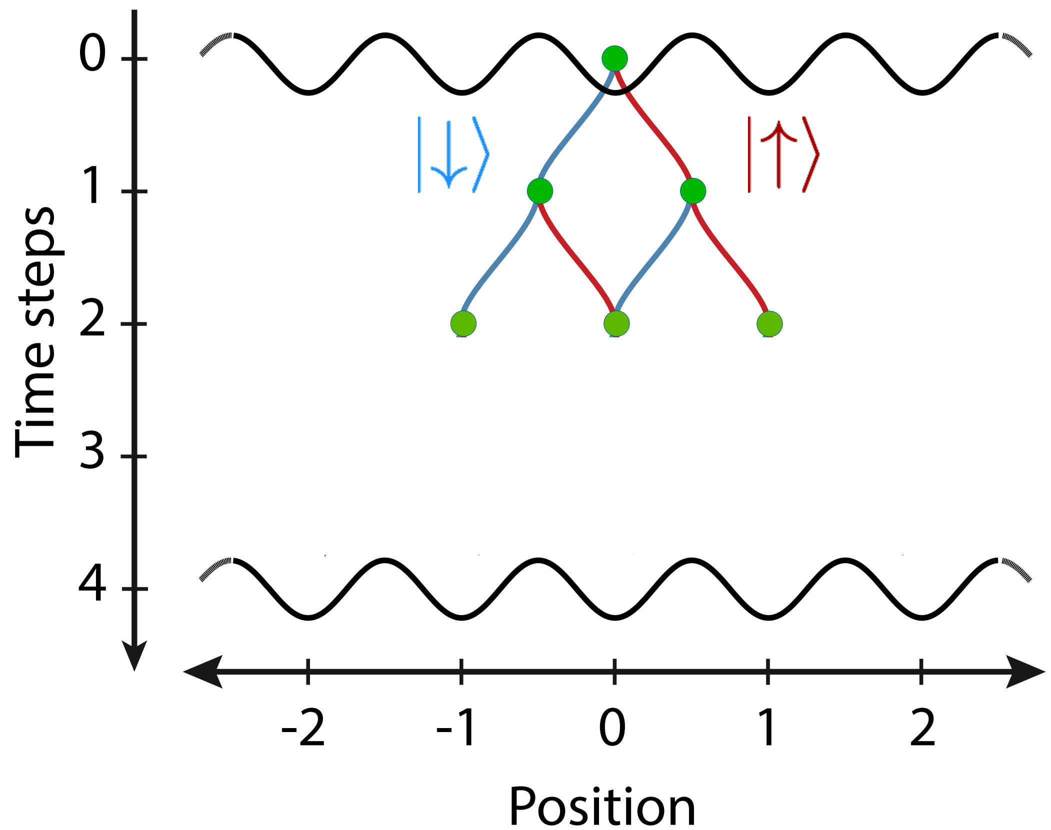

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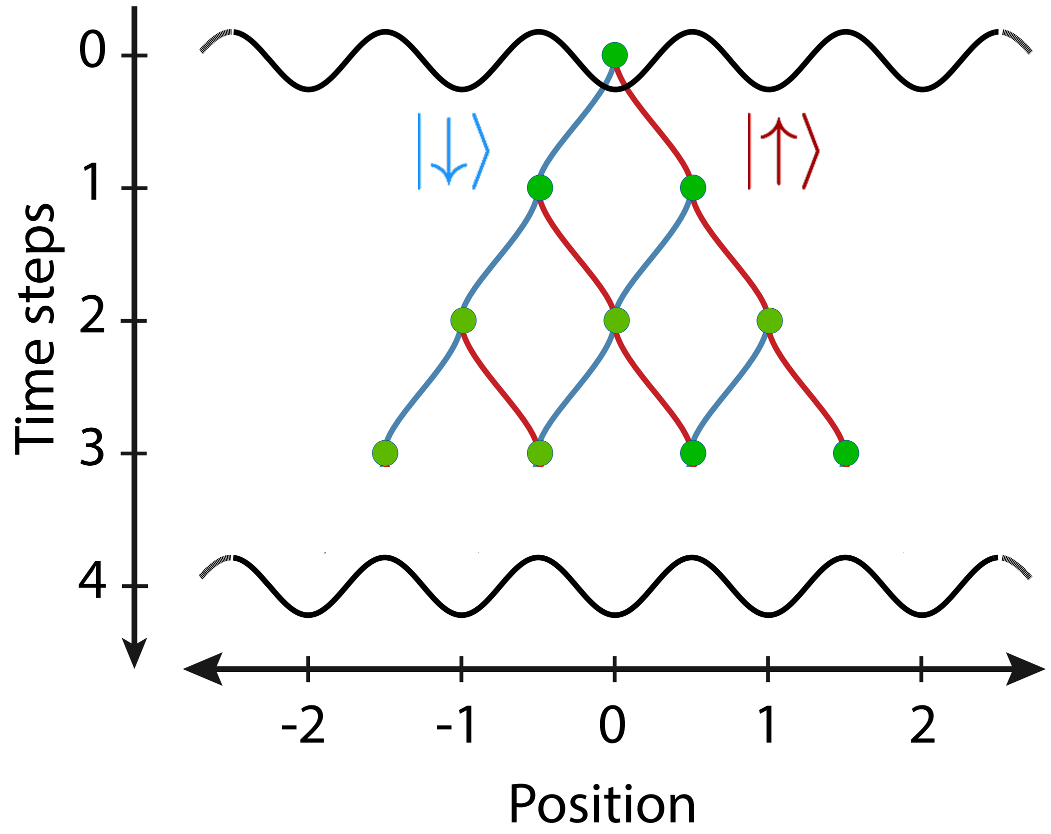

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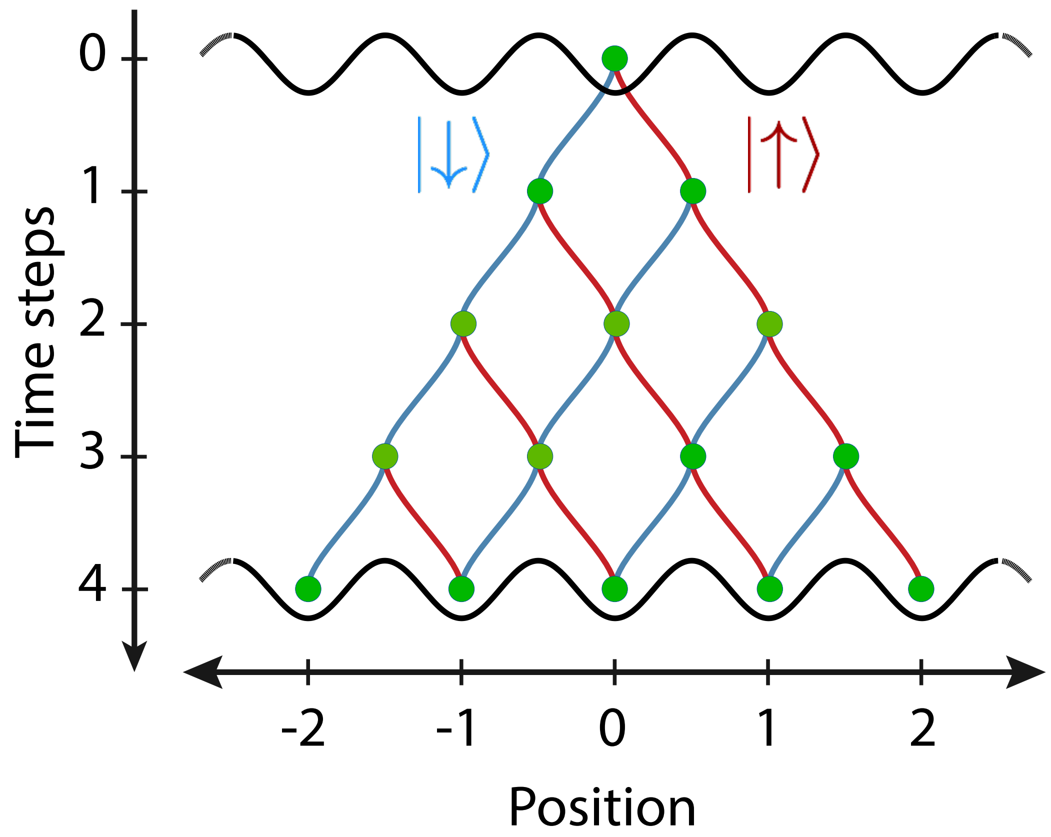

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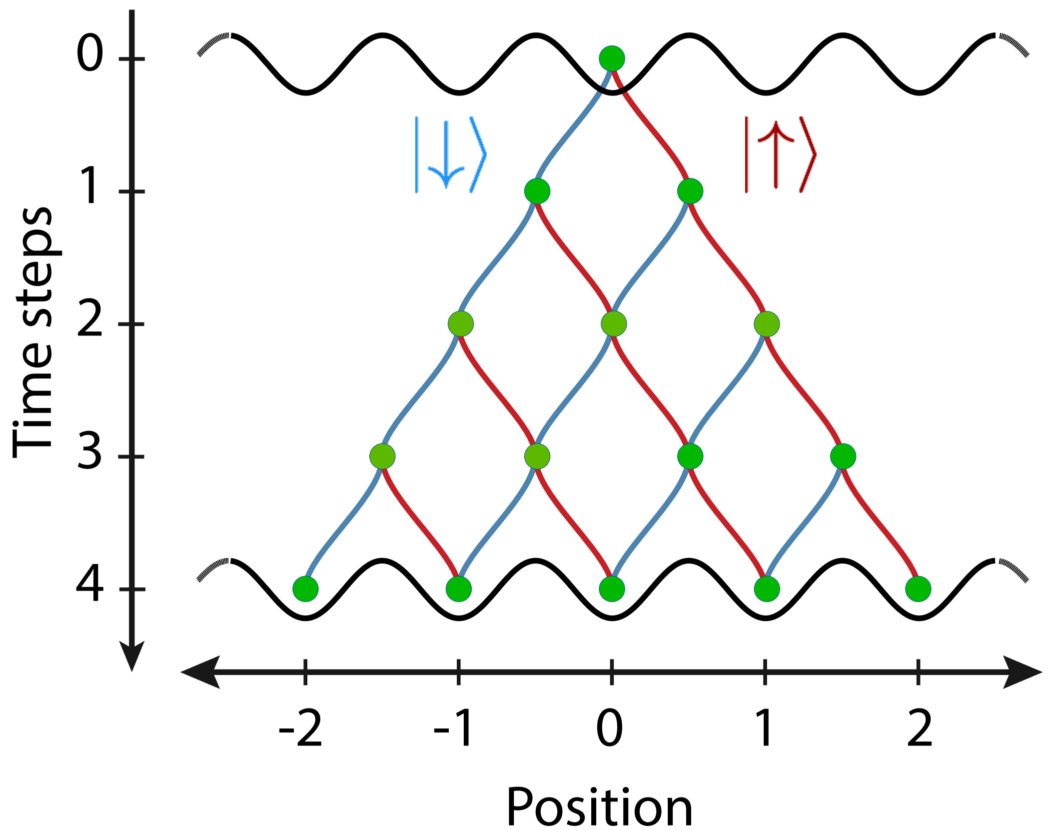

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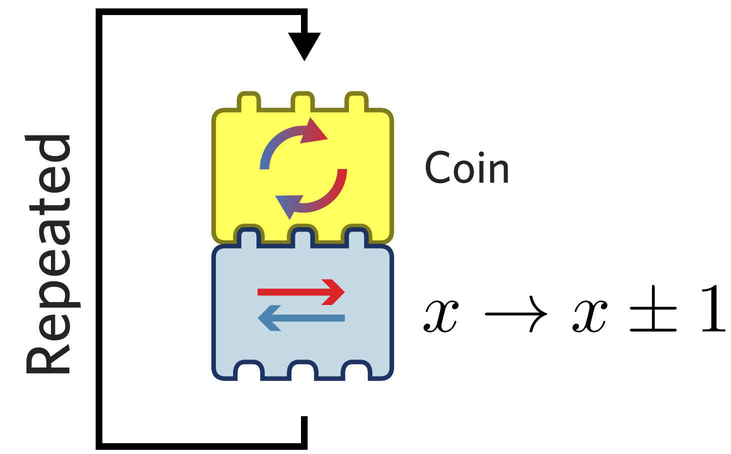

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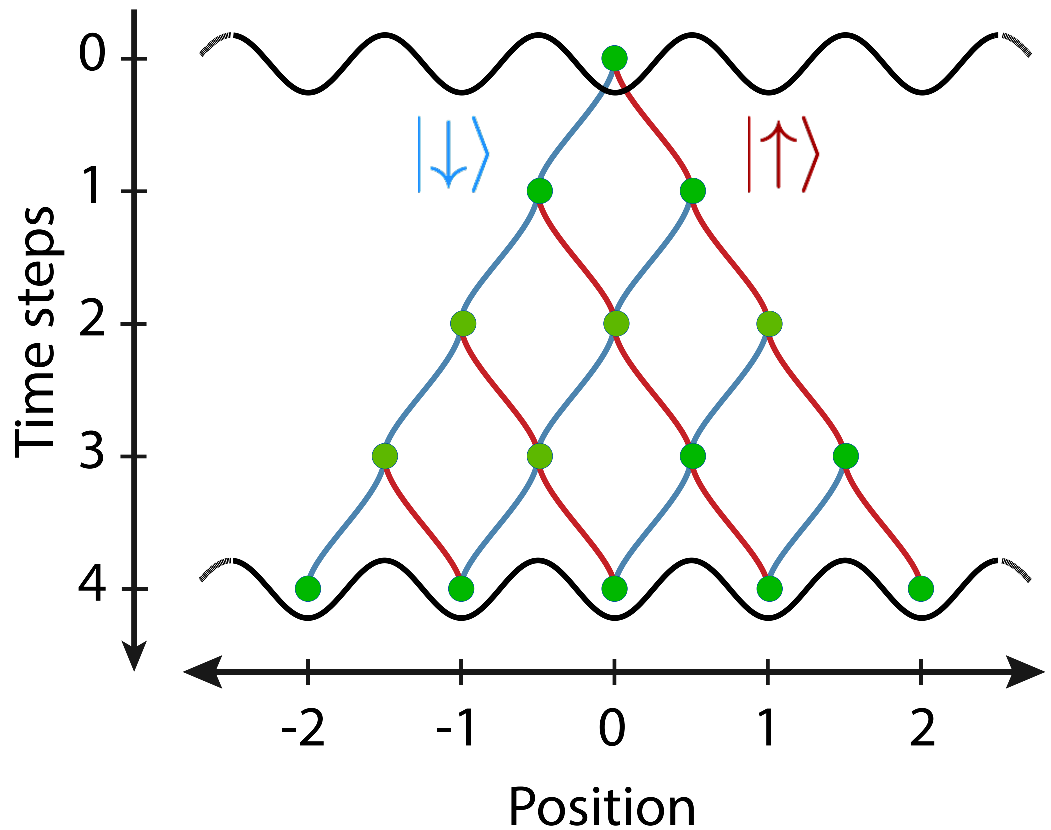

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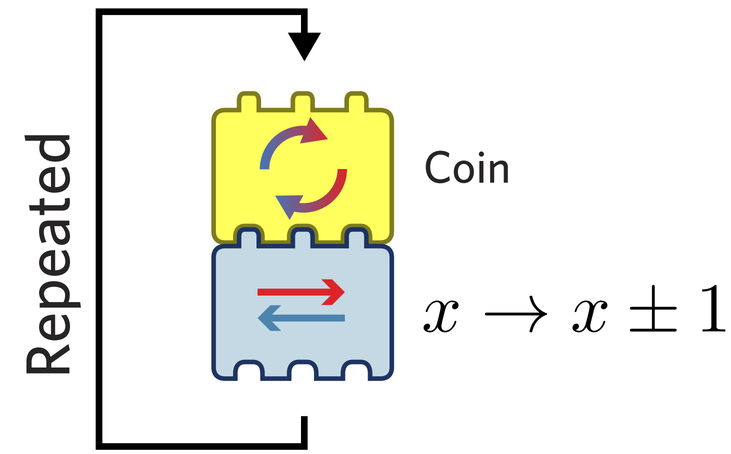

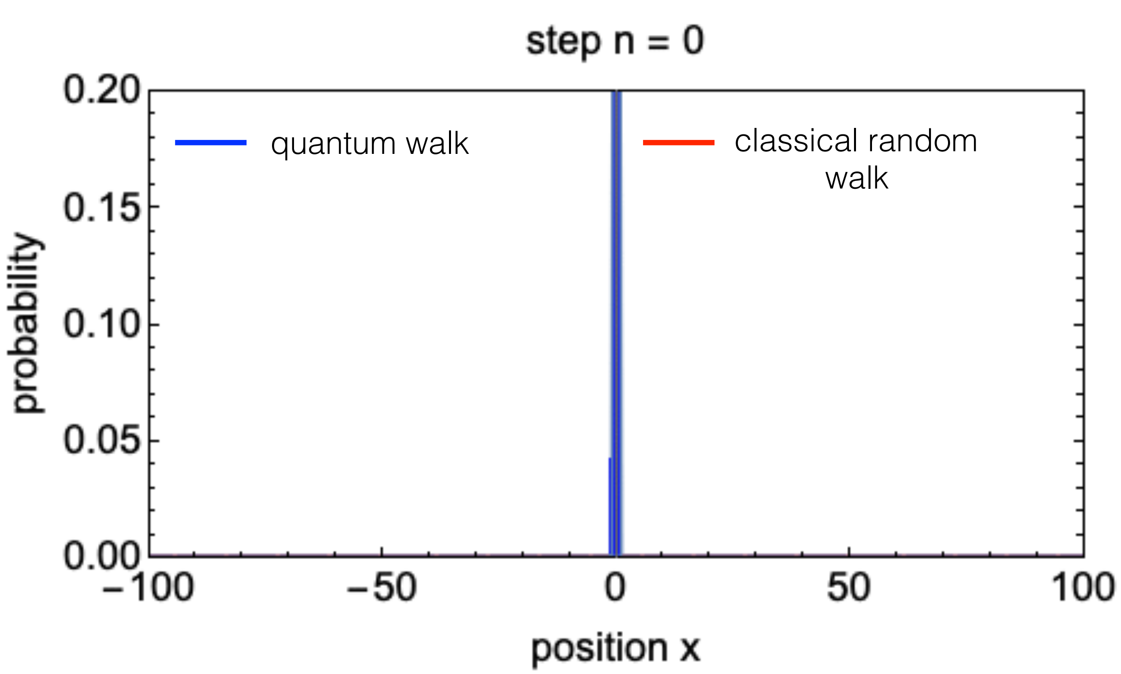

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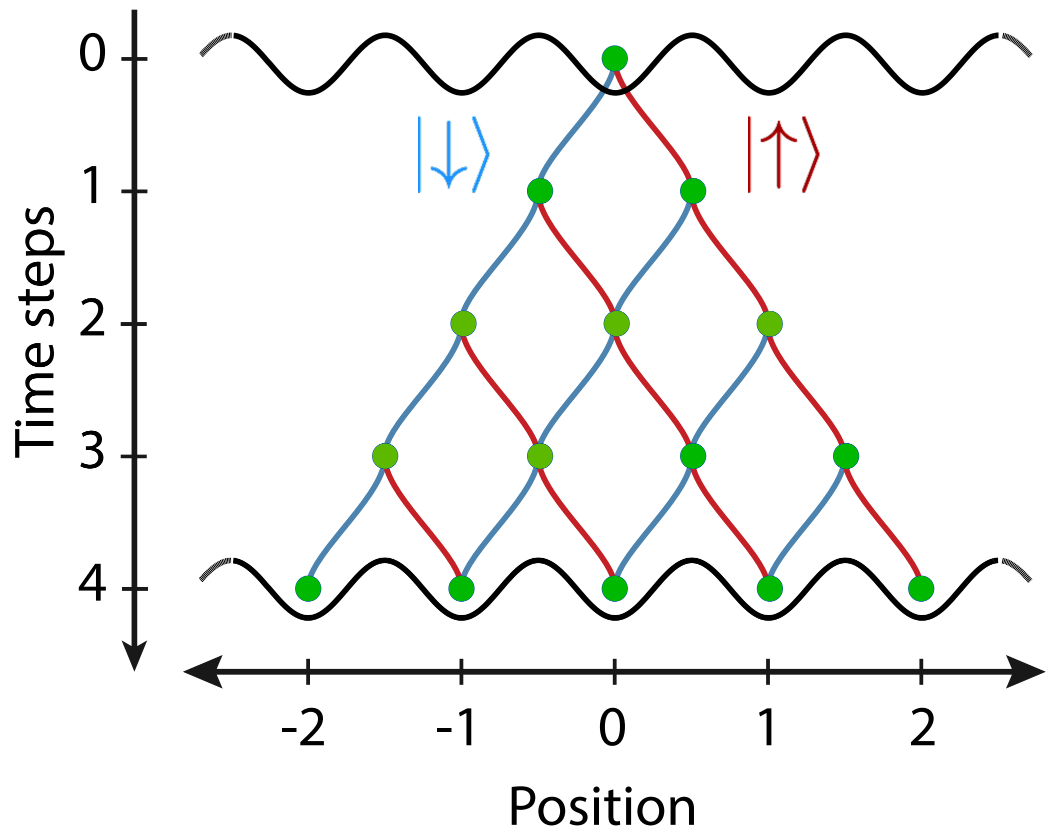

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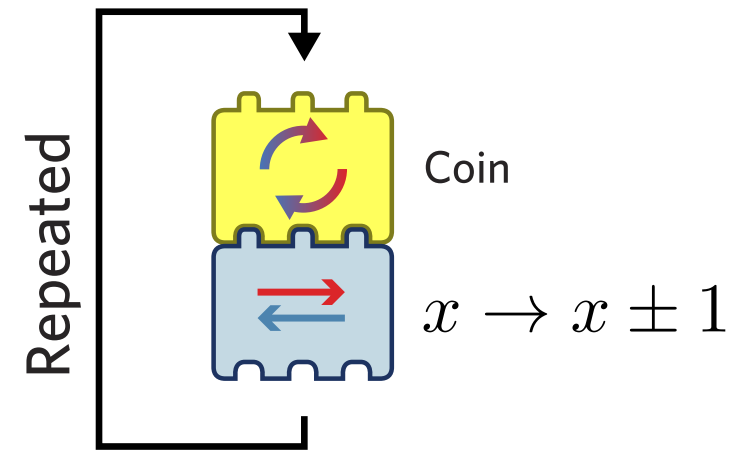

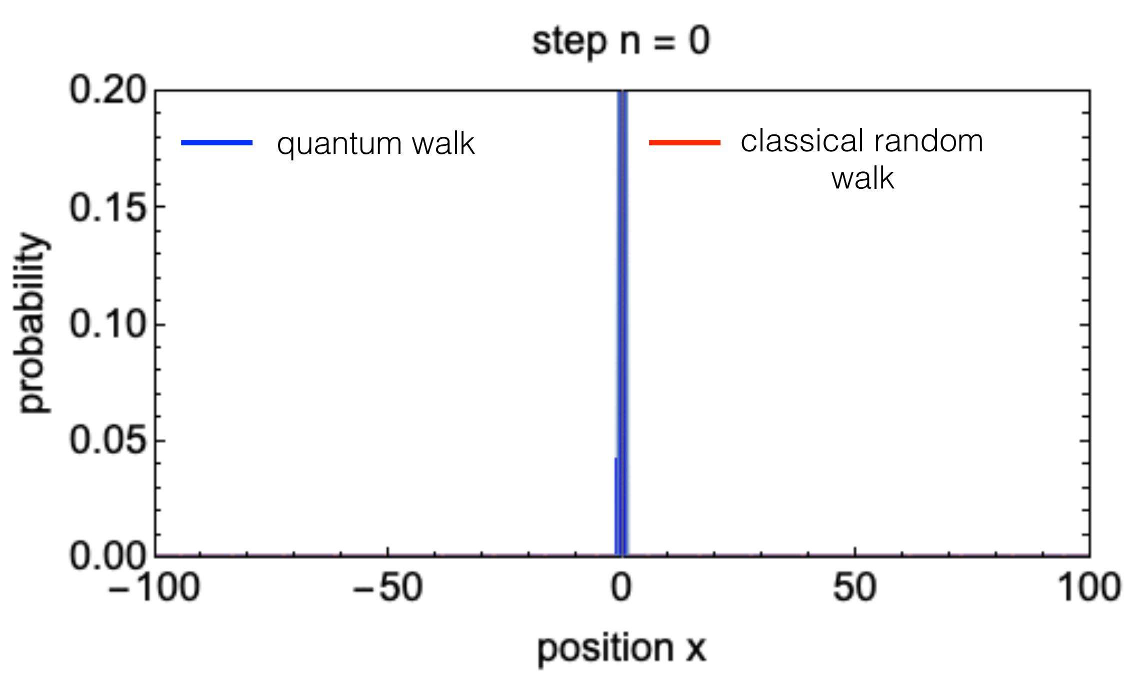

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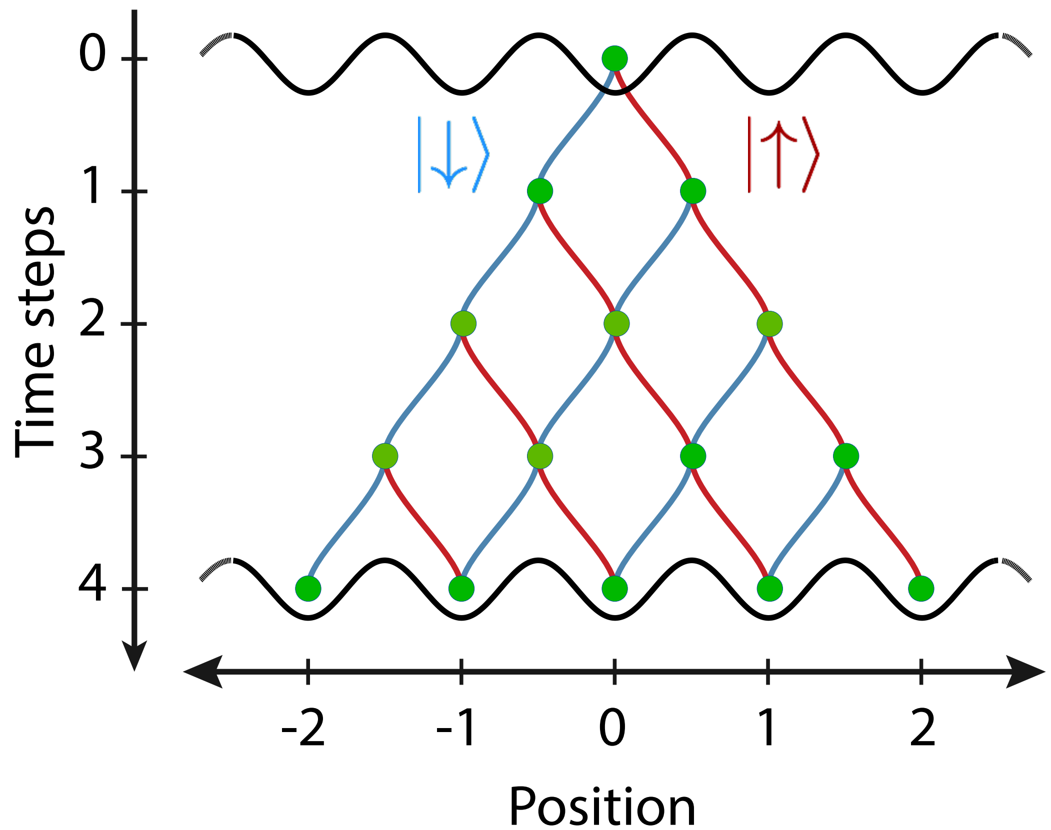

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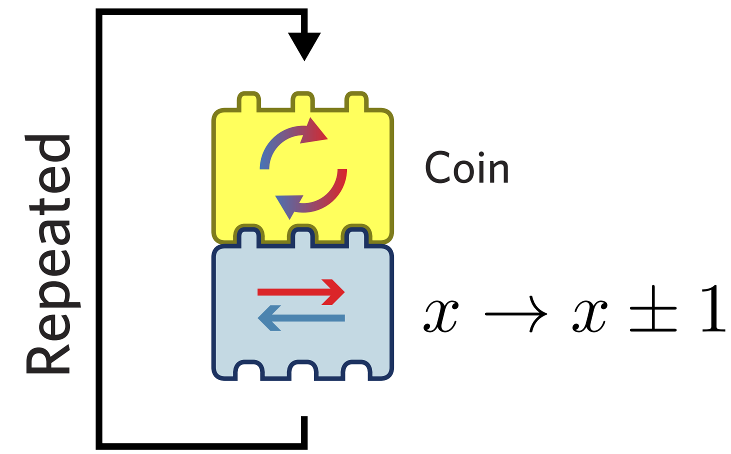

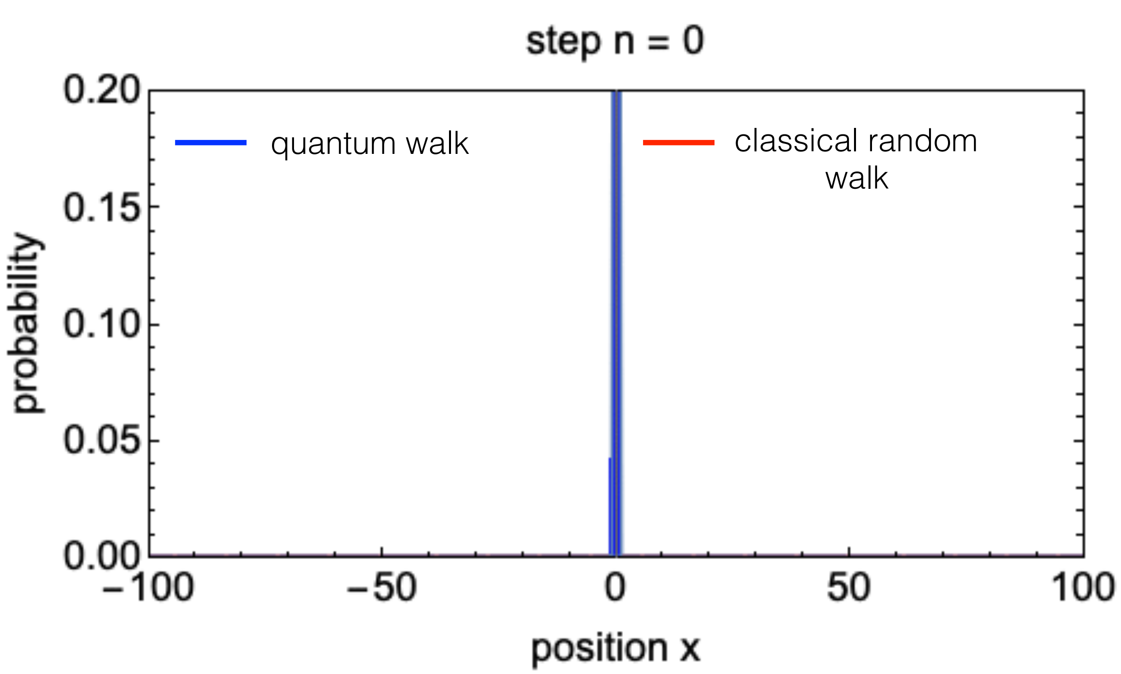

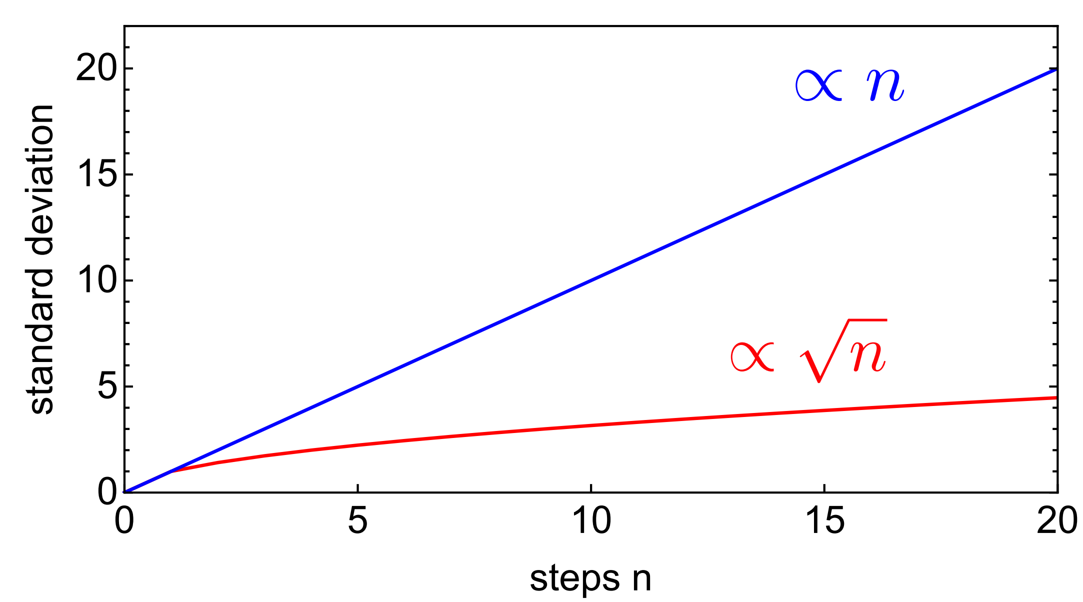

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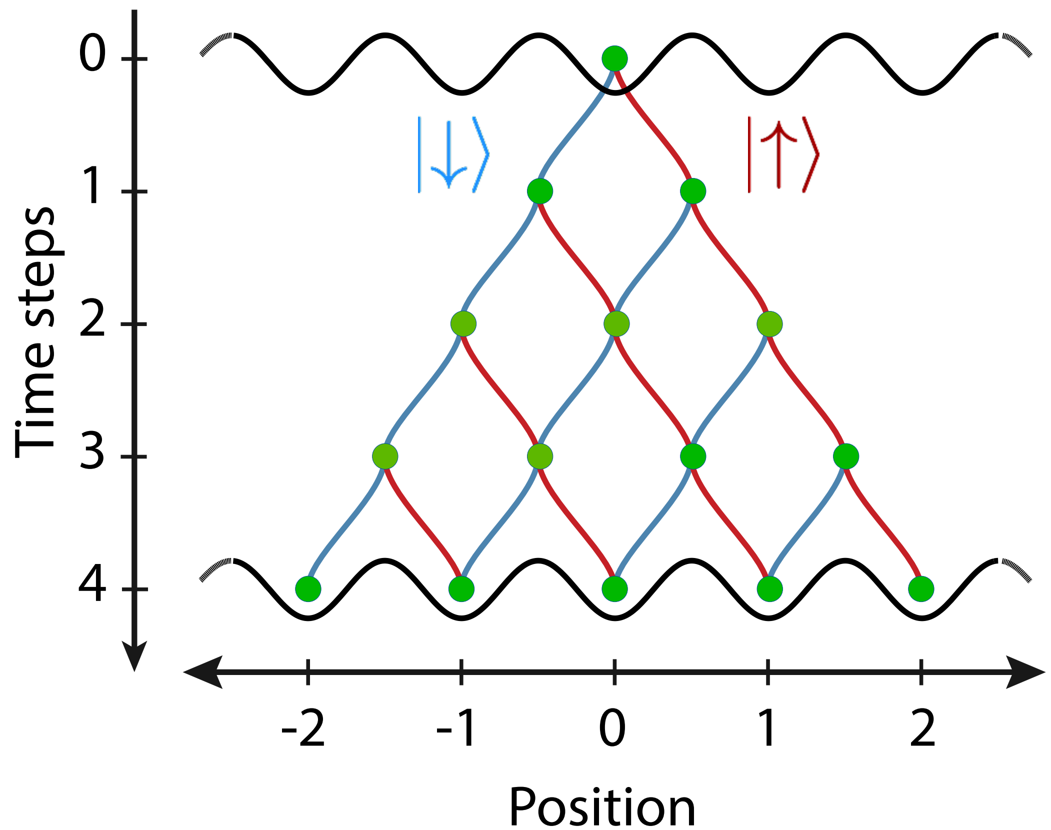

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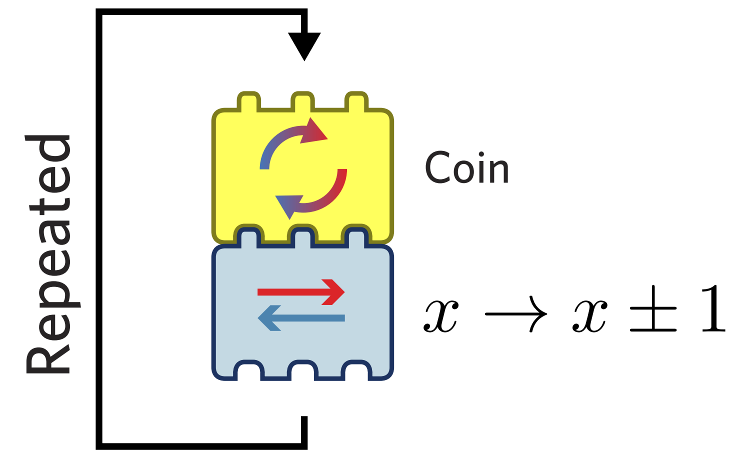
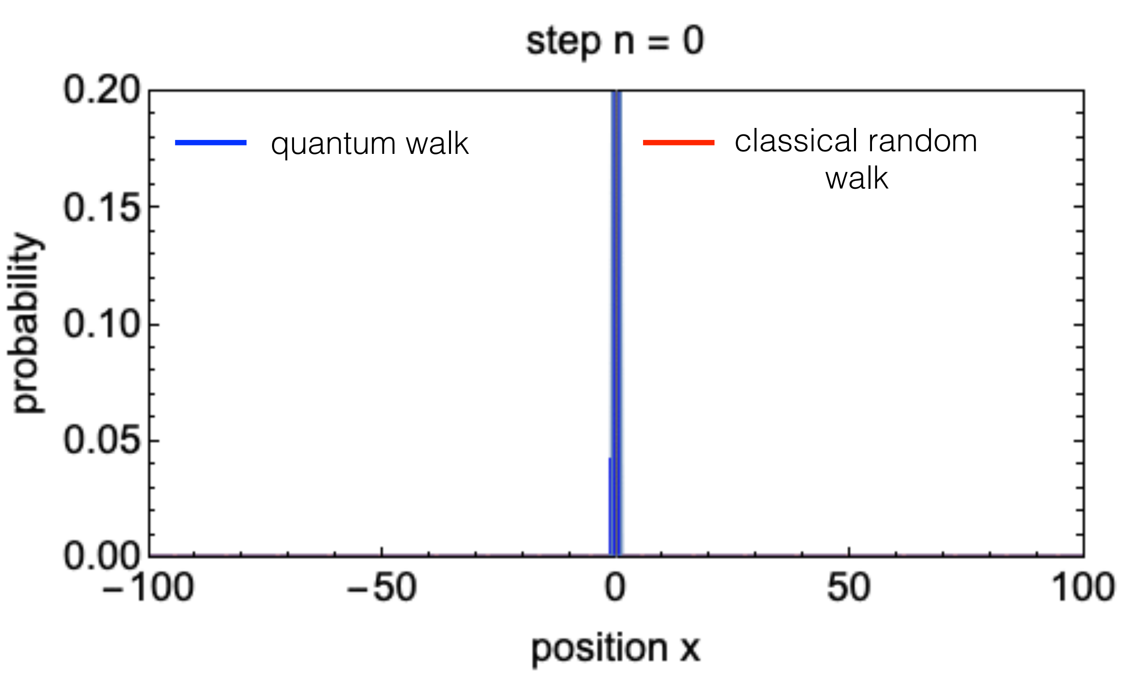
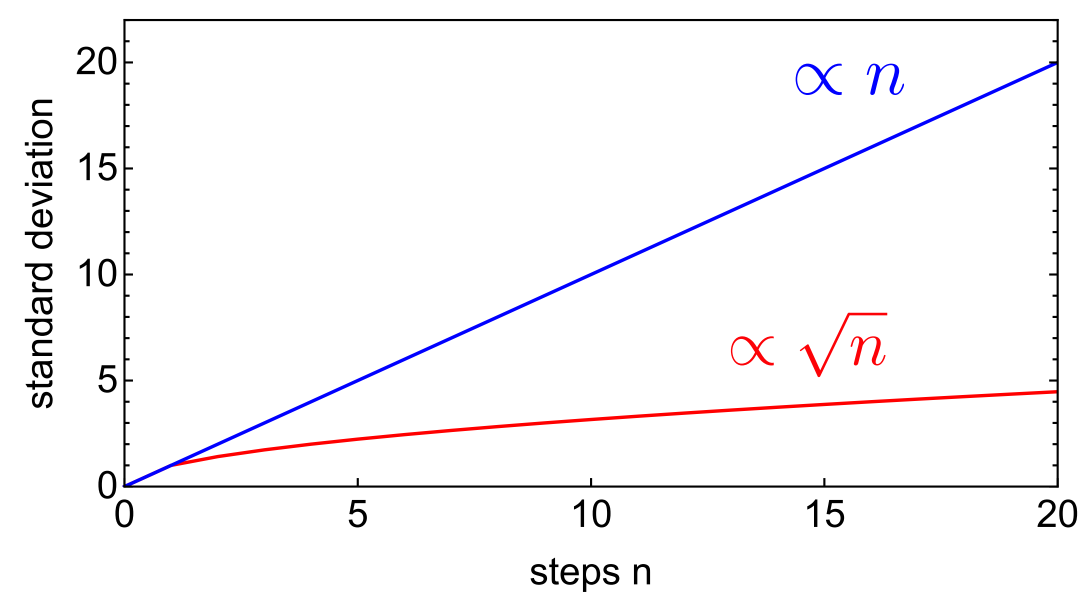




quantum walk spread ballistically faster than the classical one



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Applications

1: Quantum information

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- Quantum walks can be employed to develop efficient algorithms

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2: Physics (quantum simulation)

- A versatile platform to simulate different physical phenomena, e.g. topological phenomena, Anderson localization, etc.

Applications

Quantum Simulation:

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Quantum Simulation:

Using a controllable quantum system to study another less controllable or accessible quantum system is known as quantum simulation.

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- Neutral Atoms trapped in optical lattices
- Ions
- Superconducting circuits
- Cavities
- Quantum dots

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Applications

Floquet engineering: Design different walk protocols, simulate different physics

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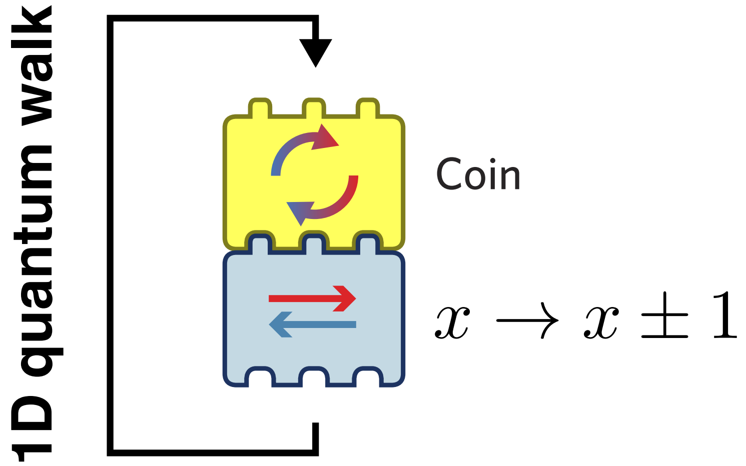

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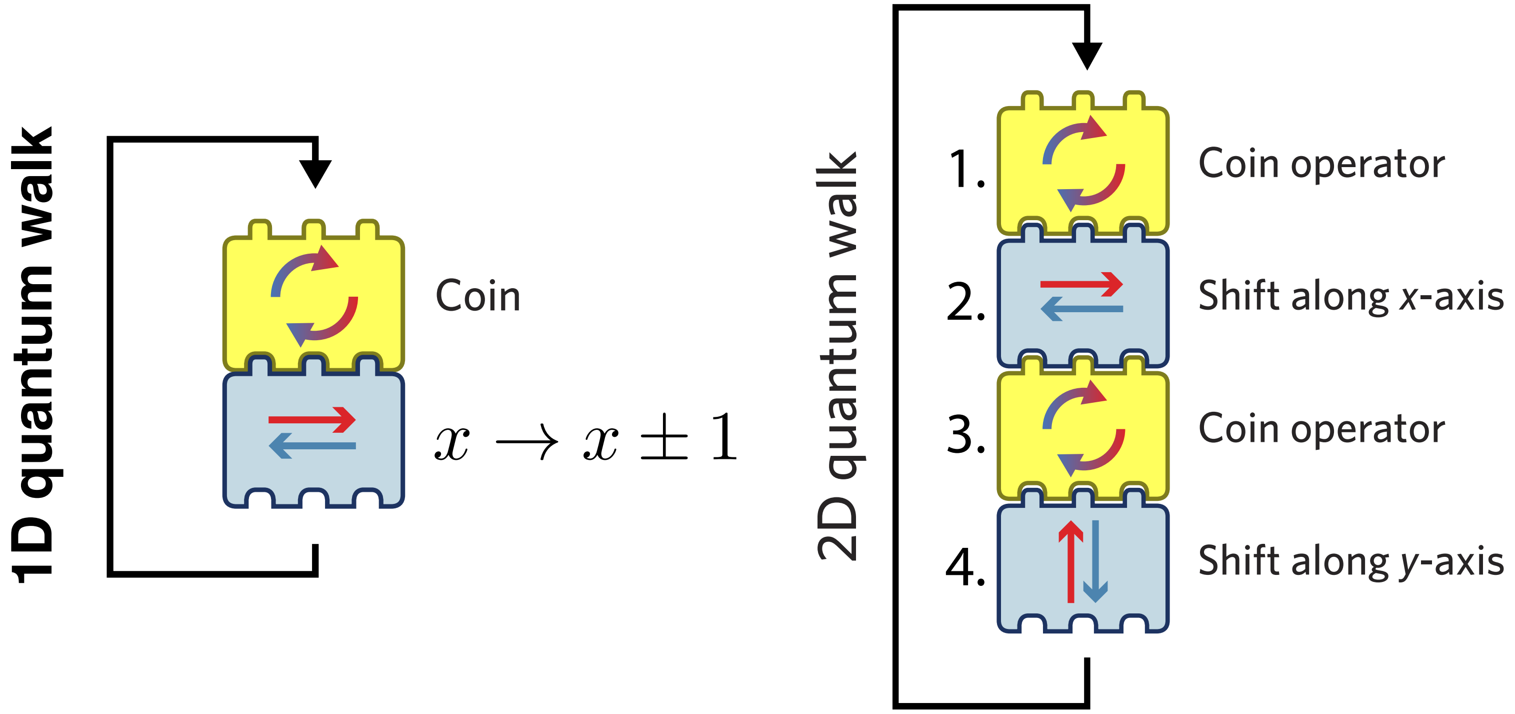

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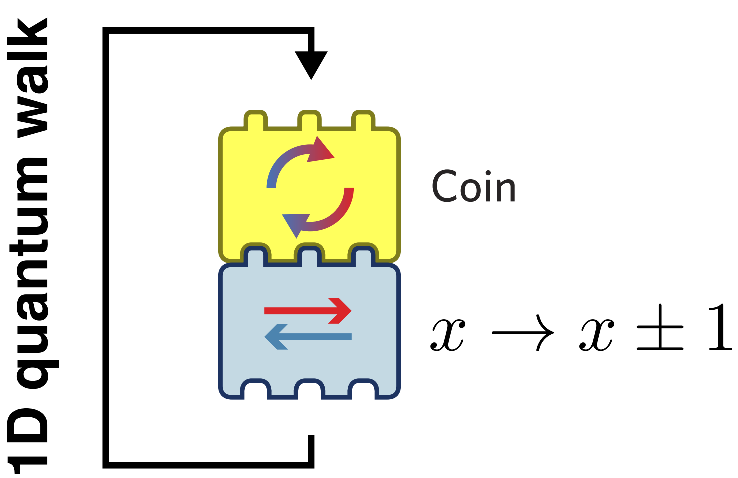

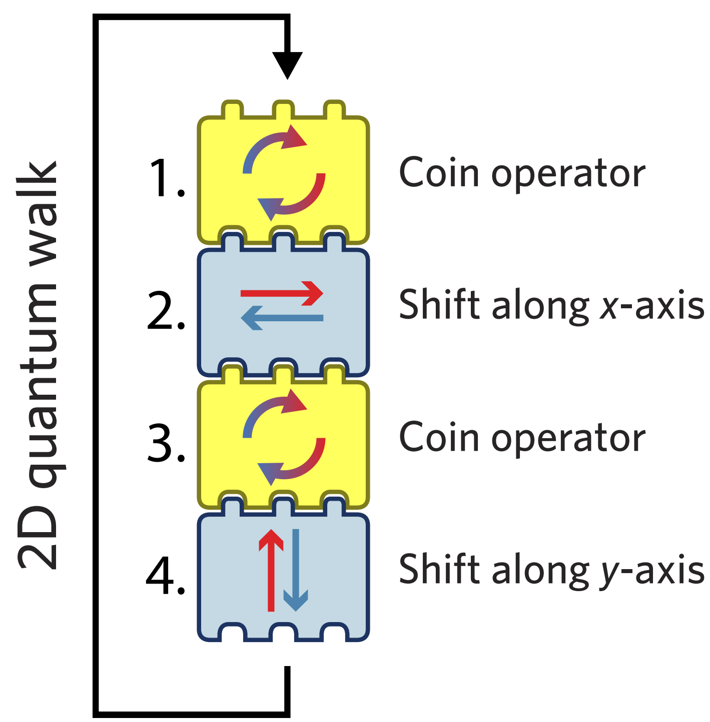

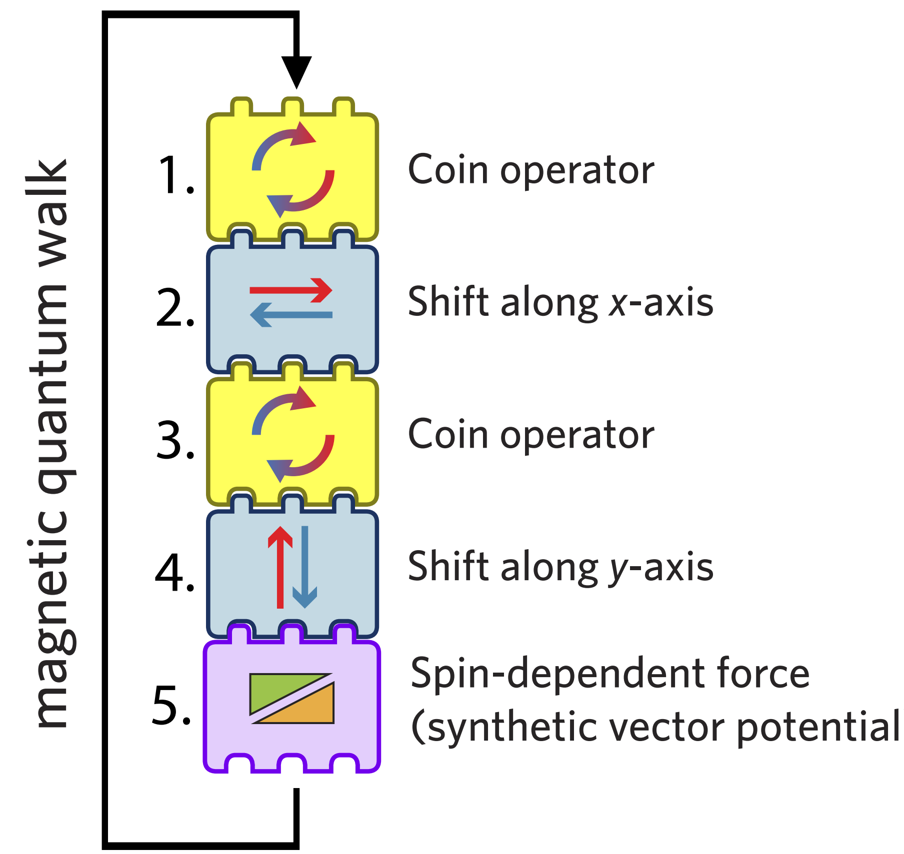

Topological Phenomena

1D quantum walk

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$$|\psi(\tau)\rangle = \hat{W}\,|\psi(0)\rangle = e^{-i\hat{\mathcal{H}}_{\text{eff}}\tau/\hbar}\,|\psi(0)\rangle$$

τ : time duration of a single step of the walk

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quasienergy spectrum

$$\hat{\mathcal{H}}_{\text{eff}} = \varepsilon(k_x)\big(\hat{\text{n}}(k_x)\cdot\vec{\sigma}\big); \qquad \varepsilon \in [-\pi,\pi] \quad \text{Floquet zone}$$

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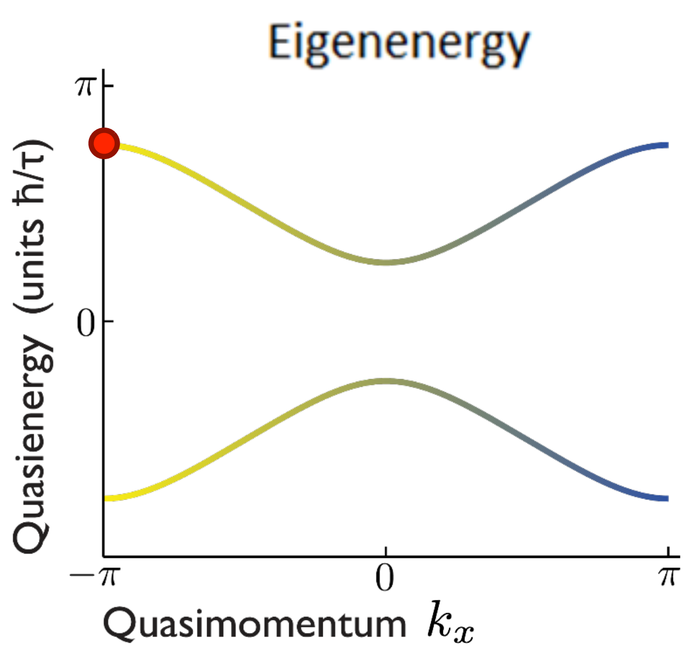

Slide courtesy: Andrea Alberti

Topological Phenomena

Topological invariant

$$\hat{\mathcal{H}}_{\text{eff}} = \varepsilon(k_x)\big(\hat{\mathrm{n}}(k_x)\cdot\vec{\sigma}\big); \qquad \varepsilon \in [-\pi, \pi]$$ Floquet zone

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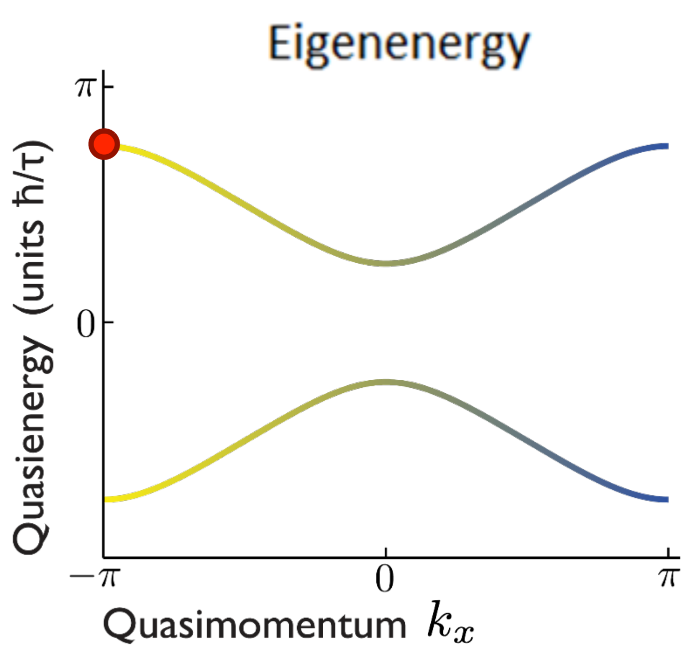

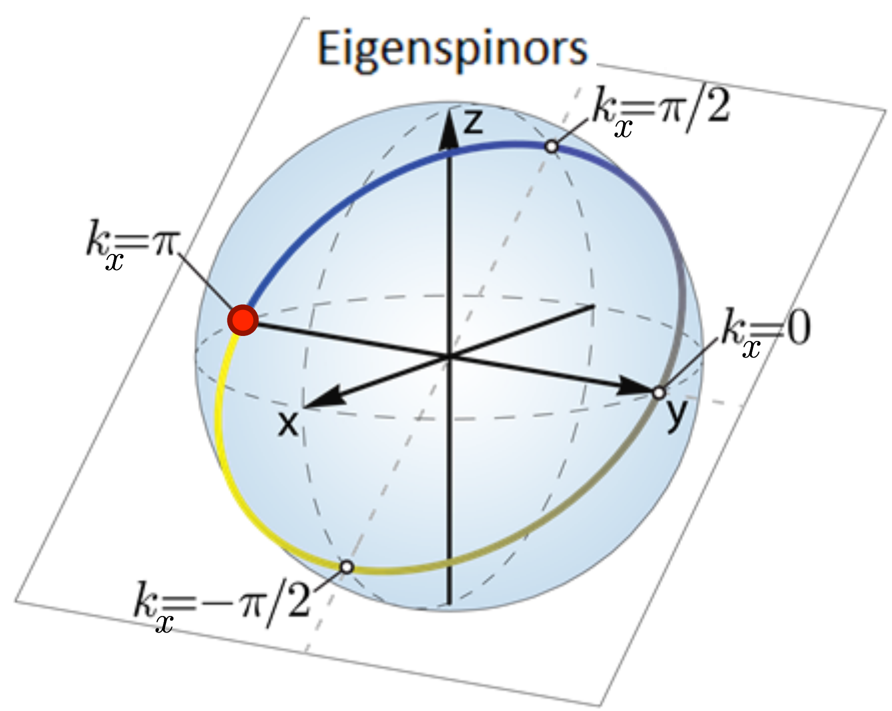

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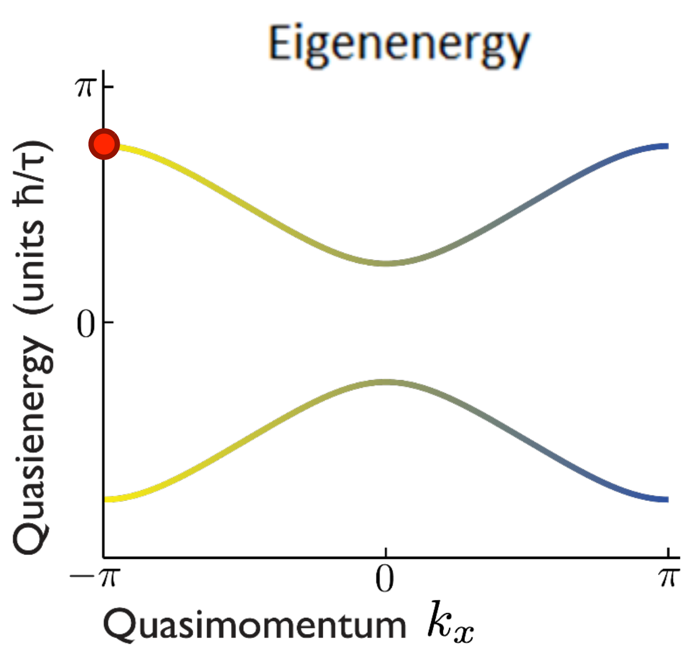

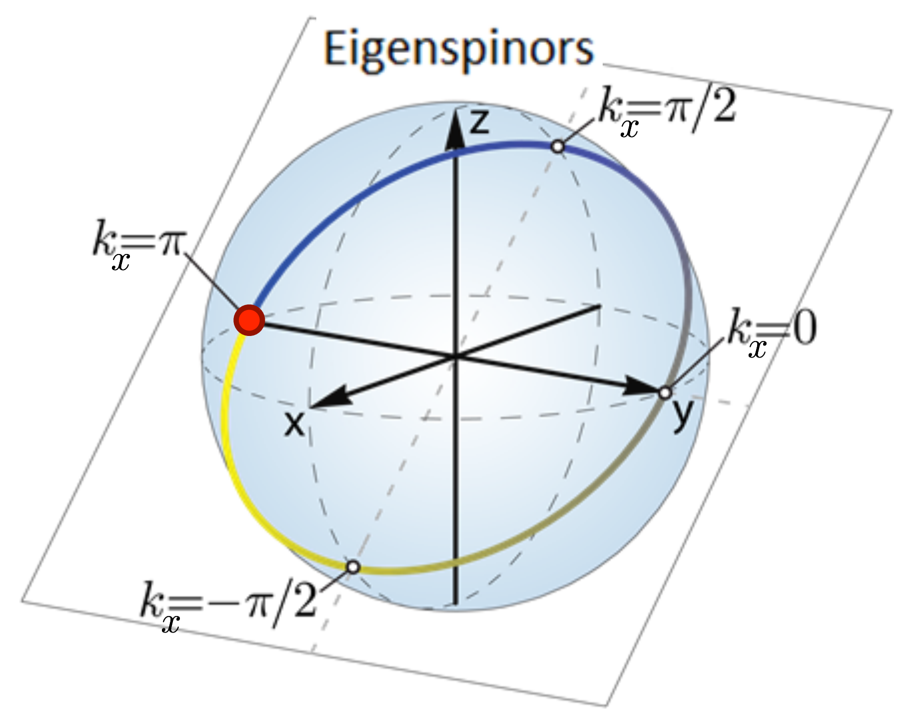

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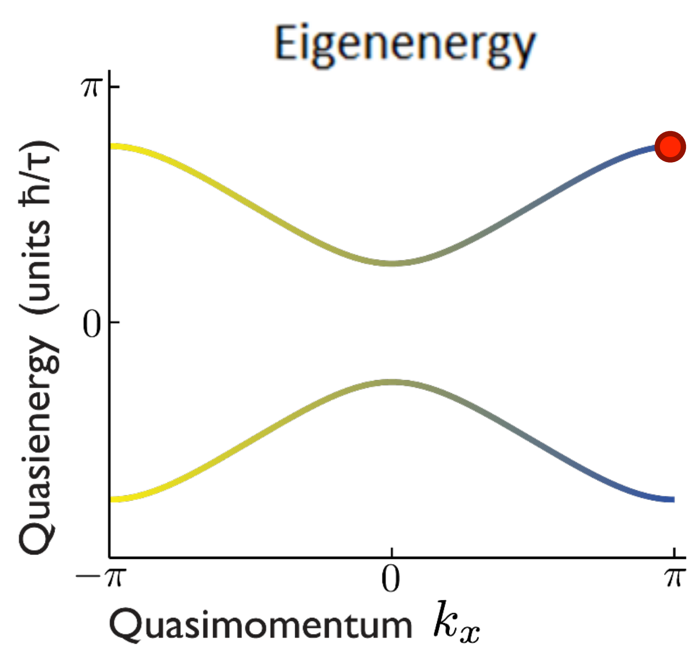

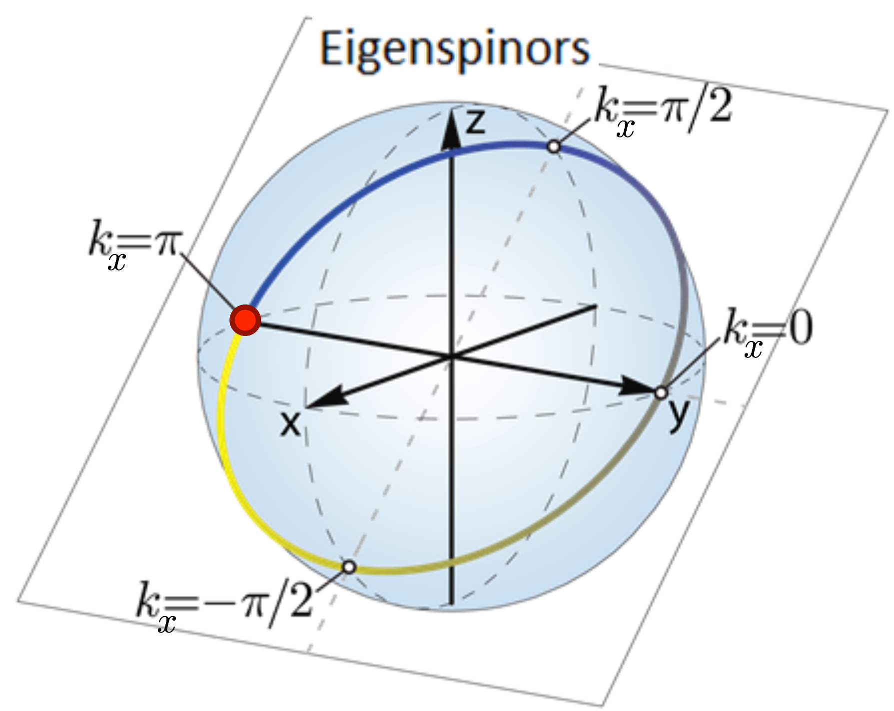

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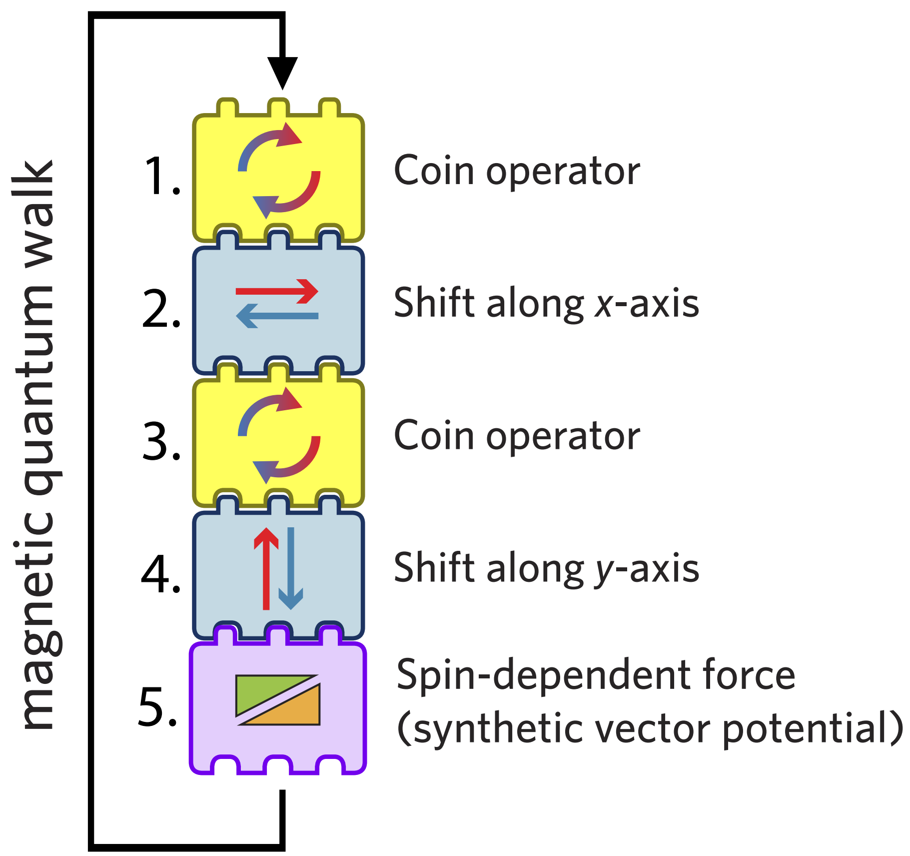
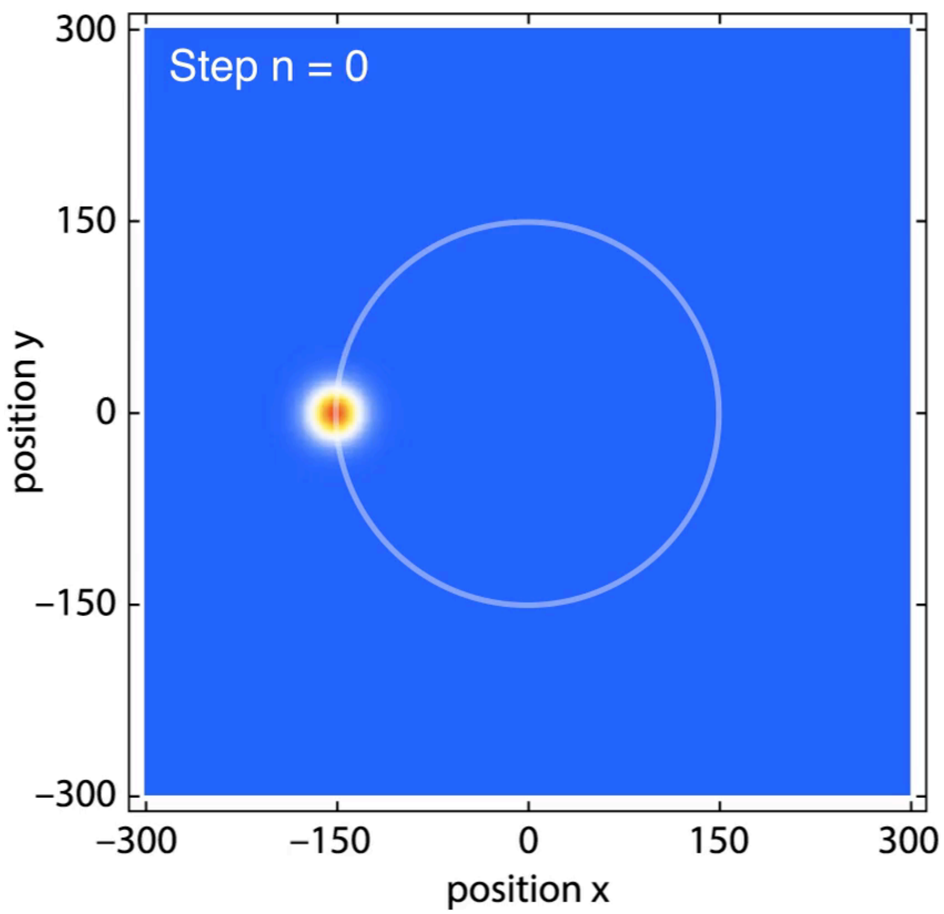
Quantum Hall Physics

Quantum transport in a magnetic field





M. Sajid et al., PRB **99**, 214303

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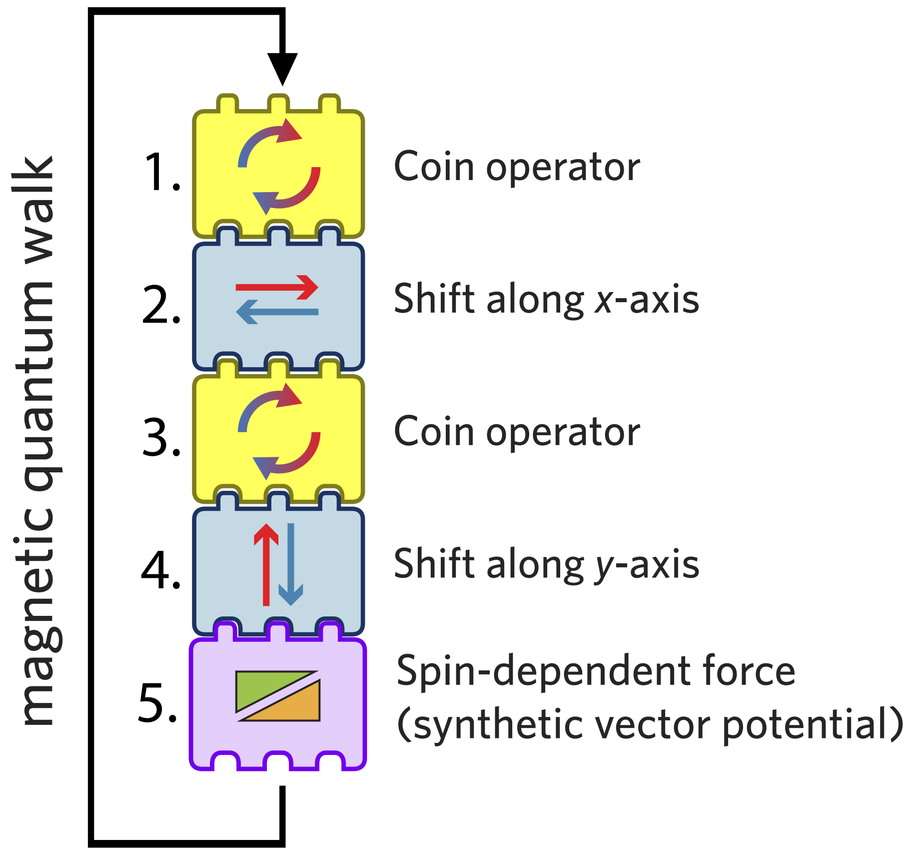
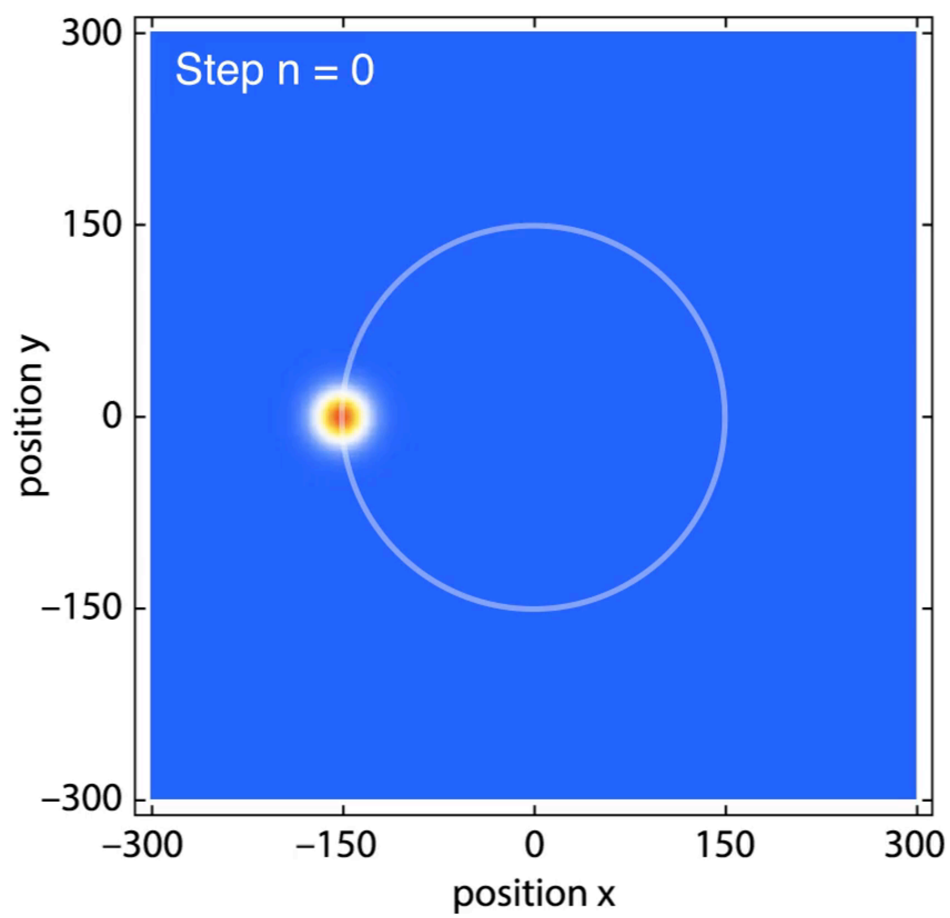
Long wavelength approximation





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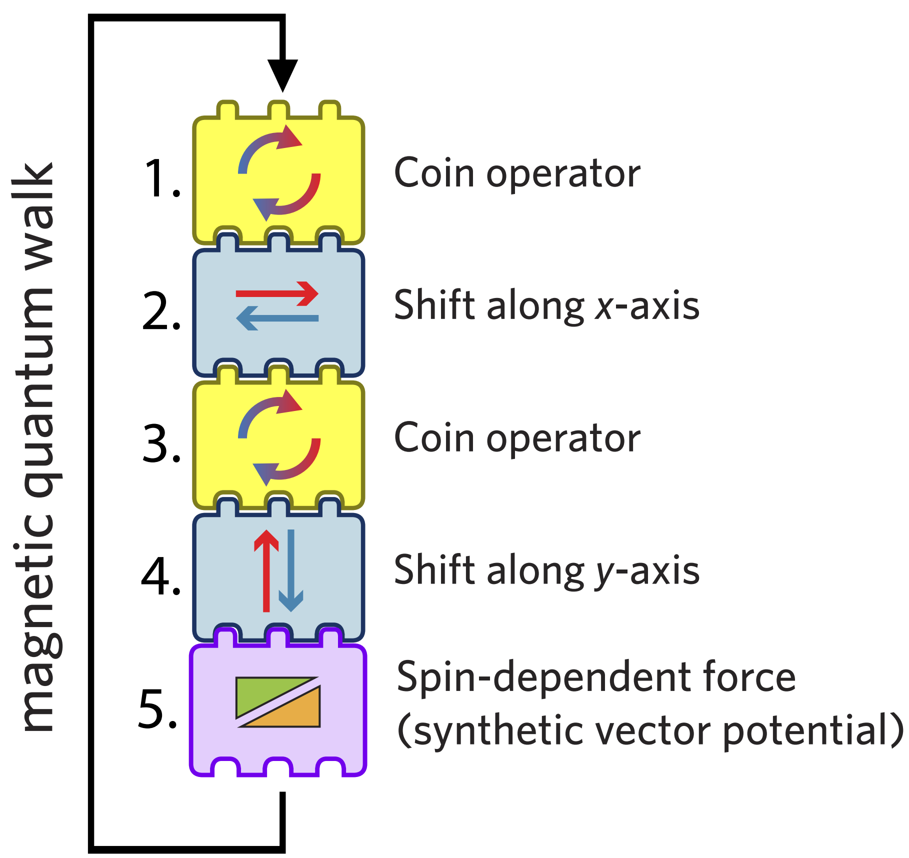
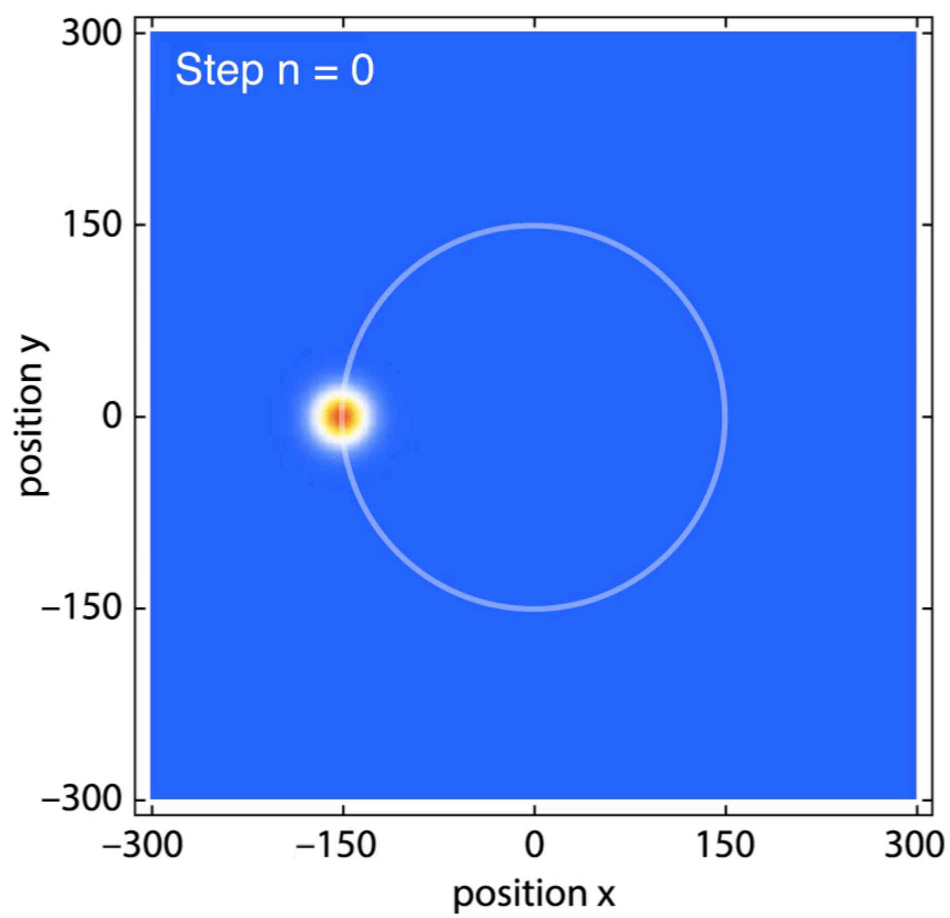
Long wavelength approximation





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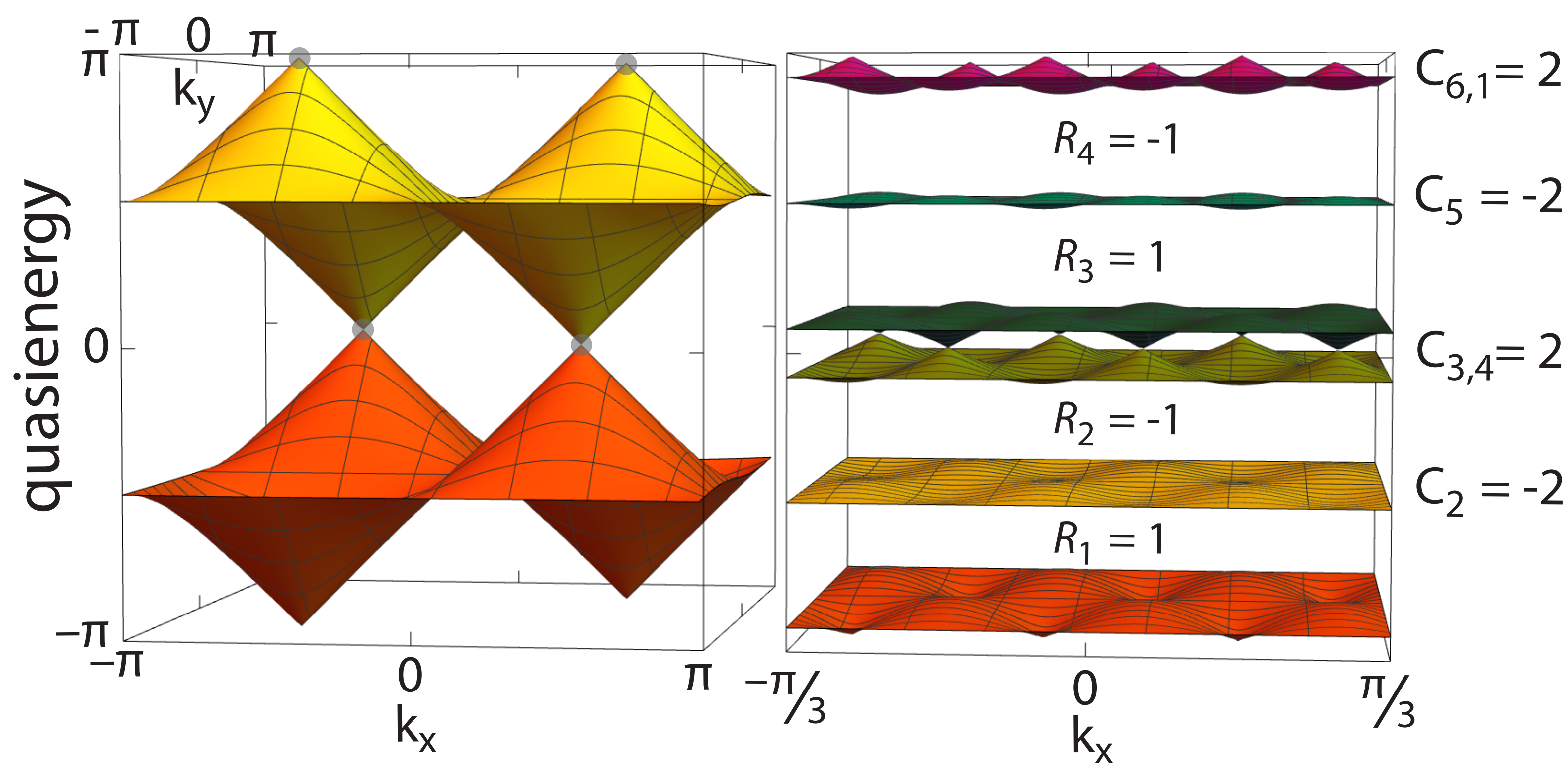
Magnetic quantum walks

$p/q = 0$ $p/q = 1/3$

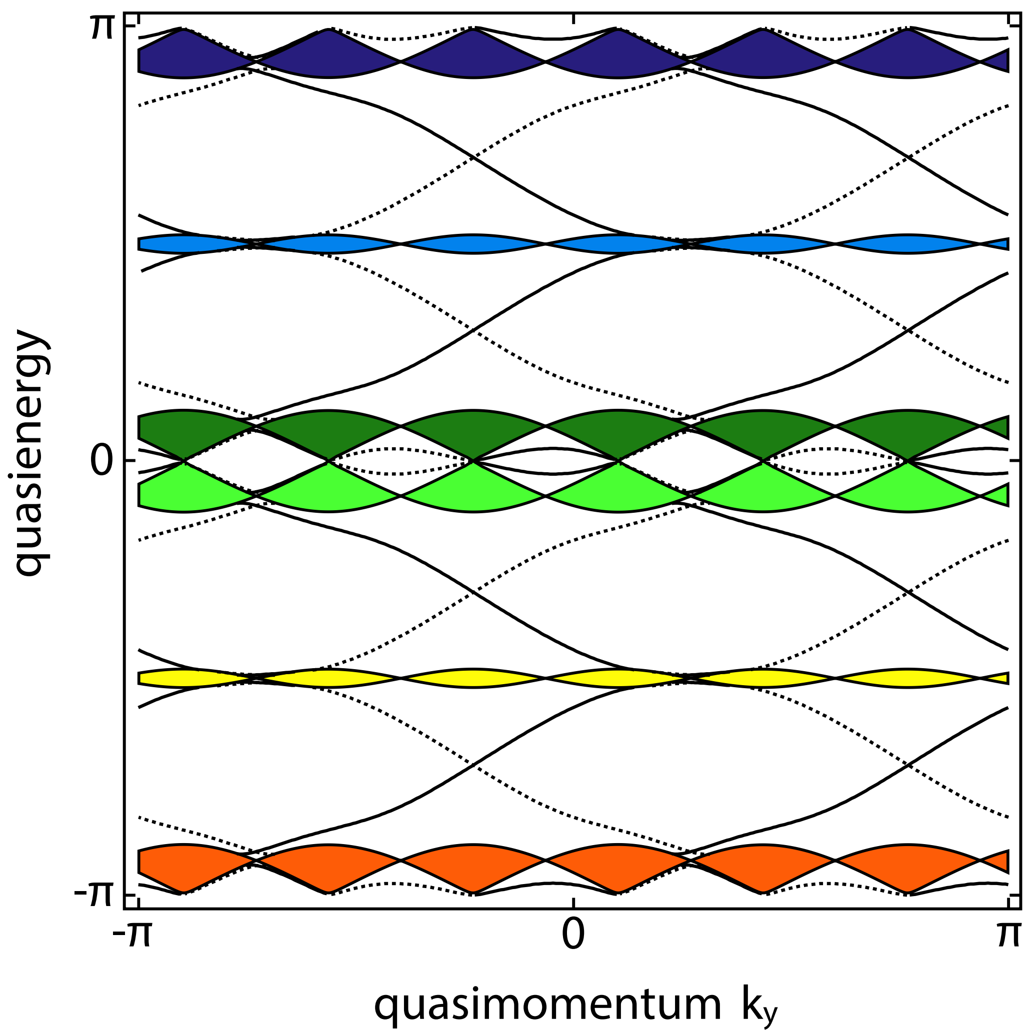


Topologically trivial

Insulating, non-trivial topology

Quantum Hall Physics

Topologically Protected Edge States



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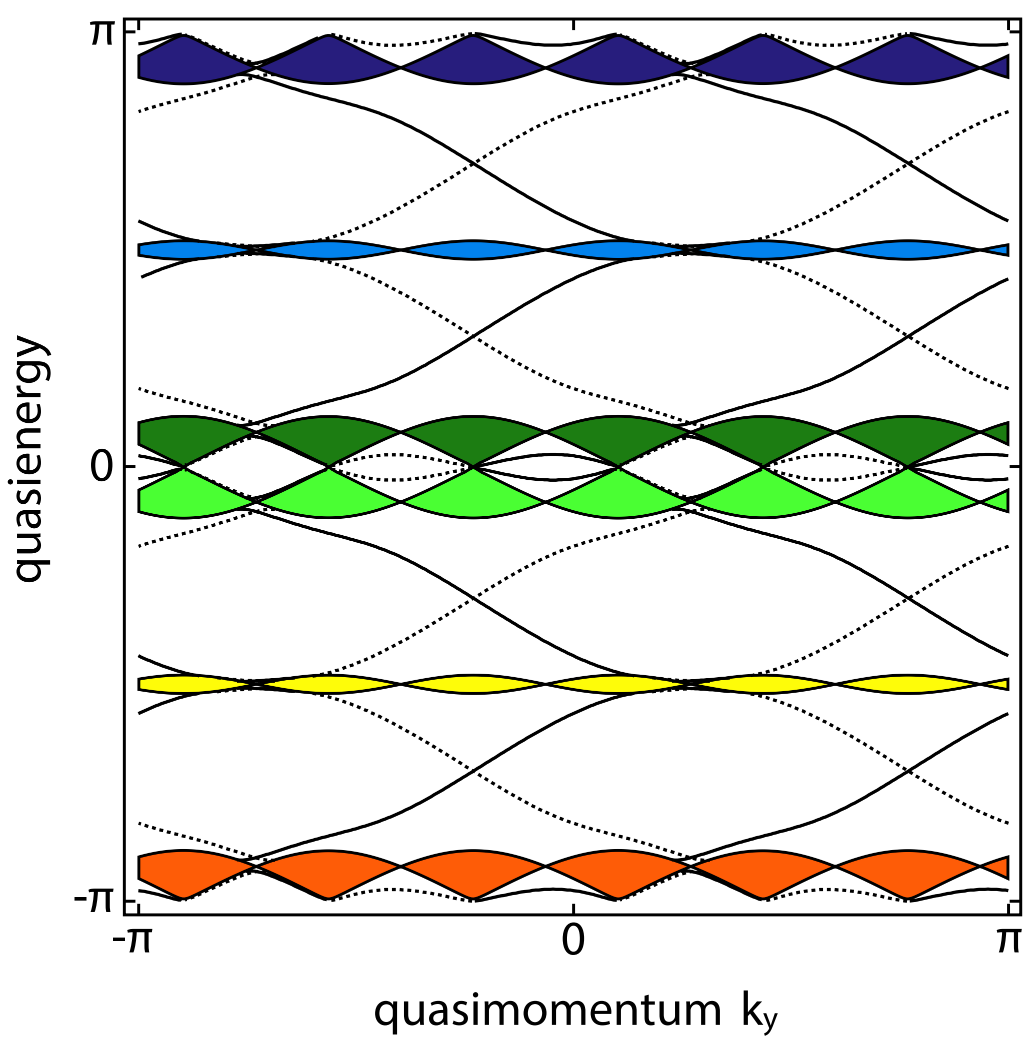
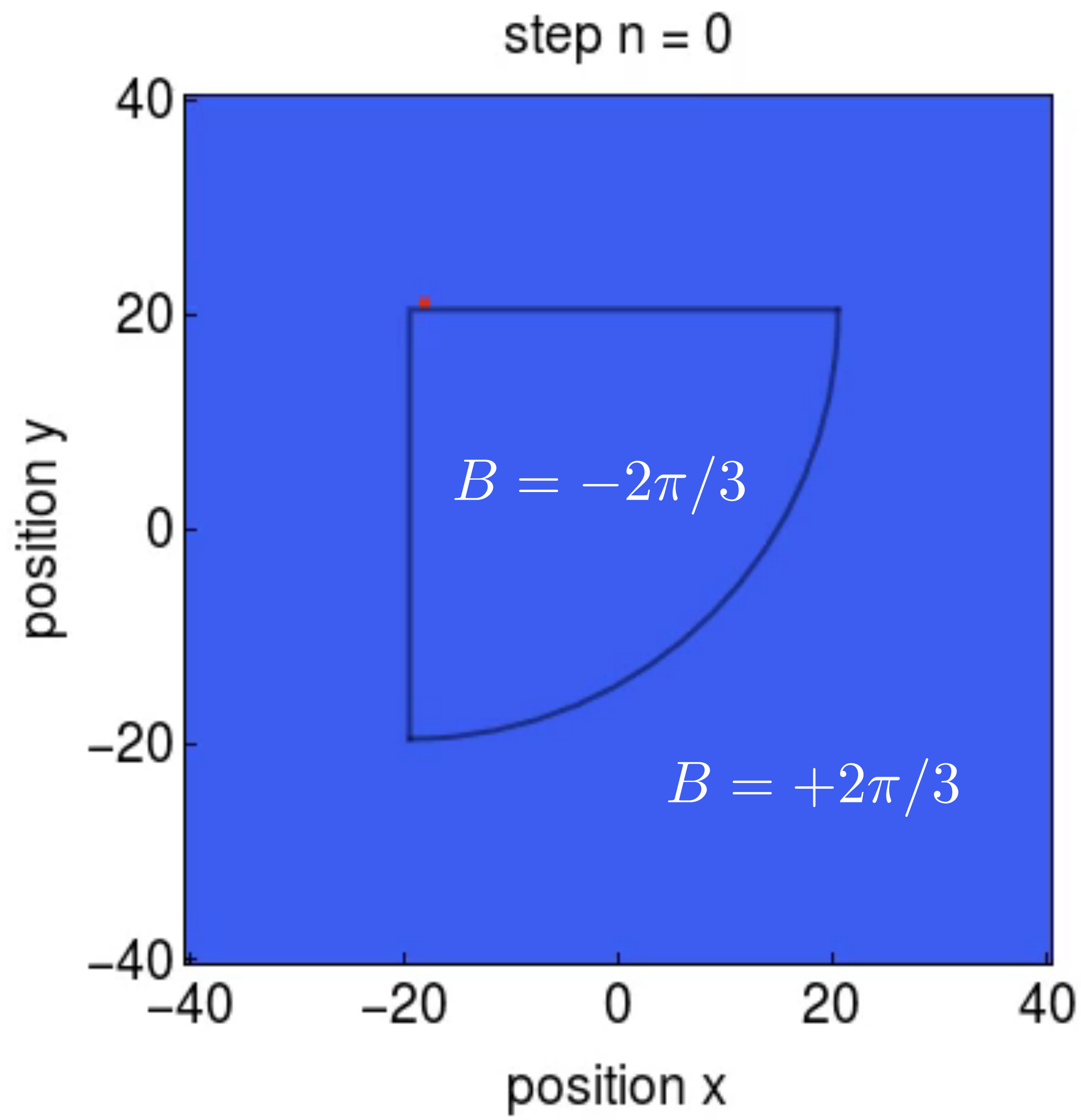
Quantum Hall Physics

Topologically Protected Edge States





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Quantum Hall Physics

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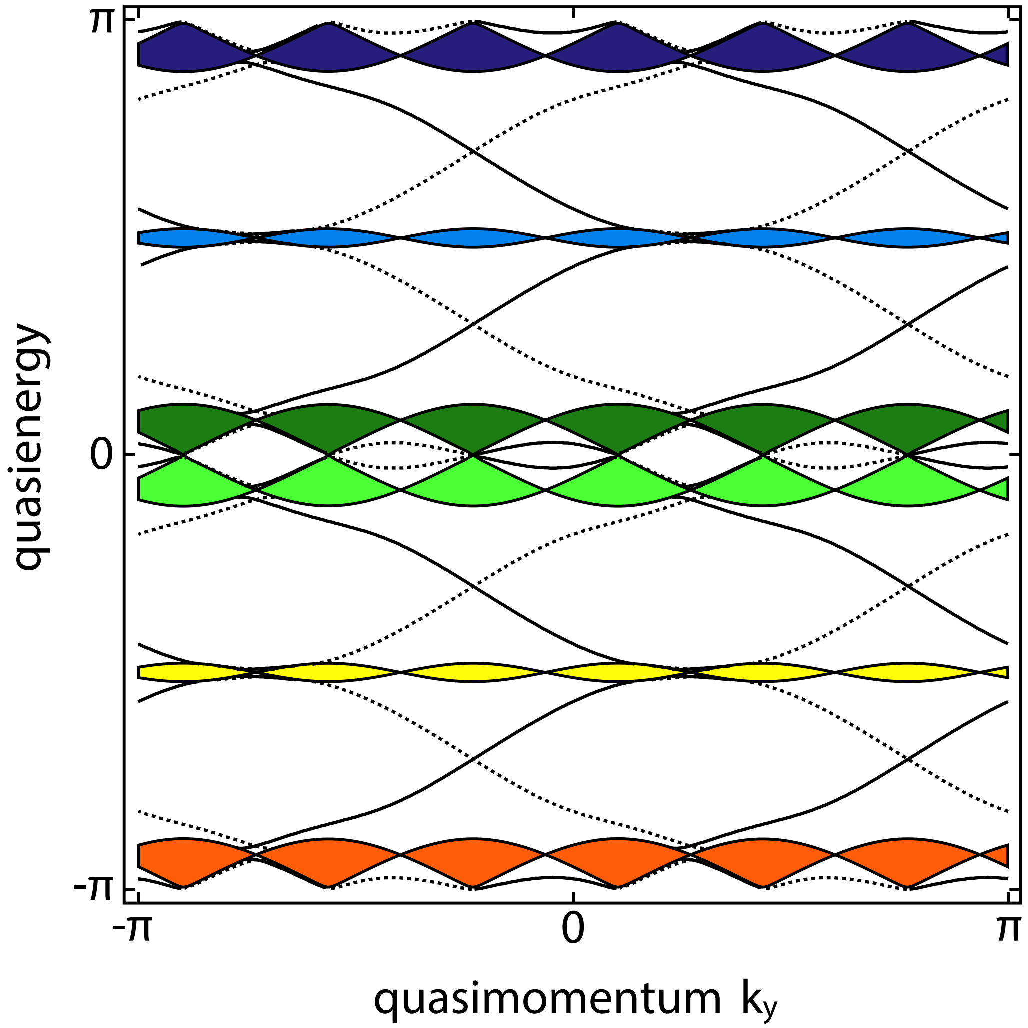

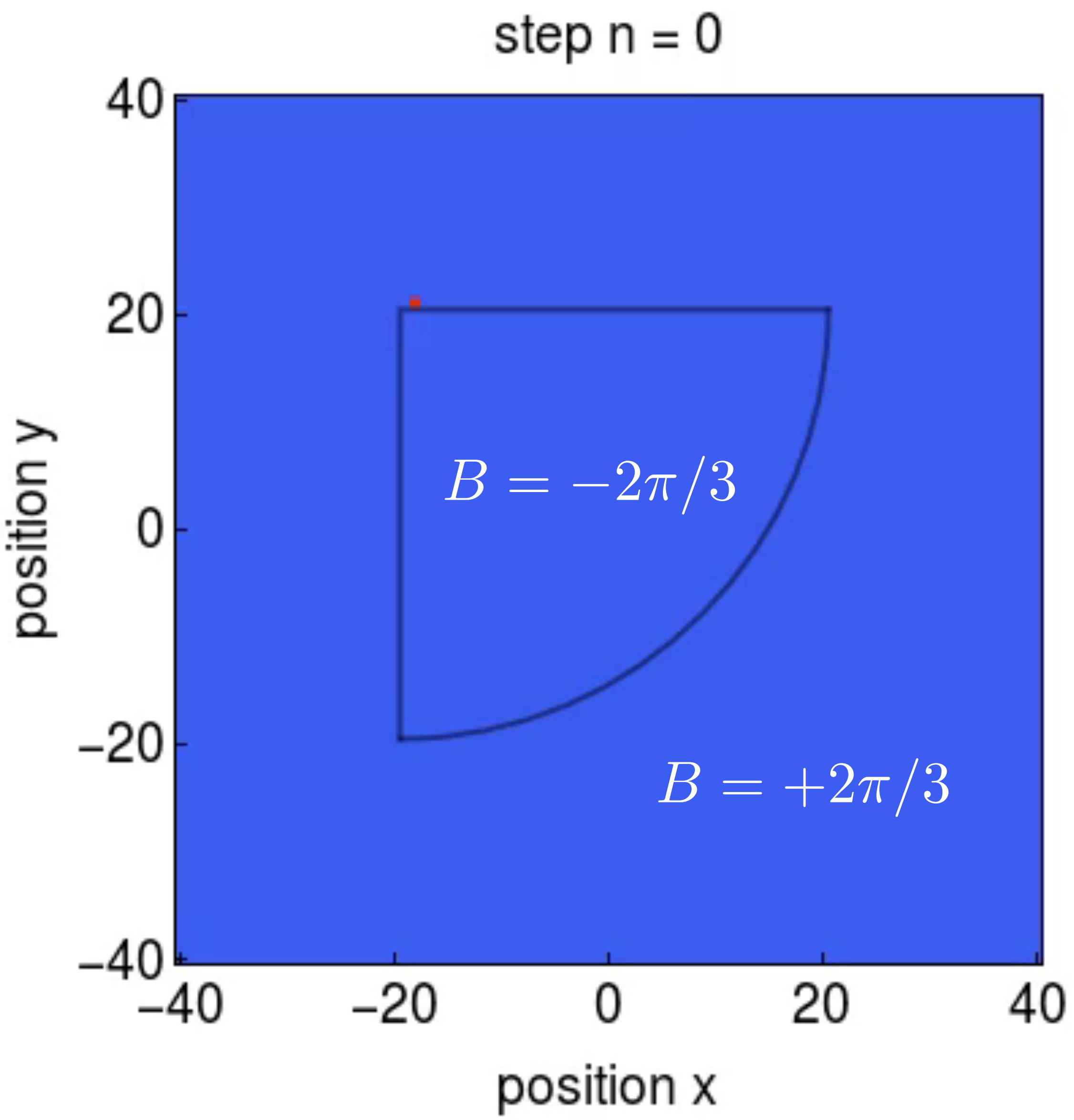

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Outline

- Introduction to quantum walks
- Applications: A versatile tool for quantum simulation
- Quantum walks with artificial electric field

Electric quantum walks

Split-step protocol:

$$\hat{W}_{ss,\phi}(n) = \hat{F}(\phi n)\, \hat{S}_x^{\downarrow}\, \hat{C}_2(\theta_2)\, \hat{S}_x^{\uparrow}\, \hat{C}_1(\theta_1).$$

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$$\hat{S}_x^{\downarrow} = \sum_x \Big[\left|\uparrow\right\rangle \left\langle\uparrow\right| \otimes \left|x\right\rangle \left\langle x\right| + \left|\downarrow\right\rangle \left\langle\downarrow\right| \otimes \left|x-1\right\rangle \left\langle x\right| \Big]$$

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$$\phi = qEd\tau/\hbar$$

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$$\delta\theta = (\theta_1 - \theta_2)/2$$

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Electric quantum walks

Evolution of the walk:

$$|\Psi_n(\phi)\rangle = \hat{W}_{\phi}^{t=n}(n)\,|\Psi_{\mathrm{i}}\rangle$$

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Electric quantum walks

Evolution of the walk:

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Probability distribution:

$$P(x,n) = \sum_{s=0,1} \langle \Psi_n(\phi)|\Psi_n(\phi)\rangle$$

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$$P(x = x_{\mathrm{i}}, n) = \sum_{s=0,1} \langle \Psi_{\mathrm{i}}|\Psi_n(\phi)\rangle$$

Standard deviation:

$$\sigma(n) = \sqrt{< x^2 > - < x >^2}.$$

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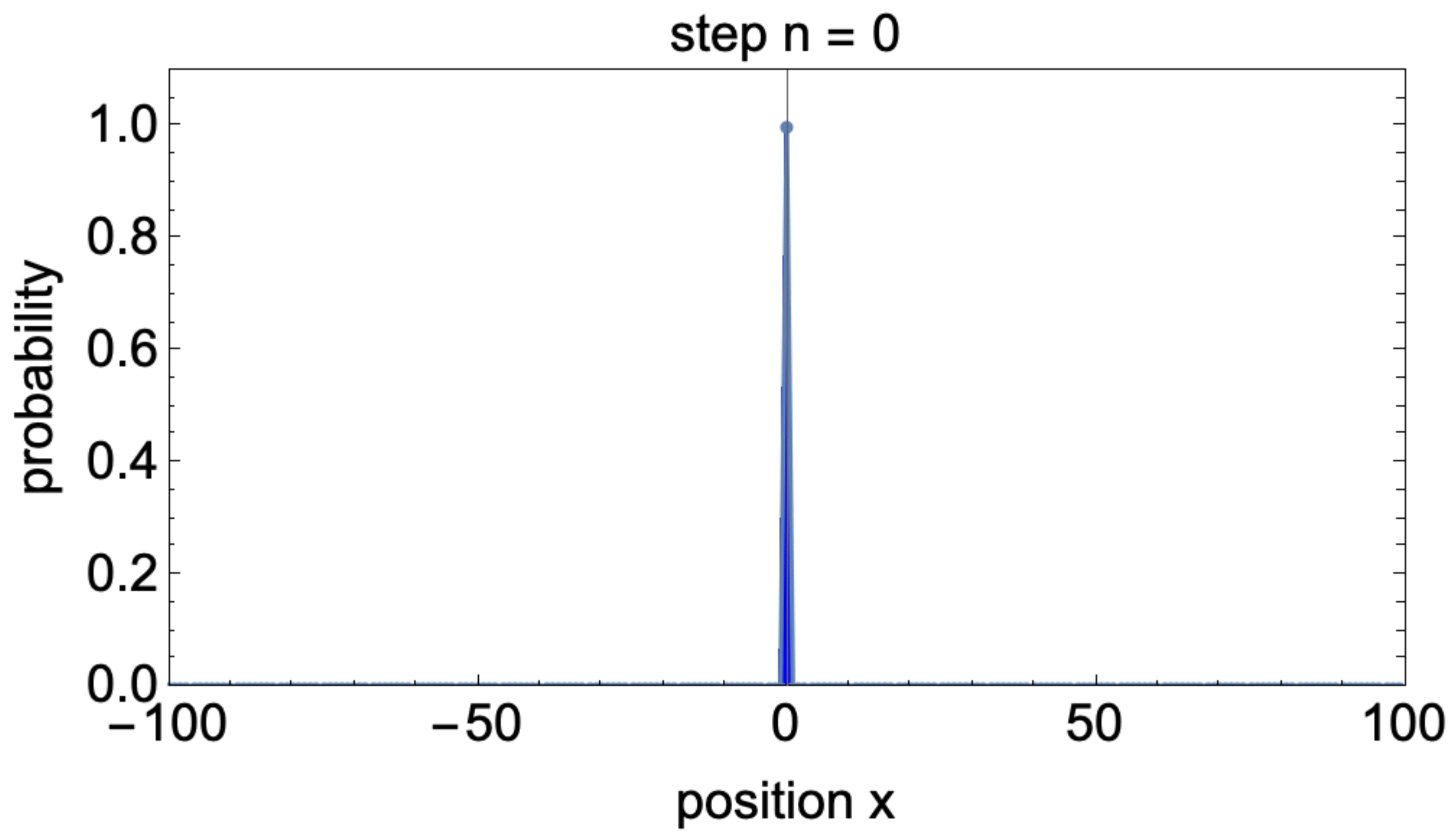
Electric quantum walks

Revivals:



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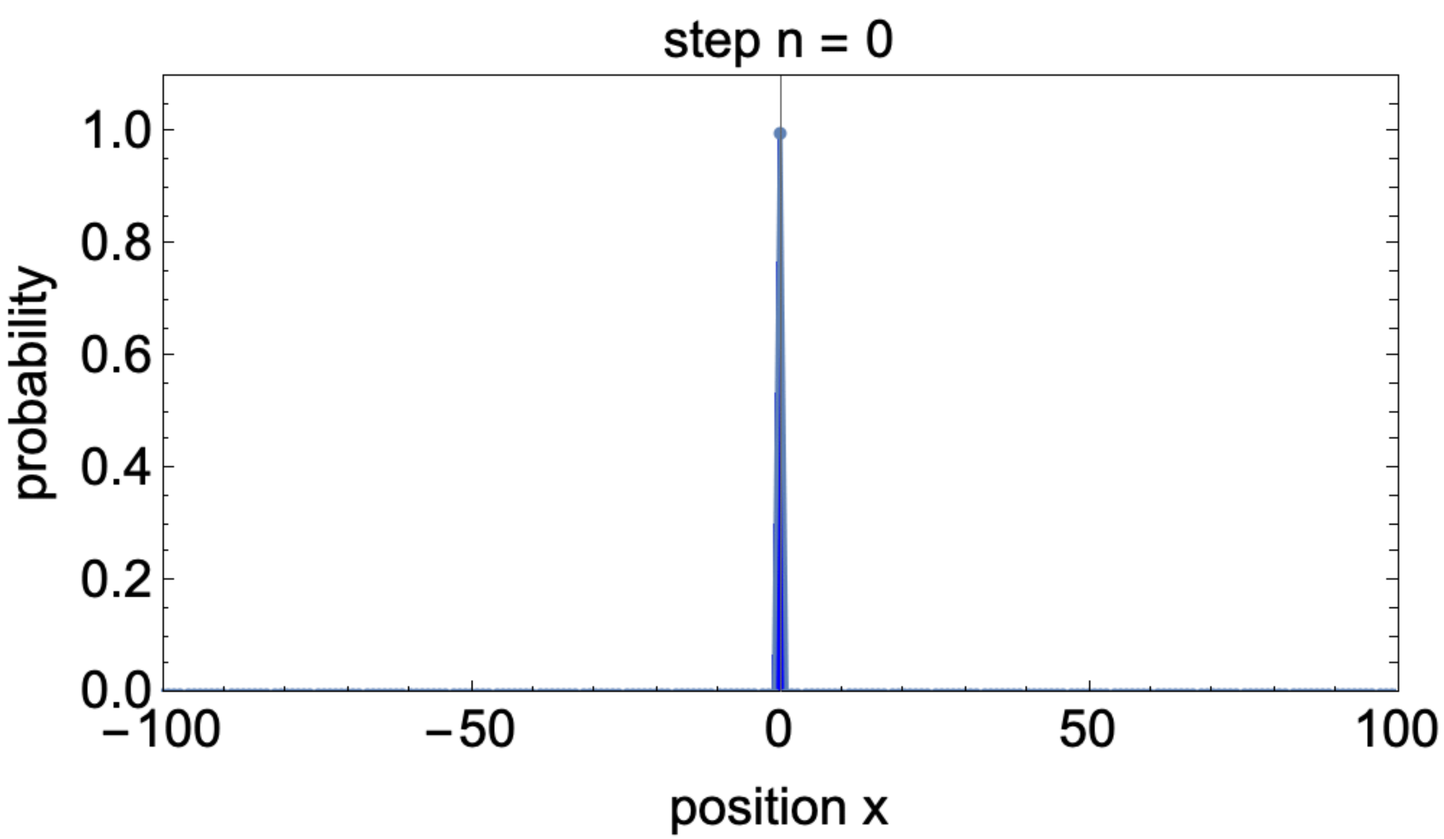
Electric quantum walks

Revivals: $P(x = x_{\rm i}, n) = \sum_{s=0,1} \langle \Psi_{\rm i} | \Psi_n(\phi) \rangle \; = 1$



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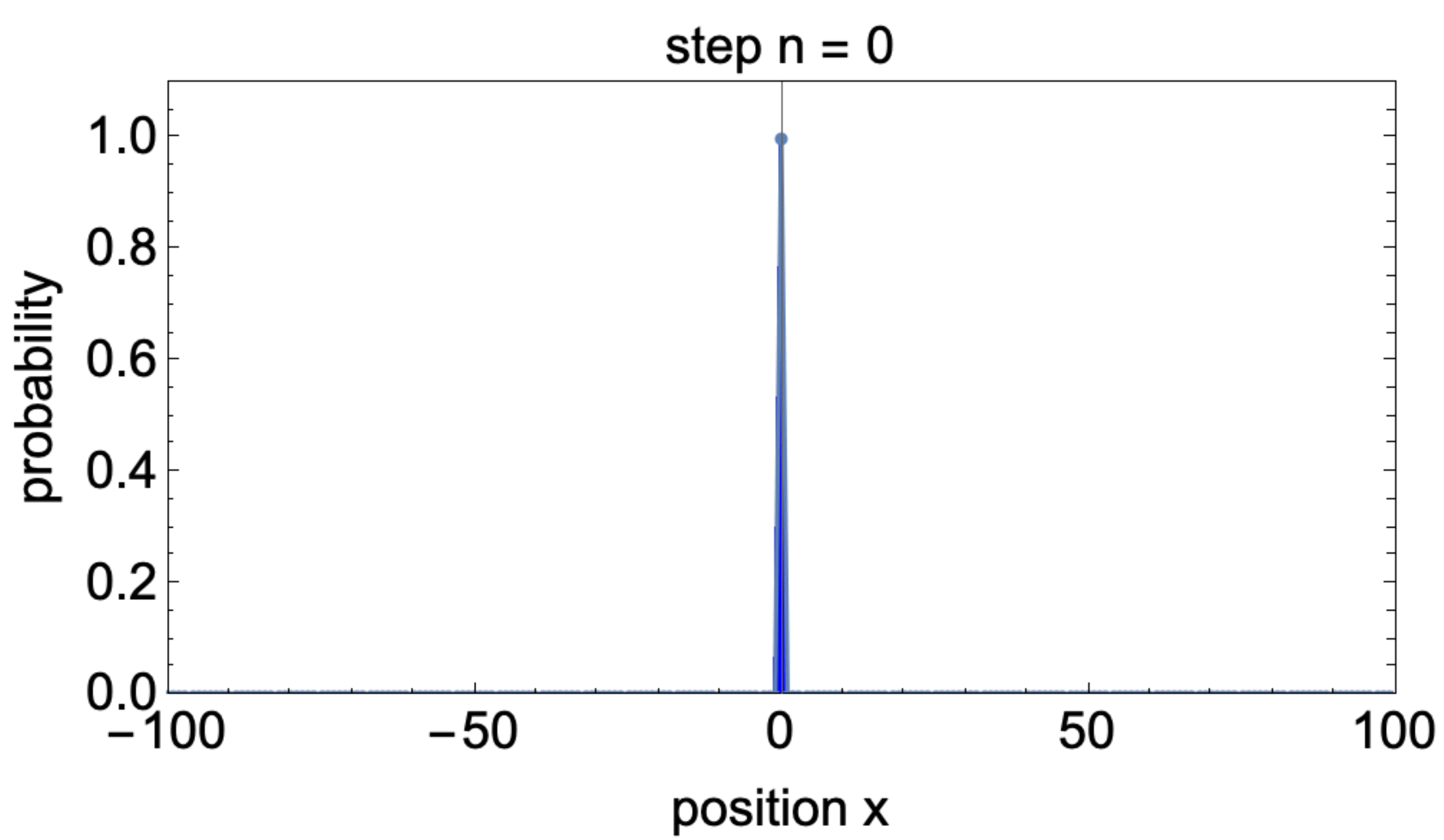
Electric quantum walks

Revivals: $$P(x=x_{\mathrm{i}},n)=\sum_{s=0,1}\langle\Psi_{\mathrm{i}}|\Psi_n(\phi)\rangle \;=1$$

$$\sigma(n)=\sqrt{<x^2>-<x>^2}. \;=0$$



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Electric quantum walks

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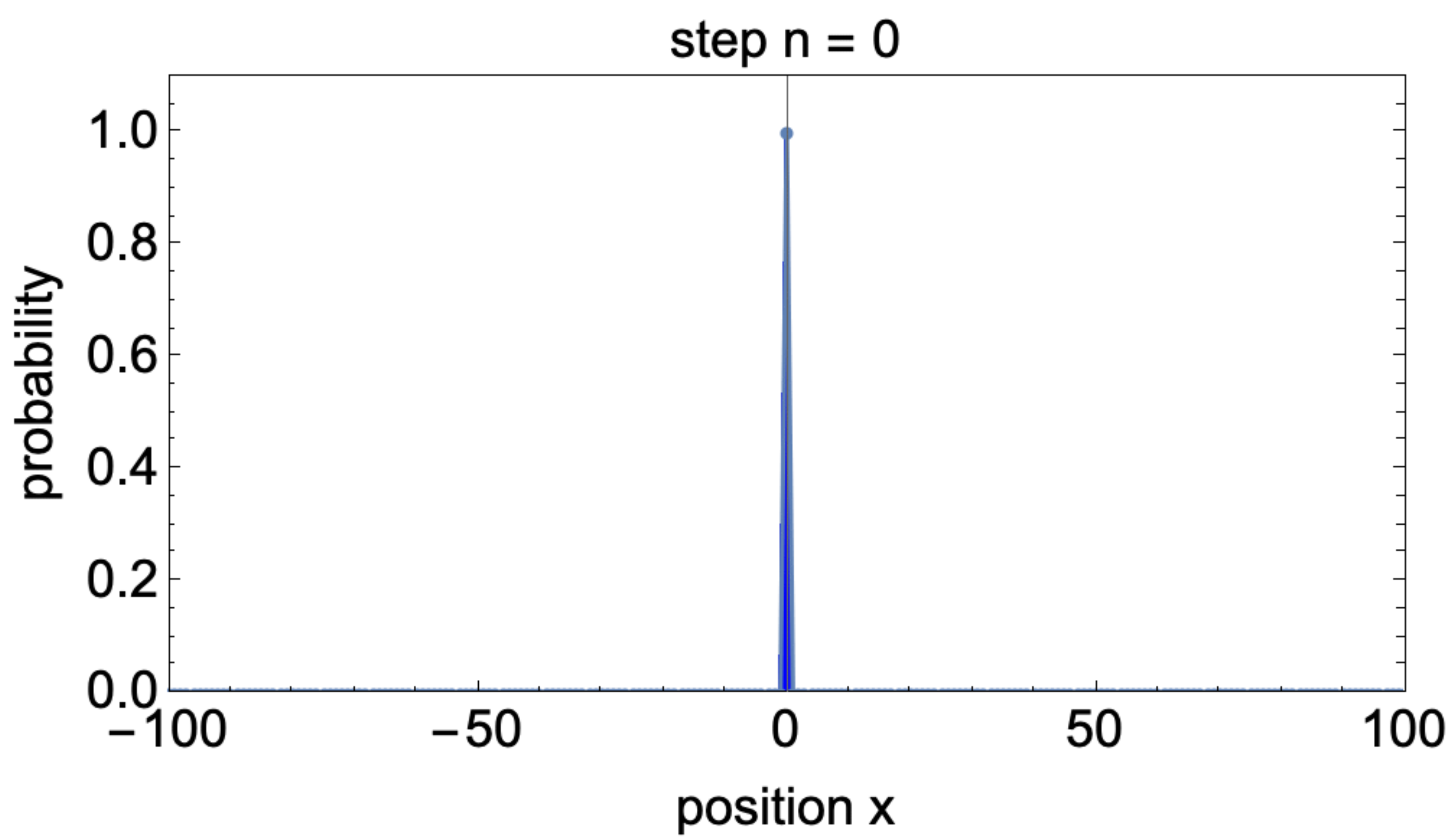
$$\sigma(n) = \sqrt{< x^2 > - < x >^2}. \quad = 0$$



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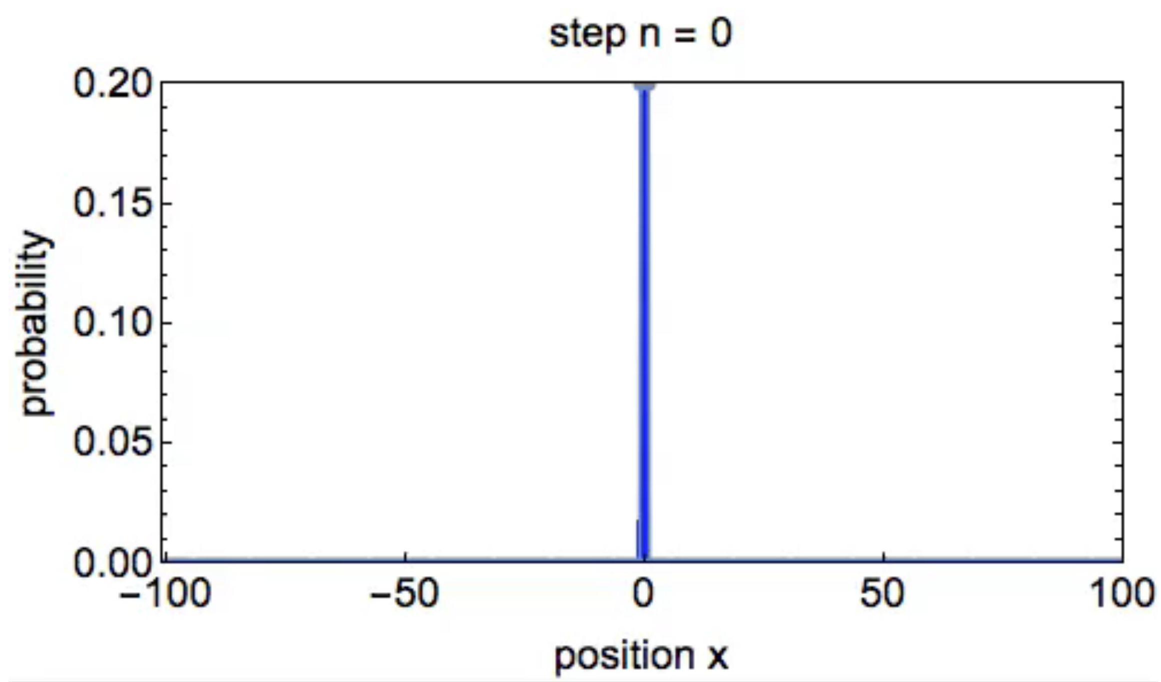
Electric quantum walks

Ballistic spread:



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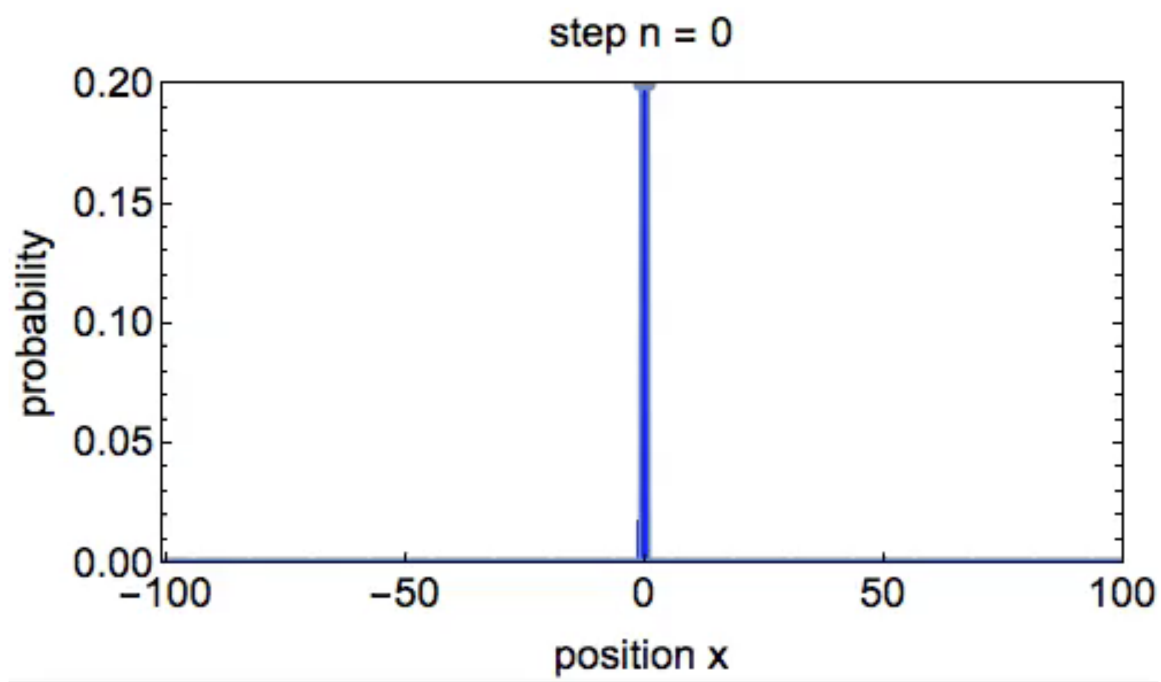
Electric quantum walks

Ballistic spread: $\sigma(n) = \sqrt{<x^2> - <x>^2}. \quad \propto n$



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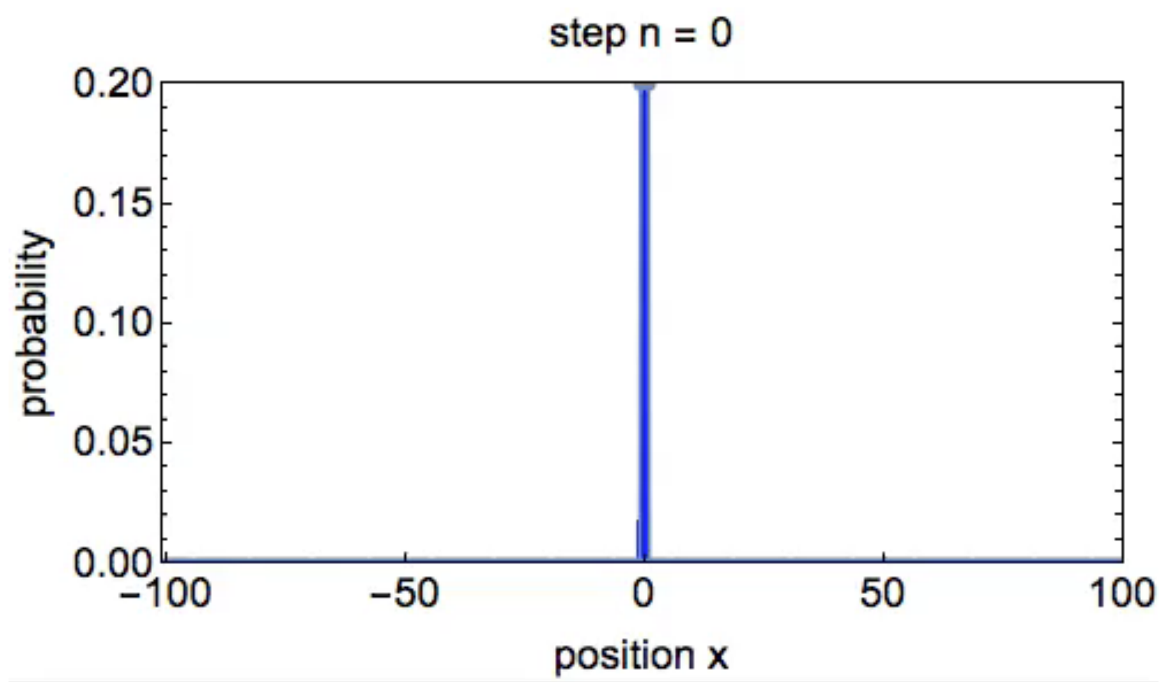

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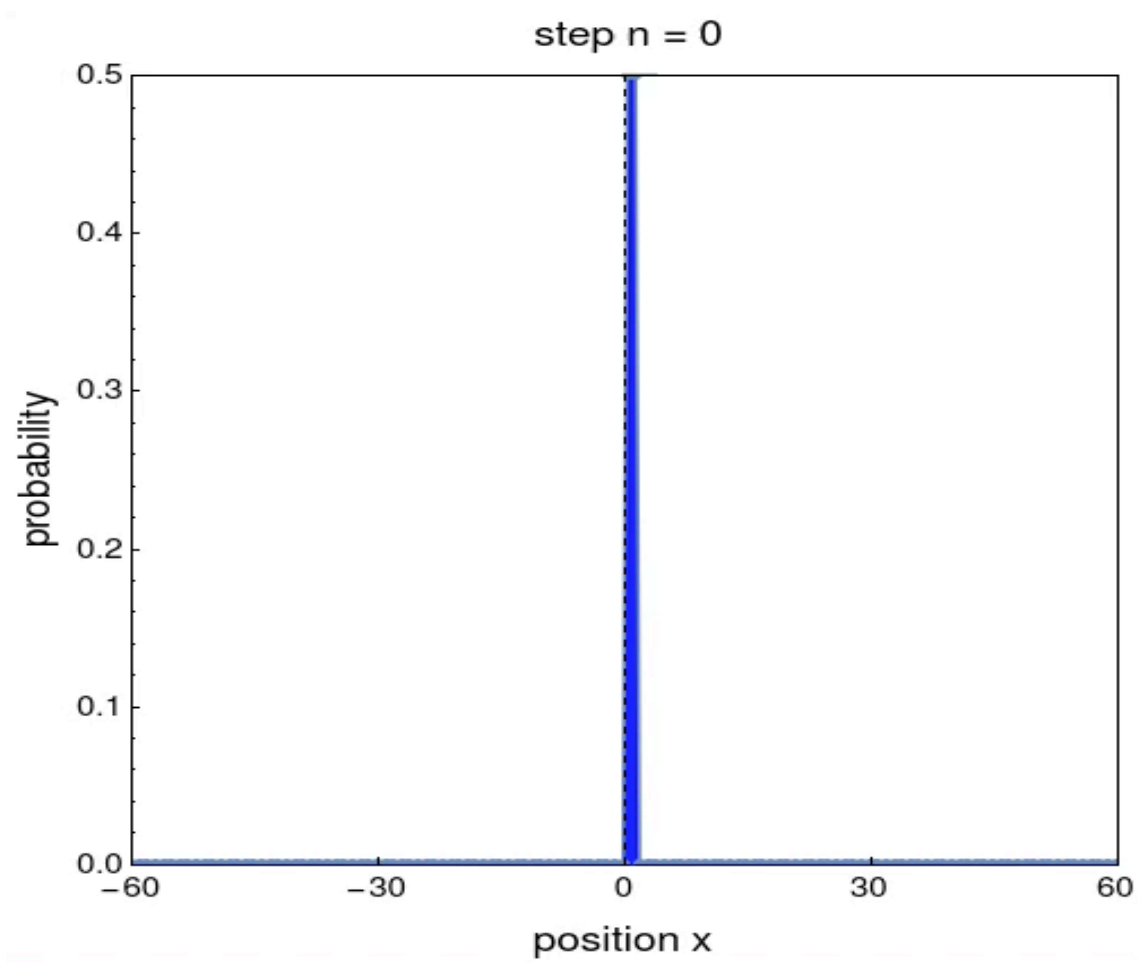
Electric quantum walks

Localization:



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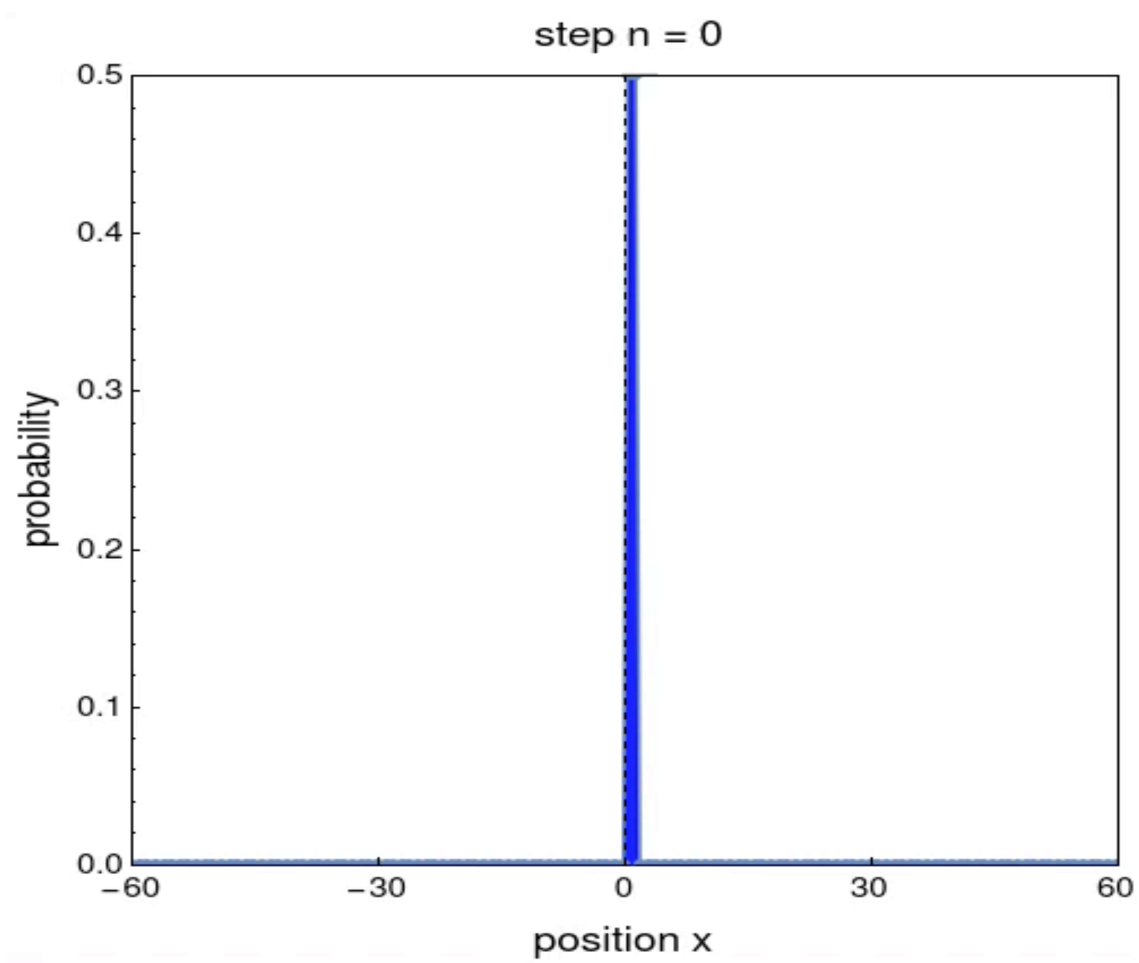
Electric quantum walks

Localization: $\sigma(n) = \sqrt{< x^2 > - < x >^2}.$ → constant



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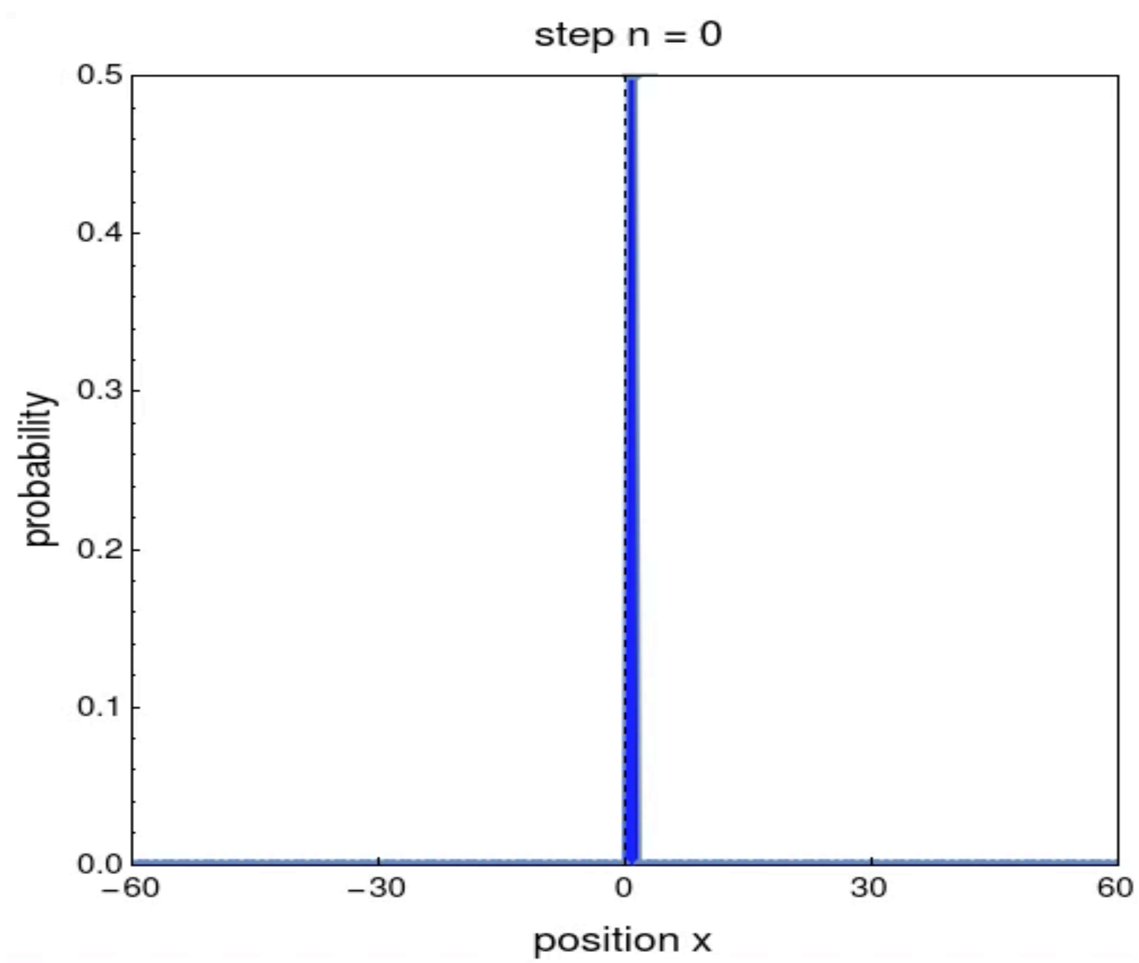

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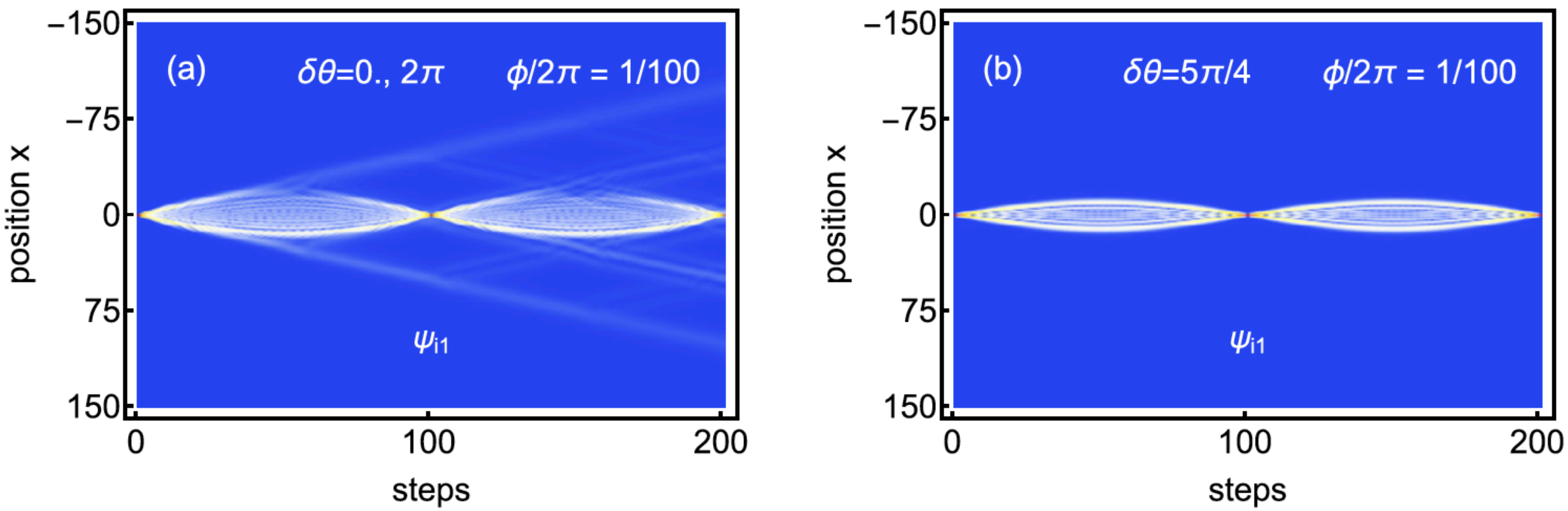
Revivals, Rational Electric Field



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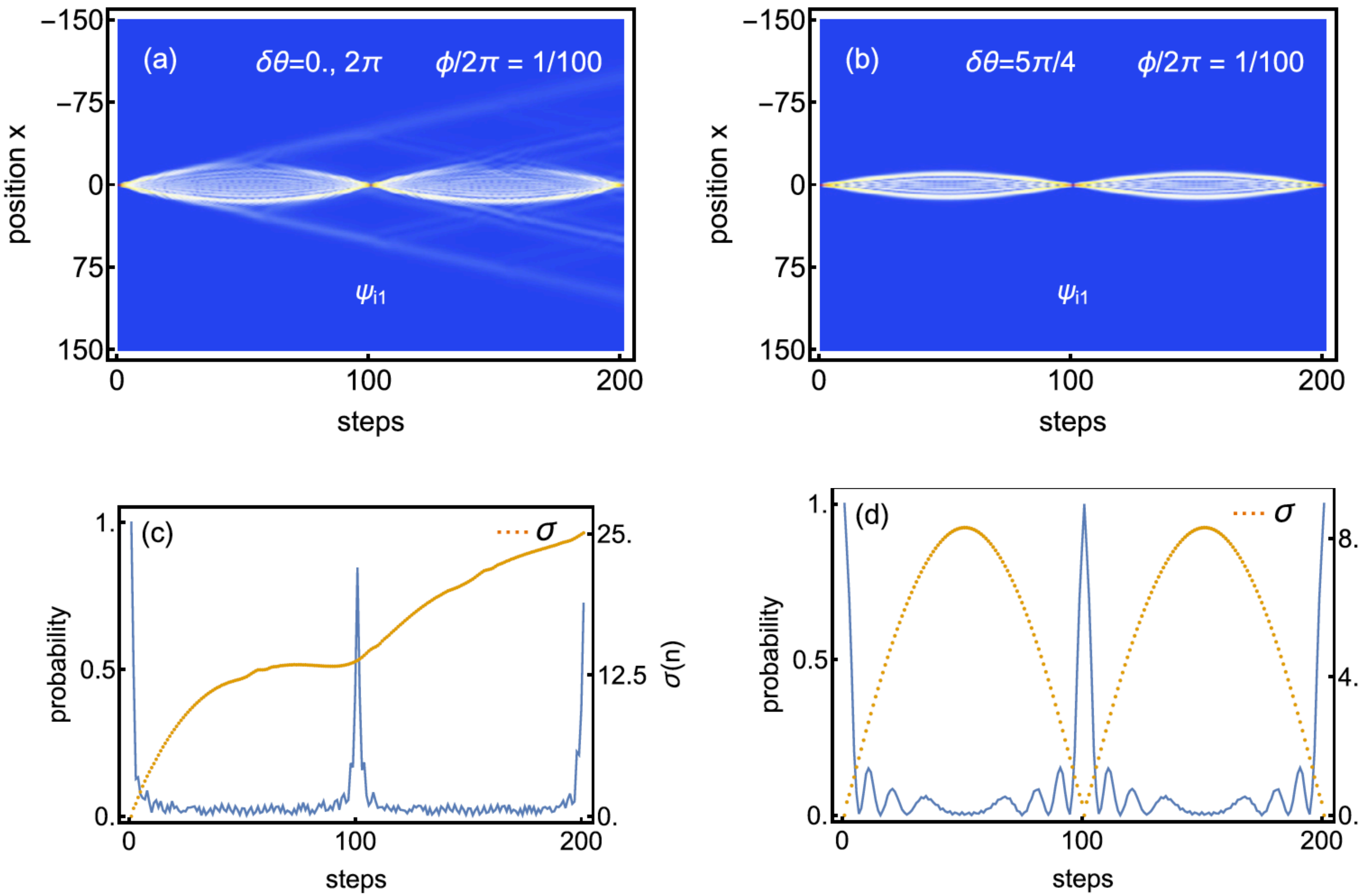

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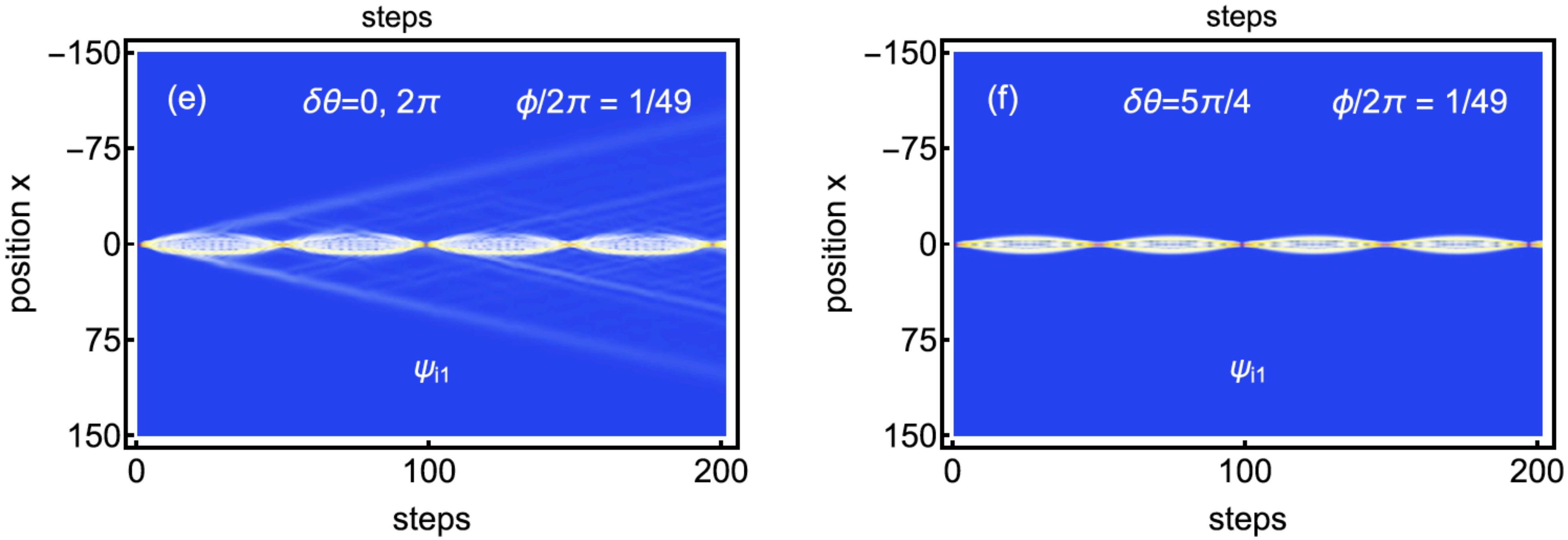

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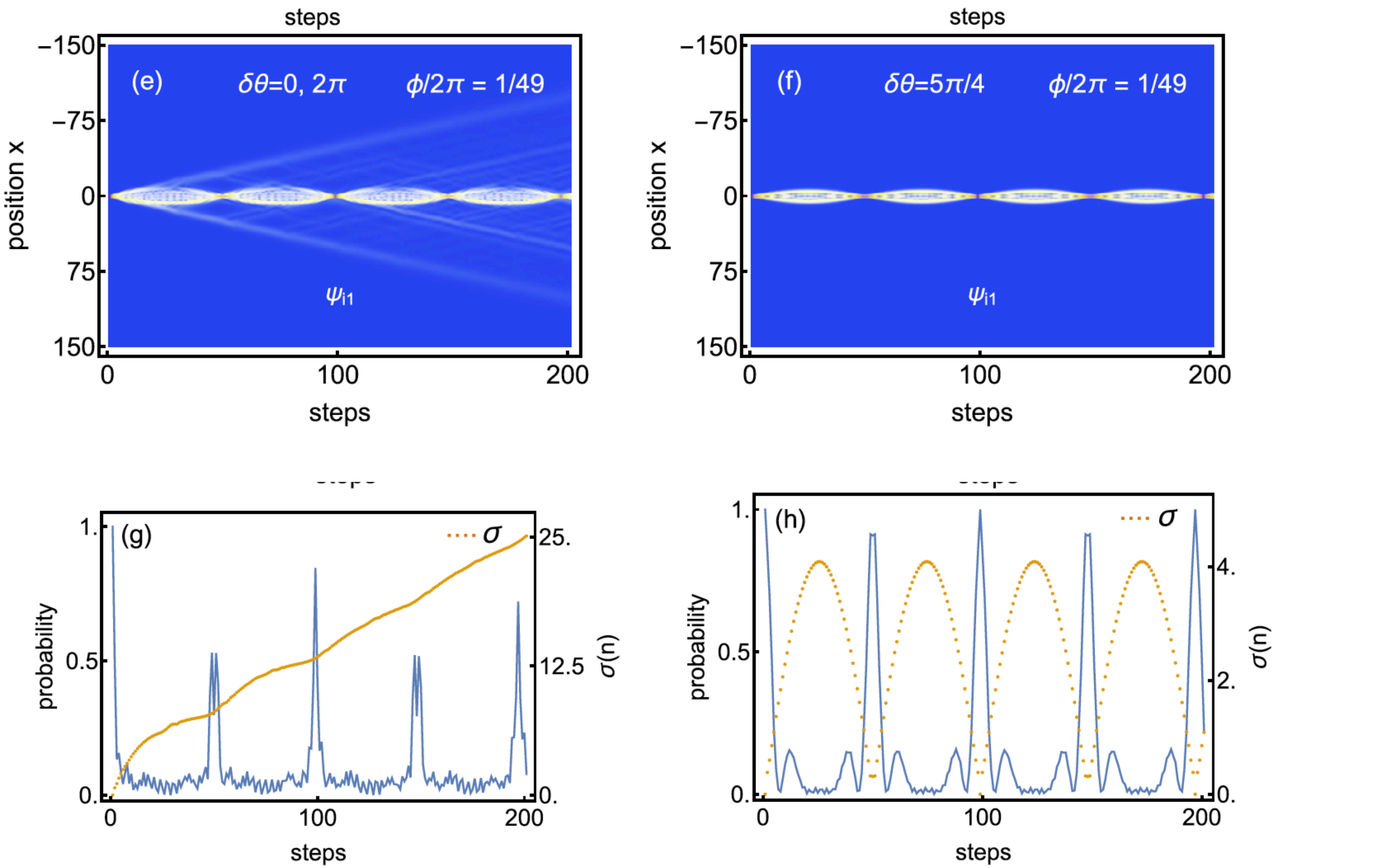

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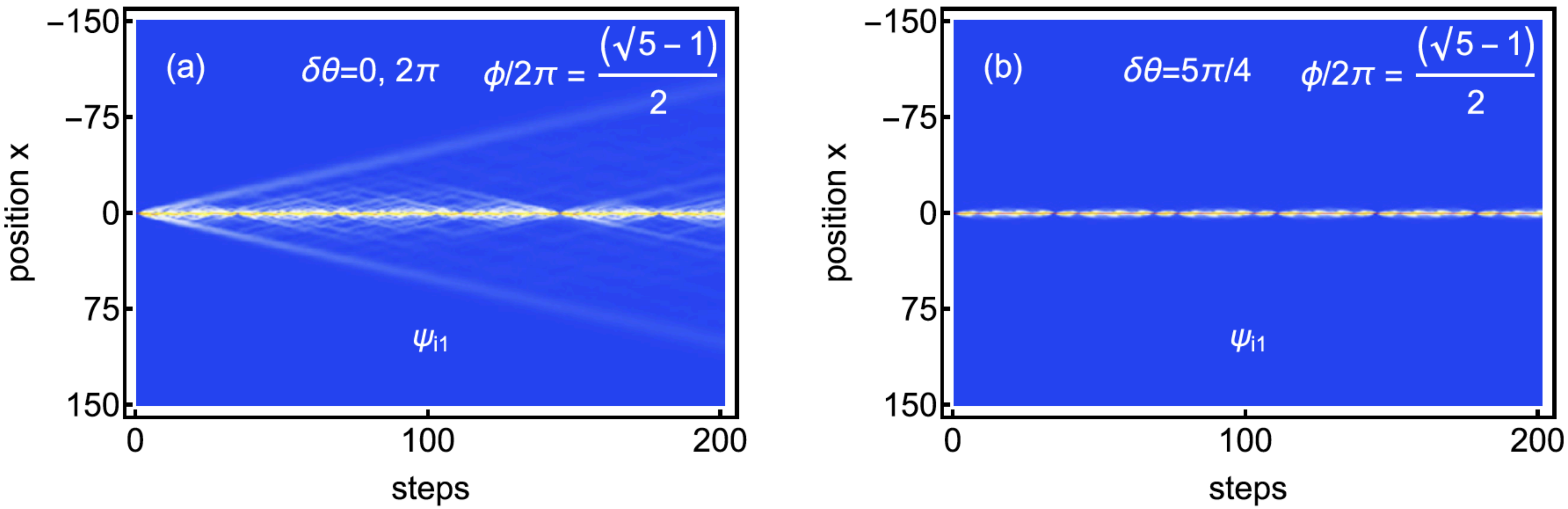
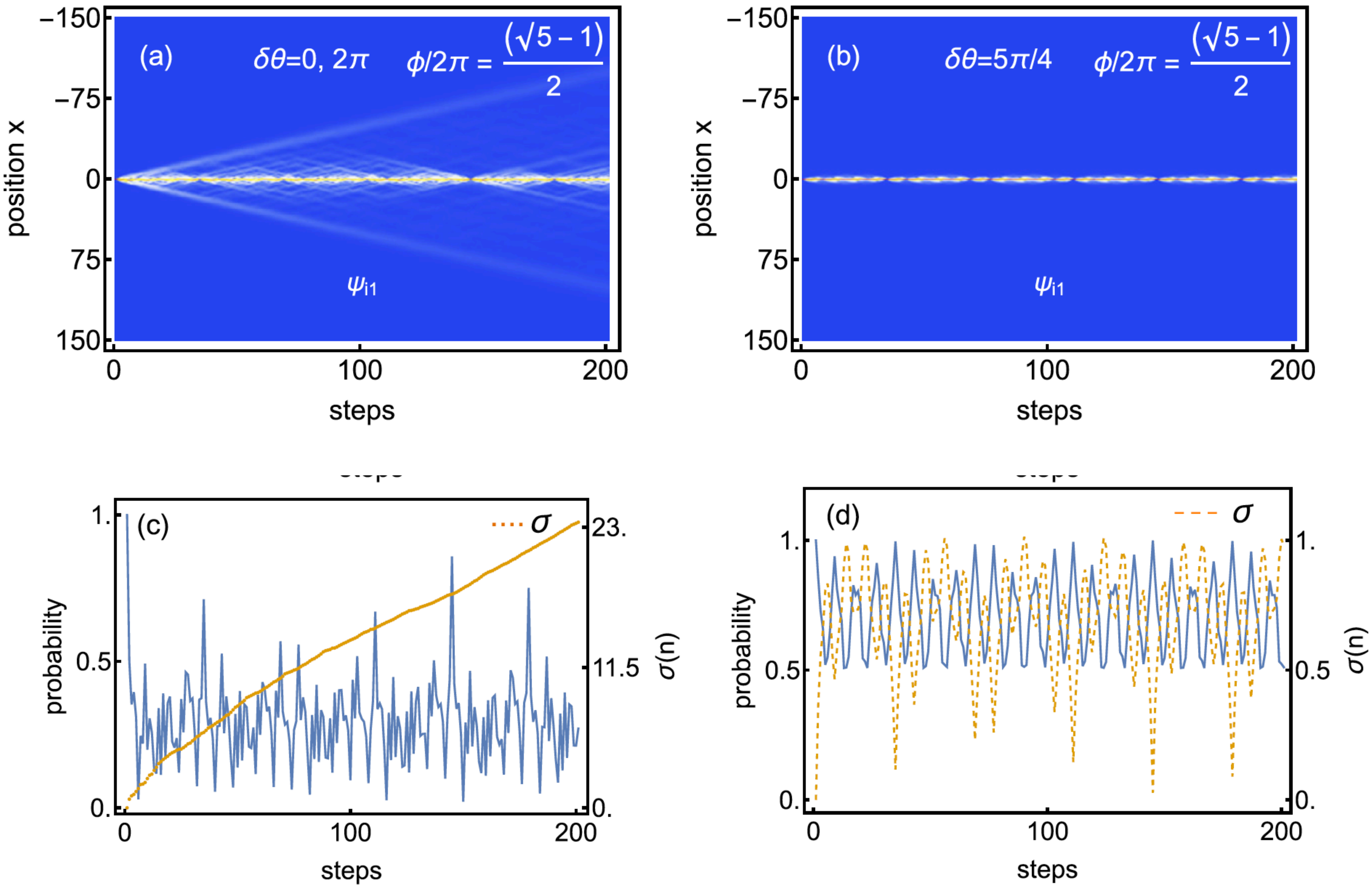
Localization, Irrational Electric Field



Localization, Irrational Electric Field



Ballistic spread, Noise in the Electric Field

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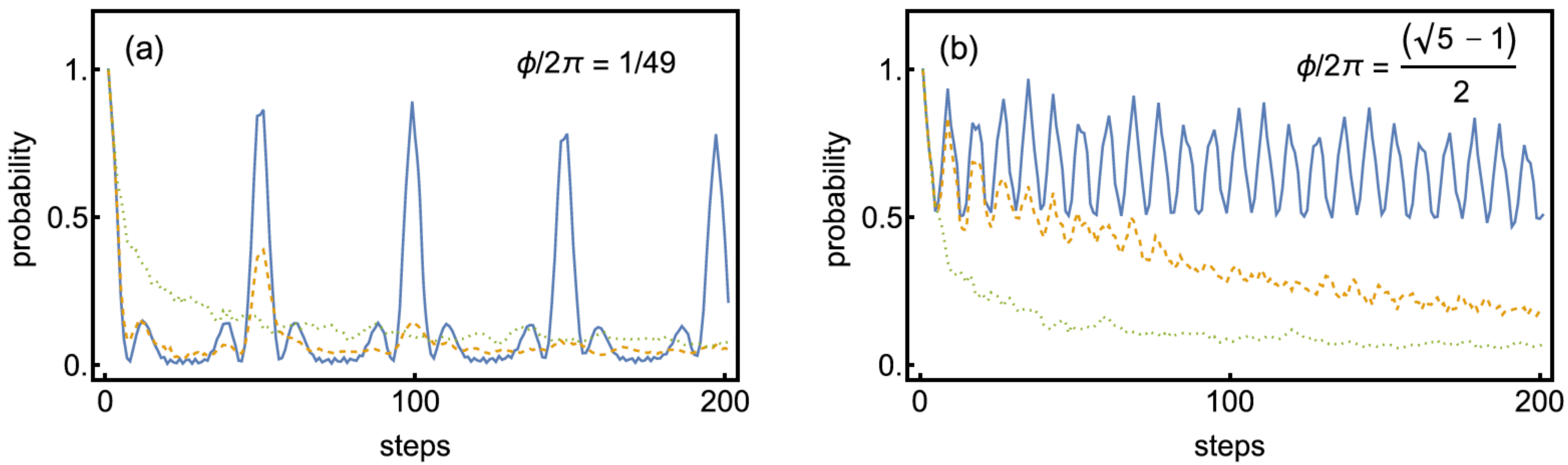

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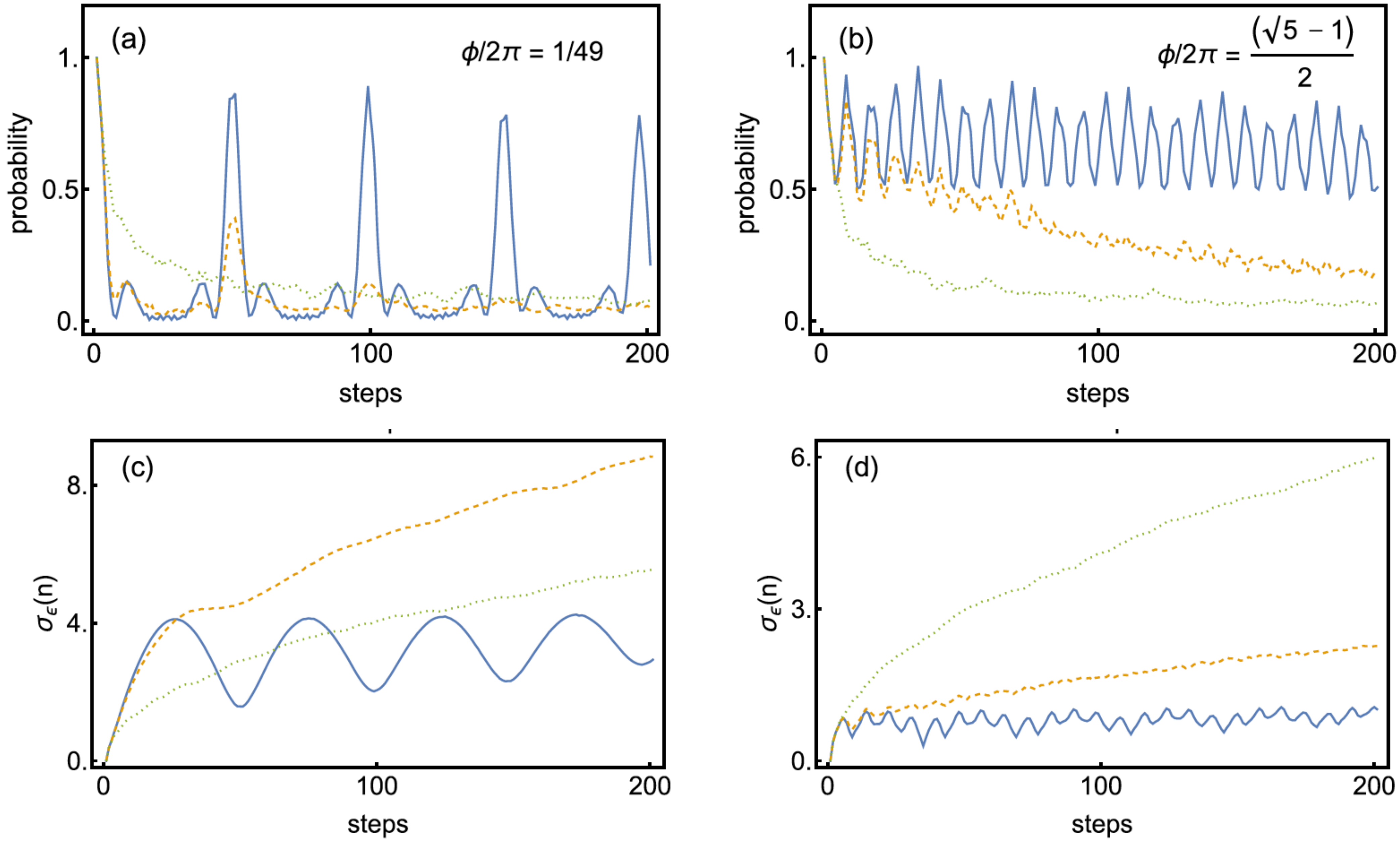

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