“Exploring Neutrino Masses and Mixing in the Seesaw Model with $L_e - L_\tau$ Gauged Symmetry"

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Abstract

1. We have studied neutrino phenomenology with the extension of Standard Model via $U(1)_{L_e-L_\tau}$ gauged symmetry in the framework of type-(I+II) seesaw mechanism.
2. We have added three right handed neutrinos, one singlet and one scalar triplet to study neutrino oscillation, lepton flavor violation and neutrinoless double beta decay m_{ee}.
3. Upper limits of branching ratio for $\tau \to e\gamma, \tau \to \mu\bar{\mu}\mu$ lepton flavor violations are well explained by our model.

Model description

	Particles	$SU(2)_L \times U(1)_Y$	$U(1)_{L_e-L_\tau}$
Fermions	$l_{eL}, l_{\mu L}, l_{\tau L}$	$(\mathbf{2}, -1)$	$1, 0, -1$
	e_R, μ_R, τ_R	$(\mathbf{1}, -2)$	$1, 0, -1$
	$\nu_{eR}, \nu_{\mu R}, \nu_{\tau R}$	$(\mathbf{1}, 0)$	$1, 0, -1$
Scalars	H	$(\mathbf{2}, 1)$	0
	S	$(\mathbf{1}, 0)$	1
	Δ	$(\mathbf{3}, -2)$	-1

Table 1: Particle contents in $U(1)_{L_e-L_\tau}$ model.

The scalar potential for the model is:

$$\begin{aligned} V(S,H,\Delta) = & -\mu_H^2 H^\dagger H + \lambda_H (H^\dagger H)^2 - \mu_S^2 S^\dagger S + \lambda_S (S^\dagger S)^2 + \frac{1}{2} M_\Delta^2 \mathrm{Tr}(\Delta^\dagger \Delta) \\ & + (\zeta_1 S H^{\mathrm{T}} \mathrm{i}\sigma_2 \Delta H + \mathrm{h.c.}) + \frac{\zeta_2}{2}(H^\dagger H)\mathrm{Tr}(\Delta^\dagger \Delta) \\ & + \frac{\zeta_3}{4}(H^\dagger \sigma_i H)\mathrm{Tr}(\Delta^\dagger \sigma_i \Delta) + \frac{\zeta_4}{2}(S^\dagger S)\mathrm{Tr}(\Delta^\dagger \Delta) + \frac{\zeta_5}{2}(S^\dagger S)(H^\dagger H) \\ & + \frac{\zeta_6}{4}\mathrm{Tr}\left[(\Delta^\dagger \Delta)^2\right] + \frac{\zeta_7}{4}\left[\mathrm{Tr}(\Delta^\dagger \Delta)\right]^2 , \end{aligned} \quad (1)$$

The Lagrangian for leptonic part for our model can be written as,

$$\begin{aligned} \mathcal{L}_{\text{lepton}} = & -y_\alpha^l \overline{\ell_{\alpha \mathrm{L}}} \alpha_{\mathrm{R}} H - \frac{1}{2} y_\Delta \left(\overline{\ell_{\mu \mathrm{L}}} \Delta \mathrm{i}\sigma_2 \ell_{\tau \mathrm{L}}^{\mathrm{C}} + \overline{\ell_{\tau \mathrm{L}}} \Delta \mathrm{i}\sigma_2 \ell_{\mu \mathrm{L}}^{\mathrm{C}}\right) - y_\nu^\nu \overline{\ell_{\alpha \mathrm{L}}} \tilde{H} \nu_{\alpha \mathrm{R}} \\ & - \frac{1}{2} y_S^{\mu\tau} \left(\overline{\nu_{\mu \mathrm{R}}^{\mathrm{C}}} \nu_{\tau \mathrm{R}} + \overline{\nu_{\tau \mathrm{R}}^{\mathrm{C}}} \nu_{\mu \mathrm{R}}\right) S - \frac{1}{2} y_S^{e\mu} \left(\overline{\nu_{e \mathrm{R}}^{\mathrm{C}}} \nu_{\mu \mathrm{R}} + \overline{\nu_{\mu \mathrm{R}}^{\mathrm{C}}} \nu_{e \mathrm{R}}\right) S^\dagger \\ & - \frac{1}{2}\left[m_{\mathrm{R}}^{\mu\mu} \overline{\nu_{\mu \mathrm{R}}^{\mathrm{C}}} \nu_{\mu \mathrm{R}} + m_{\mathrm{R}}^{e\tau} \left(\overline{\nu_{e \mathrm{R}}^{\mathrm{C}}} \nu_{\tau \mathrm{R}} + \overline{\nu_{\tau \mathrm{R}}^{\mathrm{C}}} \nu_{e \mathrm{R}}\right)\right] + \mathrm{h.c.} \, . \end{aligned} \quad (2)$$

Active Neutrino Masses

After sponteneous symmetry breaking, H, Δ, and S will have vev as $< H^0 >= v_H/\sqrt{2}, < \Delta^0 >= v_\Delta, < S >= v_S/\sqrt{2}$ respectively.
From the Lagrangian, mass matices can be written as [1, 2] :

$$M_l = \frac{v_H}{\sqrt{2}} \begin{pmatrix} y_l^e & 0 & 0 \\ 0 & y_l^\mu & 0 \\ 0 & 0 & y_l^\tau \end{pmatrix} , \quad (3)$$

$$M_{\mathrm{D}} = \frac{v_H}{\sqrt{2}} \begin{pmatrix} y_\nu^e & 0 & 0 \\ 0 & y_\nu^\mu & 0 \\ 0 & 0 & y_\nu^\tau \end{pmatrix} , \quad M_{\mathrm{L}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & y_\Delta v_\Delta \\ 0 & y_\Delta v_\Delta & 0 \end{pmatrix} , \quad (4)$$

,

$$M_{\mathrm{R}} = \begin{pmatrix} 0 & |y_S^{e\mu}| \frac{v_S}{\sqrt{2}} e^{i\phi} & m_R^{e\tau} \\ |y_S^{e\mu}| \frac{v_S}{\sqrt{2}} e^{i\phi} & m_R^{\mu\mu} & y_s^{\mu\tau} \frac{v_s}{\sqrt{2}} \\ m_R^{e\tau} & y_s^{\mu\tau} \frac{v_s}{\sqrt{2}} & 0 \end{pmatrix} . \quad (5)$$

where M_R is the mass matrix of right-handed neutrinos.
The active neutrino mass matrix can be obtained by taking type-(I+II) seesaw mechanism and is a function of M_D, M_R, M_L as:-

$$M_\nu = M_L - M_D M_R^{-1} M_D^T \quad (6)$$

Putting the expressions of M_R, M_D, M_L, and taking the assumption $|y_S^{e\mu}| >> |y_S^{\mu\tau}|$, the form of M_ν will be,

$$M_\nu = \begin{pmatrix} 0 & 0 & -\frac{v_H^2 y_\nu^e y_\nu^\tau}{2 m_R^{e\tau}} \\ 0 & \frac{v_H^2 (y_\nu^\mu)^2}{2 m_R^{\mu\mu}} & y_\Delta v_\Delta + \frac{v_H^2 |y_S^{e\mu}| v_S y_\nu^\mu y_\nu^\tau}{2\sqrt{2} m_R^{e\tau} m_R^{\mu\mu}} e^{i\phi} \\ -\frac{v_H^2 y_\nu^e y_\nu^\tau}{2 m_R^{e\tau}} & y_\Delta v_\Delta + \frac{v_H^2 |y_S^{e\mu}| v_S y_\nu^\mu y_\nu^\tau}{2\sqrt{2} m_R^{e\tau} m_R^{\mu\mu}} e^{i\phi} & -\frac{v_S^2}{2} \frac{|y_S^{e\mu}|^2 v_H^2 (y_\nu^\tau)^2}{(m_R^{e\tau})^2 m_R^{\mu\mu}} e^{2\mathrm{i}\phi} \end{pmatrix} . \quad (7)$$

which is a two-zero A_1 texture.

Neutrino flavor mixing

Right handed neutrinos and $m_R^{\mu\mu}, |y_S^{e\mu}| v_S, |y_S^{\mu\tau}| v_S, m_R^{e\tau}$ have the relation [3]:

$$m_{\mathrm{R}}^{\mu\mu} = M_3 (1 - y_{\mathrm{R}} + x_{\mathrm{R}} y_{\mathrm{R}}) , \quad (8)$$

$$\frac{|y_S^{e\mu}| v_S}{\sqrt{2}} = M_3 \left[\frac{y_{\mathrm{R}}(1 - x_{\mathrm{R}})(1 - y_{\mathrm{R}})(1 + x_{\mathrm{R}} y_{\mathrm{R}})}{1 - y_{\mathrm{R}} + x_{\mathrm{R}} y_{\mathrm{R}}}\right]^{1/2} , \quad (9)$$

$$m_{\mathrm{R}}^{e\tau} = M_3 \left(\frac{x_{\mathrm{R}} y_{\mathrm{R}}^2}{1 - y_{\mathrm{R}} + x_{\mathrm{R}} y_{\mathrm{R}}}\right)^{1/2} , \quad (10)$$

where $x_r = \frac{M_1}{M_2}, y_R = \frac{M_2}{M_3}$ with range [0, 1).
With the help of the above equations, the flavor unitary matrix U for right-handed neutrinos can be expressed as:

$$\begin{aligned} U_{e1} &= e^{\mathrm{i}\phi} \left[\frac{x_{\mathrm{R}}(1 - y_{\mathrm{R}})}{(1 + x_{\mathrm{R}})(1 - x_{\mathrm{R}} y_{\mathrm{R}})}\right]^{1/2} , \\ U_{e2} &= \mathrm{i} e^{\mathrm{i}\phi} \left[\frac{1 + x_{\mathrm{R}} y_{\mathrm{R}}}{(1 + x_{\mathrm{R}})(1 + y_{\mathrm{R}})}\right]^{1/2} , \\ U_{e3} &= e^{\mathrm{i}\phi} \left[\frac{y_{\mathrm{R}}(1 - x_{\mathrm{R}})}{(1 - x_{\mathrm{R}} y_{\mathrm{R}})(1 + y_{\mathrm{R}})}\right]^{1/2} , \\ U_{\mu1} &= -\left[\frac{x_{\mathrm{R}} y_{\mathrm{R}}(1 - x_{\mathrm{R}})(1 + x_{\mathrm{R}} y_{\mathrm{R}})}{(1 + x_{\mathrm{R}})(1 - x_{\mathrm{R}} y_{\mathrm{R}})(1 - y_{\mathrm{R}} + x_{\mathrm{R}} y_{\mathrm{R}})}\right]^{1/2} , \\ U_{\mu2} &= -\mathrm{i} \left[\frac{y_{\mathrm{R}}(1 - x_{\mathrm{R}})(1 - y_{\mathrm{R}})}{(1 + x_{\mathrm{R}})(1 + y_{\mathrm{R}})(1 - y_{\mathrm{R}} + x_{\mathrm{R}} y_{\mathrm{R}})}\right]^{1/2} , \\ U_{\mu3} &= \left[\frac{(1 - y_{\mathrm{R}})(1 + x_{\mathrm{R}} y_{\mathrm{R}})}{(1 - x_{\mathrm{R}} y_{\mathrm{R}})(1 + y_{\mathrm{R}})(1 - y_{\mathrm{R}} + x_{\mathrm{R}} y_{\mathrm{R}})}\right]^{1/2} , \\ U_{\tau1} &= e^{-\mathrm{i}\phi} \left[\frac{1 - y_{\mathrm{R}}}{(1 + x_{\mathrm{R}})(1 - x_{\mathrm{R}} y_{\mathrm{R}})(1 - y_{\mathrm{R}} + x_{\mathrm{R}} y_{\mathrm{R}})}\right]^{1/2} , \\ U_{\tau2} &= -\mathrm{i} e^{-\mathrm{i}\phi} \left[\frac{x_{\mathrm{R}}(1 + x_{\mathrm{R}} y_{\mathrm{R}})}{(1 + x_{\mathrm{R}})(1 + y_{\mathrm{R}})(1 - y_{\mathrm{R}} + x_{\mathrm{R}} y_{\mathrm{R}})}\right]^{1/2} , \\ U_{\tau3} &= e^{-\mathrm{i}\phi} \left[\frac{x_{\mathrm{R}} y_{\mathrm{R}}^3 (1 - x_{\mathrm{R}})}{(1 - x_{\mathrm{R}} y_{\mathrm{R}})(1 + y_{\mathrm{R}})(1 - y_{\mathrm{R}} + x_{\mathrm{R}} y_{\mathrm{R}})}\right]^{1/2} , \end{aligned} \quad (11)$$

We have taken experimental data of oscillation parameter from NuFit [4] at 3σ interval. Oscillation data and model parameters are listed as:

$$\begin{aligned} &\Delta m^2_{\text{atm}} = [2.47, 2.63] \times 10^{-3}\ \text{eV}^2, \ \Delta m^2_{\text{sol}} = [6.94, 8.14] \times 10^{-5}\ \text{eV}^2, \\ &\sin^2\theta_{13} = [0.0200, 0.02405], \ \sin^2\theta_{23} = [0.434, 0.610], \ \sin^2\theta_{12} = [0.271, 0.369] \, . \end{aligned} \quad (12)$$

$$\{y_\nu^e, y_\nu^\mu, y_\nu^\tau\} \in \ [10^{-6}, 10^{-7}], \quad y_\Delta v_\Delta \in [0.01, 0.1] \quad (13)$$

And for sum of active neutrino mass Σm_i, we have taken range $\Sigma m_i < 0.12\ eV$ from Planck cosmological data [5].

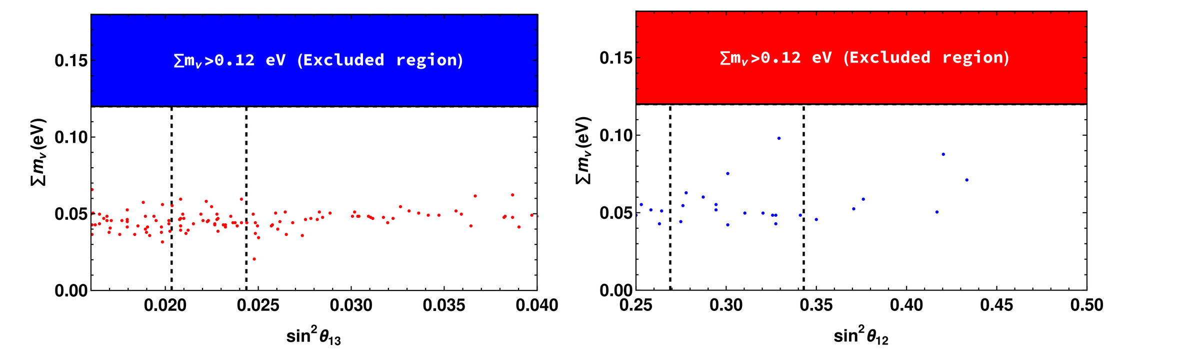

Figure 1: Left(right) panel shows the variation of $sin^2\theta_{13}(sin^2\theta_{12})$ with respect to sum of active neutrino masses (Σm_ν), the blue(red) shaded region indicates excluded portion of Σm_ν.

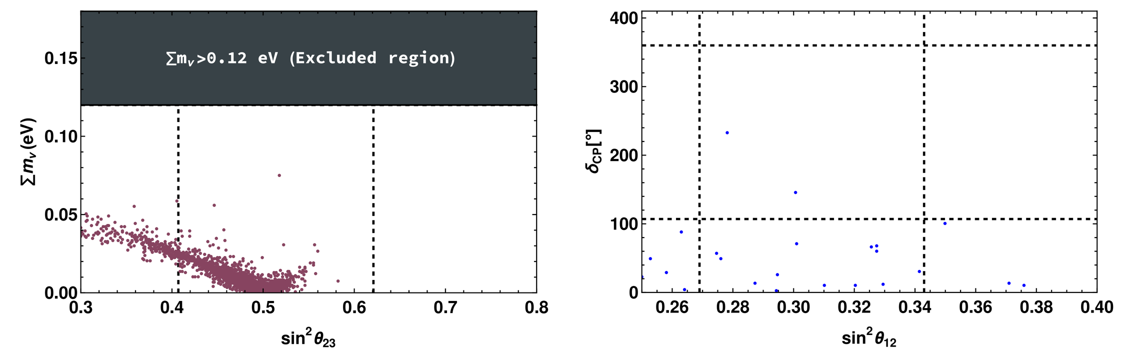

Figure 2: Left panel shows the variation of $sin^2\theta_{23}$ with respect to the sum of active neutrino masses Σm_ν where magenta shade indicates excluded region for Σm_ν, right panel shows the variation of $sin^2\theta_{12}$ with respect to δ_{CP}.

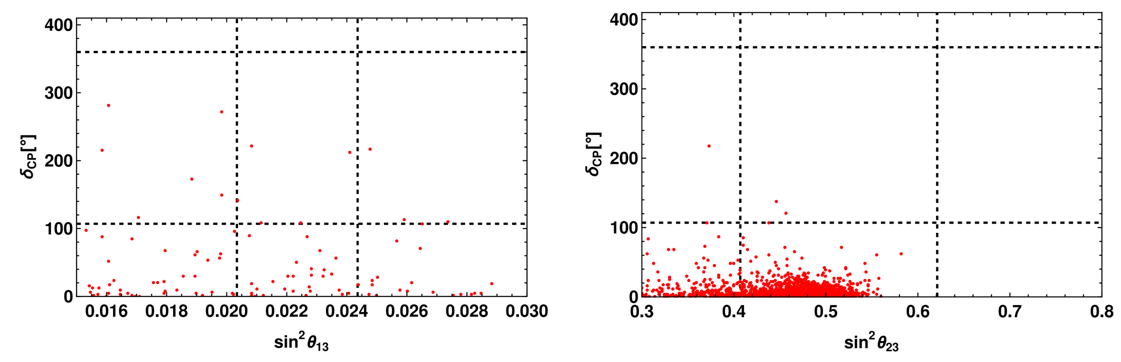

Figure 3: Left(right) panel shows the variation of $sin^2\theta_{13}(sin^2\theta_{23})$ with respect to δ_{CP}.

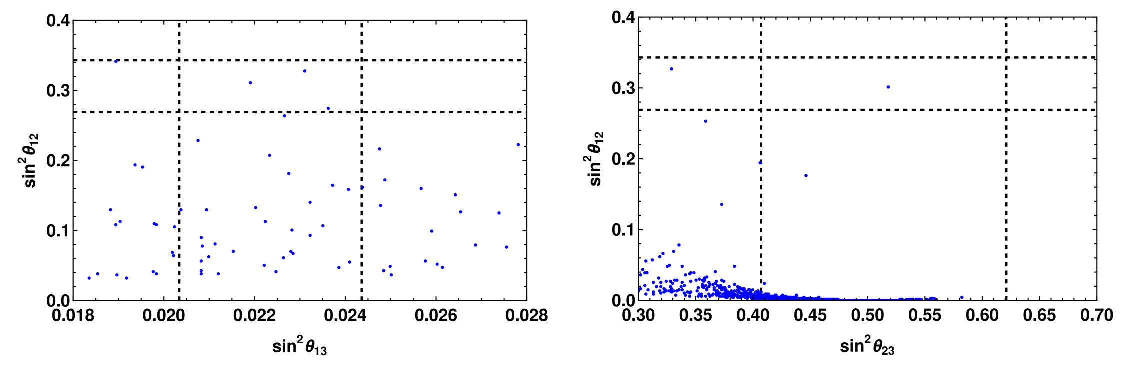

Figure 4: Left(right) panel shows the variation of $sin^2\theta_{12}$ with respect to $sin^2\theta_{13}$ ($sin^2\theta_{12}$ with respect to $sin^2\theta_{23}$).

Lepton flavor violation

One of the conclusions of Standard Model is that, lepton numbers in any nuclear reaction will be conserved independently and separately. But recent experiments shows some of the lepton flavor violation reactions with some upper limit uncertainities. Our $L_e - L_\tau$ model have a sizable contribution in that section also. Our model have succeded to show the acceptable upper limit of branching ratio to the lepton flavor violation equations like: $\tau \to e\gamma, \tau \to \mu\bar{\mu}\mu$.

Implication on LFV

The braching ratios for the lepton flavor violating decay modes:- $\tau \to e\gamma, \tau \to \mu\bar{\mu}\mu$ are [6], [7]

$$\mathrm{Br}(\tau \to e\gamma) = \frac{27\alpha_{em} |\langle m^2 \rangle_{e\tau}|^2}{256\pi G_F^2 v_\Delta^4 M_\Delta^4} < 5.6 \times 10^{-8} , \quad (14)$$

$$\mathrm{Br}(\tau \to \mu\bar{\mu}\mu) = \frac{|m_{\tau\mu}|^2 |m_{\mu\mu}|^2}{16 G_F^2 v_\Delta^4 M_\Delta^4} < 3.2 \times 10^{-8} , \quad (15)$$

where α_{em} is the fine stucture constant=$\frac{1}{137}$ and $G_F = 1.17 \times 10^{-5}\ GeV^2$ =Fermi coupling.
In the equations, branching ratios are the function of $m_{e\tau}, m_{\mu\mu}, m_{\mu\tau}$ respectively.
Following figures show the dependency of branching ratio on $m_{e\tau}$ and $m_{\mu\mu}, m_{\mu\tau}$.

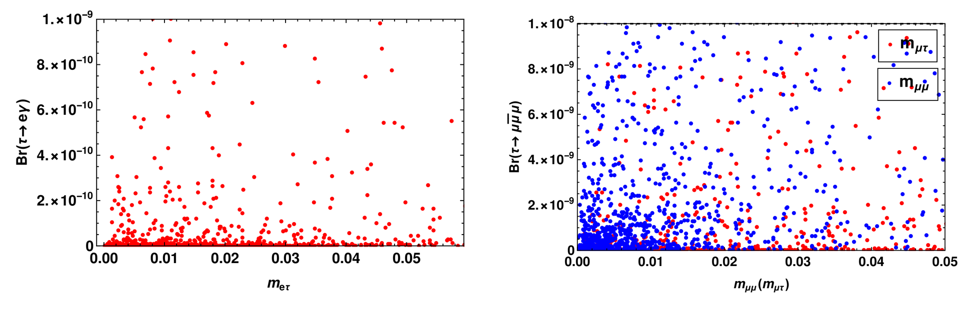

Figure 5: Left(right) panel shows the correlation between the branching ratio of $\tau \to e\gamma$ ($\tau \to \mu\bar{\mu}\mu$) and the mass element $m_{e\tau}$ ($m_{\mu\mu}$ and $m_{\mu\tau}$).

$0\nu\beta\beta$ decay mass

One of the most important decay for confirming the nature of neutrino (is it Dirac or Majorana) is $0\nu\beta\beta$ decay. In the decay, two neutrons are going to decay into protons and two electrons (without emiting any neutrino):

$$2n \to 2p + 2e^- \quad (16)$$

The existence of $0\nu\beta\beta$ deacy will confirm the Majorana nature of neutrino as for that, neutrino is its own antiparticle.
The expression for $0\nu\beta\beta$ decay mass is:

$$m_{ee} = |m_1 |U_{e1}|^2 + m_2 |U_{e2}|^2 e^{i\alpha_{21}} + m_3 |U_{e3}|^2 e^{i(-2\delta_{CP} + \alpha_{31})}| \quad (17)$$

where $m_i, i = 1, 2, 3$ are the mass of active neutrinos and U_{ij} are the flavor mixing matrix elements for active neutrino. Our model has a consistent upper limit of m_{ee} with experimental data.

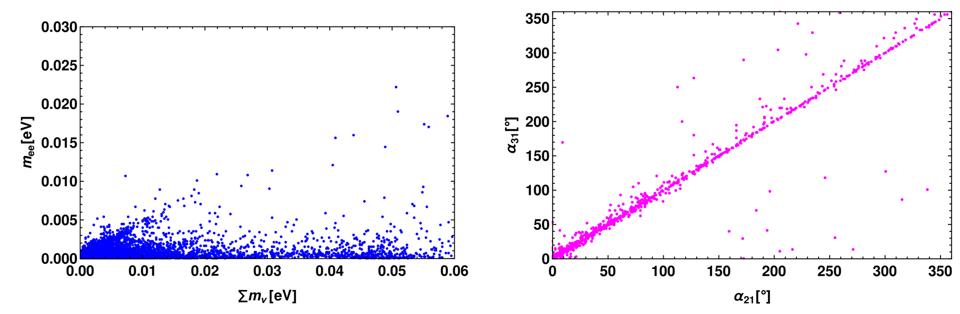

Figure 6: Left panel signify the correlation between the effective neutrinoless double beta decay mass parameter $\langle m_{ee} \rangle$ w.r.t sum of active neutrino mass Σm_i whereas right panel shows the correlation between the Majorana phases i.e.α_{21} and α_{31}.

N.B.-All the experimental data of neutrino parameters are taken from NuFIT current experimental table (July 2020).

Conclusion

- We have established the neutrino model with $e - \tau$ gauged symmetry which can explain neutrino flavor mixing phenomenology.
- By taking three right-handed neutrinos, one scalar singlet and one scalar triplet in our model, we have shown the consistent upper limit of branching ratios for lepton flavor violating reactions: $\tau \to e\gamma, \tau \to \mu\bar{\mu}\mu$.
- The mass of $0\nu\beta\beta$ decay is found to be within experimental limit.

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Figure 7: QR code for actual paper of the presentation