Collins-Soper kernel with lattice QCD

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in collaboration with
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PRD 108 (2023) 11, 114505 [2307.12359]
PRL 132 (2024) 23, 231901 [2402.06725]
+ ongoing work

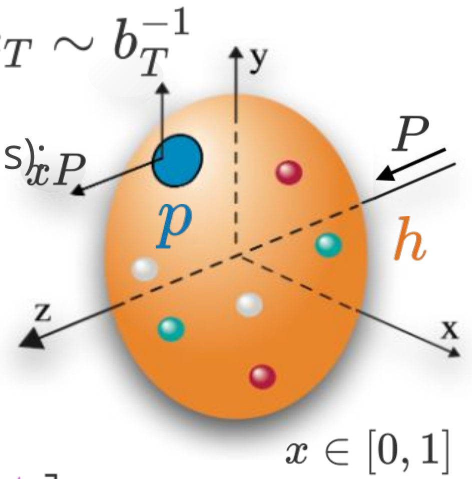
1 Massachusetts Institute of Technology
2 Fermilab
3 Argonne National Laboratory

Meeting on Lattice Parton Physics from Large Momentum Effective Theory (LaMET)

University of Maryland, College Park

August 11–14, 2024

The Collins-Soper (CS) kernel



Based on Fig. 1.1 in
TMD Handbook, 2304.03302

- Related to TMDs (transverse-momentum-dependent distributions): a generalization of e.g. PDFs:

PDFs $f_{q/h}(x, \mu)$ $\longrightarrow$ TMD PDFs $f_{q/h}(x, b_T, \mu, \zeta)$

- Describes RG evolution of TMDs along ζ:

$$f_{p/h}(x, b_T, \mu, \zeta) = f_{p/h}(x, b_T, \mu, \zeta_0) \exp\left[\frac{1}{2}\gamma_p(b_T, \mu) \ln \frac{\zeta}{\zeta_0}\right]$$

- $\Rightarrow$ Computed as a ratio of TMDs at different ζ:

- Independent of hadronic state
- Different for $p \in \{q, g\}$
- Non-perturbative at large b_T (for any μ)

$$\gamma_q(b_T, \mu) = \frac{2}{\ln(\zeta_1/\zeta_2)} \ln \frac{f_{q/h}(x, b_T, \mu, \zeta_1)}{f_{q/h}(x, b_T, \mu, \zeta_2)}$$

Proportional to hadron momentum P

- Encoded by light-like matrix elements
- Matched onto space-like matrix elements with LaMET **⇒ computable in LQCD**

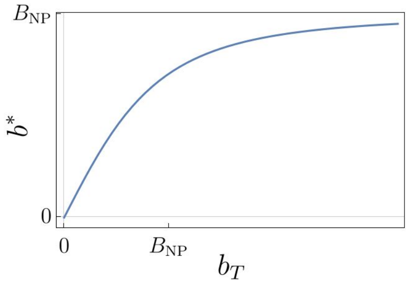
CS kernel: pheno. models and input from LQCD

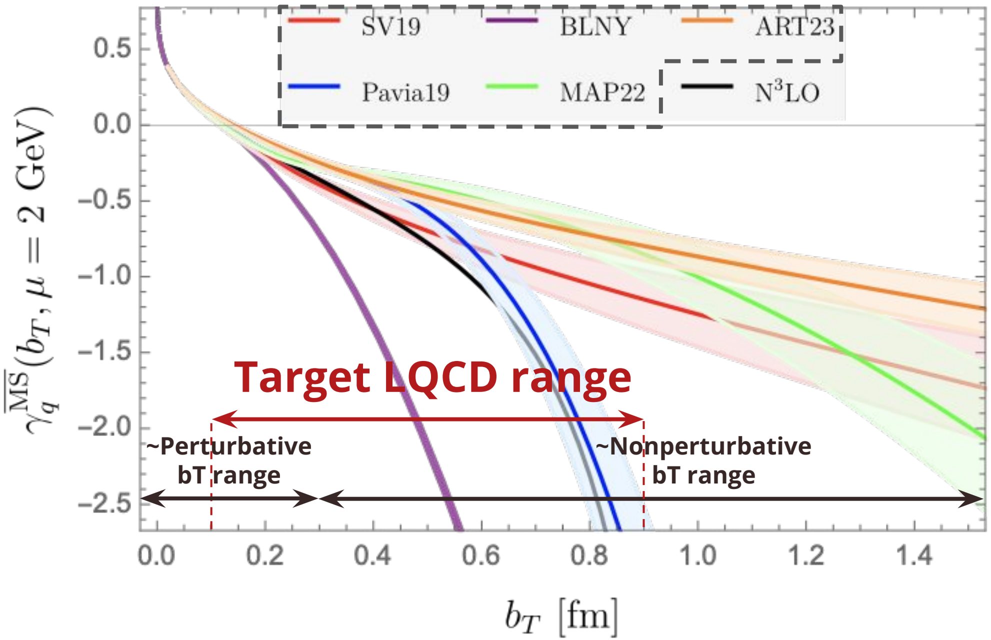
Pheno. models fit to experimental data (SIDIS, DY)

$$-\frac{1}{2}\gamma_q^{\text{param.}}(b_T,\mu;B_{\text{NP}},\{c_i\})$$ **free parameters**

$$=\boxed{\mathcal{D}_{\text{pert.}}(b^*,\mu)}+\boxed{\mathcal{D}_{\text{NP}}(b_T,b^*;B_{\text{NP}},\{c_i\})}$$

Perturbative term
controlled by $b^*(b_T;B_{\text{NP}})$:





Non-perturbative term
modeled b_T dependence, e.g.

$$\mathcal{D}_{\text{NP}}(b_T,b^*;B_{\text{NP}},\{c_i\})$$
$$=b_Tb^*\left[c_0+c_1\ln\frac{b^*}{B_{\text{NP}}}\right]$$

or $\mathcal{D}_{\text{NP}}(b_T;g_2)=-g_2\frac{b_T^2}{2}$

LQCD goal: sufficient precision for direct comparison

BLNY: PRD 67 (2003), [hep-ph/0212159]
SV19: JHEP 06, 137 [1912.06532]
Pavia19: JHEP 07, 117, [1912.07550]
MAP22: JHEP 10, 127, [2206.07598]
ART23: [2305.07473]

Other recent results (not plotted):
Isaacson, Fu, Yuan [2311.09916]
Bury et. al JHEP 10, 118 (2022) [2201.07114]
MAP24 [2405.13833]

LQCD results directly comparable with pheno. models

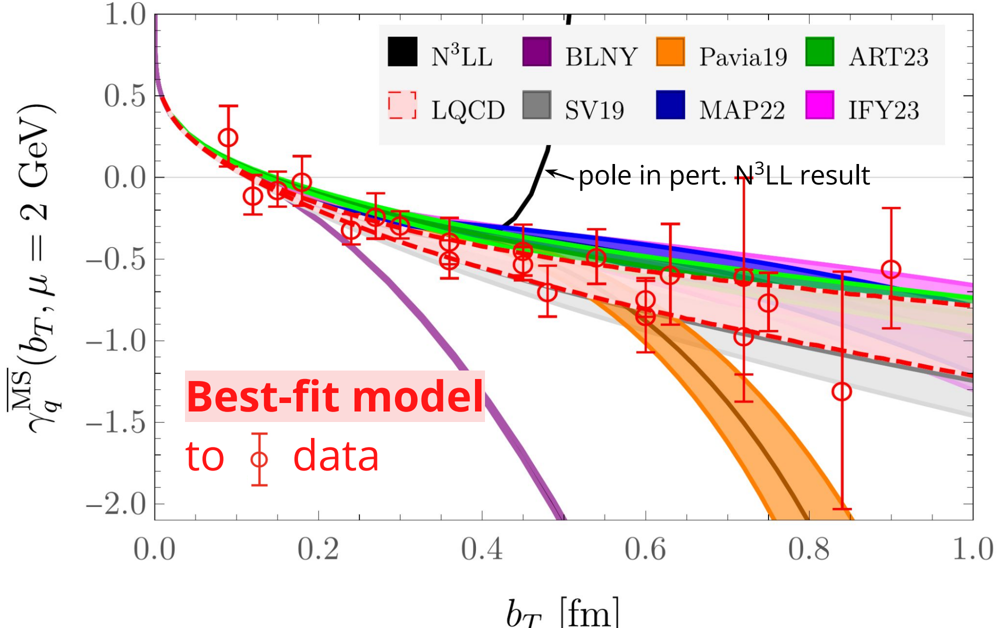
Fit CS parameterization to **LQCD data** from 3 lattice spacings

pheno. parameters

$$-\frac{1}{2}\left(\gamma_q^{\text{param.}}(b_T,\mu;B_{\text{NP}},\{c_i\})-k_1\frac{a}{b_T}-k_2\frac{a^2}{b_T^2}\right)$$
$$=\mathcal{D}_{\text{pert.}}(b^*;\mu)+\mathcal{D}_{\text{NP}}(b_T,b^*;B_{\text{NP}},\{c_i\})$$

discretization effects

Consistent with:
SV19: JHEP 06, 137, [1912.06532]
MAP22: JHEP 10, 127, [2206.07598]
ART23: [2305.07473]
IFY23: Isaacson, Fu, Yuan [2311.09916]



Disfavors at large bT:
BLNY: PRD 67 (2003), [hep-ph/0212159]
Pavia19: JHEP 07, 117, [1912.07550]

CS kernel from LQCD: outline

X. Ji et. al., Phys. Lett. B 811 [1911.03840]

5. Continuum extrapolation

2. Fourier transform

1. Quasi-TMDs in position space

$$\gamma_q(b_T,\mu) = \boxed{\lim_{a\to 0}} \frac{1}{\ln(P_1^z/P_2^z)} \ln\left[\frac{\int db^z e^{ib^z x P_1^z} P_1^z N_\Gamma(P_1^z) \sum_{\Gamma'} Z_{\Gamma\Gamma'}(\mu) \lim_{\ell\to\infty} W_{\Gamma'}^{(0)}(b^z, b_T, \ell, P_1^z)}{\int db^z e^{ib^z x P_2^z} P_2^z N_\Gamma(P_2^z) \sum_{\Gamma'} Z_{\Gamma\Gamma'}(\mu) \lim_{\ell\to\infty} W_{\Gamma'}^{(0)}(b^z, b_T, \ell, P_2^z)}\right] + \delta\gamma_q(\mu, P_1^z, P_2^z) \boxed{+\,\mathrm{p.\,c.}}$$

3. EFT matching

4. Power corrections

X. Ji et. al, PRD91 (2015);
Ebert et. al, PRD99 (2019), JHEP09 (2019) 037;

3 lattice spacings,
Mπ ≈ 150 – 180 MeV,
P^z = 0.77 – 2.32 GeV

2+1+1 fermion flavors
Wilson-flowed
Clover-on-HISQ fermions,

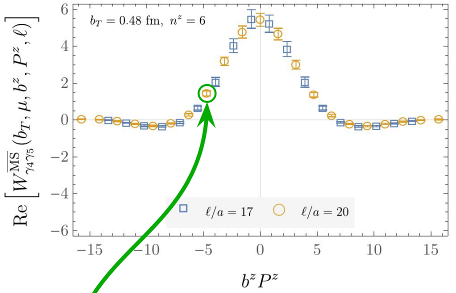
(1) 48^3 x 64, **a=0.12 fm**,

new: (2) 32^3 x 48, **a=0.15 fm**

(3) 64^3 x 96, **a=0.09 fm**

CS kernel from LQCD: outline

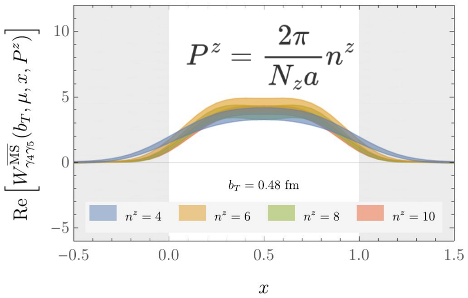
Calculate position-space MEs



$\sum_{\Gamma'} Z_{\Gamma\Gamma'}(\mu) \lim_{\ell\to\infty} W^{\Gamma'}_{\mathcal{O}}(b^z, b_\mathrm{T}, \ell, P^z_1)$

Each point is a separate matrix element calculation

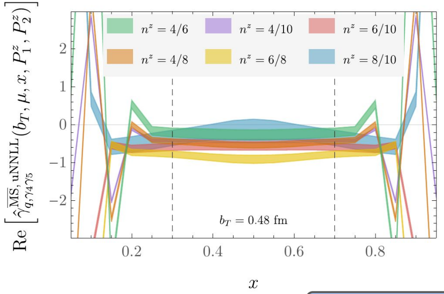
Fourier transform (FT) to momentum-space MEs



$\int \mathrm{d}b^z e^{ib^z x P^z_1} P^z_1 N_\Gamma(P^z_1)$

$\sum_{\Gamma'} Z_{\Gamma\Gamma'}(\mu) \lim_{\ell\to\infty} W^{\Gamma'}_{\mathcal{O}}(b^z, b_\mathrm{T}, \ell, P^z_1)$

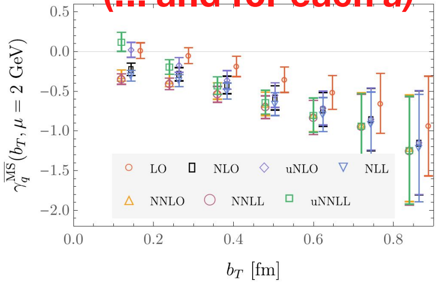
Form ratios of MEs + match + fit in x



$$\gamma_q(b_T,\mu) = \boxed{\lim_{a\to 0}} \frac{1}{\ln(P^z_1/P^z_2)} \ln \left[\frac{\int \mathrm{d}b^z e^{ib^z x P^z_1} P^z_1 N_\Gamma(P^z_1) \sum_{\Gamma'} Z_{\Gamma\Gamma'}(\mu) \lim_{\ell\to\infty} W^{\Gamma'}_{\mathcal{O}}(b^z, b_\mathrm{T}, \ell, P^z_1)}{\int \mathrm{d}b^z e^{ib^z x P^z_2} P^z_2 N_\Gamma(P^z_1) \sum_{\Gamma'} Z_{\Gamma\Gamma'}(\mu) \lim_{\ell\to\infty} W^{\Gamma'}_{\mathcal{O}}(b^z, b_\mathrm{T}, \ell, P^z_2)}\right] + \delta\gamma_q(\mu, P^z_1, P^z_2) \boxed{+\,\mathrm{p.\,c.}}$$

...repeat for each bT

(... and for each *a*)

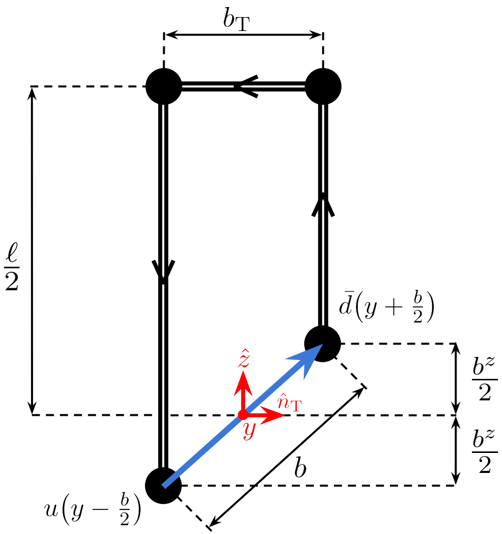


Position-space quasi-TMDs

- Compute **quasi-TMD wavefunctions (WFs)**

$$\phi_\Gamma(b_T, b^z, P^z, \ell) = \langle 0 | \mathcal{O}_\Gamma(b_T, b^z, 0, \ell) | \pi(P^z) \rangle$$

- Operators $\mathcal{O}_\Gamma(b_T, b^z, y, \ell)$ with staple-shaped Wilson lines

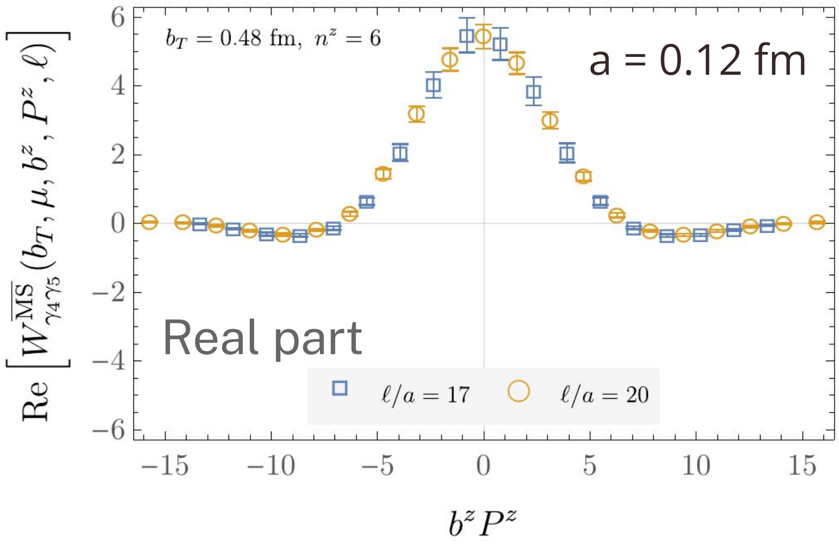


- Separately for each **b^z, P^z, bT, ℓ**, Dirac (**Γ**) structure

- Matrix elements have divergences $\sim \ell + b_T$

- Subtract divergences in **quasi-TMD WF ratios**

$$W_\Gamma^{(0)}(b_T, b^z, P^z, \ell) = \frac{\tilde{\phi}_\Gamma(b_T, b^z, P^z, \ell)}{\tilde{\phi}_{\gamma^4\gamma^5}(b_T, 0, 0, \ell)}$$



Position-space quasi-TMD WFs

- Mixing effects included via **RIxMOM** scheme

$$W_{\Gamma}^{\overline{\rm MS}}(b_T,\mu,b^z,P^z,\ell) :$$

$$= \sum_{\Gamma'} Z_{\Gamma\Gamma'}^{\overline{\rm MS}}(\mu)\, W_{\Gamma}^{(0)}(b_T,b^z,P^z,\ell)$$

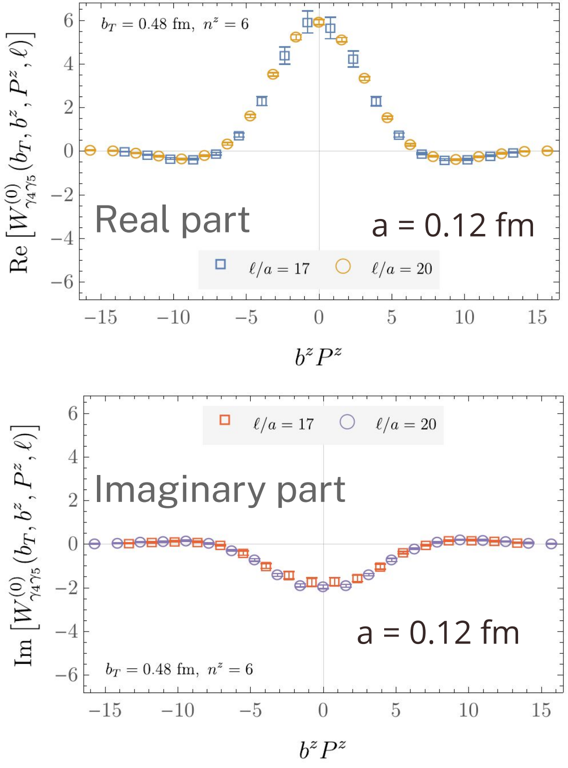
$$\Gamma \in \{\gamma_4\gamma_5, \gamma_3\gamma_5\}$$

- Shown for bT = 0.48 fm, Pz = 1.29 GeV

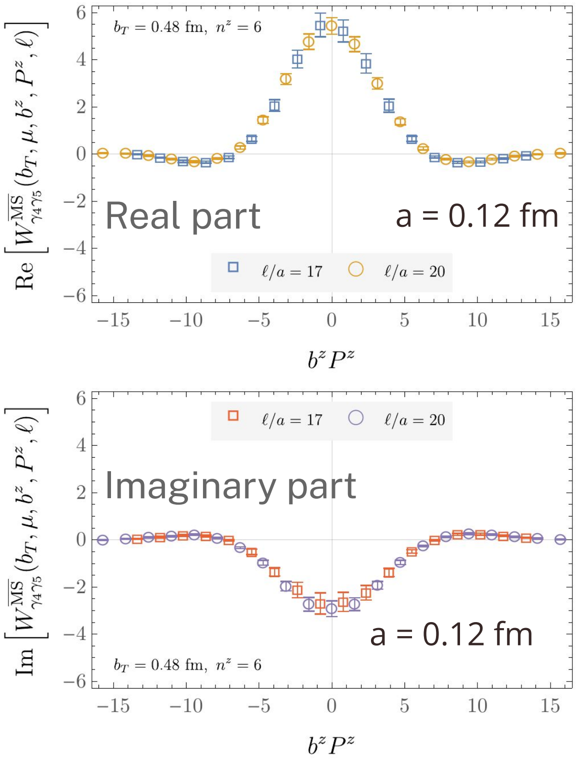
- Consistent between different staple lengths ℓ.

- Decay to zero within computed bz ranges.

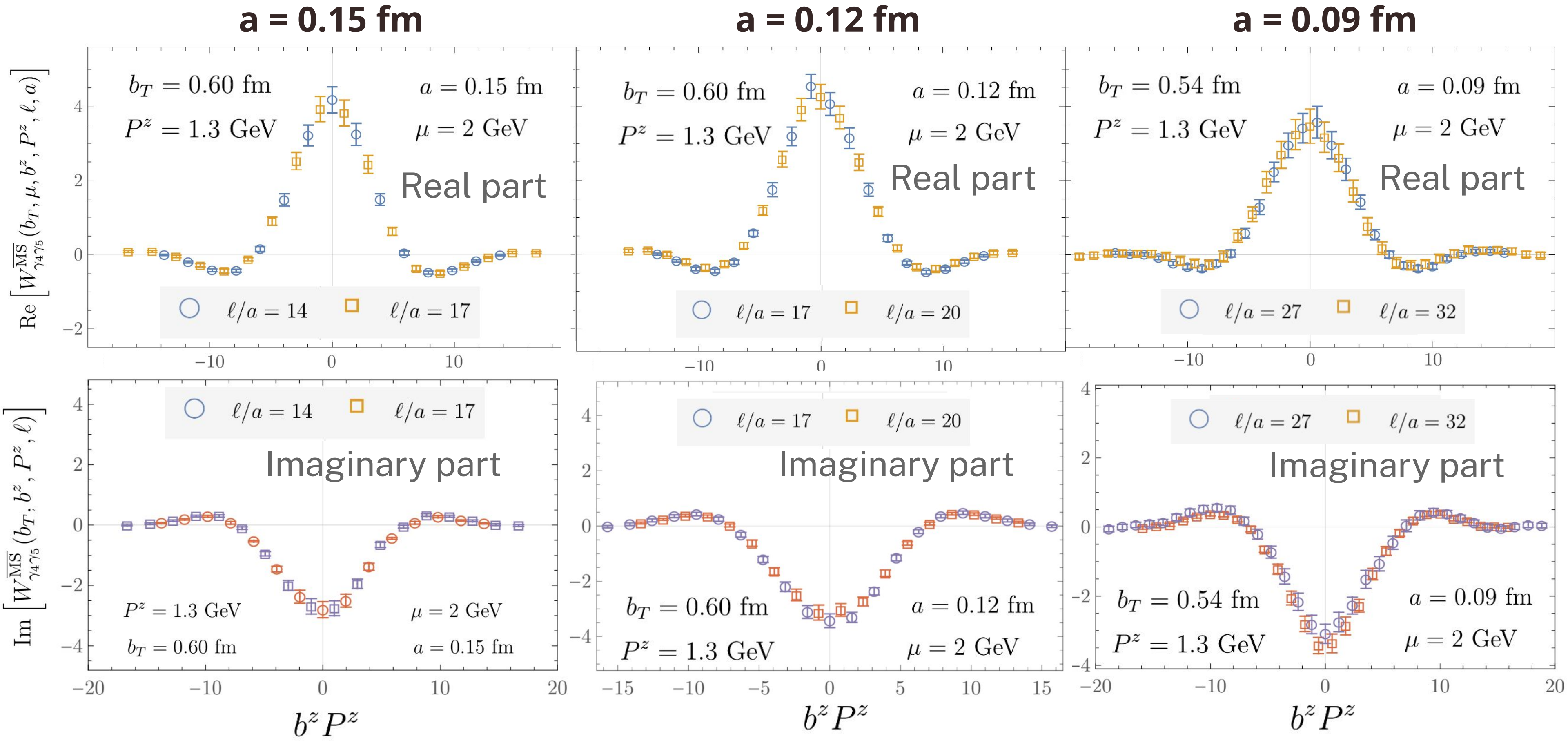
without mixing



with mixing (via **RIxMOM**)

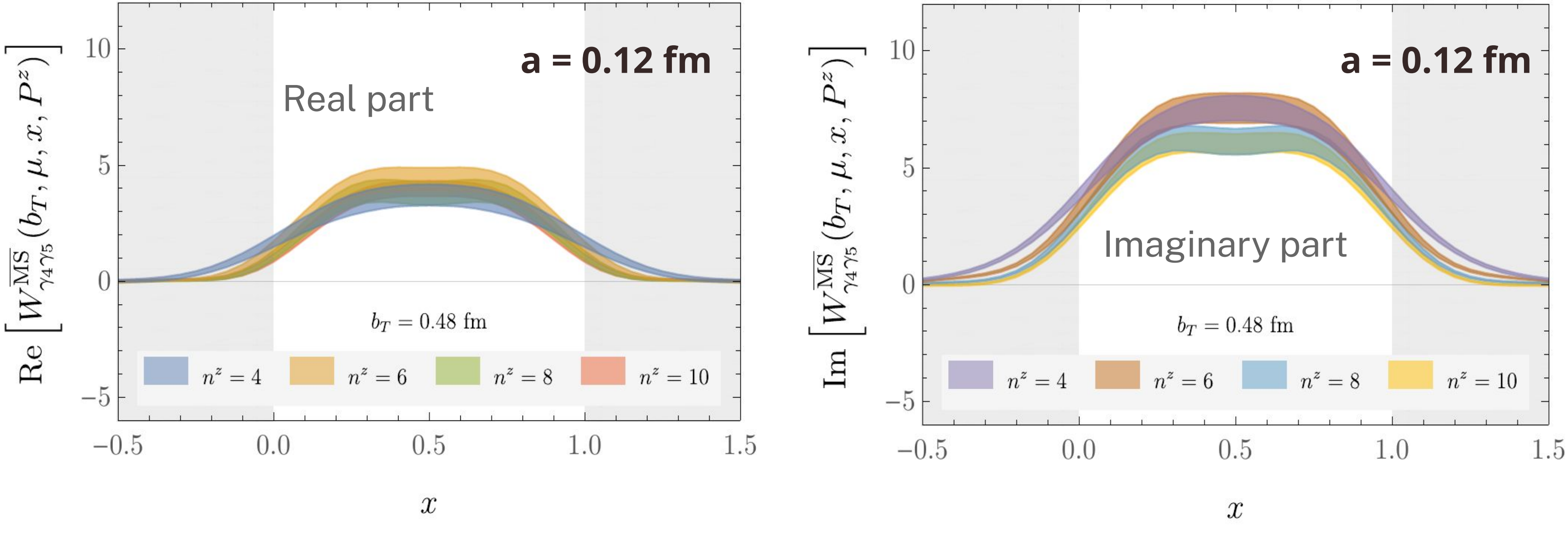


Position-space quasi-TMD WFs at 3 lattice spacings



Momentum-space quasi-TMDs

- Have support outside $x \in [0, 1]$, as expected.
- Converge to physical range $x \in [0, 1]$ with increasing $P^z = \frac{2\pi}{L} n^z$.



CS kernel estimate

$$\hat{\gamma}_{\Gamma}^{\overline{\mathrm{MS}}}(b_T, x, P_1^z, P_2^z, \mu)$$
$$= \frac{1}{\ln(P_1^z/P_2^z)} \ln\left[\frac{W_{\Gamma}^{\overline{\mathrm{MS}}}(b_T, x, P_1^z, \ell)}{W_{\Gamma}^{\overline{\mathrm{MS}}}(b_T, x, P_2^z, \ell)}\right]$$
$$+\delta\gamma_q^{\overline{\mathrm{MS}}}(x, P_1^z, P_2^z, \mu)$$

- Separate for each momentum pair, bT, Dirac / Γ, and matching.

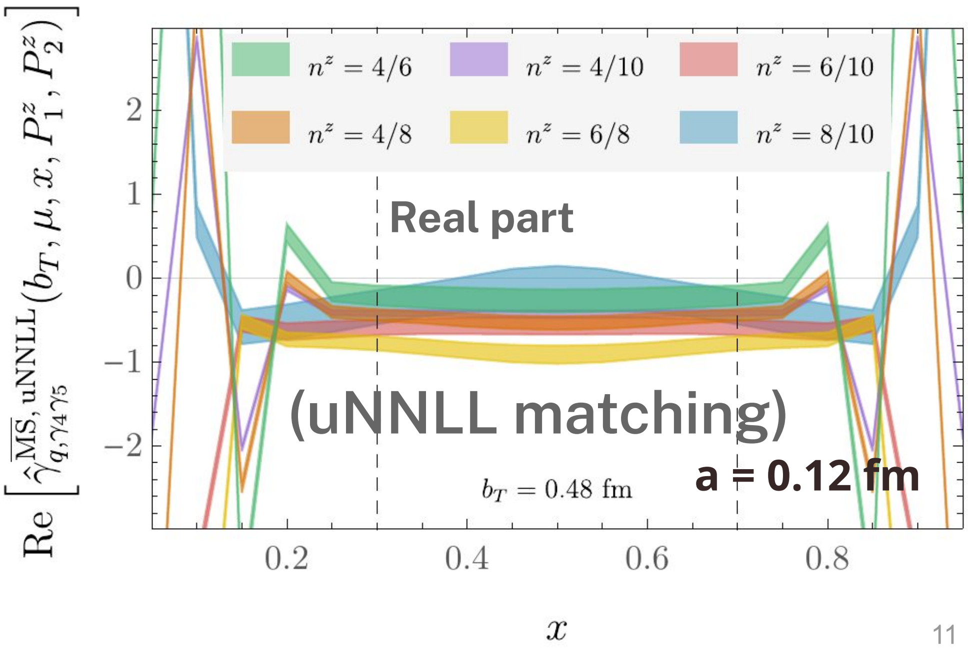
X. Ji et. al., Phys. Lett. B 811 [1911.03840]
X. Ji and Y. Liu, PRD 105, [2106.05310]
Z.-F. Deng et. al, JHEP 09, [2207.07280]

- Differ by power corrections:

$$\boxed{\mathcal{O}\left(\frac{1}{(xP^z b_T)^2}, \frac{m_\pi^2}{(xP^z)^2}\right) + (x \leftrightarrow 1-x)}$$

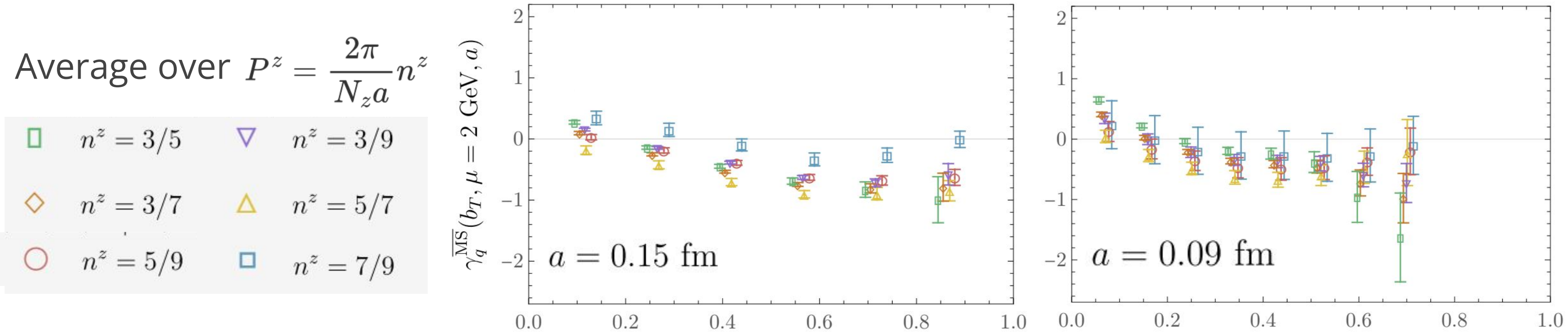
- Corrections ~P1^z, ~P2^z partially cancel in ratios — **insufficient precision to fit & subtract power corrections**

- ⇒ Fit each estimator separately to a constant in $x \in [0.3, 0.7]$, then average fits at fixed bT and matching accuracy.

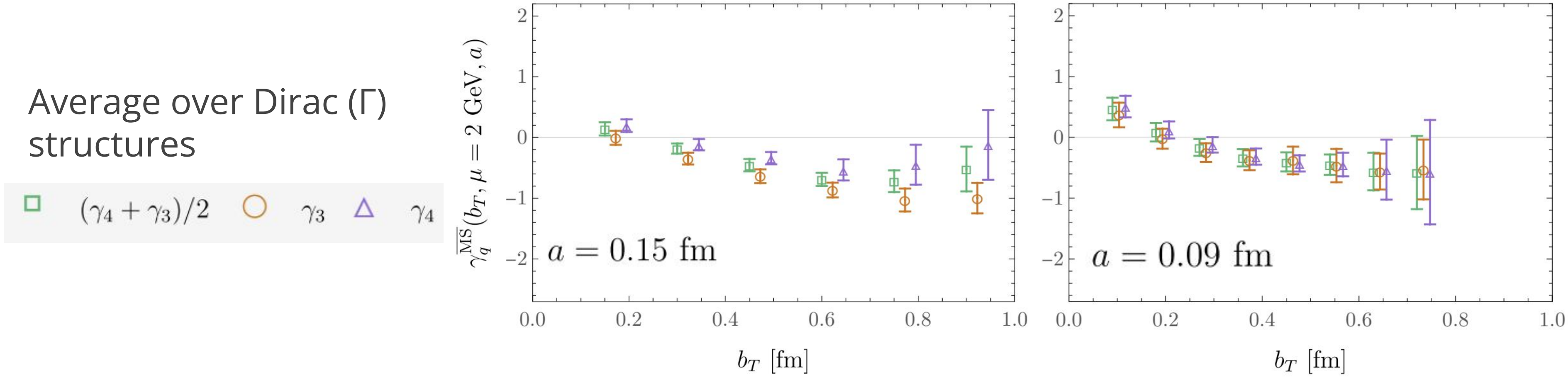


Weighted averages account for power corrections

Average over $P^z = \frac{2\pi}{N_z a} n^z$



Average over Dirac (Γ) structures

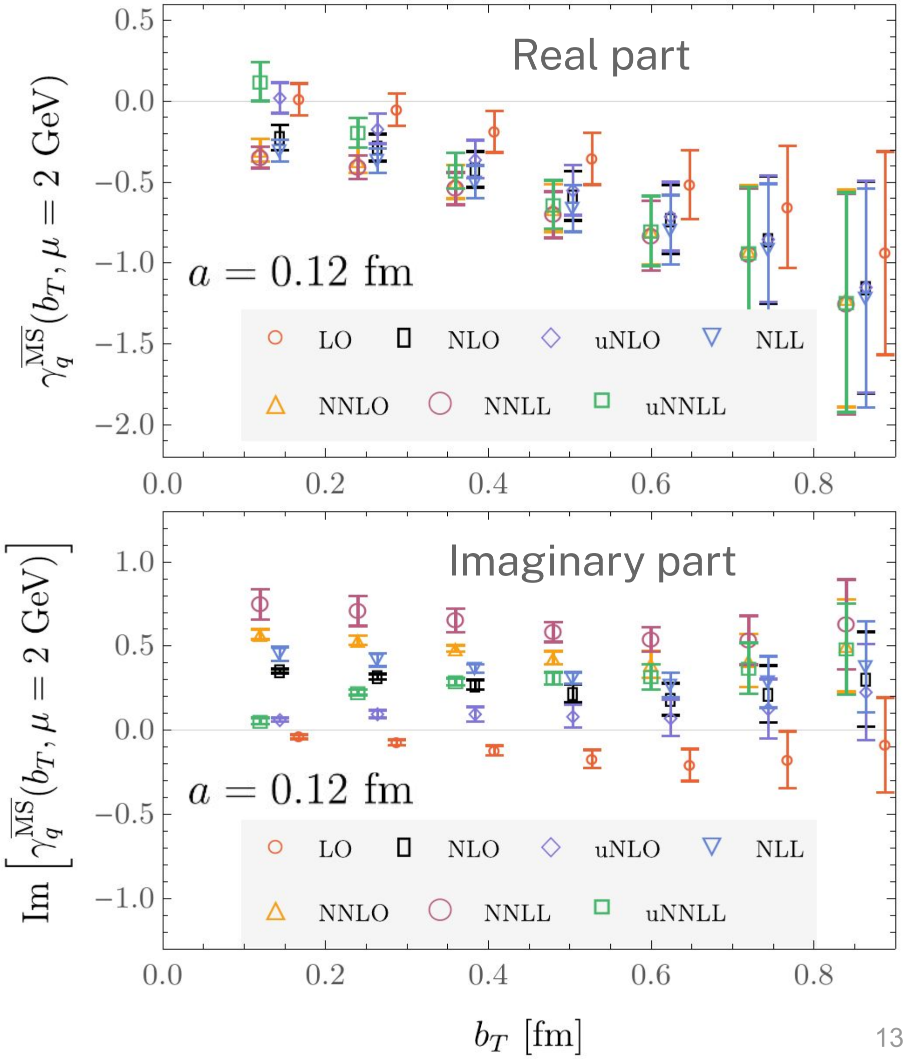


Study of LaMET matching
using **Re+Im** parts of CS kernel estimate

- The CS kernel is real-valued.
- The CS kernel *estimate* has a **nonzero imaginary part** due to
  - poor perturbative convergence of matching coefficients
  - bT power corrections

  ⇒ **not treated as an independent systematic**

  M.-H. Chu et al. (LPC), PRD 106, 034509, [2204.00200]
  M.-H. Chu et al. (LPC), [2302.09961]
  M.-H. Chu et al. (LPC), [2306.06488]
- Re+Im parts used to characterize matching



Why bT-unexpanded matching

1. **Re part**: differs from expanded matching at small bT (beyond **LO**)

$\frac{1}{(xP^z b_T)^2}$ (**Power corrections** break down TMD factorization at small bT)

$$\mathbf{C}_\phi(p^z, b_T, \mu) = C_\phi(p^z, \mu) + \delta C_\phi(p^z, b_T)$$
$$p^z \in xP^z, \bar{x}P^z$$

uNLO

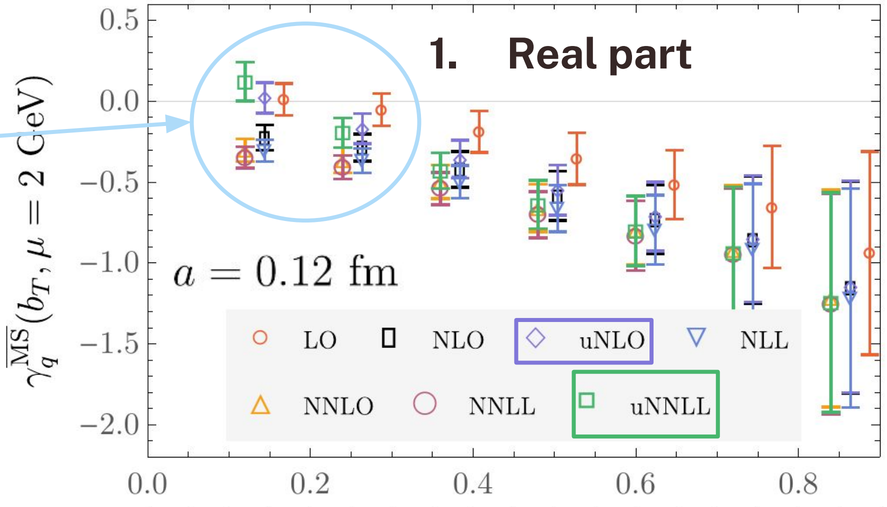
$\delta C_\phi(p^z, b_T)$ contains bT-dependent terms on $x \in (-\infty, \infty)$ suppressed in $P^z b_T$

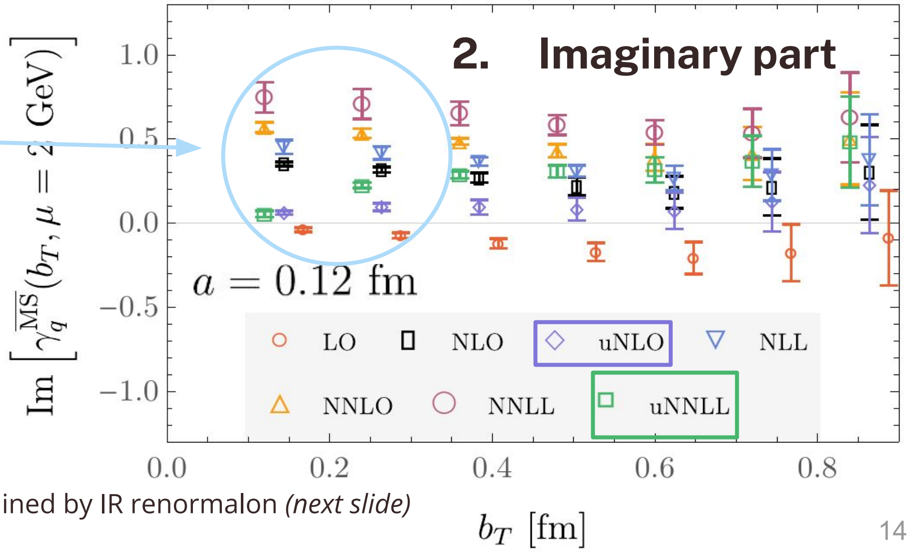
2. **Im part**:

M. A. Ebert et. al,, JHEP 09, 037, [1901.03685]
Z.-F. Deng et. al, JHEP 09, [2207.07280]

- Reduced by bT-unexpanded matching at small bT.
- Still nonzero at large bT:
  - even for **LO (= no matching)**
  - *esp.* for resummed logs (*LL)

However: no impact on the real part
⇒ no impact on CS kernel estimate





explained by IR renormalon *(next slide)*

Leading infrared renormalon

- Asymptotic series in matching coefficient

$$R(p^z,\mu) = i\boxed{N_m}\frac{\mu}{p^z}\sum_{n=0}^{\infty}\boxed{\beta_0^n}\alpha_s^{n+1}(\mu)n!$$

N_m = 0.552 for 4 flavors; β_0^n: lowest-order β-function

$p^z \in xP^z, \bar{x}P^z$

- Leads to slow perturbative convergence in the imaginary part of CS kernel.
  Y. Liu, Y. Su (LPC), JHEP02(204), [2311.06907]

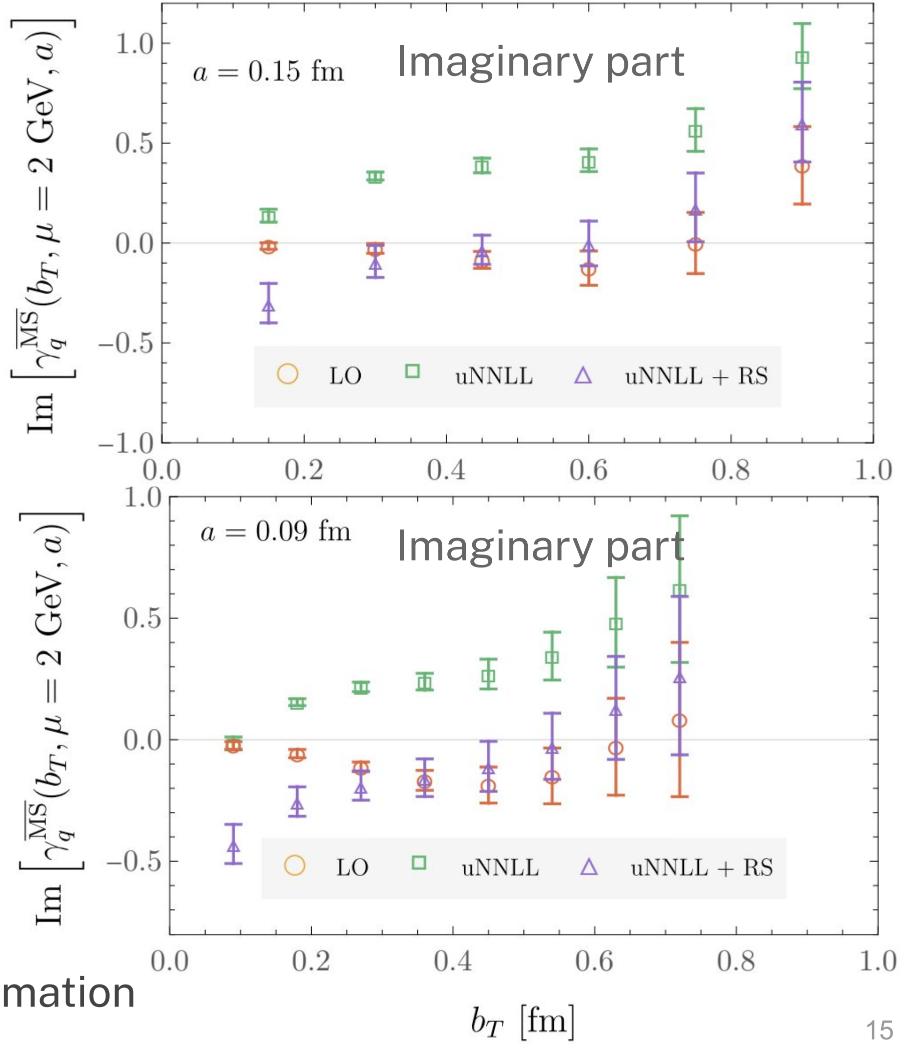
- **Leading renormalon resummation** (LRR):

$$C^{\rm LRR}(p^z,\mu) = C(p^z,\mu) - R(p^z,\mu) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{p^z}\right)$$

  - **Suppresses remaining p.c. at large bT in the imaginary part for uNNLL**
  - Not expected to work at small bT (IR effect)

- ⇒ no unexplained systematics in **uNNLL**

Final determination: uNNLL = uNLO + resummation



CS kernel parameterization and continuum extrapolation

- Pheno params + **Discretization effects** modeled together:

$$\gamma_q^{\text{param.}}(b_T,\mu;B_{\text{NP}},\{c_i\},a) = -2\left[\mathcal{D}_{\text{pert.}}(b^*,\mu)+\mathcal{D}_{\text{NP}}(b_T,b^*;B_{\text{NP}},\{c_i\})\right]+k_1\frac{a}{b_T}+k_2\frac{a^2}{b_T^2}$$

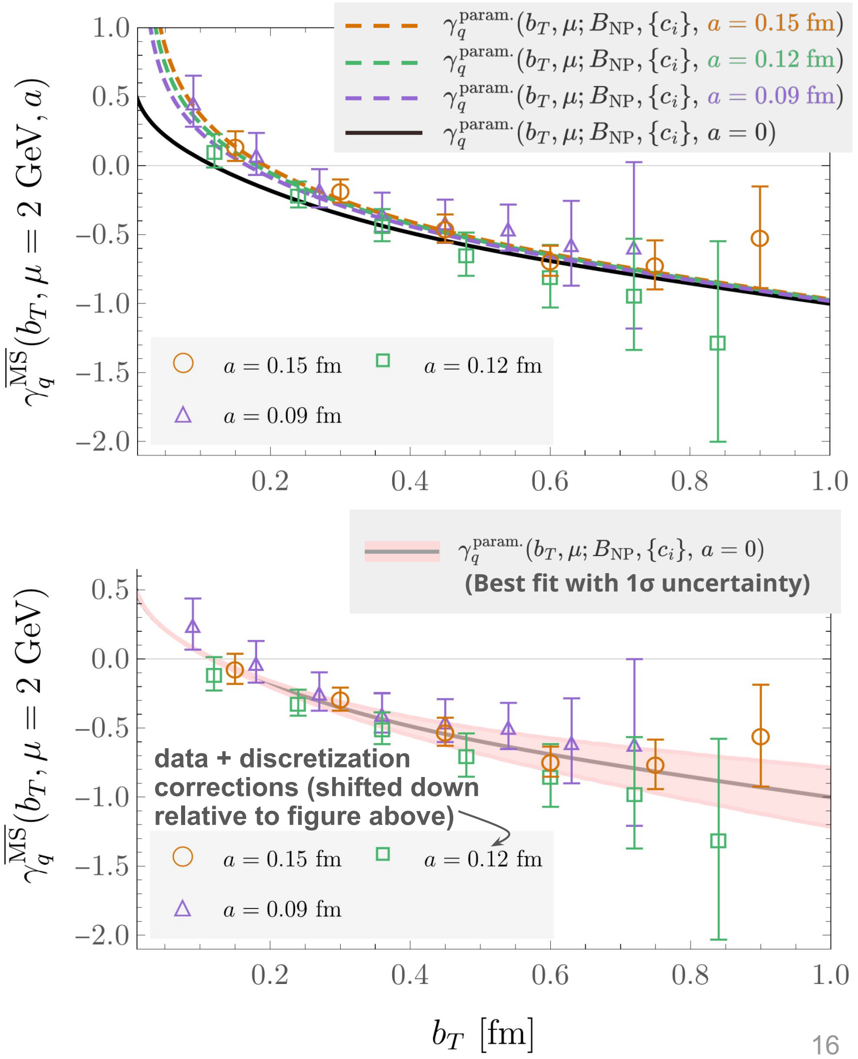
best fit $b_T b^*\left[c_0+c_1\ln\frac{b^*}{B_{\text{NP}}}\right]$

- Fits to different **NP models** consistent, directly comparable with params from pheno. results.

- **Discretization effects:**
  - **~20%** from **best fit**:
    1. Fix subsets of params to **ref. values**
    2. **Optimize** others
    3. Use AIC to pick **best fit**
    (Akaike Information Criterion)

    $c_0 = 0.032(12)$, $k_1 = 0.22(8)$,

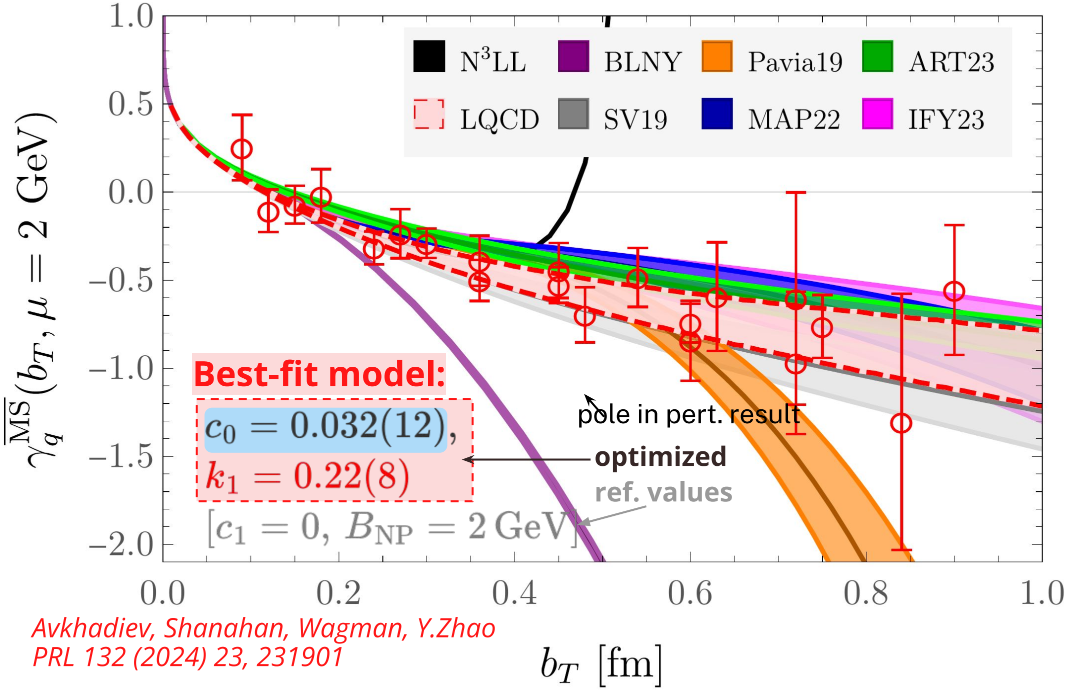
    $[c_1 = 0,\ B_{\text{NP}} = 2\,\text{GeV},\ k_2 = 0]$
  - **~10–30% range** as **NP models** are varied



Summary and outlook

- **Systematic control** over quark mass, operator mixing, and discretization effects
- Sufficiently precise to **discriminate between some pheno models**
- Still not fitting power corrections, using weighted averages
- Improvements expected from
  - Coulomb Gauge fixing for stapleless calculations
  - bT-dependent matching, incl. in collinear factorization limit

Avkhadiev, Shanahan, Wagman, Y.Zhao
PRD 108 (2023) 11, 114505



Avkhadiev, Shanahan, Wagman, Y.Zhao
PRL 132 (2024) 23, 231901

X. Gao(Argonne), W-Y. Liu, Y. Zhao, PRD109 (2024) [2306.14960]
Y. Zhao [2311.01391]
Bollweg, X. Gao, Mukherjee, Y. Zhao, PLB 852 (2024) [2403.00664]

Talks: Tue Y. Zhao [9.20 am], X. Gao [9.50 am]
Wed Y.B. Yang [8.30 am], J. He [9.00 am]

Calculations of quark vs. gluon CS kernel differ by operator and matrix element, otherwise analogous

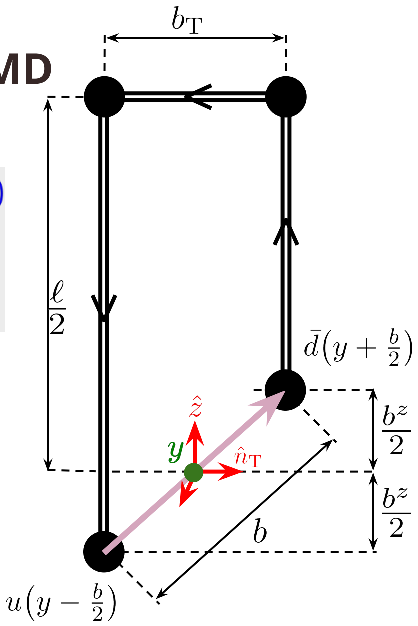
Quark CS kernel — completed

Computed using **quasi-TMD wavefunctions (WFs)**:

$$W_\Gamma(b_T, b^z, \ell, P^z) \quad (\Gamma \in \{\gamma^4\gamma^5, \gamma^5\}) = \frac{\langle 0|\mathcal{O}_\Gamma(b_T, b^z, \ell, P^z)\,|\pi(P^z)\rangle}{\boxed{\langle 0|\mathcal{O}_{\gamma^4\gamma^5}(b_T, 0, \ell, 0)\,|\pi(0)\rangle}}$$

Pz=0, bz=0 matrix elt to subtract divergences linear in ℓ

Account for renormalization-induced mixing between Γ structures



Gluon CS kernel — ongoing

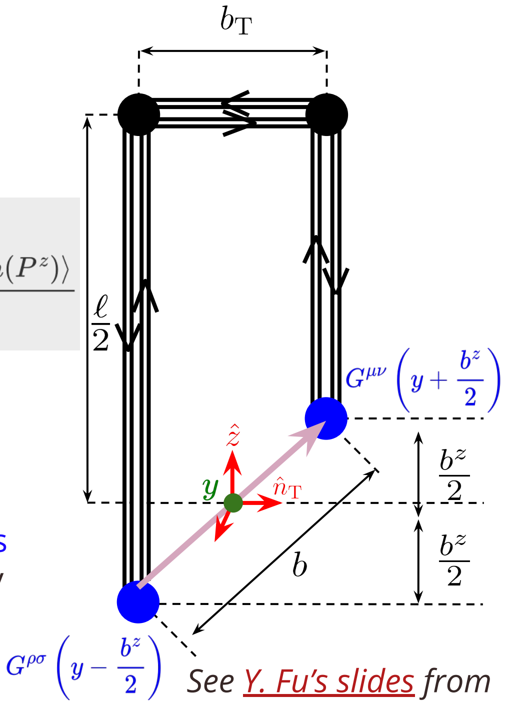
Computed using **quasi-TMD beam functions**:

$$P^\mu P^\nu \tilde{B}^{\rho\sigma}_{g/h}(b^z, b_T, P^z, \ell) = \frac{\langle h(P^z)|\mathcal{O}_g^{\mu\nu\rho\sigma}(b_T, b^z, 0, \ell)\,|h(P^z)\rangle}{\boxed{\sqrt{Z_A(b_T, \ell)}}}$$

Square root of adjoint Wilson loop to subtract divergences linear in ℓ

Can choose Lorentz indices that lead to multiplicatively renormalizable operators

J-H. Zhang, X. Ji, Schäfer, W. Wang, S. Zhao, PRL 122 (2019) [1808.10824]



See Y. Fu's slides from Lattice 2024

Gluon calculation will require ~30x stats of the quark project

Quark CS kernel — completed

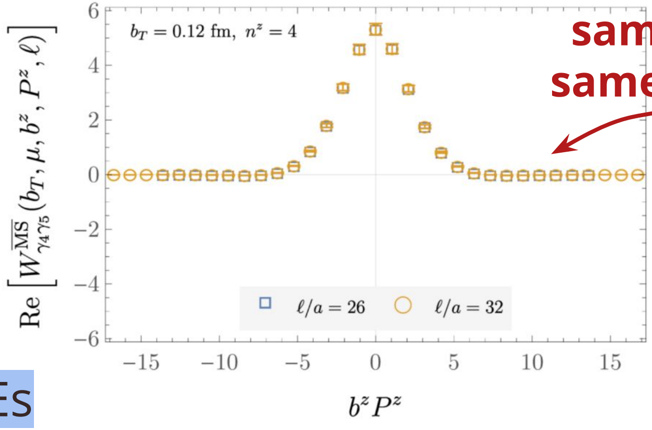
Gluon CS kernel — ongoing

Position space MEs

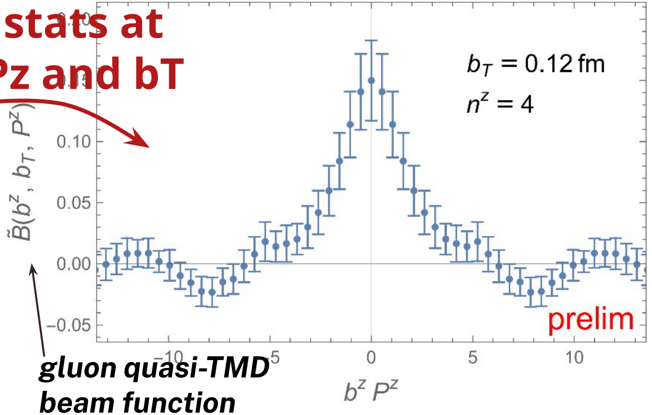
- Quark ME has non-zero imaginary part
- Gluon ME real, and exactly bz-symmetric after averaging +/- bT

Fourier-transformed MEs

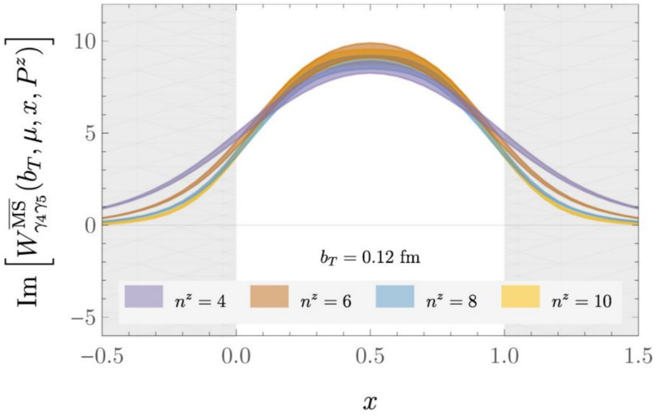
- Quark ME symmetric wrt x → 1-x
- Gluon ME symmetric wrt x → -x, power corrections still most suppressed around |x| ~ 0.5

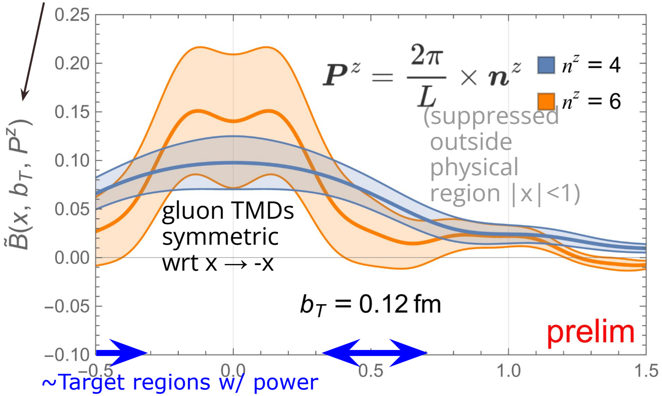


same stats at same Pz and bT



gluon quasi-TMD beam function





~Target regions w/ power corrections to LaMET matching controlled

Artur Avkhadiev, MIT

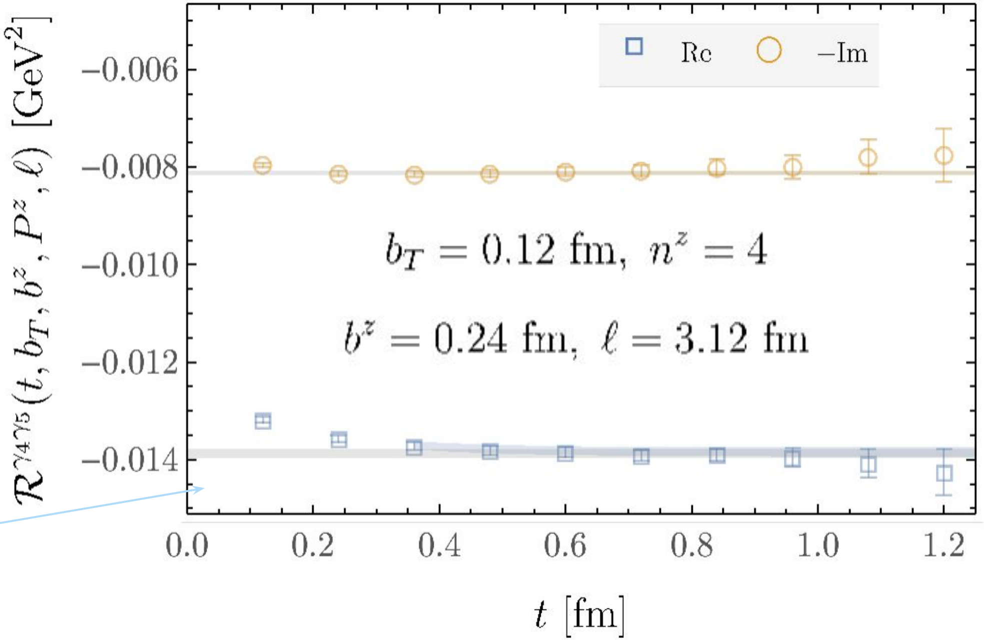
Backup

Unsubtracted quasi-TMD WFs: examples

- Extracted from correlation functions

$$\sum_{\mathbf{y}} e^{i\mathbf{P}\cdot\mathbf{y}} \left\langle \mathcal{O}_\Gamma(b_T, b^z, y, \ell)\chi^\dagger_{\mathbf{P}}(0) \right\rangle$$
$$\xrightarrow{t \gg 0} \frac{Z^S_\pi(\mathbf{P})}{2E_\pi(\mathbf{P})} \tilde{\phi}_\Gamma(b_T, b^z, \mathbf{P}, \ell) e^{-E_\pi(\mathbf{P})t}$$

- Momentum-smeared interpolators $\chi^\dagger_{\mathbf{P}}$
- $E_\pi(\mathbf{P})$ and $Z^S_\pi(\mathbf{P})$ fit and cancelled in ratios $\mathcal{R}^\Gamma(t, b_T, b^z, P^z, \ell)$:
- Plateau gives $\tilde{\phi}_\Gamma(b_T, b^z, \mathbf{P}, \ell)$.
- Repeated for each $P^z,\ b_T,\ b^z,\ \ell$.

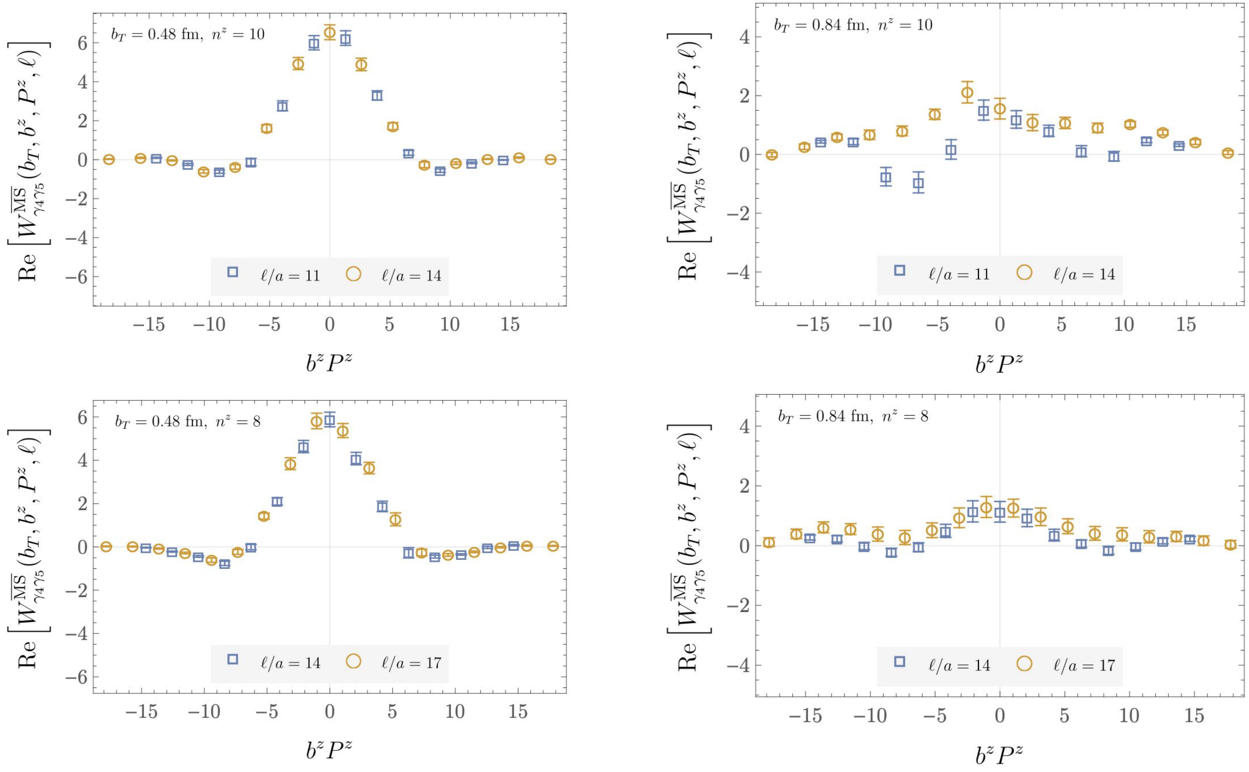


- A range of time windows chosen systematically
- Covariance matrix from bootstrap + linear shrinkage
- Correlated determinations between staple geometries
- AIC-preferred fits (1 + 2 state)
- Further selection cuts + combine in weighted average

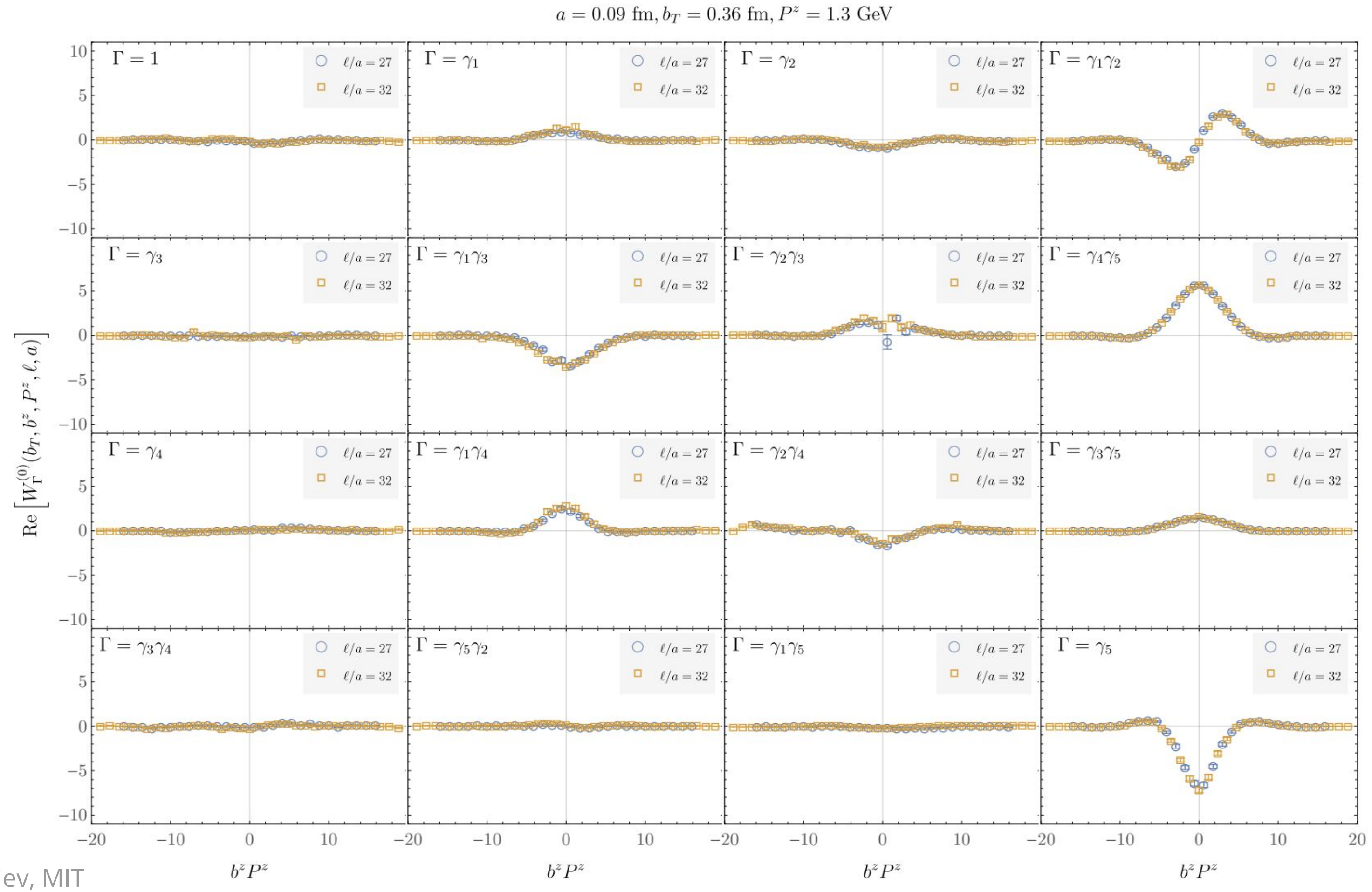
TMD WFs in position space

Statistical noise makes computation challenging for large $P^z, \ell,$ and b_T

- $b_T = 0.48\,\mathrm{fm},\ 0.84\,\mathrm{fm}$ left to right
- $P^z = 2.15\,\mathrm{GeV},\ 1.72\,\mathrm{GeV}$ top to bottom
- Our group's previous calculation had $b_T^{\max} = 0.48\,\mathrm{fm}$, $P^z_{\max} = 1.51\,\mathrm{GeV}$



MEs with all 16 Dirac structures calculated



Mixing effects quantified with RIxMOM

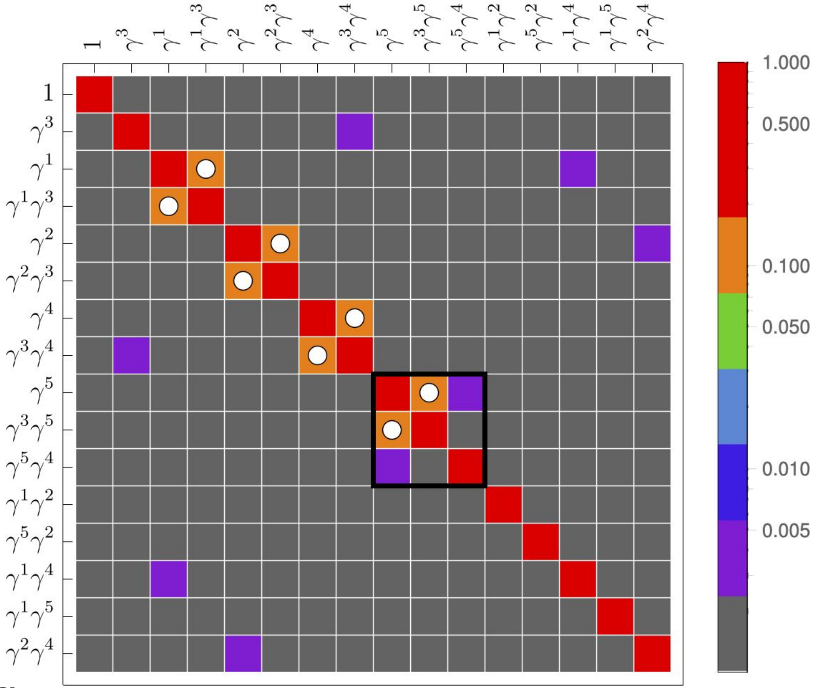
- Calculation of mixing effects in RIxMOM independent of staple geometry.

$$W_{\Gamma}^{\overline{\mathrm{MS}}}(b_T,\mu,b^z,P^z,\ell)=\sum_{\Gamma'} Z_{\Gamma\Gamma'}^{\overline{\mathrm{MS}}}(\mu)\,W_{\Gamma}^{(0)}(b_T,b^z,P^z,\ell)$$

- Full 16x16 mixing matrix computed

$$\mathcal{M}_{\Gamma\Gamma'}^{\mathrm{RI/xMOM}}(p_{\mathrm{R}},\,\xi_{\mathrm{R}},a) \equiv \frac{\mathrm{Abs}[Z_{\Gamma\Gamma'}^{\mathrm{RI/xMOM}}(p_{\mathrm{R}},\xi_{\mathrm{R}},a)]}{\frac{1}{16}\sum_{\Gamma}\mathrm{Abs}[Z_{\Gamma\Gamma}^{\mathrm{RI/xMOM}}(p_{\mathrm{R}},\xi_{\mathrm{R}},a)]}$$

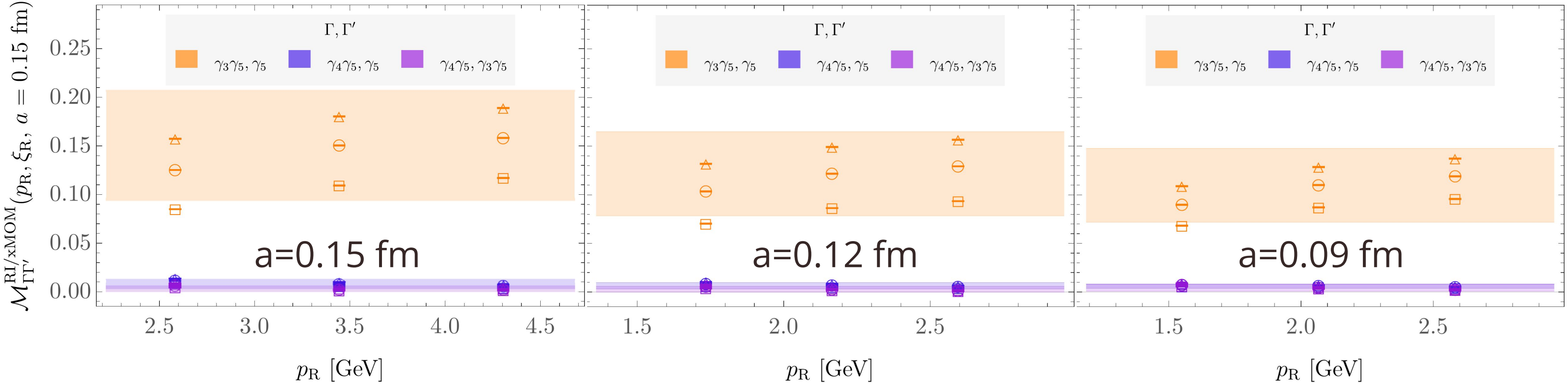
- Dominant mixings consistent with lattice perturbation theory at 1-loop.*



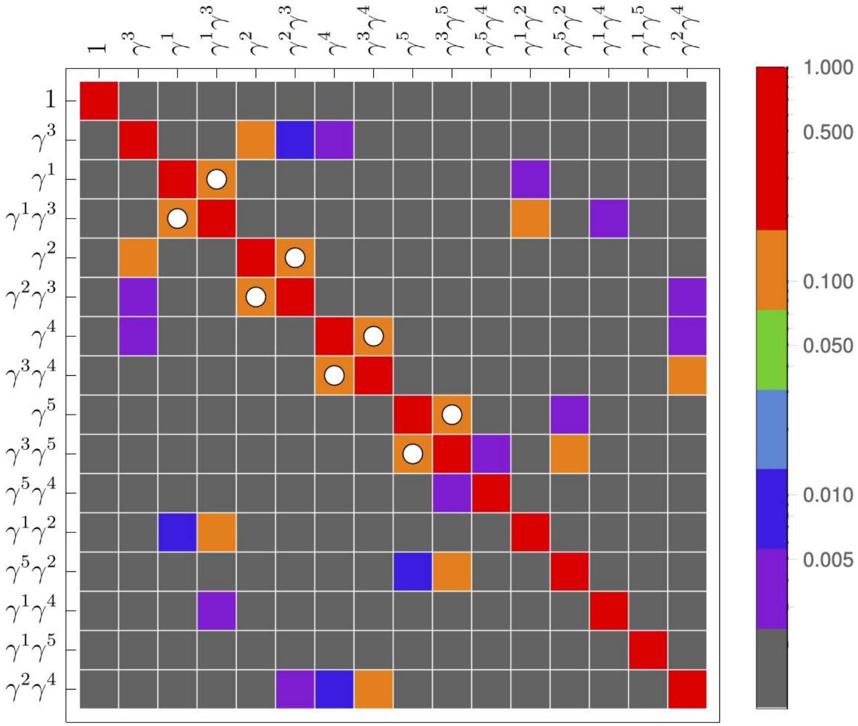
$$p_{\mathrm{R}}^{\mu}=\frac{2\pi}{L}\times(0,0,10,0),\ \xi=0.24\ \mathrm{fm}$$

X. Ji, et. al, PRL 120 (2018), [1706.08962]
J. Green et. al, PRL 121 (2018), [1707.07152]
J. Green et. al, PRD 101 (2020), [2002.09408]
*M. Constantinou et al., PRD 99 (2019), [1901.03862]
Y. Ji et. al., PRD 104 (2021), [2104.13345]
C. Alexandrou et al., [2305.11824]

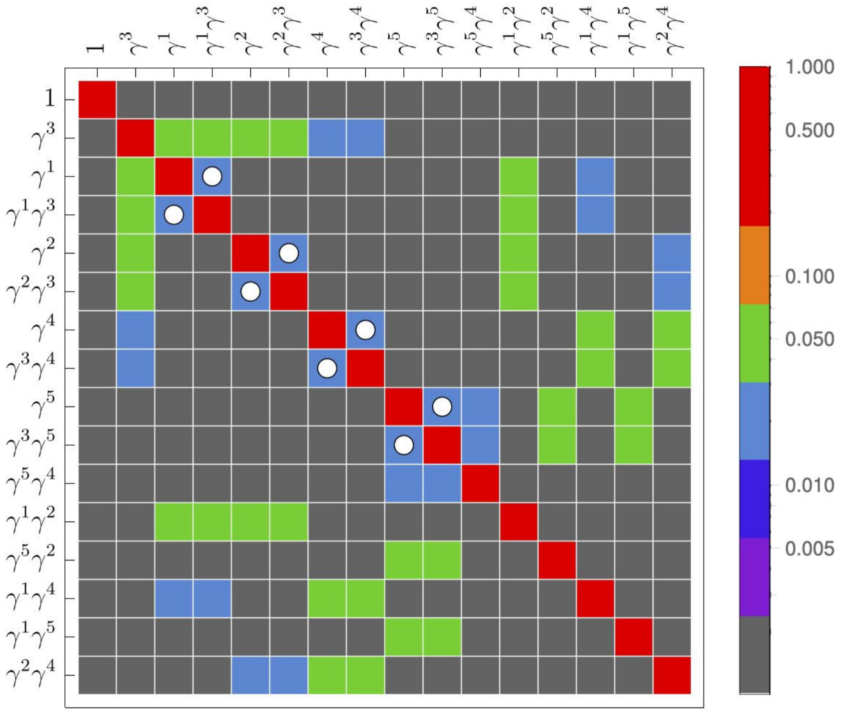
Mixing reduced at finer lattice spacings, as expected



Scheme dependence of mixing patterns



$$p^{\mu}_{\mathrm{R}} = \frac{2\pi}{L} \times (0, 10, 0, 0),\ \xi = 0.24\ \mathrm{fm}$$



$$p^{\mu}_{\mathrm{R}} = \frac{2\pi}{L} \times (6, 6, 6, 6),\ \xi = 0.24\ \mathrm{fm}$$

TMD WFs in momentum space

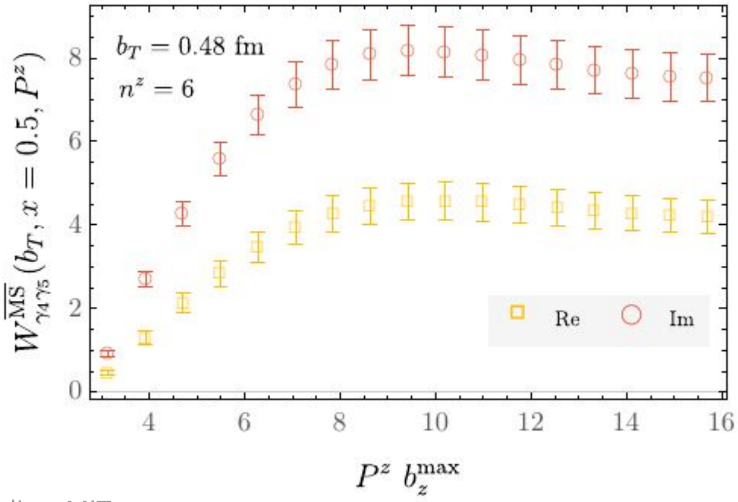
$$\gamma_q(\mu, b_\mathrm{T}) = \lim_{a\to 0} \frac{1}{\ln(P_1^z/P_2^z)} \ln\left[\frac{\int \mathrm{d}b^z e^{ib^z x P_1^z} P_1^z \sum_{\Gamma'} Z_{\Gamma\Gamma'}(\mu,a) \lim_{\ell\to\infty} W_{\mathcal{O}}^{\Gamma'}(b^z, b_\mathrm{T}, \ell, P_1^z, a)}{\int \mathrm{d}b^z e^{ib^z x P_2^z} P_2^z \sum_{\Gamma'} Z_{\Gamma\Gamma'}(\mu,a)\lim_{\ell\to\infty} W_{\mathcal{O}}^{\Gamma'}(b^z,b_\mathrm{T},\ell,P_2^z,a)}\right] + \delta\gamma_q(\mu,b_\mathrm{T},P_1^z,P_2^z) + \mathcal{O}\left(\frac{1}{(xP^zb_\mathrm{T})^2}, \frac{M^2}{(xP^z)^2}, \frac{\Lambda^2_{\mathrm{QCD}}}{(xP^z)^2}\right)$$

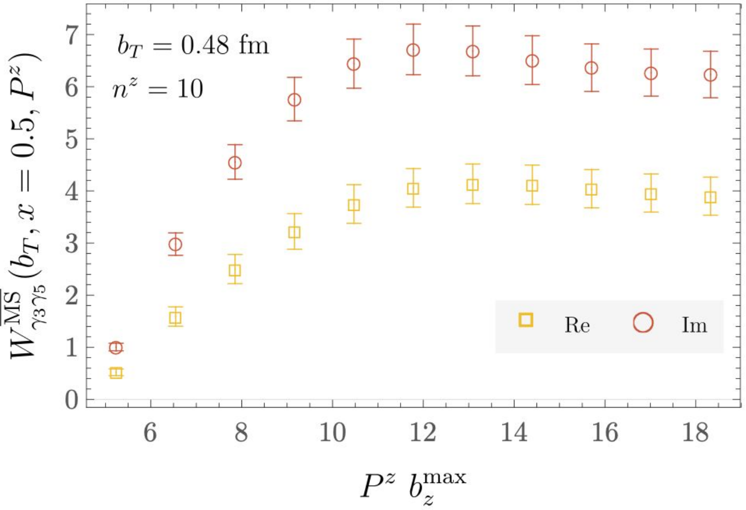
bz range sufficient to use a Discrete Fourier Transform

$$\bar{W}_\Gamma^{\overline{\mathrm{MS}}}(b_T, \mu, x, P^z) = \frac{P^z}{2\pi} N_\Gamma(P) \sum_{|b_z|\le b_z^{\max}} e^{i\left(x-\frac{1}{2}\right)P^z b^z} \bar{W}_\Gamma^{\overline{\mathrm{MS}}}(b_T, \mu, b^z, P^z)$$

Normalization factor to compare between Dirac structures

The DFT is stable to decreasing the range in $b_T^{\max}$:

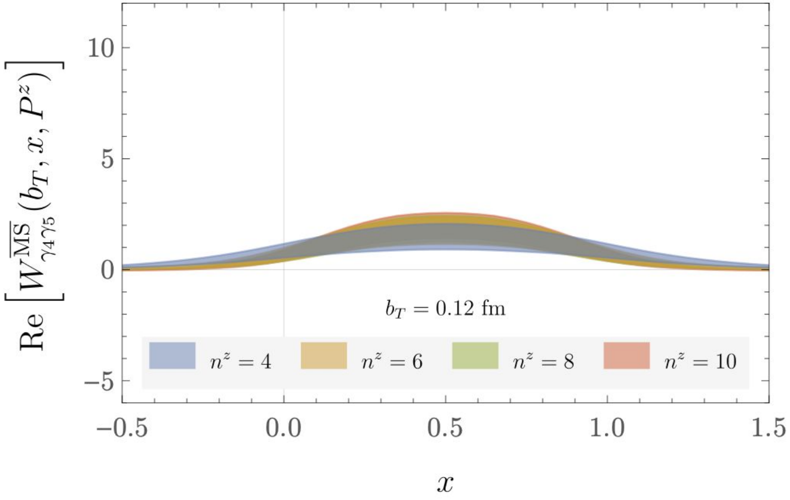


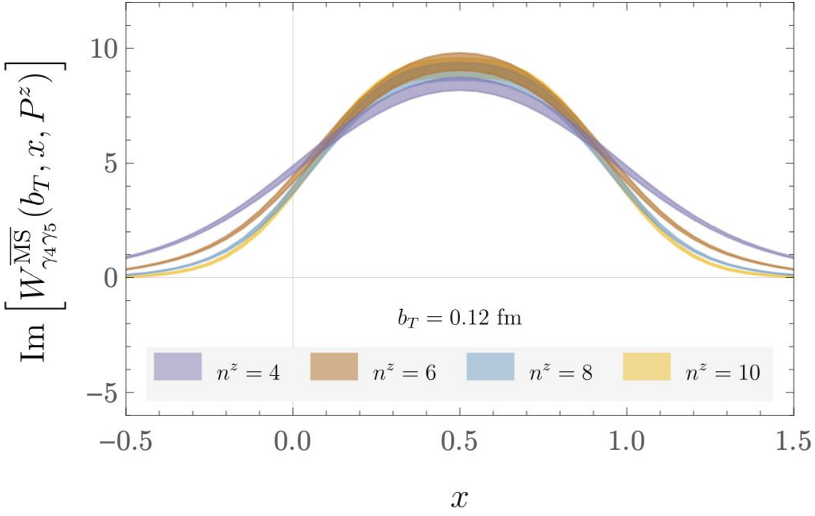


TMD WFs in momentum space

$$\gamma_q(\mu, b_\mathrm{T}) = \boxed{\lim_{a\to 0}} \frac{1}{\ln(P_1^z/P_2^z)} \ln\left[\frac{\int \mathrm{d}b^z e^{ib^z x P_1^z} P_1^z \sum_{\Gamma'} Z_{\Gamma\Gamma'}(\mu,a) \lim_{\ell\to\infty} W_{\mathcal{O}}^{\Gamma'}(b^z, b_\mathrm{T}, \ell, P_1^z, a)}{\int \mathrm{d}b^z e^{ib^z x P_2^z} P_2^z \sum_{\Gamma'} Z_{\Gamma\Gamma'}(\mu,a) \lim_{\ell\to\infty} W_{\mathcal{O}}^{\Gamma'}(b^z, b_\mathrm{T}, \ell, P_2^z, a)}\right]$$
$$+\delta\gamma_q(\mu, b_\mathrm{T}, P_1^z, P_2^z) + \boxed{\mathcal{O}\left(\frac{1}{(xP^z b_\mathrm{T})^2}, \frac{M^2}{(xP^z)^2}, \frac{\Lambda_\mathrm{QCD}^2}{(xP^z)^2}\right)}$$

See convergence to the physical range $x \in [0,1]$ with increasing $P^z = \frac{2\pi}{L} n^z$

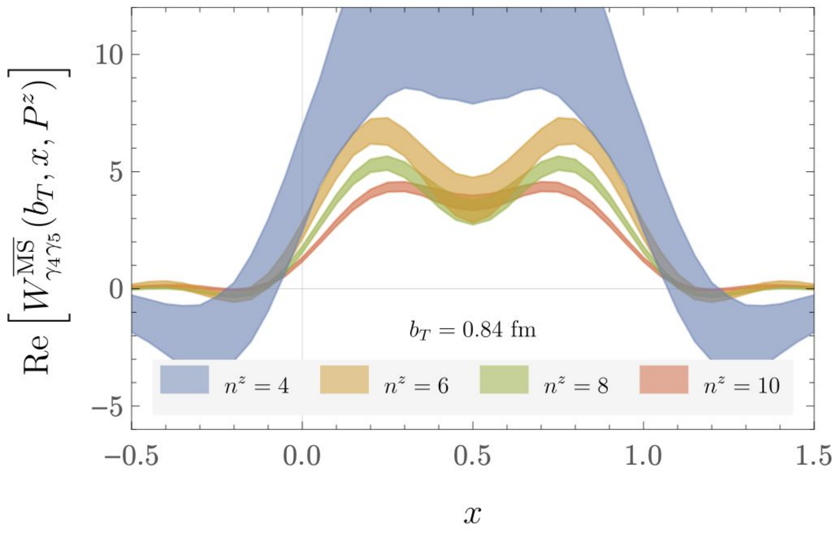


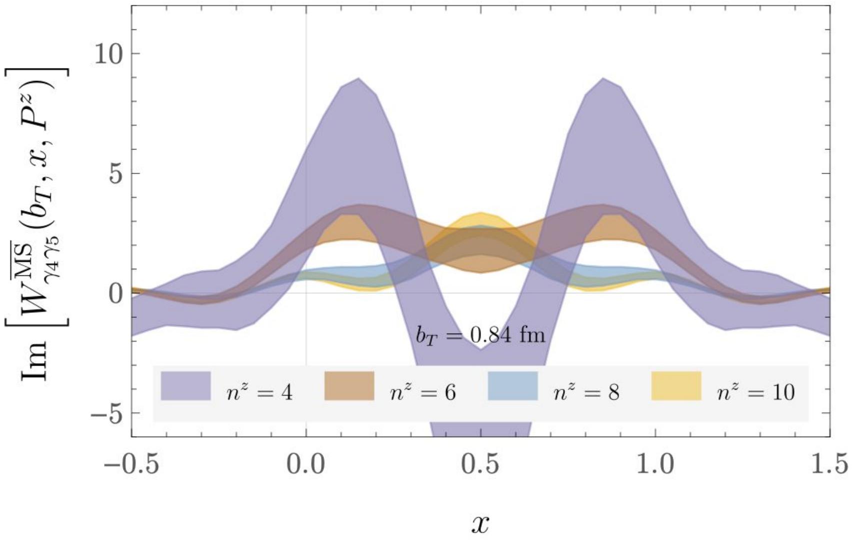


TMD WFs in momentum space

$$\gamma_q(\mu, b_T) = \boxed{\lim_{a\to 0}} \frac{1}{\ln(P_1^z/P_2^z)} \ln\left[\frac{\int db^z e^{ib^z x P_1^z} P_1^z \sum_{\Gamma'} Z_{\Gamma\Gamma'}(\mu,a) \lim_{\ell\to\infty} W_{\mathcal{O}}^{\Gamma'}(b^z, b_T, \ell, P_1^z, a)}{\int db^z e^{ib^z x P_2^z} P_2^z \sum_{\Gamma'} Z_{\Gamma\Gamma'}(\mu,a) \lim_{\ell\to\infty} W_{\mathcal{O}}^{\Gamma'}(b^z, b_T, \ell, P_2^z, a)}\right] + \delta\gamma_q(\mu, b_T, P_1^z, P_2^z) + \boxed{\mathcal{O}\left(\frac{1}{(xP^z b_T)^2}, \frac{M^2}{(xP^z)^2}, \frac{\Lambda_{QCD}^2}{(xP^z)^2}\right)}$$

See convergence to the physical range $x \in [0,1]$ with increasing $P^z = \frac{2\pi}{L} n^z$





NLO, NNLO, and resummations

The correction is given by coefficients

$$\delta\gamma_q(x, P_1^z, P_2^z, \mu) \equiv \frac{1}{\ln(P_1^z/P_2^z)}\left(\ln\frac{C_\phi(xP_2^z,\mu)}{C_\phi(xP_1^z,\mu)} + (x \leftrightarrow \bar{x})\right)$$

$C_\phi(p^z, \mu)$ appear in the TMD WF matching formula and are computed perturbatively as

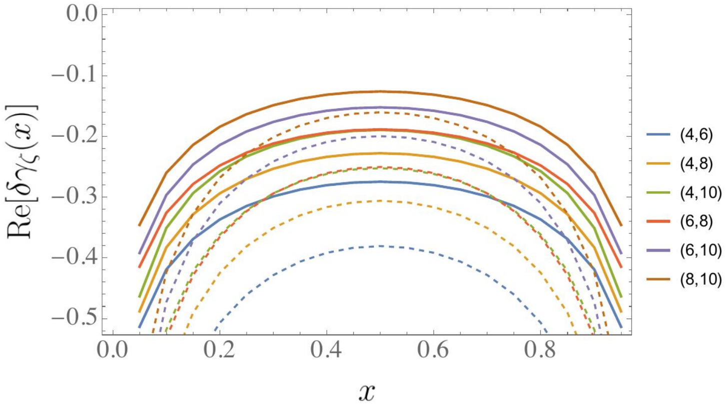
$$C_\phi(p^z,\mu) = 1 + \sum_{n=1}\left(\frac{\alpha_s(\mu)}{4\pi}\right)^n C_\phi^{(n)}(p^z,\mu)$$

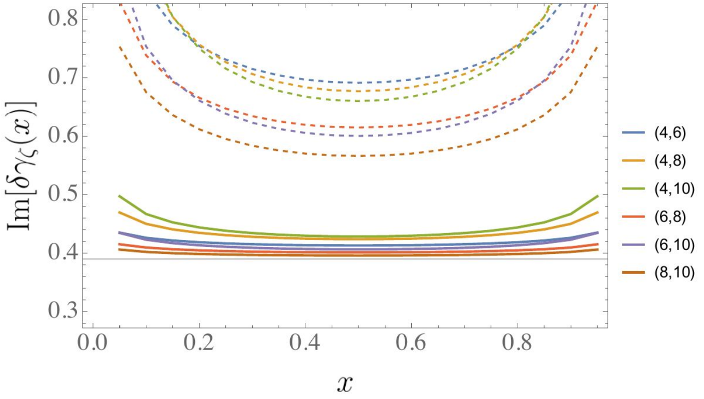
at LO, NLO and recently at NNLO, and resummed as

O. del Río and A. Vladimirov, [2304.14440]
X. Ji et. al, [2305.04416]

Resummation kernel

$$C_\phi(p^z,\mu) = C_\phi(p^z, 2p^z) \times \exp[K_\phi(p^z, 2p^z)]$$





NLO (solid) and NNLO (dashed);
No convergence in the imaginary part

NLL and NNLL

X. Ji et. al., Phys. Lett. B 811 [1911.03840]
Ebert et. al, JHEP 04 (2022), [2201.08401]

Resummation kernel is $K_\phi(2p^z,\mu) = 2K_\Gamma(2p^z,\mu) - K_{\gamma_\mu}(2p^z,\mu) - i\pi\eta(2p^z,\mu)$

$$K_{\gamma_\mu}(\mu_0,\mu) = \int_{\alpha_s(\mu_0)}^{\alpha_s(\mu)} \frac{\mathrm{d}\alpha_s}{\beta(\alpha_s)}\gamma_\mu\left(\alpha_s\right),$$

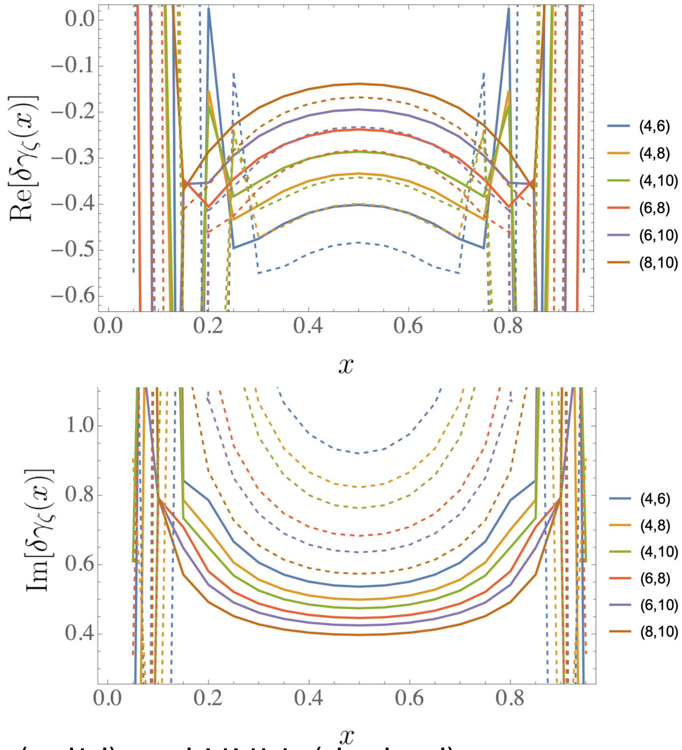
$$K_\Gamma(\mu_0,\mu) = \int_{\alpha_s(\mu_0)}^{\alpha_s(\mu)} \frac{\mathrm{d}\alpha_s}{\beta(\alpha_s)}\Gamma_{\mathrm{cusp}}(\alpha_s)\int_{\alpha_s(\mu_0)}^{\alpha_s} \frac{\mathrm{d}\alpha_s'}{\beta(\alpha_s')},$$

$$\eta_\Gamma(\mu_0,\mu) = \int_{\alpha_s(\mu_0)}^{\alpha_s(\mu)} \frac{\mathrm{d}\alpha_s}{\beta(\alpha_s)}\Gamma_{\mathrm{cusp}}(\alpha_s)$$

where $\Gamma_{\mathrm{cusp}}(\alpha_s(\mu)) = \dfrac{\mathrm{d}\gamma_\mu\left(p^z,\mu\right)}{\mathrm{d}\ln p^z}$ and $\gamma_\mu(p^z,\mu) \equiv \dfrac{d\ln C_\phi(p^z,\mu)}{d\ln\mu}$

are computed perturbatively at following loop orders for each resummation accuracy:

	K_Γ	K_{γ_C}	K_{γ_μ}	η	C_ϕ
NLL	2	1	1	1	0
NNLL	3	2	2	2	1



NLL (solid) and NNLL (dashed)
No convergence in the imaginary part

bT-dependent matching

Matching coefficients C include are a $P^z b_T \gg 1$ limit of

$$\boxed{\mathbf{C}_\phi(p^z, b_T, \mu) = C_\phi(p^z, \mu) + \delta C_\phi(p^z, b_T)}$$

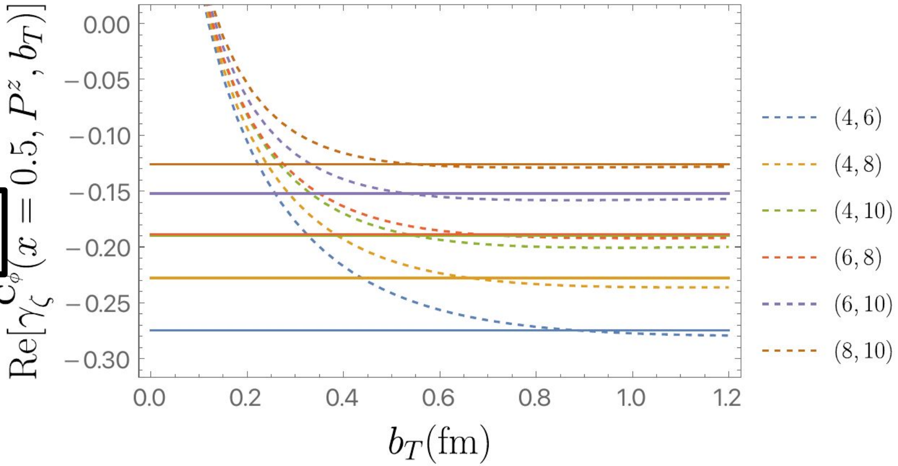
uNLO

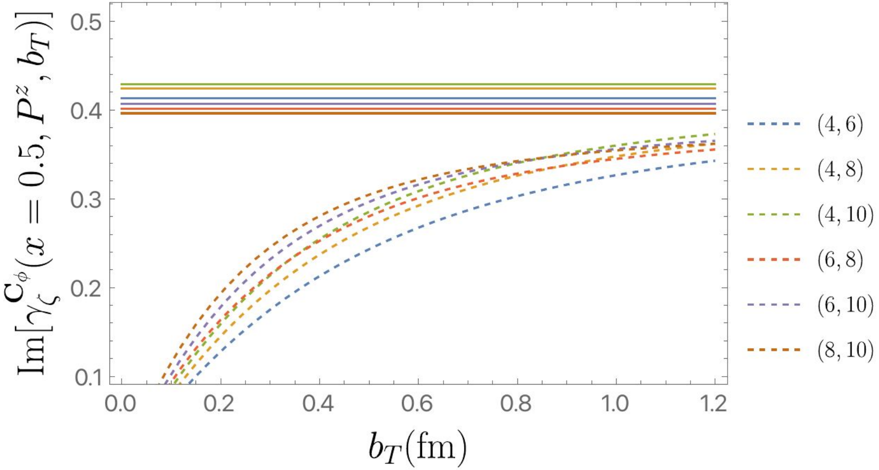
$$p^z \in xP^z, \bar{x}P^z$$

- $\delta C_\phi(p^z, b_T)$ contains bT-dependent terms on $x \in (-\infty, \infty)$ suppressed in $P^z b_T$
- Has been computed at NLO.
- Corresponding unexpanded (in bT) matching correction reveals power corrections in 1/Pz bT.

M. A. Ebert et. al,, JHEP 09, 037, [1901.03685]
Z.-F. Deng et. al, JHEP 09, [2207.07280]

- Imaginary part more sensitive to power corrections => <u>not taken as a systematic uncertainty directly.</u>

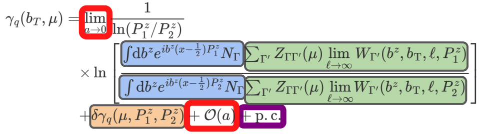




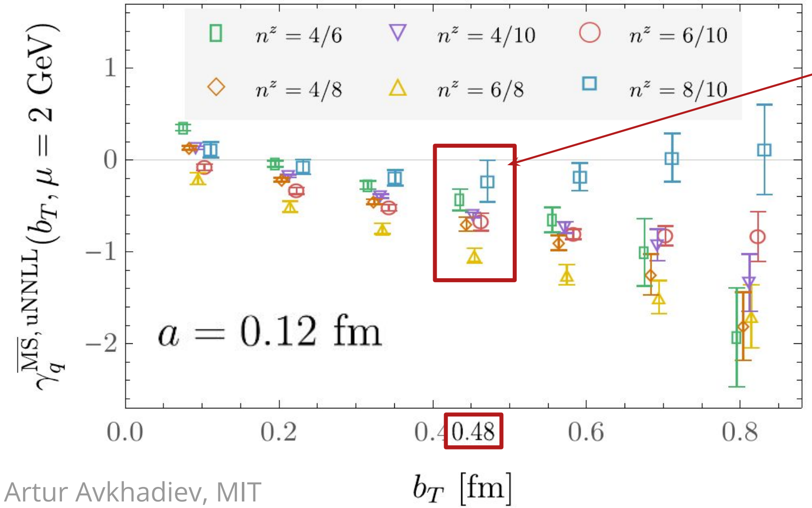
M.-H. Chu et al. (LPC), PRD 106, 034509, [2204.00200]
M.-H. Chu et al. (LPC), [2302.09961]
M.-H. Chu et al. (LPC), [2306.06488]

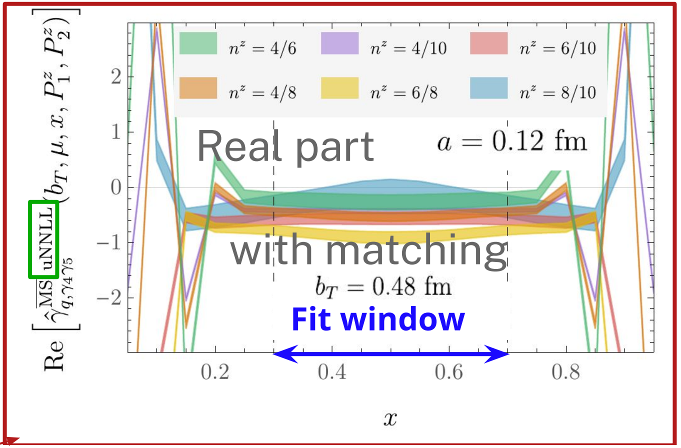
Dashed: **uNLO**, solid: NLO.

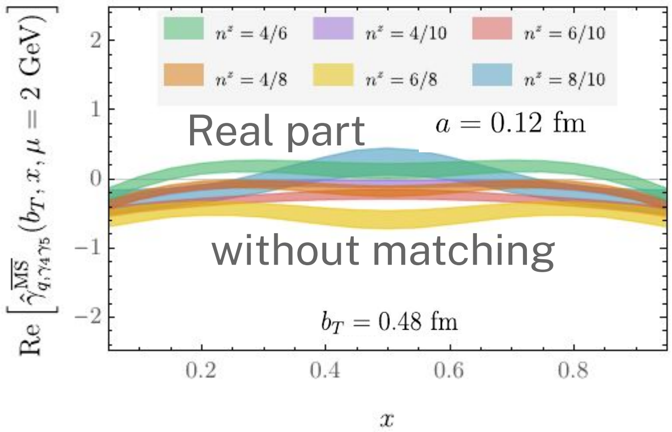
Power corrections expected to decrease with higher-order LaMET matching



- **Matching applied before weighted averaging** to ratios of MEs for each bT, Pz pair, Dirac (Γ) structure, and *a*
- Final results use b_T-unexpanded, next-to-next-to-leading log (**uNNLL**) matching **with leading renormalon subtraction**



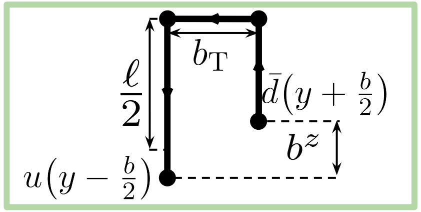




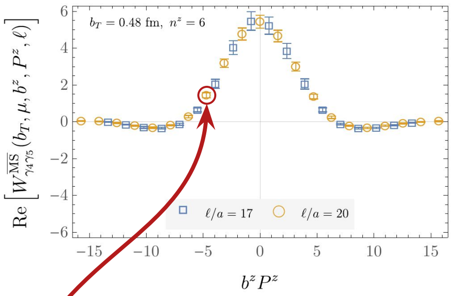
Fit each band separately to a constant in $x \in [0.3, 0.7]$ to get points at fixed bT (**power corrections** enhanced at x~0 and x~1)

$$\mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{xP^z}, \frac{1}{b_T(xP^z)}\right) + (x \leftrightarrow 1 - x)$$

Rest of LQCD analysis remains as before — now w/ calculations at 2 additional lattice spacings



Calculate position-space MEs

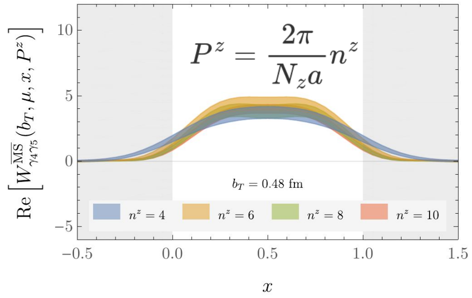


$$\sum_{\Gamma'} Z_{\Gamma\Gamma'}(\mu) \lim_{\ell\to\infty} W^{\Gamma'}_{\mathcal{O}}(b^z, b_\mathrm{T}, \ell, P^z_1)$$

$$W_\Gamma(b_T, b^z, \ell, P^z) \quad (\Gamma \in \{\gamma^4\gamma^5, \gamma^5\}) = \frac{\langle 0|\mathcal{O}_\Gamma(b_T, b^z, \ell, P^z)\,|\pi(P^z)\rangle}{\langle 0|\mathcal{O}_{\gamma^4\gamma^5}(b_T, 0, \ell, 0)\,|\pi(0)\rangle}$$

Each plotted point is a separate ME calculation (defined by **b^z, P^z, bT, ℓ**, and Dirac (**Γ**) structure.

Fourier transform (FT) to momentum-space MEs

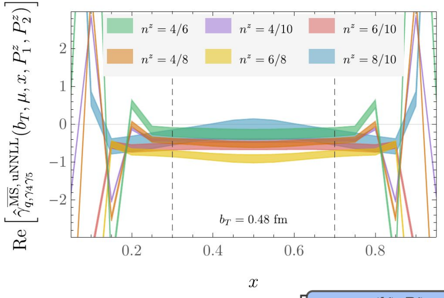


$$\int db^z e^{ib^z x P^z_1} P^z_1 N_\Gamma(P^z_1)$$

$$\sum_{\Gamma'} Z_{\Gamma\Gamma'}(\mu) \lim_{\ell\to\infty} W^{\Gamma'}_{\mathcal{O}}(b^z, b_\mathrm{T}, \ell, P^z_1)$$

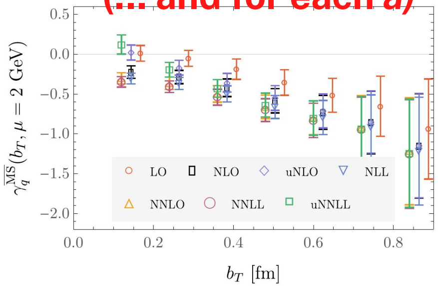
Discrete FT needs sufficient range in **$b^z P^z$** to avoid truncation effects in x space (each **P^z, bT, Γ**)

Form ratios of MEs + match + fit in x



$$\gamma_q(b_T,\mu) = \boxed{\lim_{a\to 0}} \frac{1}{\ln(P^z_1/P^z_2)} \ln\left[\frac{\int db^z e^{ib^z x P^z_1} P^z_1 N_\Gamma(P^z_1) \sum_{\Gamma'} Z_{\Gamma\Gamma'}(\mu) \lim_{\ell\to\infty} W^{\Gamma'}_{\mathcal{O}}(b^z, b_\mathrm{T}, \ell, P^z_1)}{\int db^z e^{ib^z x P^z_2} P^z_2 N_\Gamma(P^z_1) \sum_{\Gamma'} Z_{\Gamma\Gamma'}(\mu) \lim_{\ell\to\infty} W^{\Gamma'}_{\mathcal{O}}(b^z, b_\mathrm{T}, \ell, P^z_2)}\right] + \delta\gamma_q(\mu, P^z_1, P^z_2) \boxed{+\,\mathrm{p.c.}}$$

...repeat for each bT (... and for each *a*)



Needs careful choices of:

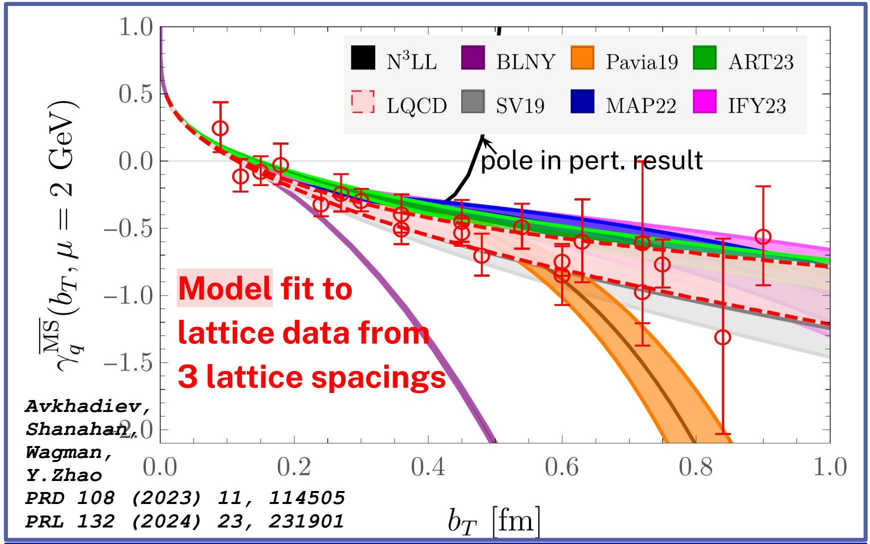
- **Pz:** several pairs to characterize **p.c.**, large enough **(p.c.)**, small enough (disc. effects, stat. noise).
- **bT:** large enough to see NP region, small enough to manage stat. noise.

*(Calculation of **gluon CS kernel** will have ~same steps, only a different ME of staple-shaped op.)*

Summary and Outlook

Quark CS kernel — completed

- **Systematic control** over quark mass, operator mixing, and discretization effects
- Sufficiently precise to **discriminate between some pheno models** from global analyses
- Still not fitting power corrections, using weighted averages
- Improvements expected from Coulomb Gauge fixing, bT-dependent matching



Gluon CS kernel — ongoing

- Need **~30x more stats** than in quark project
- No global analysis results yet, but expected with EIC data — **lattice QCD + LaMET will provide a prediction**