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Non-zero θ_{13} and δ_{CP} phase with A_4 Flavor Symmetry and Deviations to Tri-Bi-Maximal mixing via $z_2 \times z_2$ invariant perturbation in the Neutrino sector.

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Presented by:
Gayatri Ghosh*

Department of Physics, Gauhati University, Jalukbari, Assam-781015,
Department of Physics, Pandit Deendayal Upadhayay Mahavidyalaya,
Assam University, Assam-788720, India
[*gayatrighsh@gmail.com](mailto:gayatrighsh@gmail.com)

Motivation and Introduction

- In this work we propose a A_4 family symmetry - the symmetry group of even permutations of 4 objects or equivalently that of a tetrahedron, which is used here to obtain neutrino mixing predictions within fundamental theories of neutrino mass.

- Persuaded by the prerequisite for departing from the simplest first-order form for the TBM ansatz,

$$U_{PMNS} \simeq \begin{bmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix} = U_{TBM}$$

➢ ***We propose here a generalized version of the TBM ansatz in which the new ansatz is realised in a model based on A_4 group as suggested in literaure by breaking A_4 symmetry spontaneously to Z_2 in the neutrino sector, which correctly accounts for the non-zero value of θ_{13} and introduces CP violation.***

➢*We then incorporate a real $Z_2 \times Z_2$ perturbations in the neutrino sector leading to feasible values of θ_{13} and δ_{CP} . This results in predictions of neutrino oscillation parameters and leptonic CPV phase that will be tested at upcoming neutrino experiments.*

THEORETICAL FRAMEWORK

Model: Type I SeeSaw model based on A_4 symmetry

- ***Let us limit ourselves to only leptonic sector. The field consists of three left handed $SU(2)_L$ gauge doublets, three right handed charged gauge singlets, three right handed neutrino gauge singlets. In addition there exists also four Higgs doublets φ_i (i = 1; 2; 3) and φ_0 and three scalar singlets.***

The Yukawa Lagrangian of the leptonic fields of the model $G_{SM} \times A_4 \times U(1)_X$ is

$$L = L_{Charged\ leptons\ Dirac} + L_{Neutrino\ Dirac} + L_{Neutrino\ Majorana}$$

- *where $G_{SM} \rightarrow$ the standard model gauge symmetry, $G_{SM} = U(1)_Y \times SU(2)_L \times SU(3)_C$.*

THEORETICAL FRAMEWORK

Fields	$SU(2)_L$	$U(1)_Y$	A_4	Representation
Left Handed Doublets	$\frac{1}{2}$	$Y=-1$	$\underline{3}$	Y_{iL}
Right Handed Charged Lepton Singlets	0	$Y=-2$	$\underline{1}\oplus\underline{1}^{'}\oplus 1^{''}$	l_{iR}
Right Handed Neutrino Singlets	0	$Y=0$	$\underline{3}$	ν_{iR}
Higgs Doublet	$\frac{1}{2}$	$Y=1$	$\underline{3}$	ϕ_i
Higgs Doublet	$\frac{1}{2}$	$Y=1$	$\underline{1}$	ϕ_0
Real Gauge Singlet	0	$Y=0$	$\underline{3}$	F_i

Table I: Allocations under various irreducible representations of $SU(2)_L$, $U(1)_Y$ and A_4.

$$L_{Charged\ leptons\ Dirac} = -\left[h_1\left(\bar{Y_{1L}}\phi_1\right)l_{1R} + h_1\left(\bar{Y_{2L}}\phi_2\right)l_{1R} + h_1\left(\bar{Y_{3L}}\phi_3\right)l_{1R} + h_2\left(\bar{Y_{1L}}\phi_1\right)l_{2R}\right.$$
$$+\omega^2\left\{h_2\left(\bar{Y_{2L}}\phi_2\right)l_{2R}\right\} + \omega\left\{h_2\left(\bar{Y_{3L}}\phi_3\right)l_{2R}\right\} + h_3\left(\bar{Y_{1L}}\phi_1\right)l_{3R}$$
$$\left.+\omega\left\{h_3\left(\bar{Y_{2L}}\phi_2\right)l_{3R}\right\} + \omega^2\left\{h_3\left(\bar{Y_{3L}}\phi_3\right)l_{3R}\right\}\right] + hc$$

$$L_{Neutrino\ Dirac} = -h_0\left(\bar{Y_{1L}}\nu_{1R}\right)\bar{\phi_0} - h_0\left(\bar{Y_{2L}}\nu_{2R}\right)\bar{\phi_0} - h_0\left(\bar{Y_{3L}}\nu_{2R}\right)\bar{\phi_0} + h.c$$

$$L_{Neutrino\ Majorana} = -\frac{1}{2}[\{M\nu_{1R}^T C^{-1}\nu_{1R} + M\nu_{2R}^T C^{-1}\nu_{2R} + M\nu_{3R}^T C^{-1}\nu_{3R}\} + h.c + h_s F_1 \nu_{2R}^T C^{-1}\nu_{3R}$$
$$+ h_s F_1 \nu_{3R}^T C^{-1}\nu_{2R} + h_s F_2 \nu_{3R}^T C^{-1}\nu_{1R}$$
$$+ h_s F_2 \nu_{1R}^T C^{-1}\nu_{3R} + h_s F_3 \nu_{1R}^T C^{-1}\nu_{2R} + h_s F_3 \nu_{2R}^T C^{-1}\nu_{1R}]$$

PERTURBATIONS IN NEUTRINO SECTOR

- The general $Z_2 \times Z_2$ invariant perturbation is of the form;

$$h_1 Y_L M_1 \phi l_{1R} + h_2 Y_L M_2 \phi l_{2R} + h_3 Y_L M_3 \phi l_{3R}$$

- Constraining U_w to be unitary, we limit Z as $Z = -1 \pm \sqrt{1 - S^2} + iS$

For Z < 1 → Perturbations → order of S → S = Sin α

$$U_\omega = \frac{1}{\sqrt{3}} \begin{bmatrix} e^{i\alpha} & 1 & e^{-i\alpha} \\ e^{i\alpha} & \omega & \omega^2 e^{-i\alpha} \\ e^{i\alpha} & \omega^2 & \omega e^{-i\alpha} \end{bmatrix}$$

PERTURBATIONS IN NEUTRINO SECTOR

- We choose the perturbation as

Where $1/\rho\ e^{-i\varphi}$ characterises the soft breaking of A_4

$$M\nu_R^T C^{-1} \begin{bmatrix} \frac{1}{\rho}e^{-i\varphi} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\rho}e^{-i\varphi} \end{bmatrix} \nu_R$$

THE PERTURBED MATRIX IS NOW →

$$\begin{bmatrix} M + \frac{1}{\rho}e^{-i\varphi}M & 0 & M' \\ 0 & M & 0 \\ M' & 0 & M - \frac{1}{\rho}e^{-i\varphi}M \end{bmatrix} \nu_R$$

We can diagonalise it with rotation angle *x*

$$tan\ 2x = \frac{M'}{\frac{1}{\rho}e^{-i\varphi}M}$$

DATA SIMULATION

- PMNS matrix after perturbation →

$$U_{PMNS} = \frac{1}{\sqrt{3}}\begin{bmatrix} e^{i\alpha} & 1 & e^{-i\alpha} \\ e^{i\alpha} & \omega & \omega^2 e^{-i\alpha} \\ e^{i\alpha} & \omega^2 & \omega e^{-i\alpha} \end{bmatrix}\begin{bmatrix} Cosx & 0 & -Sinx \\ 0 & 1 & 0 \\ Sinx & 0 & Cosx \end{bmatrix}$$

- After computation matching with the actual PMNS matrix

$$\mathrm{Sin}\theta_{13} e^{-i\delta_{CP}} = \frac{1}{\sqrt{3}}\left(e^{-i\alpha}\mathrm{Cosx} - e^{i\alpha}\mathrm{Sinx}\right) = \frac{1}{\sqrt{3}}\left(\mathrm{Cos}\alpha\mathrm{Cosx} - \mathrm{Cos}\alpha\mathrm{Sinx} - i\mathrm{Sin}\alpha\mathrm{Cosx} - i\mathrm{Sin}\alpha\mathrm{Sinx}\right)$$

- Therefore ⇒

$$\mathrm{Sin}\theta_{13} Cos\delta_{CP} = \frac{1}{\sqrt{3}}\left(\mathrm{Cos}\alpha\mathrm{Cosx} - \mathrm{Cos}\alpha\mathrm{Sinx}\right)$$

DATA SIMULATION

- *And* $\mathrm{Sin}\theta_{13} Sin\delta_{CP} = \frac{1}{\sqrt{3}}(\mathrm{Sin}\alpha\mathrm{Cosx} + \mathrm{Sin}\alpha\mathrm{Sinx})$
- *Perturbations in neutrino sector is* $\kappa = \frac{1}{\rho} e^- i\varphi \frac{M}{M'} = Cot2x$
- *Thus in terms of soft breaking parameters one gets*

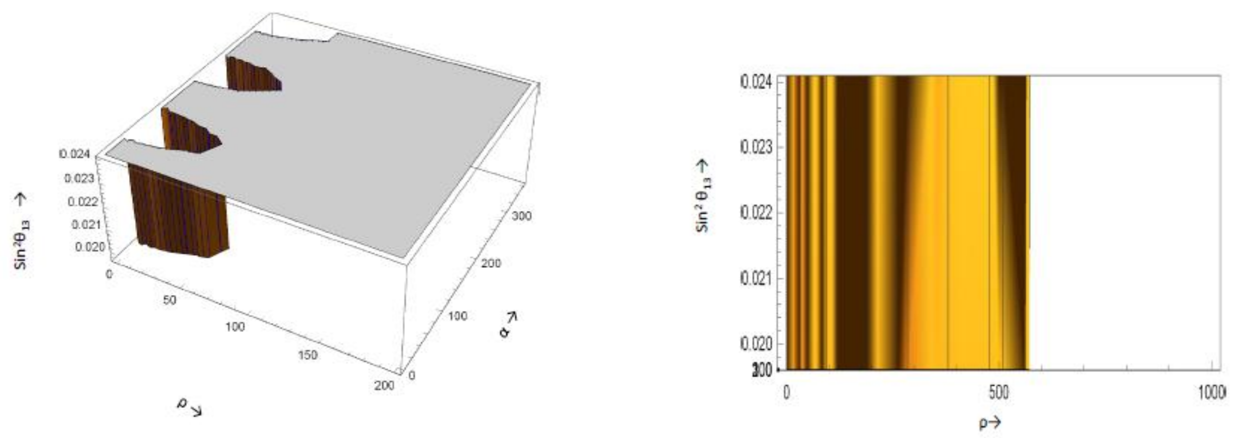
$$Sin^2\theta_{13} = \frac{1}{6\rho^2} e^- 2i\varphi \frac{M^2}{M'^2} + \frac{2}{3} Sin^2\ \alpha - \frac{1}{3\rho^2} e^- 2i\varphi \frac{M^2}{M'^2}^2 Sin^2\ \alpha$$

- *Similarly one can find expression for* $\theta_{12}\,\theta_{23}$ *in terms of ρ, φ and α*

Analysis Strategy

- ***Finally***

$$Cos\,\delta_{CP} = \sqrt{1 - \frac{(-\kappa)^2}{4Sin^2\alpha + \kappa^2 - 16\frac{\kappa^2 Sin^2\alpha}{3}}} = \sqrt{1 - \frac{\frac{1}{\rho^2}e^{-2i\varphi}\frac{M^2}{M'^2}{}^2}{4Sin^2\alpha + \frac{1}{\rho^2}e^{-2i\varphi}\frac{M^2}{M'^2}{}^2 - 16\frac{\frac{1}{\rho^2}e^{-2i\varphi}\frac{M^2}{M'^2}{}^2 Sin^2\alpha}{3}}}$$



(a) (b)

Figure 1: Left panel: The points in $\rho - \alpha$ space which satisfy the 3σ constraints on $Sin^2\theta_{13}$ for $\frac{M'}{M} = 10^{-2}$. Right Panel: The points in $Sin^2\theta_{13} - \rho$ space corresponding to the 3σ bounds on $Sin^2\theta_{13}$ for $\frac{M'}{M} = 10^{-3}$.

Analysis Strategy

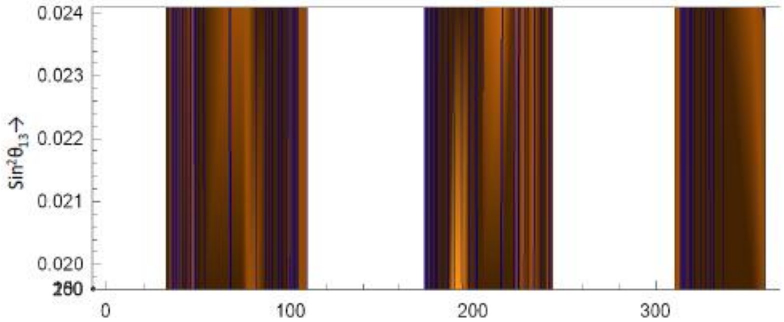
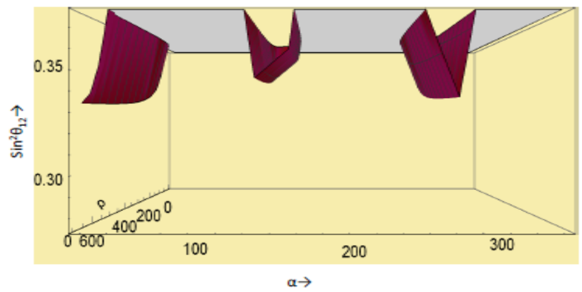
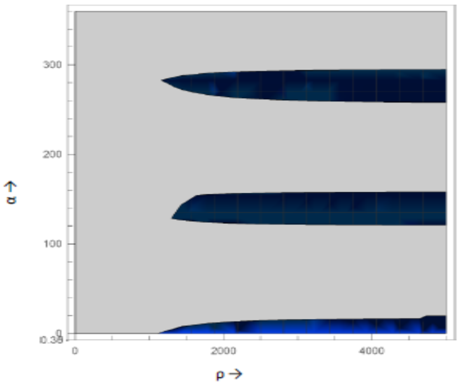


Figure 2: The predicted dependence of $Sin^2\theta_{13}$ on the parameter α which comes from varying θ_{13} within its 3σ range



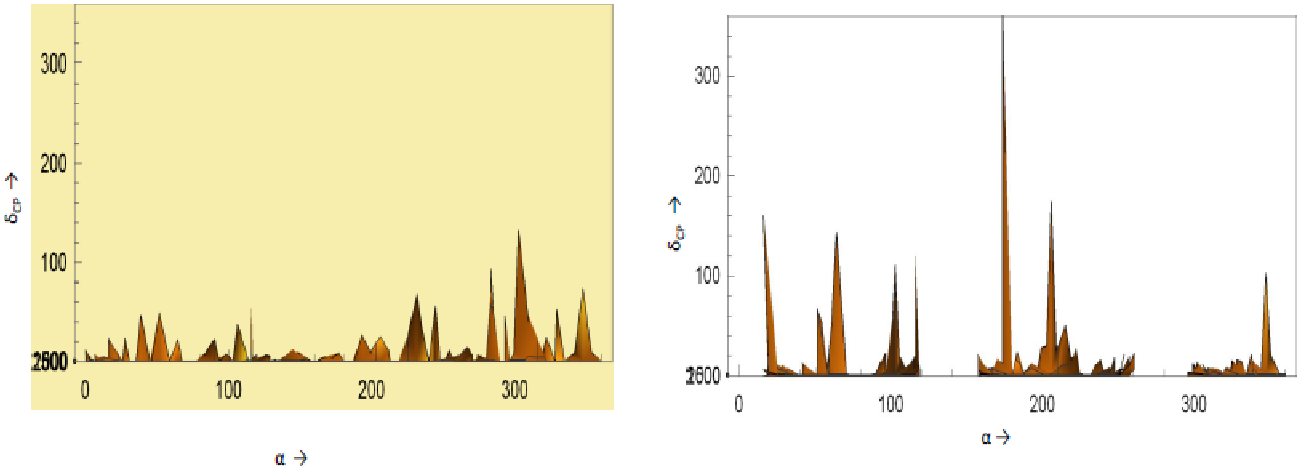
(a)



(b)

Figure 3: Left panel: The allowed range of $\rho - \alpha$ space which satisfy the 3σ constraints on $Sin^2\theta_{12}$ for $\frac{M'}{M} = 10^{-2}$. Right Panel: The correlation between ρ and α space corresponding to the 3σ bounds on $Sin^2\theta_{12}$ for $\frac{M'}{M} = 10^{-3}$.

Analysis Strategy



(a) (b)

Figure 4: Left Panel : The values of δ_{CP} within its 3σ bounds phase for different regions in α space for $\frac{M'}{M} = 10^{-2}$. Right Panel: The values of δ_{CP} within its 3σ bounds as indicated by current neutrino oscillation global fit [48] for different regions in α space for $\frac{M'}{M} = 10^{-3}$.

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Analysis Strategy

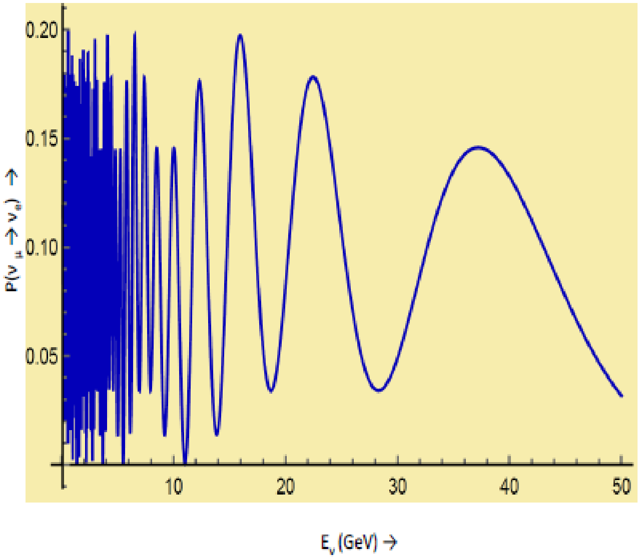


Figure 5: The allowed range of electron neutrino appearance probability at T2K covers a more restricted region.

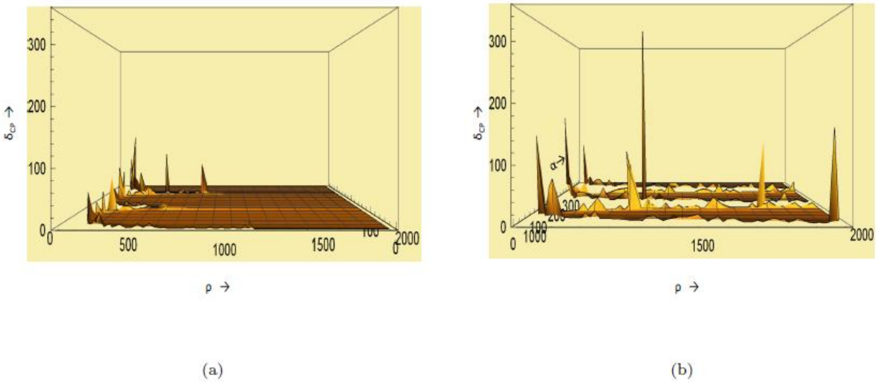
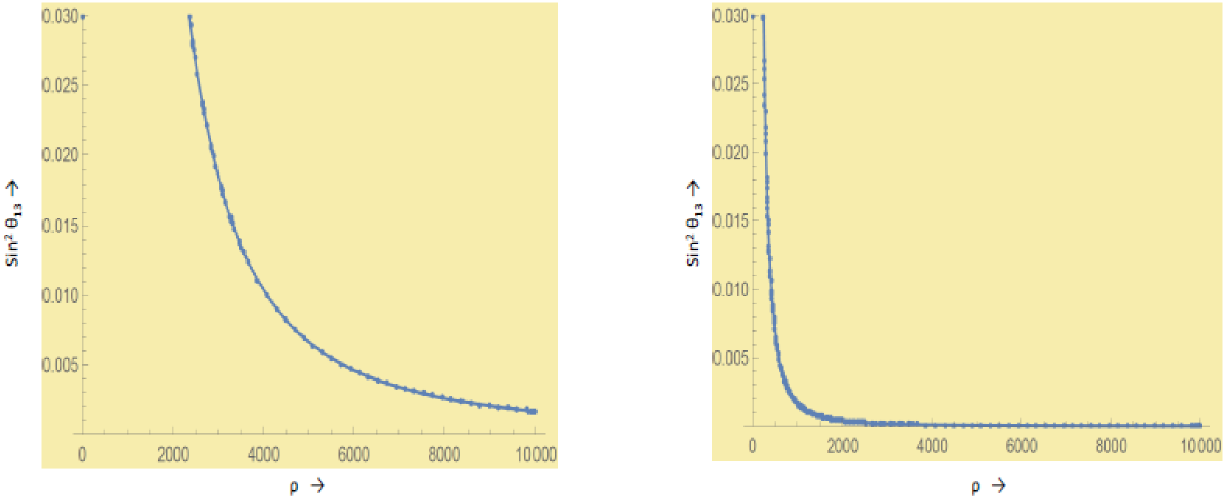


Figure 6: The values of δ_{CP} within its 3σ bounds phase for different regions in ρ space for $\frac{M'}{M} = 10^{-2}$. Right Panel: The values of δ_{CP} within its 3σ bounds as indicated by current neutrino oscillation global fit [48], for different regions in ρ space for $\frac{M'}{M} = 10^{-3}$.

Analysis Strategy



(a) (b)

Figure 7: The plot of sine squared values of mixing angles for maximal δ_{CP} through a $Z_2 \times Z_2$invariant perturbation in neutrino sector. The figure in the left and right panel corresponds to $\frac{M'}{M} = 10^{-2}$ and $\frac{M'}{M} = 10^{-3}$ respectively.

Analysis Strategy

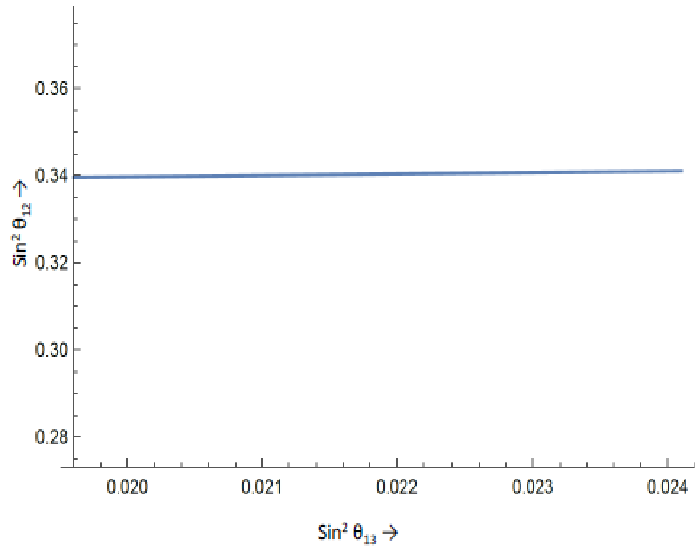


Figure 8: The plot of sine squared values of mixing angles for maximal δ_{CP} through a $Z_2 \times Z_2$invariant perturbation in neutrino sector.

Analysis Strategy

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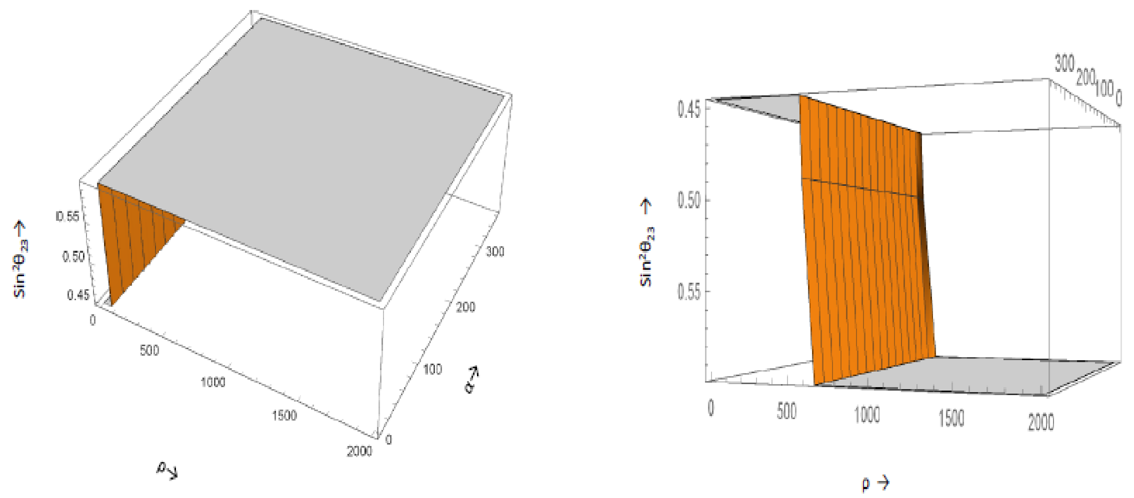

(a) (b)

Figure 9: The points in $\rho - \alpha$ space which satisfy the 3σ constraints on $Sin^2\theta_{23}$ for $\frac{M'}{M} = 10^{-2}$. Right Panel: The points in $Sin^2\theta_{23} - \rho$ space corresponding to the 3σ bounds on $Sin^2\theta_{23}$ for $\frac{M'}{M} = 10^{-3}$.

Analysis Strategy

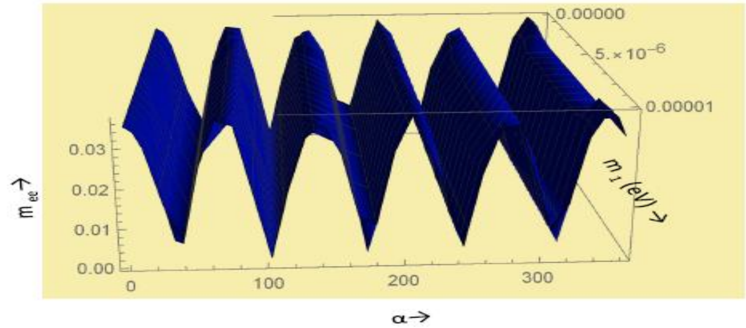

Figure 10: $|m_{ee}|$ prediction for lightest neutrino mass m_1 (eV) and α space in our model.

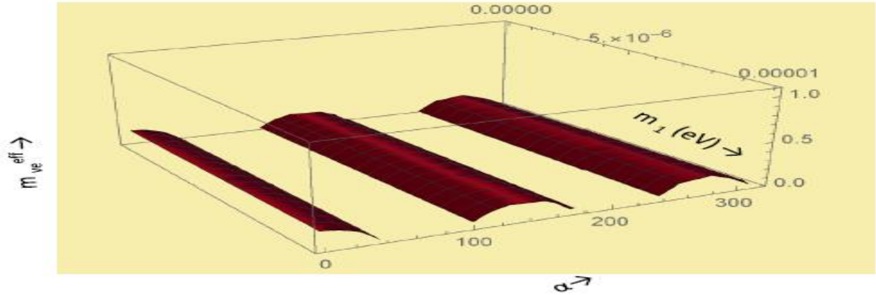

Figure 11: $|m^{eff}_{\nu_e}|$ prediction for lightest neutrino mass m_1 (eV) and α space in our model.

Analysis Strategy

We consider a model based on A_4 symmetry which gives the corrections to the TBM form for the leading order neutrino mixing matrix. We present here the phenomenology of a model with A_4symmetry which envisage the tribimaximal form for the PMNS matrix. In this model, we have instigated a $Z_2 \times Z_2$ invariant perturbations in both the charged lepton in the form of $Sin\alpha$ and the neutrino sectors in the form of κ. We perceive that perturbations in the neutrino sector leads to allowable values of non zero θ_{13} varying within its 3σ range as indicated by current neutrino oscillation global fit [48] and maximal CP violation for $Sin\alpha = 0$. The desired value of the CP violating phase δ_{CP} lying within its 3σ range can be procured by choosing the fitting and pertinent values for $Sin\alpha$ term and for the perturbations in neutrino sectors. However, there is a strain in obtaining large values of δ_{CP} phase and best fit experimental value of $Sin^2\theta_{23}$. For $\kappa > 2Sin\alpha$ we have large CPV phase, and that keeps the value of $Sin^2\theta_{23}$ close to the TBM value of 0.5. Also, our analysis of δ_{CP} phase $\sim 144°$ in this model exactly coincides with the preferred

Results and Discussions

- ***The CP violating phase δ_{CP} is around 144 degree in this model exactly coincides with the* value of CP phase ~ 0.8 pi by the analysis of NovA results.**

We have considered leading order corrections in the form of $Z_2 \times Z_2$ invariant perturbations in neutrino sector after spontaneous breaking of A_4 symmetry. The neutrino mixing angles, thus obtained are found to be within the 3σ ranges of their experimental values. The CP violating phase δ_{CP} is around $\sim 144^\circ$ in this model. We also studied the variation of the the neutrino oscillation probability $P(\nu_\mu \to \nu_e)$, the effective Majorana mass $|m_{ee}|$ and $|m_{\nu e}^{eff}|$ with the lightest neutrino mass m_1 in the case of normal hierarchy and found its value to be lower than the experimental upper limit for all allowed values of $m_1 \in [0\ eV, 10^{-5}\ eV]$.

- The $Z_2 \times Z_2$ invariant perturbations in neutrino sector is charactererised by 3 independent parameters *ρ, φ and α, which determines all the 3 mixing angles and CPV phase leading to several testable predictions.*

Results and Discussions

- *A more comprehensive version of the generalized CP methodology and its potential to produce other hypothetically and realistic ansatz forms for the lepton mixing matrix will be presented in our future work.*

THANK YOU

TAU LEPTON PHYSICS