Sequences & Series

1. Sequeces, Series, Convergence
2. Binomial Series
3. Power Series
4. Taylor Series
5. Fourier Series

UMD Summer Enrichment, 2024

5. Fourier Series

Fourier Series Formalism

- A periodic function $f(x)$ with period $2L$ can be represented as a Fourier series in the form:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{L} + b_n \sin \frac{n\pi x}{L} \right)$$

- An often used function is the Kronecker delta: $\delta_{nm} = \begin{cases} 1 \text{ if } m = n \\ 0 \text{ if } m \neq n \end{cases}$
- Some useful orthogonality relations are the following:
  - $\int_{-L}^{L} \cos\left(\frac{m\pi x}{L}\right) \cos\left(\frac{n\pi x}{L}\right) dx = L\delta_{nm}$
  - $\int_{-L}^{L} \sin\left(\frac{m\pi x}{L}\right) \sin\left(\frac{n\pi x}{L}\right) dx = L\delta_{nm}$
  - $\int_{-L}^{L} \cos\left(\frac{m\pi x}{L}\right) \sin\left(\frac{n\pi x}{L}\right) dx = 0$

- **Q:**

A. Prove the orthogonality relations

B. Show that $f(x + 2L) = f(x)$

C. Show that a_0 is twice the average value of $f(x)$

D. Find formulas for a_n and b_n

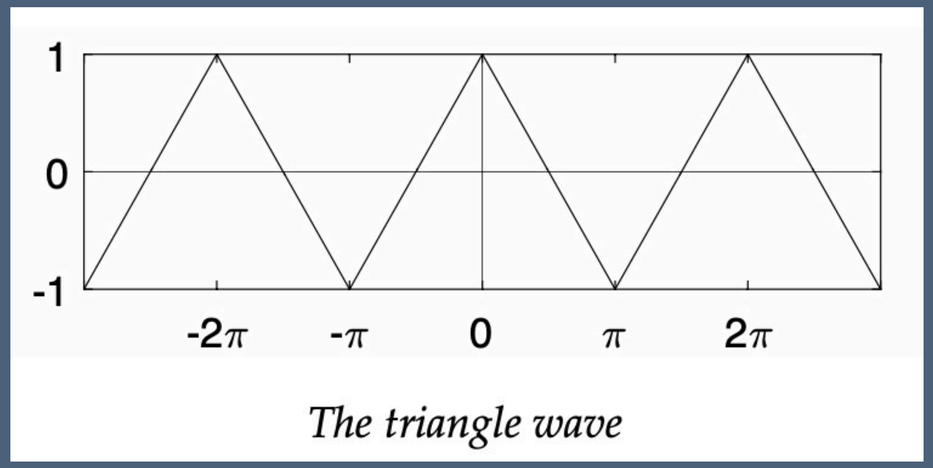
E. What is the Fourier series expansion for an even function

F. What is the Fourier series expansion for an odd function

G. Show that the Fourier series expansion for an odd function satisfies $f(0) = 0$ and that of an even function satisfies $f'(0) = 0$

Fourier Series Applications

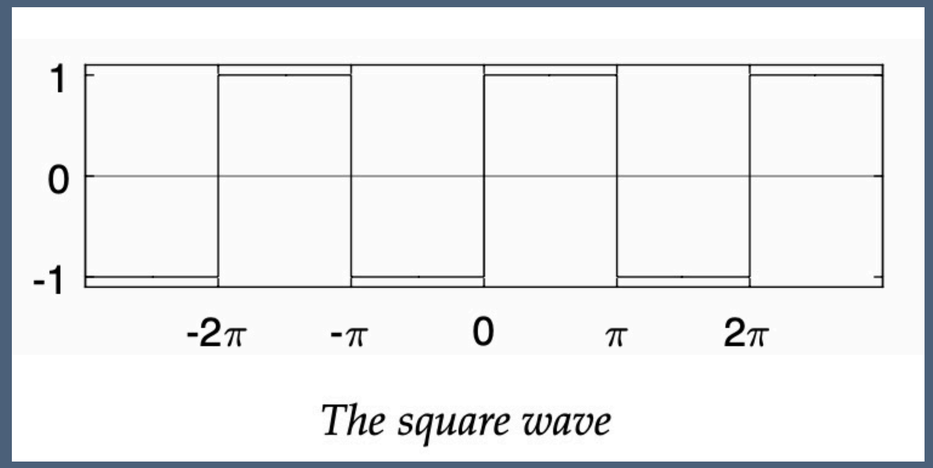
- Determine the Fourier series of the triangle wave $f(x) = 1 - \frac{2x}{\pi}, \quad 0 < x < \pi$



The triangle wave

- Use your result to find a series for π^2

- Determine the Fourier series of the square wave



The square wave

- Use your result to find a series for π