HVP contributions to the muon's anomalous magnetic moment from lattice QCD

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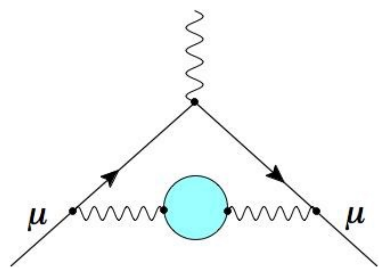
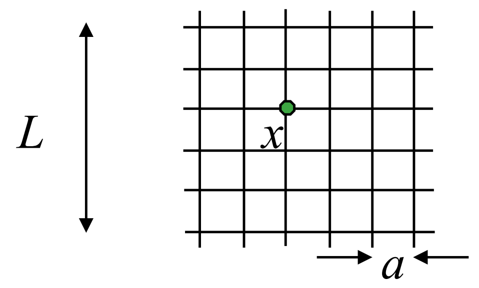
University of Illinois

16th International Workshop on Tau Lepton Physics
(TAU 2021, virtual edition)
Indiana University



HVP Comparison



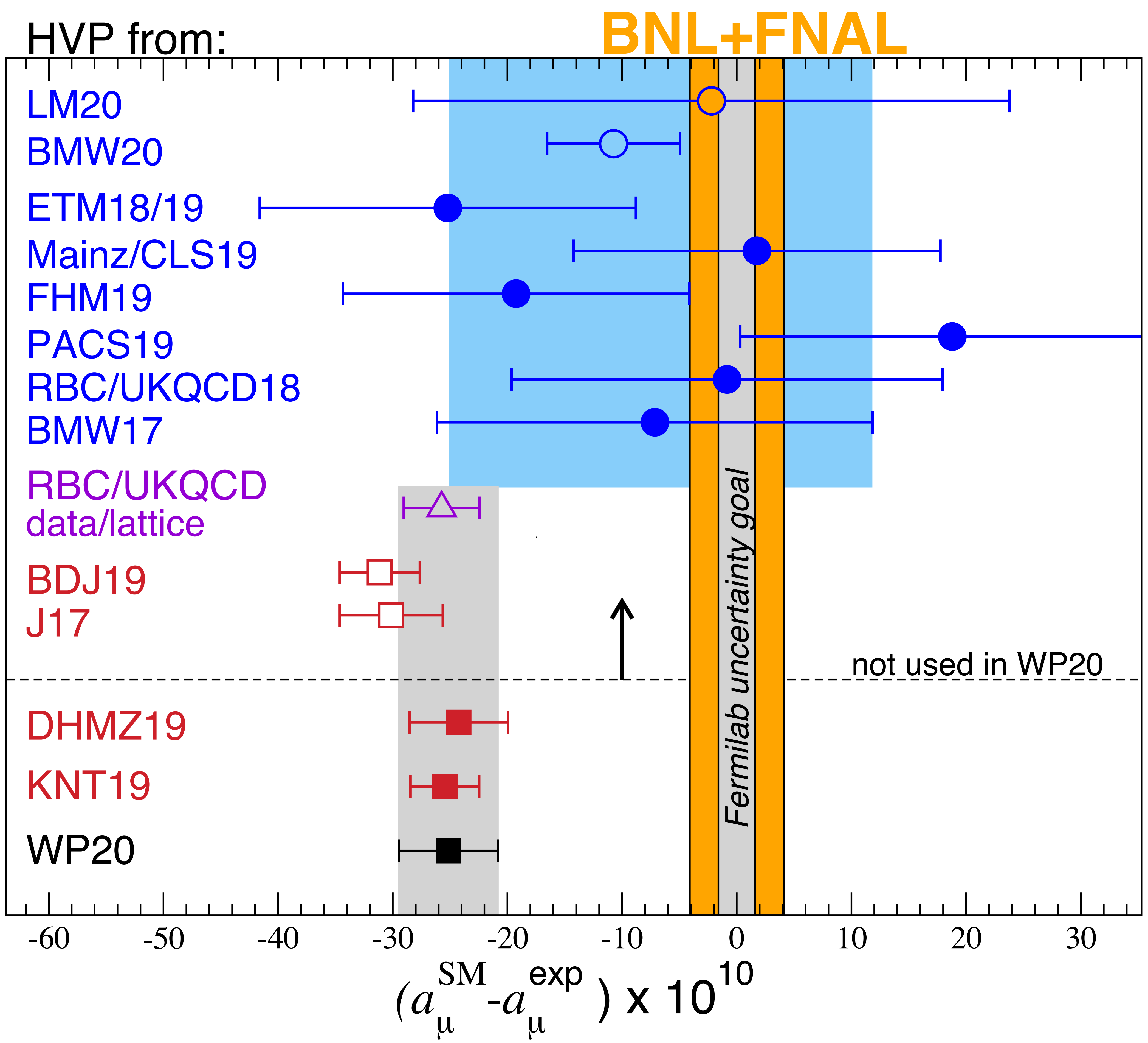
$$a_\mu^{\rm SM} \quad \left< \; a_\mu^{\rm HVP} + [a_\mu^{\rm QED} + a_\mu^{\rm Weak} + a_\mu^{\rm HLbL}] \right.$$

Lattice QCD + QED

hybrid: combine data & lattice

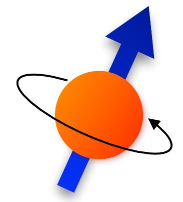
data driven

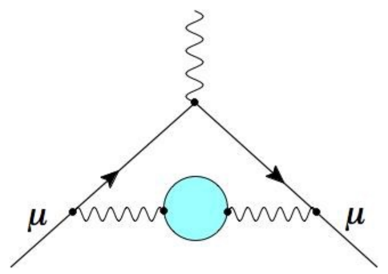
+ unitarity/analyticity constraints



- Introduction to LQCD
- How to compute HVP with LQCD
- Systematic errors
- Results for each contribution
- Windows in Euclidean time ➠ detailed comparisons
- Summary and Outlook: a path forward

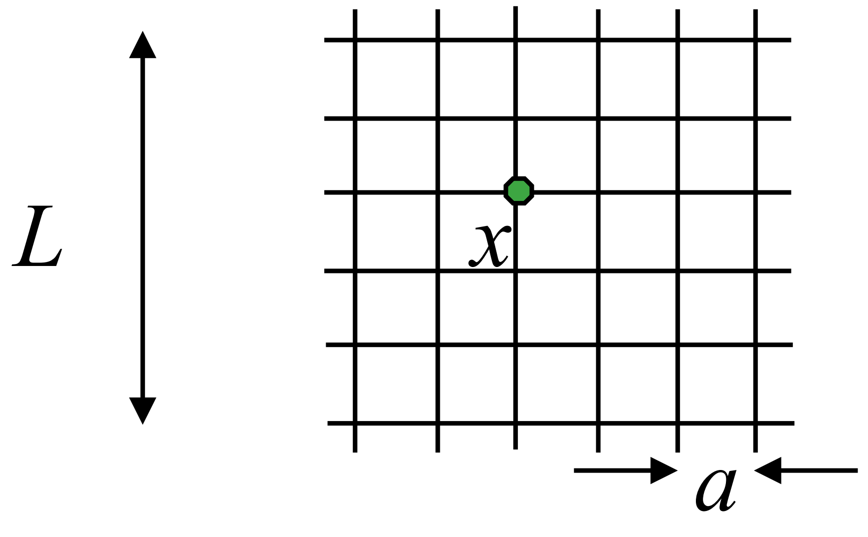
[T. Aoyama et al, arXiv:2006.04822, Phys. Repts. 887 (2020) 1-166.]





Lattice QCD Introduction

$$\mathcal{L}_{\mathrm{QCD}} = \sum_f \bar{\psi}_f (\not{D} + m_f)\psi_f + \frac{1}{4}\mathrm{tr}F_{\mu\nu}F^{\mu\nu}$$

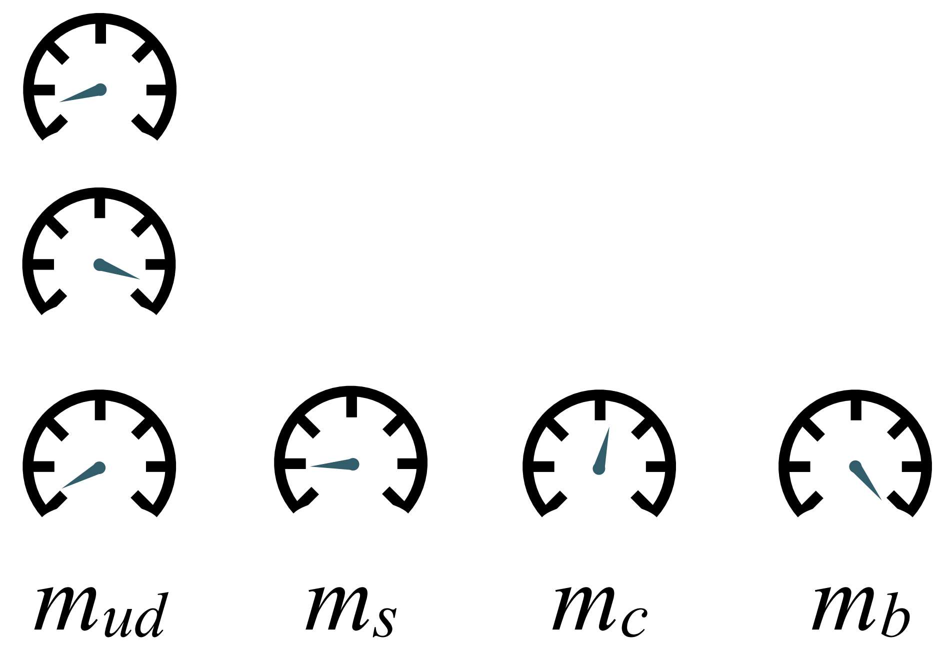


- discrete Euclidean space-time (spacing a)
  derivatives ➙ difference operators, etc…
- finite spatial volume (L)
- finite time extent (T)

Integrals are evaluated numerically using monte carlo methods.

adjustable parameters

- lattice spacing: $a \to 0$
- finite volume, time: $L \to \infty,\ T > L$
- quark masses (m_f): $M_{H,\mathrm{lat}} = M_{H,\mathrm{exp}}$
  tune using hadron masses $m_f \to m_{f,\mathrm{phys}}$
  extrapolations/interpolations



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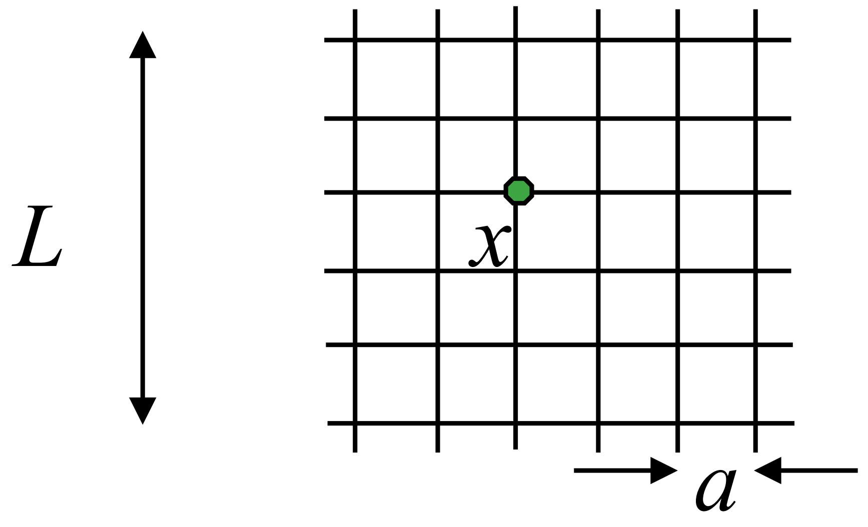

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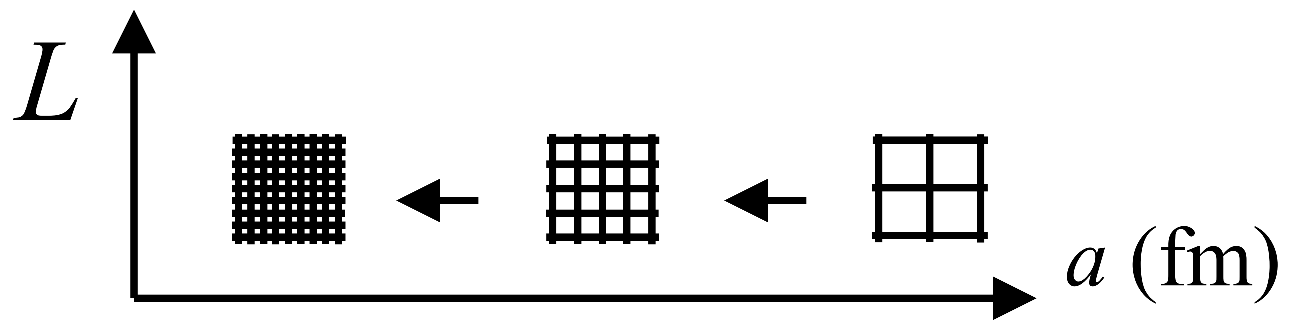

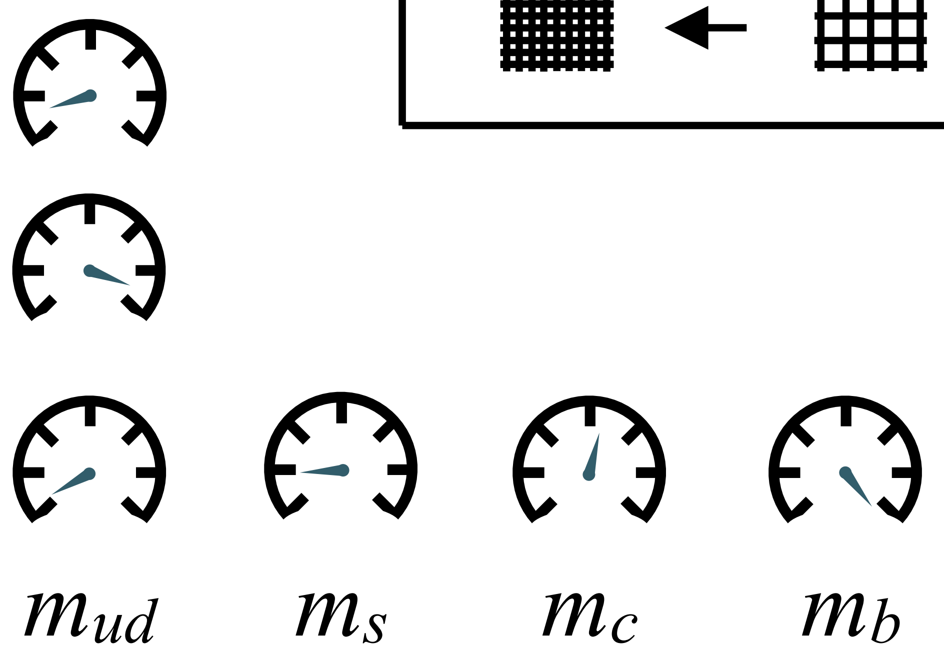

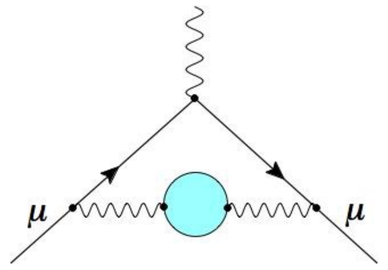

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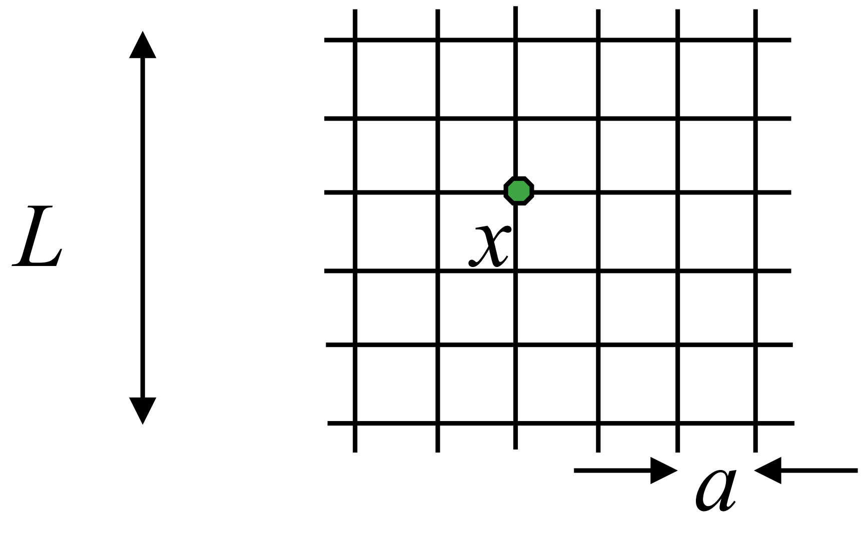

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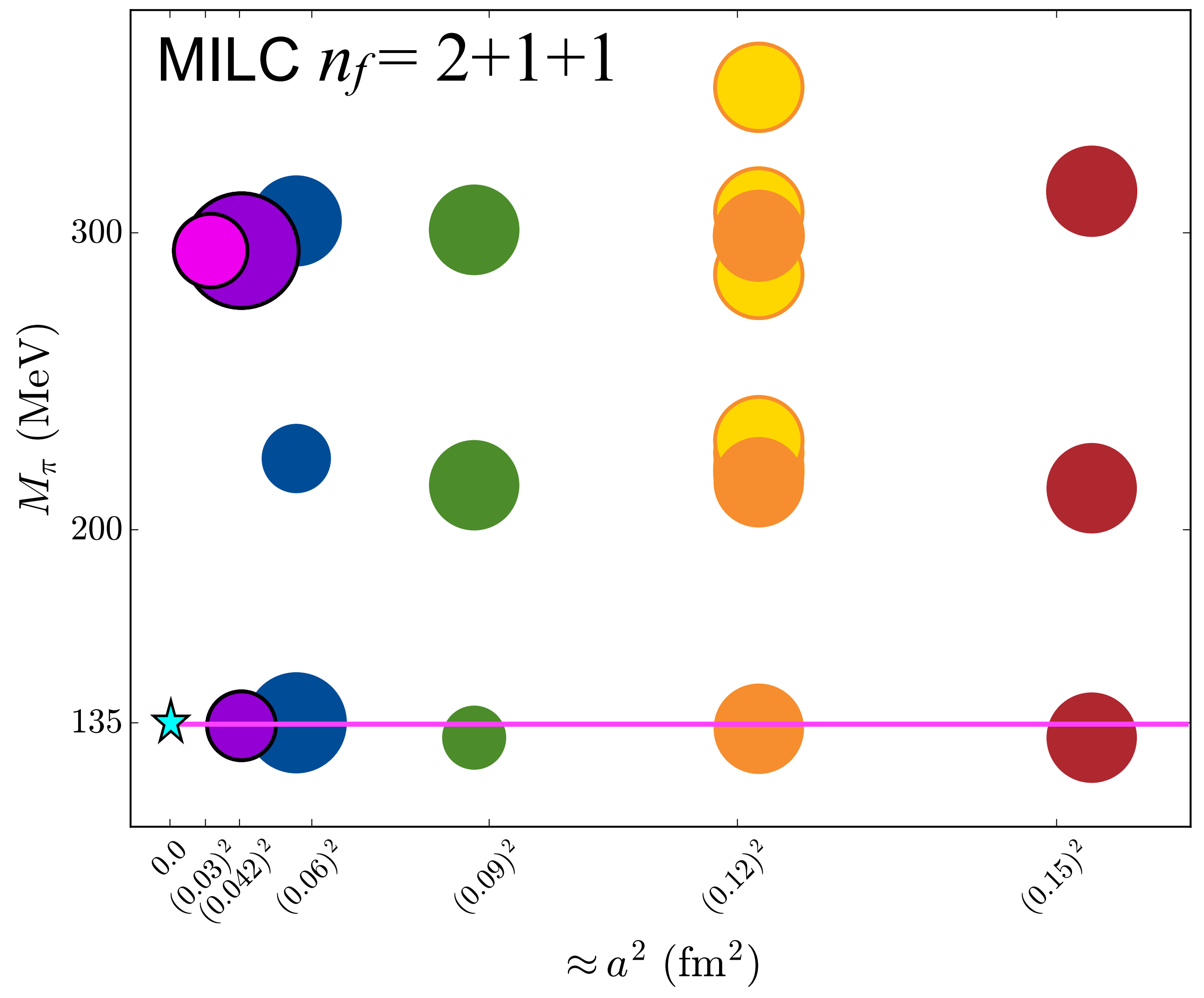

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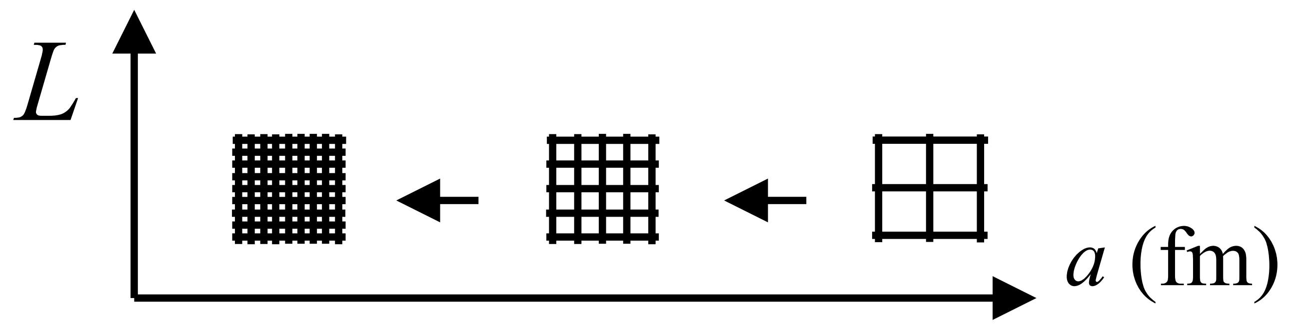
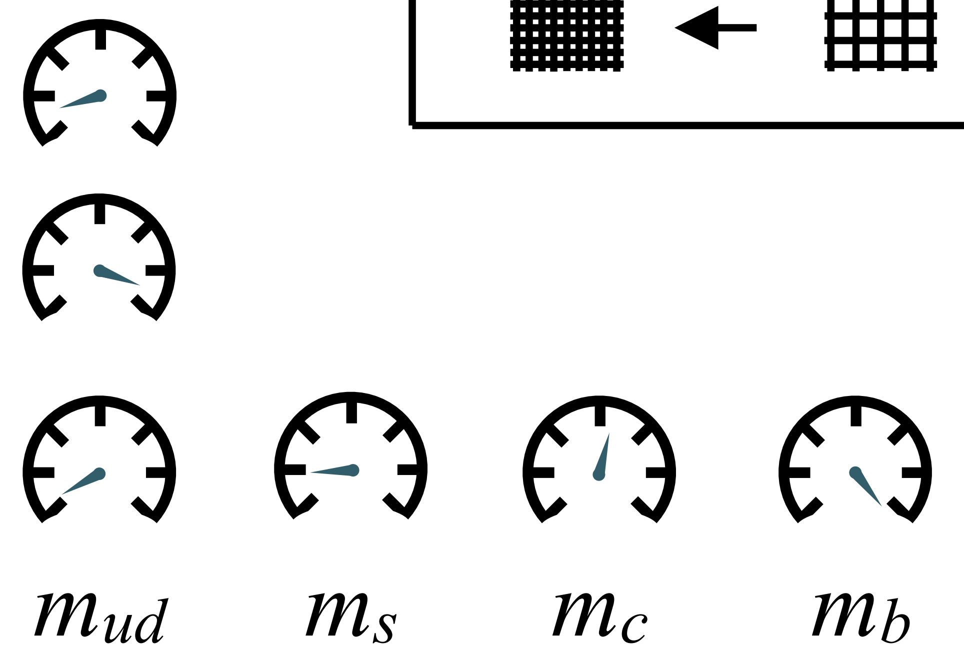
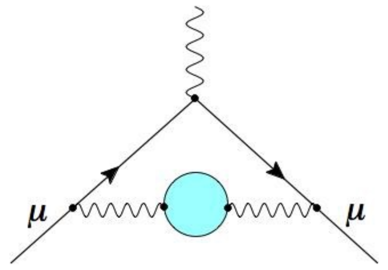
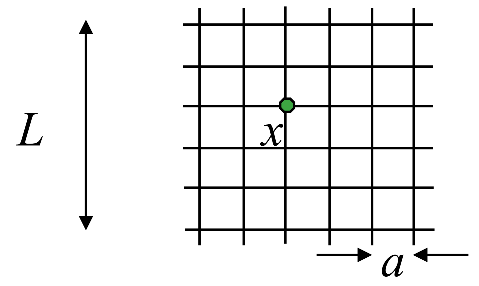






Lattice QCD Introduction



The State of the Art

Lattice QCD calculations of simple quantities (with at most one stable meson in initial/final state) that **quantitatively account for all systematic effects** (discretization, finite volume, renormalization,…) in some cases with

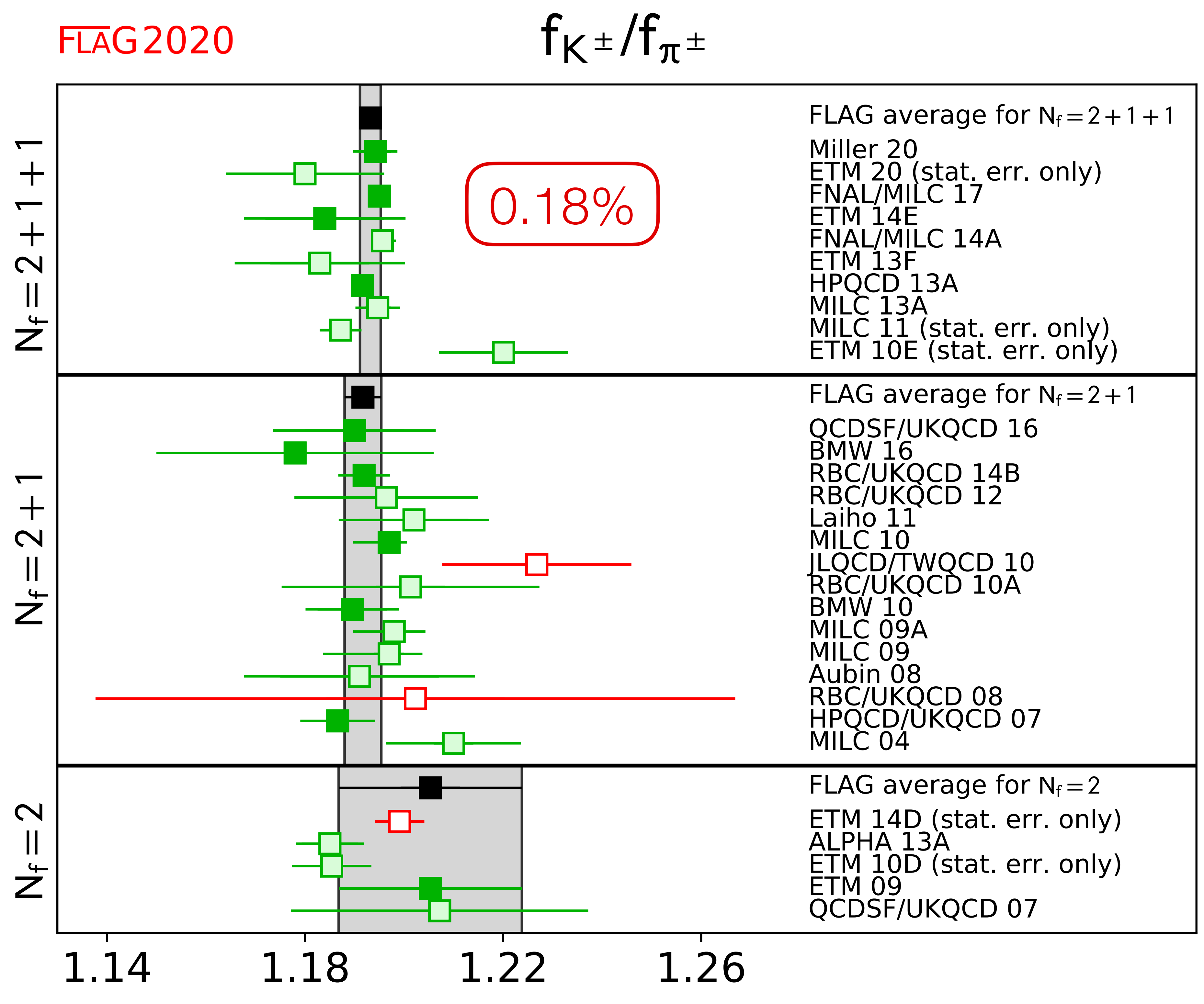
- sub percent precision.
- total errors that are commensurate (or smaller) than corresponding experimental uncertainties.

Scope of LQCD calculations is increasing due to continual development of new methods:

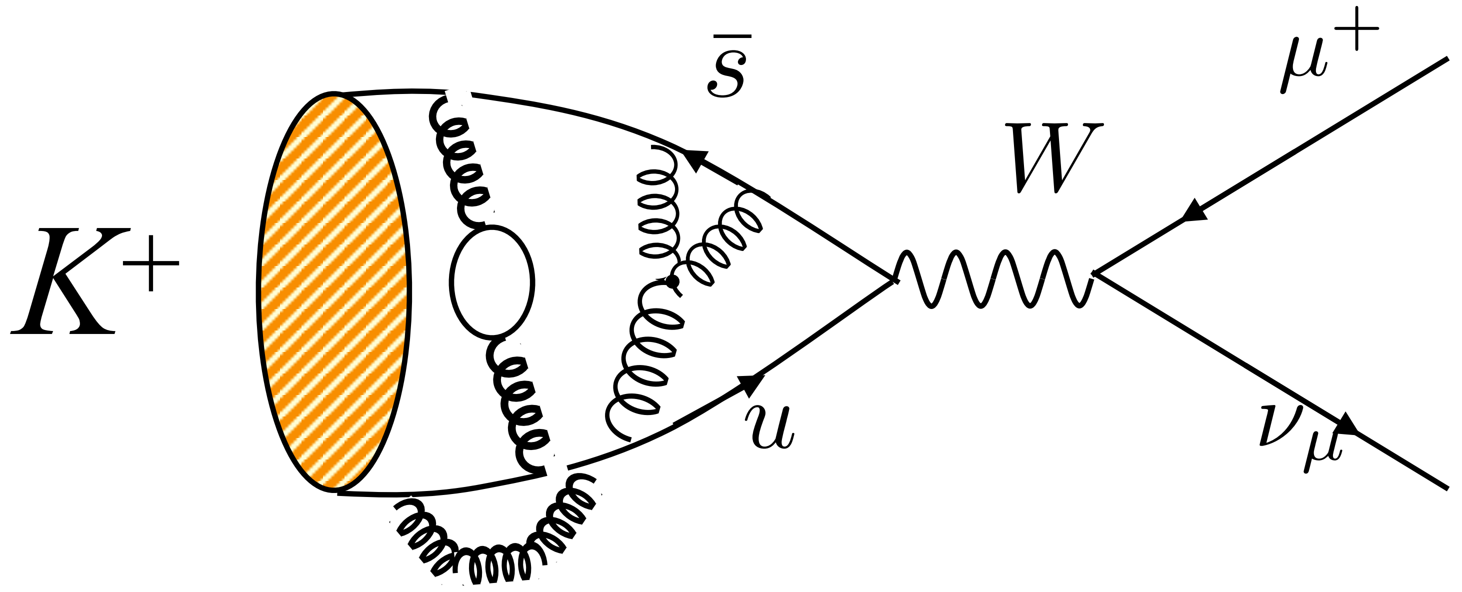
- nucleons and other baryons
- nonleptonic decays ($K \to \pi\pi$, …)
- resonances, scattering, long-distance effects, …
- QED effects
- radiative decay rates …

The State of the Art

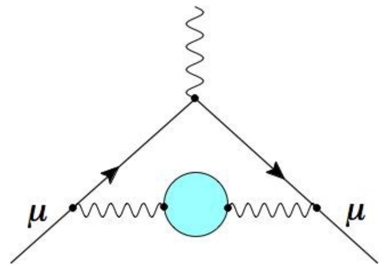
f_{K^+}/f_{π^+}



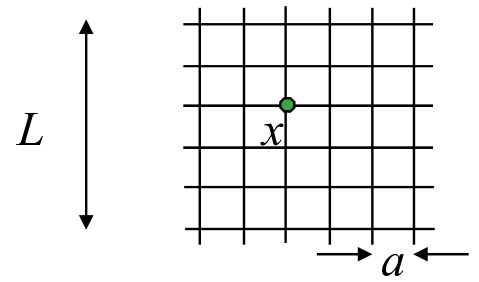
[S. Aoki et al, arXiv:1902.08191, EPJC 2020]



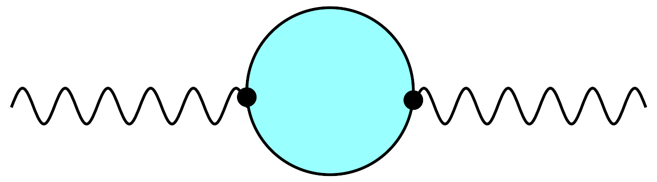
small errors due to
- ✦ ensembles at **physical light quark masses**
- ✦ improved light-quark actions
- ✦ large ensemble set (range of $a, L, m_i, \ldots$)
- ✦ NPR or no renormalization



Lattice HVP: Introduction



[B. Lautrup, A. Peterman, E. de Rafael, Phys. Rep 1972; E. de Rafael, Phys. Let. B 1994; T. Blum, PRL 2002]



$\hat{\Pi}(q^2)$

Leading order HVP correction:

$$a_\mu^{\mathrm{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int dq^2 \omega(q^2)\, \hat{\Pi}(q^2)$$

- Calculate $a_\mu^{\mathrm{HVP,LO}}$ in Lattice QCD

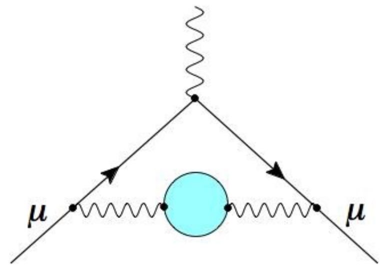
Compute correlation function: $C(t) = \frac{1}{3} \sum_{i,x} \langle j_i(x,t) j_i(0,0) \rangle$

and $\hat{\Pi}(Q^2) = 4\pi^2 \int_0^\infty dt\, C(t) \left[t^2 - \frac{4}{Q^2} \sin^2\left(\frac{Qt}{2}\right)\right]$

[D. Bernecker and H. Meyer, arXiv:1107.4388, EPJA 2011]

Obtain $a_\mu^{\mathrm{HVP,LO}}$ from an integral over Euclidean time:

$$a_\mu^{\mathrm{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt\, \tilde{w}(t)\, C(t)$$



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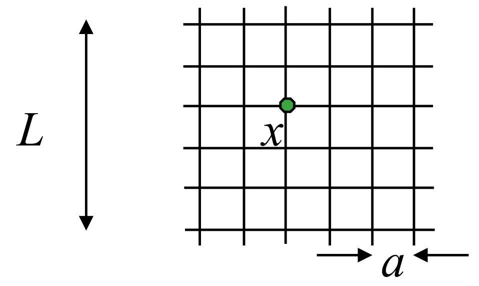
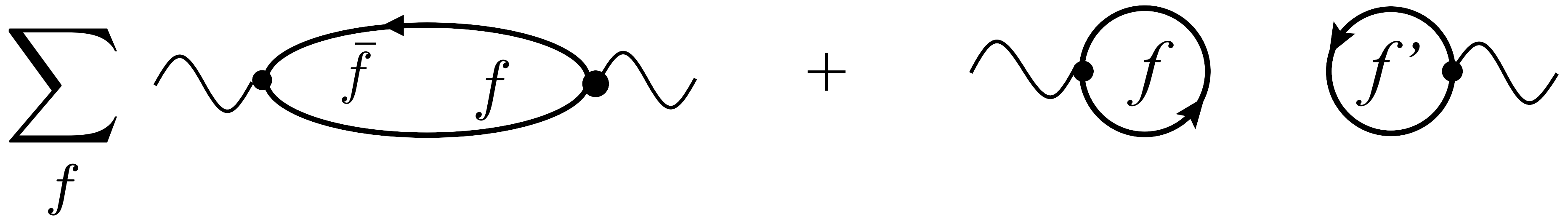


Calculate $a_\mu^{\rm HVP}$ in Lattice QCD:

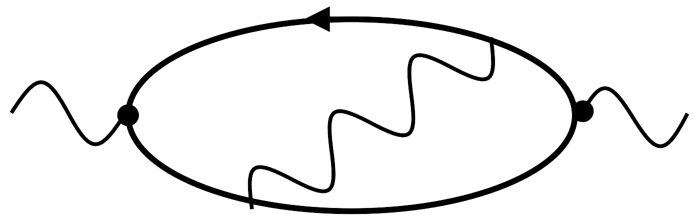
$$a_\mu^{\rm HVP,LO} = \sum_f a_{\mu,f}^{\rm HVP,LO} + a_{\mu,{\rm disc}}^{\rm HVP,LO}$$

- Separate into connected for each quark flavor + disconnected contributions (gluon and sea-quark background not shown in diagrams)
  Note: almost always $m_u = m_d$



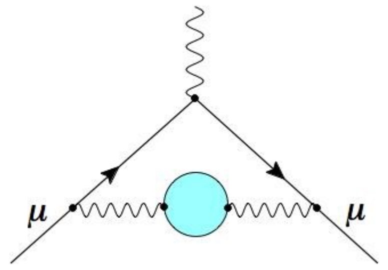
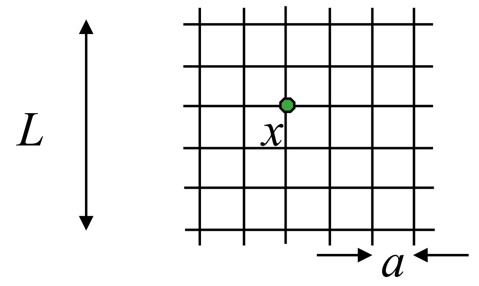
f= ud, s, c, b

- need to add QED and strong isospin breaking ($\sim m_u - m_d$) corrections:



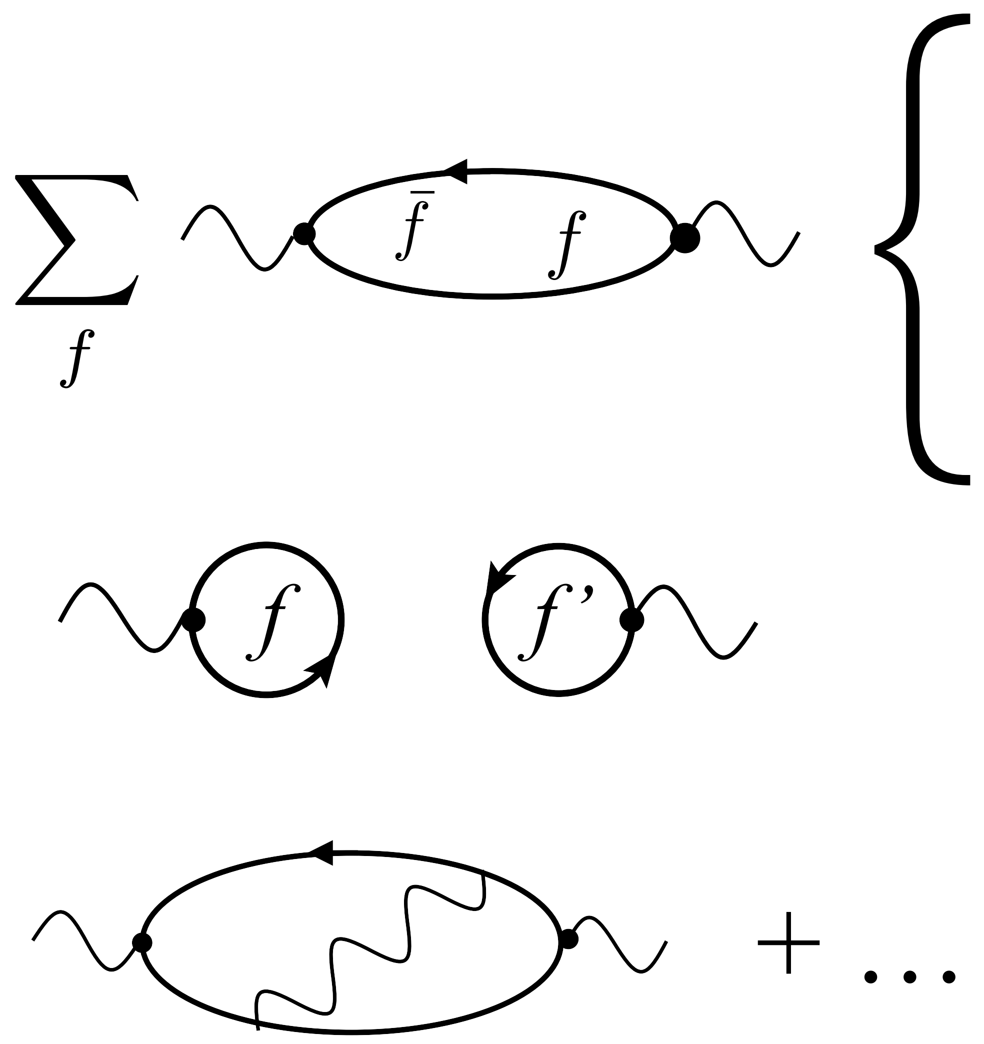
$+ \ldots$

- either perturbatively on isospin symmetric QCD background
- or by using QCD + QED ensembles with $m_u \neq m_d$

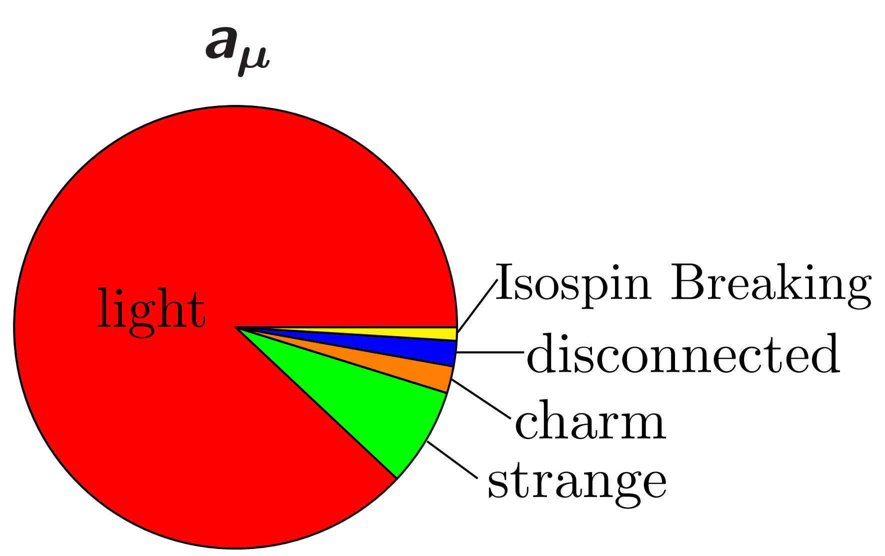
Lattice HVP: Introduction

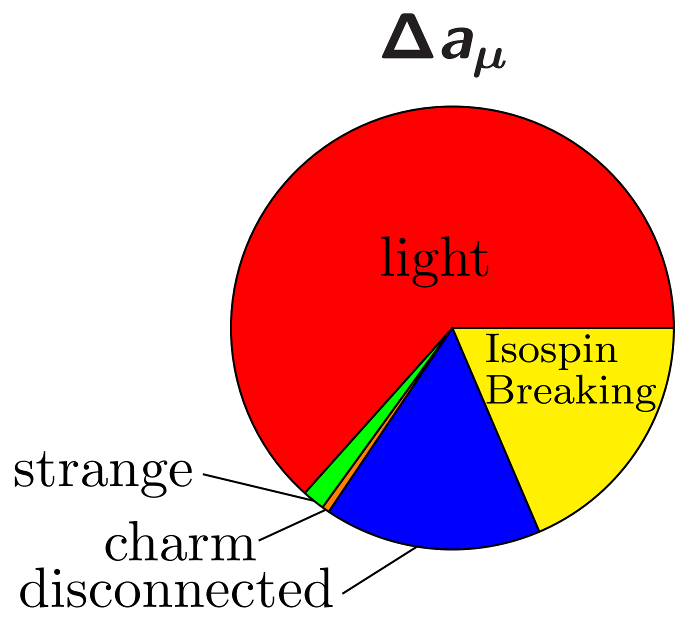
- Target: ~ 0.2% total error



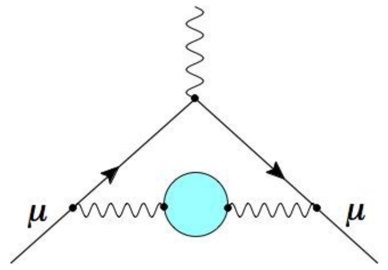
- light-quark connected contribution:
  $a_\mu^{\rm HVP,LO}(ud)$ ~90% of total
- *s*,*c*,*b*-quark contributions
  $a_\mu^{\rm HVP,LO}(s,c,b)$ ~8%, 2%, 0.05% of total
- disconnected contribution:
  $a_{\mu,\rm disc}^{\rm HVP,LO}$ ~2% of total
- Isospinbreaking (QED + $m_u \neq m_d$) corrections:
  $\delta a_\mu^{\rm HVP,LO}$ ~1% of total

V. Gülpers @ Lattice HVP workshop





$$a_\mu^{\rm HVP,LO} = a_\mu^{\rm HVP,LO}(ud) + a_\mu^{\rm HVP,LO}(s) + a_\mu^{\rm HVP,LO}(c) + a_{\mu,\rm disc}^{\rm HVP,LO} + \delta a_\mu^{\rm HVP,LO}$$



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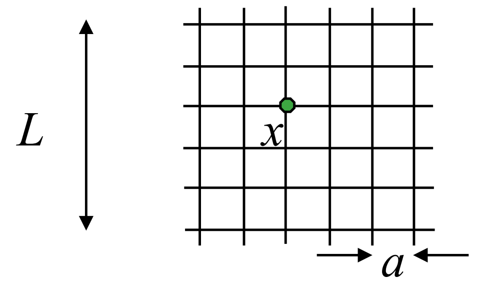
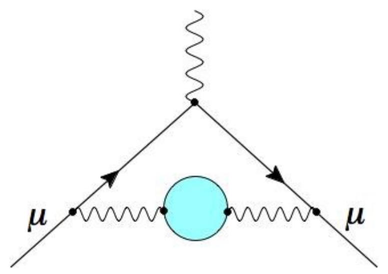
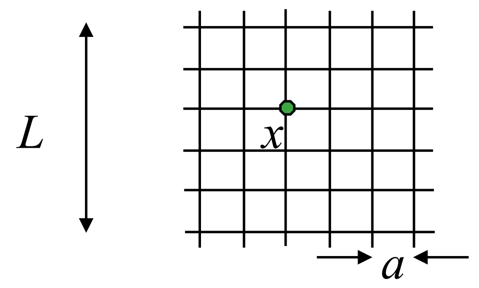


- Target: ~ 0.2% total error
- Challenges:
  - ✓ needs ensembles with (light sea) quark masses at their physical values
  - ✓ finite volume corrections
  - growth of statistical errors at long-distances
  - Continuum extrapolation
  - scale setting
  - disconnected contribution
  - QED and strong isospin breaking corrections ($m_u \neq m_d$)
- Focus on windows in Euclidean times [T. Blum et al, arXiv:1801.07224, 2018 PRL]
  - disentangle systematics/statistics from long distance/FV and discretization effects
    ➠ valuable cross checks
  - intermediate window easy to compute & compare with disperse methods



g-2 & related talks/posters @ Lattice 2021



QCD in searches for New Physics

https://indico.cern.ch/event/1006302/

(light-quark) connected HVP, windows
- Monday, 13:00-15:00 US EDT
  - Finn Stokes (BMWc) FV effects
  - Kalman Szabo (BMWc) cont. limit
  - Shaun Lahert (Fermilab-HPQCD-MILC)
  - Chris Aubin (Aubin et al)
- Tuesday, 5:00-8:00 US EDT
  - Hartmut Wittig (Mainz)
  - Christoph Lehner (RBC/UKQCD)
  - Davide Giusti (ETMc)

IB corrections, disc. HVP
- Tuesday, 5:00-8:00 US EDT
  - Andreas Risch (Mainz)
  - Letizia Parato (BMWc)
- Poster, Wednesday, 8:00-9:00 US EDT
  - C. McNeile (Fermilab-HPQCD-MILC)

$\Delta\alpha$ and $\Delta \sin^2\theta_W$
- Tuesday, 5:00-8:00 US EDT
  - Teseo San Jose (Mainz)
  - Kohtaroh Miura (Mainz)

HLbL contributions, PS transition form factors
- Tuesday, 5:00-8:00 US EDT
  - Willem Verplanke (BMWc)
  - Sebastian Burri (ETMc)
  - En-Hung Chao (Mainz) complete HLbL

Scale Setting
- Monday, 13:00-15:00 US EDT
  - Lukas Varnhorst (BMWc)
- Thursday 5:00-8:00 US EDT / Friday, 5:00–8:00 US EDT
  - Alexander Segner (Mainz) / Ben Strassberger (Mainz)

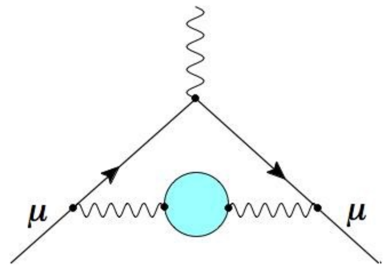
Hadron spect.

SM params

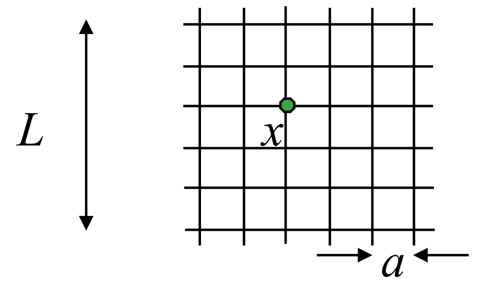
Cut-off effects
- Tuesday, 5:00-8:00 US EDT
  - Tim Harris (NEPhEU QCD) thermal observables
- Friday, 5:00-8:00 US EDT
  - Nicolai Husung (DESY) log corrections

Variance reduction
- Tuesday, 5:00-8:00 US EDT
  - Leonardo Giusti (Milan) Multi-level integration
- Tuesday, 13:00-15:00 US EDT
  - Tej Kanwar (MIT) contour deformation

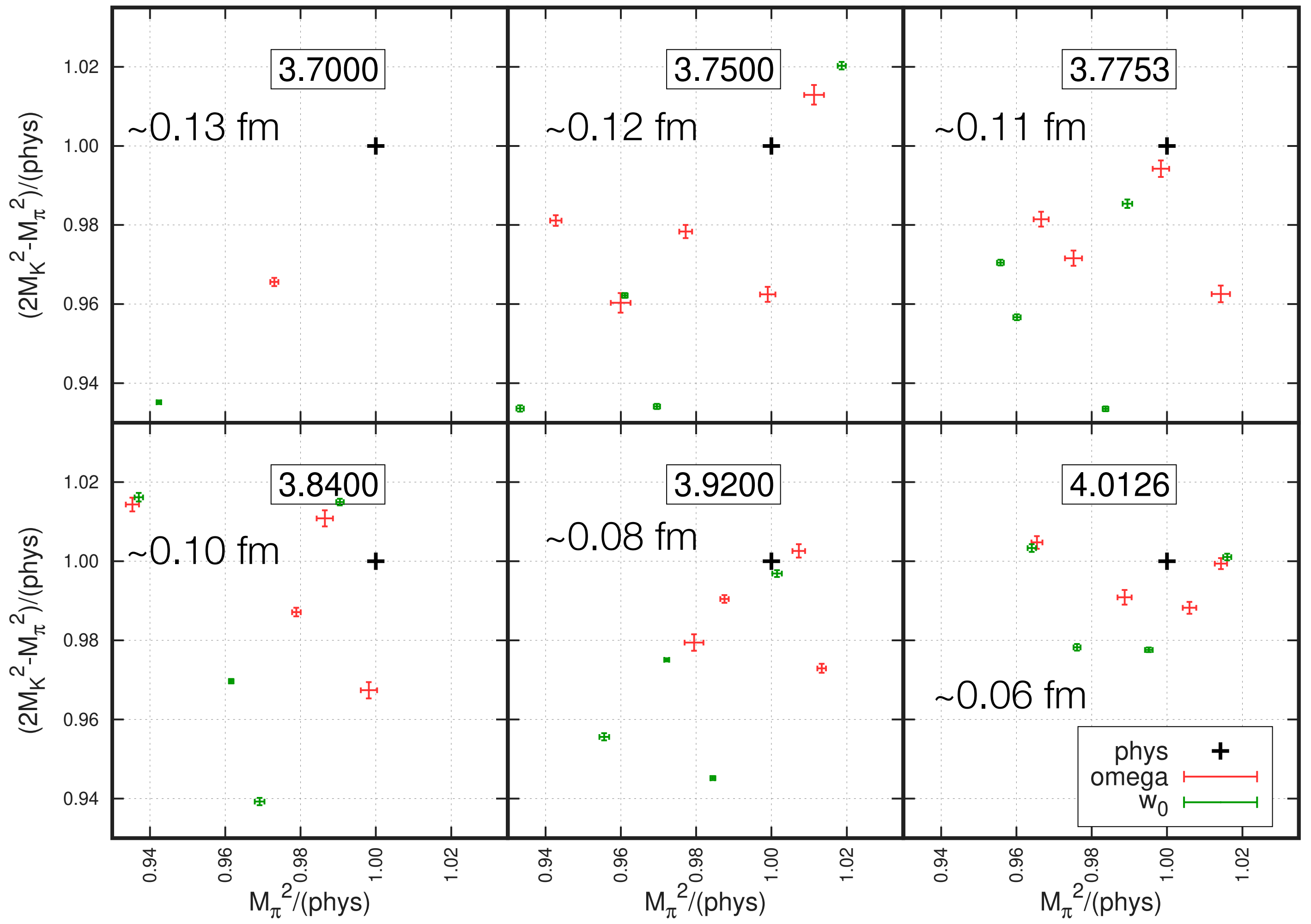
Algorithms



Ensemble parameters

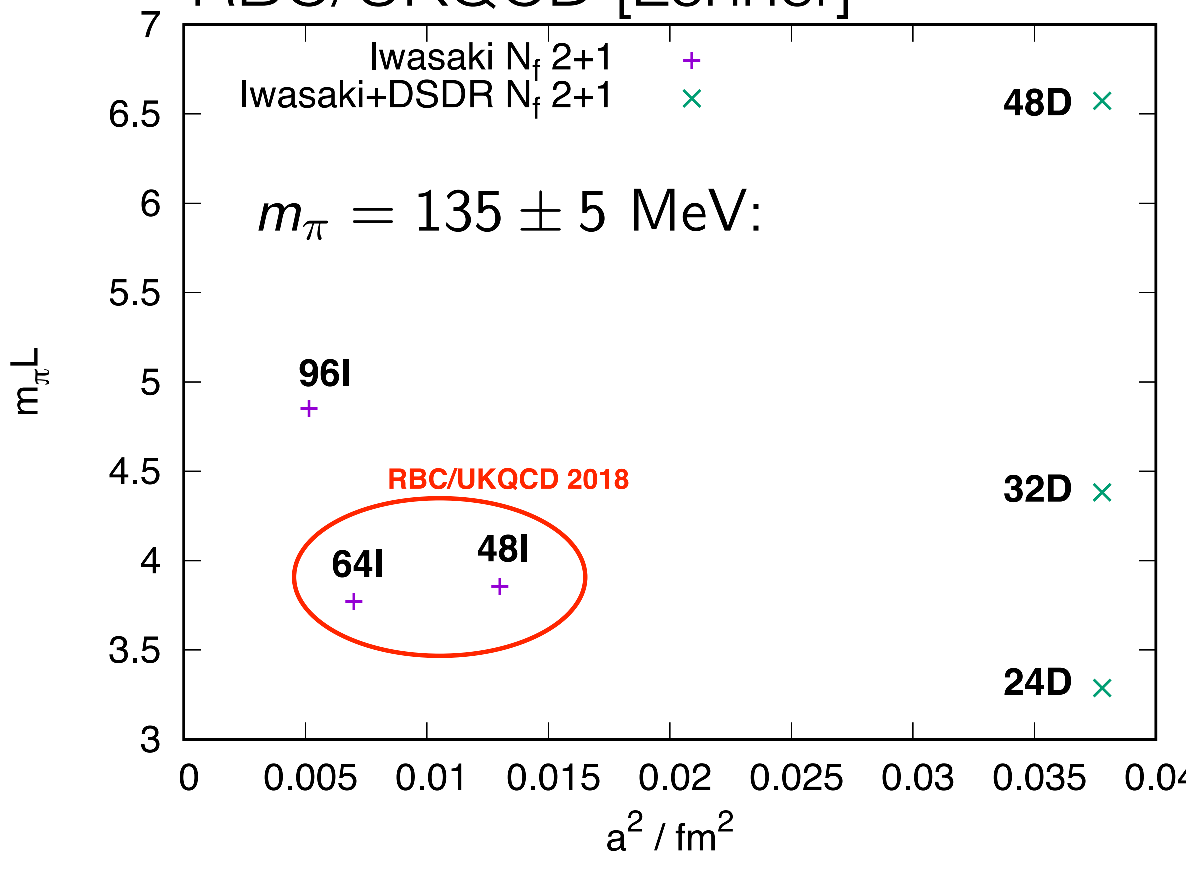


BMWc [K. Szabo, F. Stokes, L. Parato]



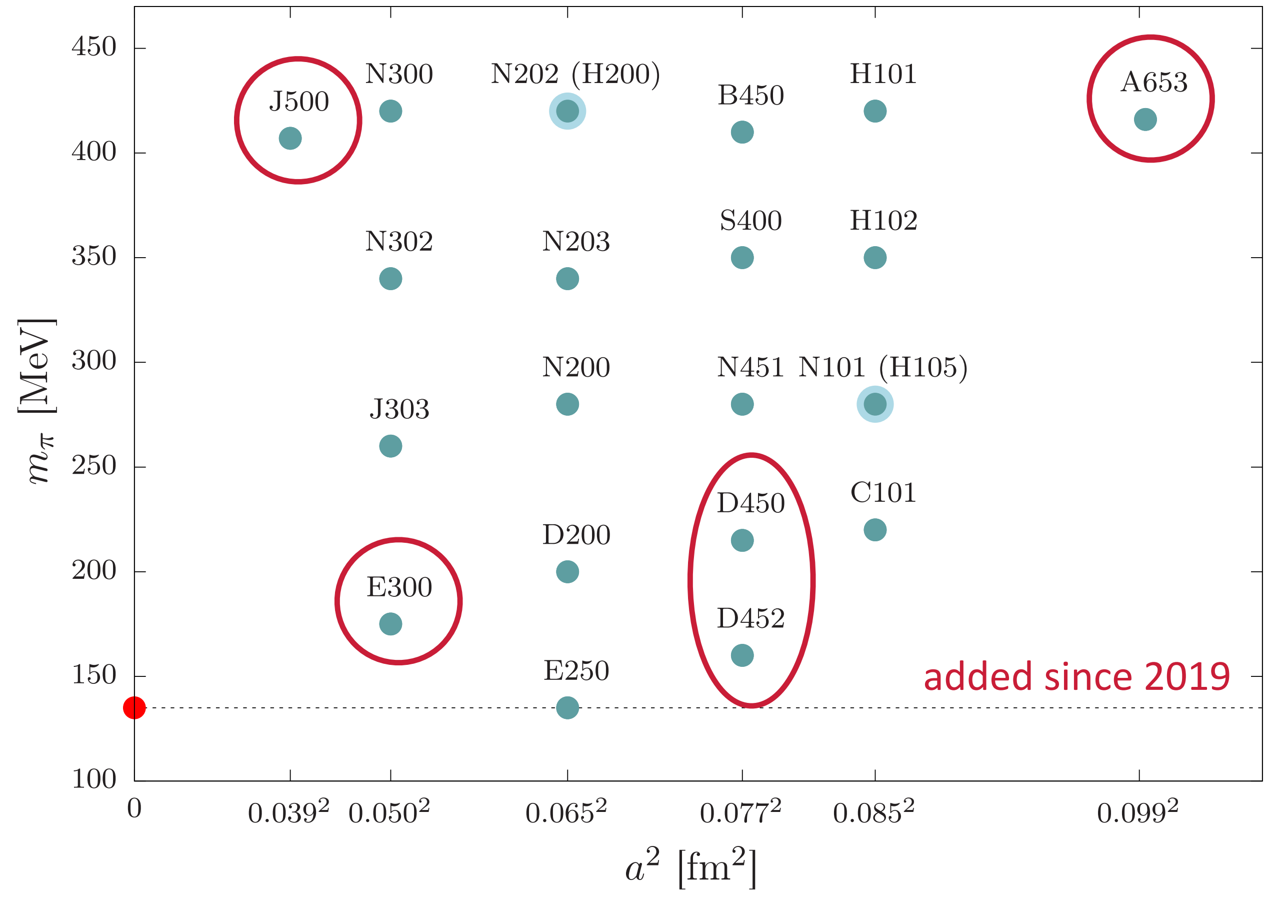
- Stout-smeared staggered fermions 2+1+1
- $L \sim 6 - 11$ fm

RBC/UKQCD [Lehner]



- Domain Wall fermions

Mainz [Wittig, Risch, Miura, San Jose, Chao]



- Wilson-clover fermions, 2+1
- $L \sim 5 - 6$ fm

ETMc [D. Giusti]

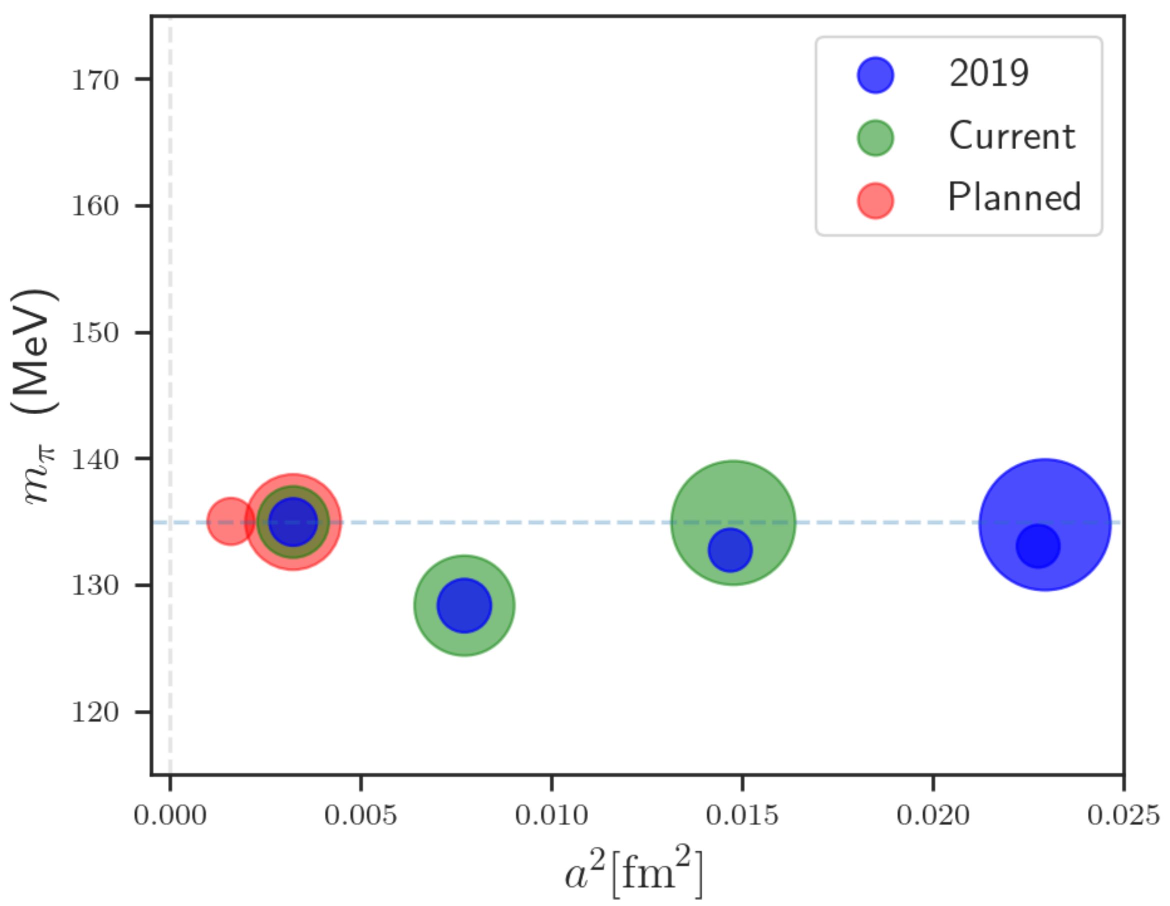
Pion masses in the range 220 - 490 MeV

4 volumes @ $M_\pi \simeq 320$ MeV and $a \simeq 0.09$ fm

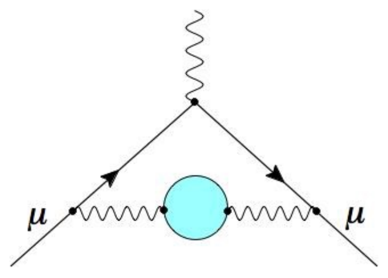
$M_\pi L \simeq 3.0 \div 5.8$

- Twisted-mass Wilson fermions, 2+1+1
- Plan to include phys. mass ensemble in future

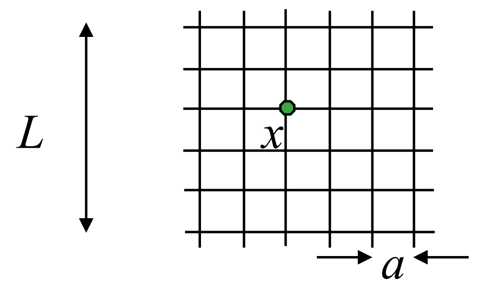
Fermilab-HPQCD-MILC [Lahert, McNeile]



- Highly Improved Staggered Quarks (HISQ) 2+1+1+1
- $L \sim 5 - 6$ fm
- Subset also used by Aubin et al



Finite volume



Finn Stokes (BMWc) @ Lattice 2021

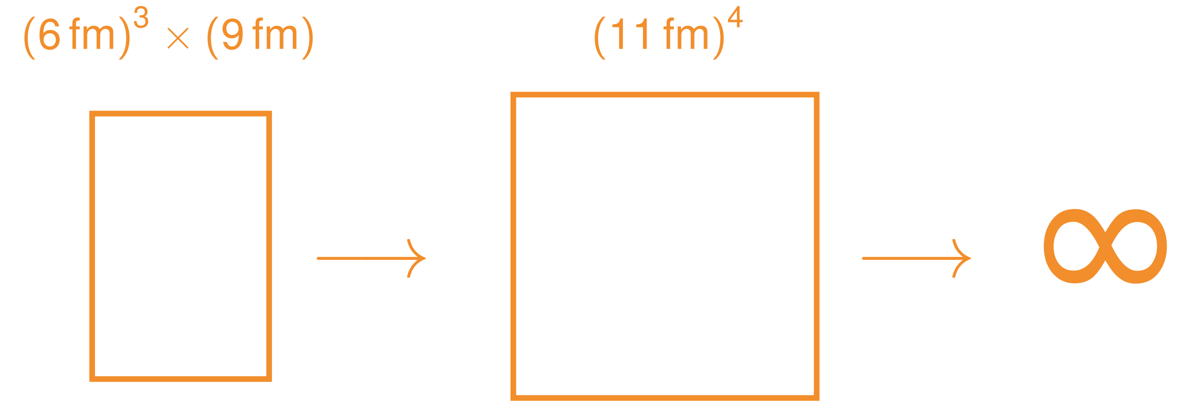
Model comparison

- Two more models for finite L (but not T)
  - Generic field-theory approach [Hansen & Patella '19, '20] (HP) relates the finite-size effect to $F_\pi(k)$
  - Rho-pion-gamma model [Chakraborty et al '17] (RHO) incorporates the $\rho(770)$ resonance directly into a χPT-like framework

- Compare finite L corrections for reference volume in infinite-T limit
- All four models agree within $\sim 2.5 \times 10^{-10}$

NNLO χPT	16.7	$\times 10^{-10}$
MLLGS	18.8	$\times 10^{-10}$
HP	17.7	$\times 10^{-10}$
RHO	16.2	$\times 10^{-10}$

Residual correction



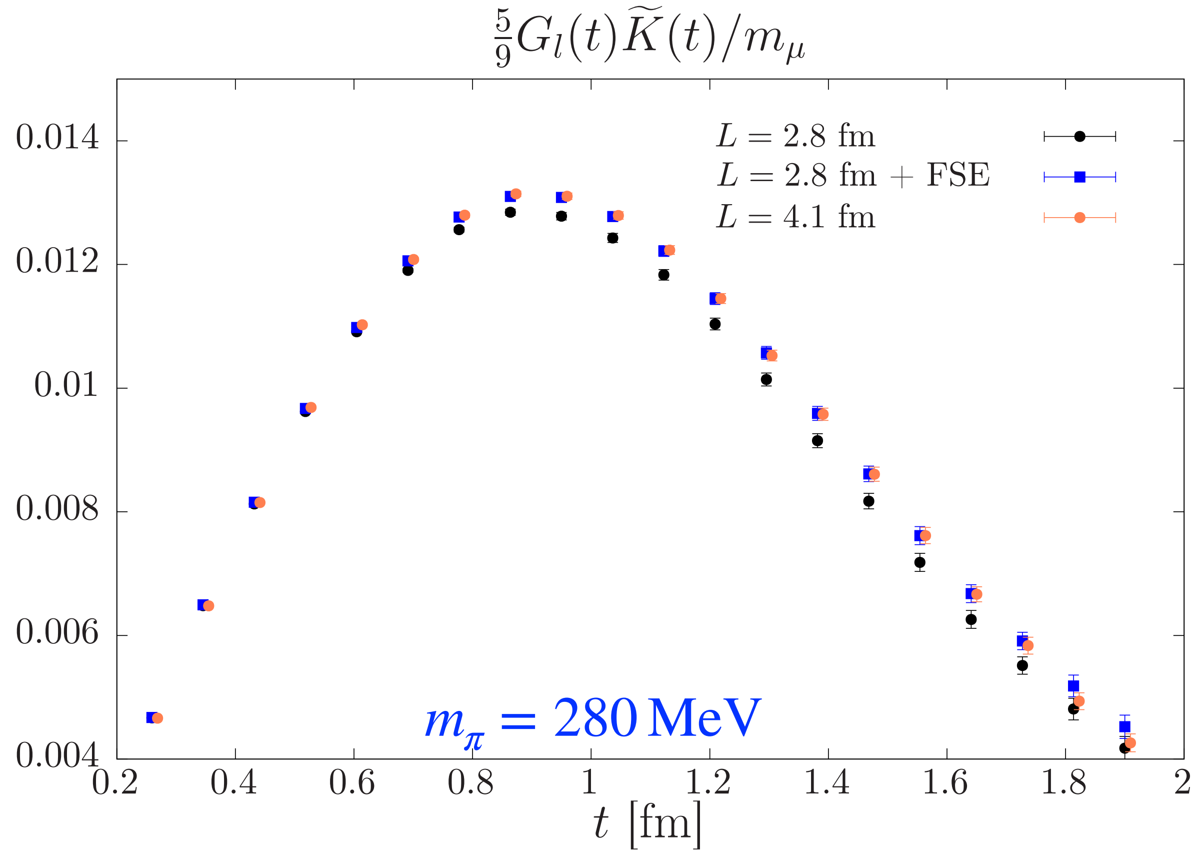
4HEX	18.1 ± 2.4		$\times 10^{-10}$
NNLO χPT	15.7	0.6 ± 0.3	$\times 10^{-10}$
MLLGS	17.8		$\times 10^{-10}$

Hartmut Wittig (Mainz) @ Lattice 2021

Mainz method (aka MLL): $G(t,L) \overset{t\to\infty}{=} \sum_n |A_n|^2 \, e^{-\omega_n t}$ $\quad G(t,\infty) = \int_0^\infty d\omega\, \omega^2 \rho(\omega^2)\, e^{-\omega|t|}$

Both $|A_n|$ and $\rho(\omega^2)$ can be related to the pion form factor $F_\pi(\omega)$ $\Rightarrow$ $G(t,\infty) - G(t,L)$

[Meyer 2011, Francis et al. 2013, Della Morte et al. 2017; Lellouch & Lüscher 2001]



Full HVP contribution:

Explicit verification at $m_\pi = 280\,\mathrm{MeV}$:

$\rightarrow$ method works remarkably well

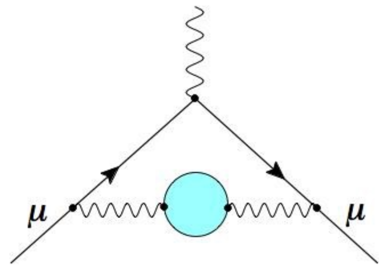
$m_\pi L = 4 \quad (L = 6.2\,\mathrm{fm})$

$\Rightarrow \quad \Delta_{\mathrm{FV}} a_\mu^{\mathrm{hvp}} = 22.6 \cdot 10^{-10} \quad (3\%)$

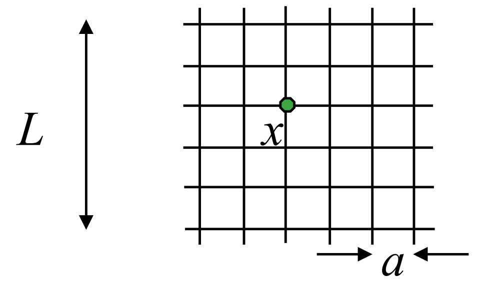
In agreement with direct lattice calculations: PACS, BMWc

[Shintani & Kuramashi 2019, BMWc 2020]

- Other direct calculations by RBC/UKQCD [Lehner @ 2019 INT workshop, Shintani & Kuramashi, PRD 2019] are consistent (but with larger errors).



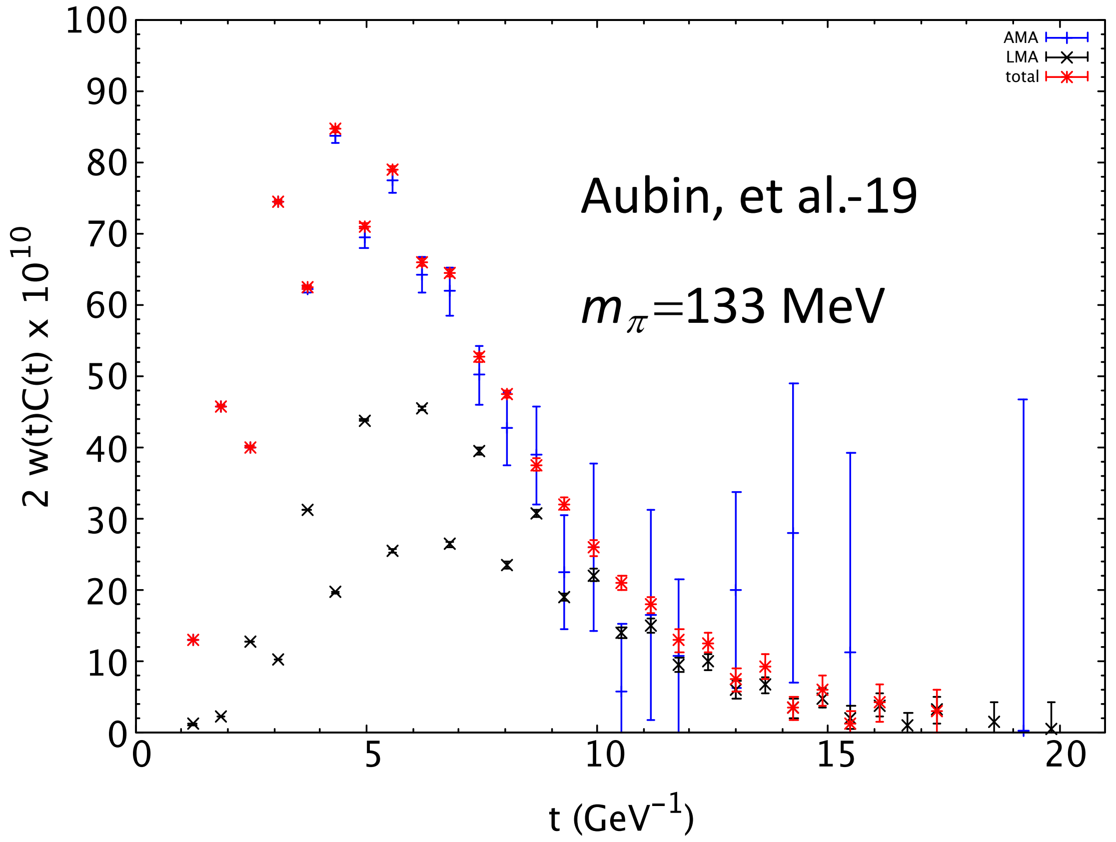
Long-distance tail



$$G(t) = \frac{1}{3}\sum_{i,x}\langle j_i(x,t)\, j_i(0,0)\rangle$$

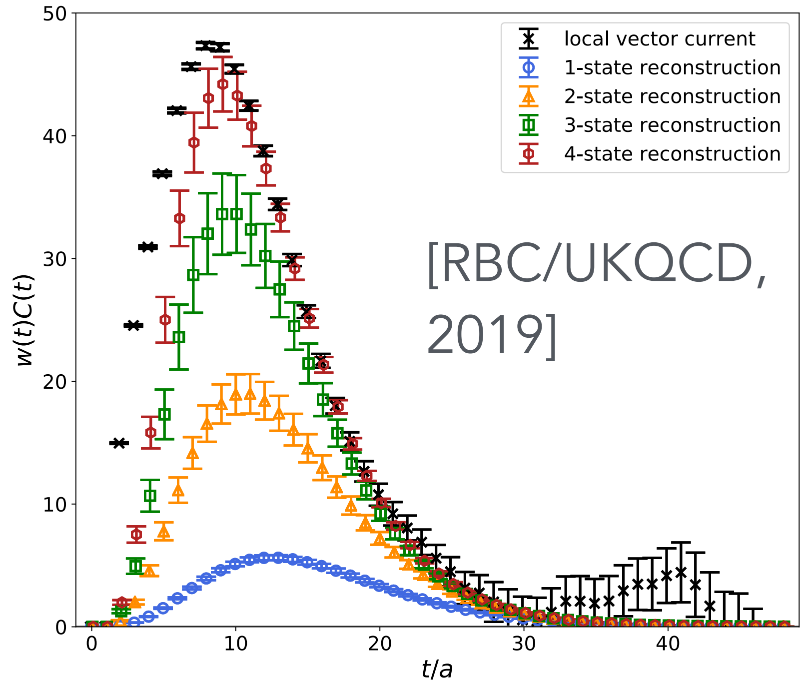
- Use noise reduction methods (AMA, LMA,…):

  Aubin et al, RBC/UKQCD, BMWc, Mainz, …



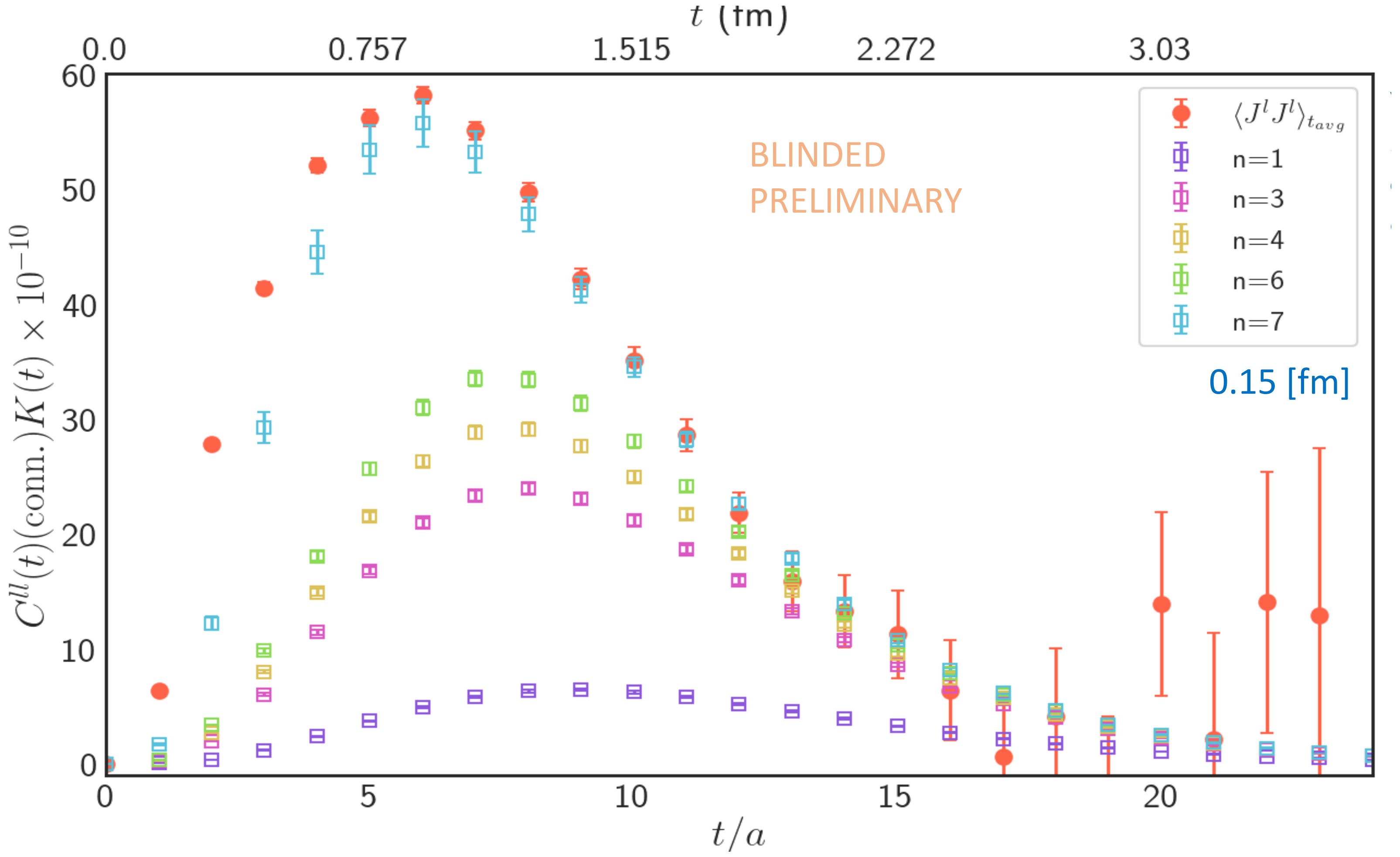
- Spectral reconstruction (RBC/UKQCD, Mainz):
  - ✦ obtain low-lying finite-volume spectrum (E_n, A_n) in dedicated study using additional operators that couple to two-pion states
  - ✦ use to reconstruct $G(t > t_c)$
  - ✦ can be used to improve bounding method:

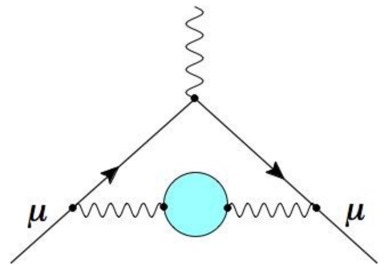
$$G(t) \to G(t) - \sum_{n=0}^{N} A_n^2\, e^{-E_n t}$$



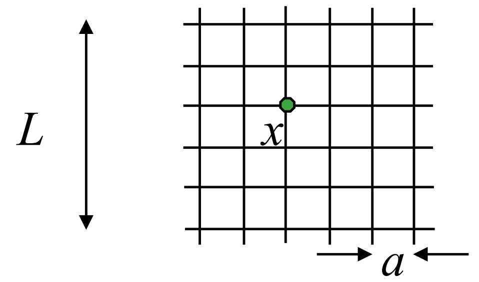
> Shaun Lahert (Fermilab-HPQCD-MILC) @ Lattice 2021

- First calculation with staggered multi-pion operators





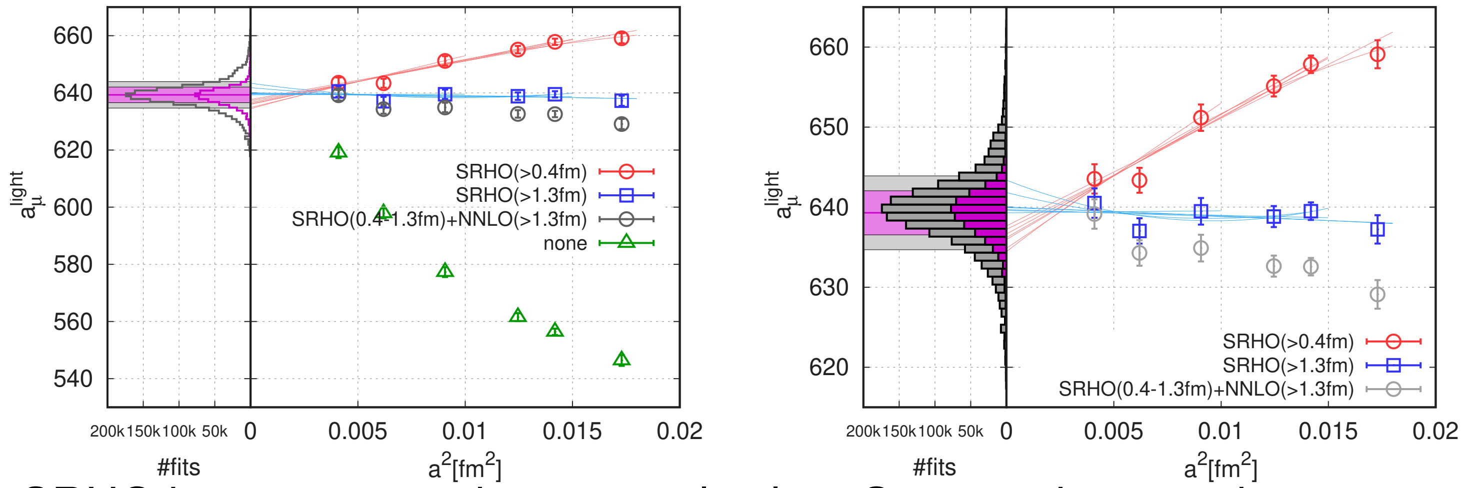
Continuum extrapolation



Kalman Szabo (BMWc) @ Lattice 2021

Taste improvement II

- $a_\mu(a) \to a_\mu(a) - a_\mu^{\rm SRHO}(a) + a_\mu^{\rm RHO}$
- reduces lattice artefact, also makes a^2 dependence linear



SRHO improvement gives central value. Systematic errors by:

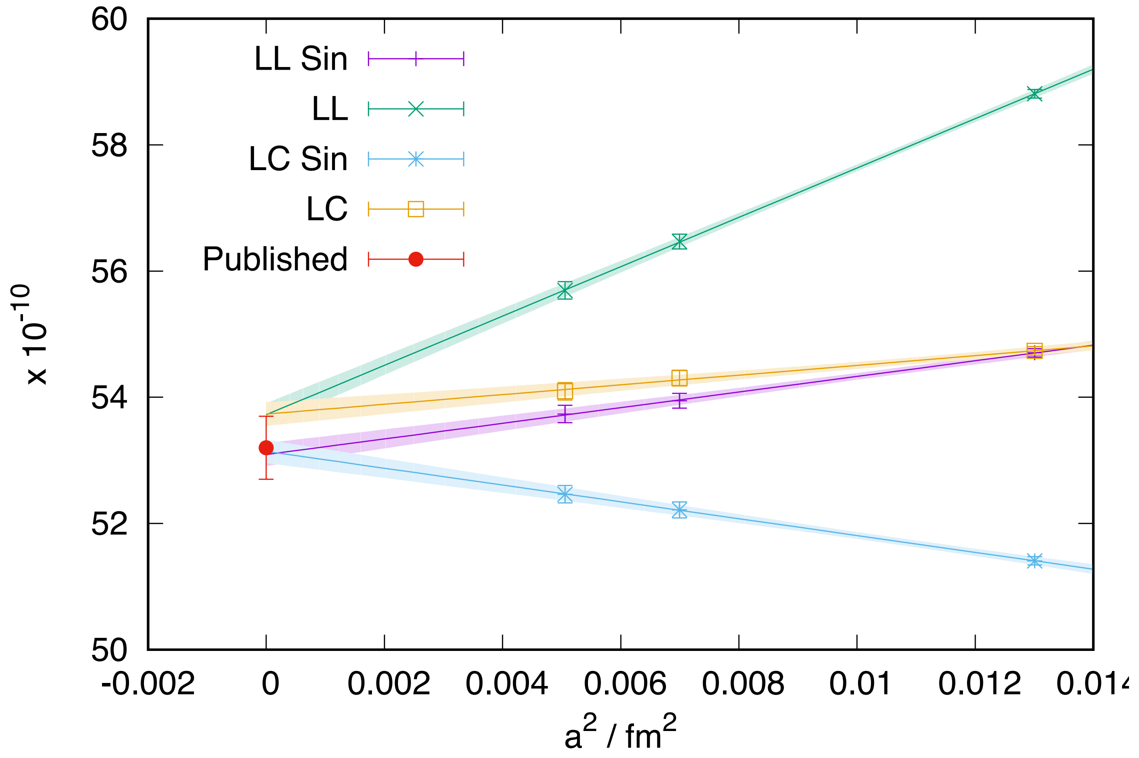
1. change starting point of improvement $t = 0.4 \to 1.3$ fm
2. skip coarse lattices
3. change $\Gamma = 0$ and $\Gamma = 3$
4. replace SRHO by NNLO SXPT above 1.3 fm

- Large taste-breaking effects with BMW set-up ➠ uncorrected data not easily fit to power series, i.e.

1. $A_0 + A_1\left[a^2\right] + A_2\left[a^2\right]^2$
2. $A_0 + A_1\left[a^2\alpha_s^3(\tfrac{1}{a})\right] + A_2\left[a^2\alpha_s^3(\tfrac{1}{a})\right]^2$

Christoph Lehner (RBC/UKQCD) @ Lattice 2021

- Third lattice spacing for strange data ($a^{-1} = 2.77$ GeV with $m_\pi = 234$ MeV with sea light-quark mass corrected from global fit):

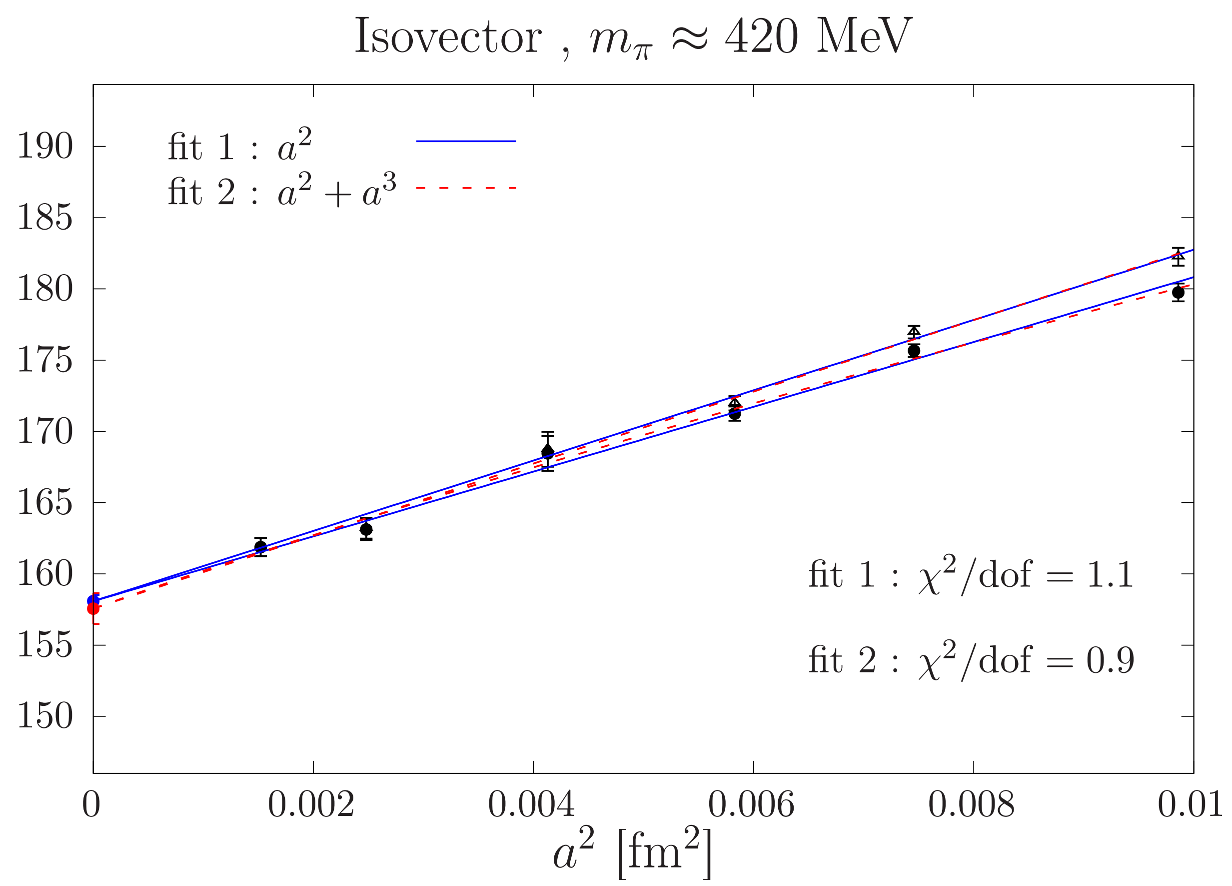


- For light quark use new 96I ensemble at physical pion mass. Data still being generated on Summit in USA and Booster in Germany ($a^{-1} = 2.77$ GeV with $m_\pi = 139$ MeV)

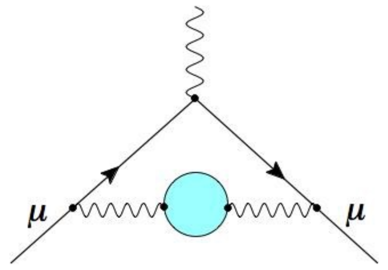
- RBC:
  Currently adding add a third lattice spacing

- Fermilab-HPQCD-MILC:
  planning to add a 5th lattice spacing (0.042 fm).

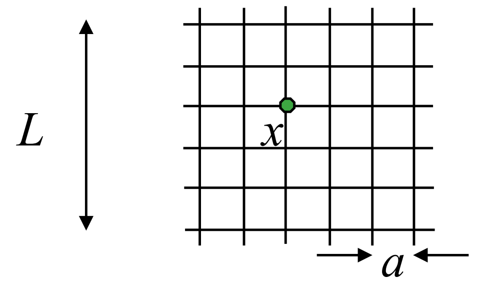
Hartmut Wittig (Mainz) @ Lattice 2021



- Mainz and ETMc perform combined chair and continuum extrapolaiton



Scale Setting

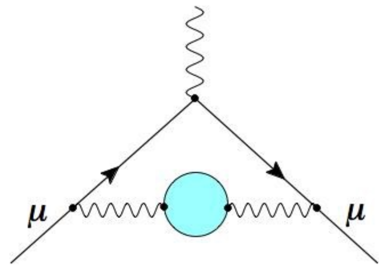


- a_μ is dimensionless, but depends on the lattice indirectly, through masses in lattice units in the Kernel. In particular, am_μ:

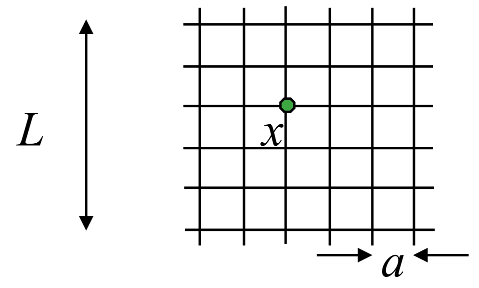
$$\frac{\delta a_\mu^{\rm hvp}}{a_\mu^{\rm hvp}} = \underbrace{\frac{1}{a_\mu^{\rm hvp}} \left| a \frac{da_\mu^{\rm hvp}}{da} \right|}_{\approx\, 1.8} \frac{\delta a}{a}$$

[H. Wittig @ 1st Muon g-2 Theory Initiative workshop; Della Morte et al, Lattice 2017]

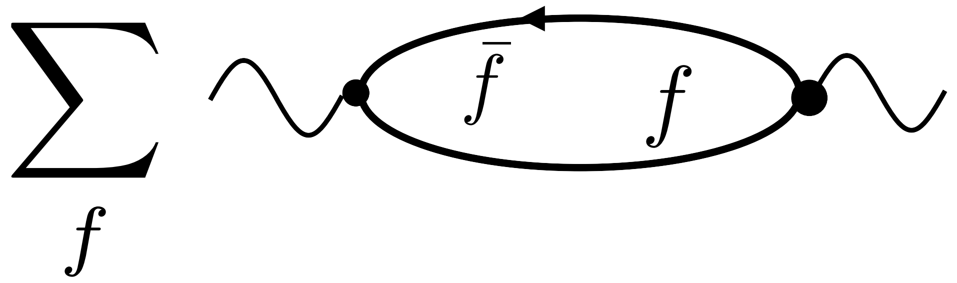
- need a good physical quantity to determine lattice spacing to high precision (< 0.2%). Currently in use:
  - f_π — depends on V_{ud} and requires radiative QED corrections
  - Ω baryon mass (RBC/UKQCD, BMW) also being adopted by Mainz, Fermilab-HPQCD-MILC

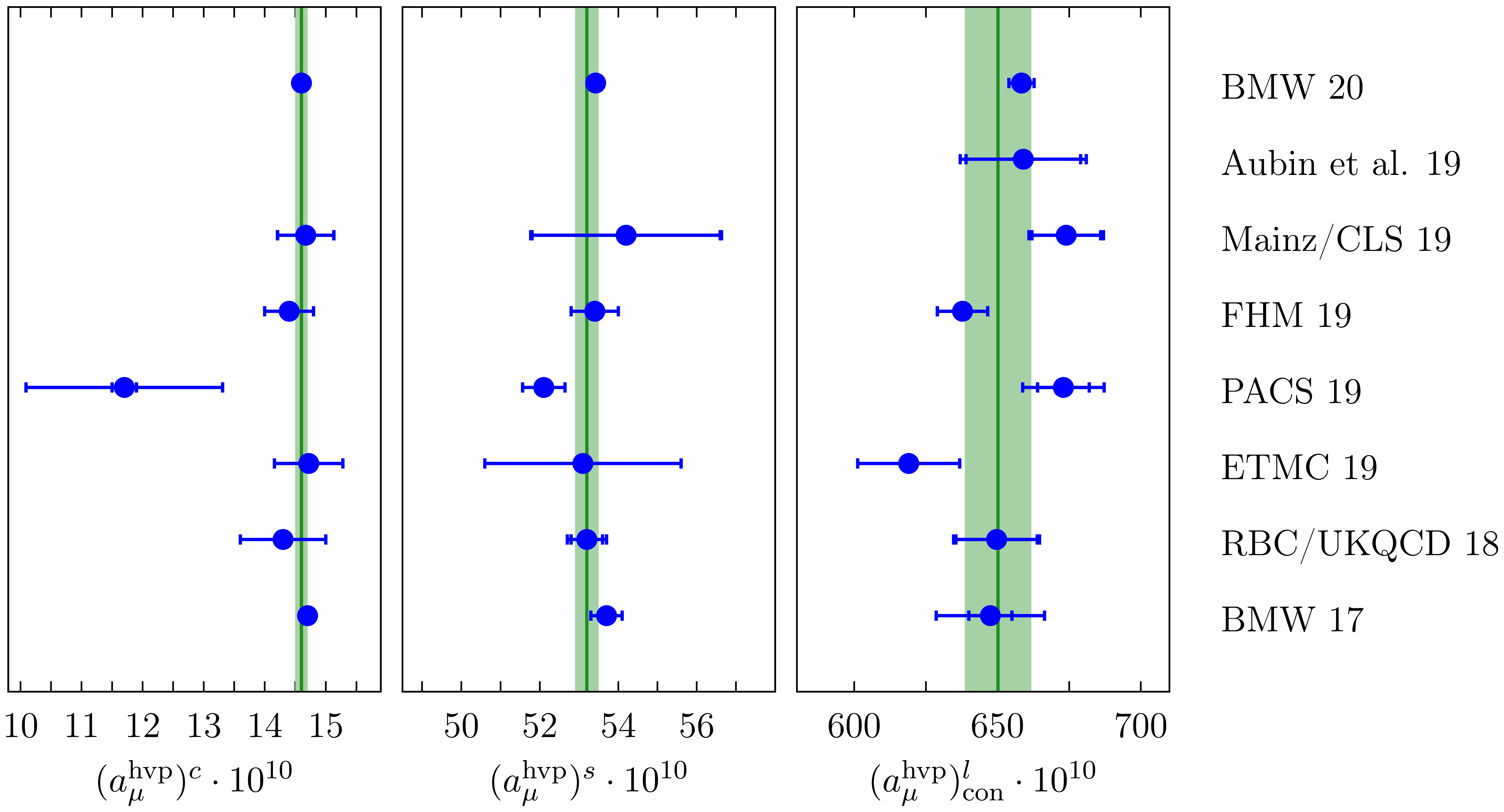


Connected contributions by flavor (ℓ, s, c)

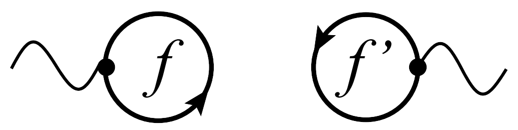


H. Wittig @ Lattice HVP workshop

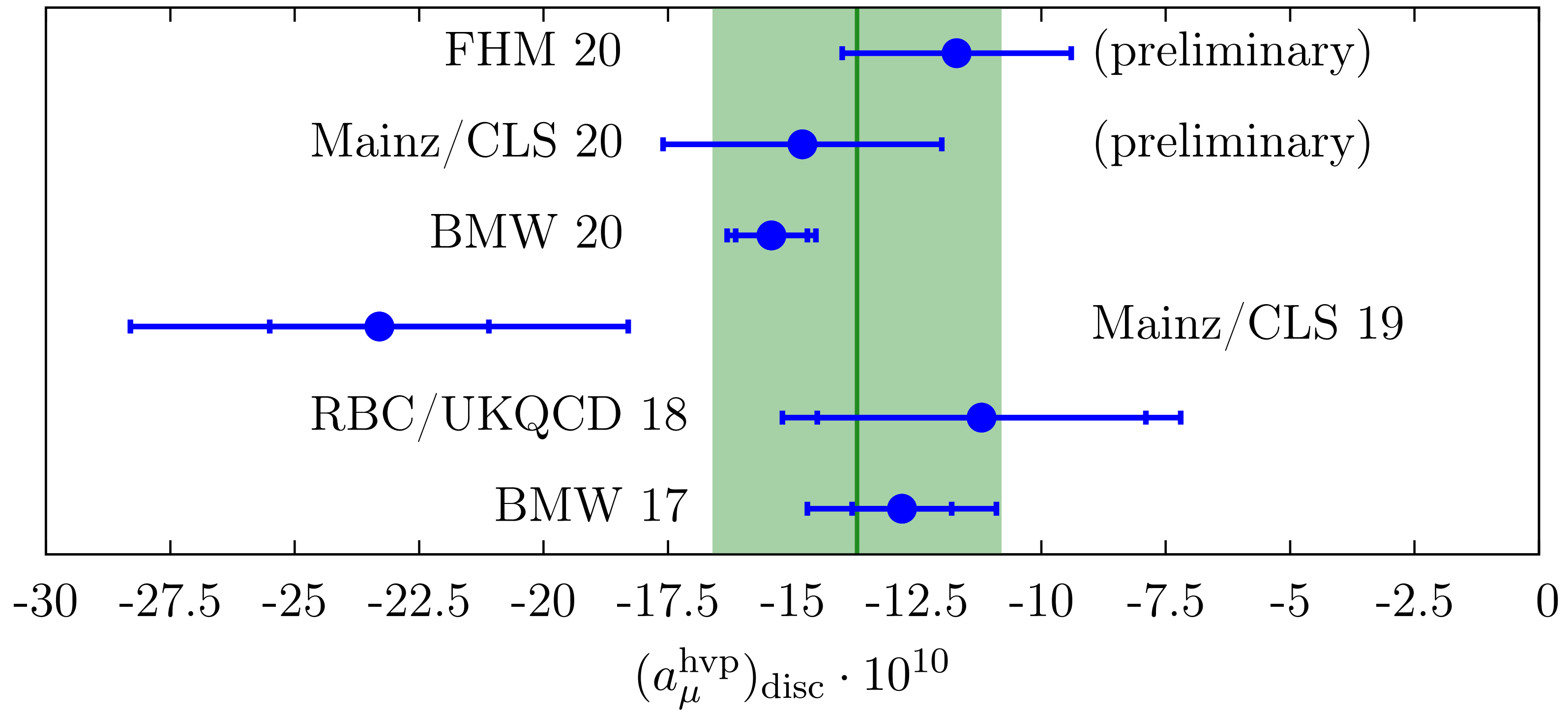




Disconnected contribution

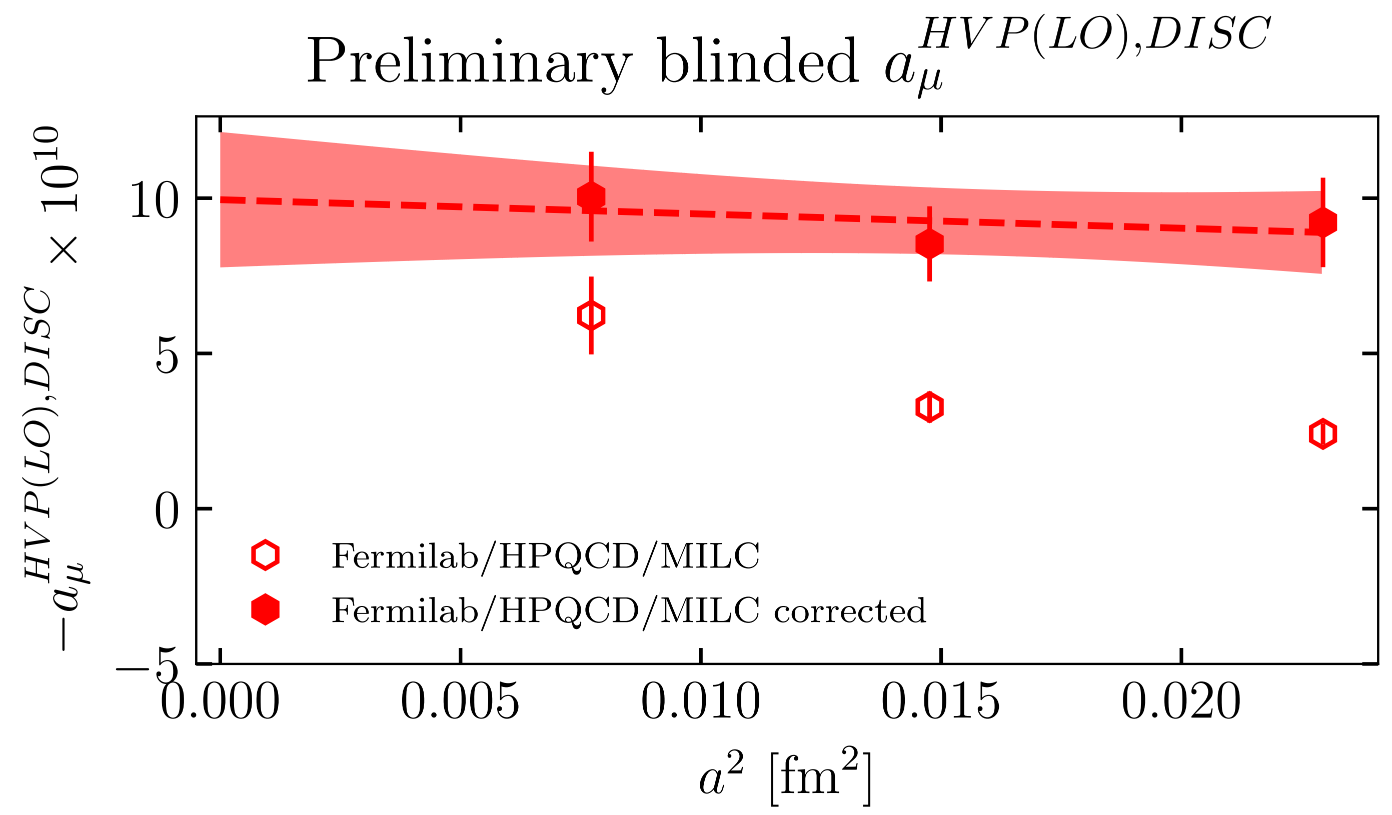


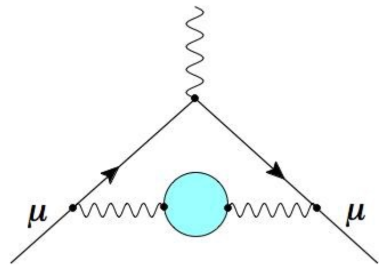
H. Wittig @ Lattice HVP workshop



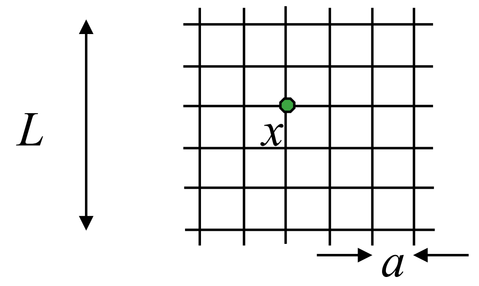
Convergence towards precise & consistent results

- C. McNeile (Fermilab-HPQCD-MILC) @ Lattice 2021



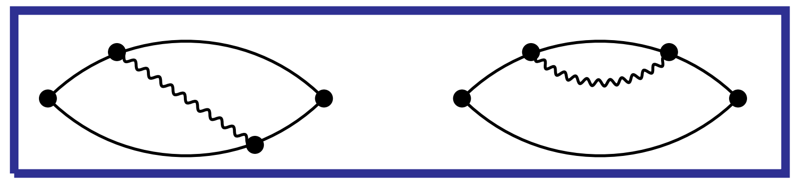


Isospin-breaking corrections

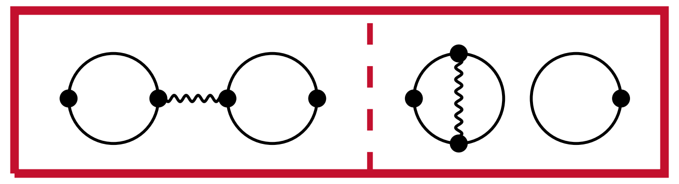


V. Gülpers @ Lattice HVP workshop

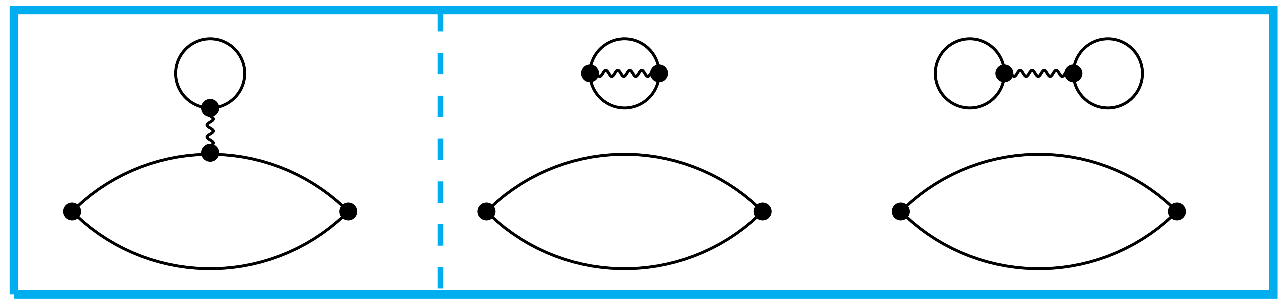
Overview of published results - contributions to $a_\mu \times 10^{10}$



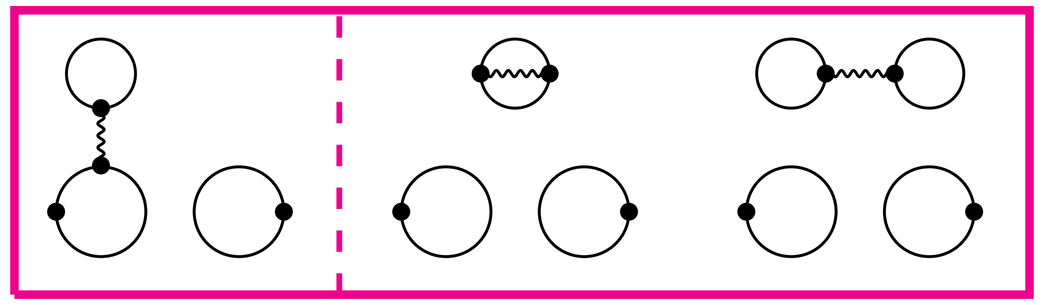
BMW $-1.27(40)(33)$
RBC/UKQCD $5.9(5.7)(1.7)$
ETM $1.1(1.0)$



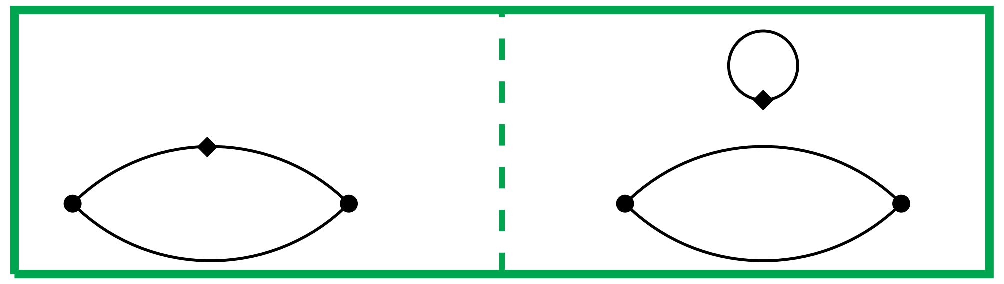
$-0.55(15)(11)$ BMW
$-6.9(2.1)(2.0)$ RBC/UKQCD



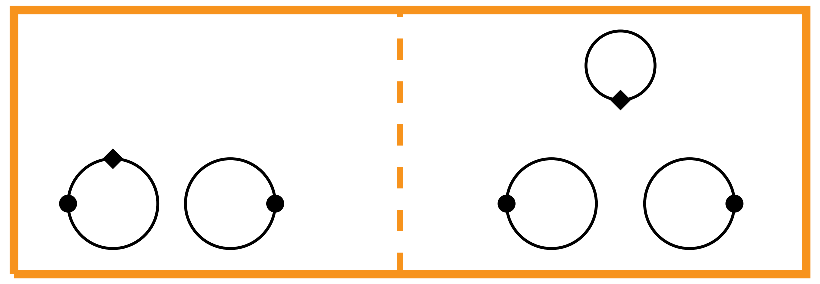
$-0.0095(86)(99)$ $0.42(20)(19)$ BMW



$0.011(24)(14)$ $-0.047(33)(23)$ BMW



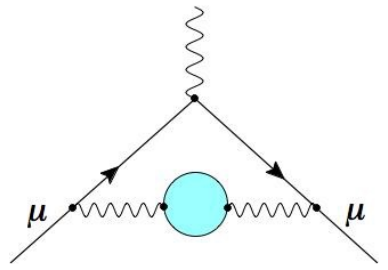
$6.59(63)(53)$ BMW
$10.6(4.3)(6.8)$ RBC/UKQCD
$6.0(2.3)$ ETM
$7.7(3.7)$ $9.0(2.3)$ FHM
$9.0(0.8)(1.2)$ LM



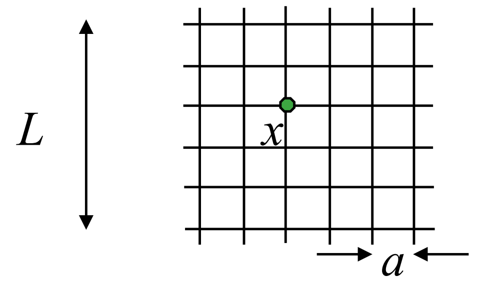
$-4.63(54)(69)$ BMW

BMW [arXiv:2002.12347]
RBC/UKQCD [Phys.Rev.Lett. 121 (2018) 2, 022003]
ETM [Phys. Rev. D 99, 114502 (2019)]
FHM [Phys.Rev.Lett. 120 (2018) 15, 152001]
LM [Phys.Rev.D 101 (2020) 074515]

- Some tensions between lattice results for individual contributions.
- Large cancellations between individual contributions:
  $\delta a_\mu^{\mathrm{IB}} \lesssim 1\,\%$
- Ongoing efforts presented by Mainz and Fermilab-HPQCD-MILC @ Lattice 2021.



Windows in Euclidean time



[T. Blum et al, arXiv:1801.07224, 2018 PRL]

$$a_\mu^{\mathrm{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt\, \tilde{w}(t)\, C(t)$$

- Use windows in Euclidean time to study the different time regions

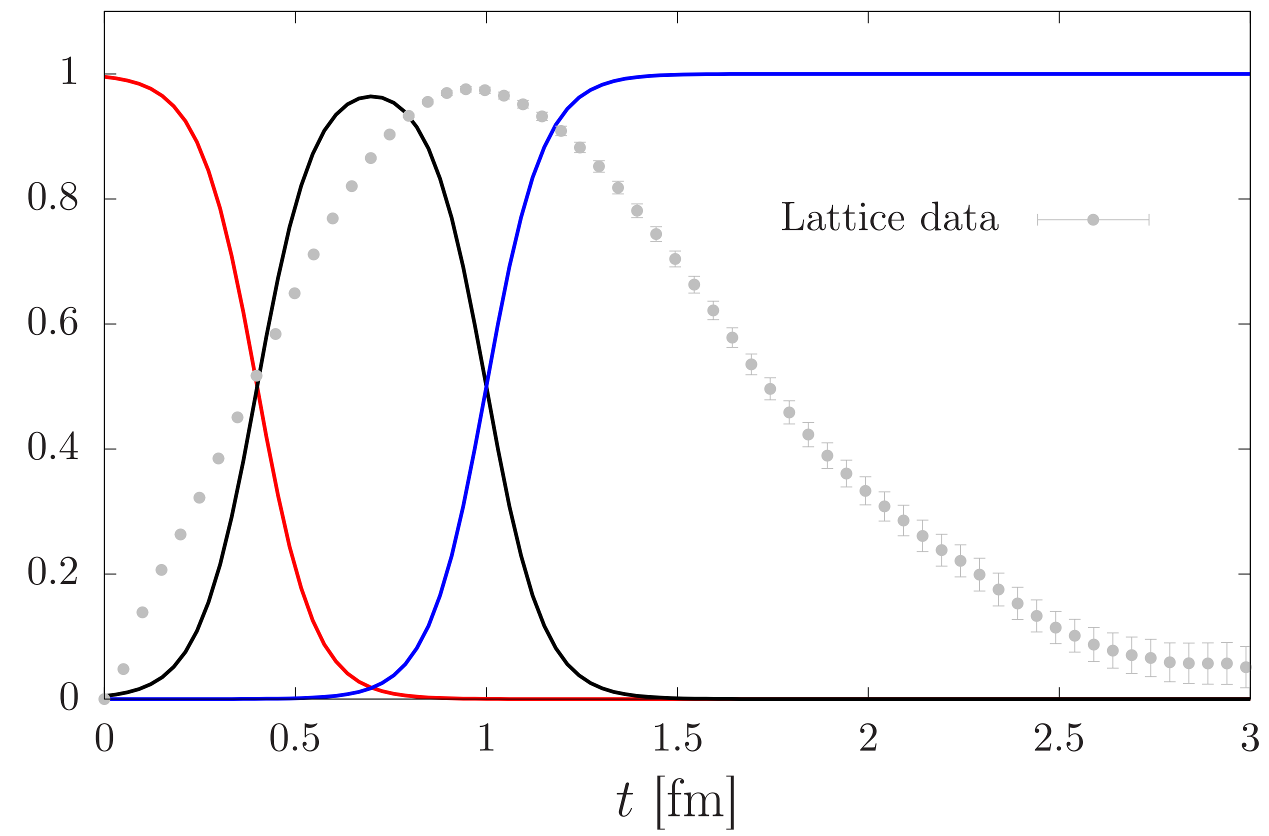
Short Distance (SD) $t : 0 \to t_0$

Intermediate (W) $t : t_0 \to t_1$

Long Distance (LD) $t : t_1 \to \infty$

$t_0 = 0.4\,\mathrm{fm}, t_1 = 1.0\,\mathrm{fm}$

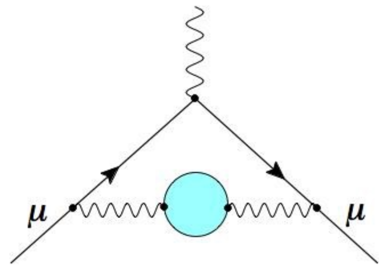
H. Wittig @ Lattice 2021



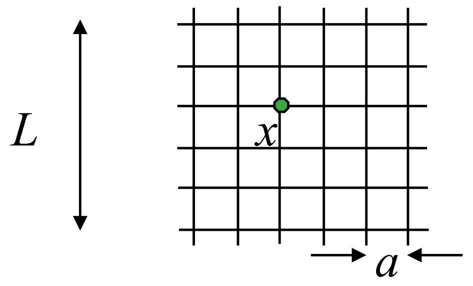
Allows separation of short- and long-distance systematic effects (discretization, FV, …)

- Compute each window separately (in continuum, infinite volume limits,…) and combine

$$a_\mu = a_\mu^{\mathrm{SD}} + a_\mu^{\mathrm{W}} + a_\mu^{\mathrm{LD}}$$

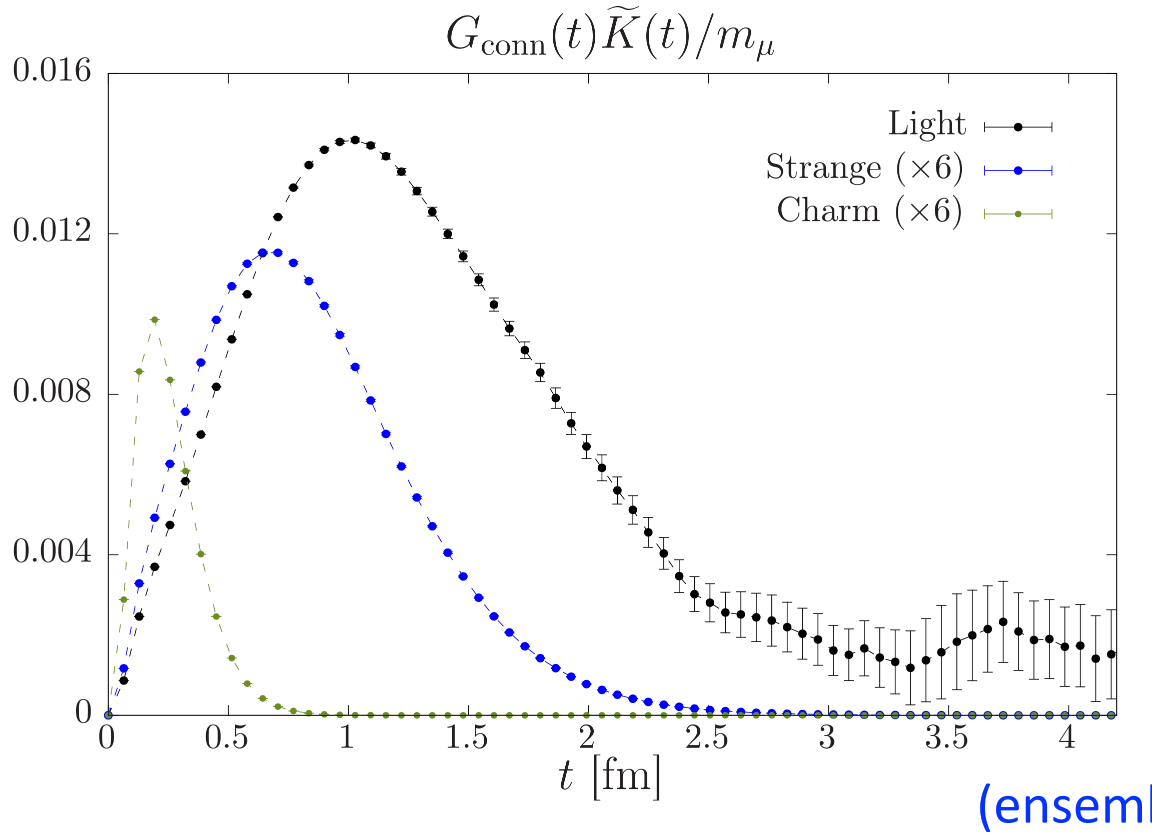


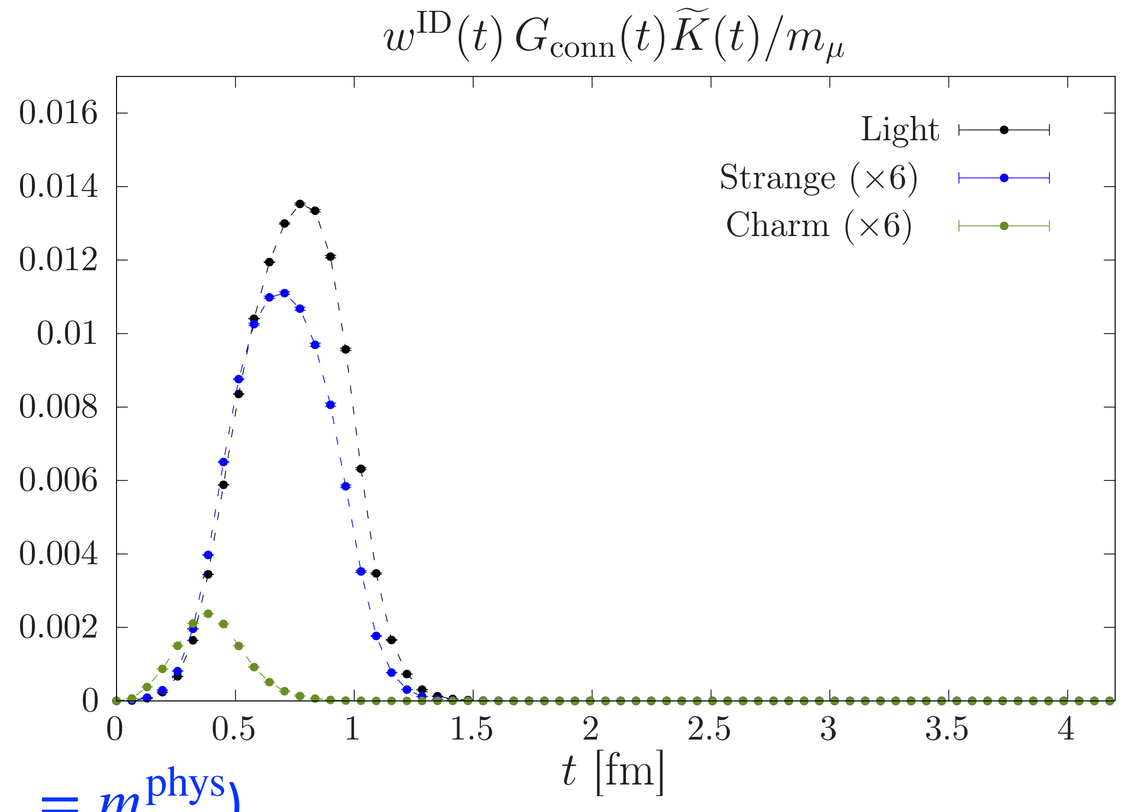
Intermediate window



- The long-distance tail is suppressed by the intermediate window weight function:

Hartmut Wittig (Mainz) @ Lattice 2021



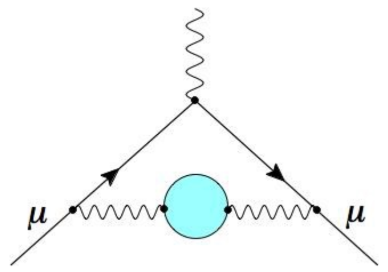


(ensemble E250: $m_\pi = m_\pi^{\rm phys}$)

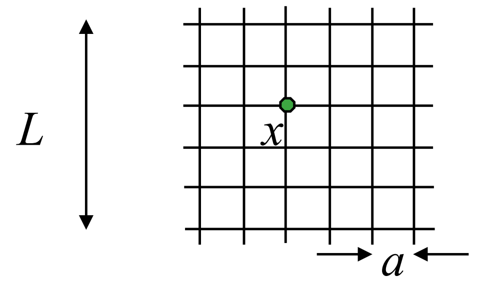
➠ can be computed efficiently with small statistical uncertainty

- Expect reduced finite volume and discretization effects

➠ commensurate uncertainties compared to dispersive evaluations



Windows in Euclidean time

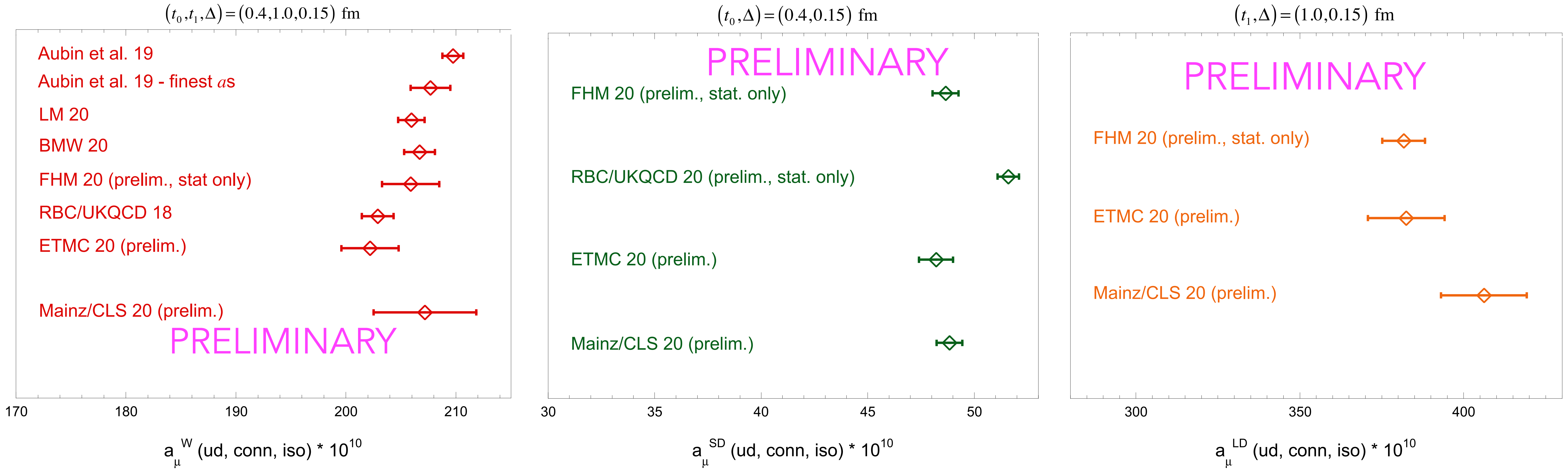


H. Wittig @ Lattice HVP workshop

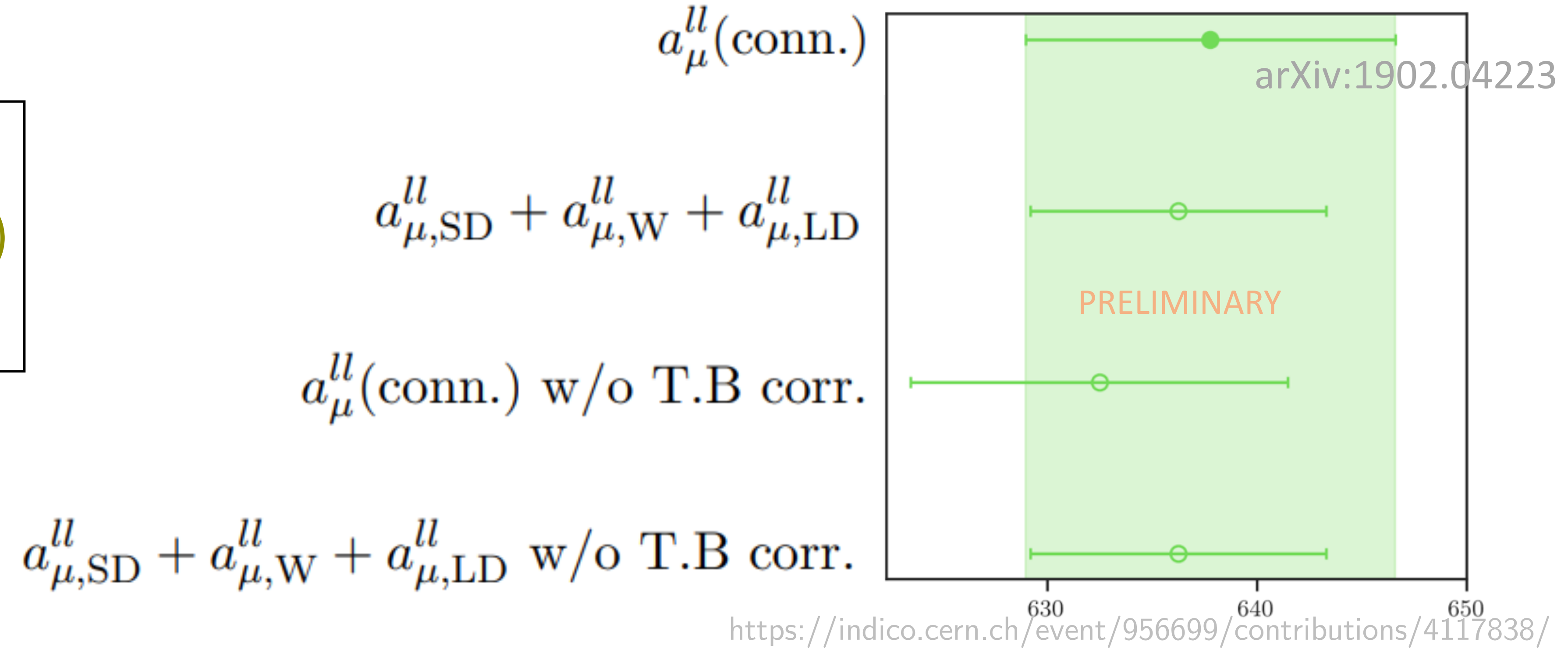
$$a_\mu = a_\mu^{\rm SD} + a_\mu^{\rm W} + a_\mu^{\rm LD}$$

"Window" quantities

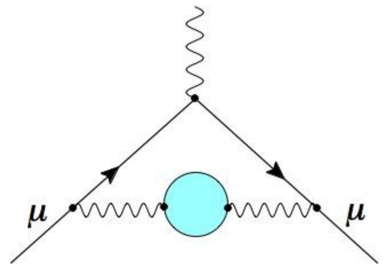
(Plots from Davide Giusti)



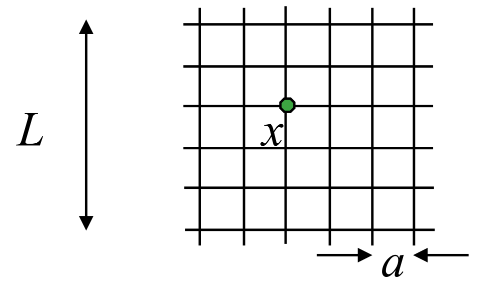
- Shaun Lahert (Fermilab-HPQCD-MILC) @ Lattice 2021



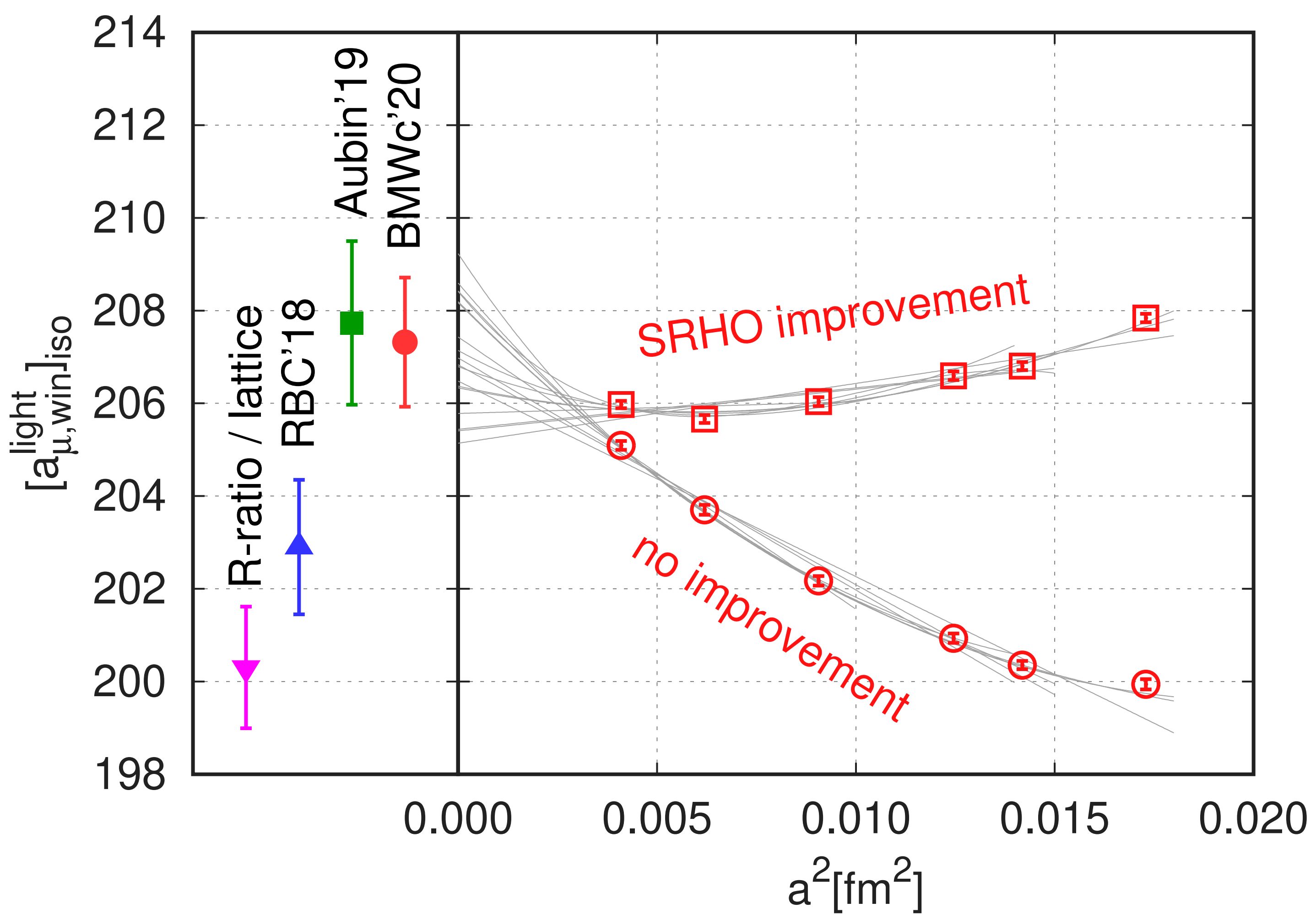
https://indico.cern.ch/event/956699/contributions/4117838/



Intermediate window (ud)



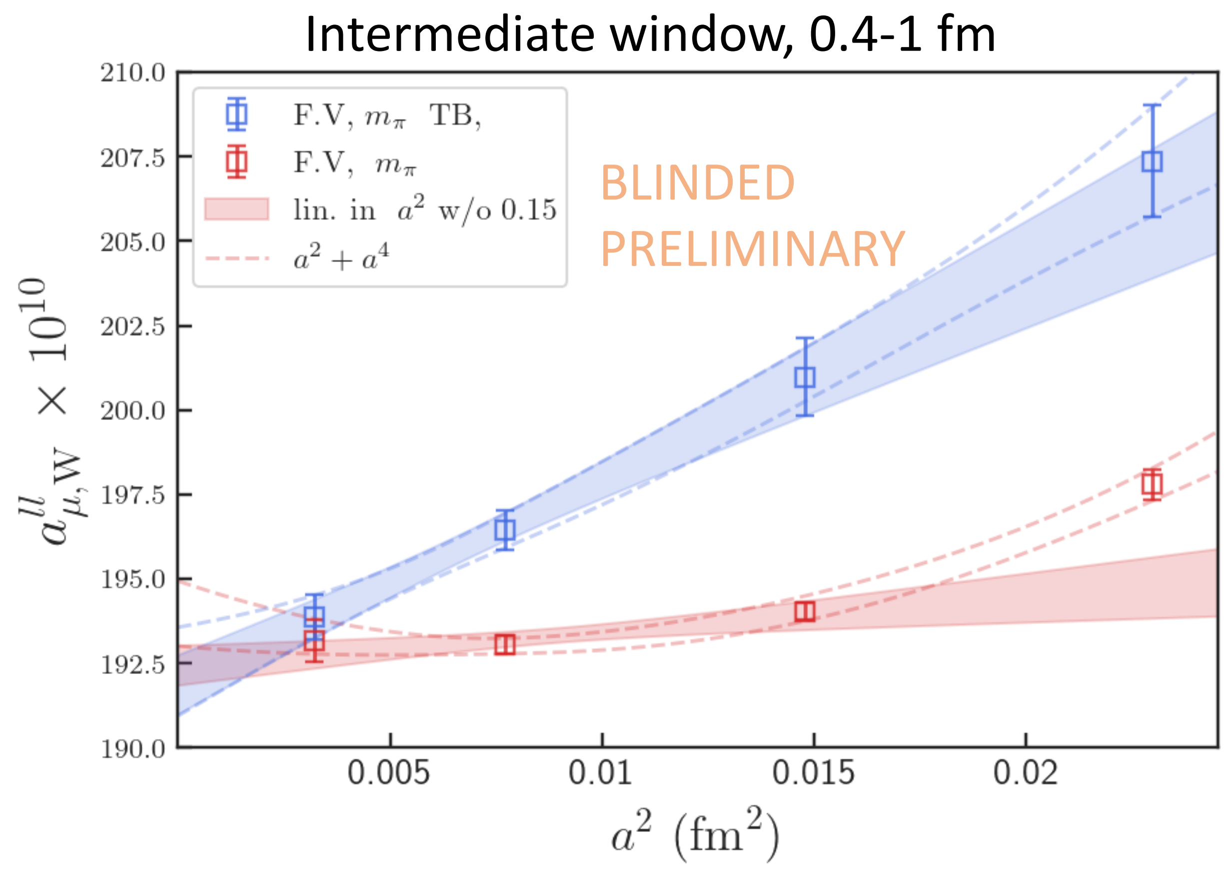
Kalman Szabo (BMWc) @ Lattice 2021



-3.7 σ tension with data-driven evaluation

-2.2 σ tension with RBC/UKQCD18

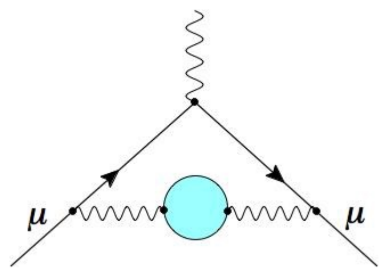
Shaun Lahert (Fermilab-HPQCD-MILC) @ Lattice 2021



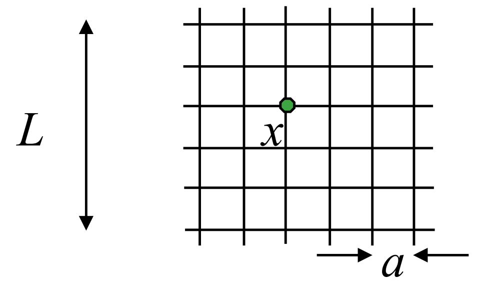
- Corrections from $\rho - \gamma - \pi\pi$ model (leading order).
- Good consistency between extrapolations of data with(out) discretization effect corrections.

Ongoing work:

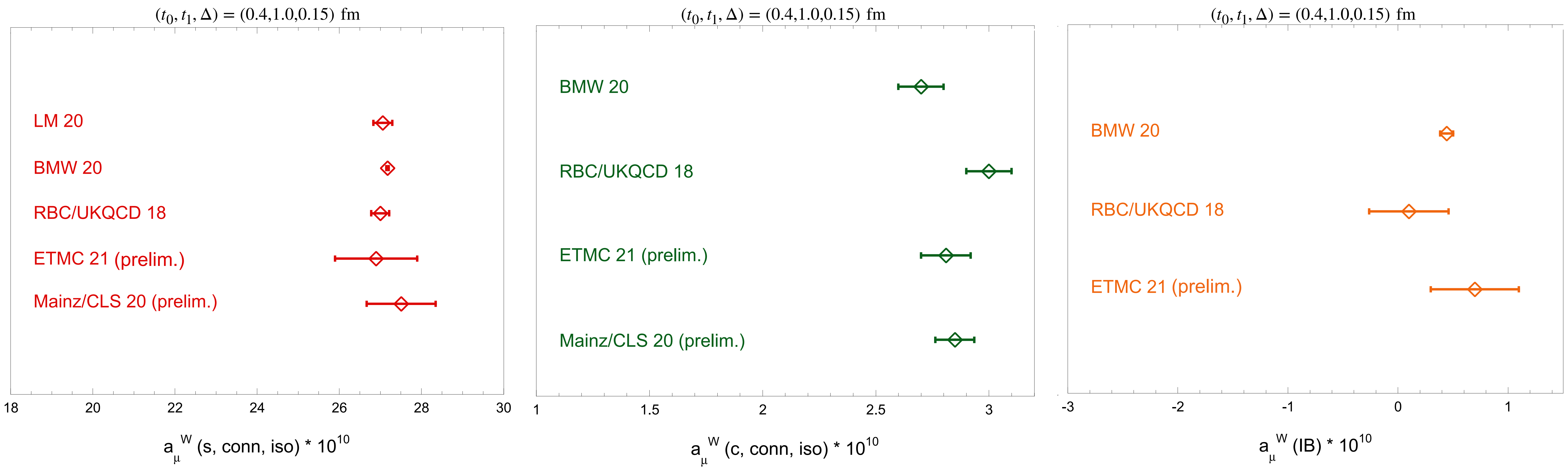
- vary functional form of extrapolation, e.g. include $\alpha_s^m a^{2n}$ terms in continuum extrapolation
- RBC/UKQCD adding a 3rd lattice spacing
- Blind analyses by Fermilab-HPQCD-MILC and RBC/UKQCD



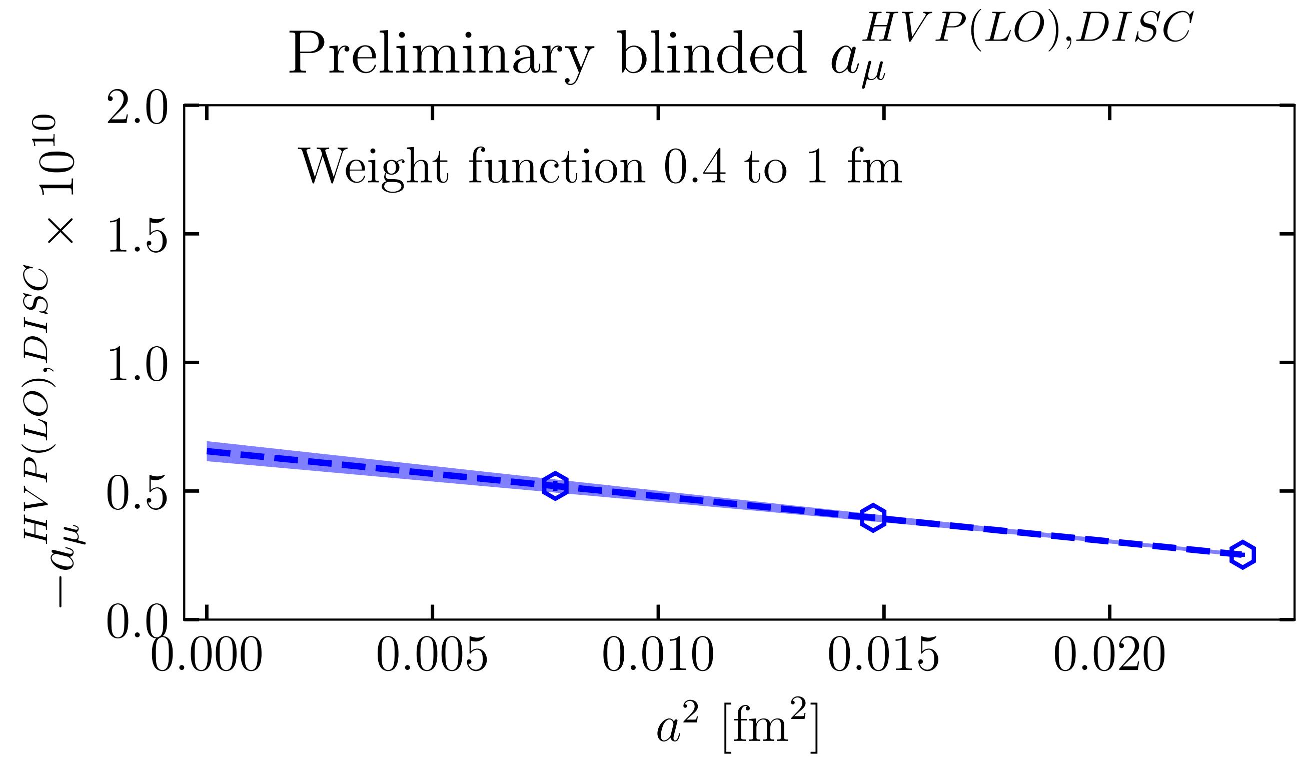
Intermediate windows: other contributions



Davide Giusti (ETMc) @ Lattice 2021



C. McNeile (Fermilab-HPQCD-MILC) @ Lattice 2021

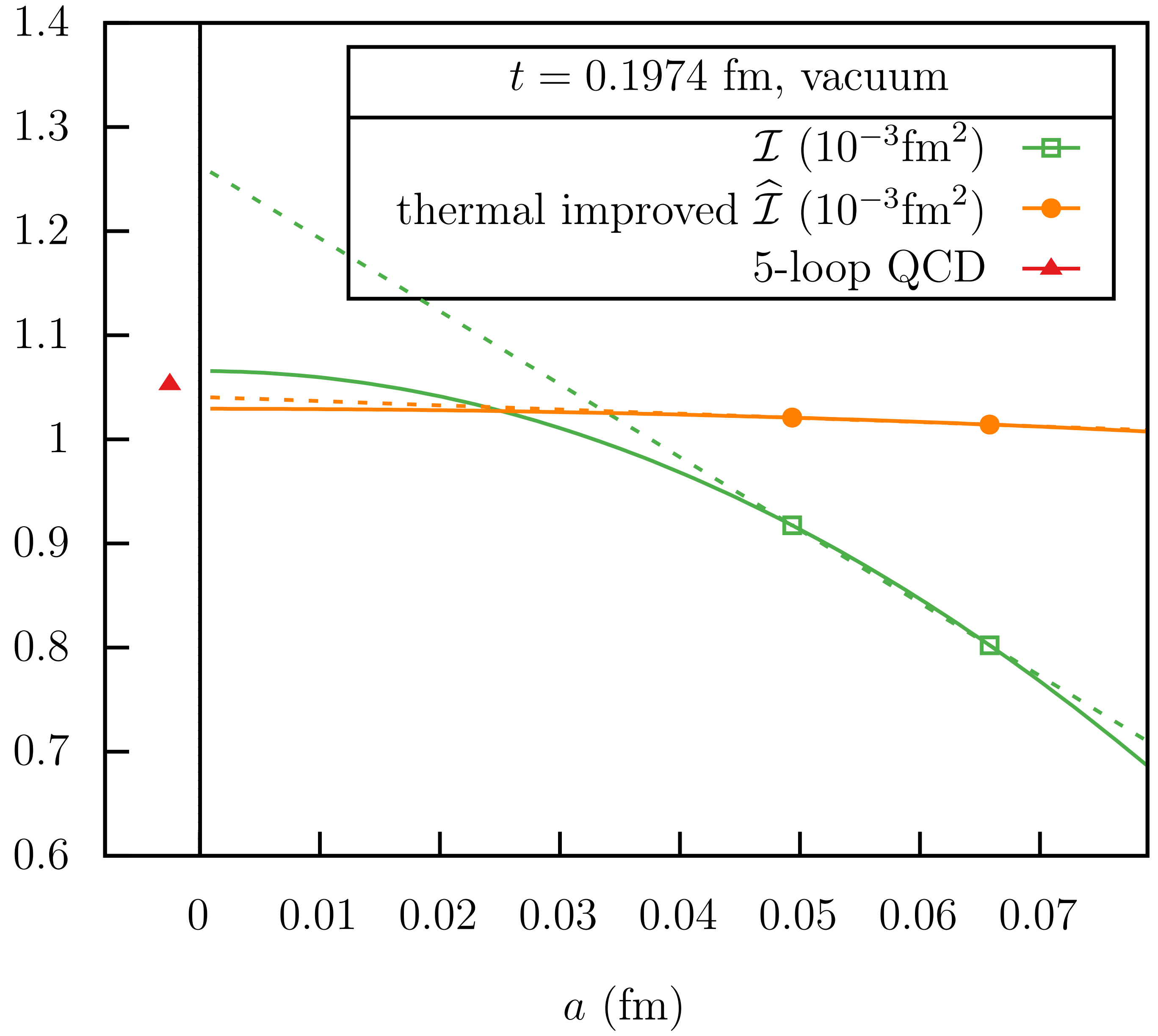


Hartmut Wittig (Mainz) @ Lattice 2021

Short-distance corrections

Tim Harris (NEPhEU QCD) @ Lattice 2021

- Use thermal observables and gauge ensembles at finite temperature and very fine lattice spacings to resolve discretization effects in calculations of short-distance quantities
- Construct improved observables with smaller discretization effects
- Can be applied to SD window and $\Delta\alpha$
- New step-scaling method for computing $\Delta\alpha$ at large Q^2



Nicolai Husung (DESY) @ Lattice 2021

- Compute anomalous dimensions of higher dimensional operators in Symanzik EFT

$$\frac{\mathcal{P}(a)}{\mathcal{P}(0)} = 1 + a^{n_{\min}} \underbrace{[\alpha(1/a)]^{\hat{\Gamma}}}_{=1,\text{ classically}} \text{const.} + \mathrm{O}(a^{n_{\min}+1}, a^{n_{\min}} \alpha^{\hat{\Gamma}+1}(1/a), \ldots)$$

$\neq 1$, due to quantum corrections

$$\alpha(1/a) \overset{a \searrow 0}{\sim} -\frac{1}{\ln(a\Lambda)}$$

- Use to guide continuum extrapolation
- In most cases studied: $\hat{\Gamma} \gtrsim 0$

Summary and Outlook

- ☆ first LQCD calculation with sub-percent (0.8%) uncertainty by BMWc but in tension with data-driven approach
- ☆ dedicated efforts by several other lattice groups
  - ➠ expect more sub-percent precision LQCD results soon.
- ☆ Theory Initiative:
  - a "WP process" for Lattice HVP and HLbL results
  - panel discussion at the KEK workshop
    - ➠ developing guidelines for assessing lattice HVP calculations
  - List of quantities to compare between lattice groups
  - prescription for defining isospin limit and separating QED & SIB contributions
- ☆ Blinding lattice calculations is good practice to avoid unintended bias

➠ Next workshop of the Muon g-2 Theory Initiative: Sep 2022