The Evolution of Parton Pseudo-Distributions

LAMET 2024

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Parton and Ioffe Time distributions

- Unpolarized Ioffe time distributions — Ioffe time: $\nu = p \cdot z$

"Ioffe time distributions instead of parton momentum distributions in description of DIS"
V. Braun, P. Gornicki, L. Mankiewicz
Phys Rev D 51 (1995) 6036-6051

- $$I_q(\nu, \mu^2) = \frac{1}{2p^+} \langle p \,|\, \bar{\psi}_q(z^-)\gamma^+ W(z^-;0)\psi_q(0) \,|\, p \rangle_{\mu^2}$$

$$z^2 = 0$$

$$I_g(\nu, \mu^2) = \frac{1}{(2p^+)^2} \langle p \,|\, F_{+i}(z^-) W(z^-;0) F^i_{+}(0) \,|\, p \rangle_{\mu^2}$$

$$i = x, y$$

- Parton Distribution Functions
  - $$I_q(\nu, \mu^2) = \int_{-1}^{1} dx\, e^{ix\nu} f_q(x, \mu^2)$$
  - $$I_g(\nu, \mu^2) = \int_{0}^{1} dx\, \cos(x\nu)\, x f_g(x, \mu^2)$$

Parton Distributions and the Lattice

- Parton Distributions are defined by operators with light-like separations

- Use space-like separations

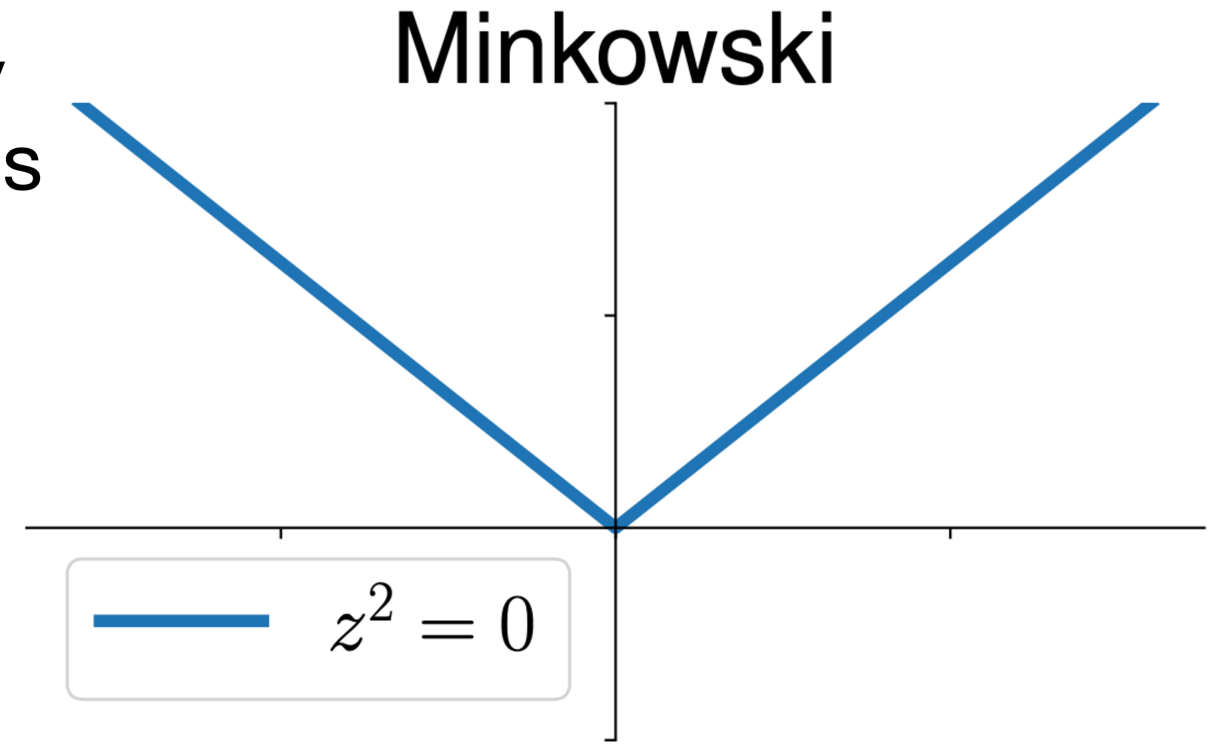
  X. Ji *Phys Rev Lett* 110 (2013) 262002

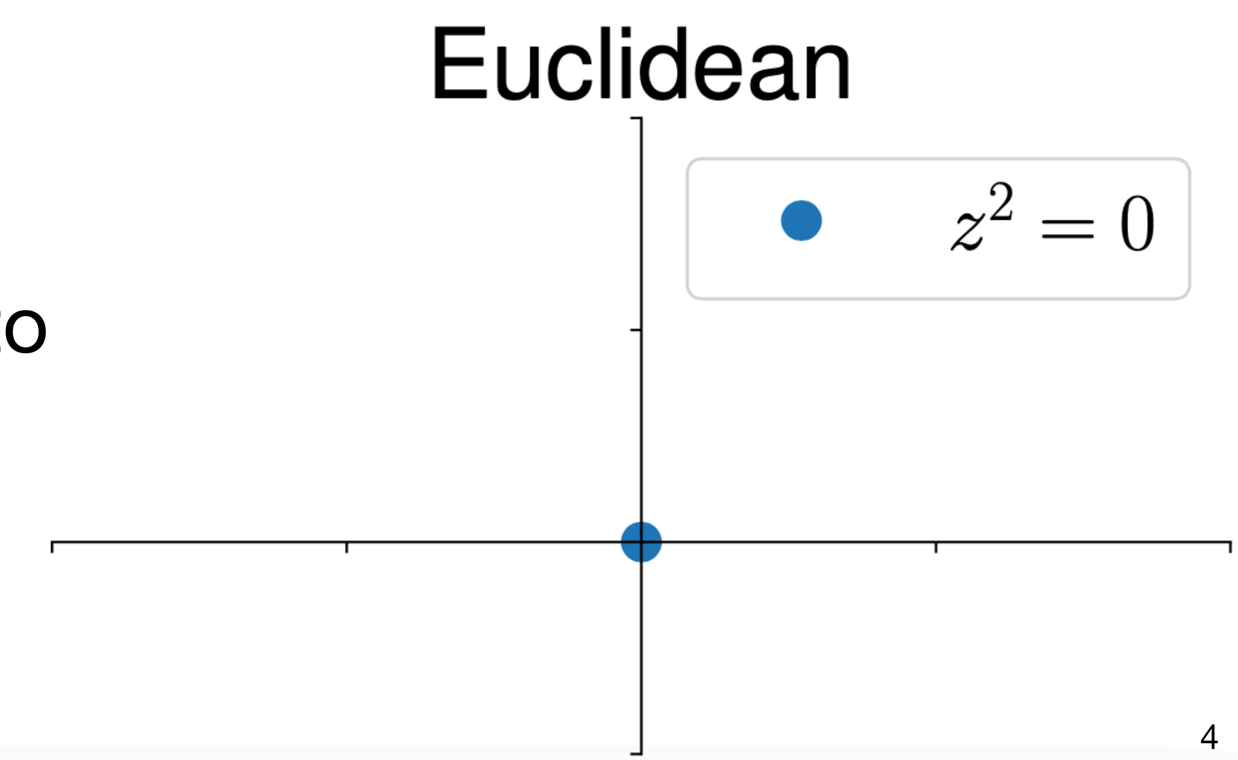
  - Wilson line operators

$$O_\Gamma^{\mathrm{WL}}(z) = \bar{\psi}(z)\Gamma W(z;0)\psi(0)$$

$$z^2 \neq 0$$

  - Factorizations exist analogous to cross sections





Wilson Line Matrix Elements

- Matrix element $M^\alpha(p,z) = \langle p \,|\, \bar{\psi}(z)\gamma^\alpha W(z;0)\psi(0) \,|\, p\rangle$ $\quad z^2 < 0$

$$= 2p^\alpha \mathscr{M}(\nu, z^2) + 2z^\alpha \mathscr{N}(\nu, z^2)$$

- Quasi-PDF: $\tilde{q}(y, p_z^2) = \frac{1}{2p_\alpha} \int dz e^{iyp_z z} M^\alpha(p_z, z)$ $\quad \alpha = t$ and $z^t = 0$

- Large Momentum Effective Theory: X. Ji *Phys. Rev. Lett.* 110 (2013) 262002

- $$\tilde{q}(y, p_z^2) = \int \frac{dx}{|x|} K\left(\frac{y}{x}, \frac{\mu^2}{(xp_z)^2}\right) q(x, \mu^2) + O\left(\frac{\Lambda^2_{\mathrm{QCD}}}{(xp_z)^2}, \frac{\Lambda^2_{\mathrm{QCD}}}{((1-x)p_z)^2}\right)$$

- Pseudo-PDF: A. Radyushkin *Phys. Rev. D* 96 (2017) 3, 034025

$$\mathscr{M}(\nu, z^2) = \int dx\, C(x\nu, \mu^2 z^2) q(x, \mu^2) + O(\Lambda^2_{\mathrm{QCD}} z^2)$$

$$= \int du C'(u, \mu^2 z^2) I_q(u\nu, \mu^2) + O(\Lambda^2_{\mathrm{QCD}} z^2)$$

The Role of Separation and Momentum

- In Structure Functions, quasi-PDF, and pseudo-PDF, variables have common roles

Scale:

$Q^2 \,/\, x^2 p_z^2 \,/\, z^2$

Dynamical variable:

$x_B \,/\, z \,/\, p_z$, or $\nu = p \cdot z$

- Scale for factorization to PDF
- Scale in power expansion
- Keep away from $\Lambda^2_{\rm QCD}$
- Technically only requires single value, use many to study systematics

- Variable describes non-perturbative dynamics
- Can take large or small value
- Want as many as are available
- Wider range improves the inverse problem

Renormalization

Primary z dependence of bare matrix element

- Wilson line divergence gives dominant power divergence
- Other terms contribute $O(10\%)$ of total

Diagram z/a	0	1	2	3	4	5	6	7	8
Sunset	0	0.97346(2)	2.32308(7)	3.7762(1)	5.2709(2)	6.7871(3)	8.3163(4)	9.8550(5)	11.3998(6)
Sail	0	0	0.54974(3)	0.98654(6)	1.2110(1)	1.3580(1)	1.4518(2)	1.5226(2)	1.5776(2)
Vertex	1.4339(6)	0.5959(6)	0.1216(6)	-0.0847(6)	-0.2047(6)	-0.2717(6)	-0.3307(6)	-0.3725(6)	-0.4115(7)
Total Γ_{LPT}	1.4339(6)	1.5694(6)	2.9944(6)	4.6780(6)	6.2772(6)	7.8734(6)	9.4374(6)	11.005(6)	12.5659(7)

TABLE IV. The integrals of 1-loop diagrams in lattice perturbation theory for various z/a.

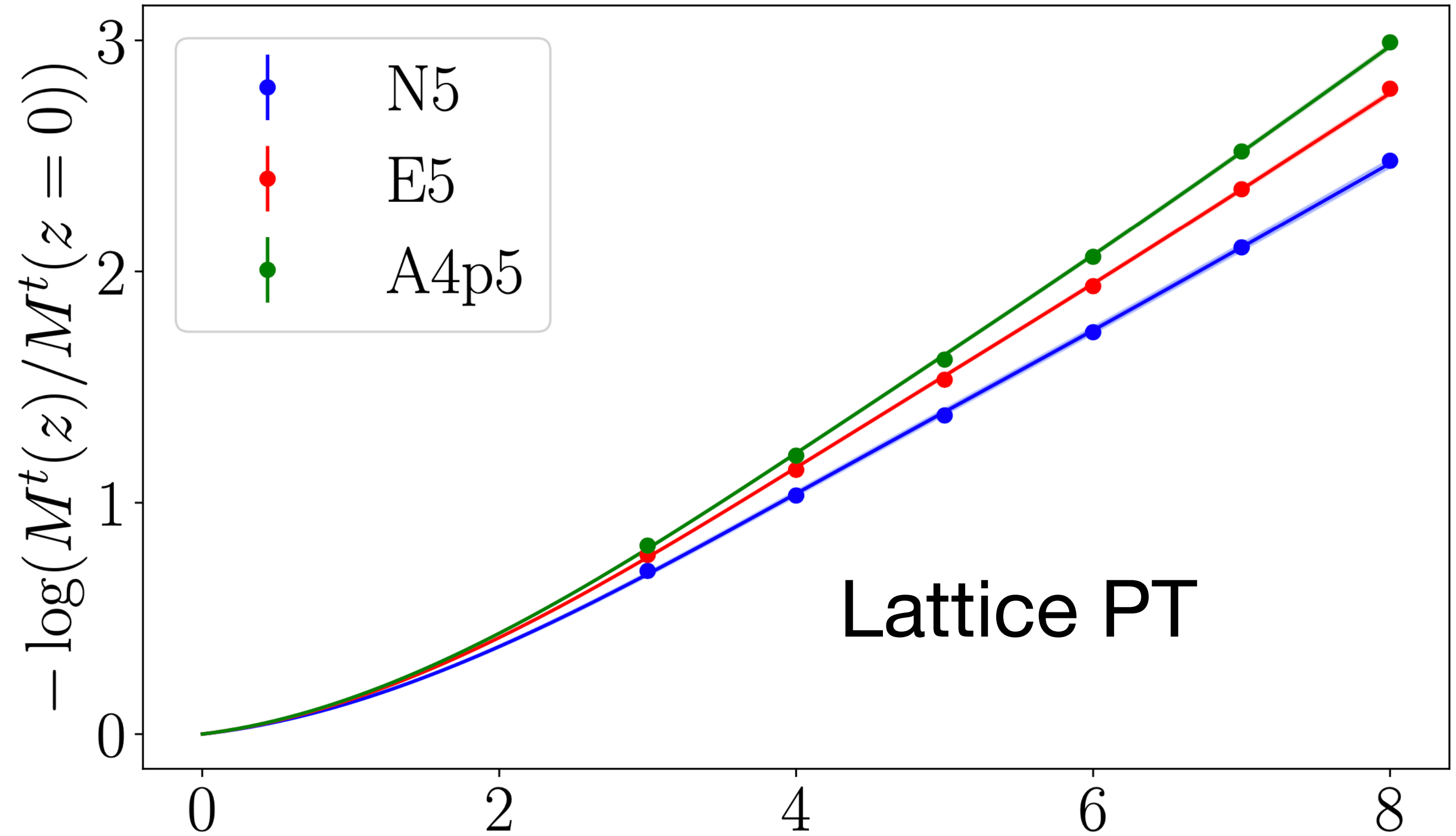
- Polyakov Scheme has spatial cutoff like lattice

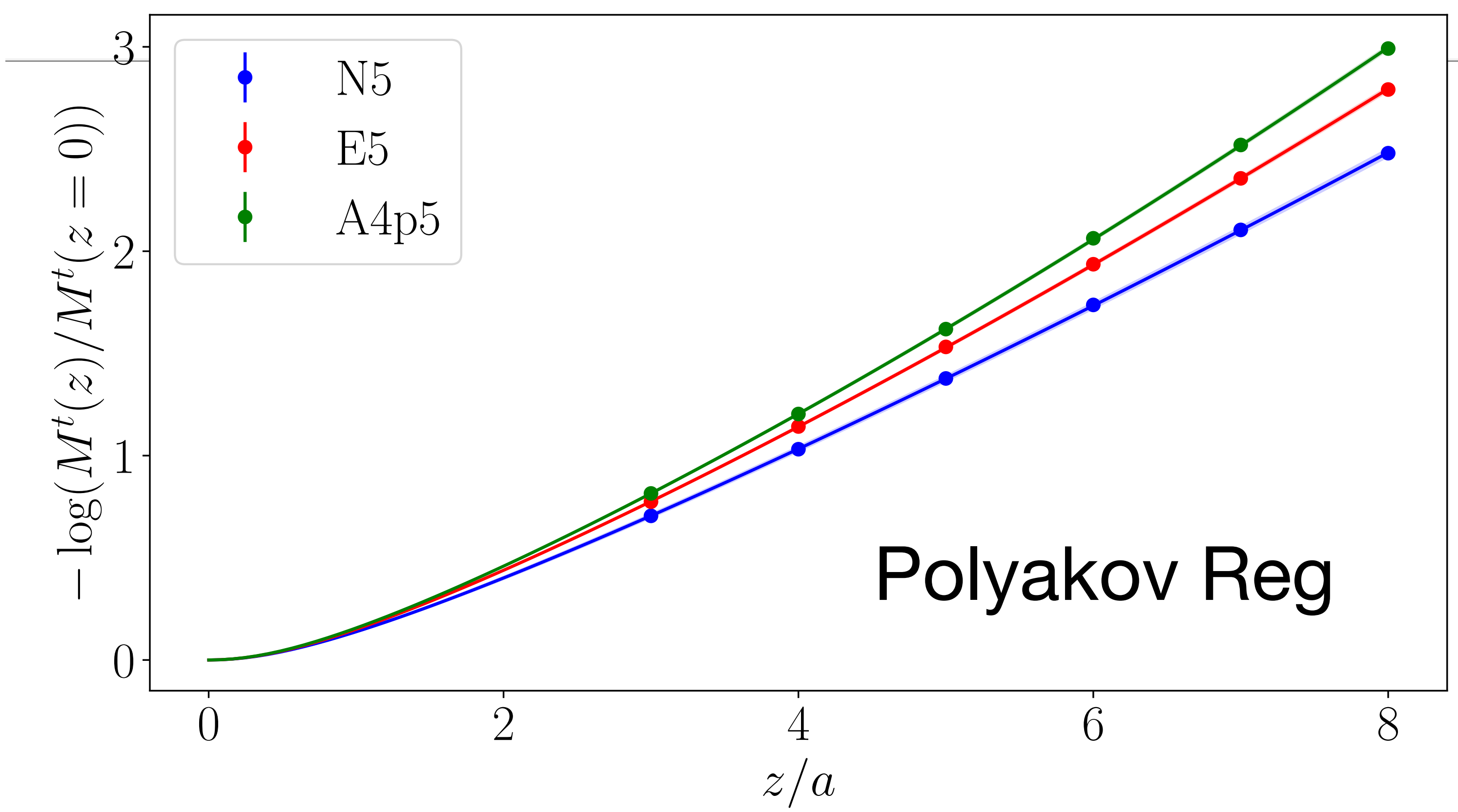
$$\frac{1}{z^2} \to \frac{1}{z^2 + a^2}$$

JK, C. Monahan, A. Radyushkin arXiv:2407.16577

Rest Frame Matrix Element

- Studied 3 lattice spacings at sub-precision stat error
  $a \sim 0.045, 0.065, 0.075\,\mathrm{fm}$
- Agrees to 1-loop perturbation theory up to few percent
- Scale is $\mu \sim \frac{\pi}{a}$
- Inclusion of simplest finite size correction improves agreement
- Lowest $z/a \sim 1$ or 2 disagree more





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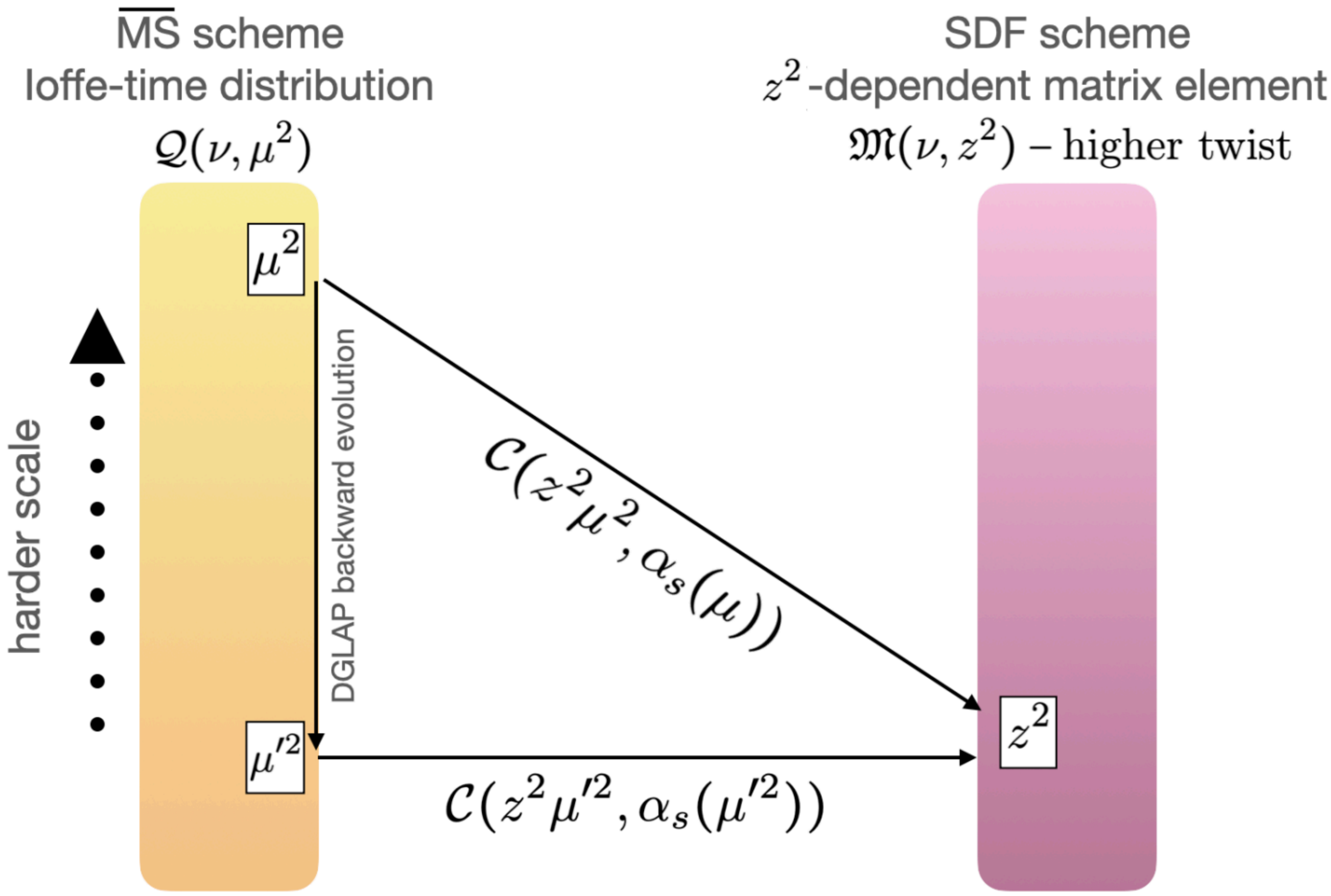
Options in truncation for matching

Equivalent only if all contributions given



H. Dutrieux, JK, C. Monahan, K. Orginos, S. Zafeiropoulos arXiv:2310.19926

Matching in Moment Space

- Ioffe time, momentum fraction, and Mellin moment space are intimately related

$$\mathfrak{M}(\nu, z^2) = \int du C(u, \mu^2 z^2) I(u\nu, \mu^2) = \int dx K(x\nu, \mu^2 z^2) q(x, \mu^2) = \sum_n \frac{(i\nu)^n}{n!} c_n(\mu^2 z^2) a_n(\mu^2)$$

- And the kernels are Mellin moments or Fourier transforms

$$c_n = \int du u^n C(u) \qquad K(x\nu) = \int du C(u) e^{iu\nu x}$$

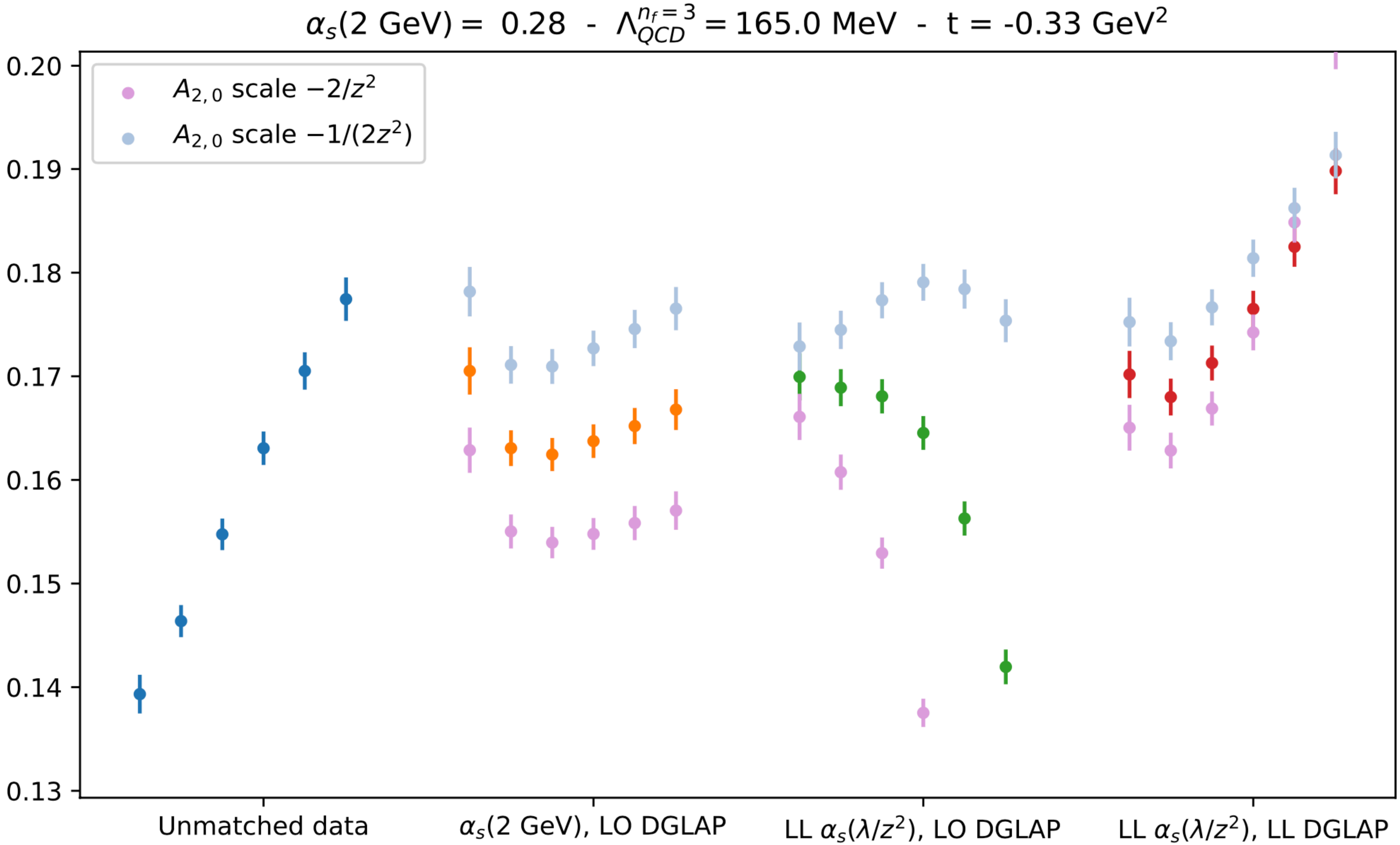
- How to evaluate $C(u), c_n, K(x\nu)$?

$\alpha_s(\mu) C_{NLO}(u, \mu^2)$ $\qquad$ $\alpha_s(1/z^2) C_{NLO}(u, \mu^2)$ $\qquad$ $\alpha_s(1/z^2) C_{LL}(u, 1/z^2)$

H. Dutrieux et al (HadStruc) arXiv:2405.10304

Matching to MS-Bar scheme

Very truncation dependent



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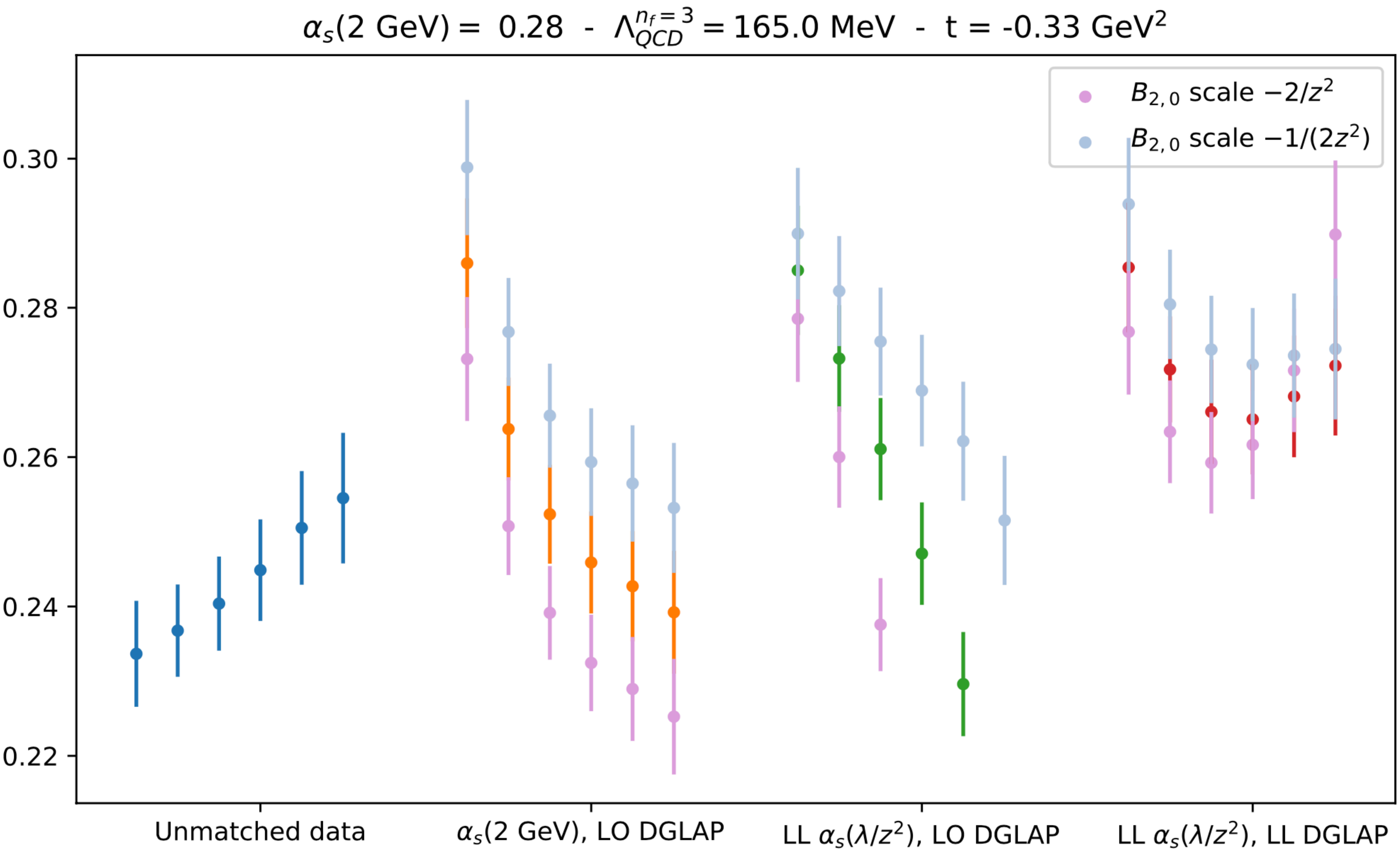
Matching to MS-Bar scheme



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Evolution of parton distributions

- Standard DGLAP evolution
  - Parton model: Splitting of partons into smaller x

$$\mu^2 \frac{d}{d\mu^2} q(x,\mu^2) = \int_x^1 dy\, P_{qq}(y) q(\frac{x}{y},\mu^2)$$

- MSbar Step Scaling function
  - Integrated or discretized version of evolution

$$q(x,\mu^2) = \int_x^1 dy\, \mathscr{E}(y,\mu^2,\mu_0^2) q(\frac{x}{y},\mu_0^2)$$

PDF at high scale $\mu \sim Q$

PDF at low input scale $\mu_0 \sim m_c$

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$$q(x,\mu^2) = \int_x^1 dy\, \mathscr{E}(y,\mu^2,\mu_0^2) q(\frac{x}{y},\mu_0^2)$$

- pseudo-PDF evolution
  - $\mathfrak{M}(\nu,z^2) = \int_0^1 du\, C(u,\mu^2 z^2)\, I(u\nu,\mu^2) + O(z^2)$
  - Data does not know about MSbar scale

$$\mu^2 \frac{d}{d\mu^2} \mathfrak{M}(\nu,z^2) = 0$$

$$\Downarrow$$

$$z^2 \frac{d}{dz^2} \mathfrak{M}(\nu,z^2) = \int_0^1 d\alpha \mathscr{P}(\alpha,z^2) \mathfrak{M}(\alpha\nu,z^2) + O(z^2)$$

$$\mathfrak{M}(\nu,z^2) = \int_0^1 d\alpha\, \Sigma(\alpha,z^2,z_0^2) \mathfrak{M}(\alpha\nu,z_0^2) + O(z^2,z_0^2)$$

$$\mathscr{E}(\mu^2,\mu_0^2) = C^{-1}(\mu^2 z^2) \otimes \Sigma(z^2,z_0^2) \otimes C(\mu_0^2 z_0^2)$$

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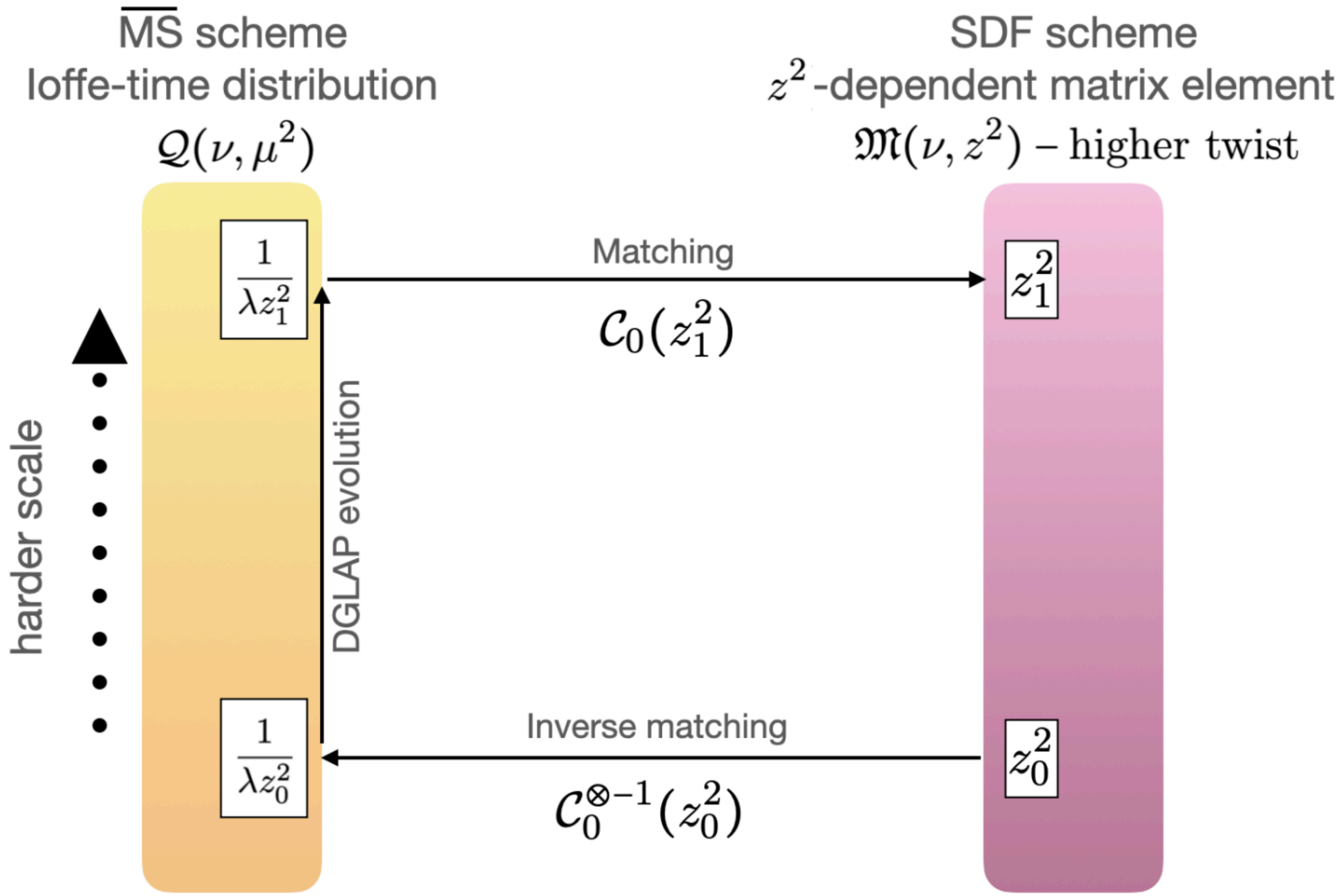
Flow of evolution



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Evolution of parton distributions

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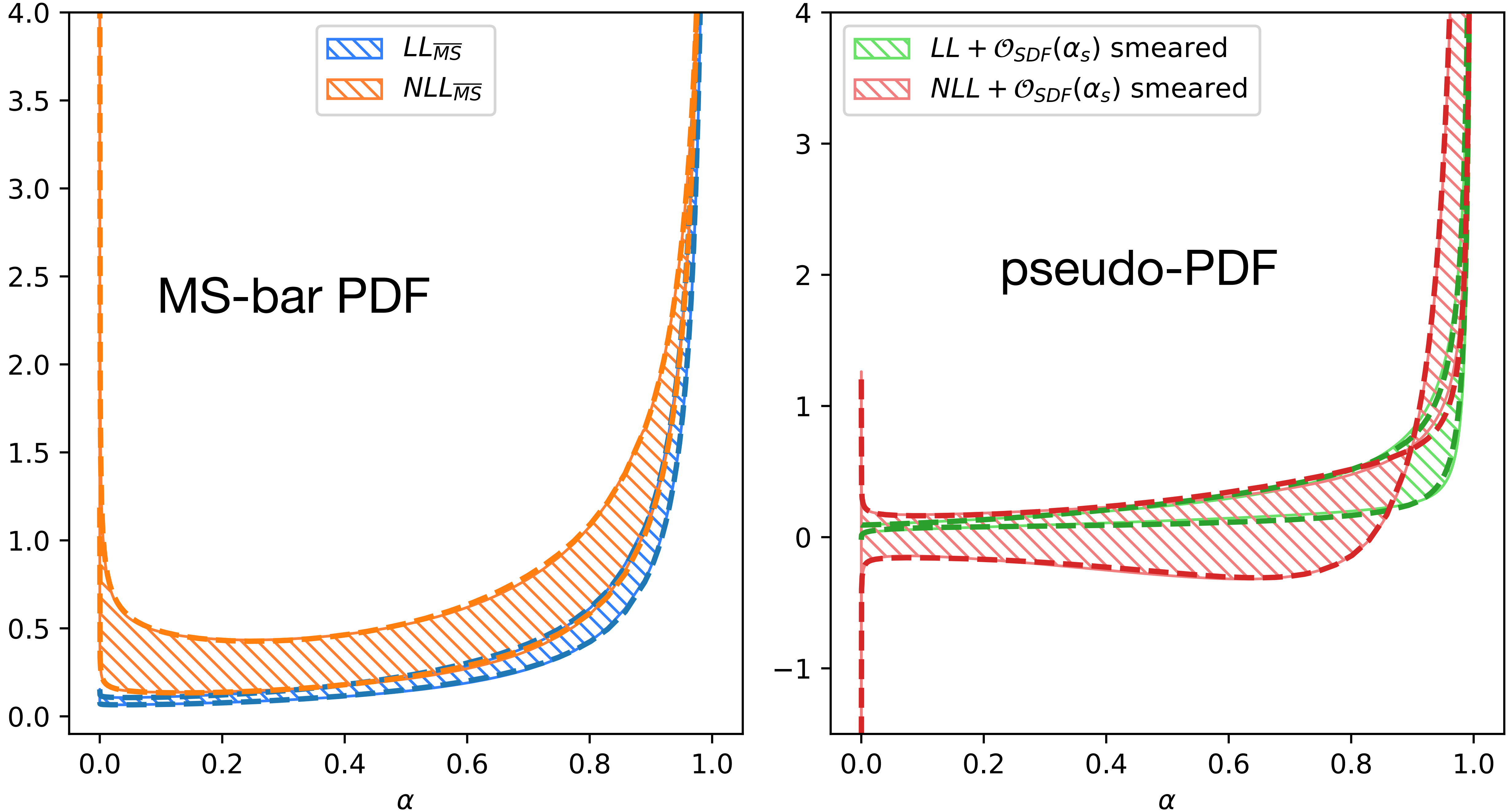
- Perturbative evolution from ~700 MeV (0.282 fm) to ~1GeV (0.188 fm)
- Bands from varying scale by factor of 2 to estimate higher order effects



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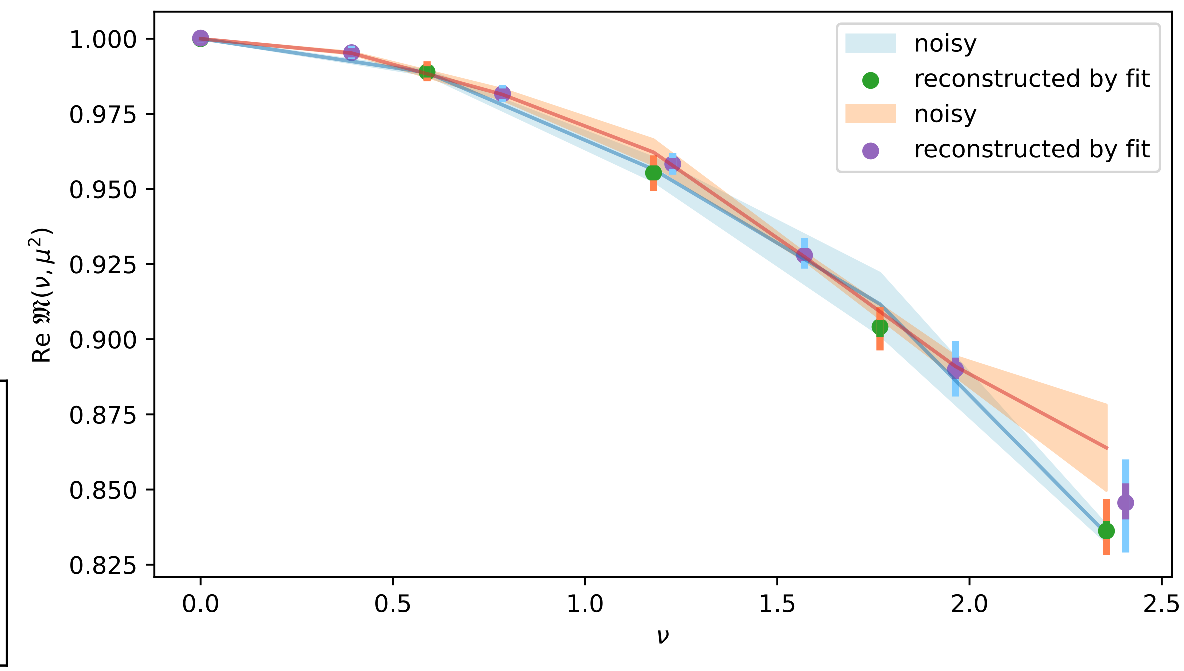
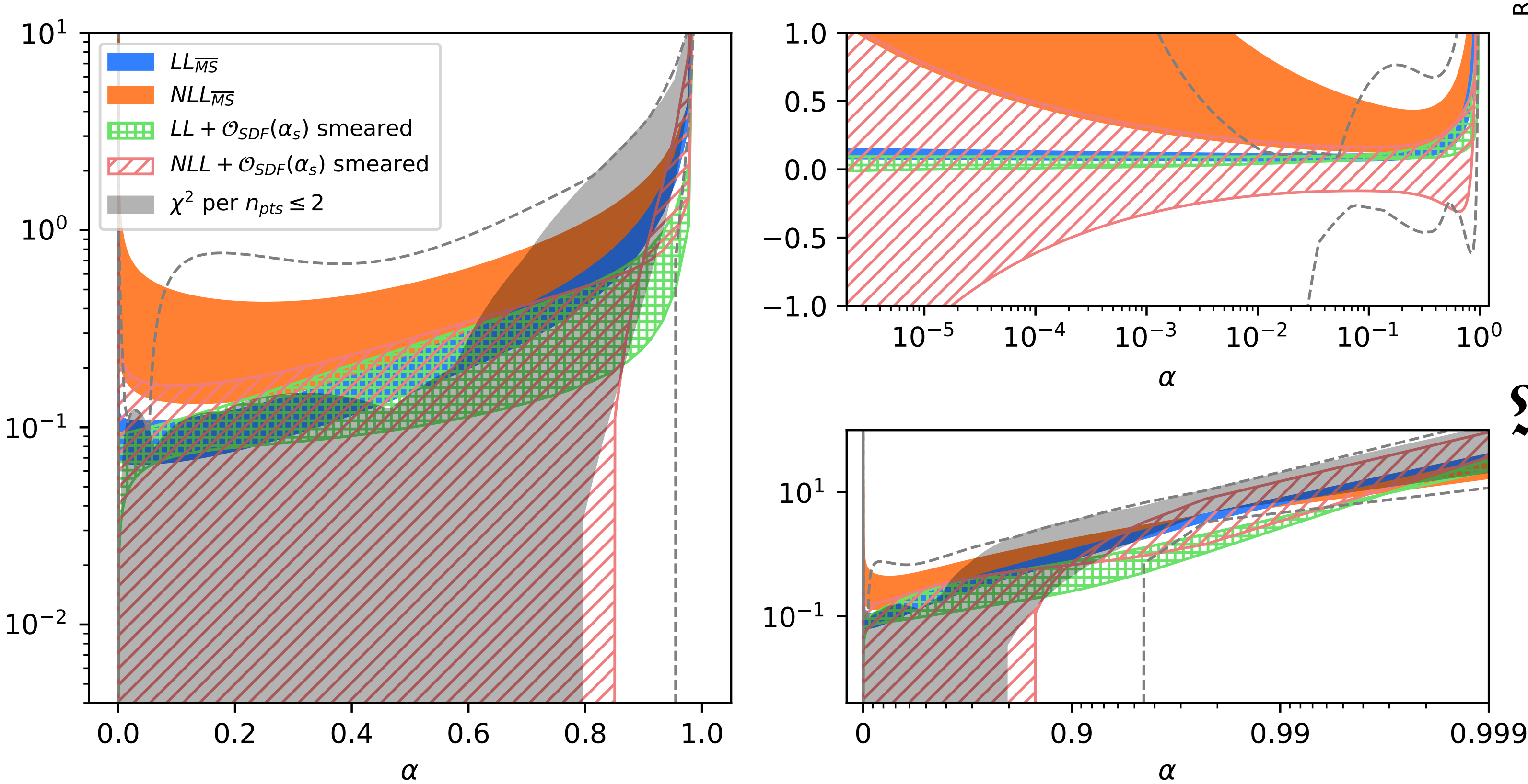
Step Scaling from the lattice

- Requires data in same range of ν and different z

- Model Function



$$\Sigma(\alpha) = A\alpha^{-\delta}(1 + r\alpha) + B(-\ln(\alpha))^{-\eta}\ln^2(1-\alpha) + \sigma\alpha(1-\alpha)$$



$$\mathfrak{M}(\nu, z^2) = \int_0^1 d\alpha\, \Sigma(\alpha, z^2, z_0^2)\mathfrak{M}(\alpha\nu, z_0^2)$$

- Catch: Requires assumption of leading twist dominance and ranges of ν are limited
  - Need very fine lattices to study systematics
  - Test universality by studying pion, kaon, nucleon, quark (in fixed gauge)

Non-Parametric Bayesian inferences

- Take advantage of single dimension and limited range
- Approximate unknown by value on grid and interpolate for integrals
- Maximize the posterior distribution

$$P\left[q\,|\,\mathfrak{M},I\right] \propto P\left[\mathfrak{M}\,|\,q,I\right]P\left[q\,|\,I\right]$$

- Add prior information to regulate the inverse problem

$$P\left[q\,|\,I\right] \propto \exp[-S(q)]$$

Y. Burnier and A. Rothkopf (2013) 1307.6106

Shannon-Jaynes entropy

$$S(q) = \alpha\int_0^1 dx\,\left(q(x) - m(x) - q(x)\log(\frac{q(x)}{m(x)})\right)$$

Burnier-Rothkopf

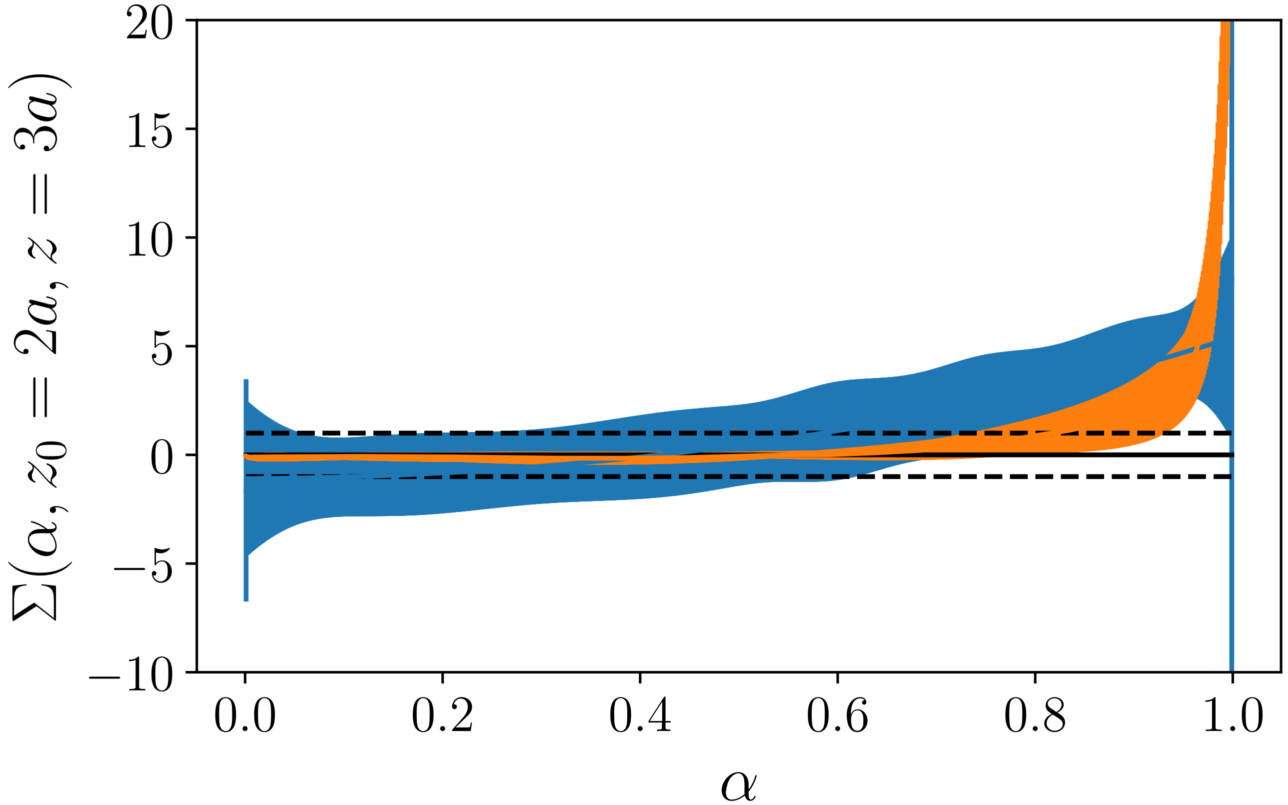
$$S(q) = \alpha\int_0^1 dx\,\left(1 - \frac{q(x)}{m(x)} + \log(\frac{q(x)}{m(x)})\right)$$

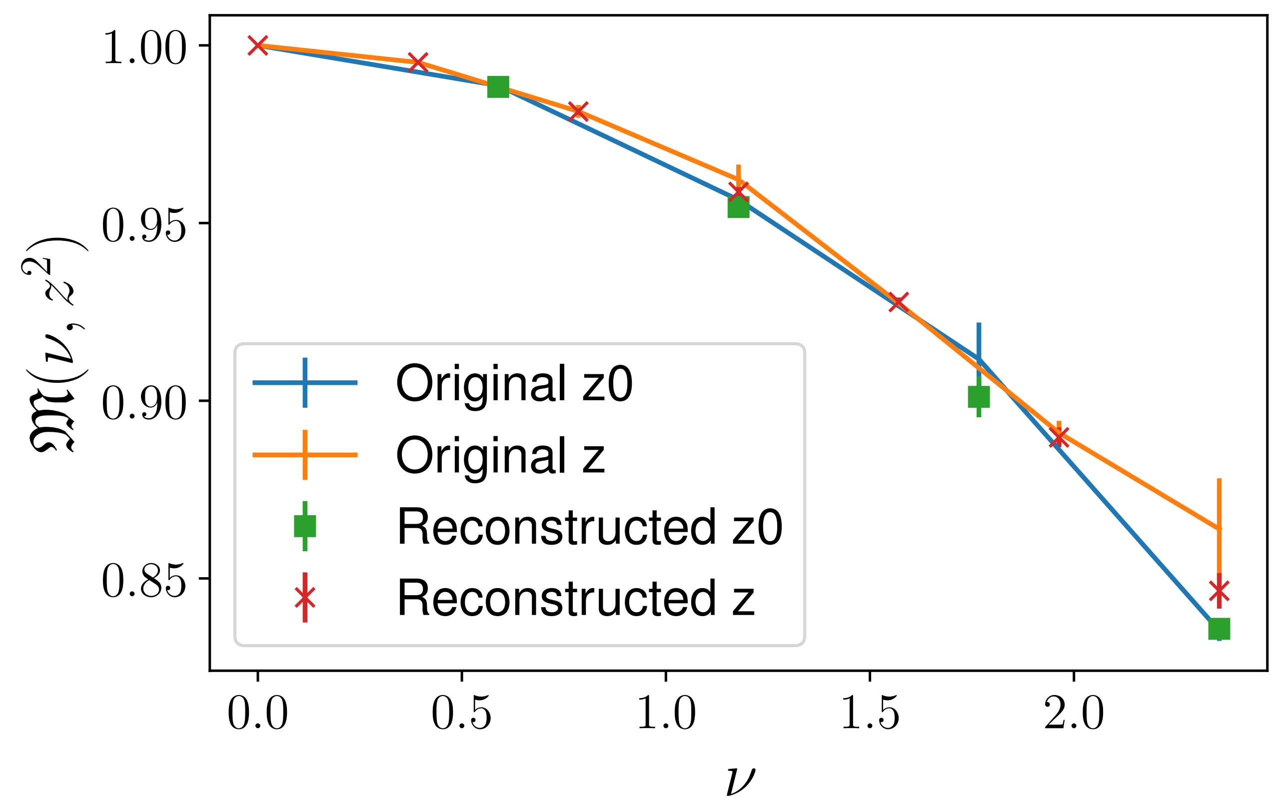
Non-Parametric Bayesian inferences

- Use different priors to study model dependencies

- First prior with easily understood biases

  - Quadratic Difference Ratio (QDR) $S(\Sigma) = u \int_0^1 d\alpha \, \frac{\left(\Sigma(\alpha) - h(\alpha)\right)^2}{\sigma(\alpha)^2}$





$$u = 1 \quad h(\alpha) = 0 \quad \sigma(\alpha) = 1$$

  - Large errors from prior with no correlations at different α
    Need for better choices

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Non-Parametric Bayesian inferences

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- First prior with easily understood biases
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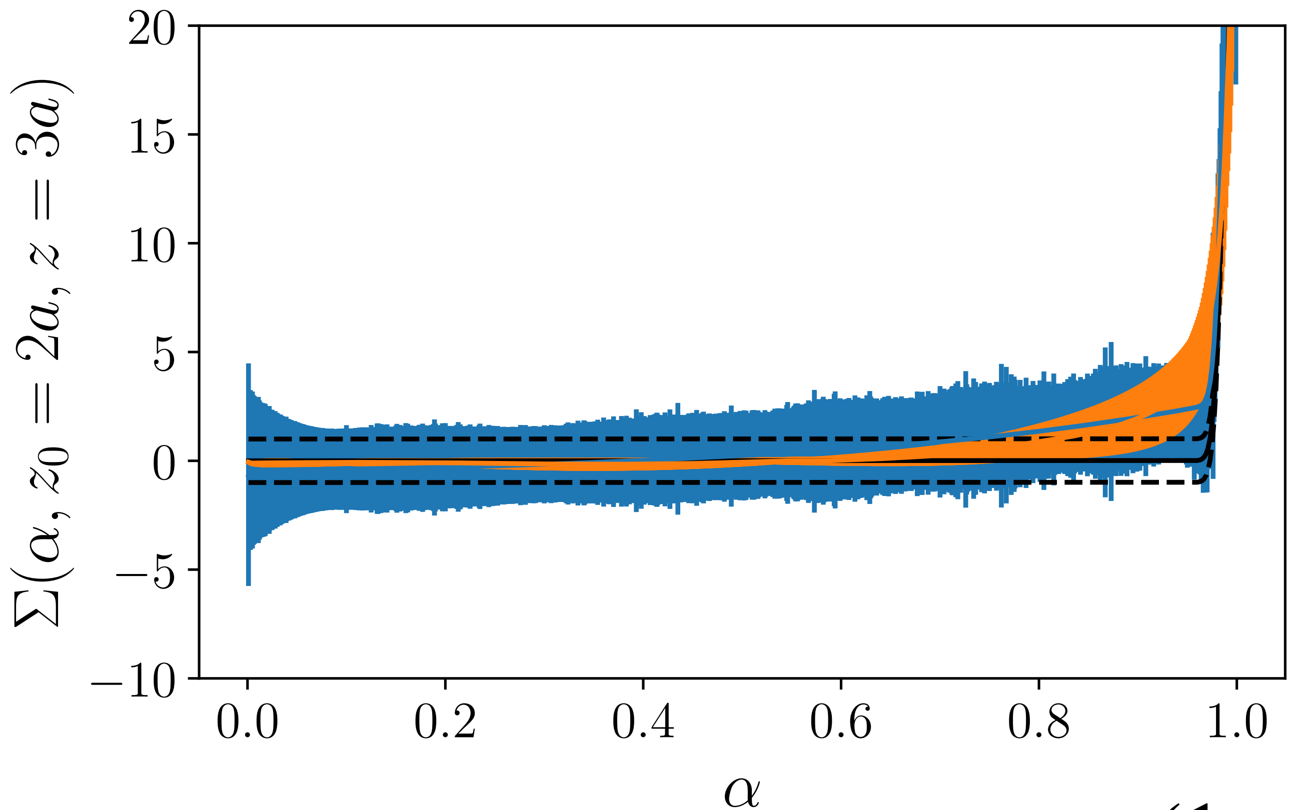

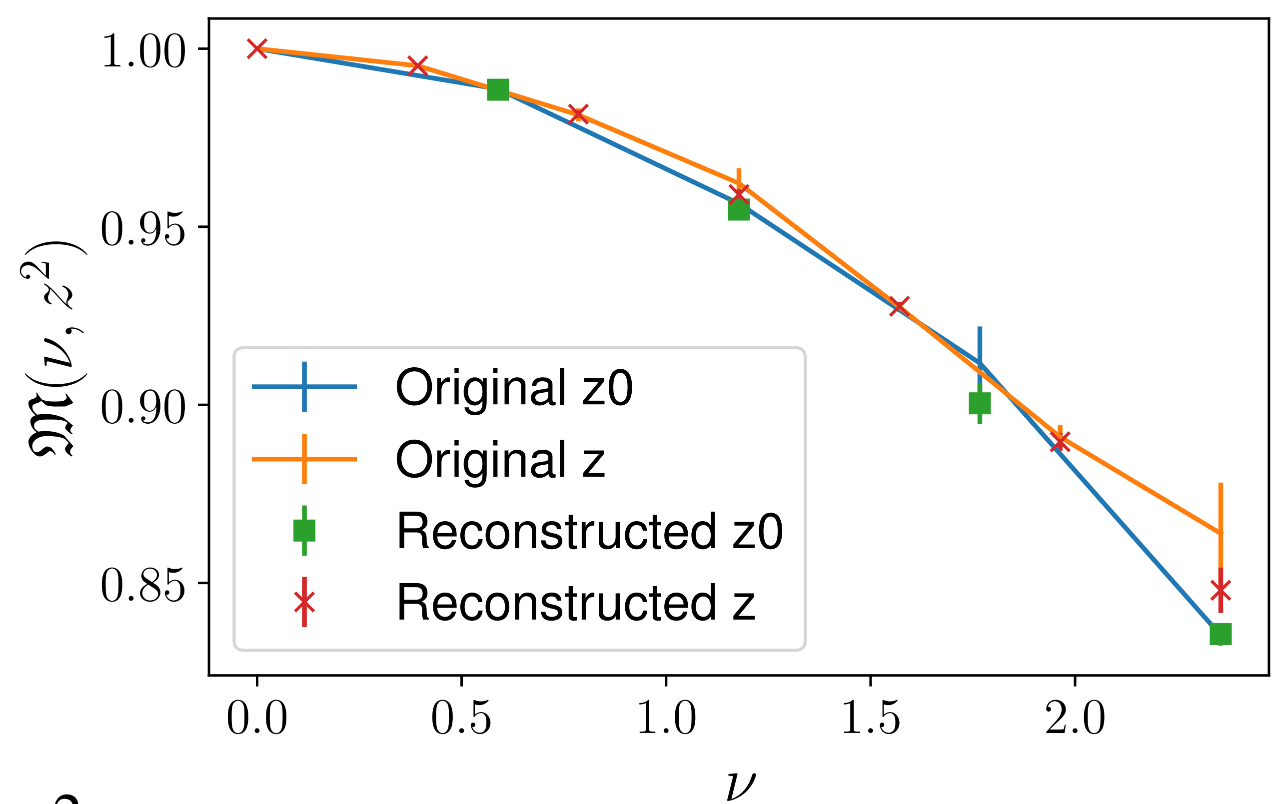

$$u = 1 \quad h(\alpha) = \exp(-\frac{(1-\alpha)^2}{w^2})/(w\sqrt{2\pi}) \quad \sigma(\alpha) = 1$$

$$w = 0.01$$

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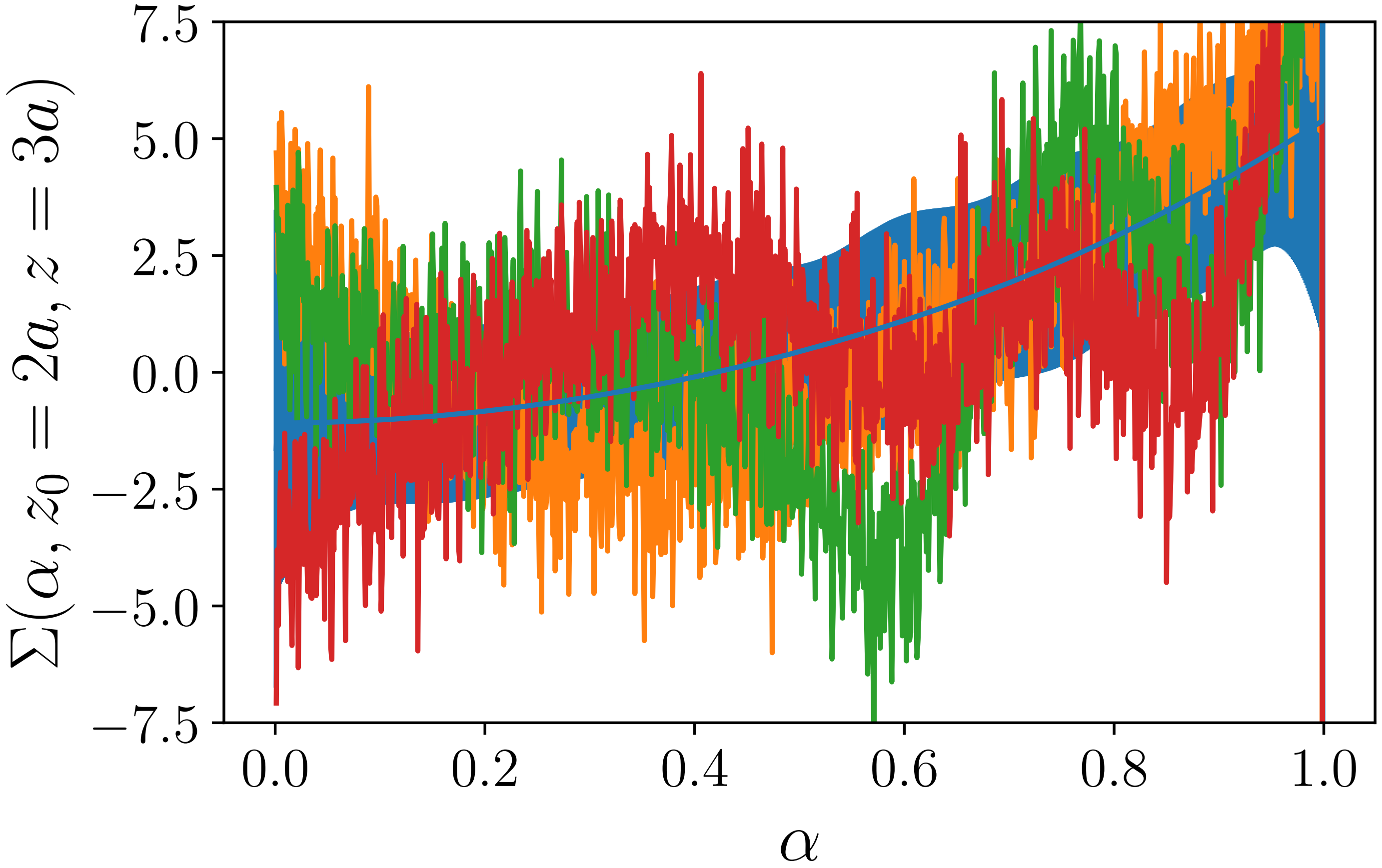
“I’m sorry, Nature hates Wiggles”

-A. Radyushkin

- Characteristic curves from fit
- QDR has no correlations between neighbors
- Need better priors!



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“I’m sorry, Nature hates Wiggles”

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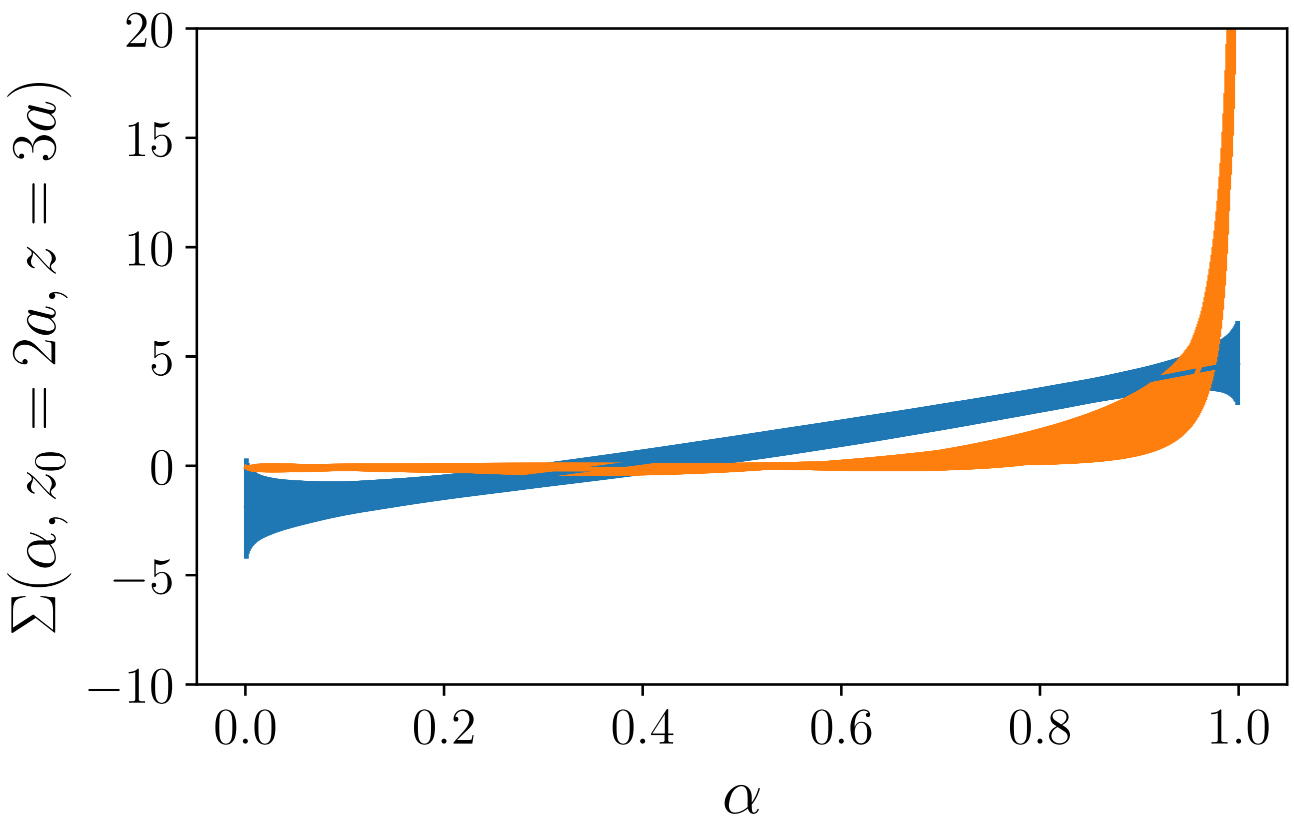
- Use different priors to study model dependencies
- Can we remove the wiggles?
  - A smoothing prior

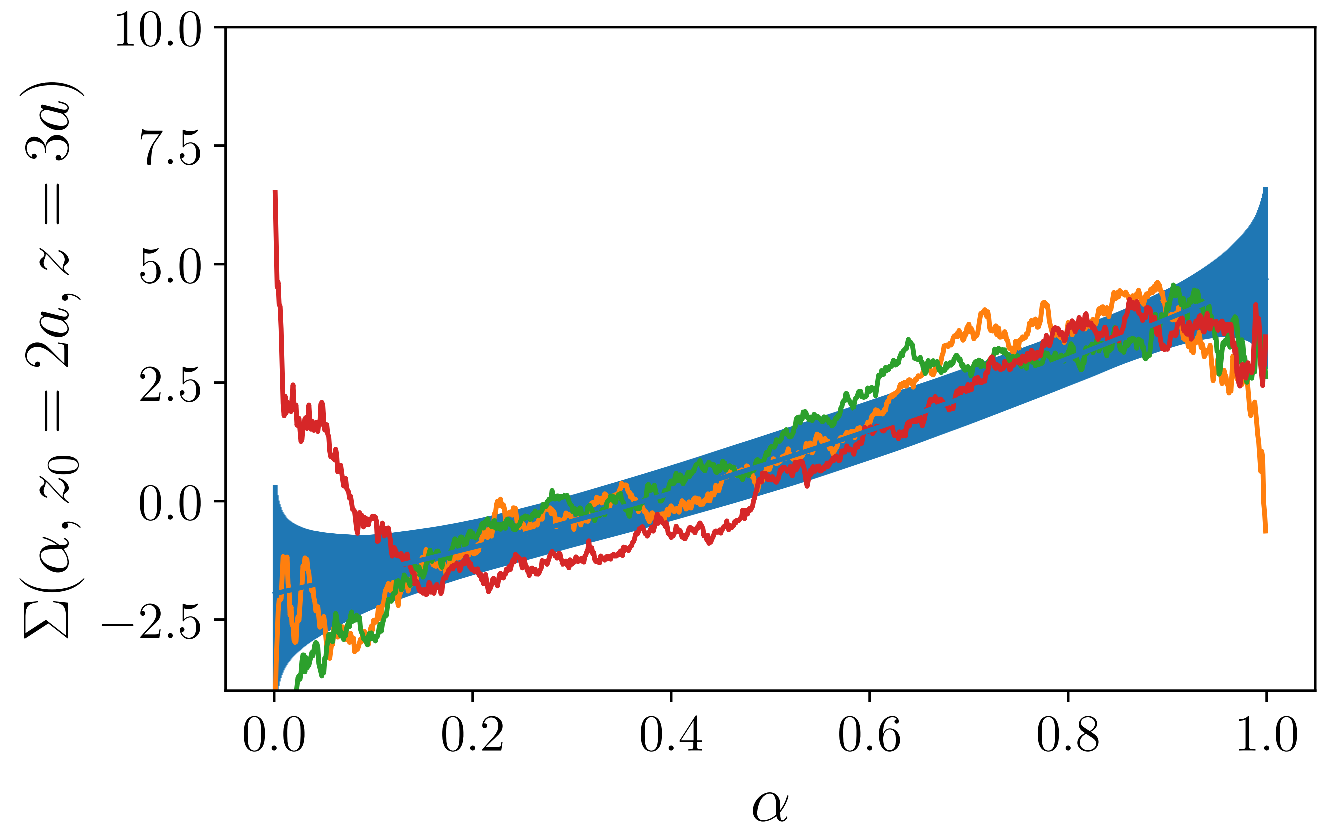
$$S(\Sigma) = u \int_0^1 d\alpha\, \alpha(1-\alpha)\left(\frac{\partial \Sigma}{\partial \alpha}\right)^2$$

- Set u too large and it forces a flat result.

$$u = 1$$

- Alternative to correlate α’s is to use Gaussian Processes





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Conclusions

- Lattice matrix elements can be related to PDFs and their calculation have matured over the decade

- Scale dependence fundamental

- Non-perturbative PDF evolution can be determined from lattice data

- All lessons can be extended to TMDs and GPDs