The pseudoscalar poles contributions to $(g-2)_\mu$

based on P. Masjuan, PS: Phys.Rev. D95 (2017)

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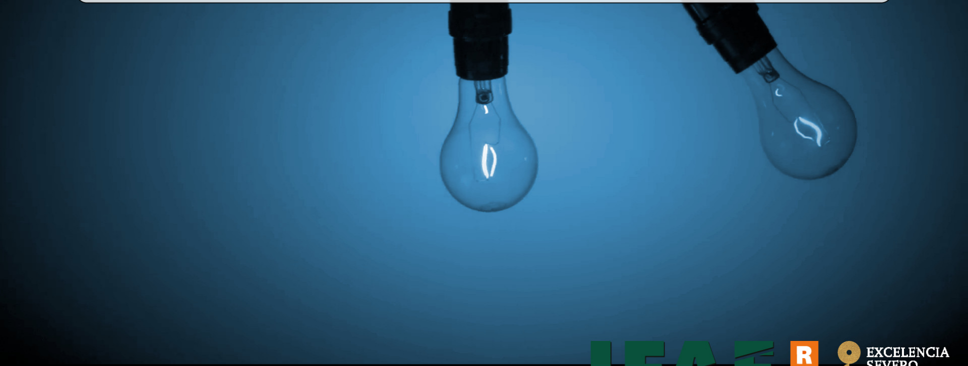

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BIST

Outline

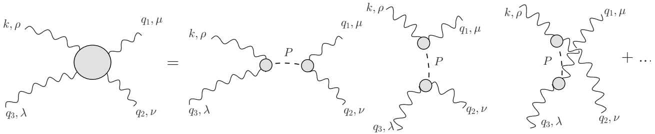
1. Pseudoscalar poles contribution to a_μ^{HLbL}: the essentials
2. Describing the pseudoscalar form factors: Padé approximants
3. Application to HLbL: results
4. Summary

Section 1

Pseudoscalar poles contribution to a_μ^{HLbL}: the essentials

The pseudoscalar poles in brief

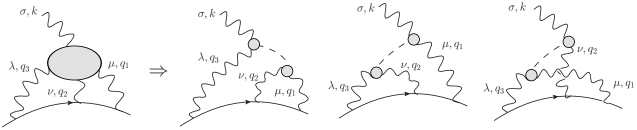
- We want the pseudoscalar (π^0, η, η') pole contributions to a_μ^{HLbL}



unambiguosly identified (see G. Colangelo's talk)

The pseudoscalar poles in brief

- Plugging it into the a_μ^{HLbL} contribution



The pseudoscalar poles in brief

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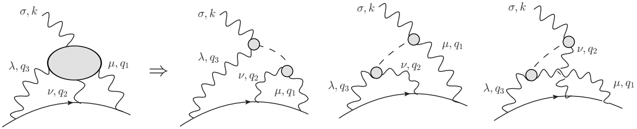
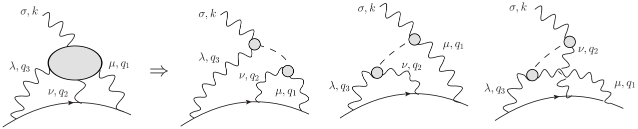


- Result as weighted integral over spacelike form factors [Phys.Rept., 477 (2009)]

$$a_\ell^{\mathrm{HLbL};P} = \frac{-2\pi}{3}\left(\frac{\alpha}{\pi}\right)^3 \int_0^\infty dQ_1 dQ_2 \int_{-1}^{+1} dt \sqrt{1-t^2}\, Q_1^3 Q_2^3 \times \left[\frac{F_{P\gamma^*\gamma^*}(Q_1^2, Q_3^2) F_{P\gamma^*\gamma}(Q_2^2, 0) I_1(Q_1, Q_2, t)}{Q_2^2 + m_P^2} + \frac{F_{P\gamma^*\gamma^*}(Q_1^2, Q_2^2) F_{P\gamma^*\gamma}(Q_3^2, 0) I_2(Q_1, Q_2, t)}{Q_3^2 + m_P^2} \right]$$

The pseudoscalar poles in brief

- Plugging it into the a_μ^{HLbL} contribution



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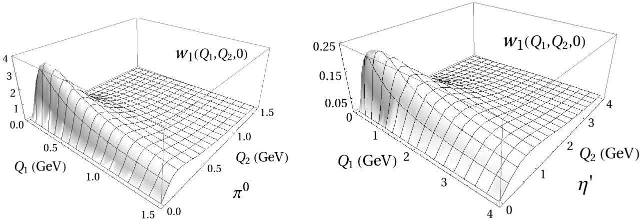
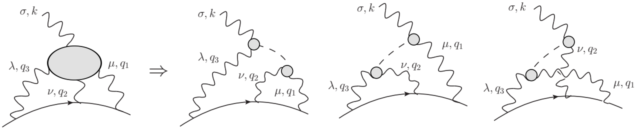


notice the peaks at the relevant low energies

The pseudoscalar poles in brief

- Plugging it into the a_μ^{HLbL} contribution



- Solved if form factor known precisely at Q^2 and HE extrapolation appropriate

Whitepaper criteria [Phys.Rept. 887 (2020) 1-166]

- In addition to the normalization [...], also HE constraints must be fulfilled
- At least the SL data [...] must be reproduced
- Systematic uncertainties must be assessed [...]

$\Rightarrow$ Padé approximants, Dispersion relations, Lattice QCD

Here I will focus on Padé approximants

Section 2

Describing the pseudoscalar form factors: Padé approximants

Padé approximants: the philosophy

—The challenge—

Get a description for the SL TFF

—The philosophy—

Nature has solved QCD: make use of data

—The problem—

Inter(extra)polate the data in the SL region while (WP criteria):

- Incorporating QCD constraints
- Assessing systematics from procedure

—Our proposal—

Padé & Canterbury approximants: not a model but a mathematical toolkit

Padé approximants: singly virtual

- How to approximate (not model) non-perturbative hadronic functions?

  Taylor series: $F_{\pi\gamma\gamma^*}(q^2) = F_{\pi\gamma\gamma}(1 + b_P q^2 + ...)$ ✗ poles(cuts)

Padé approximants: singly virtual

- How to approximate (not model) non-perturbative hadronic functions?

Laurent exp.: $F_{\pi\gamma\gamma^*}(q^2) = \sum_{n=-1} c_n(q^2 - M^2)^n$ ✗ next pole(cuts)

Padé approximants: singly virtual

- How to approximate (not model) non-perturbative hadronic functions?

$$\text{PAs: } F_{\pi\gamma\gamma^*}(q^2) = P_M^N = \frac{Q_N(q^2)}{R_M(q^2)} = F_{\pi\gamma\gamma}\left(1 + b_P q^2 + \ldots + \mathcal{O}(q^{N+M+1})\right)$$

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- Convergence to meromorphic and Stieltjes in the SL $\Rightarrow$ beyond large-N_c

Convergence applies to sequences: $P_1^N, P_N^N, P_{N+1}^N, \ldots$

Sequence convergence: model-independence and systematics

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Convergence applies to sequences: $P^N_1, P^N_N, P^N_{N+1}, \ldots$

Sequence convergence: model-independence and systematics

- Mixed $q^2 = 0$ and $q^2 = \infty$ expansions if required $\rightarrow$ pQCD constraints

$$P^0_1(q^2) = \frac{F_{P\gamma\gamma^*}(0)}{1 - b_P Q^2} = F_{P\gamma\gamma^*}(0)(1 + b_P Q^2 + \mathcal{O}(q^4)), \quad b_P \neq M_V^{-2}!!!$$

Can incorporate relevant QCD constraints; $q^2 = 0$ relevant for a_μ^{HLbL}

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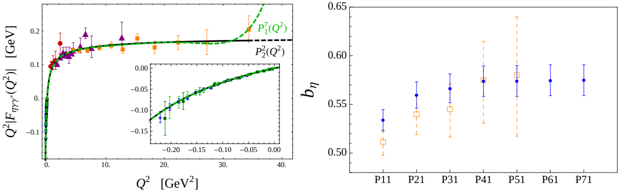
Can incorporate relevant QCD constraints; $q^2 = 0$ relevant for a_μ^{HLbL}

- Obtain the derivatives from data (nor spectroscopy nor direct fitting!!!)

Inputs from data, PAs from derivatives

- Inputs: sequences data fitting (not just fitting models)



Inputs from data, PAs from derivatives

- Inputs: sequences data fitting (not just fitting models)

$$\begin{array}{c|lll} \text{Reduced} & P^N_{N+1}: & P_1^{0;\text{fit}} \to b_P\ ,\ \ldots\ ,\ P_2^{1;\text{fit}}\ ✗ \\ \text{Data Set} & P^N_1: & P_1^{0;\text{fit}} \to b_P\ ,\ \ldots\ ,\ P_1^{4;\text{fit}} \to^{b_P}_{c_P},\ P_1^{5;\text{fit}}\ ✗ \end{array}$$

- Let's see impact on $a_\mu^{\text{HLbL};\eta}$ assuming $F_{P\gamma^*\gamma^*}(q_1^2, q_2^2) = F_{P\gamma\gamma^*}(q_1^2)F_{P\gamma\gamma^*}(q_2^2)$

	Fit
P_1^0	14.5
P_2^1	—

Inputs from data, PAs from derivatives

- Inputs: sequences data fitting (not just fitting models)

Reduced	P^N_{N+1} :	$P_1^{0;\mathrm{der}} \to b_P$, ... , $P_2^{1;\mathrm{der}}$ ✓
Data Set	P^N_1 :	$P_1^{0;\mathrm{fit}} \to b_P$, ... , $P_1^{4;\mathrm{fit}} \to {}^{b_P}_{c_P}$; $P_1^{5;\mathrm{fit}}$ ✗

- Let's see impact on $a_\mu^{\mathrm{HLbL};\eta}$ assuming $F_{P\gamma^*\gamma^*}(q_1^2, q_2^2) = F_{P\gamma\gamma^*}(q_1^2) F_{P\gamma\gamma^*}(q_2^2)$

	Fit	Der
P_1^0	14.5	13.2
P_2^1	—	13.3

Inputs from data, PAs from derivatives

- Inputs: sequences data fitting (not just fitting models)

$$\begin{array}{c|ll} \text{New larger} & P^N_{N+1}: & P^{0;\text{fit}}_1 \to b_P \;, \;\ldots\;, \; P^{1;\text{fit}}_2, \; P^{2;\text{fit}}_3 \;✗ \\ \text{Data Set} & P^N_1: & P^{0;\text{fit}}_1 \to b_P \;, \;\ldots\;, \; P^{7;\text{fit}}_1 \to^{b_P}_{c_P}, \; P^{8;\text{fit}}_1 \;✗ \end{array}$$

- Let's see impact on $a_\mu^{\text{HLbL};\eta}$ assuming $F_{P\gamma^*\gamma^*}(q_1^2, q_2^2) = F_{P\gamma\gamma^*}(q_1^2)F_{P\gamma\gamma^*}(q_2^2)$

	Fit	Der	New Fit
P^0_1	14.5	13.2	14.0
P^1_2	—	13.3	13.4

Inputs from data, PAs from derivatives

- Inputs: sequences data fitting (not just fitting models)

$$\begin{array}{r|ll} \text{New larger} & P^N_{N+1}: & P_1^{0;\text{der}} \to b_P \;,\; \ldots \;,\; P_2^{1;\text{der}} \checkmark \\ \text{Data Set} & P^N_1: & P_1^{0;\text{fit}} \to b_P \;,\; \ldots \;,\; P_1^{7;\text{fit}} \to {}^{b_P}_{c_P} \;,\; P_1^{8;\text{fit}} \text{✗} \end{array}$$

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	Fit	Der	New Fit	New Der
P_1^0	14.5	13.2	14.0	13.1
P_2^1	—	13.3	13.4	13.3

Inputs from data, PAs from derivatives

- Inputs: sequences data fitting (not just fitting models)

New larger Data Set	
P^N_{N+1} :	$P_1^{0;\text{der}} \to b_P$, ... , $P_2^{1;\text{der}}$ ✓
P^N_1 :	$P_1^{0;\text{fit}} \to b_P$, ... , $P_1^{7;\text{fit}} \to {}^{b_P}_{c_P}$, $P_1^{8;\text{fit}}$ ✗

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	Fit	Der	New Fit	New Der
P_1^0	14.5	13.2	14.0	13.1
P_2^1	—	13.3	13.4	13.3

- Advantage of PAs: interrelation, acceleration of convergence, systematics
- Cannot overmphasize: not directly fitting functions, nor spectroscopy!!!

P. Masjuan '12; R. Escribano, P. Masjuan, P. S (& S. Gonzalez) '14 '15 (&16)

Padé approximants: doubly virtual

- Commonly referred to as Canterbury approximants

$$F_{\pi\gamma^*\gamma^*}(q_1^2,q_2^2) = C_M^N = \frac{Q_N(q_1^2,q_2^2)}{R_M(q_1^2,q_2^2)}; \quad Q_N(R_M) = \sum_{i,j}^{N(M)} c_{n,m}^{Q(R)} q_1^{2i} q_2^{2j}$$

Again, match derivatives to get $c_{ij}^{Q,R}$*'s*

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- Again nice convergence properties, as for instance, pQCD models

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- Again nice convergence properties, as for instance, pQCD models

$$F_{P\gamma^*\gamma^*}^{\log}(Q_1^2,Q_2^2) = \frac{F_{P\gamma\gamma}M^2}{Q_1^2-Q_2^2}\ln\left(\frac{1+Q_1^2/M^2}{1+Q_2^2/M^2}\right)$$

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$$F_{P\gamma^*\gamma^*}^{\log}(Q_1^2,Q_2^2) = F_{P\gamma\gamma} M^2 \int_0^1 dx \frac{1}{xQ_1^2 + (1-x)Q_2^2 + M^2}$$

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- Simple example with appropriate power-like behavior

$$C_1^0 = \frac{1}{1 + c_{0,1}^R(q_1^2+q_2^2) + c_{1,1}^R q_1^2 q_2^2} \to \frac{1}{1 + c_{0,1}^R(q_1^2+q_2^2)}$$

Again, benefits of not doing spectroscopy!

- Unfortunately not available data yet (except for η'; expected for π^0)

See to believe: toy models and systematics

—a_μ^π: Regge Model—

$$F^{\mathrm{Regge}}_{\pi^0\gamma^*\gamma^*}(Q_1^2,Q_2^2) = \frac{aF_{P\gamma\gamma}}{Q_1^2-Q_2^2}\frac{\left[\psi^{(0)}\left(\frac{M^2+Q_1^2}{a}\right)-\psi^{(0)}\left(\frac{M^2+Q_2^2}{a}\right)\right]}{\psi^{(1)}\left(\frac{M^2}{a}\right)}$$

	C_1^0	C_2^1	C_3^2	C_4^3
LE	55.2	59.7	60.4	60.6
OPE	65.7	60.8	60.7	60.7
Fit^{OPE}	66.3	62.7	61.1	60.8
Exact		60.7		

—a_μ^π: Logarithmic Model—

$$F^{\log}_{\pi^0\gamma^*\gamma^*}(Q_1^2,Q_2^2) = \frac{F_{P\gamma\gamma}M^2}{Q_1^2-Q_2^2}\ln\left(\frac{1+Q_1^2/M^2}{1+Q_2^2/M^2}\right)$$

	C_1^0	C_2^1	C_3^2	C_4^3
LE	56.7	64.4	66.1	66.8
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Fit^{OPE}	79.6	71.9	69.3	68.4
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P. Masjuan & P. Sanchez Phys.Rev. D95, 054026 (2017)

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$$F^{\rm log}_{\pi^0\gamma^*\gamma^*}(Q_1^2,Q_2^2) = \frac{F_{P\gamma\gamma}M^2}{Q_1^2-Q_2^2}\ln\left(\frac{1+Q_1^2/M^2}{1+Q_2^2/M^2}\right)$$

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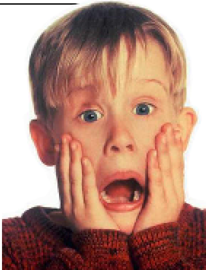
- The convergence result is excellent!
- The OPE choice seems the best → high energy matters
- Still, low energies provide a good performance
- Error ∼ difference among elements → Systematics!



P. Masjuan & P. Sanchez Phys.Rev. D95, 054026 (2017)

Section 3

Application to HLbL and results

Reconstructing $F_{P\gamma^*\gamma^*}(Q_1^2, Q_2^2)$

- Simplest approach: $C_1^0(Q_1^2, Q_2^2)$

$$C_1^0(Q_1^2, Q_2^2) = \frac{F_{P\gamma\gamma}(0,0)}{1 + b_P(Q_1^2 + Q_2^2)}$$

Reconstructing $F_{P\gamma^*\gamma^*}(Q_1^2, Q_2^2)$

- Simplest approach: $C_1^0(Q_1^2, Q_2^2)$

$$C_1^0(Q_1^2, Q_2^2) = \frac{F_{P\gamma\gamma}(0,0)}{1 + b_P(Q_1^2 + Q_2^2)}$$

- Next element: $C_2^1(Q_1^2, Q_2^2)$

$$C_2^1(Q_1^2, Q_2^2) = \frac{F_{P\gamma\gamma}(0,0)(1 + \alpha_1(Q_1^2 + Q_2^2) + \alpha_{1,1}Q_1^2 Q_2^2)}{1 + \beta_1(Q_1^2 + Q_2^2) + \beta_2(Q_1^4 + Q_2^4) + \beta_{1,1}Q_1^2 Q_2^2 + \beta_{2,1}Q_1^2 Q_2^2(Q_1^2 + Q_2^2) + \beta_{2,2}Q_1^4 Q_2^4}$$

Reconstructing $F_{P\gamma^*\gamma^*}(Q_1^2, Q_2^2)$

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Reconstruction

1.Reduce to Padé Approximants
$F_{P\gamma\gamma}(0,0), \alpha_1, \beta_1, \beta_2 \to$ from PAs

Reconstructing $F_{P\gamma^*\gamma^*}(Q_1^2, Q_2^2)$

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Reconstruction

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$F_{P\gamma\gamma}(0,0), \alpha_1, \beta_1, \beta_2 \rightarrow$ from PAs

Reconstructing $F_{P\gamma^*\gamma^*}(Q_1^2, Q_2^2)$

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Reconstruction

1.Reduce to Padé Approximants

2.Reproduce the OPE behavior (high energies)

$$F_{\pi\gamma^*\gamma^*} = \frac{1}{3Q^2}(2F_\pi)\left(1 - \frac{8}{9}\frac{\delta^2}{Q^2} + \mathcal{O}(\alpha_s(Q^2))\right) \Rightarrow \beta_{2,2} = 0, \alpha_{1,1}, \beta_{2,1}$$

Reconstructing $F_{P\gamma^*\gamma^*}(Q_1^2, Q_2^2)$

- Simplest approach: $C_1^0(Q_1^2, Q_2^2)$

$$C_1^0(Q_1^2, Q_2^2) = \frac{F_{P\gamma\gamma}(0,0)}{1 + b_P(Q_1^2 + Q_2^2)}$$

- Next element: $C_2^1(Q_1^2, Q_2^2)$

$$C_2^1(Q_1^2, Q_2^2) = \frac{F_{P\gamma\gamma}(0,0)(1 + \alpha_1(Q_1^2 + Q_2^2) + \alpha_{1,1} Q_1^2 Q_2^2)}{1 + \beta_1(Q_1^2 + Q_2^2) + \beta_2(Q_1^4 + Q_2^4) + \beta_{1,1} Q_1^2 Q_2^2 + \beta_{2,1} Q_1^2 Q_2^2 (Q_1^2 + Q_2^2)}$$

Reconstruction

1.Reduce to Padé Approximants

2.Reproduce the OPE behavior (high energies)

3.Reproduce the low energies ($a_{P;1,1}$)

Reconstructing $F_{P\gamma^*\gamma^*}(Q_1^2, Q_2^2)$

- Simplest approach: $C_1^0(Q_1^2, Q_2^2)$

$$C_1^0(Q_1^2, Q_2^2) = \frac{F_{P\gamma\gamma}(0,0)}{1 + b_P(Q_1^2 + Q_2^2)}$$

- Next element: $C_2^1(Q_1^2, Q_2^2)$

$$C_2^1(Q_1^2, Q_2^2) = \frac{F_{P\gamma\gamma}(0,0)(1 + \alpha_1(Q_1^2 + Q_2^2) + \alpha_{1,1} Q_1^2 Q_2^2)}{1 + \beta_1(Q_1^2 + Q_2^2) + \beta_2(Q_1^4 + Q_2^4) + \beta_{1,1} Q_1^2 Q_2^2 + \beta_{2,1} Q_1^2 Q_2^2 (Q_1^2 + Q_2^2)}$$

Reconstruction

1.Reduce to Padé Approximants

2.Reproduce the OPE behavior (high energies)

3.Reproduce the low energies ($a_{P;1,1}$)
Be generous: all configurations with no poles $\Rightarrow a_{P;1,1}^{\min} < a_{P;1,1} < a_{P;1,1}^{\max}$

Reconstructing $F_{P\gamma^*\gamma^*}(Q_1^2, Q_2^2)$

- Simplest approach: $C_1^0(Q_1^2, Q_2^2)$

$$C_1^0(Q_1^2, Q_2^2) = \frac{F_{P\gamma\gamma}(0,0)}{1 + b_P(Q_1^2 + Q_2^2)}$$

- Next element: $C_2^1(Q_1^2, Q_2^2)$

$$C_2^1(Q_1^2, Q_2^2) = \frac{F_{P\gamma\gamma}(0,0)(1 + \alpha_1(Q_1^2 + Q_2^2) + \alpha_{1,1} Q_1^2 Q_2^2)}{1 + \beta_1(Q_1^2 + Q_2^2) + \beta_2(Q_1^4 + Q_2^4) + \beta_{1,1} Q_1^2 Q_2^2 + \beta_{2,1} Q_1^2 Q_2^2 (Q_1^2 + Q_2^2)}$$

Reconstruction

1.Reduce to Padé Approximants

2.Reproduce the OPE behavior (high energies)

3.Reproduce the low energies ($a_{P;1,1}^{\min} < a_{P;1,1} < a_{P;1,1}^{\max}$)

Pseudoscalar-pole contribution: Final results

$$—C_1^0(Q_1^2, Q_2^2)—$$

$a_\mu^{\mathrm{HLbL};P} \times 10^{11}$	OPE ($a_{P;1,1} = 2b_P^2$)	Fact ($a_{P;1,1} = b_P^2$)
π^0	$65.3(1.4)_F(2.4)_{b_\pi}[2.8]_t$	$54.3(1.5)_F(2.2)_{b_\pi}[2.5]_t$
η	$17.1(0.6)_F(0.2)_{b_\eta}[0.6]_t$	$13.0(0.4)_F(0.2)_{b_\eta}[0.5]_t$
η'	$16.0(0.5)_F(0.3)_{b_{\eta'}}[0.6]_t$	$12.0(0.4)_F(0.3)_{b_{\eta'}}[0.5]_t$
Total	$98.4[2.9]_t$	$79.3[2.6]_t$

DR: *Hoferichter et al*, PRL121 (2018); lattice: *Gérardin et al*, PRD100 (2019)

Pseudoscalar-pole contribution: Final results

—$C_1^0(Q_1^2, Q_2^2)$—

$a_\mu^{\mathrm{HLbL};P} \times 10^{11}$	OPE ($a_{P;1,1} = 2b_P^2$)	Fact ($a_{P;1,1} = b_P^2$)
π^0	$65.3(1.4)_F(2.4)_{b_\pi}[2.8]_t$	$54.3(1.5)_F(2.2)_{b_\pi}[2.5]_t$
η	$17.1(0.6)_F(0.2)_{b_\eta}[0.6]_t$	$13.0(0.4)_F(0.2)_{b_\eta}[0.5]_t$
η'	$16.0(0.5)_F(0.3)_{b_{\eta'}}[0.6]_t$	$12.0(0.4)_F(0.3)_{b_{\eta'}}[0.5]_t$
Total	$98.4[2.9]_t$	$79.3[2.6]_t$

—$C_2^1(Q_1^2, Q_2^2)$—

$a_\mu^{\mathrm{HLbL};P} \times 10^{11}$	$a_{P;1,1}^{\min}$	$a_{P;1,1}^{\max}$
π^0	$64.1(1.3)_L(0)_\delta[1.3]_t$	$63.0(1.1)_L(0.5)_\delta[1.2]_t$
η	$16.3(0.8)_L(0)_\delta[0.8]_t$	$16.2(0.8)_L(0.6)_\delta[1.0]_t$
η'	$14.7(0.7)_L(0)_\delta[0.7]_t$	$14.3(0.5)_L(0.5)_\delta[0.7]_t$
Total	$95.1[1.7]_t$	$93.5[1.7]_t$

DR: *Hoferichter et al*, PRL121 (2018); lattice: *Gérardin et al*, PRD100 (2019)

Pseudoscalar-pole contribution: Final results

—$C_1^0(Q_1^2, Q_2^2)$—

$a_\mu^{\mathrm{HLbL};P} \times 10^{11}$	OPE ($a_{P;1,1} = 2b_P^2$)	Fact ($a_{P;1,1} = b_P^2$)
π^0	$65.3(1.4)_F(2.4)_{b_\pi}[2.8]_t$	$54.3(1.5)_F(2.2)_{b_\pi}[2.5]_t$
η	$17.1(0.6)_F(0.2)_{b_\eta}[0.6]_t$	$13.0(0.4)_F(0.2)_{b_\eta}[0.5]_t$
η'	$16.0(0.5)_F(0.3)_{b_{\eta'}}[0.6]_t$	$12.0(0.4)_F(0.3)_{b_{\eta'}}[0.5]_t$
Total	$98.4[2.9]_t$	$79.3[2.6]_t$

—$C_2^1(Q_1^2, Q_2^2)$—

$a_\mu^{\mathrm{HLbL};P} \times 10^{11}$	$a_{P;1,1}^{\min}$	$a_{P;1,1}^{\max}$
π^0	$64.1(1.3)_L(0)_\delta[1.3]_t\{1.2\}_{\mathrm{sys}}$	$63.0(1.1)_L(0.5)_\delta[1.2]_t\{2.3\}_{\mathrm{sys}}$
η	$16.3(0.8)_L(0)_\delta[0.8]_t$	$16.2(0.8)_L(0.6)_\delta[1.0]_t$
η'	$14.7(0.7)_L(0)_\delta[0.7]_t$	$14.3(0.5)_L(0.5)_\delta[0.7]_t$
Total	$95.1[1.7]_t$	$93.5[1.7]_t$

DR: *Hoferichter et al*, PRL121 (2018); lattice: *Gérardin et al*, PRD100 (2019)

Pseudoscalar-pole contribution: Final results

—$C_1^0(Q_1^2, Q_2^2)$—

$a_\mu^{\mathrm{HLbL};P} \times 10^{11}$	OPE ($a_{P;1,1} = 2b_P^2$)	Fact ($a_{P;1,1} = b_P^2$)
π^0	$65.3(1.4)_F(2.4)_{b_\pi}[2.8]_t$	$54.3(1.5)_F(2.2)_{b_\pi}[2.5]_t$
η	$17.1(0.6)_F(0.2)_{b_\eta}[0.6]_t$	$13.0(0.4)_F(0.2)_{b_\eta}[0.5]_t$
η'	$16.0(0.5)_F(0.3)_{b_{\eta'}}[0.6]_t$	$12.0(0.4)_F(0.3)_{b_{\eta'}}[0.5]_t$
Total	$98.4[2.9]_t$	$79.3[2.6]_t$

—$C_2^1(Q_1^2, Q_2^2)$—

$a_\mu^{\mathrm{HLbL};P} \times 10^{11}$	$a_{P;1,1}^{\mathrm{min}}$	$a_{P;1,1}^{\mathrm{max}}$
π^0	$64.1(1.3)_L(0)_\delta[1.3]_t\{1.2\}_{\mathrm{sys}}$	$63.0(1.1)_L(0.5)_\delta[1.2]_t\{2.3\}_{\mathrm{sys}}$
η	$16.3(0.8)_L(0)_\delta[0.8]_t\{0.8\}_{\mathrm{sys}}$	$16.2(0.8)_L(0.6)_\delta[1.0]_t\{0.9\}_{\mathrm{sys}}$
η'	$14.7(0.7)_L(0)_\delta[0.7]_t\{1.3\}_{\mathrm{sys}}$	$14.3(0.5)_L(0.5)_\delta[0.7]_t\{1.7\}_{\mathrm{sys}}$
Total	$95.1[1.7]_t\{3.3\}_{\mathrm{sys}}$	$93.5[1.7]_t\{4.9\}_{\mathrm{sys}}$

DR: *Hoferichter et al*, PRL121 (2018); lattice: *Gérardin et al*, PRD100 (2019)

Pseudoscalar-pole contribution: Final results

—$C_1^0(Q_1^2, Q_2^2)$—

$a_\mu^{\mathrm{HLbL};P} \times 10^{11}$	OPE ($a_{P;1,1} = 2b_P^2$)	Fact ($a_{P;1,1} = b_P^2$)
π^0	$65.3(1.4)_F(2.4)_{b_\pi}[2.8]_t$	$54.3(1.5)_F(2.2)_{b_\pi}[2.5]_t$
η	$17.1(0.6)_F(0.2)_{b_\eta}[0.6]_t$	$13.0(0.4)_F(0.2)_{b_\eta}[0.5]_t$
η'	$16.0(0.5)_F(0.3)_{b_{\eta'}}[0.6]_t$	$12.0(0.4)_F(0.3)_{b_{\eta'}}[0.5]_t$
Total	$98.4[2.9]_t$	$79.3[2.6]_t$

—$C_2^1(Q_1^2, Q_2^2)$—

$a_\mu^{\mathrm{HLbL};P} \times 10^{11}$	$a_{P;1,1}^{\min}$	$a_{P;1,1}^{\max}$
π^0	$64.1(1.3)_L(0)_\delta[1.3]_t\{1.2\}_{\mathrm{sys}}$	$63.0(1.1)_L(0.5)_\delta[1.2]_t\{2.3\}_{\mathrm{sys}}$
η	$16.3(0.8)_L(0)_\delta[0.8]_t\{0.8\}_{\mathrm{sys}}$	$16.2(0.8)_L(0.6)_\delta[1.0]_t\{0.9\}_{\mathrm{sys}}$
η'	$14.7(0.7)_L(0)_\delta[0.7]_t\{1.3\}_{\mathrm{sys}}$	$14.3(0.5)_L(0.5)_\delta[0.7]_t\{1.7\}_{\mathrm{sys}}$
Total	$95.1[1.7]_t\{3.3\}_{\mathrm{sys}}$	$93.5[1.7]_t\{4.9\}_{\mathrm{sys}}$

—Final Result (combining errors just for clarity)

$$a_\mu^{\pi,\eta,\eta'} = (63.6(2.7) + 16.3(1.3) + 14.5(1.8)) \times 10^{-11} = 94.3(5.3) \times 10^{-11}$$

DR: *Hoferichter et al*, PRL121 (2018); lattice: *Gérardin et al*, PRD100 (2019)

Pseudoscalar-pole contribution: Final results

—$C_1^0(Q_1^2, Q_2^2)$—

$a_\mu^{\mathrm{HLbL};P} \times 10^{11}$	OPE ($a_{P;1,1} = 2b_P^2$)	Fact ($a_{P;1,1} = b_P^2$)
π^0	$65.3(1.4)_F(2.4)_{b_\pi}[2.8]_t$	$54.3(1.5)_F(2.2)_{b_\pi}[2.5]_t$
η	$17.1(0.6)_F(0.2)_{b_\eta}[0.6]_t$	$13.0(0.4)_F(0.2)_{b_\eta}[0.5]_t$
η'	$16.0(0.5)_F(0.3)_{b_{\eta'}}[0.6]_t$	$12.0(0.4)_F(0.3)_{b_{\eta'}}[0.5]_t$
Total	$98.4[2.9]_t$	$79.3[2.6]_t$

—$C_2^1(Q_1^2, Q_2^2)$—

$a_\mu^{\mathrm{HLbL};P} \times 10^{11}$	$a_{P;1,1}^{\min}$	$a_{P;1,1}^{\max}$
π^0	$64.1(1.3)_L(0)_\delta[1.3]_t\{1.2\}_{\mathrm{sys}}$	$63.0(1.1)_L(0.5)_\delta[1.2]_t\{2.3\}_{\mathrm{sys}}$
η	$16.3(0.8)_L(0)_\delta[0.8]_t\{0.8\}_{\mathrm{sys}}$	$16.2(0.8)_L(0.6)_\delta[1.0]_t\{0.9\}_{\mathrm{sys}}$
η'	$14.7(0.7)_L(0)_\delta[0.7]_t\{1.3\}_{\mathrm{sys}}$	$14.3(0.5)_L(0.5)_\delta[0.7]_t\{1.7\}_{\mathrm{sys}}$
Total	$95.1[1.7]_t\{3.3\}_{\mathrm{sys}}$	$93.5[1.7]_t\{4.9\}_{\mathrm{sys}}$

—Final Result (combining errors just for clarity)

$$a_\mu^{\pi,\eta,\eta'} = (63.6(2.7) + 16.3(1.3) + 14.5(1.8)) \times 10^{-11} = 94.3(5.3) \times 10^{-11}$$

- Good agreement with $a_\mu^{\mathrm{HLbL};\pi} = 63.0(2.7)/62.3(2.3)$ (DR/lattice)
- Current number in WP for η, η'

DR: *Hoferichter et al*, PRL121 (2018); lattice: *Gérardin et al*, PRD100 (2019)

Summary

- Padé approximants to reconstruct form factors
- Full use of data and theory in a systematic approach; not pheno model
- OPE for all the pseudoscalars implemented
- Bypass $\eta-\eta'$ mixing (output): non-trivial if fully theory-driven approach
- Good agreement for π with DR/lattice (th. on safe grounds)
- State of the art for η, η'

Outlook

- π re-evaluation with forthcoming BES III and PrimEx data
- Besides pseudoscalars, interesting relations with axials and $\langle VVA \rangle$ constraints [Masjuan, Roig, PSP, 2005.11761]

Section 4

Backup

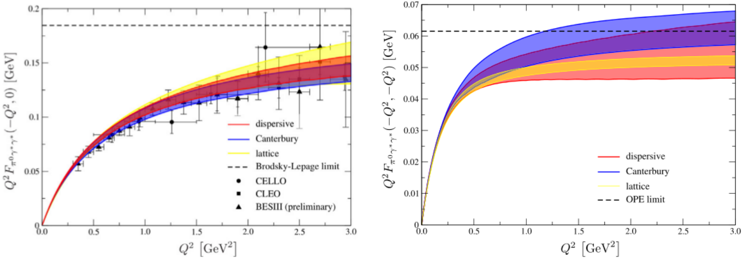


Fig. 60. Comparison of the π^0 TFF from dispersion theory [21,497] (red), CA [19] (blue), and lattice QCD [22] (yellow). We show both the singly- (left) and the doubly-virtual (right) form factors.

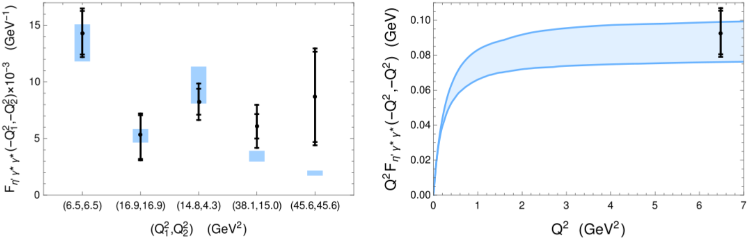
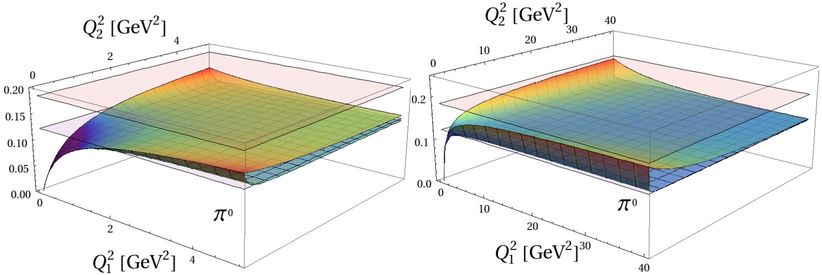
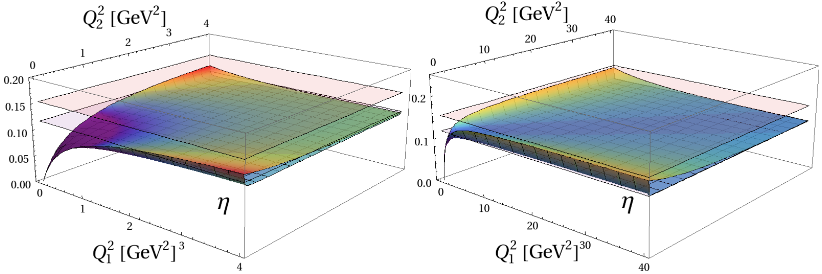


Fig. 59. Left: BABAR data points [108] with statistical errors (inner bars) and statistical and systematic combined (outer bars) in black, together with the CA prediction including errors (blue bands). Right: The analogous plot for the diagonal $Q^2 F_{\eta'\gamma^*\gamma^*}(-Q^2,-Q^2)$ TFF.

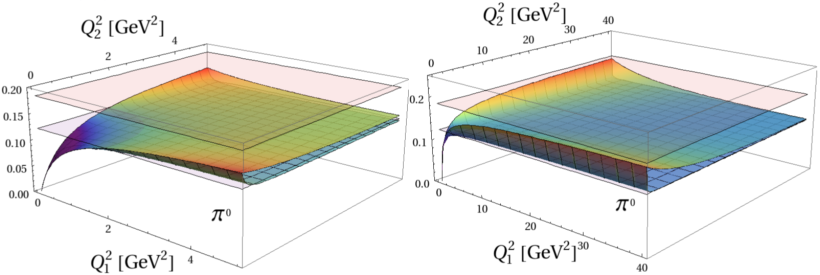
$F_{P\gamma^*\gamma^*}(Q_1^2, Q_2^2)$



The two planes: boundaries for the $a_{P;1,1}$ region



$F_{P\gamma^*\gamma^*}(Q_1^2, Q_2^2)$



The two planes: boundaries for the $a_{P;1,1}$ region