A dispersive estimate of the $f_0(980)$ contribution to $(g\text{-}2)_\mu$

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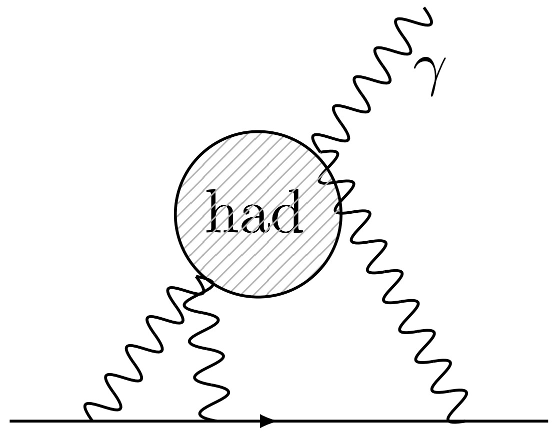
SFB 1044
THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

JG|U

QCD contributions to (g-2)$_\mu$



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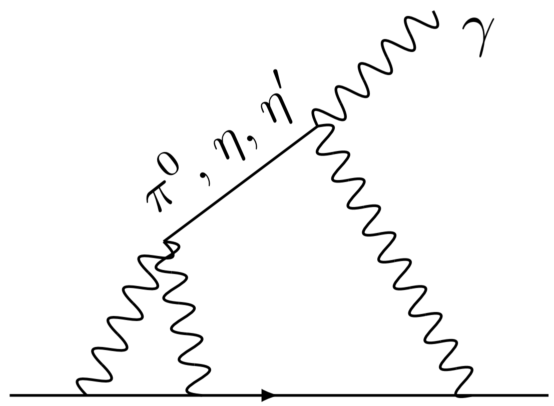

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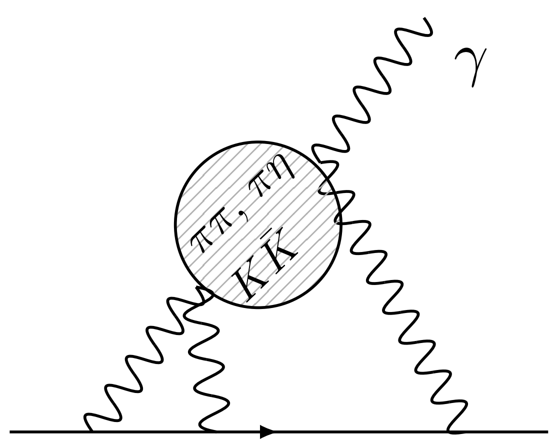
Relies on measurements of **TFF** $\pi^0\gamma^*\gamma^{(*)}, \eta\gamma^*\gamma^{(*)}, ...$
to reduce the model dependence



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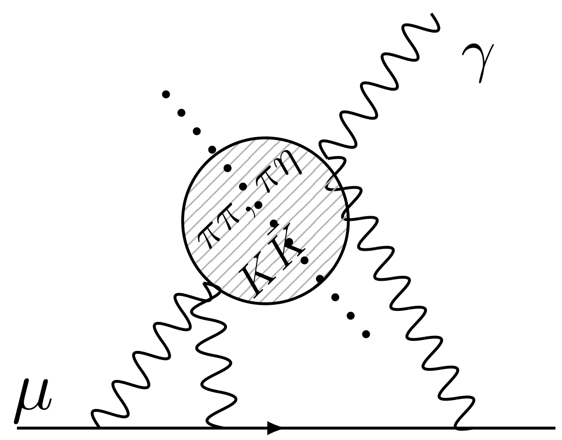
Dispersive analysis for ππ, KK, πη, … loops is needed

…

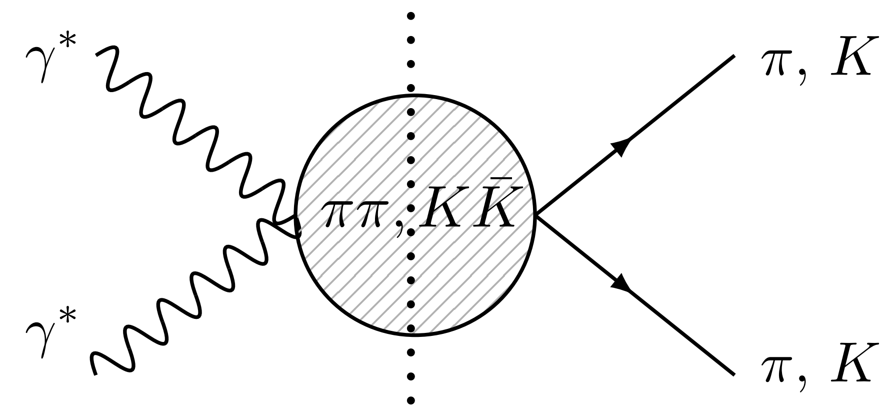
Multi-meson production

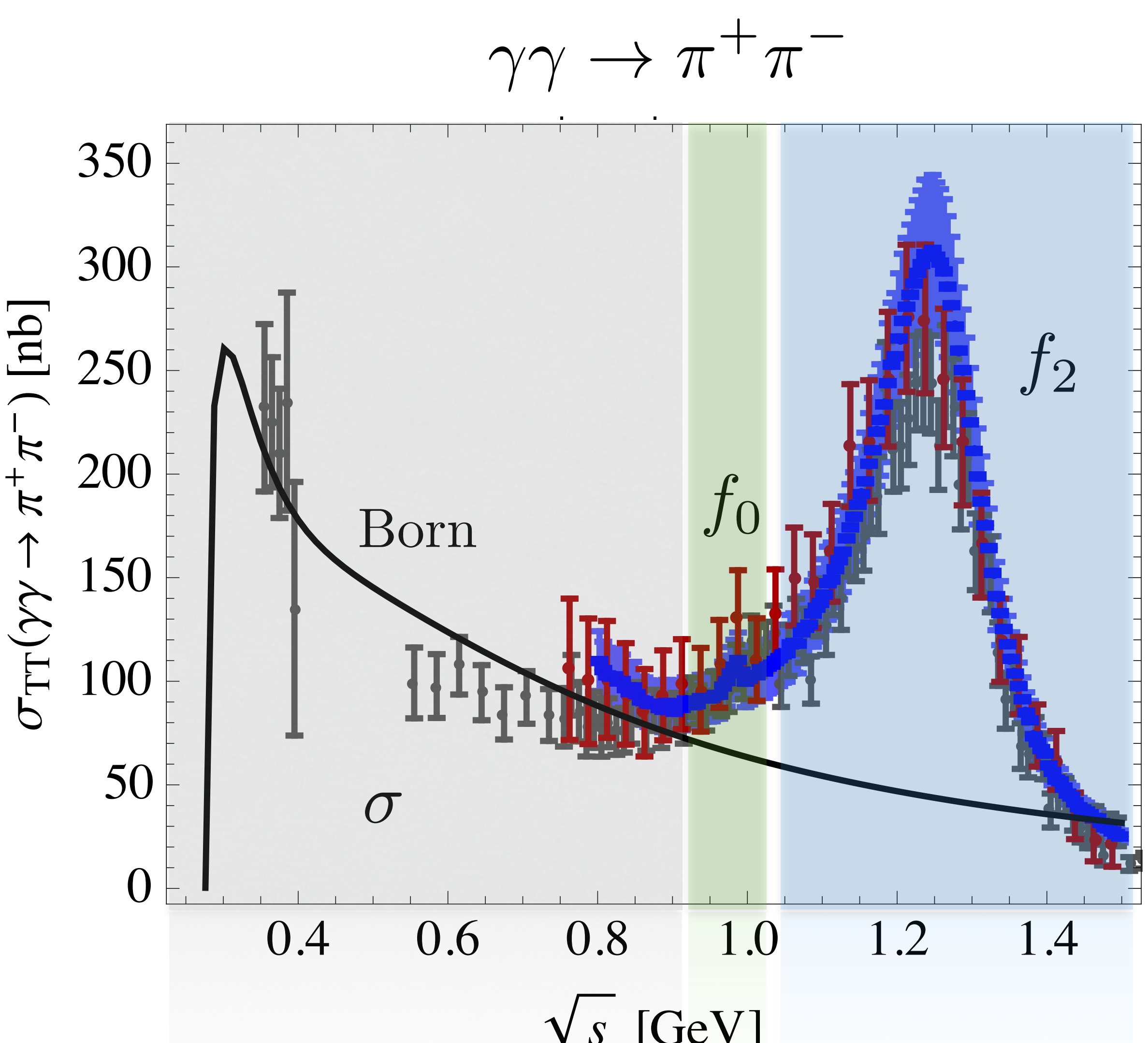


Important ingredients:

$\gamma^*\gamma^* \to \pi\pi, K\bar{K}, \ldots$

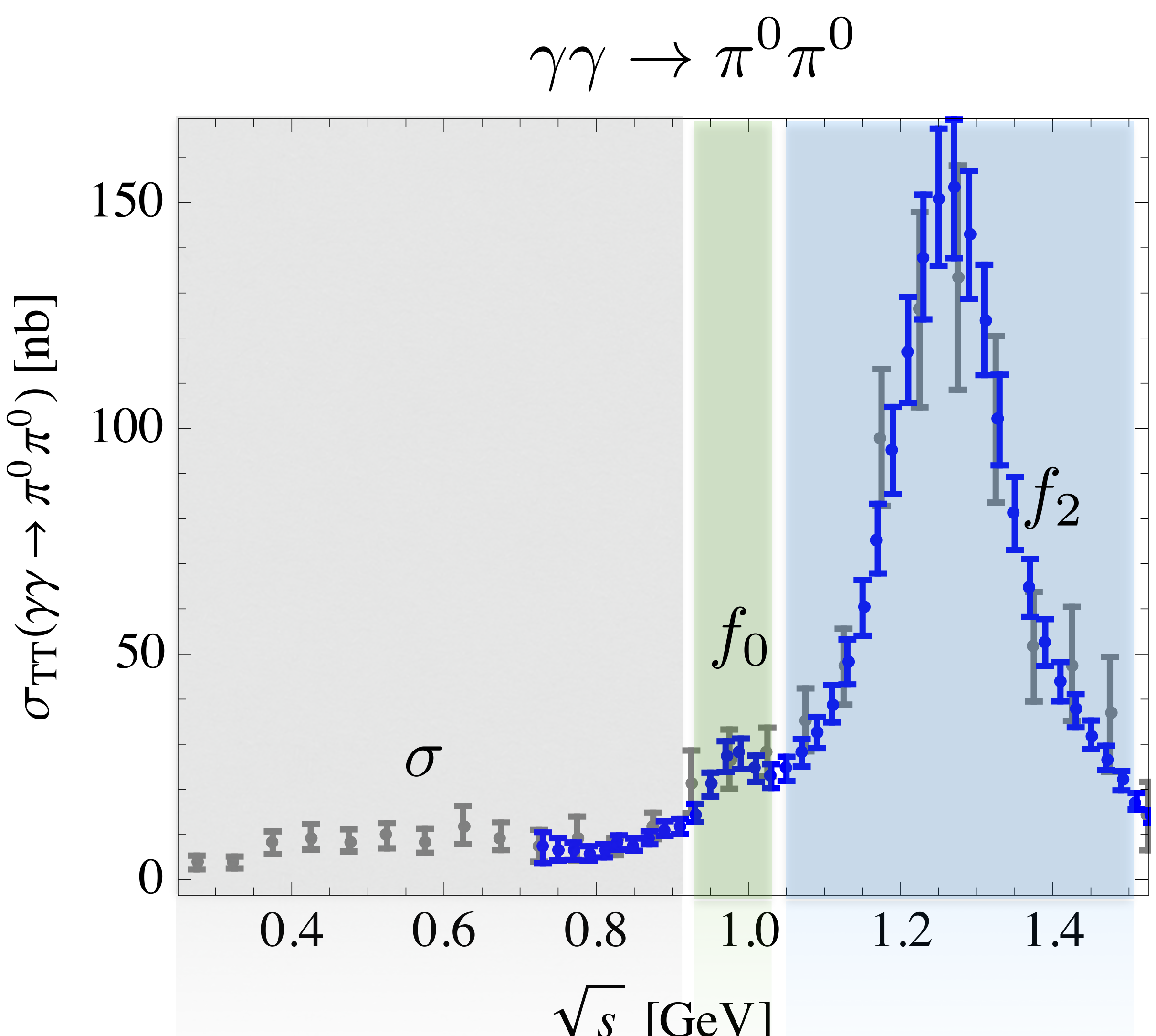
$q^2 = -Q^2 < 0$ space-like γ^*





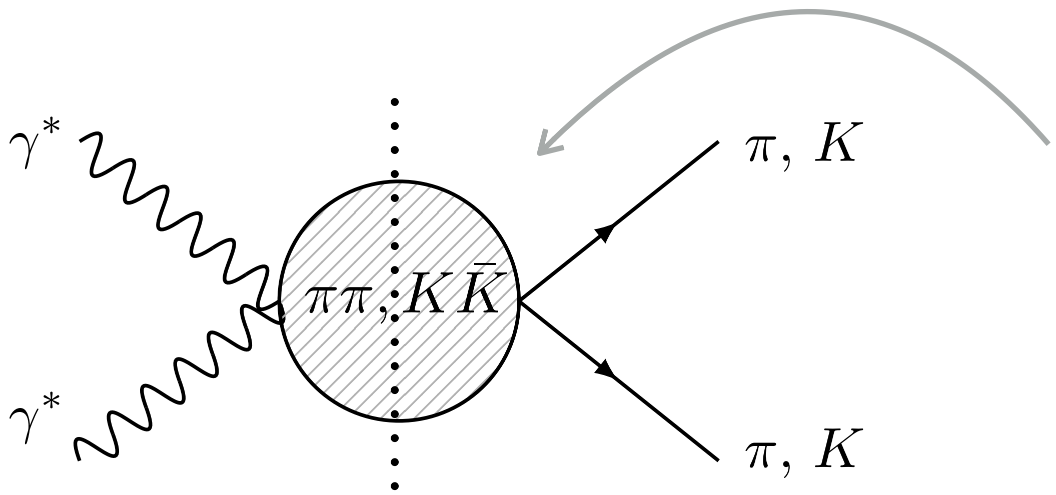
MarkII('90), CELLO ('92), Crystal Ball ('90), Belle ('07 '09)

Crystal Ball ('90), Belle ('09)



+ Belle (2016) $\gamma\gamma^* \to \pi^0\pi^0$, $Q^2 > 3.5$ GeV^2

Right-hand cuts (hadronic input)



$$\begin{pmatrix} \pi\pi \to \pi\pi & \pi\pi \to K\bar{K} \\ K\bar{K} \to \pi\pi & K\bar{K} \to K\bar{K} \end{pmatrix}_{I=0,\,J=0}$$

- **Unitarity relation** for the p.w. amplitude
  ➢ guarantees that the p.w. amplitudes behave asymptotically no worse **than a constant**

$$\mathrm{Disc}\, t_{ab}(s) = \sum_c t_{ac}(s)\,\rho_c(s)\,t^*_{cb}(s)$$

$$-\frac{1}{2\rho_1} \leq \mathrm{Re}\, t_{11}(s) \leq \frac{1}{2\rho_1}, \quad 0 < \mathrm{Im}\, t_{11}(s) \leq \frac{1}{\rho_1}$$

...

- Based on maximal analyticity principle and in accordance with the **unitarity bound** we write once subtracted **p.w. dispersion relation**

$$t_{ab}(s) = t_{ab}(0) + \frac{s}{\pi}\int_{-\infty}^{s_L} \frac{ds'}{s'}\frac{\mathrm{Disc}\ t_{ab}(s')}{s'-s} + \frac{s}{\pi}\sum_c \int_{s_{th}}^{\infty} \frac{ds'}{s'}\frac{t_{ac}(s')\,\rho_c(s')\,t^*_{cb}(s')}{s'-s}$$

N/D method

- Once-subtracted p.w. dispersion relation

$$t_{ab}(s) = \underbrace{t_{ab}(0) + \frac{s}{\pi}\int_{-\infty}^{s_L} \frac{ds'}{s'} \frac{\text{Disc } t_{ab}(s')}{s'-s}}_{U_{ab}(s)} + \frac{s}{\pi}\sum_c \int_{s_{th}}^{\infty} \frac{ds'}{s'} \frac{t_{ac}(s')\,\rho_c(s')\,t^*_{cb}(s')}{s'-s}$$

can be solved using N/D method with input from $U_{ab}(s)$ **above threshold**

Chew, Mandelstam (1960)
Luming (1964)
Johnson, Warnock (1981)

$$t_{ab}(s) = \sum_c D^{-1}_{ac}(s)\,N_{cb}(s)$$

$$N_{ab}(s) = U_{ab}(s) + \frac{s}{\pi}\sum_c \int_{s_{th}}^{\infty} \frac{ds'}{s'} \frac{N_{ac}(s')\,\rho_c(s')\,(U_{cb}(s') - U_{cb}(s))}{s'-s}$$

$$D_{ab}(s) = \delta_{ab} - \frac{s}{\pi}\int_{s_{th}}^{\infty} \frac{ds'}{s'} \frac{N_{ab}(s')\,\rho_b(s')}{s'-s} = \Omega^{-1}_{ab}(s)$$

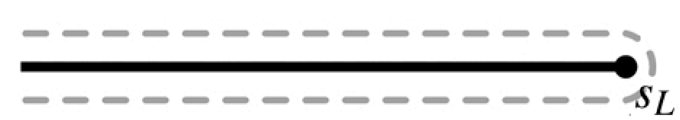
the obtained N/D solution **fulfils** the p.w. dispersion relation

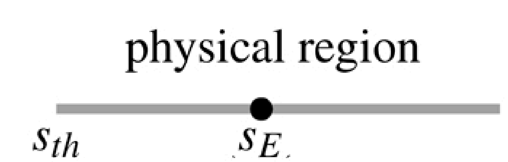
- Using the known analytical structure of left-hand cuts, one can approximate $U_{ab}(s)$ as an expansion in a **conformal mapping variable** $\xi(s)$

Gasparyan, Lutz (2010)

$$U_{ab}(s) = \sum_{n=0}^{\infty} C_{ab,n}\,(\xi_{ab}(s))^n$$

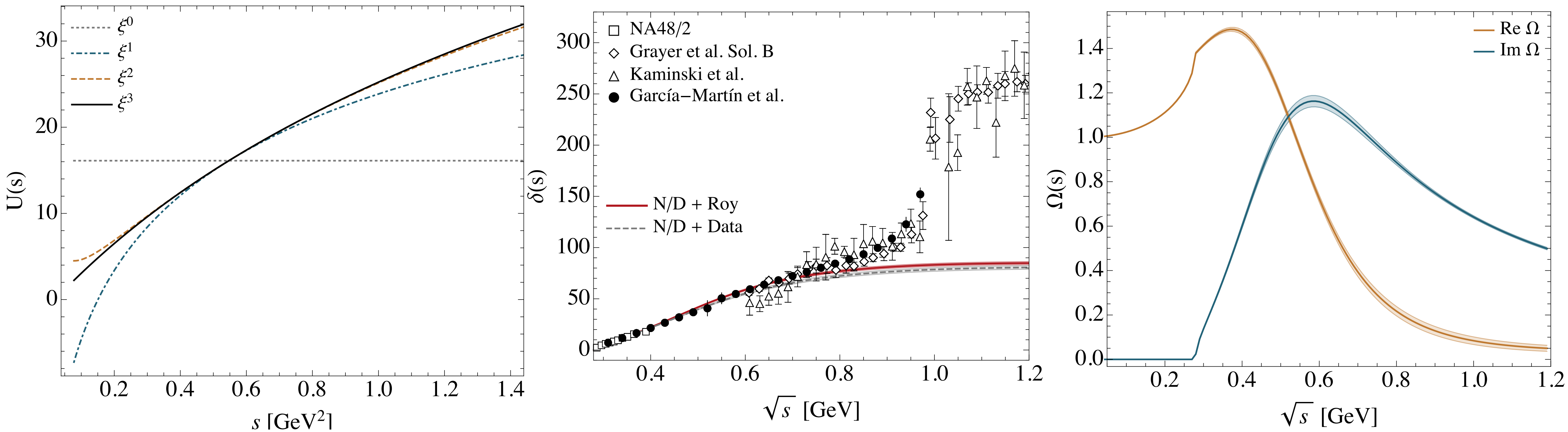
unknown coefficients fitted to data





$\xi(s_E) = 0$

single-channel {ππ}



Input: experimental data/Roy analysis + threshold parameters NNLO (a, b) + Adler zero NLO

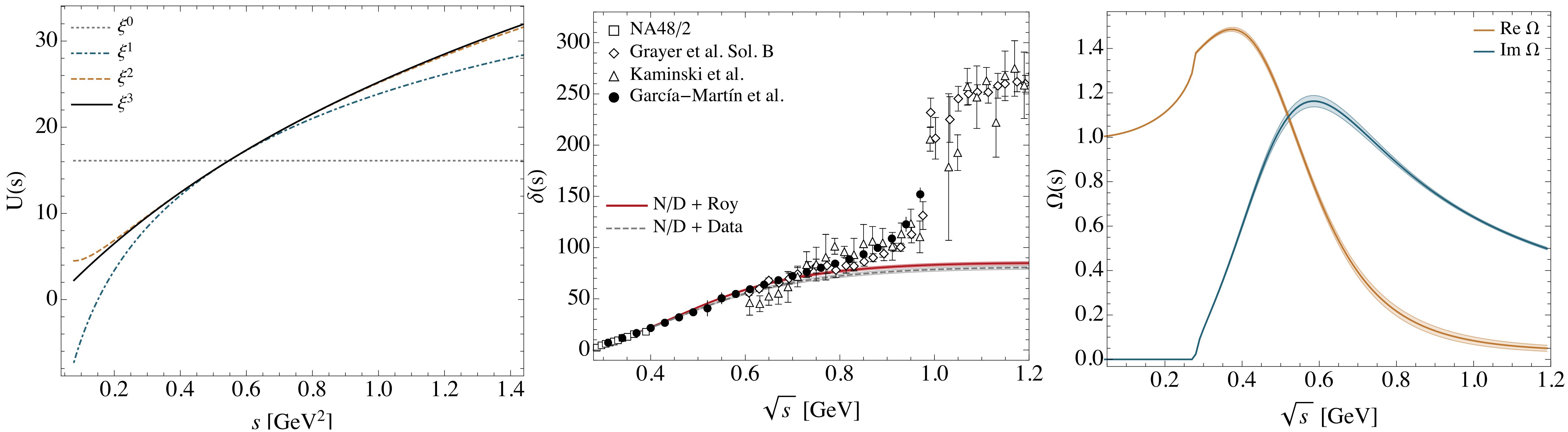
	Our results		Roy-like analyses	
	pole position, MeV	coupling, GeV	pole position, MeV	coupling, GeV
$\sigma/f_0(500)$	$458(7)^{+4}_{-10} - i\,245(6)^{+7}_{-10}$	$\pi\pi : 3.15(5)^{+0.11}_{-0.20}$	$449^{+22}_{-16} - i\,275(15)$	$\pi\pi : 3.45^{+0.25}_{-0.29}$
fit to Exp	$435(7)^{+6}_{-8} - i\,250(5)^{+6}_{-8}$			

Caprini et al. (2006)
Garcia-Martin et al. (2011)

Omnes function

$$\Omega(s) = D^{-1}(s)$$

single-channel {ππ}



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Caprini et al. (2006)
Garcia-Martin et al. (2011)

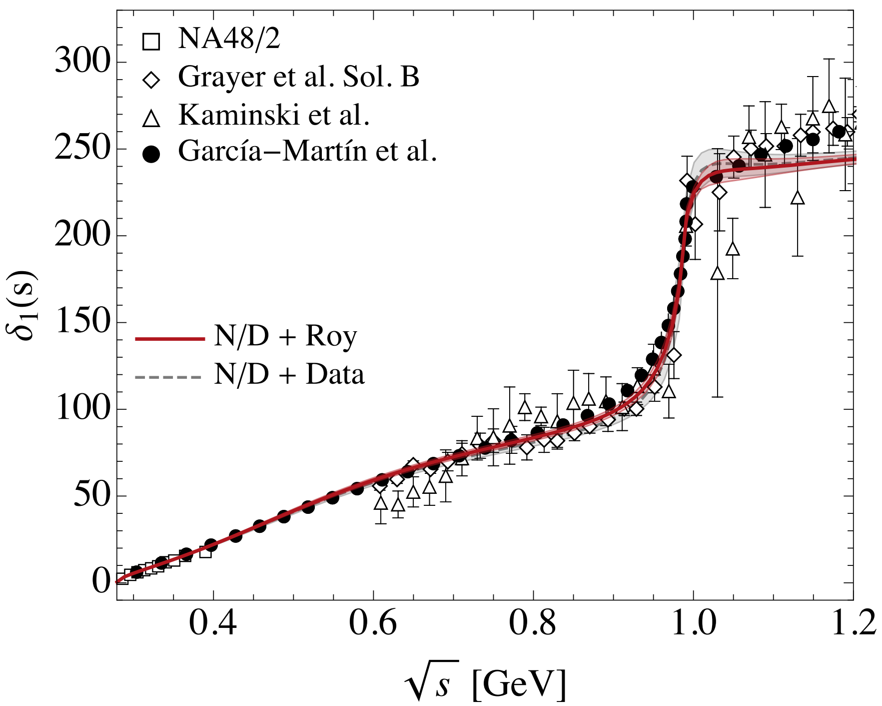
Omnes function

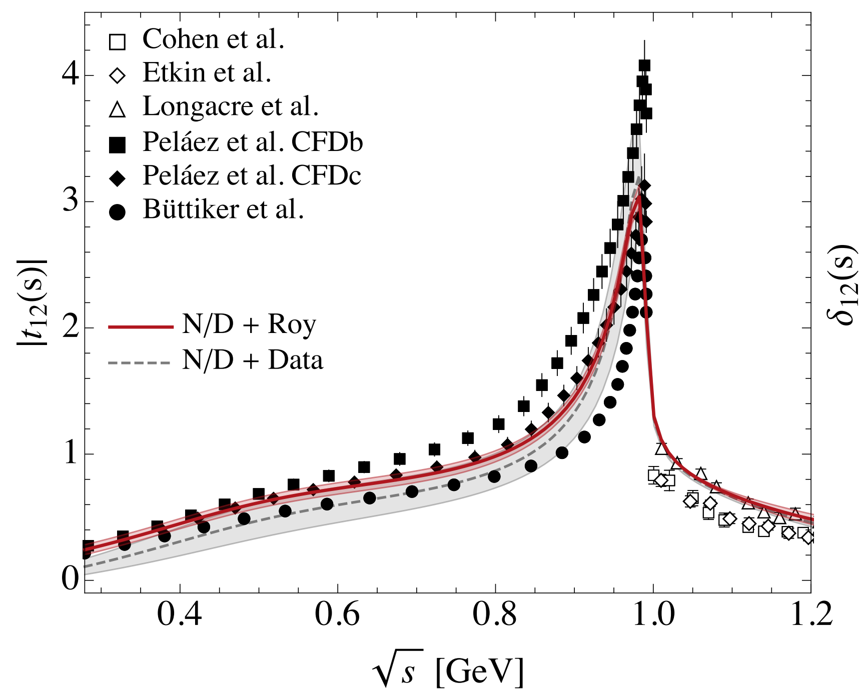
$$\Omega(s) = D^{-1}(s) = \exp\left(\frac{s}{\pi}\int_{s_{th}}^{\infty}\frac{ds'}{s'}\frac{\delta(s')}{s'-s}\right)$$

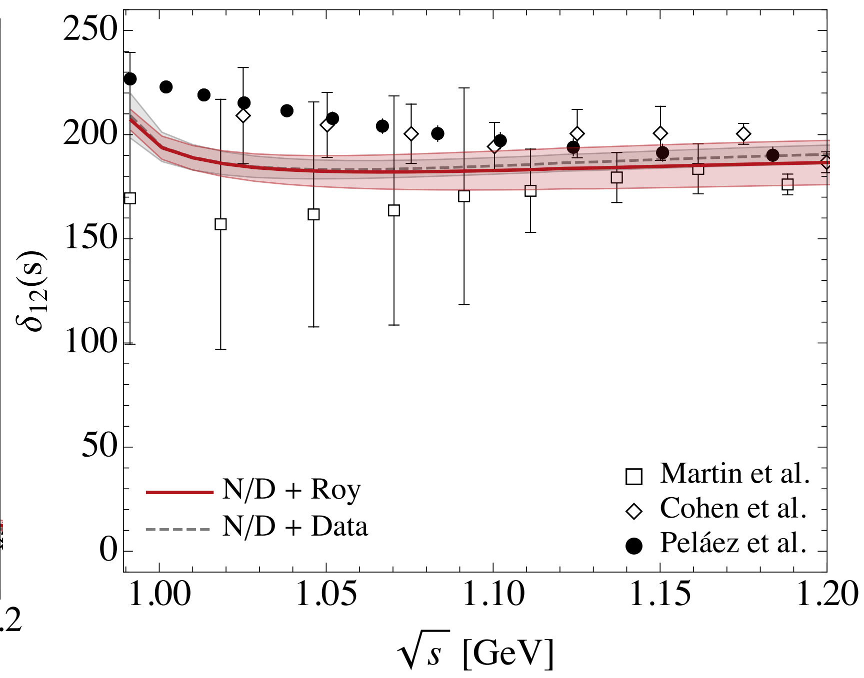
- Similar results for single-channel ππ phase-shift and Omnes function can obtained by using mIAM with the ChPT input for the left-hand cuts and subtraction constants

Gomez Nicola et al. (2008), Hanhart et al. (2008), Nebreda et al. (2010) Pelaez et al. (2010)

coupled-channel {ππ, KK}







Input: experimental data/Roy analysis + threshold parameters NNLO (a, b) + Adler zero NLO

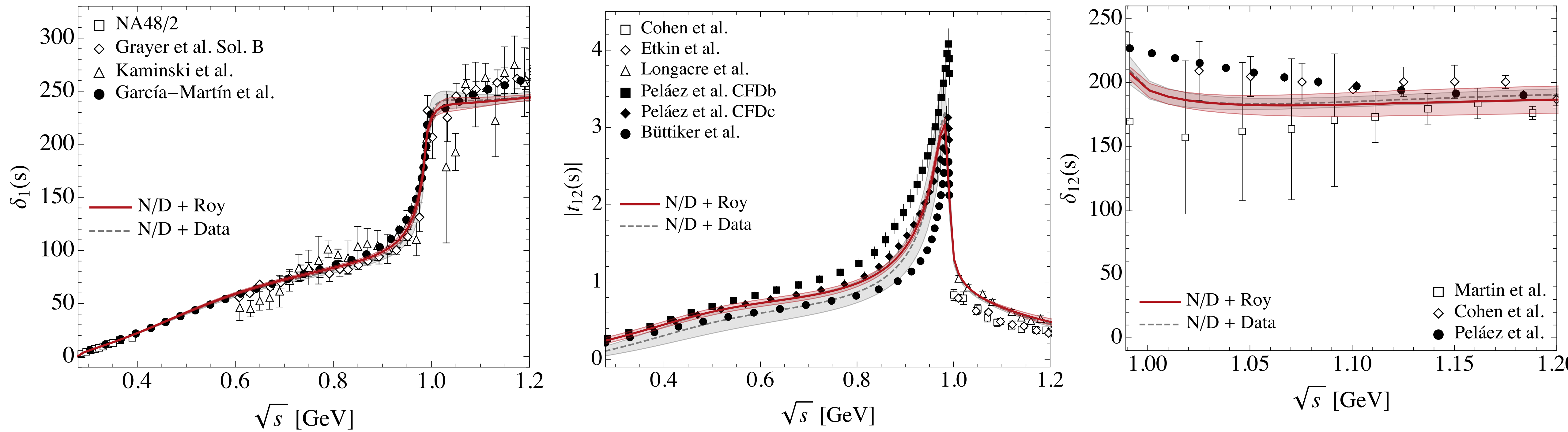
$$t_{ab}(s) = \begin{pmatrix} \frac{\eta(s)\, e^{2\,i\,\delta_1(s)}-1}{2\,i\,\rho_1(s)} & |t_{12}(s)|\, e^{\delta_{12}(s)} \\ |t_{12}(s)|\, e^{\delta_{12}(s)} & \frac{\eta(s)\, e^{2\,i\,\delta_2(s)}-1}{2\,i\,\rho_2(s)} \end{pmatrix}_{ab}$$

$$\eta(s) = \sqrt{1 - 4\,\rho_1(s)\,\rho_2(s)\,|t_{12}(s)|^2}$$

$$\delta_{12}(s) = \delta_1(s) + \delta_2(s)\,\theta(s > 4m_K^2)$$

In the **two-channel approximation** one needs to make the choice of which experimental data/Roy analysis include in the fit

coupled-channel {ππ, KK}

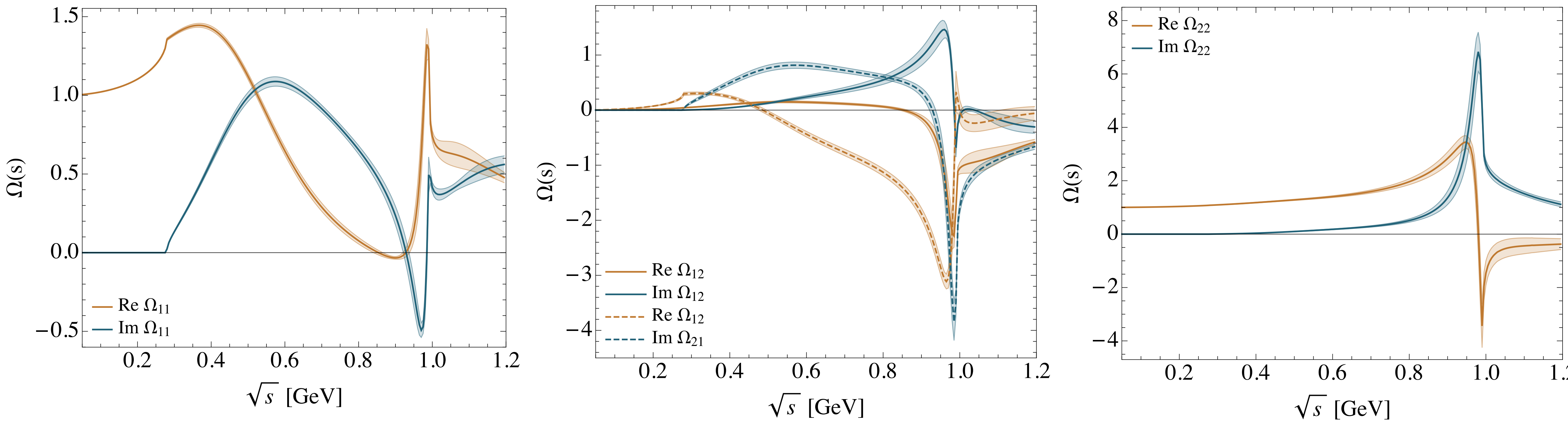


Input: experimental data/Roy analysis + threshold parameters NNLO (a, b) + Adler zero NLO

	Our results		Roy-like analyses	
	pole position, MeV	couplings, GeV	pole position, MeV	couplings, GeV
$\sigma/f_0(500)$	$458(10)^{+7}_{-15} - i\,256(9)^{+5}_{-8}$	$\pi\pi$: $3.33(8)^{+0.12}_{-0.20}$ $K\bar{K}$: $2.11(17)^{+0.27}_{-0.11}$	$449^{+22}_{-16} - i\,275(15)$	$\pi\pi$: $3.45^{+0.25}_{-0.29}$ $K\bar{K}$: –
fit to Exp	$454(12)^{+6}_{-7} - i\,262(12)^{+8}_{-12}$			
$f_0(980)$	$993(2)^{+2}_{-1} - i\,21(3)^{+2}_{-4}$	$\pi\pi$: $1.93(15)^{+0.07}_{-0.12}$ $K\bar{K}$: $5.31(24)^{+0.04}_{-0.24}$	$996^{+7}_{-14} - i\,25^{+11}_{-6}$	$\pi\pi$: $2.3(2)$ $K\bar{K}$: –
fit to Exp	$990(7)^{+2}_{-4} - i\,17(7)^{+4}_{-1}$			

Caprini et al. (2006)
Garcia-Martin et al. (2011)
Moussallam (2011)

Omnes function {ππ, KK}



Omnes function fulfils the unitarity relation on the right-hand cut and analytic everywhere else.
For the case of no bound states or CDD poles:

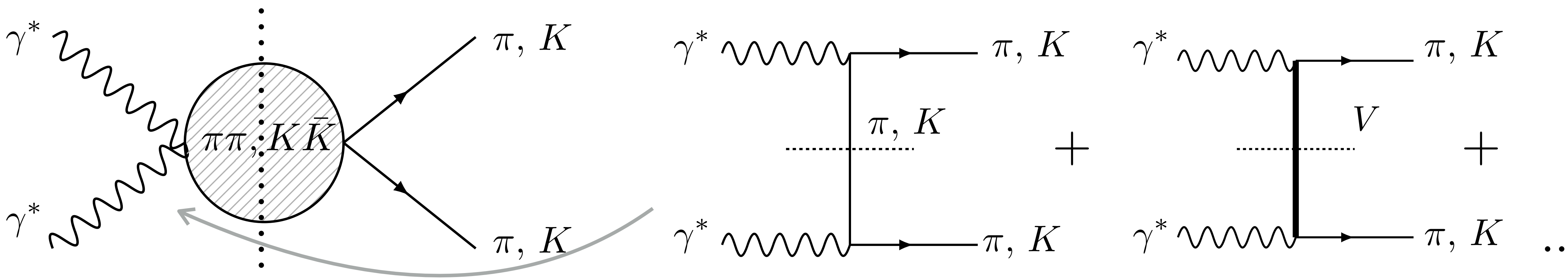
$$\Omega_{ab}(s) = D^{-1}_{ab}(s)$$

which automatically satisfies a once-subtracted dispersion relation (i.e. Ω(s) is asymptotically bounded)

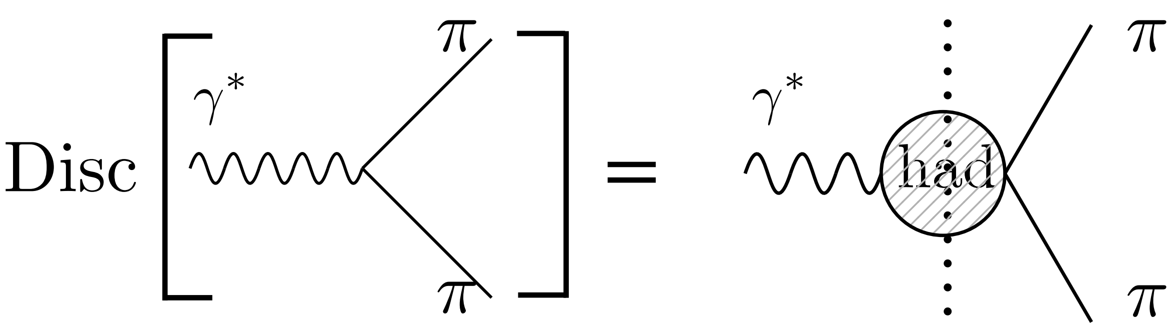
$$\Omega_{ab}(s) = \delta_{ab} + \frac{s}{\pi} \sum_c \int_{s_{th}}^{\infty} \frac{ds'}{s'} \frac{t^*_{ac}(s')\, \rho_c(s')\, \Omega_{cb}(s')}{s' - s}$$

different from
Donoghue et al. (1990)
Moussallam (2000)

Left-hand cuts (pion/kaon pole)

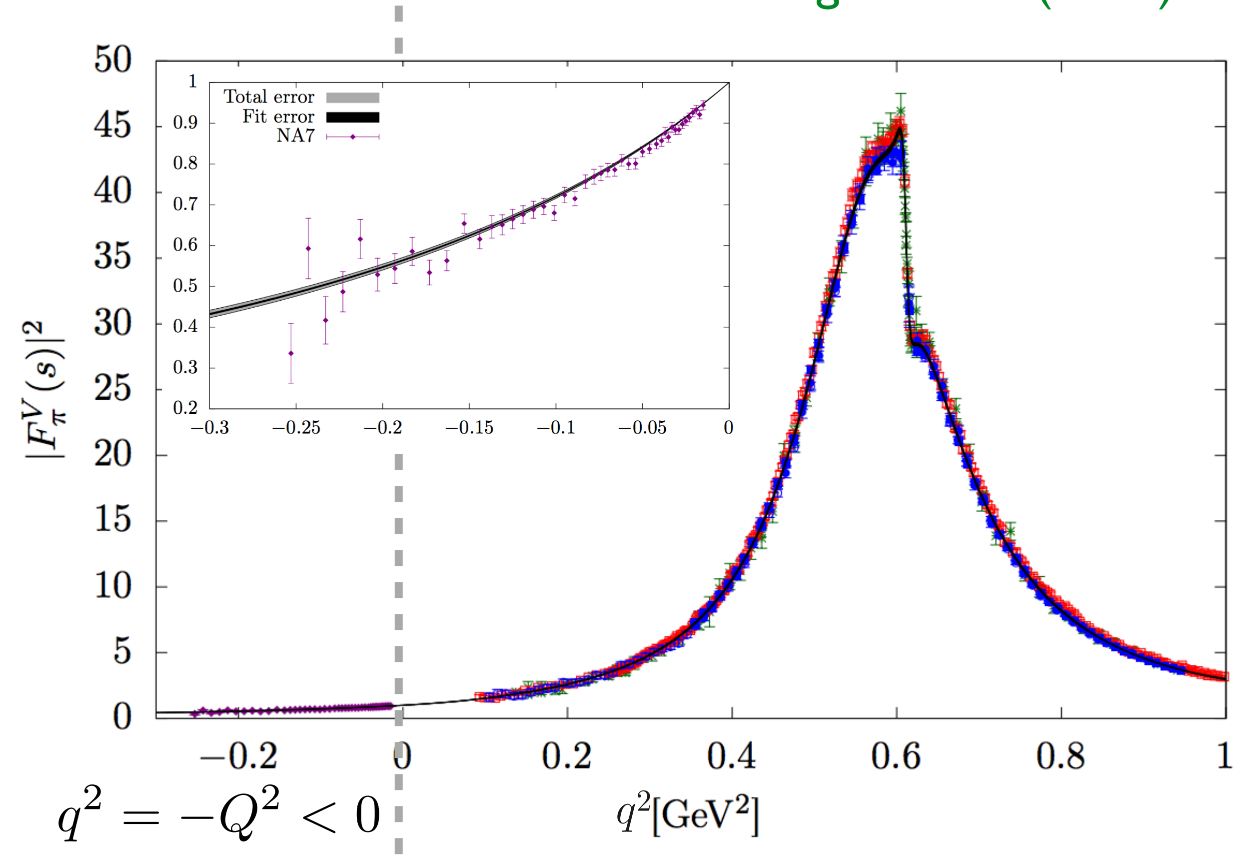


- Left-hand cuts requires knowledge from $\gamma^*\pi\pi,\ \gamma^*KK;\quad \gamma^*\pi V,\ \gamma^*KV$ form factors

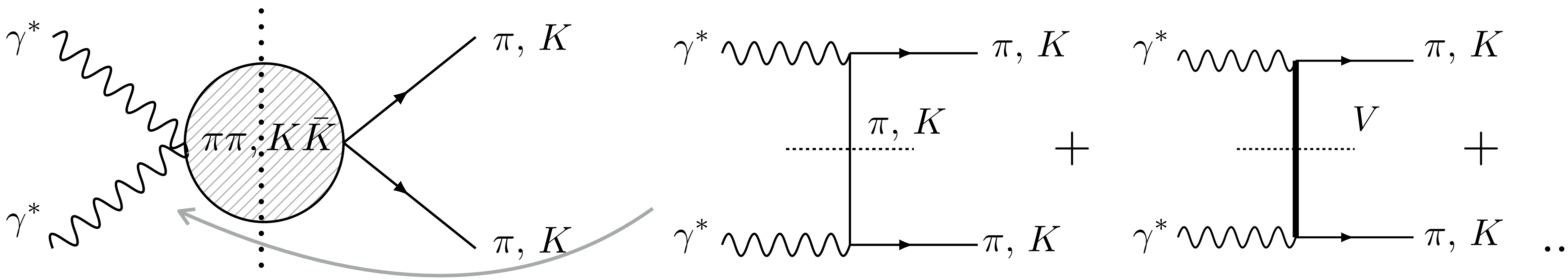


- VMD works well for the pion/kaon vector FF

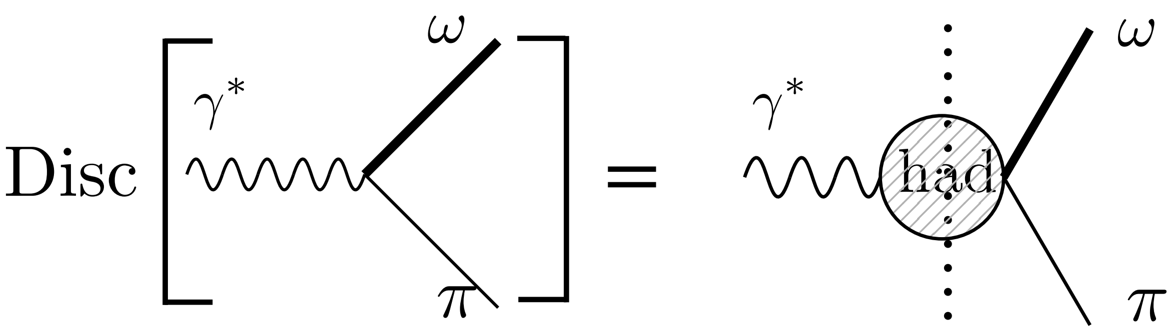
Colangelo et al. (2019)

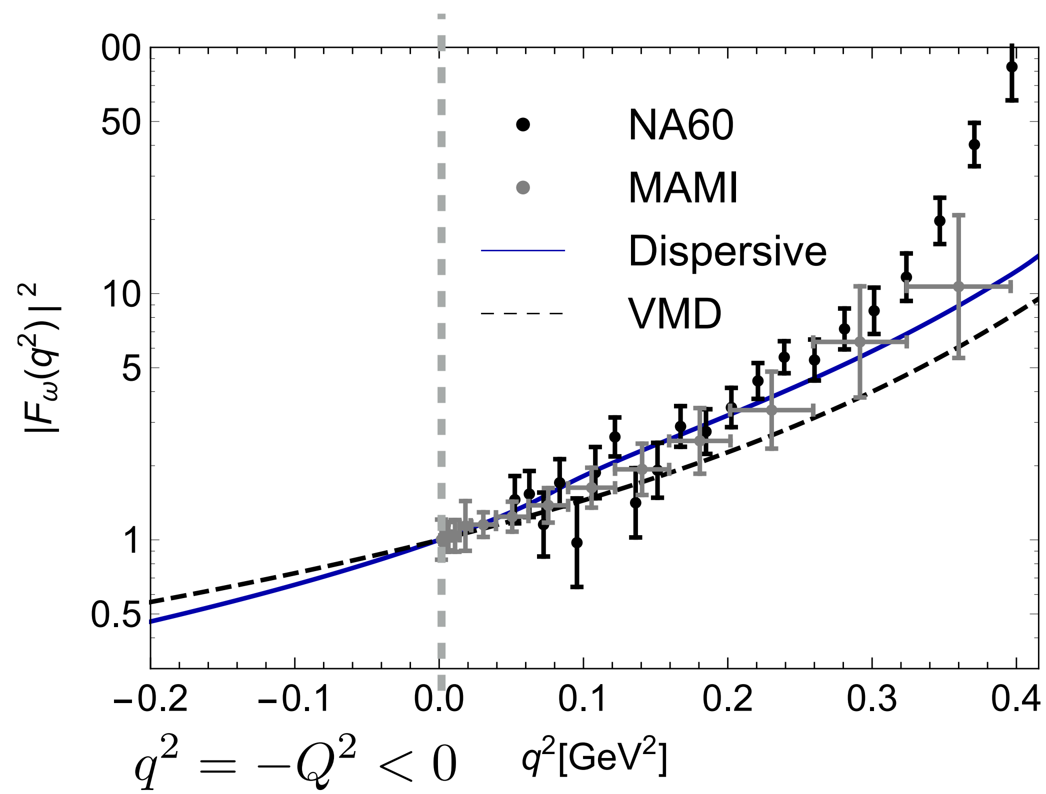


Left-hand cuts (vector poles)

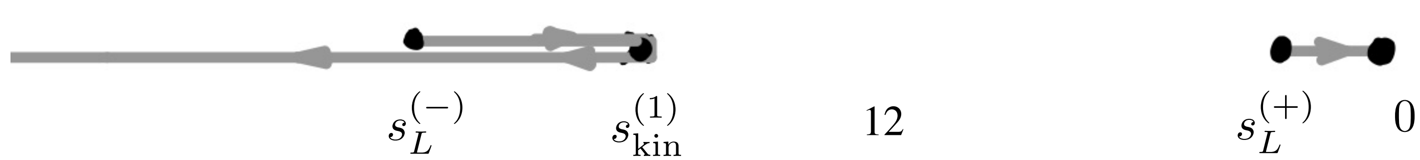


- Left-hand cuts requires knowledge from
  $\gamma^*\pi\pi,\ \gamma^* KK;\quad \gamma^*\pi V,\ \gamma^* KV$ form factors





- Left-hand cuts: "anomalous thresholds" for large virtualities $Q_1^2 Q_2^2 > (M_V^2 - m_\pi^2)^2$



Hoferichter, Stoffer (2019)
I.D., Deineka, Vanderhaeghen (2019)

Kinematic constraints

- p.w. helicity amplitudes suffer from kinematic constraints

$$h^{(J)}_{\lambda_1\lambda_2} = \int \frac{d\cos\theta}{2}\, d^J_{\lambda_1-\lambda_2,0}(\theta)\, H_{\lambda_1\lambda_2}$$

- Helicity amplitudes

$$H_{\lambda_1,\lambda_2} = \epsilon_\mu(\lambda_1)\epsilon_\nu(\lambda_2) \overbrace{\sum_{n=1}^{5} F_n(s,t)\, L_n^{\mu\nu}}^{H^{\mu\nu}}$$

Bardeen et al. (1968), Tarrach (1975)
Metz et al. (1998), Colangelo et al. (2015)

- Unconstrained basis for Born subtracted p.w. amplitudes $\bar{h}^{(J)}_i \equiv h^{(J)}_i - h^{(J),\mathrm{Born}}_i$

For S-wave

$$\bar{h}^{(0)}_{++} \pm \bar{h}^{(0)}_{00} \sim (s - s^{(\mp)}_{\text{kin}}), \quad s^{(\pm)}_{\text{kin}} \equiv -(Q_1 \pm Q_2)^2$$

$$\bar{h}^{(0)}_{i=1,2}(s) = \frac{\bar{h}^{(0)}_{++}(s) \pm \bar{h}^{(0)}_{00}(s)}{s - s^{(\mp)}_{\text{kin}}}$$

Colangelo et al. (2017)
Hoferichter, Stoffer (2019)
I.D., Deineka, Vanderhaeghen (2019)

For D-wave

$$\bar{h}^{(J)}_i = K_{ij}\, \bar{h}^{(J)}_j \qquad j \equiv \lambda_1\lambda_2 = \{++,\ +-,\ +0,\ 0+,\ 00\}$$

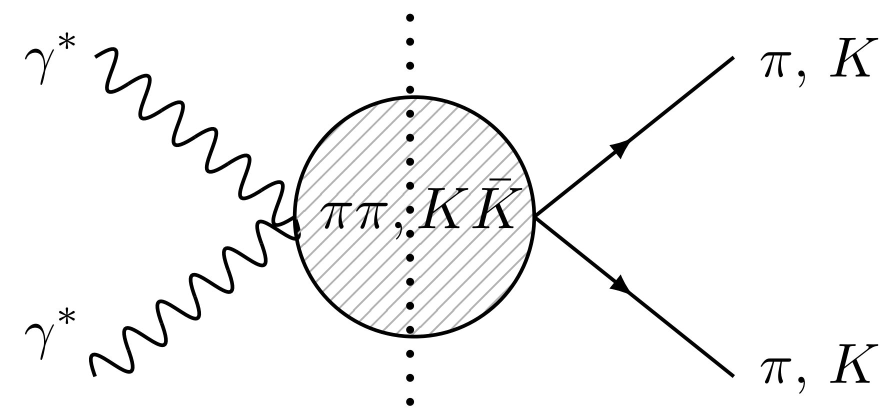
K_{ij} is 5×5 matrix

Dispersion relation

- Unsubtracted dispersion relation for kinematically unconstrained p.w. amplitudes

$$\bar{h}_i^{(J)} = \int_{-\infty}^{0} \frac{ds'}{\pi} \frac{\text{Disc}\, \bar{h}_i^{(J)}(s')}{s'-s} + \int_{4m_\pi^2}^{\infty} \frac{ds'}{\pi} \frac{\text{Disc}\, \bar{h}_i^{(J)}(s')}{s'-s}$$

$$\bar{h}_i^{(J)} \equiv h_i^{(J)} - h_i^{(J),\text{Born}}$$



which can be solved using modified MO method, i.e. by writing a dispersion relation for $\Omega^{(J)}(s)^{-1}\, \bar{h}_i^{(J)}(s)$

- For S-wave, I=0

$$\begin{pmatrix} \bar{h}_i^{(0)}(s) \\ \bar{k}_i^{(0)}(s) \end{pmatrix} = \Omega^{(0)}(s) \left[\int_{-\infty}^{0} \frac{ds'}{\pi} \frac{\Omega^{(0)}(s')^{-1}}{s'-s} \begin{pmatrix} \text{Disc}\, \bar{h}_i^{(0)}(s') \\ \text{Disc}\, \bar{k}_i^{(0)}(s') \end{pmatrix} - \int_{4m_\pi^2}^{\infty} \frac{ds'}{\pi} \frac{\text{Disc}\, \Omega^{(0)}(s')^{-1}}{s'-s} \begin{pmatrix} h_i^{(0),\text{Born}}(s') \\ k_i^{(0),\text{Born}}(s') \end{pmatrix} \right]$$

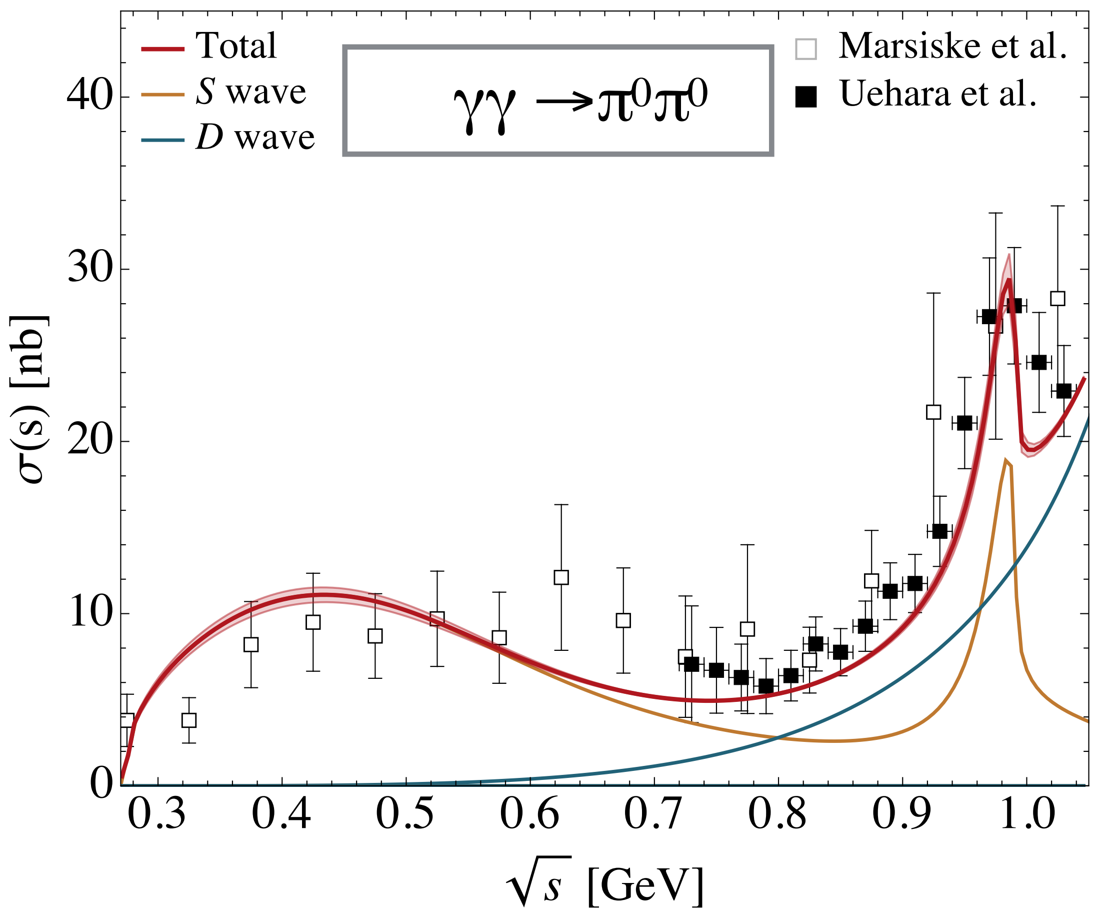
$\gamma^*\gamma^* \to \pi\pi$

$\gamma^*\gamma^* \to KK$

V-exch

Omnès (1958)
Muskhelishvili (1953)
Garcia-Martin et. al (2010)
Hoferichter et. al. (2011,19)
Dai et al. (2014)
Moussallam (2013)

γγ→ππ (postdiction)



$$\Gamma_{\gamma\gamma}[f_0(500)] = 1.37(13)^{+0.09}_{-0.06}\,\text{keV}$$
$$\Gamma_{\gamma\gamma}[f_0(980)] = 0.33(16)^{+0.04}_{-0.16}\,\text{keV}$$

consistent with

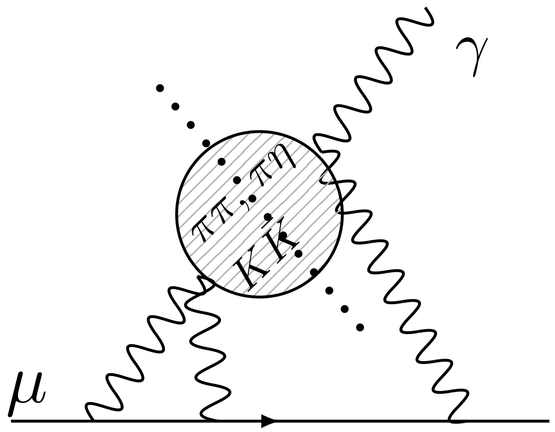
$$\Gamma^{\text{Roy-Steiner}}_{\gamma\gamma}[f_0(500)] = 1.7(4)\,\text{keV}$$ Hoferichter et. al. (2011)
$$\Gamma^{\text{MO}}[f_0(980)] = 0.29(21)^{+0.02}_{-0.07}\,\text{keV}$$ Moussallam (2011)
$$\Gamma^{\text{Ampl. analys.}}[f_0(980)] = 0.32(5)\,\text{keV}$$ Dai et al. (2014)

$$\begin{pmatrix} \bar{h}_i^{(0)}(s) \\ \bar{k}_i^{(0)}(s) \end{pmatrix} = \Omega^{(0)}(s)\left[\int_{-\infty}^{0}\frac{ds'}{\pi}\frac{\Omega^{(0)}(s')^{-1}}{s'-s}\begin{pmatrix} \text{Disc}\,\bar{h}_i^{(0)}(s') \\ \text{Disc}\,\bar{k}_i^{(0)}(s') \end{pmatrix} - \int_{4m_\pi^2}^{\infty}\frac{ds'}{\pi}\frac{\text{Disc}\,\Omega^{(0)}(s')^{-1}}{s'-s}\begin{pmatrix} h_i^{(0),\text{Born}}(s') \\ k_i^{(0),\text{Born}}(s') \end{pmatrix}\right]$$

V-exch

- This approximation can be systematically improved by adding vector-meson exchange left-hand cuts at the cost of extra subtraction when BESIII data on γγ*→ππ will be available

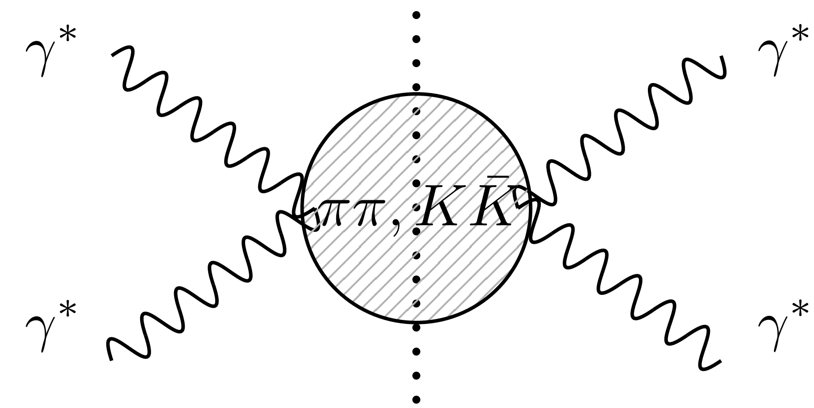
Contribution to (g-2)



Important ingredients:

$\gamma^*\gamma^* \to \pi\pi, K\bar{K}, \ldots$

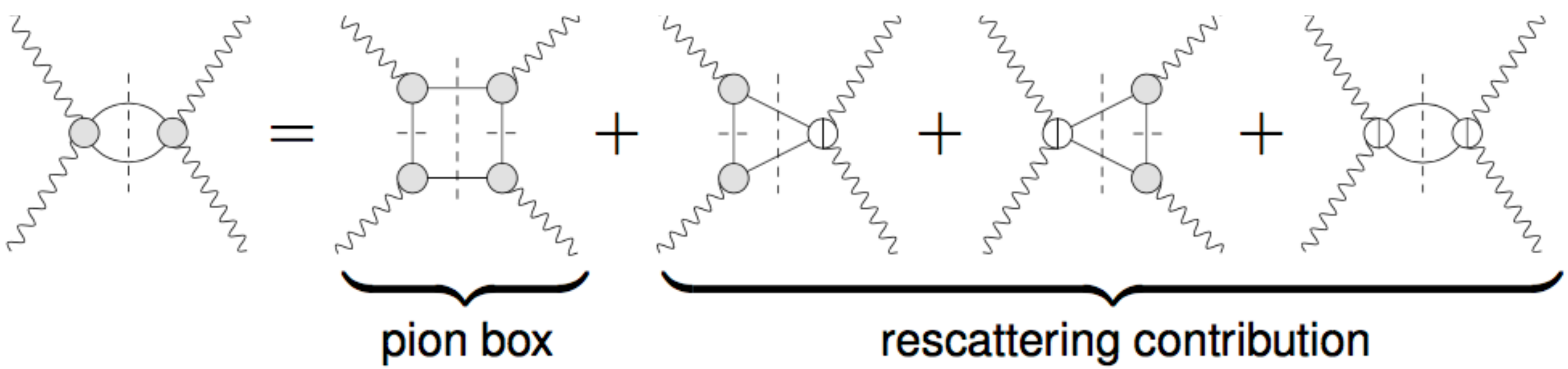
$q^2 = -Q^2 < 0 \quad$ space-like γ^*



$$a_\mu^{\mathrm{HLbL}} = \frac{2\alpha^3}{3\pi^2}\int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^1 d\tau \sqrt{1-\tau^2}\, Q_1^3\, Q_2^3 \sum_{i=1}^{12} T_i(Q_1,Q_2,Q_3)\, \bar{\Pi}_i(Q_1,Q_2,Q_3),$$

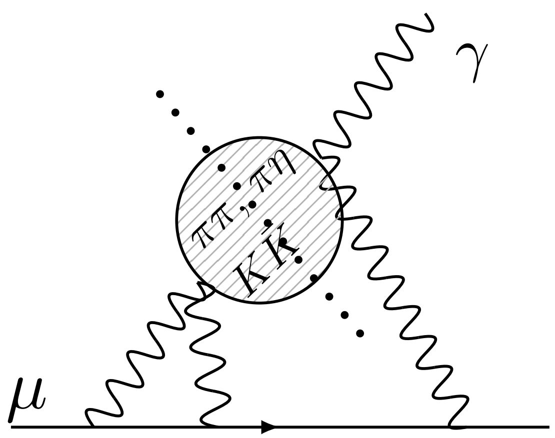
Colangelo et al. (2014-2017)

$$\Pi^{\mu\nu\lambda\sigma} = \sum_{i=1}^{54} T_i^{\mu\nu\lambda\sigma}\, \Pi_i \qquad \bar{\Pi}_i \text{ linear combination of } \Pi_i$$



$$a_\mu^{\pi\pi,KK}[\mathrm{box}] = -16.4(2)\times 10^{-11}$$

Contribution to (g-2)



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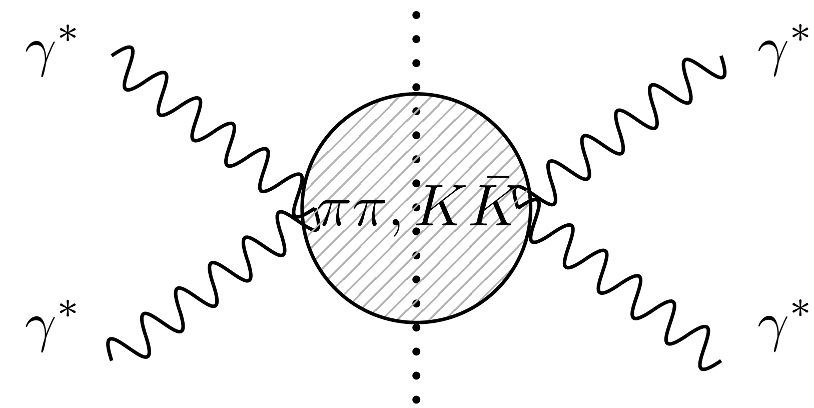


$$a_\mu^{\rm HLbL} = \frac{2\alpha^3}{3\pi^2}\int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^1 d\tau \sqrt{1-\tau^2}\, Q_1^3\, Q_2^3 \sum_{i=1}^{12} T_i(Q_1,Q_2,Q_3)\,\bar{\Pi}_i(Q_1,Q_2,Q_3),$$

Rescattering contribution ($\bar{h} \equiv h - h^{\rm Born}$) in the S-wave

$$\bar{\Pi}_3^{J=0} = \frac{1}{\pi}\int_{4m_\pi^2}^\infty ds' \frac{-2}{\lambda_{12}(s')(s'+Q_3^2)^2}\Big(4s'\,{\rm Im}\bar{h}^{(0)}_{++,++}(s') - (s'-Q_1^2+Q_2^2)(s'+Q_1^2-Q_2^2)\,{\rm Im}\bar{h}^{(0)}_{00,++}(s')\Big)$$

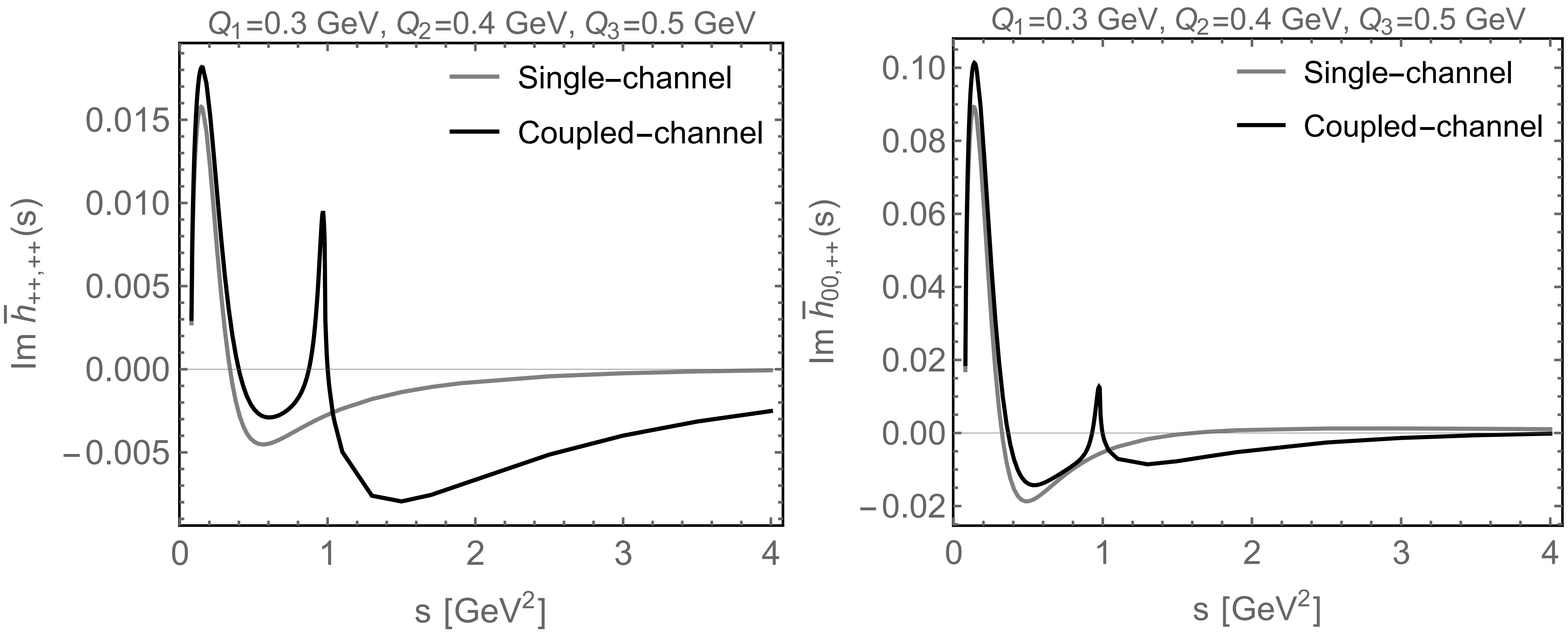
$$\bar{\Pi}_9^{J=0} = \frac{1}{\pi}\int_{4m_\pi^2}^\infty ds' \frac{4}{\lambda_{12}(s')(s'+Q_3^2)^2}\Big(2\,{\rm Im}\bar{h}^{(0)}_{++,++}(s') - (s'+Q_1^2+Q_2^2)\,{\rm Im}\bar{h}^{(0)}_{00,++}(s')\Big)$$

+crossed

Unitarity γ*γ*→γ*γ* γ*γ*→ππ γ*γ*→KK

$${\rm Im}\bar{h}^{(0)}_{\lambda_1\lambda_2,\lambda_3\lambda_4}(s) = \frac{1}{2}\,\bar{h}^{(0)}_{\lambda_1\lambda_2}(s)\,\rho_\pi(s)\,\bar{h}^{(0)*}_{\lambda_3\lambda_4}(s) + \frac{1}{2}\,\bar{k}^{(0)}_{\lambda_1\lambda_2}(s)\,\rho_K(s)\,\bar{k}^{(0)*}_{\lambda_3\lambda_4}(s)$$

Contribution to (g-2)



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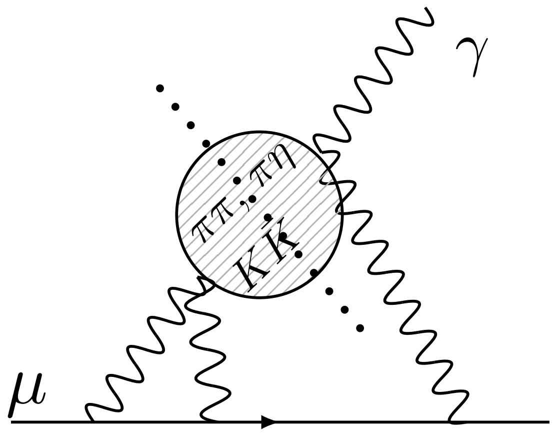
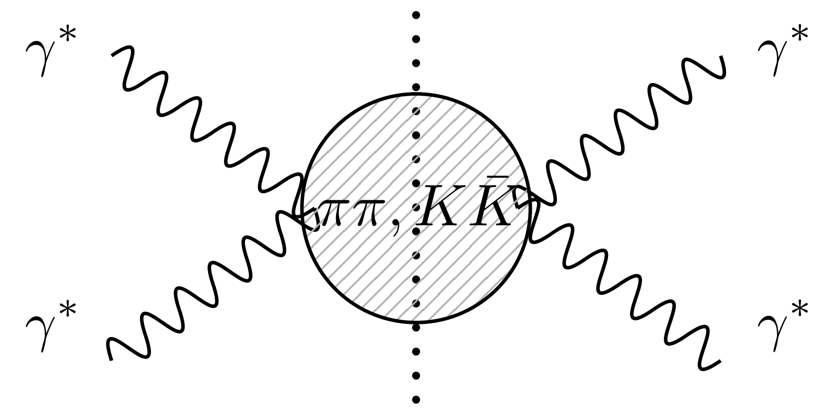
Contribution to (g-2)



Important ingredients:

$\gamma^*\gamma^* \to \pi\pi, K\bar{K}, \ldots$

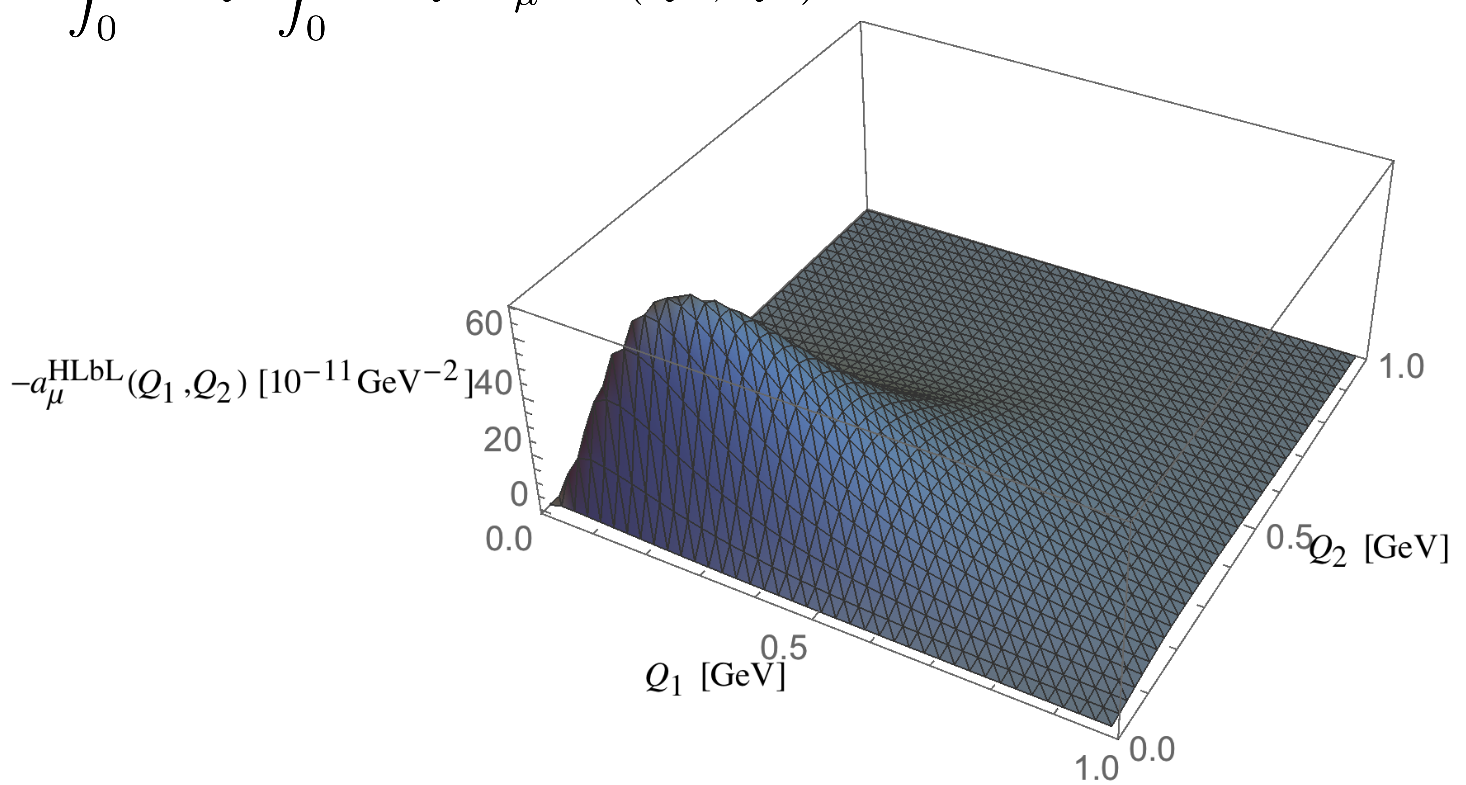
$q^2 = -Q^2 < 0 \quad$ space-like γ^*



$$a_\mu^{\text{HLbL}} = \frac{2\alpha^3}{3\pi^2}\int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^{1} d\tau \sqrt{1-\tau^2}\, Q_1^3\, Q_2^3 \sum_{i=1}^{12} T_i(Q_1,Q_2,Q_3)\, \bar{\Pi}_i(Q_1,Q_2,Q_3)$$

$$Q_3^2 = Q_1^2 + 2Q_1Q_2\tau + Q_2^2$$

$$= \int_0^\infty dQ_1 \int_0^\infty dQ_2\, a_\mu^{\text{HLbL}}(Q_1,Q_2)$$



- BESIII (under analysis)

$\gamma\gamma^* \to \pi^+\pi^-, \pi^0\pi^0$

$Q^2 = 0.1 - 3.0\,\text{GeV}^2$

Contribution to (g-2)

- Using S-wave elastic helicity amplitudes on γ*γ*→ππ, $f_0(500)$ contribution was calculated previously

$$a_\mu^{\rm HLbL}[\text{S-wave}, I=0]_{\rm rescattering} = -9.3(1)\times 10^{-11}$$

Colangelo et al. (2014-2017)

- Extending to KK channel allowed us to access energies up to ~1.2 GeV ($f_0(500)$ + $f_0(980)$ contributions)

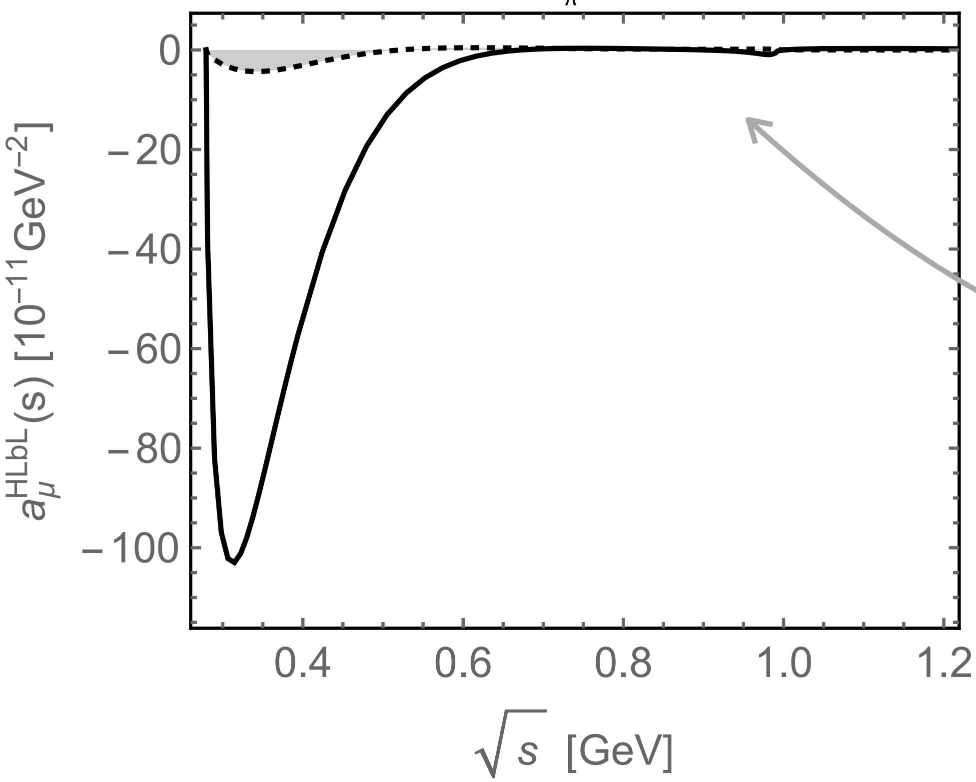
$$a_\mu^{\rm HLbL}[\text{S-wave}, I=0]_{\rm rescattering} = -9.8(1)\times 10^{-11}$$

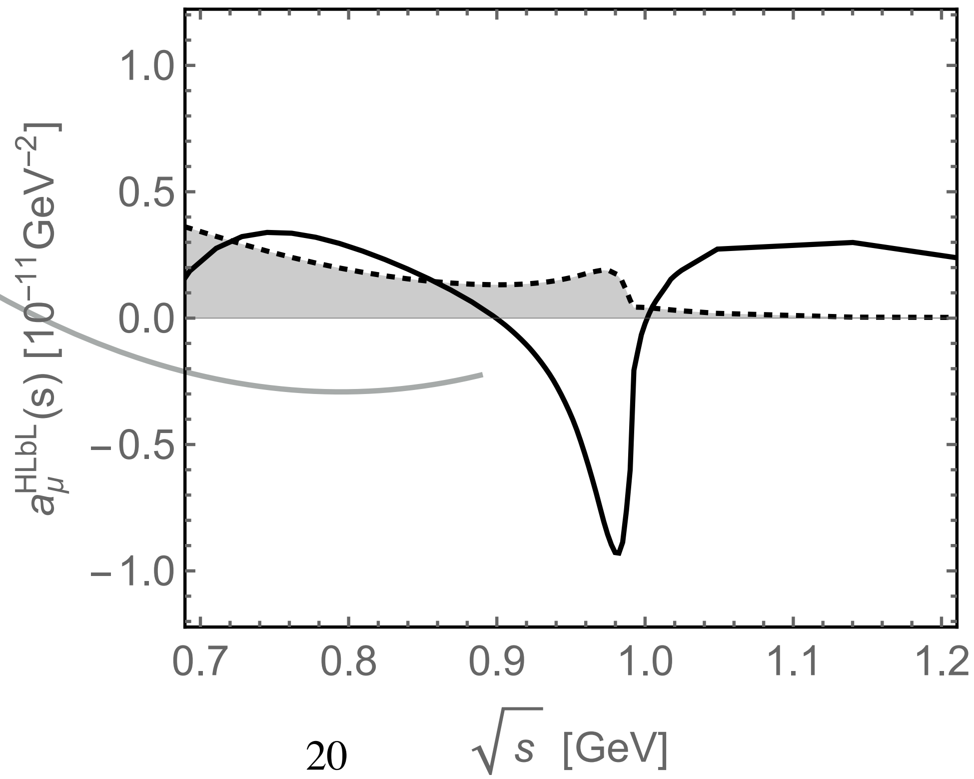
What is the contribution just from $f_0(980)$?

- One can define it as an integral over the deficit in shape

$$a_\mu^{\rm HLbL} = \int_{4m_\pi^2}^{\infty} ds'\, a_\mu^{\rm HLbL}(s')$$

$$a_\mu^{\rm HLbL}[f_0(980)]_{\rm rescattering} = -0.2(1)\times 10^{-11}$$





shaded area is a sum rule violation → result is largely basis independent

Summary and outlook

- Presented a dispersive data driven analysis of the coupled-channel I=0 {ππ,KK} contribution to (g-2)

  ➢ Main ingredient is a coupled-channel {ππ, KK} Omnes matrix

  ➢ The input from γγ*→ππ,KK can be systematically improved once BESIII data will be available

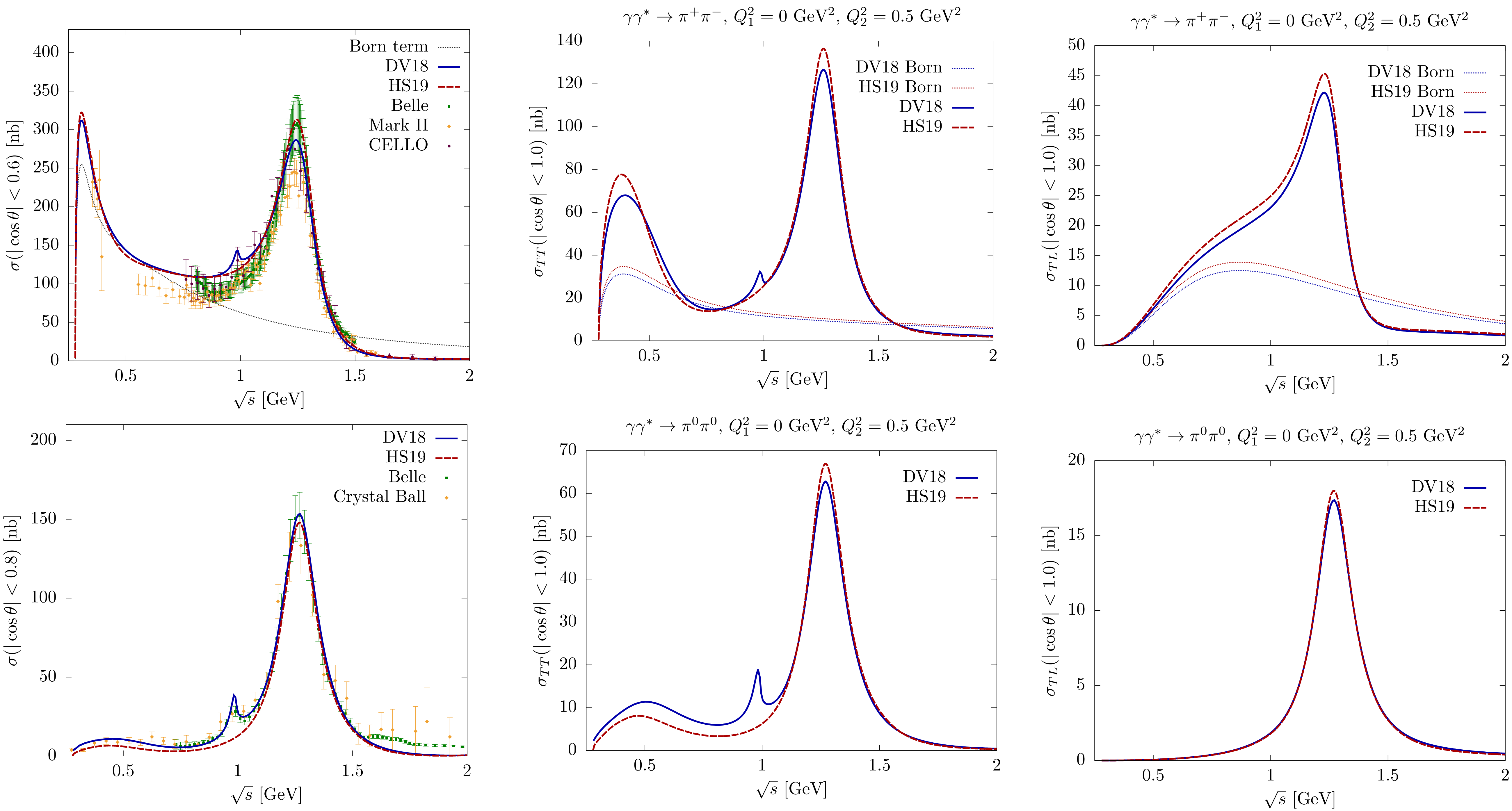
- As a next step, we plan to perform a similar dispersive analysis for the coupled-channel I=1 {πη,KK} contribution to (g-2)

  ➢ Narrow width approximation gives $a_\mu^{\mathrm{HLbL}}[a_0(980)]_{\mathrm{NWA}} = -\left([0.3, 0.6]^{+0.2}_{-0.1}\right) \times 10^{-11}$

- Also, one can calculate D-wave contribution dispersively from ππ rescattering ($f_2(1270)$) and compare the result with the narrow width approximation (need to be careful with ambiguities due to choice of the basis and sum rule violations)

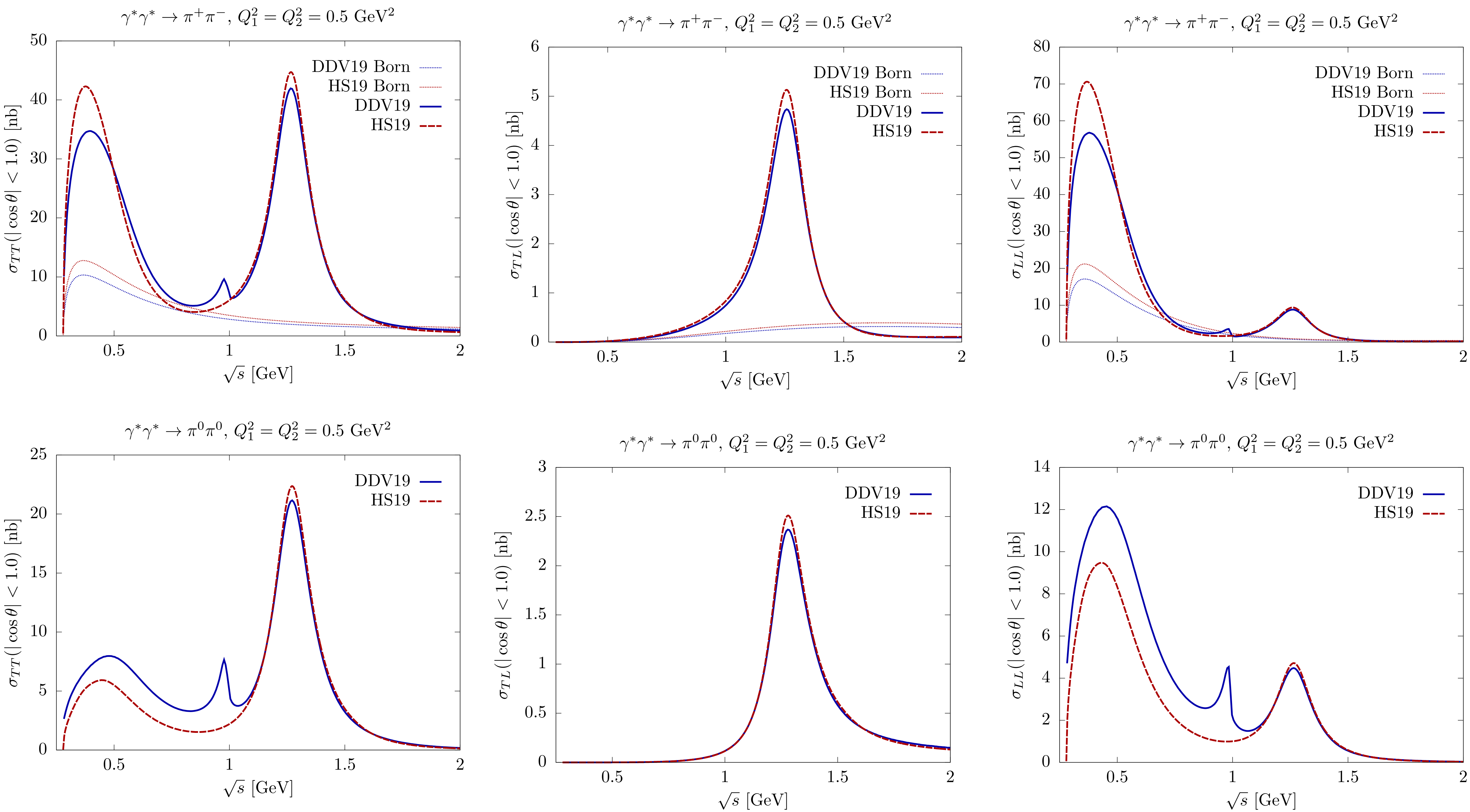
Thank you!

Results for ππ



I.D., Vanderhaeghen (2018)
Hoferichter, Stoffer (2019)

Results for ππ



$$\frac{d\sigma_{TT}}{d\cos\theta} \sim |H_{++}|^2 + |H_{+-}|^2, \quad \frac{d\sigma_{TL}}{d\cos\theta} \sim |H_{+0}|^2, \quad \frac{d\sigma_{LL}}{d\cos\theta} \sim |H_{00}|^2$$

Hoferichter, Stoffer (2019)
I.D., Deineka, Vanderhaeghen (2019)