Integral Calculus

Chris Palmer, July 9th, 2024

(Derivatives->) Integrals to remember

Indefinite integrals

- Polynomials
  - $f(x) = ax^n$
- Sine/Cosine
  - $g(t) = A\sin(\omega t)$
  - $h(\omega) = B\cos(\omega t)$
- Exponential
  - $l(T) = l_0 e^{aT}$
- Logarithms
  - $k(s) = k_0 \ln(s)$

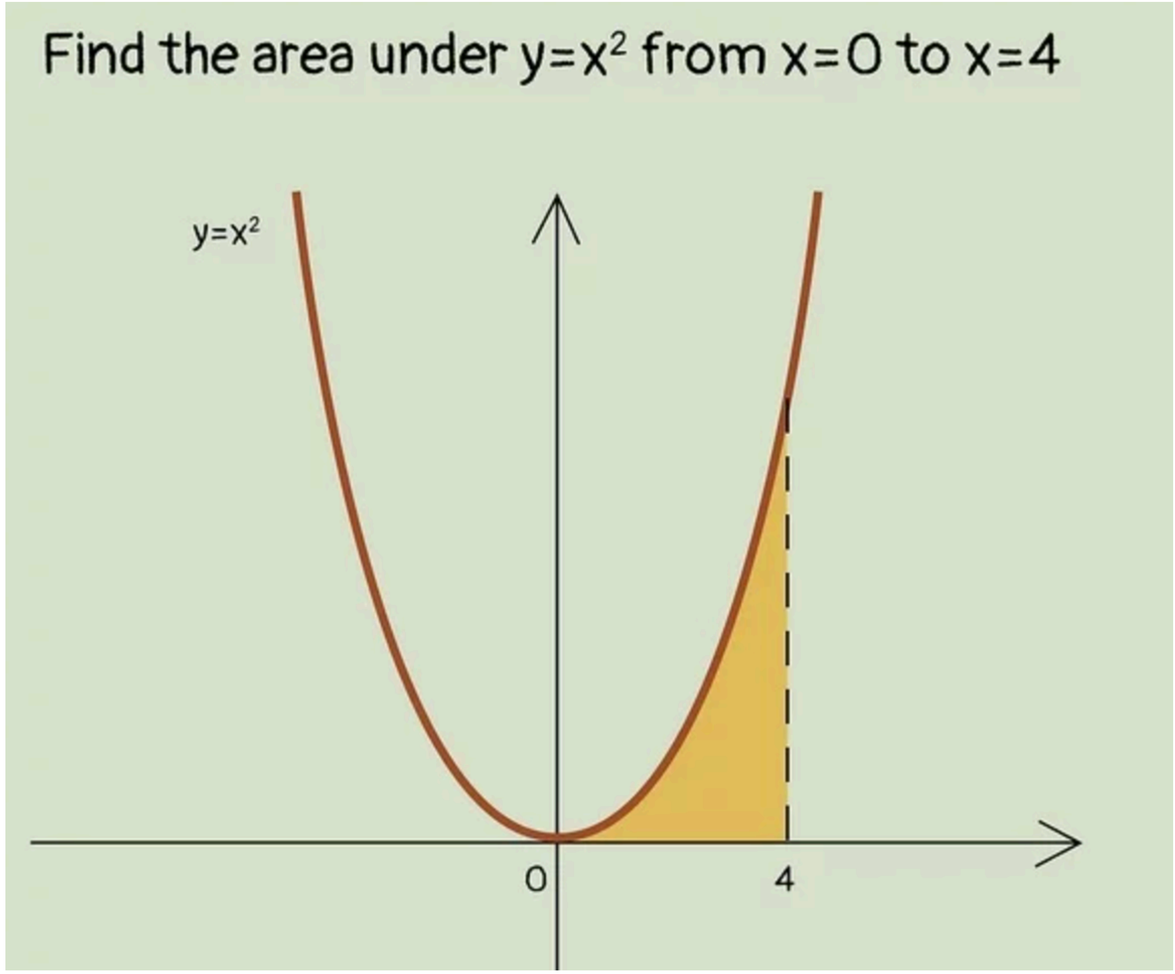
$$\int \left(x^2 + 10x\right) dx$$

$$\int \left(\sinh x\right) dx$$

$$\int \left(\frac{1}{x^2}\right) dx$$

Integration—> The Area Under Curve

- The first way you learned this was to sum up chunks and make the integration component infinitesimal.
- Riemann sum—>integral
  - Δx—>dx



Fundamental Theorem of Calculus

- Do you recall?

Fundamental Theorem of Calculus

- Integrals and derivatives are anti-functions

$$\int_{x=a}^{x=b} \frac{df}{dx} dx = f(b) - f(a)$$

Definite Integrals

- $\int_{x=x_0}^{x=x_f} g(x)\,dx =$ value
- The output will be evaluated at both ends.
- If it is a value that you for the end points, you'll get a number.
- If your end points are variables, you get a function.
- Usually just the final end point is a variable and the initial point is a defined value.

Product rule—>Integration by parts

- $h(x) = f(x)g(x); \quad \frac{dh}{dx} = \frac{df}{dx}g + f\frac{dg}{dx}$
- $d(uv) = du \cdot v + u \cdot dv \; -> u \cdot dv = d(uv) - v \cdot du$

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- $\int x \sin x dx = ?$
- $\int x^2 e^{-ax} dx = ?$
- $\int \sin(x) e^{-ax} dx = ?$

Product rule—>Integration by parts

- $d(uv) = du \cdot v + u \cdot dv \; -> v \cdot du = d(uv) - u \cdot dv$
- $\int_{x=0}^{x=\infty} xe^{-x}dx = ?$
- $\int_{x=0}^{x=\infty} \frac{\sin x}{x}dx = ?$ complex analysis?

Substitution

- Making a substitution in order to simplify an integral.
- E.g. $u = f(x); du = \frac{df}{dx}dx$
  - Where the $\frac{df}{dx}$ is part of the integrand.

$$\int \cos^2 x \sin x dx$$

$$\int \frac{x}{\left(x^2+1\right)} dx$$

$$\int_{t=0}^{t=2} t \sin\left(t^2\right) dt$$

Partial Fractions

- Sometimes impossible looking integrals can be done by factorization of the denominator and splitting into separate functions.
- $$\int_{t=1}^{t=3} \frac{23-t}{(t-5)(t+4)} dt = ?$$