Transversity PDF of nucleon using pseudo-distribution approach

LaMET '21 @ CNF

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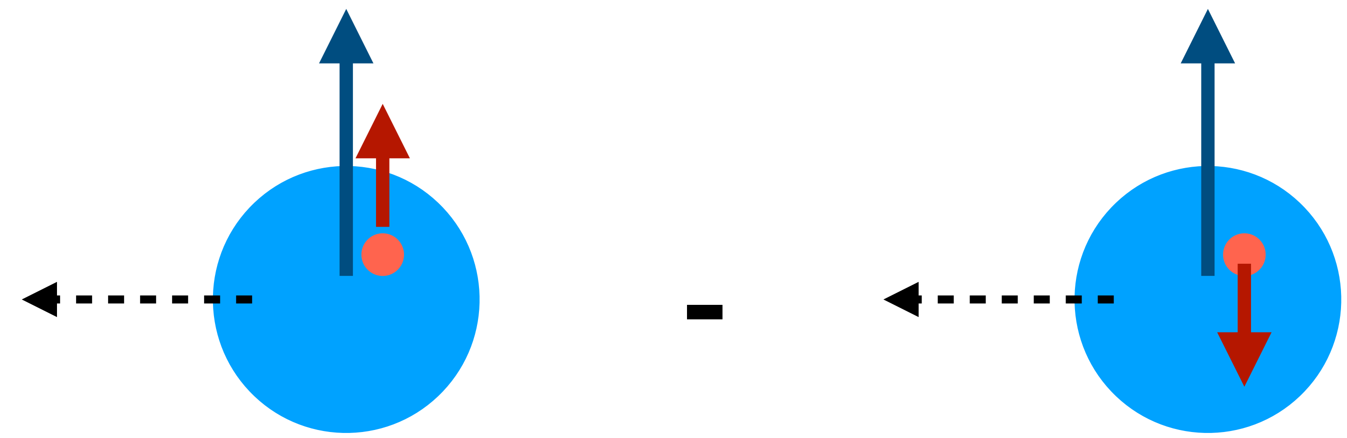
On behalf of HadStruc Collaboration

Ref: arXiv:2111.01808

C. Egerer, C. Kallidonis, J. Karpie, NK, C. J. Monahan, W. Morris,
K. Orginos, A. Radyushkin, E. Romero, R. S. Sufian, S. Zafeiropoulos

Transversity PDF

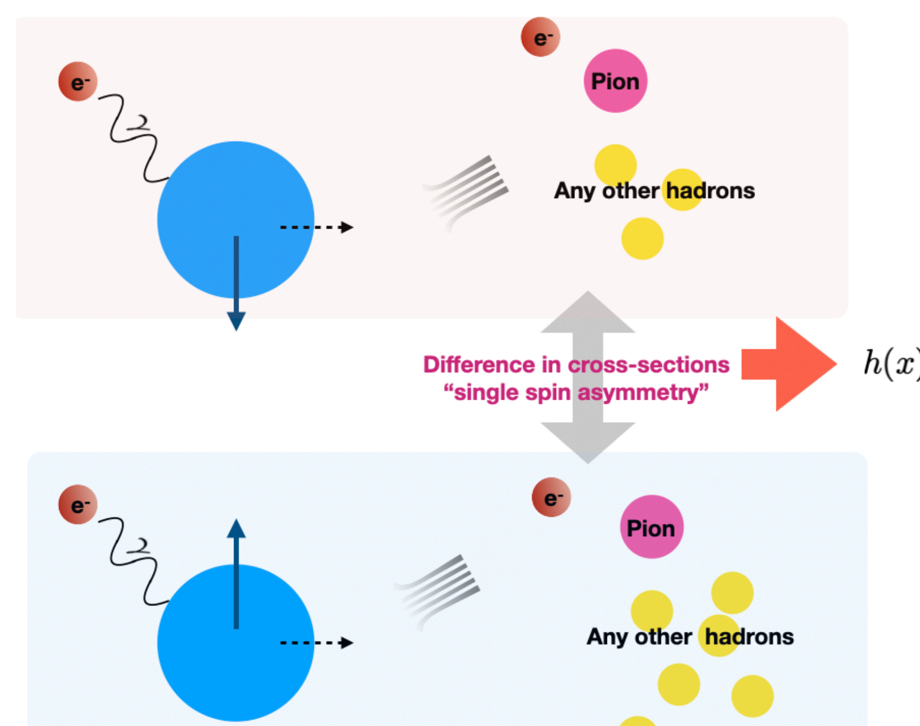
$$h_1(x) = f_\uparrow(x) - f_\downarrow(x)$$



Chiral-odd

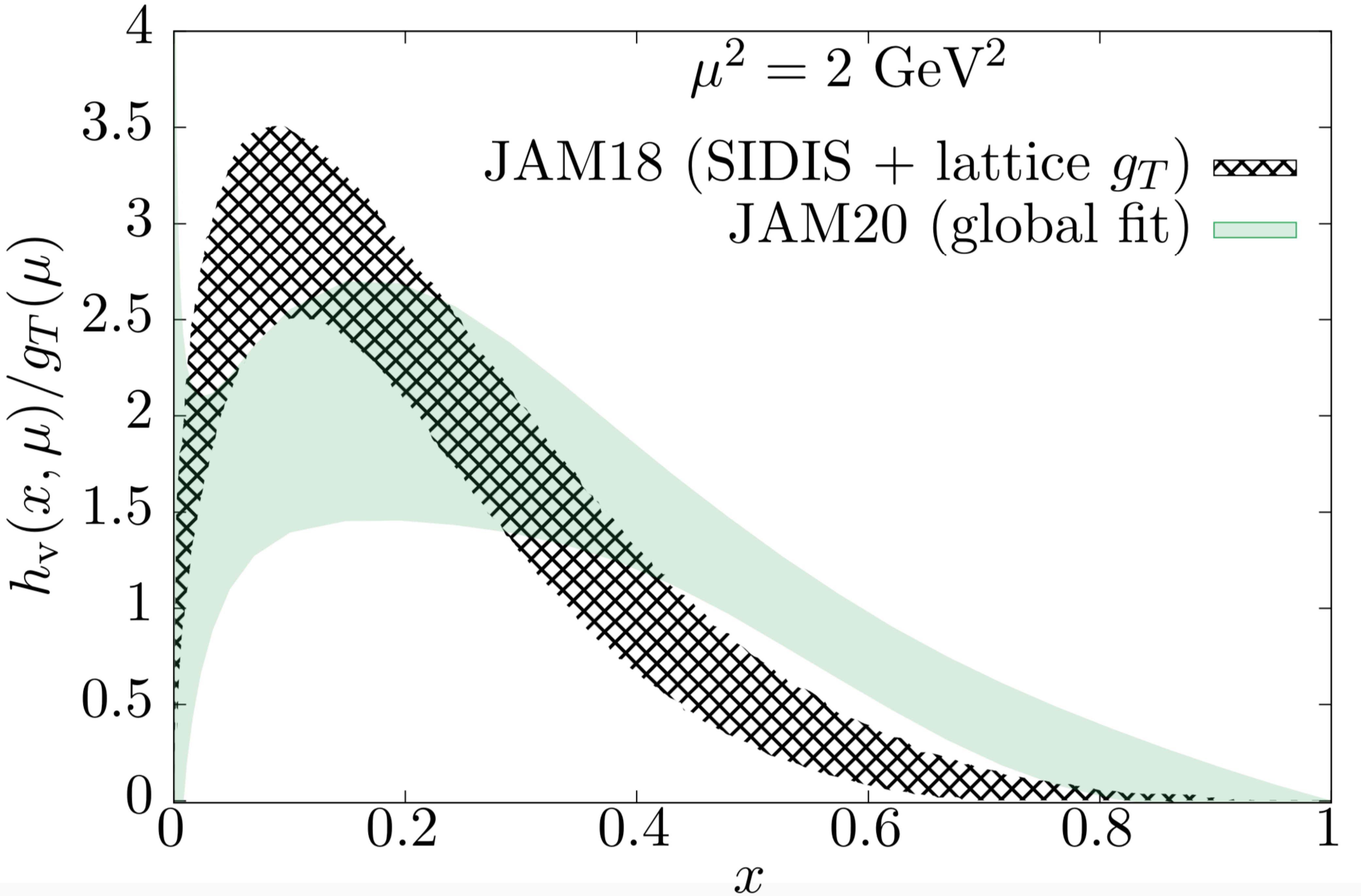
Need to couple to chiral-odd process

Accessible, for example, from Single transverse spin asymmetry in semi-inclusive DIS



This work: Obtain *x*-dependent transversity PDF using fits to bilocal M.E. (pseudo-distribution) using leading-twist OPE (+ empirically estimated corrections)

Status of experimental determination of transversity PDF



The least pheno-constrained twist-2 quark PDF → Opportunity for lattice!

Entirely within collinear framework

Framework

X.Ji '13, A. Radyushkin '17

- A good choice of kinematic variables and directions for the matrix element

$$\left\langle N; P_z, S_T \,\middle|\, \overline{\psi}(z)\gamma_5\gamma_t\gamma_T W(z,0)\tau_3\psi(0) \,\middle|\, N; P_z, S_T \right\rangle = 2ES_T\mathcal{M}(zP_z, z^2)$$

- Renormalize by ratio:

K. Orginos et al, '17

$$\mathfrak{M}(zP_z, z^2) = \frac{\mathcal{M}(z, P_z)}{\mathcal{M}(z, 0)}$$

$$\frac{\langle x^0 \rangle}{g_T}(\mu) = \int_0^1 \frac{h(x,\mu)}{g_T(\mu)}dx = 1$$

(Worry about the overall normalization g_T later)

Framework

- Capture the z_3P_3 and z_3^2 dependence via leading-twist factorization / leading-twist OPE

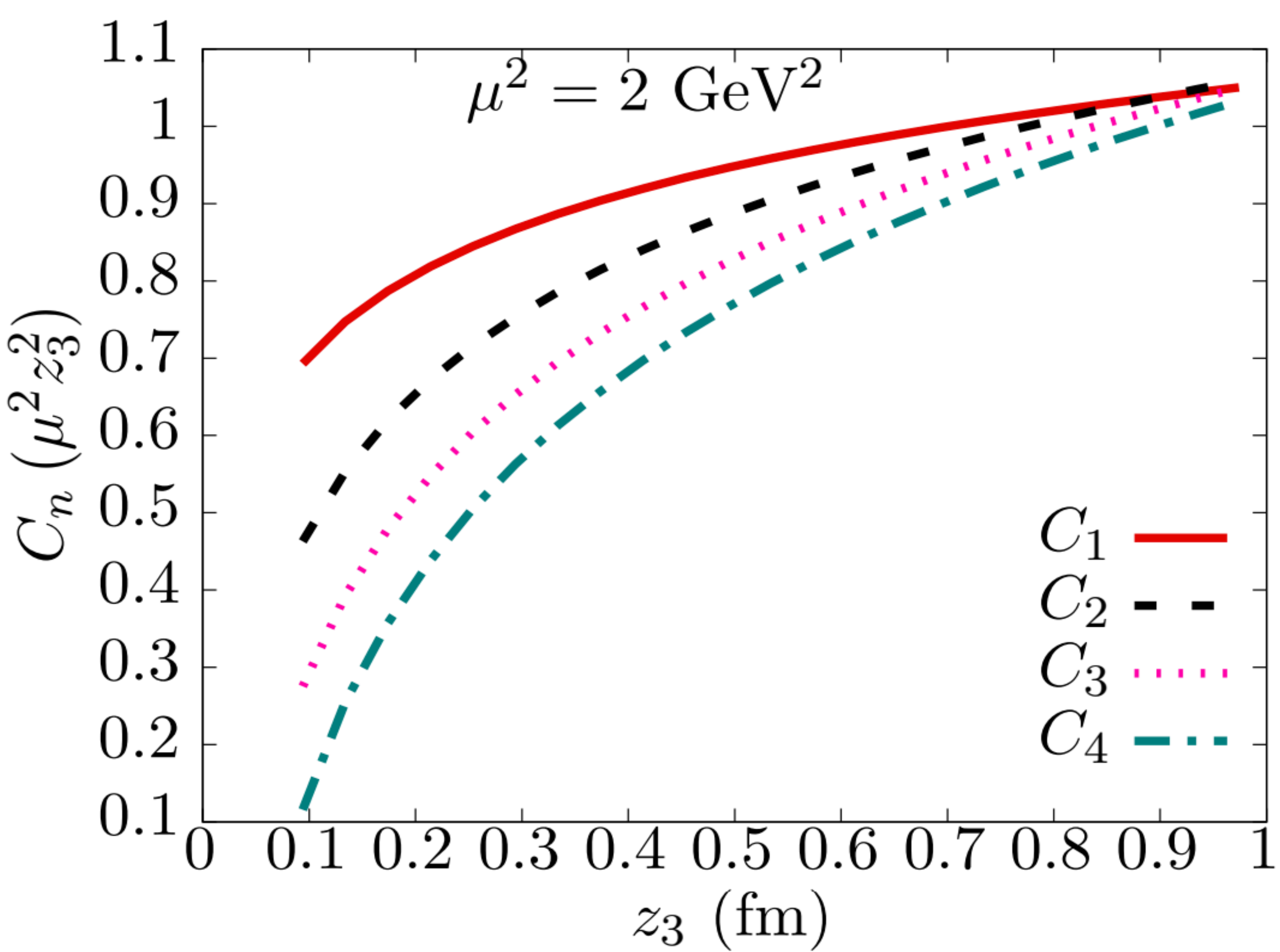
$$\mathfrak{M}^{\mathrm{twist}-2}(z_3P_3, z_3^2) = \sum_{n=0} C_n(\mu^2 z_3^2) \frac{(iz_3P_3)^n}{n!} \frac{\langle x^n \rangle(\mu)}{g_T(\mu)}$$

NLO transversity coefficients for ratio:

$$C_n\left(\mu^2 z_3^2\right) = 1 + \frac{\alpha_s C_F}{\pi}\left\{ \ln\left(\frac{z_3^2\mu^2 e^{2\gamma_E+1}}{4}\right)\sum_{k=2}^{n+1}\frac{1}{k} - \left(\sum_{k=1}^{n}\frac{1}{k}\right)^2 - \sum_{k=1}^{n}\frac{1}{k^2}\right\}.$$

This work and
Braun, Ji, Vladimirov '21

z3-dependence of
1-loop coeffs at fixed order
coupling at $\mu^2 = 2\ \mathrm{GeV}^2$



- Isotropic clover sea and valence
- 32^3 X 64 lattice, 358 cfg X 4src
- $M_\pi = 358$ MeV
- $a = 0.094$ fm
- Usage of distillation (rank 64)

(See Colin Egerer's talk)

Raw lattice data for three-point and two-point functions

↓

Extract the transversity matrix element

↓

Infer corrections to continuum leading-twist NLO just using the lattice data

← Hypothesis: lattice matrix element can be analyzed within leading-twist NLO framework to a good approximation

↓

Reconstruct x-dependent PDF, moments (Given: the hypothesis, priors, …)

Raw lattice data for three-point and two-point functions

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Extract the transversity matrix element

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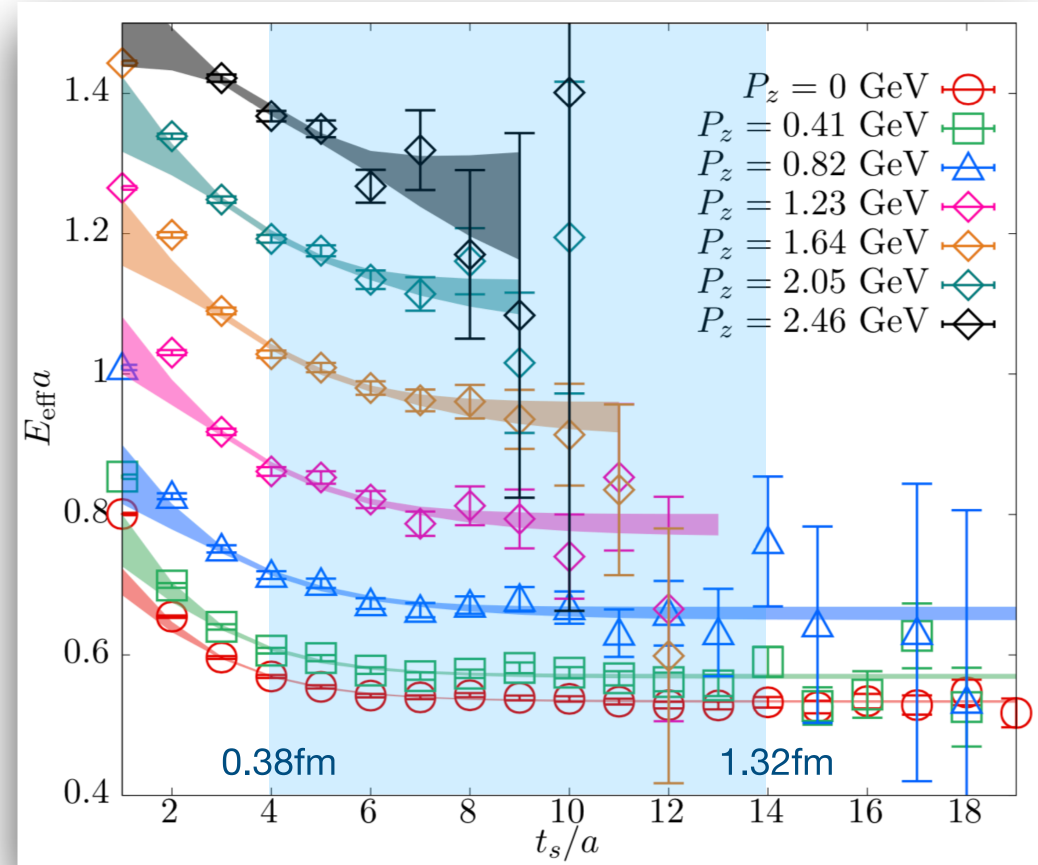
Infer corrections to continuum leading-twist NLO just using the lattice data

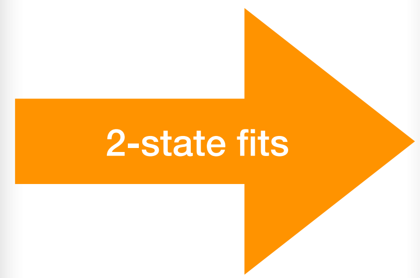
← Hypothesis: lattice matrix element can be analyzed within leading-twist NLO framework to a good approximation

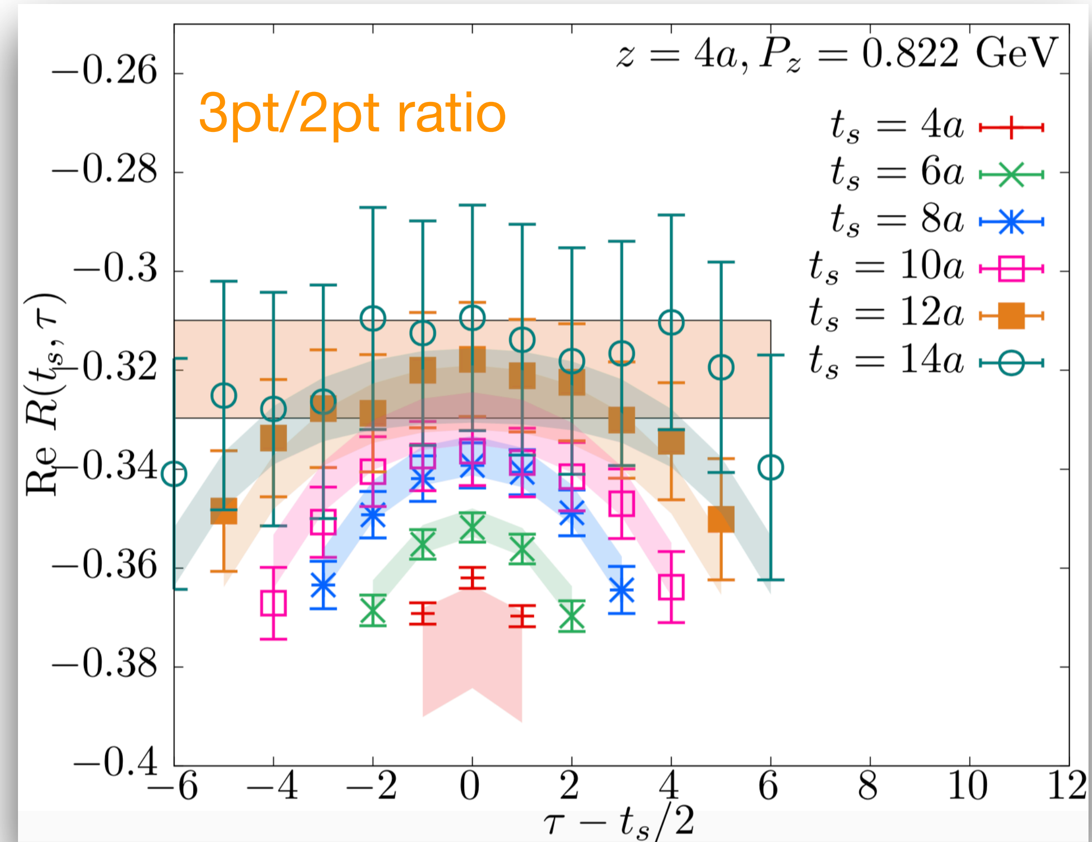
↓

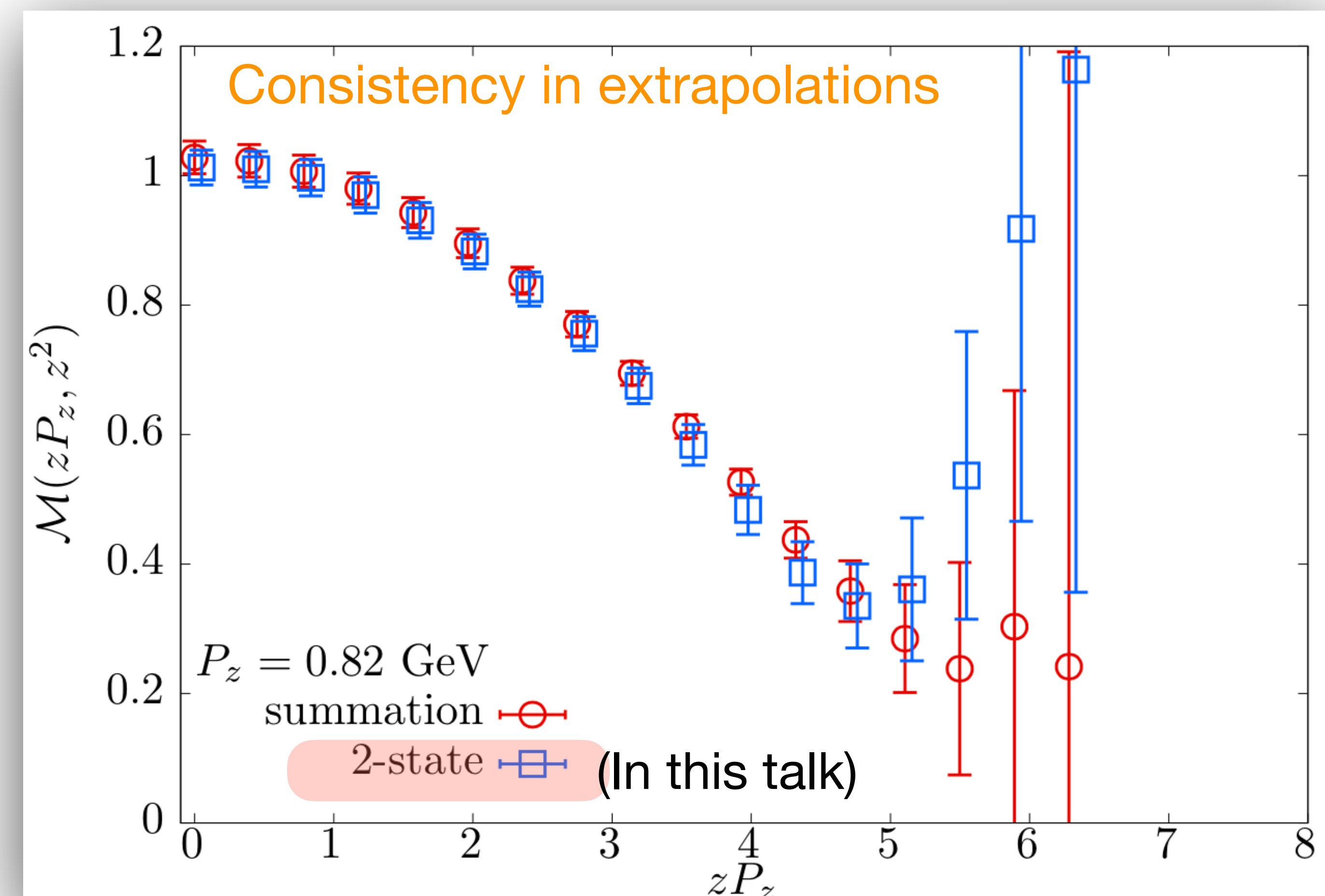
Reconstruct x-dependence, moments etc (Given the hypothesis)

Extraction of matrix elements









Raw lattice data for three-point and two-point functions

Extract the transversity matrix element

Infer corrections to continuum leading-twist NLO just using the lattice data

Hypothesis:
lattice matrix element can be analyzed within leading-twist NLO framework to a good approximation

Reconstruct x-dependent PDF, moments
(Given: the hypothesis, priors, …)

Fixed-z^2 analysis as a diagnostic tool

Fit leading-twist OPE to z_3 P_3 dependence at fixed z_3 → Extract moments Karpie et al '18

→ Analyze effectiveness of LT

Let the lattice data be [leading-twist] + [corrections] :

$$\mathfrak{M}(\nu, z_3^2) = \mathfrak{M}^{\text{twist}-2}(\nu, z_3^2) + \sum_{k,n} \left(L_{k,n} \left(\frac{a}{|z_3|} \right)^k + H_{k,n} \left(\Lambda_{\text{QCD}}^2 z_3^2 \right)^k \right) \frac{(i\nu)^n}{n!}$$

Then, an effective z_3 dependent Mellin moment becomes

$$\langle x^n \rangle_{\text{eff}}(z_3) = \langle x^n \rangle + \frac{1}{C_n(\mu^2 z_3^2)} \sum_k \left(L_{k,n} \left(\frac{a}{|z_3|} \right)^k + H_{k,n} \left(\Lambda_{\text{QCD}}^2 z_3^2 \right)^k \right)$$

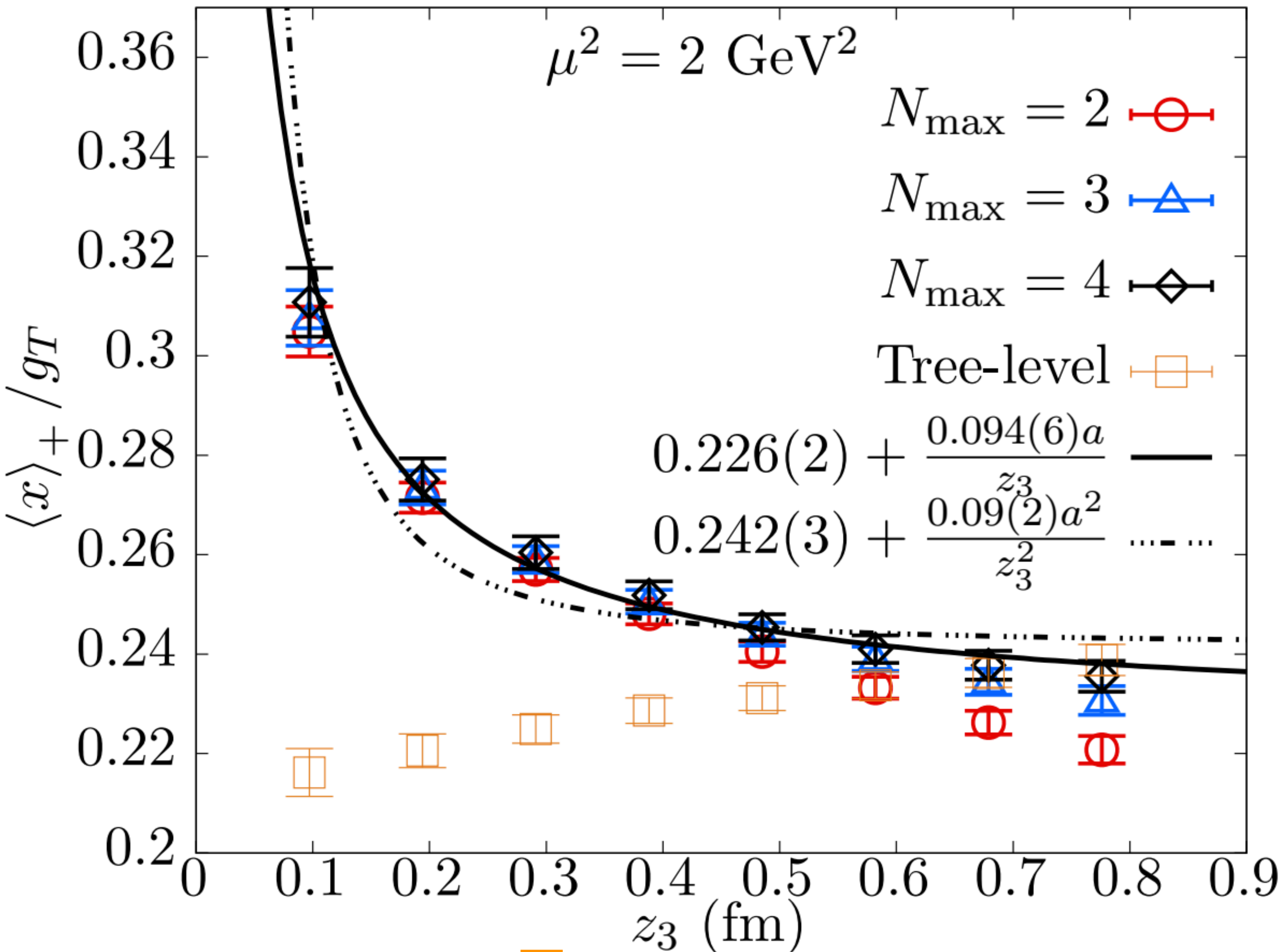
↓

See a plateau in effective Mellin moment as a function of z_3 ?

↓ yes! → Leading-twist description is good

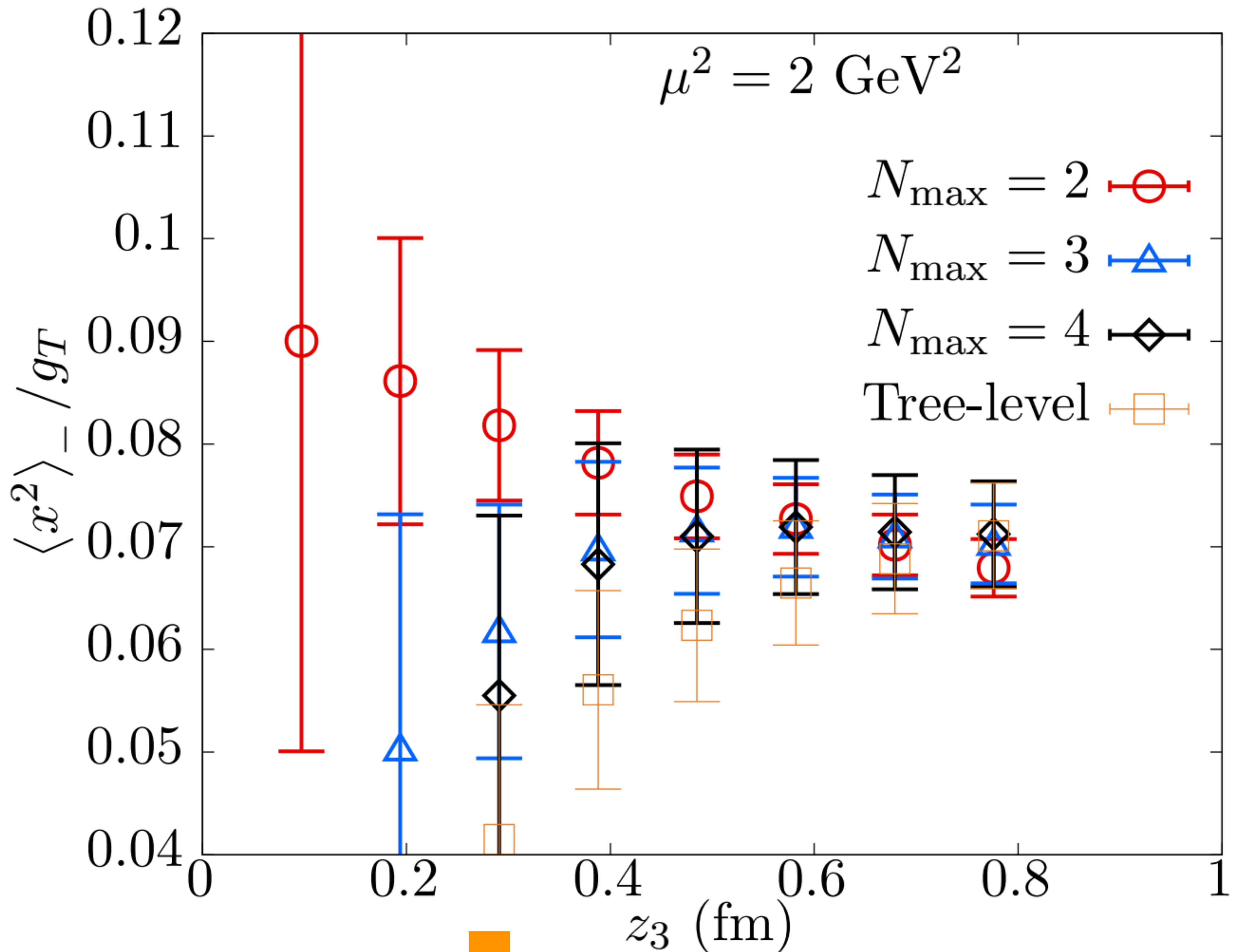
no! → Infer corrections from deviations to plateau

Deducing the corrections to twist-2 OPE : Imaginary part



$$\text{Im}\ \mathfrak{M}^{\text{twist}-2}(z_3 P_3, z_3^2) + L_{1,1}\frac{a}{|z_3|}\nu + H_{1,1}(\Lambda_{\text{QCD}}^2 z_3^2)\nu$$

Deducing the corrections to twist-2 OPE : Real part



$$\text{Re}\ \mathfrak{M}^{\text{twist}-2}(z_3 P_3, z_3^2) + L_{1,2} \frac{a}{|z_3|} \frac{\nu^2}{2} + H_{1,2}(\Lambda_{\text{QCD}}^2 z_3^2) \frac{\nu^2}{2}$$

(no visible evidence in data, “precautionary” correction terms)

Raw lattice data for three-point and two-point functions

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Extract the transversity matrix element

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Infer the corrections to continuum leading-twist NLO just using the lattice data

← Hypothesis: lattice matrix element can be analyzed within leading-twist NLO framework to a good approximation

↓

Reconstruct
x-dependent PDF, moments
(<u>Given:</u> the hypothesis, priors, …)

Methodology of Fits : towards achieving model independence

$$h_1(x) = x^\alpha (1-x)^\beta \mathcal{G}(x)$$

← Some regular slowly varying function

A common choice affecting small-x is a parametrization that empirically works:

$$\mathcal{G}(x) = 1 + \gamma\sqrt{x} + \delta x + \ldots$$

Achieve model independence — expand in complete basis — A good choice is Jacobi Polynomials:

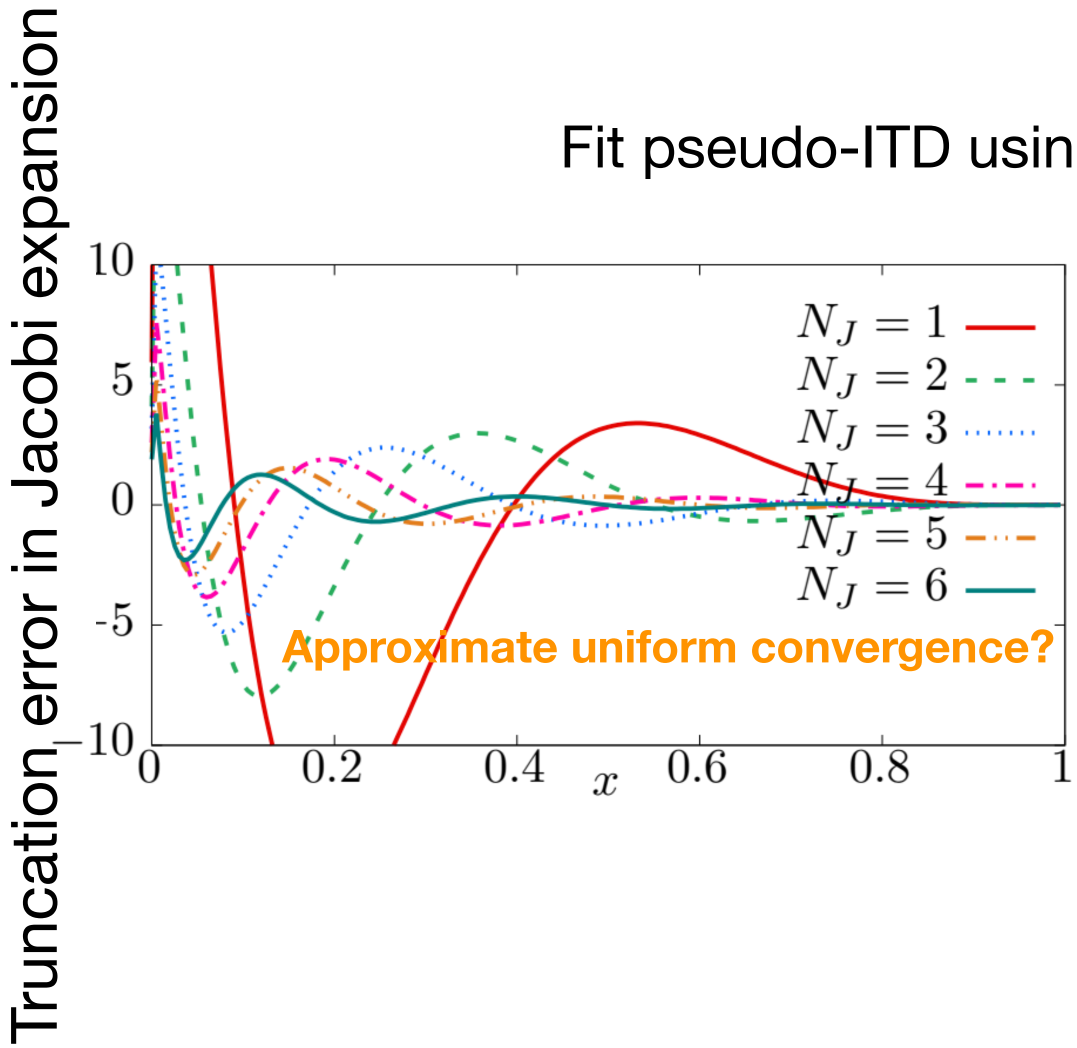
J. Karpie et al, 2105.13313

$$\mathcal{G}(x) = \sum_n^{N_{\rm max}} s_n P_n^{(\alpha,\beta)}(1-2x)$$

← Ortho-normal w.r.t $x^\alpha (1-x)^\beta$

Choosing the family of functions is crucial for convergence: E.g., VarPro method (Joe's talk)

Methodology of Fits : towards achieving model independence



Fit pseudo-ITD using $x^{\alpha}(1-x)^{\beta}(1+\gamma\sqrt{x}+\delta x)$

Expand best fit PDF in $P_n^{(\alpha,\beta)}$

s_n

Feed as prior values of s_n to full Jacobi Polynomial fit

$$\chi^2 \to \chi^2 + \sum_n \frac{(s_n - s_n^{\text{prior}})}{\sigma^2_{\text{prior}}(s_n)}$$

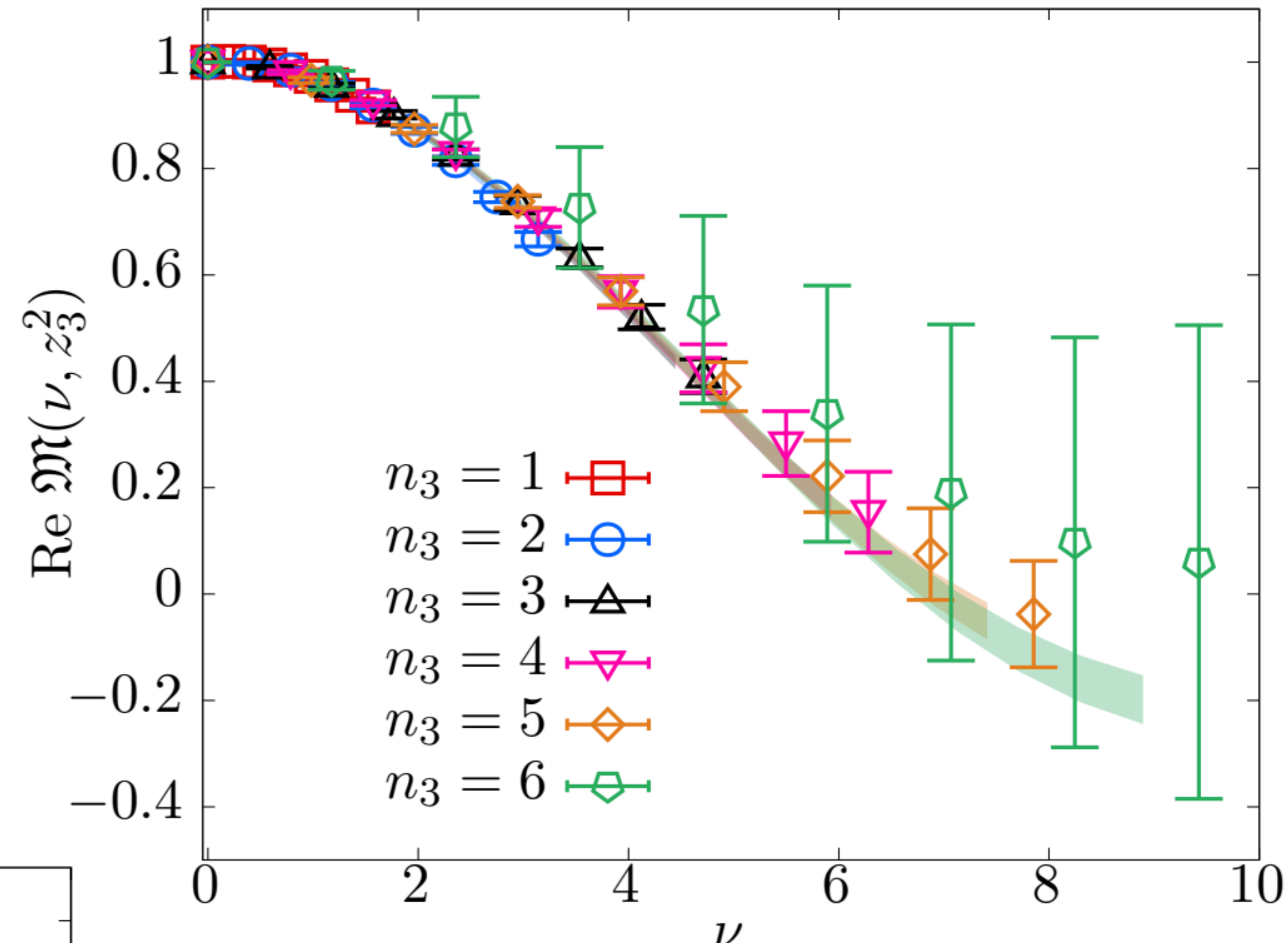
$h_1(x)$

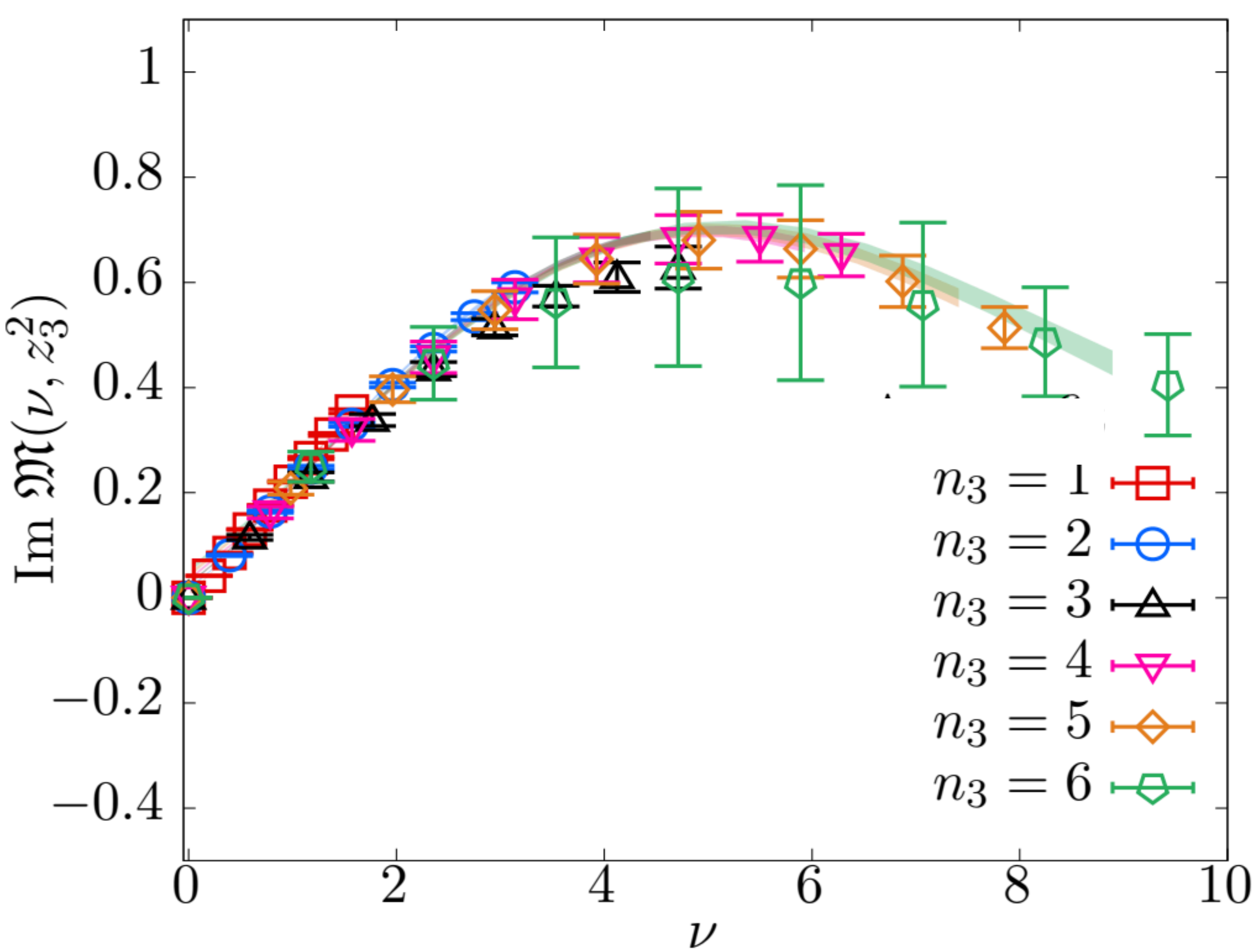
Sample-by-sample AICc average over Polynomial order N_{max} and other fit variations such as z_{min}, z_{max}

$$h_1(x) \pm \text{ stat. error } \pm \text{ syst. error}$$

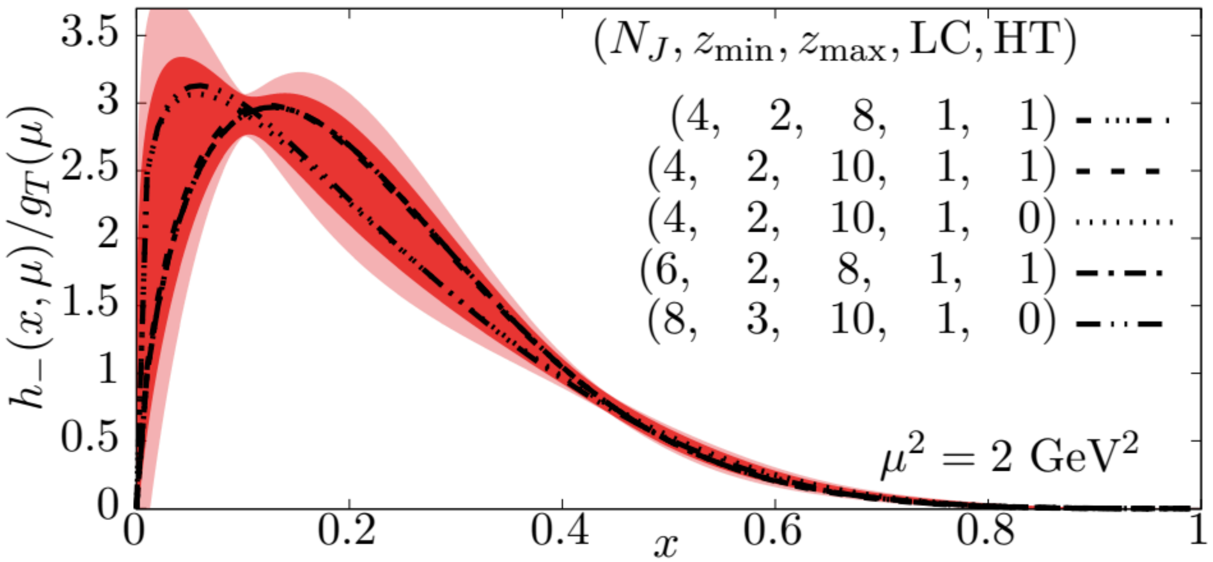
- Twist-2 OPE (+corrections) fit over a range

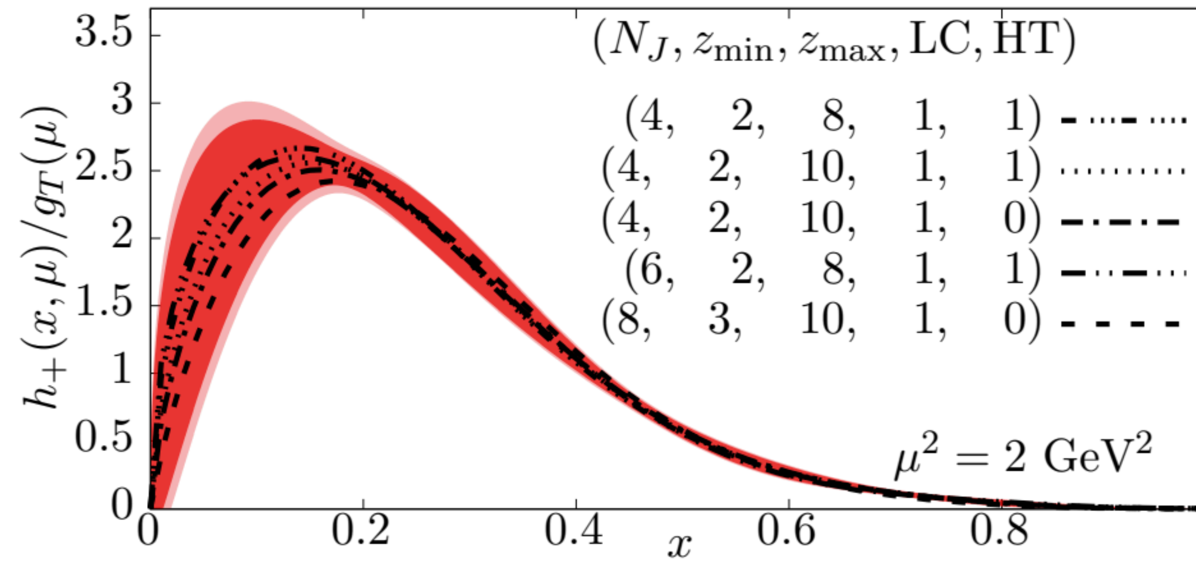
$z \in [2a, 0.56\ \text{fm}] \quad P_z \in [0.41, 2.47]\ \text{GeV}$



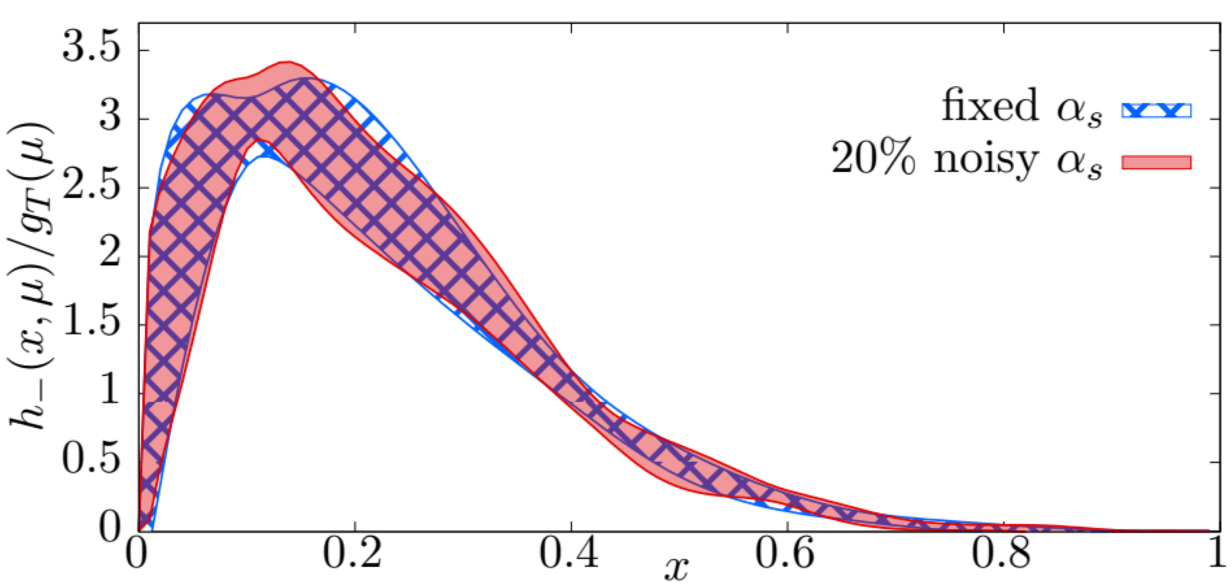


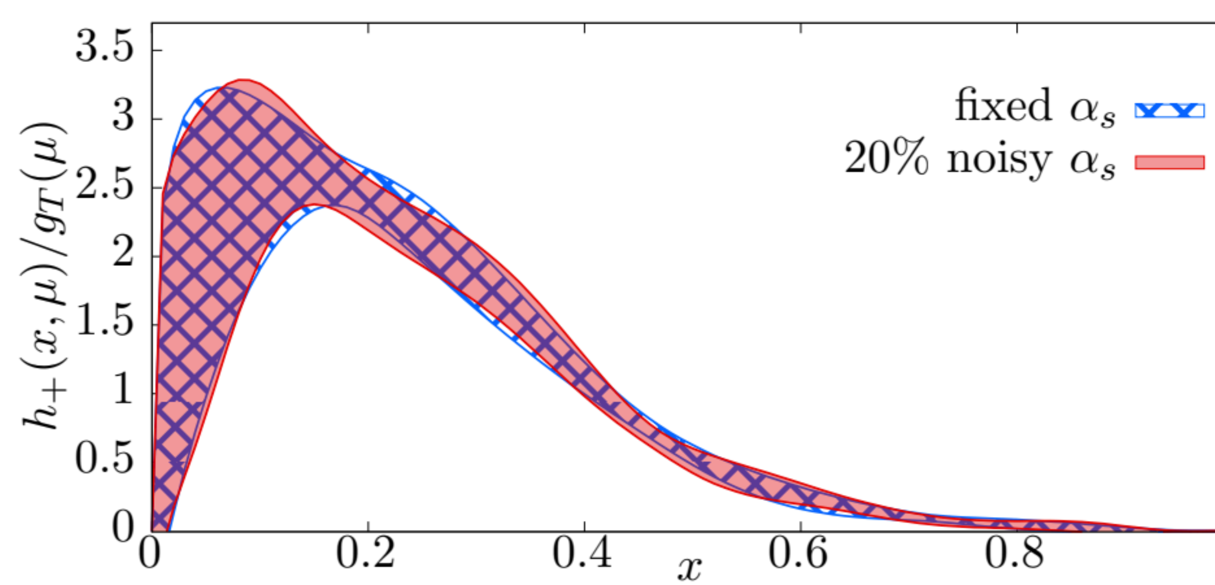
- **Systematic Error?** vary fit ranges, Jacobi polynomial order, H.T. corrections



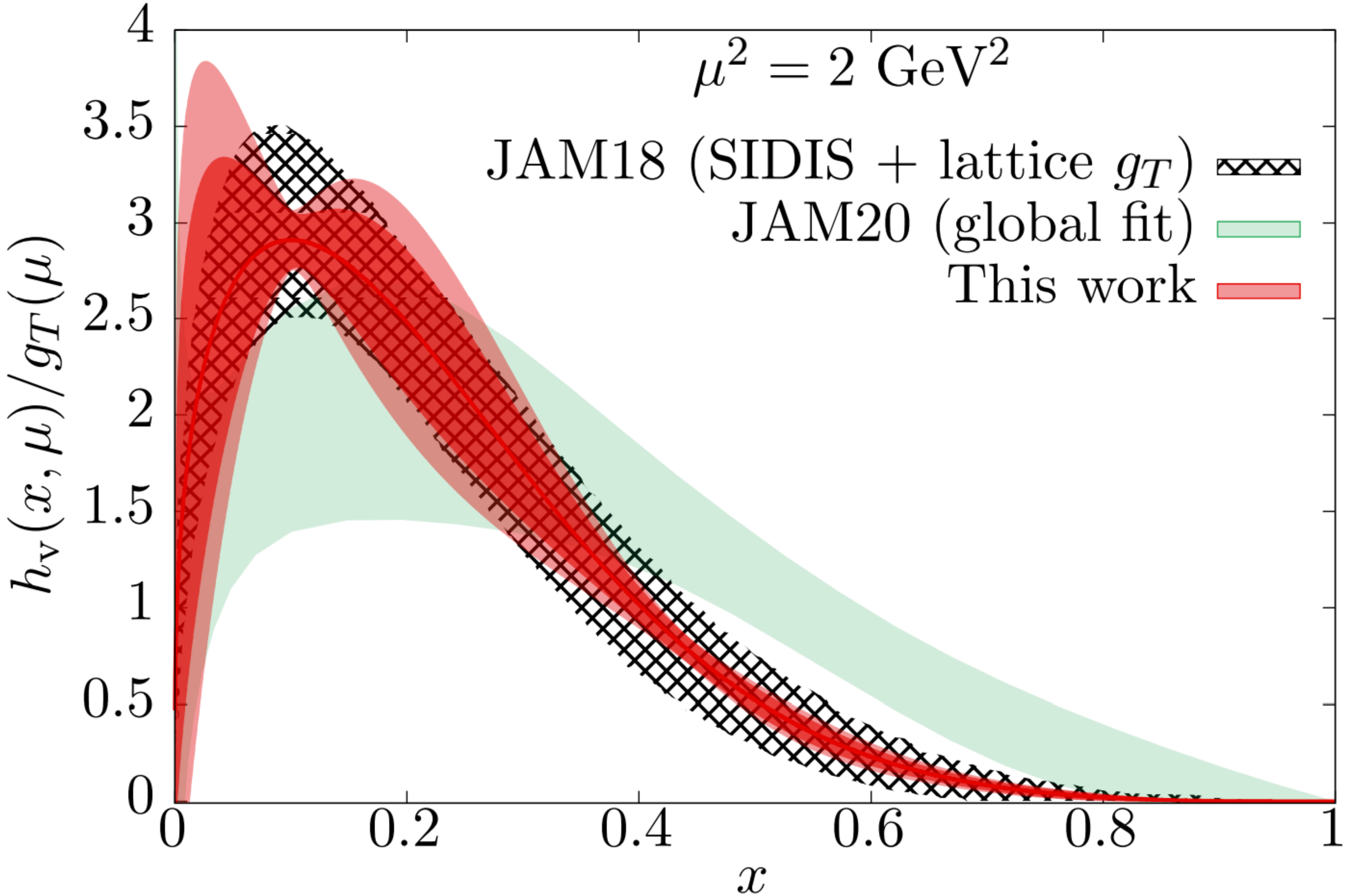


- **Higher-loop?** Add 20% Gaussian noise to α_s as a crude diagnostic

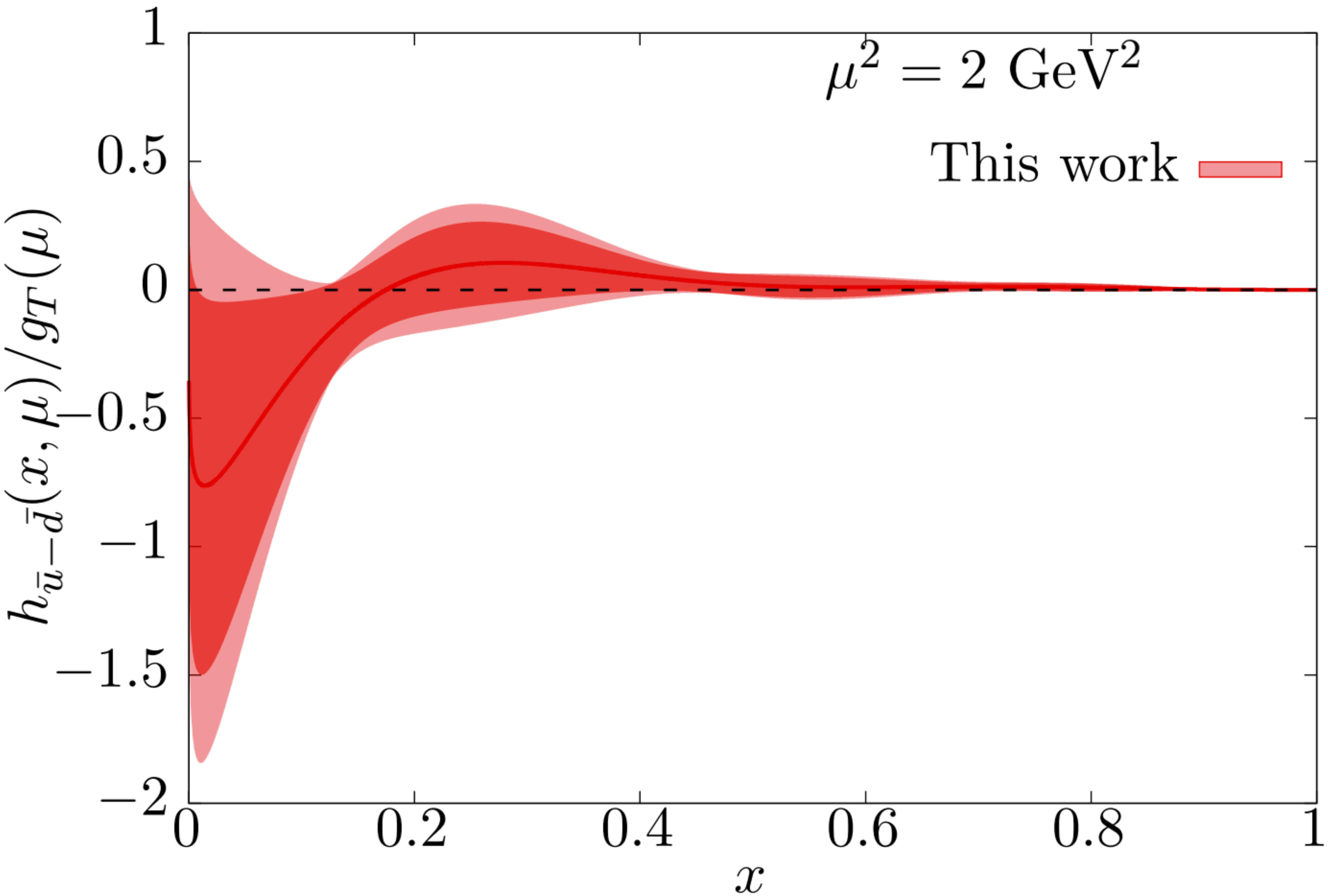




Comparison with JAM PDF determinations



Isospin symmetric intrinsic transversity sea



Summary

Raw lattice data for three-point and two-point functions

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Extracted the transversity matrix element: robust with fitting methods

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Inferred short-distance lattice corrections. HT effects could not be inferred this way.

← Hypothesis which works: lattice matrix element can be analyzed within leading-twist NLO framework to a good approximation

↓

Reconstruct x-dependent PDF, moments (Given: the hypothesis, reduced prior on model assumption)

→ Better agreement with JAM18 (lattice+SIDIS)

Can we learn about process dependence in SSA SIDIS data?