Sequences & Series

1. Sequeces, Series, Convergence
2. Binomial Series
3. Power Series
4. Taylor Series
5. Fourier Series

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2. Binomial Series

Binomial Series

- An important series that comes up often is the binomial series: $1 + px + \frac{p(p-1)}{2!}x^2 + \frac{p(p-1)(p-2)}{3!}x^3 + \cdots = (1+x)^p$
  - If p is an integer, n, then $1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots + x^n = (1+x)^n$
- **Q:**
  - Use the binomial series expression to write out $(1+x)^2, (1+x)^3$, and $(1+x)^4$
  - Use the ratio test to show when the binomial series is convergent
  - Replace x with $\frac{b}{a}$ to obtain an expression for $(a+b)^n$, where n is an integer.
    - This expression is the binomial theorem.
  - Use the binomial series expression to write out $(a+b)^3, (a+b)^4$.
  - Assume that x is so small that powers of x^3 can be safely ignored, write an approximation of $(1-x)^{1/2}$.
  - Obtain a cubic approximation of $\frac{1}{(2+x)}$. When is this approximation valid?

3. Power Series

Power Series

- As we just saw, the convergence of some series depends on x. We call a series $S(x) = \sum_{n=0}^{\infty} a_n(x - x_0)^n$ a power series
  - We can think of previous series as restricted cases of power series where $x = 1$ and $x_0 = 0$
  - In the context of power series, the a_n are called coefficients
  - For power series if the series converges for $|x - x_0| < R$ and diverges for $|x - x_0| > R$ we call R the **radius of convergence**
    - the series may or may not converge if $|x - x_0| = R$. These endpoints will not affect the radius of convergence
    - the interval of x over which thet power series converges (including the endpoints if needed) is called the **interval of convergence**

- **Q:**

  A. What is the radius of convergence of $\sum_{n=1}^{\infty} \frac{(-1)^n n}{4^n}(x+3)^n$, (hint, use the ratio test)

  B. Use the fact that $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ to find the radius of convergence of this power series, (hint, use the ratio test)

  C. Use a series expansion of $\frac{1}{1+x}$ to find an expression for $\ln(1+x)$ when $|x| < 1$

Radius of Convergence