Sequences & Series

1. Sequeces, Series, Convergence
2. Binomial Series
3. Power Series
4. Taylor Series
5. Fourier Series

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4. Taylor Series

Taylor Series

- If the function $f(x)$ can be differentiated as often as required at some $x = x_0$ then we can express it in terms of a power series known as the Taylor series: $f(x) = \sum_{n=0}^{\infty} \frac{(x - x_0)^n}{n!} f^{(n)}(x_0)$ where $f^{(n)}$ is the n-th derivative of $f(x)$

- **Q:**
  - Find the Taylor expansion of $\cos x$ about $x_0 = 0$
  - Suppose you have a system subject to some potential $V(x)$. Use Taylor expansion to show that for small perturbations about a local minimum $x_0 = a$, Hook's Law is a good approximation to describe the physics
  - The magnitude of an electric field due to a dipole is given by $E = kq\left(\frac{1}{(x-r)^2} - \frac{1}{(x+r)^2}\right)$. When $x \gg r$, find an approximation of E to the first order of r
  - In statistical mechanics, the multiplicity of a state is the number of ways you could produce that state. The multiplicity of an Einstein solid is approximately: $\Omega(N, q) \approx \frac{(q+N)!}{q!N!}$. This is difficult to calculate for large values and we resort to approximations. Find an approximation of the form x^N for $\Omega(N, q)$ when $q \gg N$. You will need Stirling's approximation: $\ln N! \approx N \ln N - N$