Volume integral of non-uniform density planet

The density of a planet is linearly declining with increasing radius in this way:

$$\rho\left(r\right) = \rho_0\left(1 - 0.5r\right)$$

Setup the integral for the total mass of the planet, *M*. The planet has a finite radius, *R*.

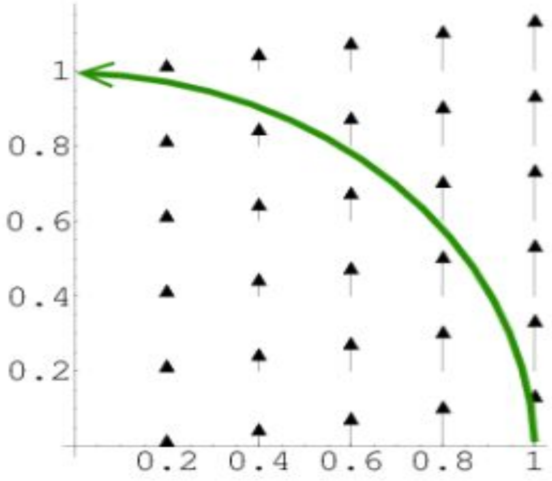
Evaluate the mass integral in spherical coordinates.

Vector Integral Examples

$$\overrightarrow{F}(x, y) = (0, x) = x\hat{y}$$

Example 1: Evaluate the line integral along the green path (which is a quarter circle).

1. Parametrize the quarter circle from (1,0) to (0,1).
2. Draw and write out $d\vec{s}$.
3. Evaluate the line integral.



$$\int_C \overrightarrow{F} \cdot d\vec{s}$$

Work example

$$W = -\int_{\vec{a}}^{\vec{b}} \vec{F} \cdot d\vec{r}$$

Example 2:

The force field acting on a two-dimensional

linear oscillator may be described by:

$$\vec{F} = -\hat{x}kx - \hat{y}ky$$

Compare the work done moving against this force field when going from (1, 1) to (4, 4) by the following straight-line paths:

(a) (1, 1) → (4, 1) → (4, 4)

(b) (1, 1) → (1, 4) → (4, 4)

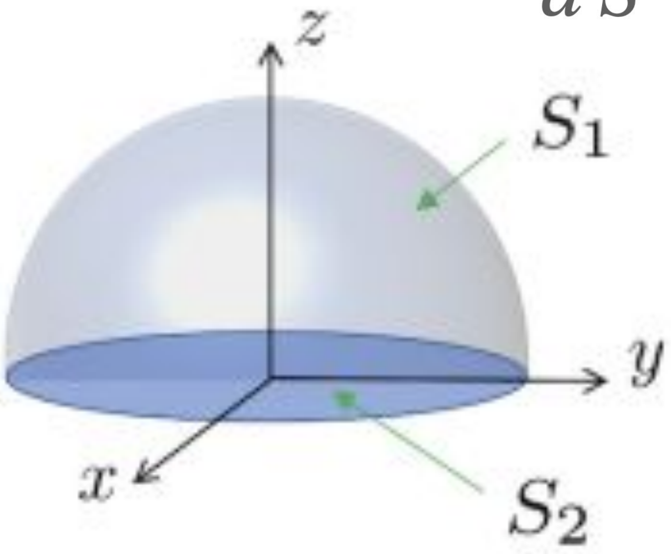
(c) (1, 1) → (4, 4) along x = y.

The Integral Theorems: Divergence/ Gauss' Theorem

$$\iiint \overrightarrow{\nabla} \cdot \overrightarrow{F}\, dV = \iint_{\text{boundary}} \overrightarrow{F} \cdot d\overrightarrow{S}$$

$$d\overrightarrow{S} = \hat{n} dA$$

- A relation between a surface integral of a vector and the volume integral of the divergence of that vector.
- Normal vector, $\hat{n}$, always points OUTSIDE the volume.



$$V : x^2 + y^2 + z^2 \leq R^2;\; z \geq 0$$

Evaluate the left- and right-hand side of the Divergence Theorem for this vector:

$$\overrightarrow{F} = (z + R)\,\hat{z}$$